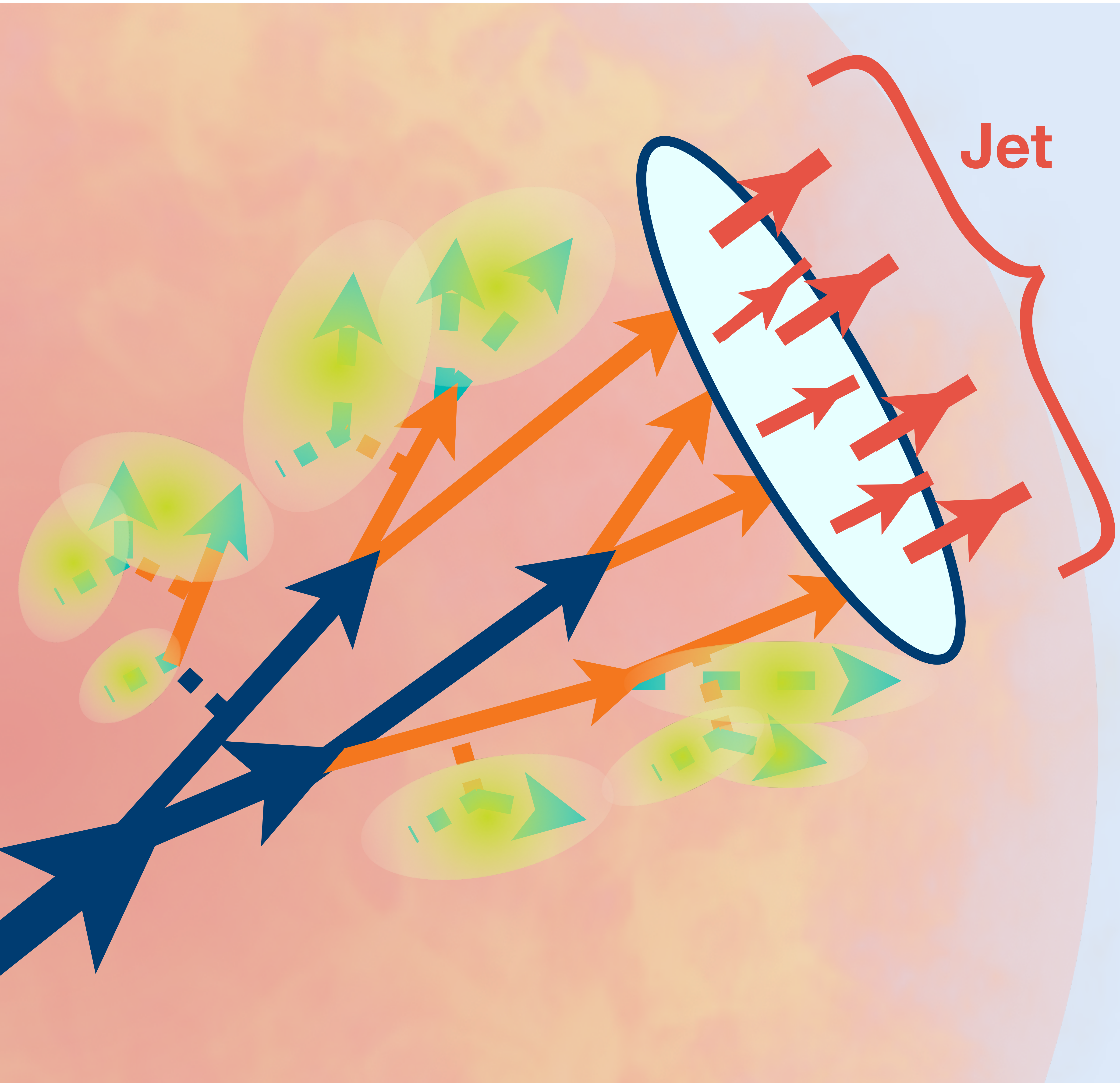
JETSCAPE Summer School 2022



Jets in QCD medium

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McGill University, Canada
July 29th, 2022

Outline

- ❑ Hard probes in heavy-ion collisions

- ❑ Factorization of soft and hard scales

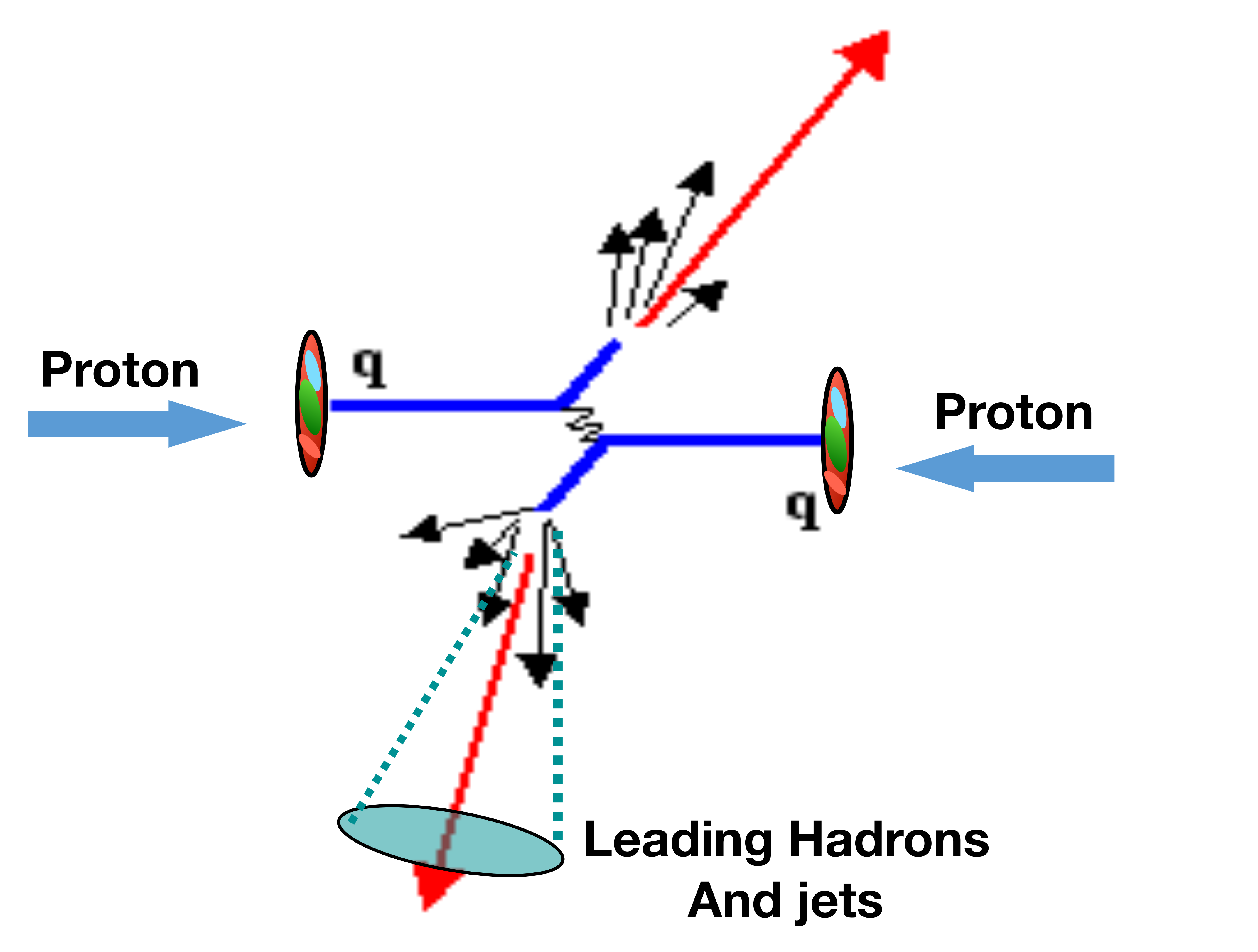
- ❑ Scale dependence of Parton distribution function and Fragmentation function

- ❑ Overview of JETSCAPE framework
- ❑ Basic review of jet energy loss in high virtuality and low virtuality phase
- ❑ MATTER, LBT and MARTINI energy loss modules

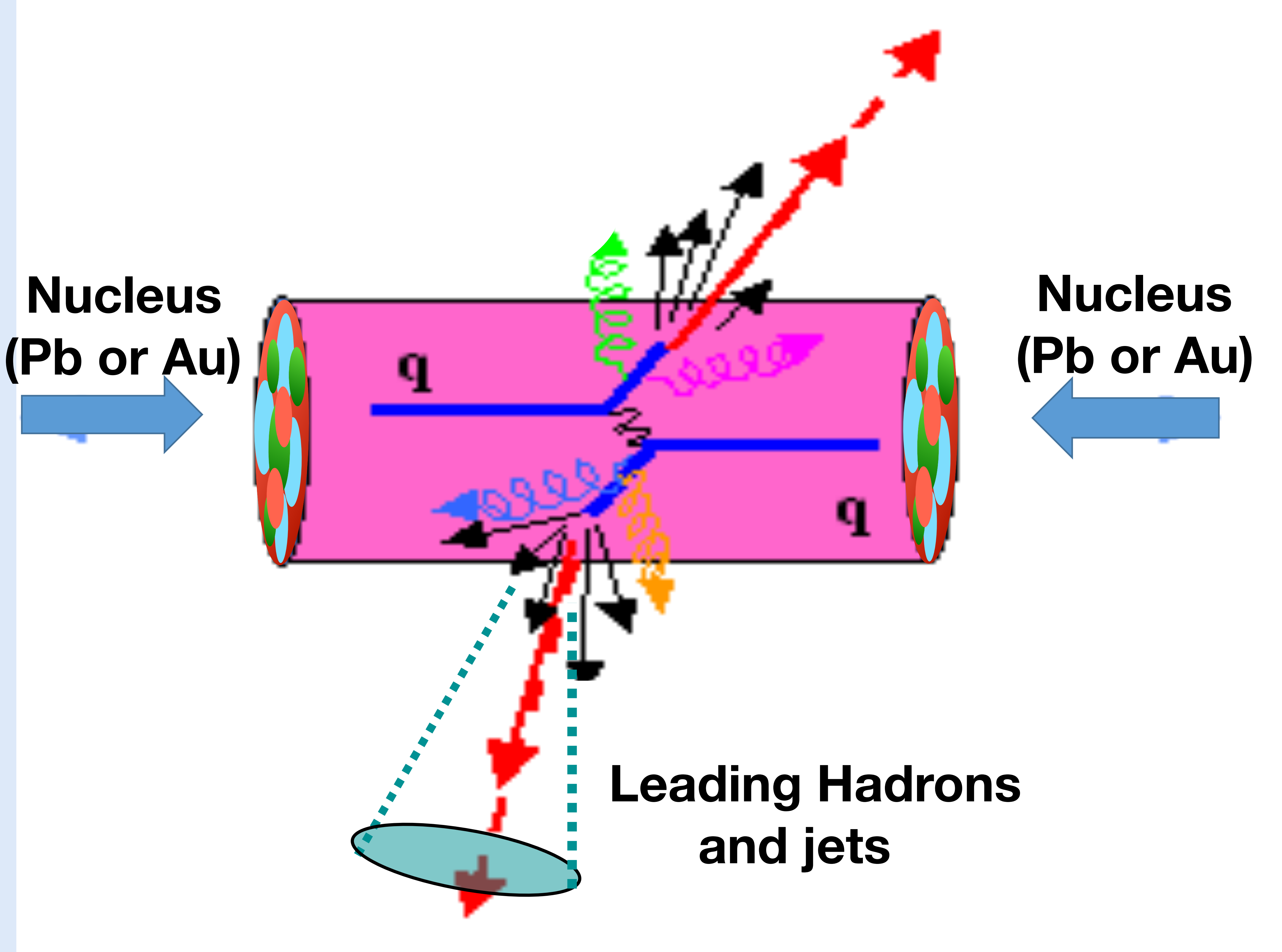
- ❑ Recent results based on multi-stage jet energy loss (MATTER+LBT) approach

Jets and leading hadron production in heavy-ion collisions

Proton-Proton Collisions

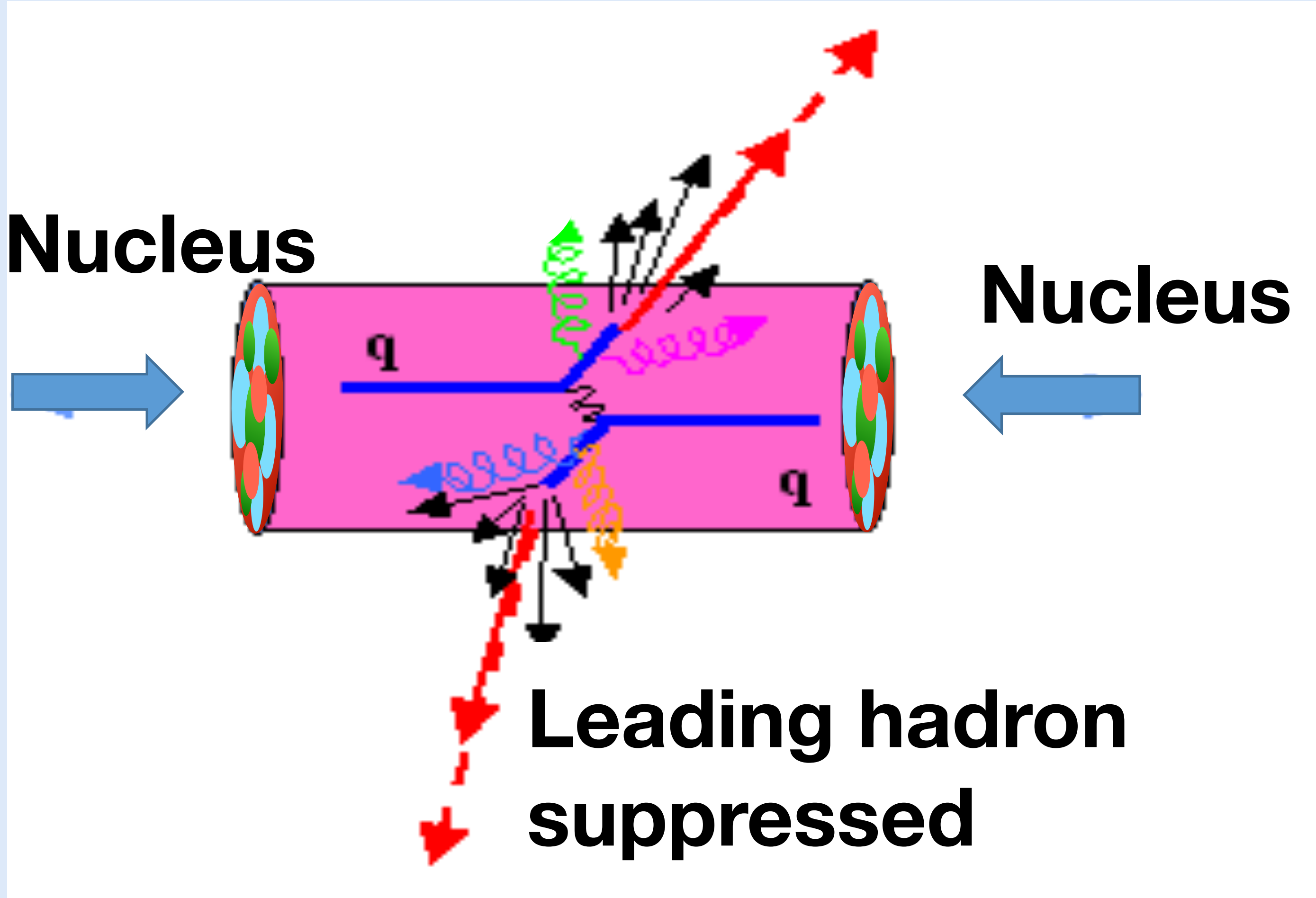


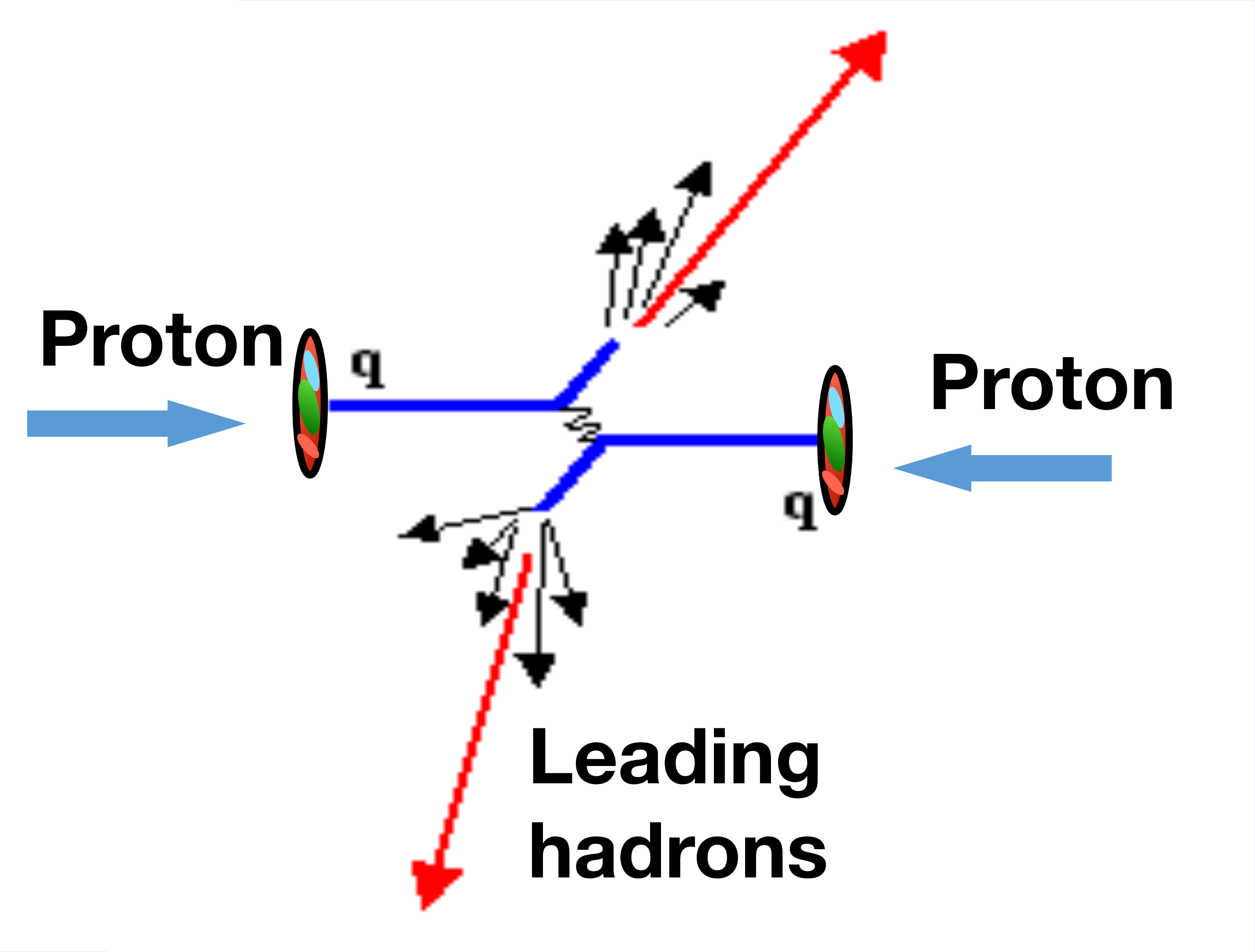
Heavy-ion Collisions



- ☐ **Initial state hard scattering $\Longrightarrow$ leading hadron and Jets**
- ☐ **Jets are collimated spray of soft and hard hadrons in a narrow cone $\Longrightarrow$ Proxy for the hard parton (After scattering)**
- ☐ **Perturbative QCD can be used to high precision**

Hard probes: Evidence for strongly interacting QGP



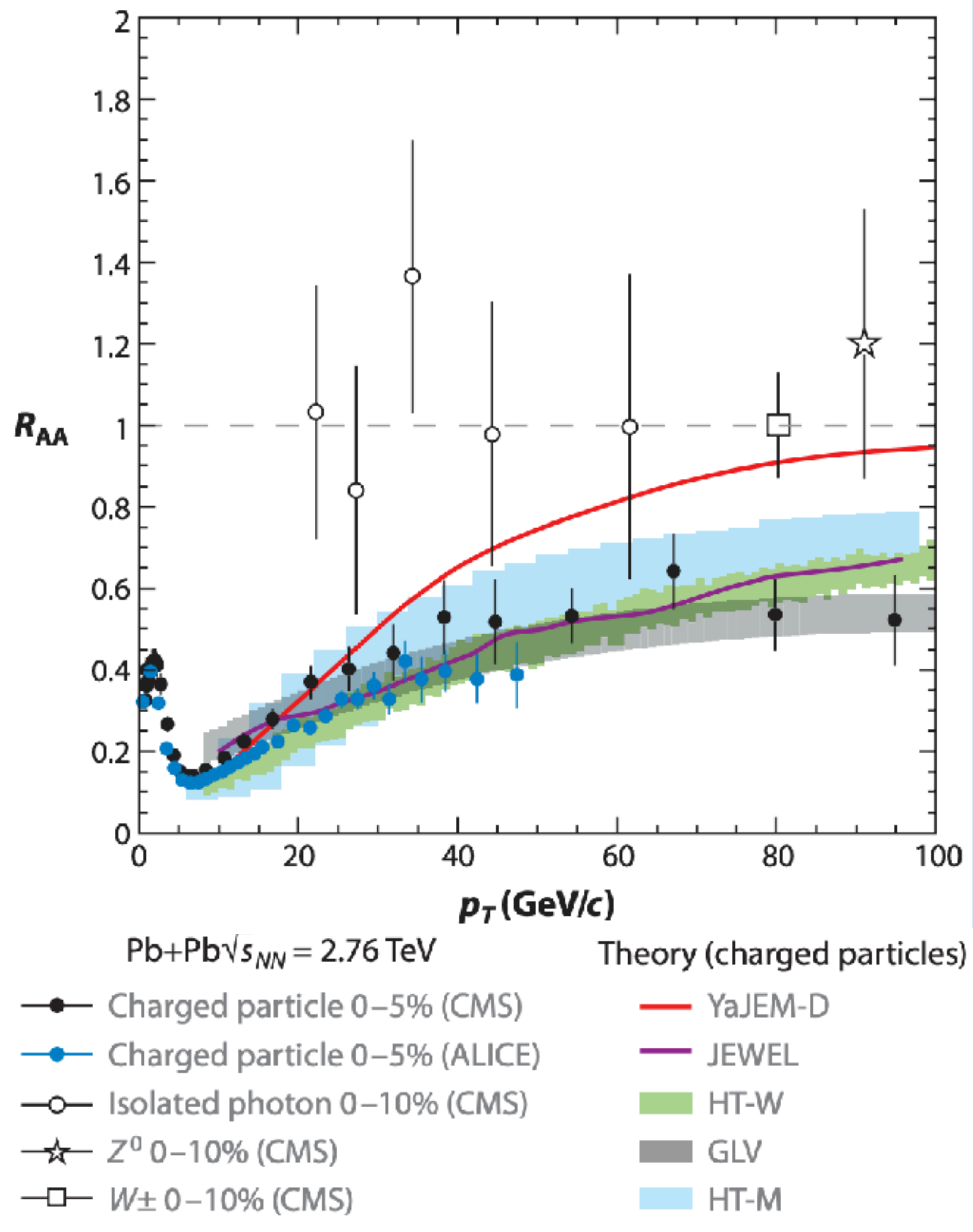


- ❑ **Nuclear modification factor R_{AA}**

$$R_{AA} \equiv \frac{d^2N^{AA}/dydp_T}{d^2N^{pp}/dydp_T \times \langle N_{coll}^{AA} \rangle}$$

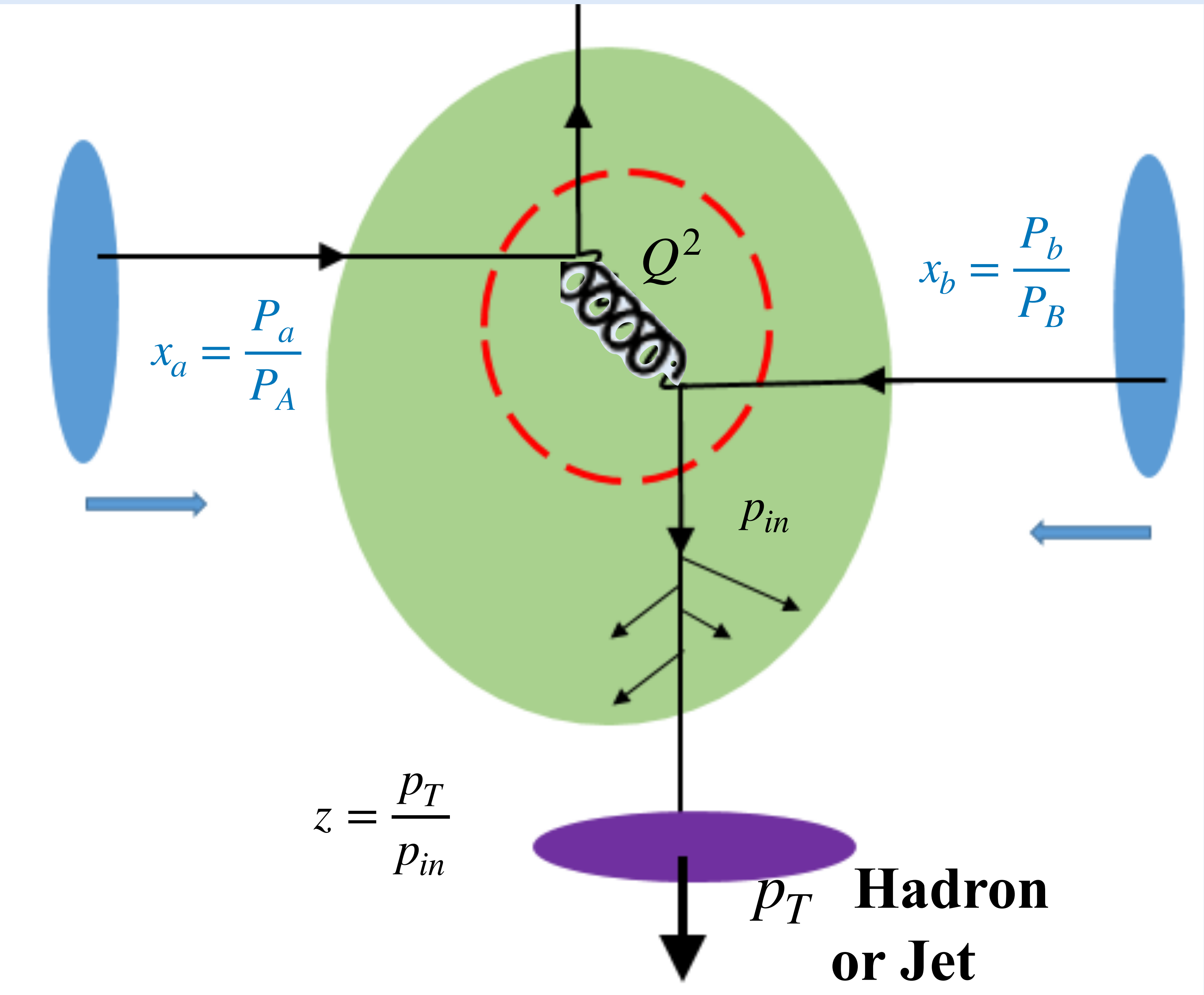
- ❑ **Hadron R_{AA} is less than 1, whereas isolated photon and Z^0 boson R_{AA} is unity.**
- ❑ **The plasma is strongly interacting**

Mueller et al., Ann. Rev. Nucl. Part. Sci. 62, 361 (2012)

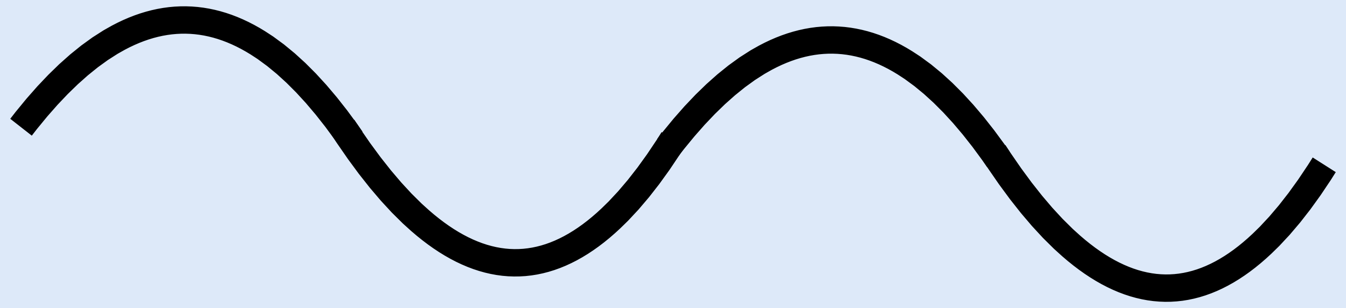


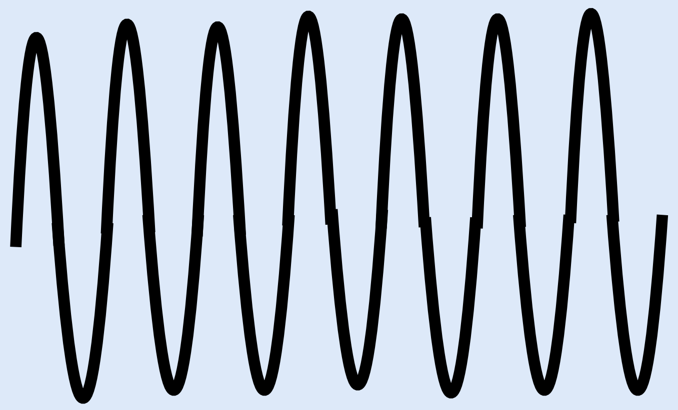
Factorization of short and long-distance physics

- Work due to Collins, Soper, Sterman for pp collision
- Factorization assumed for High p_T hadron/Jet production in Heavy-ion Collision



Soft Hadrons (LongDistance)





Hard partons (Short-distance)

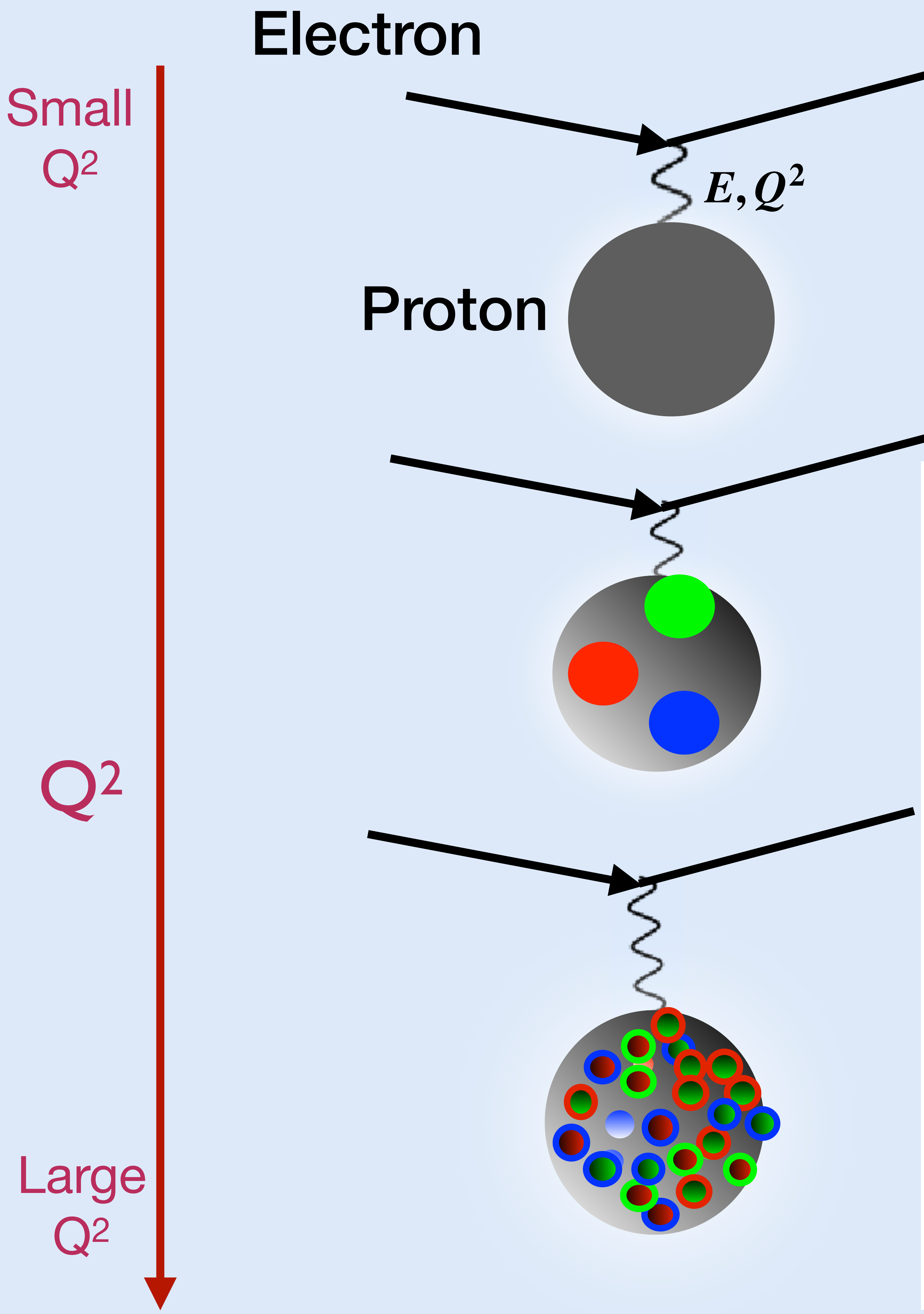
$$\frac{d\sigma^{AB\to h+X \text{ or Jet}+X}}{d^2p_T dy} \sim \int dx_a dx_b \, f_a^A(x_a, Q^2) \, f_b^B(x_b, Q^2) \, \frac{d\hat{\sigma}}{d\hat{t}} \, \tilde{D}^{med}_{modified}(z, Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

$$\updownarrow$$

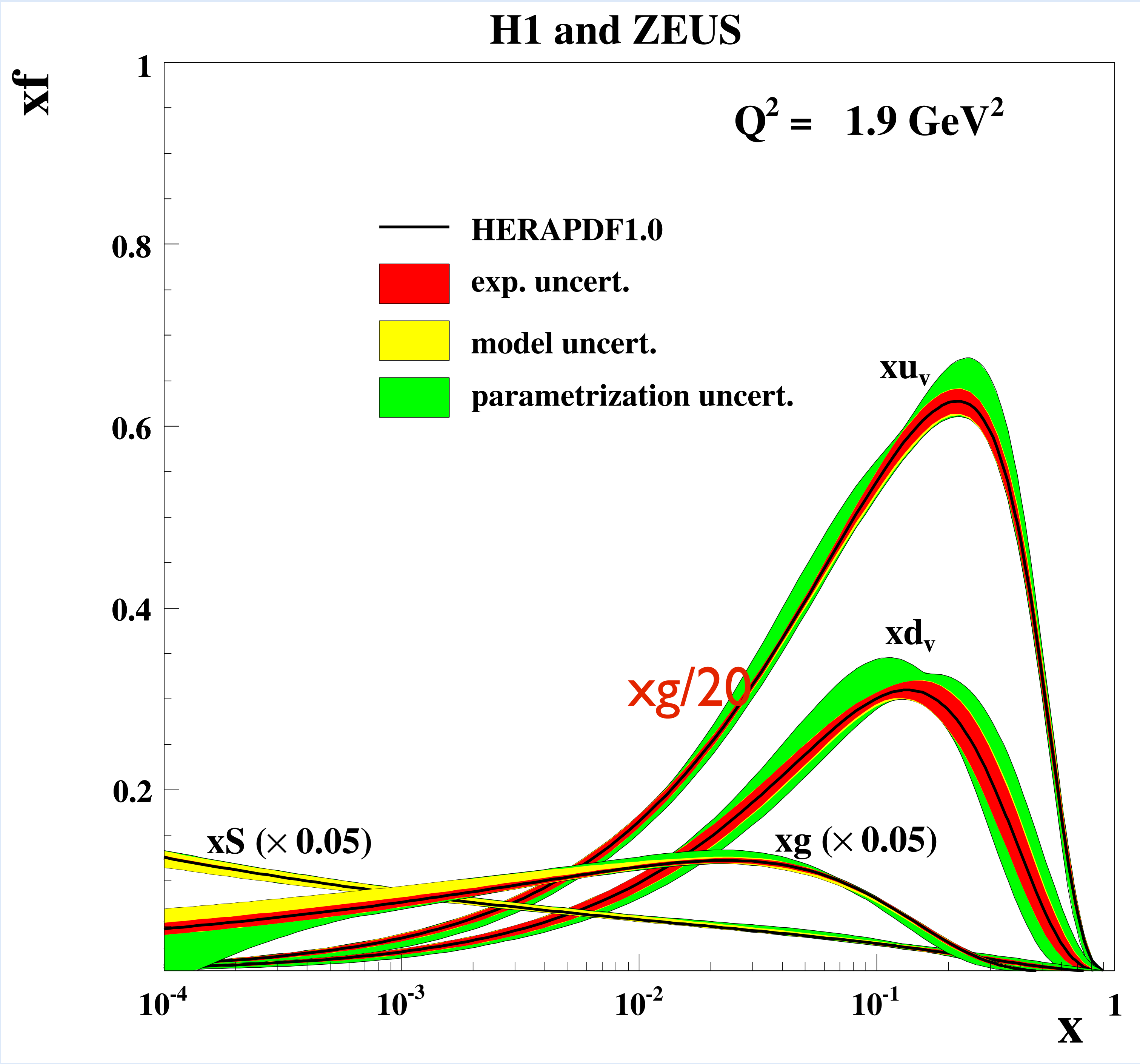
$$\tilde{J}^{med}_{modified}(z, Q^2)$$

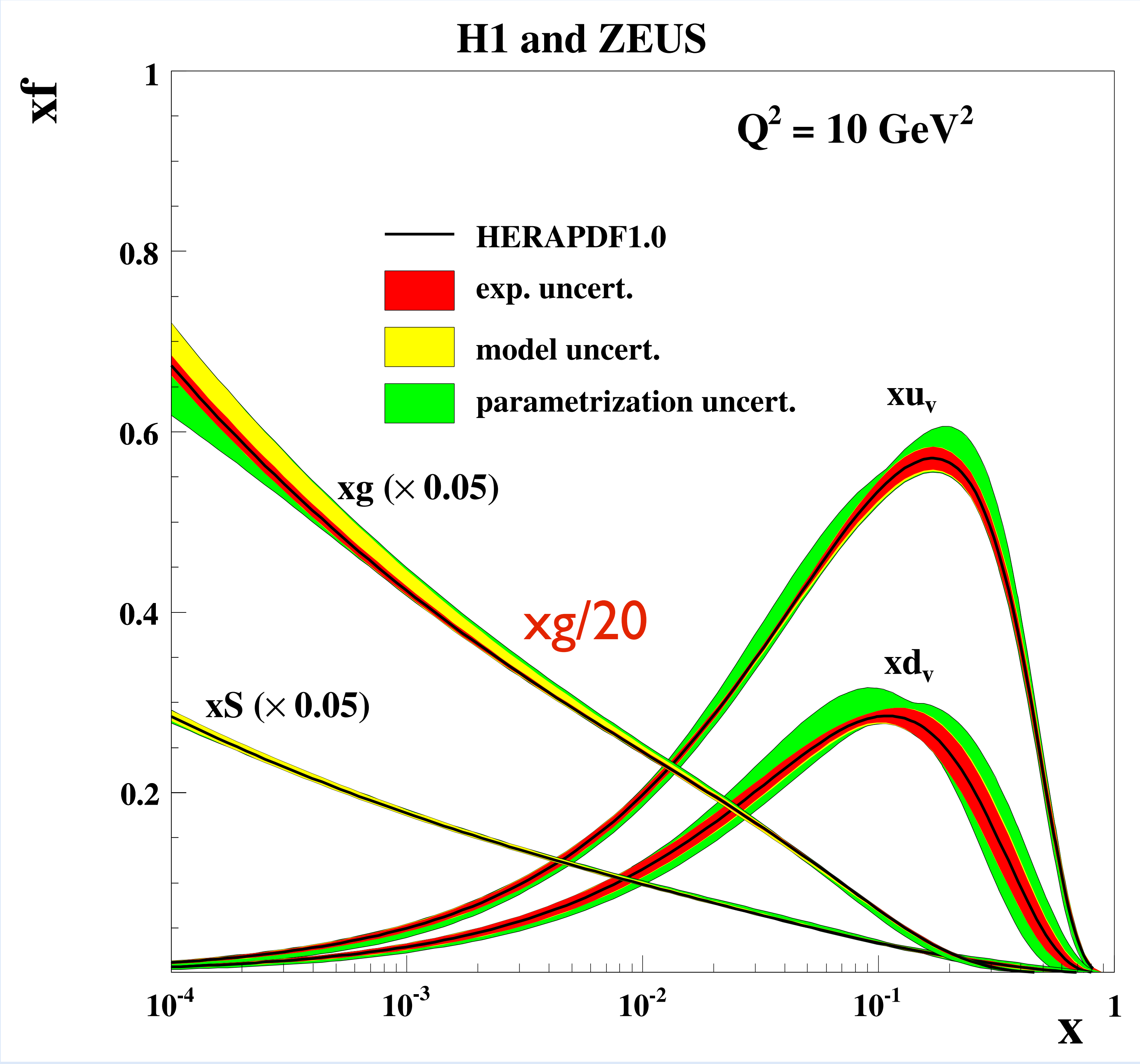
Total cross section is a product of probabilities

Scale (Q^2) evolution of parton distribution function $f(x)$



- E = Energy of photon, Q^2 = Momentum transfer
  M= Rest mass of proton
- Momentum fraction of struck parton $x_B = \dfrac{Q^2}{2M\,.\,E}$
- Parton distribution function for proton at two different scale

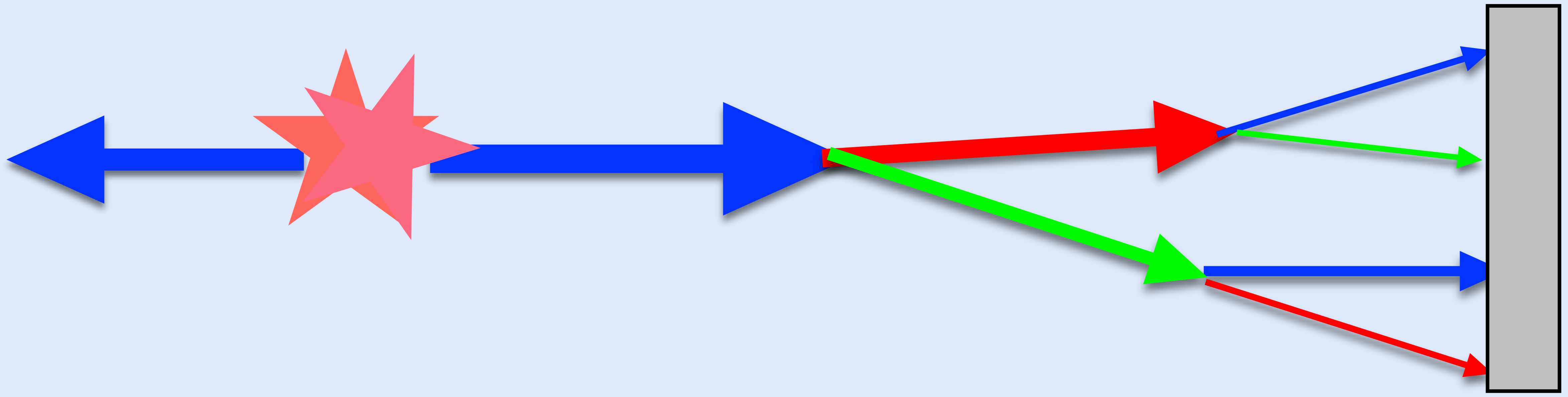


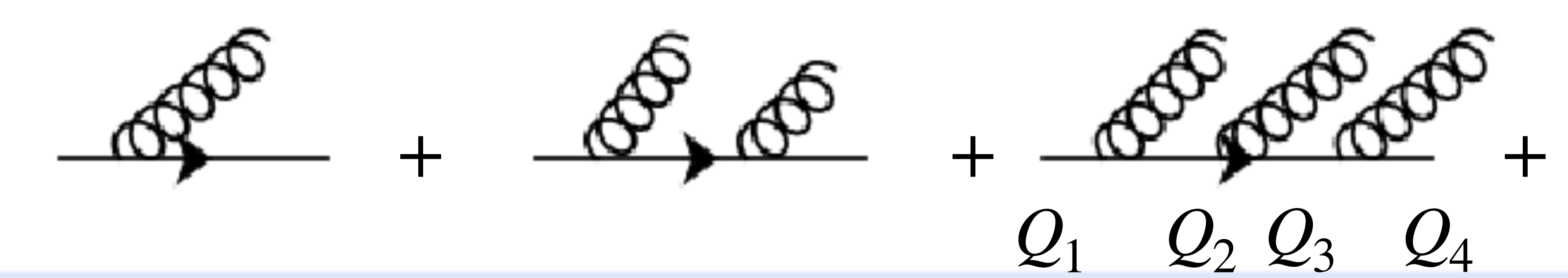


Proton structure is a scale dependent phenomenon

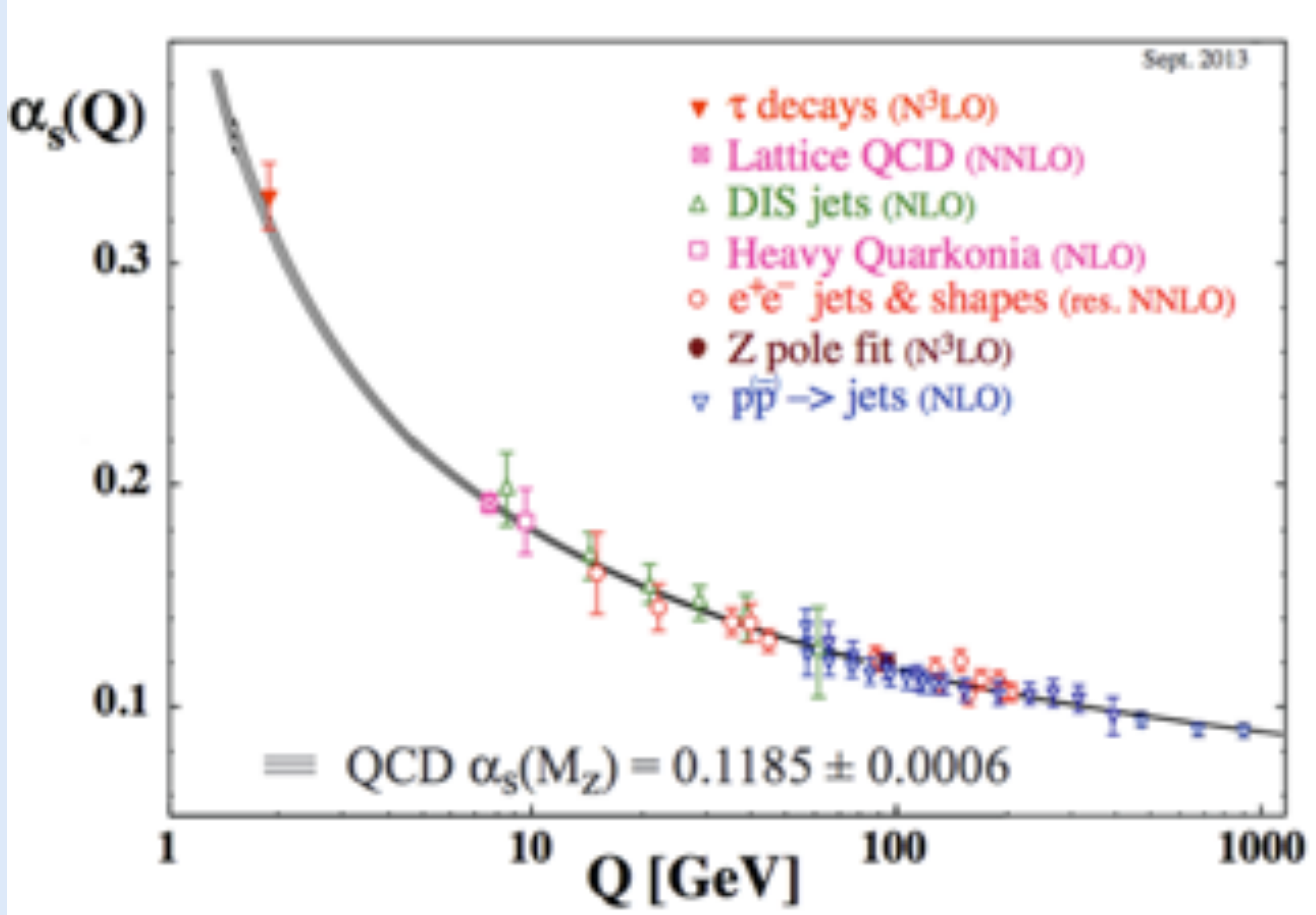
Scale (Q^2) evolution of Fragmentation function $f(x)$

❑ **Initial State Hard scattering produces are highly virtual objects**





$$Q_1^2 \geq Q_2^2 \geq Q_3^2 \ldots\ldots$$



❑ **The hard parton undergo radiative splitting which leads to decrease in the virtuality of the hard parton**

❑ **Emission process stops when the off-shell ness becomes small ($Q^2 \approx 1$ GeV^2), — In this regime perturbative description is no longer valid**

❑ **Partons undergo hadronization —detailed mechanism is unknown: Fragmentation function, PYTHIA string fragmentation**

Kinematic variables in light-cone coordinate system

- ❑ **Minkowski coordinate**

Four vector: $q = (q^t, q^x, q^y, q^z)$

Off-shellness:

$$q^2 = (q^t)^2 - (q^x)^2 - (q^y)^2 - (q^z)^2 - m_0^2$$

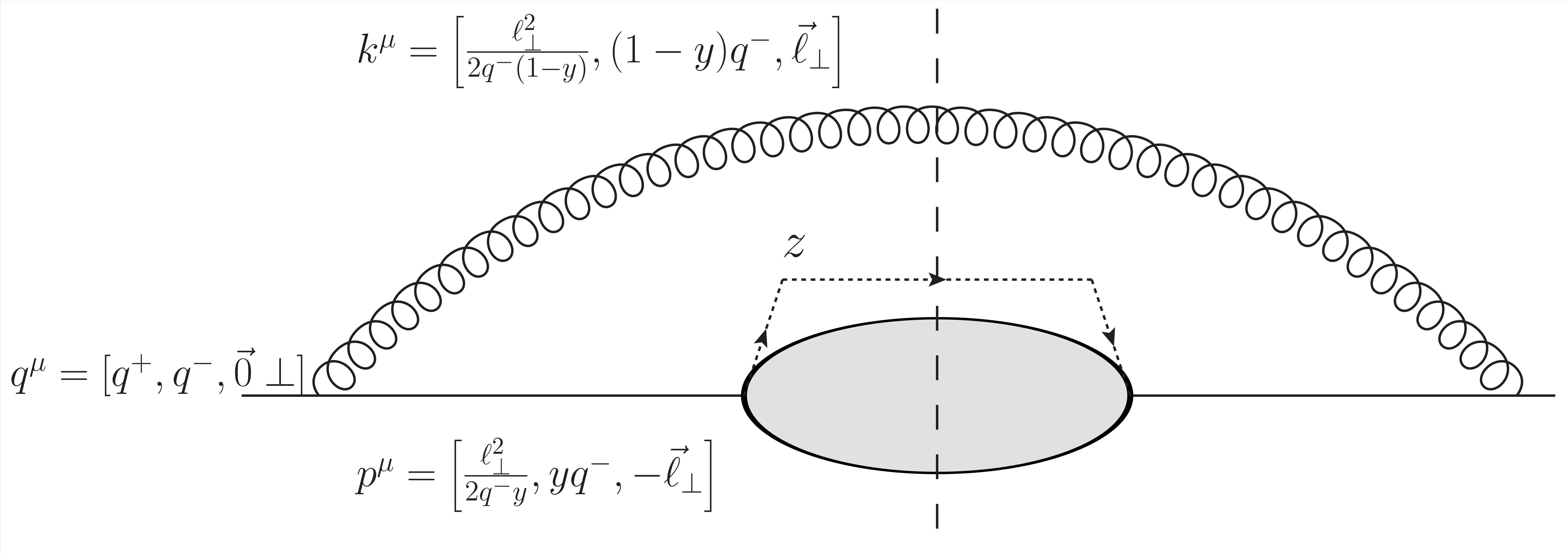
- ❑ **Light-cone coordinate**

Four vector: $q = (q^+, q^-, q_\perp^1, q_\perp^2)$

$$q^+ = \frac{q^t + q^z}{\sqrt{2}}; \quad q^- = \frac{q^t - q^z}{\sqrt{2}}; \quad q_\perp = \sqrt{(q^x)^2 + (q^y)^2}$$

Off-shellness: $q^2 = 2q^+q^- - q_\perp^2 - m_0^2$

Example: Particle traveling in -z direction $\Longrightarrow q^- \gg q^+; \quad q_\perp = 0$



Forward scattering diagram

Momentum variables

If

$p^- = yq^-$

$k^- = (1-y)q^-$

We know

$k^2 = 0$

$p^2 = 0$

$q^2 = 2q^+q^- = Q^2$

$q^\mu = k^\mu + p^\mu$

$$k^+ = \frac{l_\perp^2}{2q^-(1-y)}$$

$$p^+ = \frac{l_\perp^2}{2q^- y}$$

$$Q^2 = \frac{l_\perp^2}{y(1-y)}$$

Optical theorem

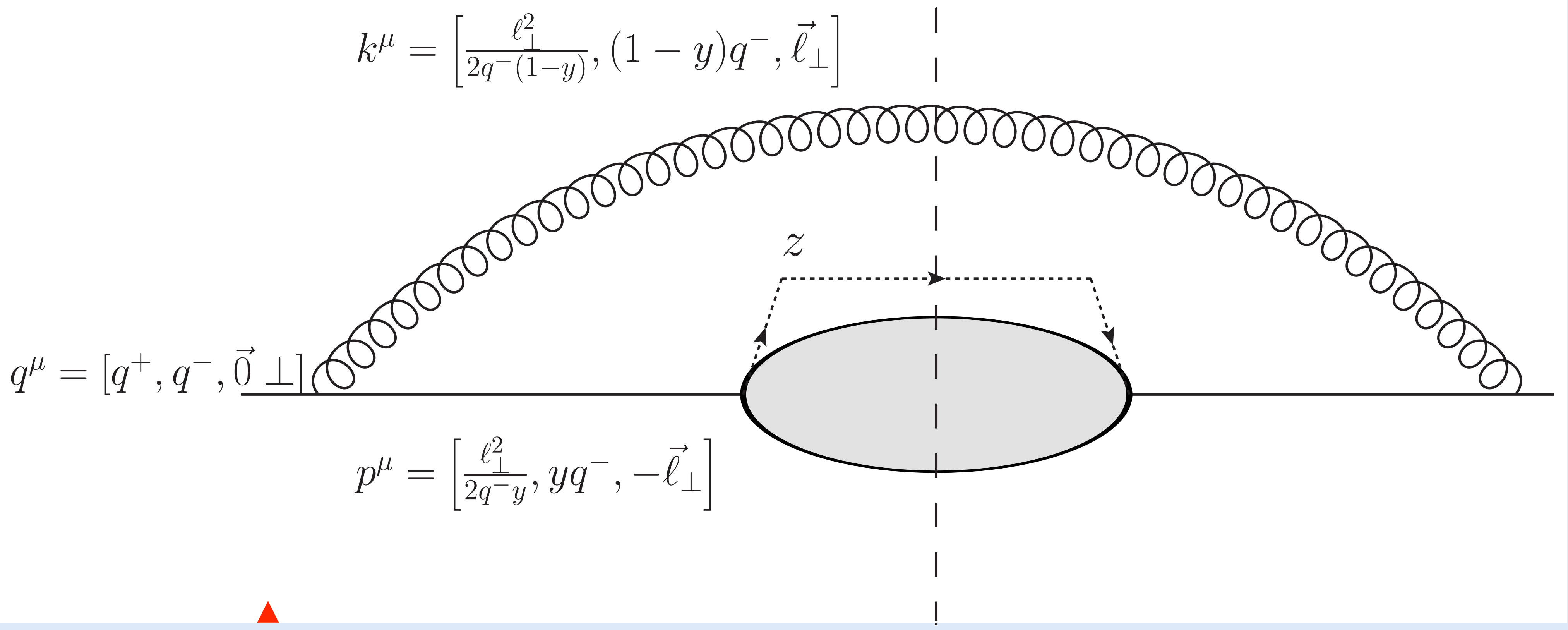
- ❑Imaginary part of the amplitude of forward scattering diagram is product of the diagram obtained by cutting the internal line

Forward scattering diagram

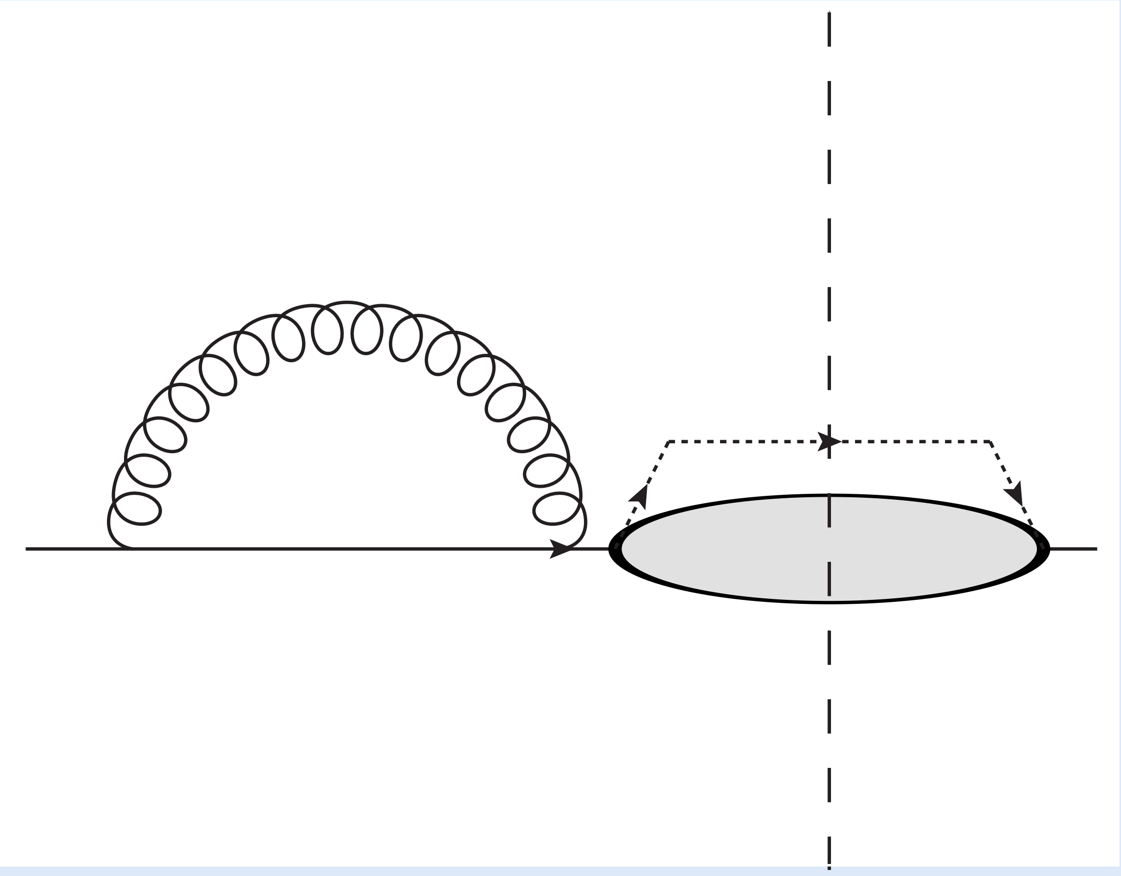
- ❑Cut-line represents final state
- ❑ Propagators on the cut-line are put on-shell

$$\frac{1}{(p^0)^2 - |\vec{p}\,|^2} \Longrightarrow \delta\left[(p^0)^2 - |\vec{p}\,|^2\right]$$

Fragmentation function: Single emission diagram and virtual correction



Real diagram



Virtual diagram

$$\frac{d\sigma}{\sigma_0} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^{Q^2}\frac{dl_\perp^2}{l_\perp^2}\int_z^1\frac{dy}{y}P(y)D\left(\frac{z}{y}\right) - \frac{\alpha_s(Q^2)}{2\pi}\int_0^{Q^2}\frac{dl_\perp^2}{l_\perp^2}D(z)\int_0^1\frac{dy}{y}P(y)$$

- ✦ **Splitting function:** $P(y) = \dfrac{1+y^2}{1-y}$
- ✦ **Soft divergence $y = 1$, canceled by the contribution from the virtual diagram**
- ✦ **Collinear divergence $l_\perp^2 \to 0$ remains and this should be included in the $D(z)$ as gluon formation happens in distant future.**

$$\frac{d\sigma}{\sigma_0} = \frac{\alpha_s(Q^2)}{2\pi}\int_0^{Q^2}\frac{dl_\perp^2}{l_\perp^2}\int_z^1\frac{dy}{y}P_+(y)D\left(\frac{z}{y}\right)$$

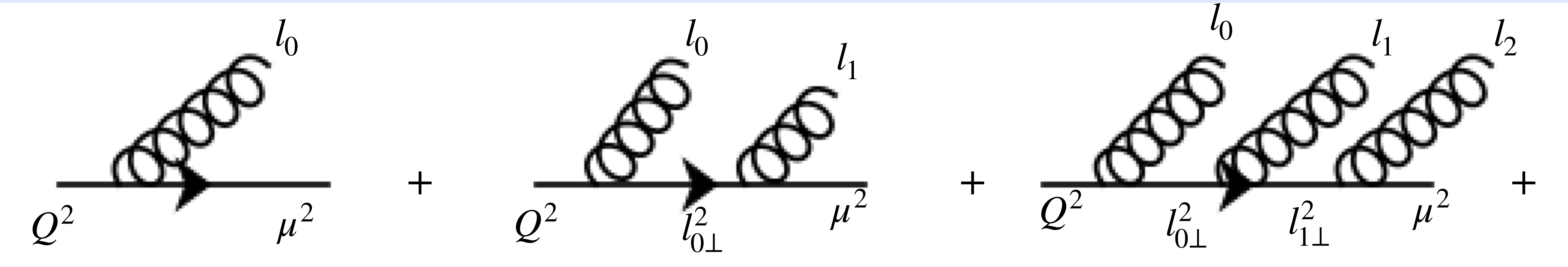
Formation time : $\tau^- = 2q^-/Q^2 = 2q^- y(1-y)/l_\perp^2$

Multiple emissions and vacuum DGLAP equation

Formulated by
V. Gribov and L. Lipatov (1972)
G. Altarelli and G. Parisi (1977)
Yu. Dokshitzer (1977)

$$\int_0^{Q^2} \frac{dl_\perp^2}{l_\perp^2} \longrightarrow \int_0^{\mu^2} \frac{dl_\perp^2}{l_\perp^2} + \int_{\mu^2}^{Q^2} \frac{dl_\perp^2}{l_\perp^2}$$

Absorb into bare fragmentation function



$$D(z,Q^2) = \left[1 + \frac{\alpha_s(Q^2)}{2\pi}\int_{\mu^2}^{Q^2}\frac{dl_{0\perp}^2}{l_{0\perp}^2}P_+(y_0) + \frac{\alpha_s(Q^2)}{2\pi}\int_{\mu^2}^{Q^2}\frac{dl_{0\perp}^2}{l_{0\perp}^2}P_+(y_0)\frac{\alpha_s(l_{0\perp}^2)}{2\pi}\int_{\mu^2}^{l_{0\perp}^2}\frac{dl_{1\perp}^2}{l_{1\perp}^2}P_+(y_1) \; + \ldots\ldots \right] * D(z,\mu^2)$$

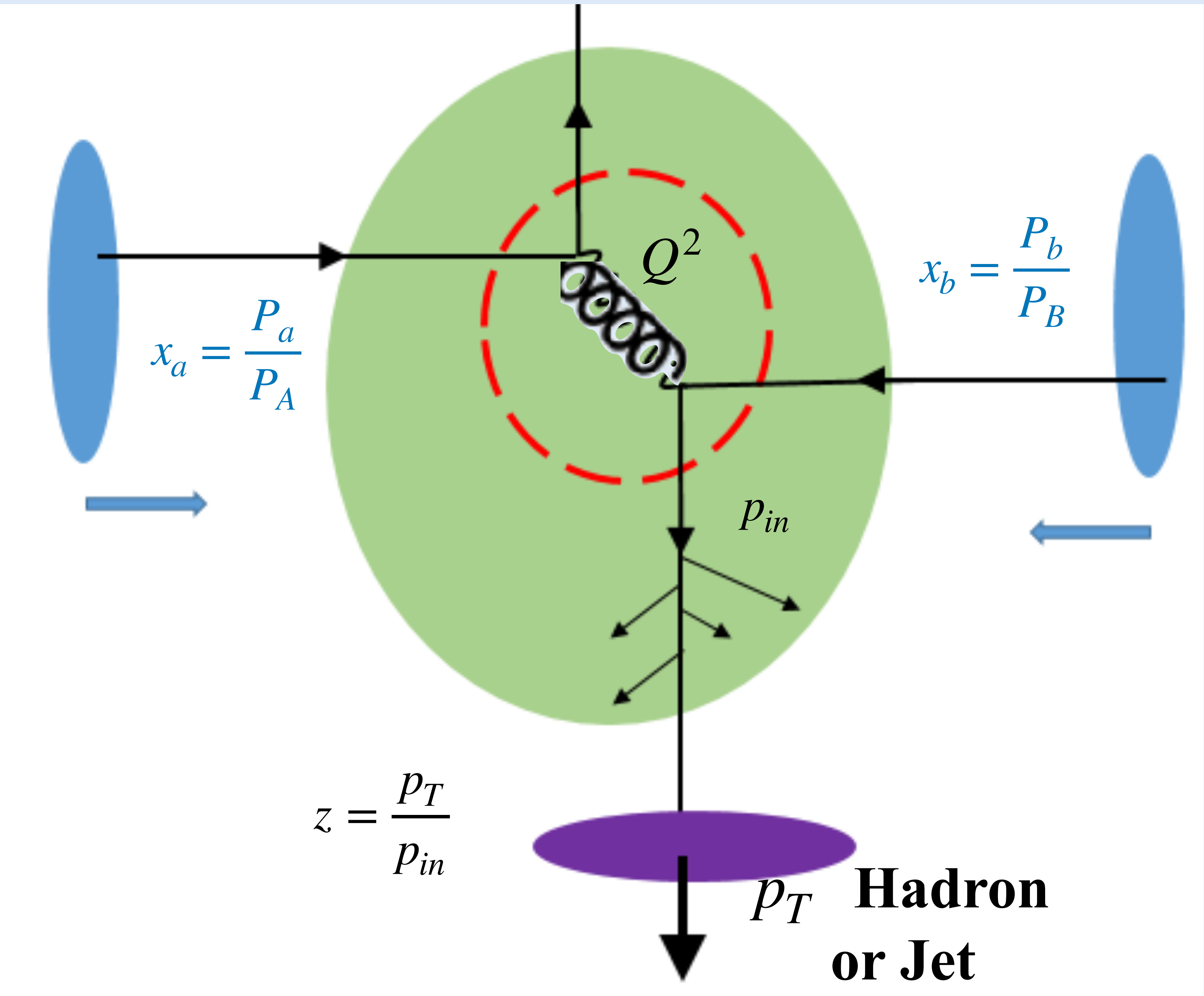
$$\int P(y) * D(z) = \int_z^1 \frac{dy}{y} P_+(y) D\left(\frac{z}{y}\right)$$

$$\frac{dD(z,Q^2)}{dQ^2} = \frac{\alpha_s}{2\pi Q^2}\int_z^1 \frac{dy}{y}P_+(y)D\left(\frac{z}{y},Q^2\right)$$

DGLAP equation is integro-differential equation

Requires Input fragmentation function at lower scale μ^2

Factorization and parton energy loss in-medium



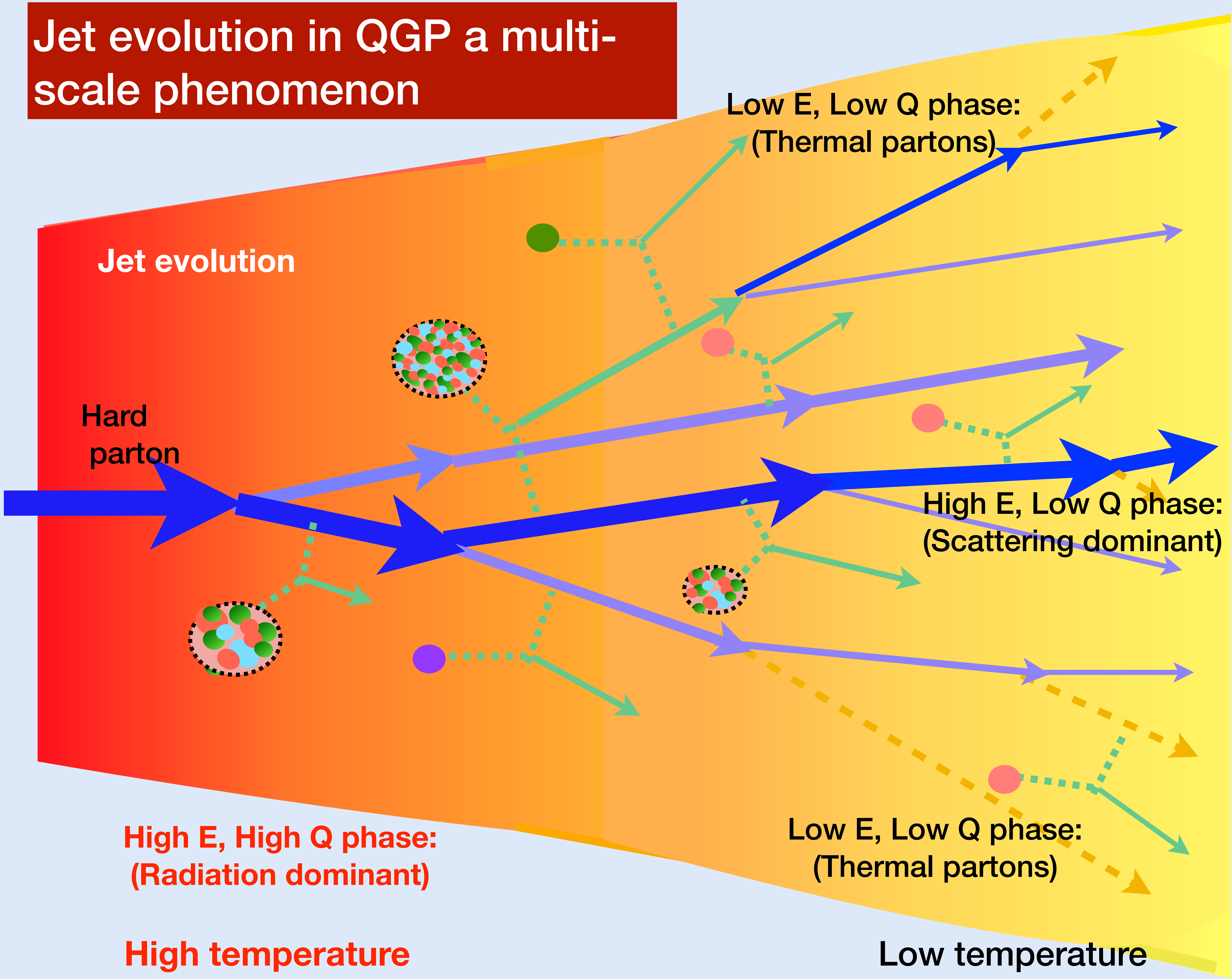
$$\frac{d\sigma^{AB\to h+X \text{ or Jet}+X}}{d^2p_{\mathrm{T}}dy} \sim \int dx_a dx_b \, f_a^A(x_a, Q^2) f_b^B(x_b, Q^2) \frac{d\hat{\sigma}}{d\hat{t}} \tilde{D}^{med}_{modified}(z, Q^2) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{Q^2}\right)$$

$$\updownarrow$$

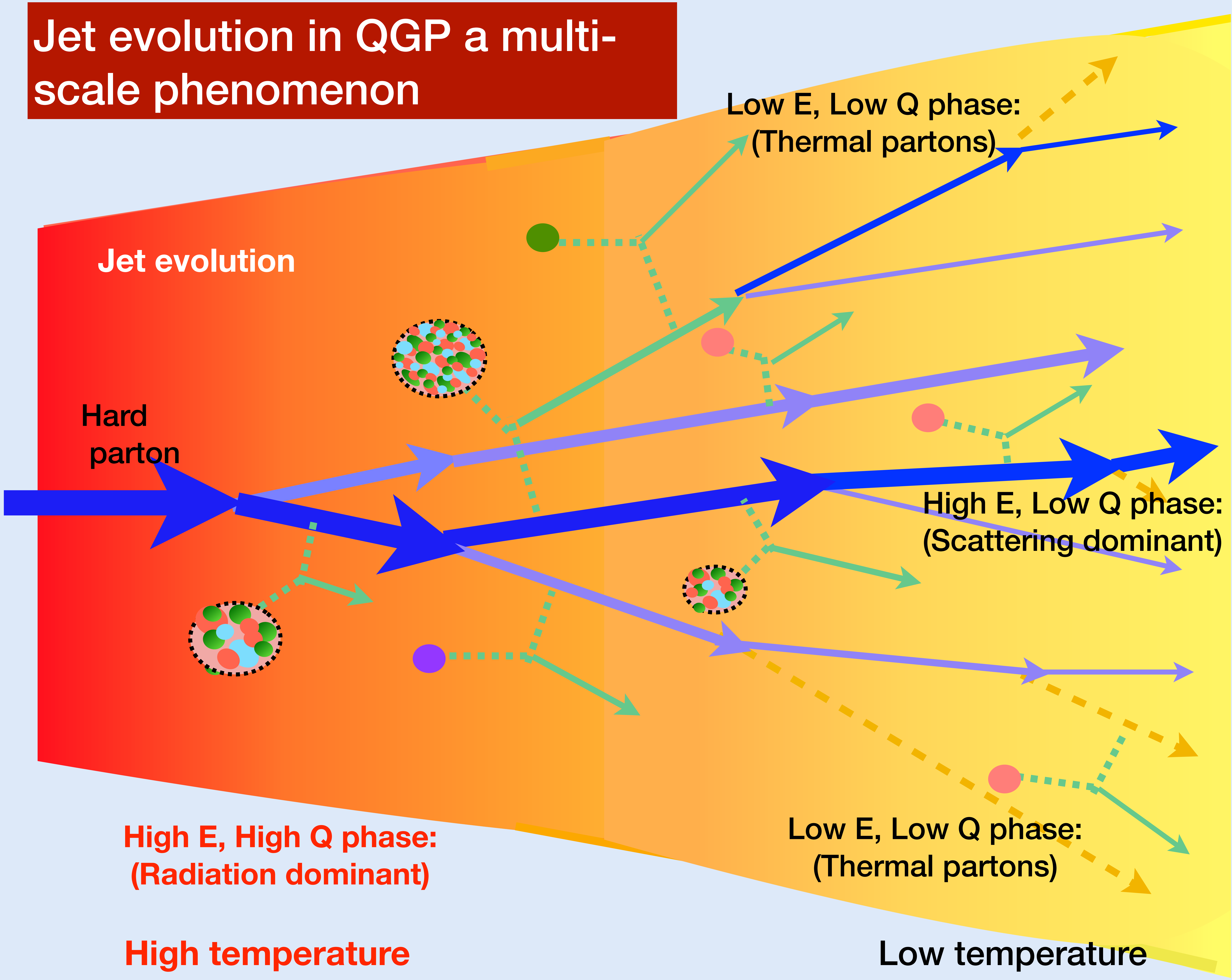
$$\tilde{J}^{med}_{modified}(z, Q^2)$$

Total cross section is a product of probabilities

Jet evolution in QGP a multi-scale phenomenon



Jet evolution in QGP a multi-scale phenomenon



Relevant theoretical framework

High E, High Q:

Higher-twist approach
MATTER

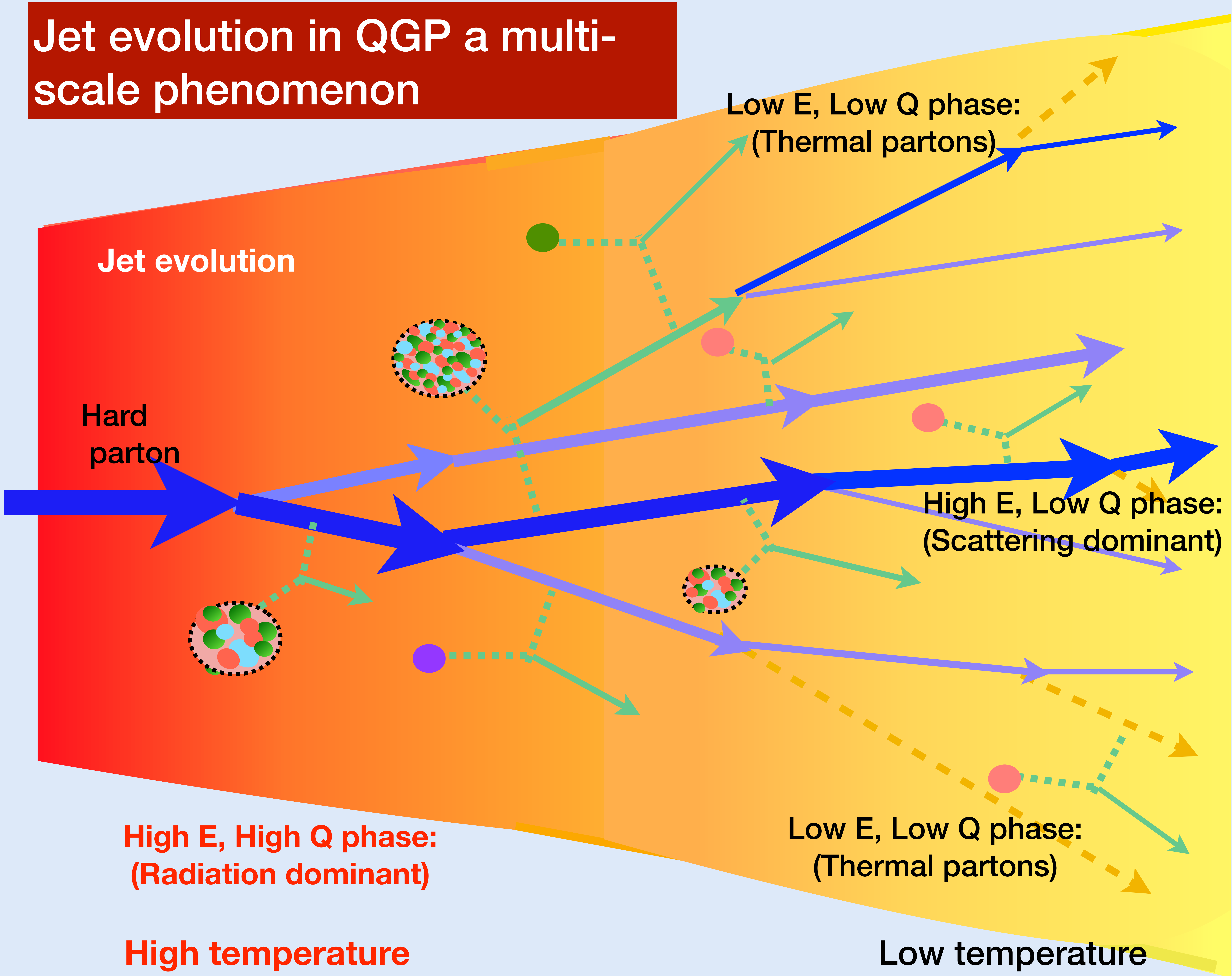
High E, low Q:

On-shell parton transport model
AMY, BDMPS approach
LBT, MARTINI

Low E, low Q:

Strong coupling formalism
AdS-CFT

Jet evolution in QGP a multi-scale phenomenon



Outstanding questions:

What is the microscopic structure of QGP ? Are there quasi-particles?

How does jet energy thermalizes in the plasma?

Jet substructure modifications?

Modification to Quark-gluon fractions ?

What can we learn about jet transport coefficients?

JETSCAPE: Framework to simulate all aspects of heavy-ion collisions

- ☐ **Modular, extensible and task-based event generator**
- ☐ **Framework is modular to "multi-stage", "energy-loss"**

✦ **JETSCAPE framework (arXiv:1903.07706)**
JETSCAPE pp19 tune (arXiv:1910.05481)
JETSCAPE AA (arXiv:2204.01163)

GitHub **JETSCAPE is available on GitHub:** github.com/JETSCAPE

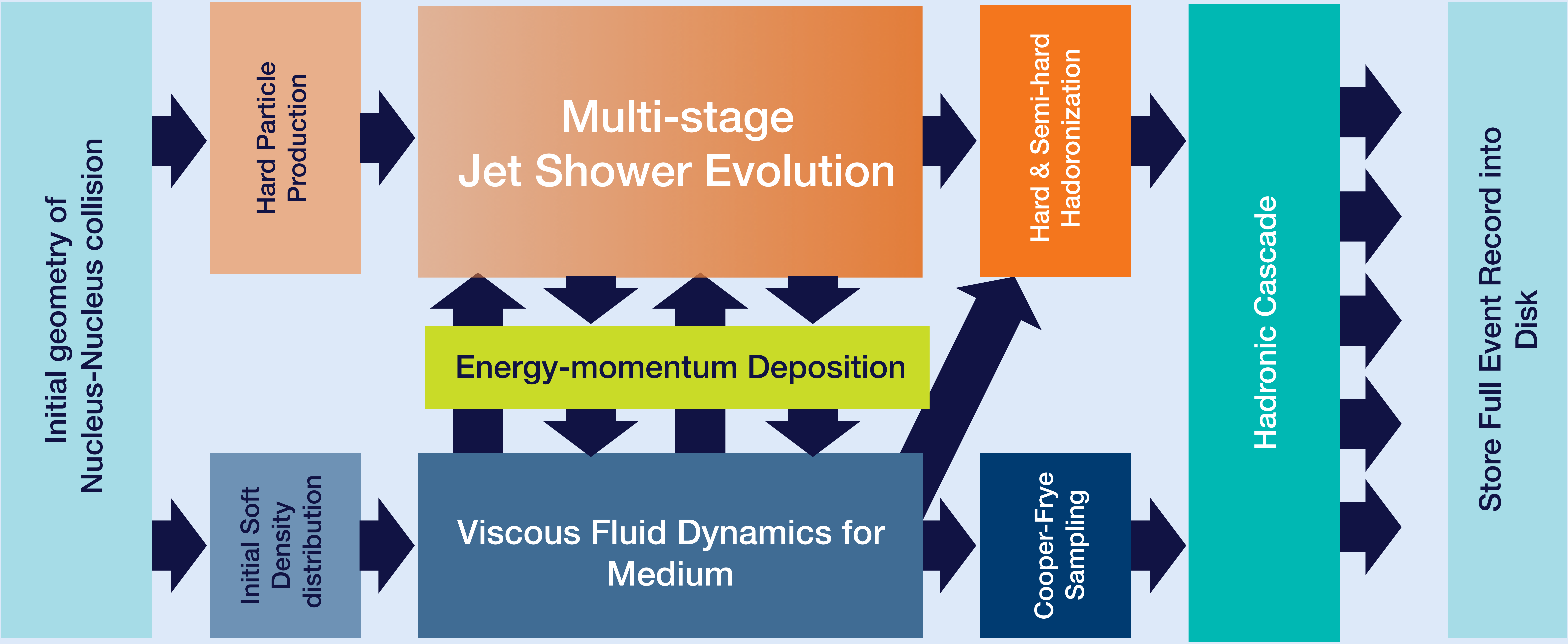

Initial geometry of Nucleus-Nucleus collision
Hard Particle Production
Initial Soft Density distribution
Multi-stage Jet Shower Evolution
Energy-momentum Deposition
Viscous Fluid Dynamics for Medium
Hard & Semi-hard Hadoronization
Cooper-Frye Sampling
Hadronic Cascade
Store Full Event Record into Disk

Diagram by:
Y. Tachibana

In this session, we focus on multi-stage jet energy loss formalism

JETSCAPE: Framework to simulate all aspects of heavy-ion collisions

- ☐ **Modular, extensible and task-based event generator**
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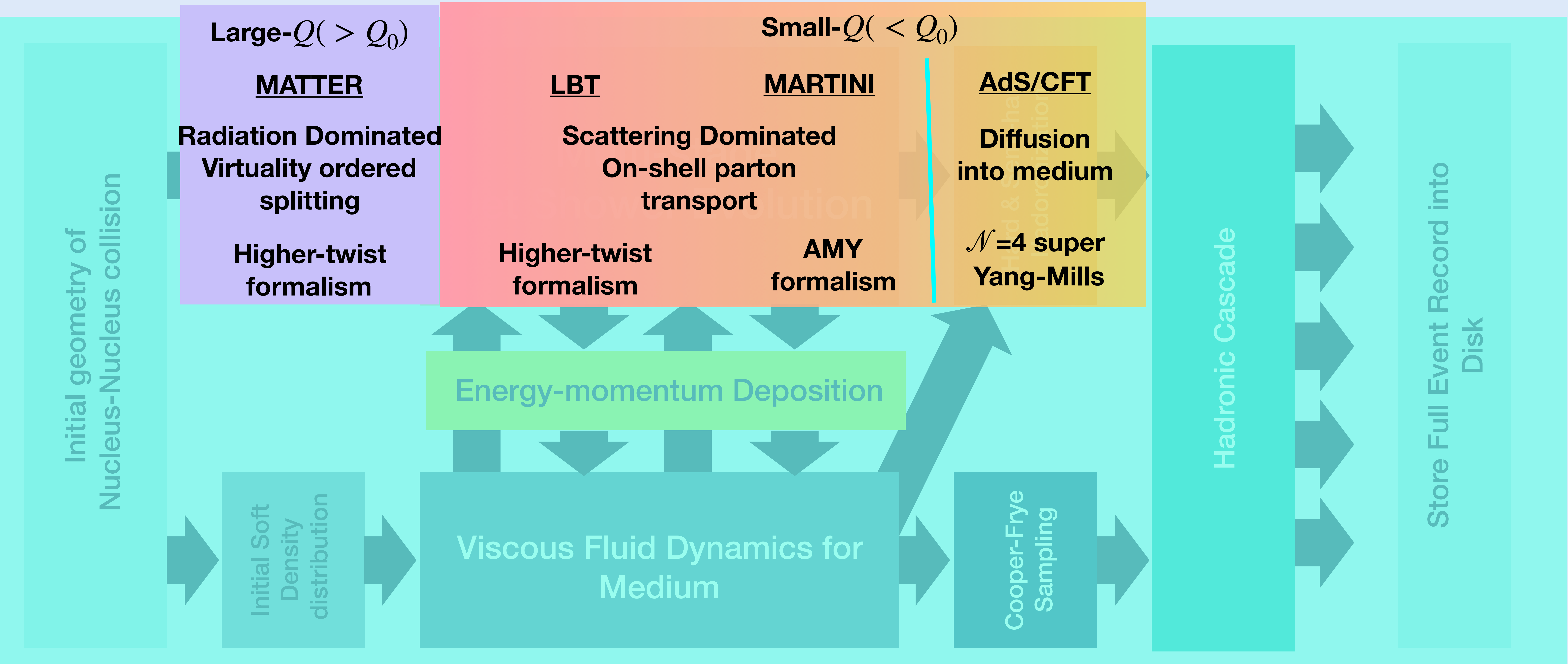
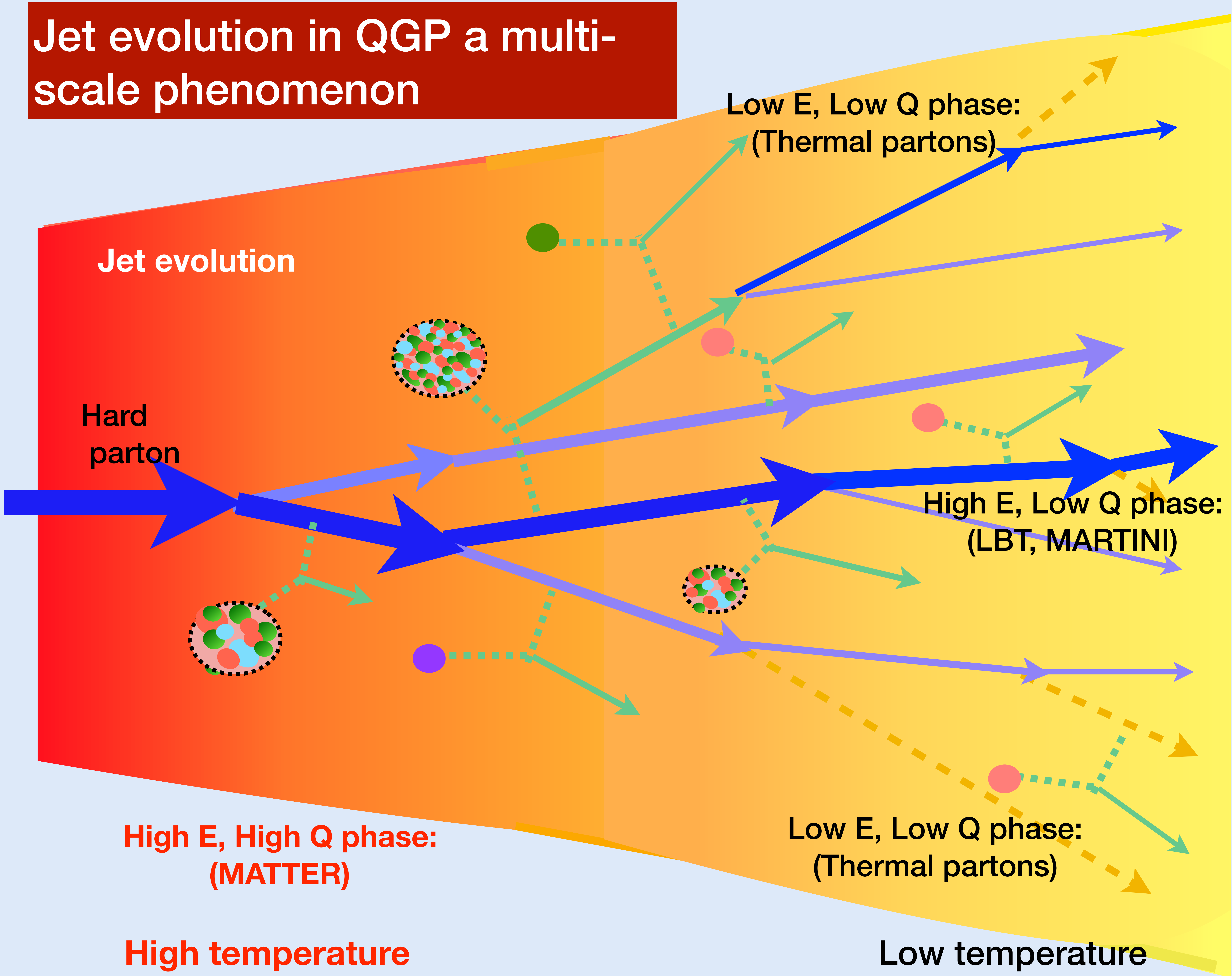


Large-$Q(>Q_0)$	Small-$Q(<Q_0)$		
MATTER	**LBT**	**MARTINI**	**AdS/CFT**
Radiation Dominated Virtuality ordered splitting	Scattering Dominated On-shell parton transport		Diffusion into medium
Higher-twist formalism	Higher-twist formalism	AMY formalism	$\mathcal{N}$=4 super Yang-Mills

Diagram by:
Y. Tachibana

In this session, we focus on parton energy loss for light-flavors

Jet evolution in QGP a multi-scale phenomenon



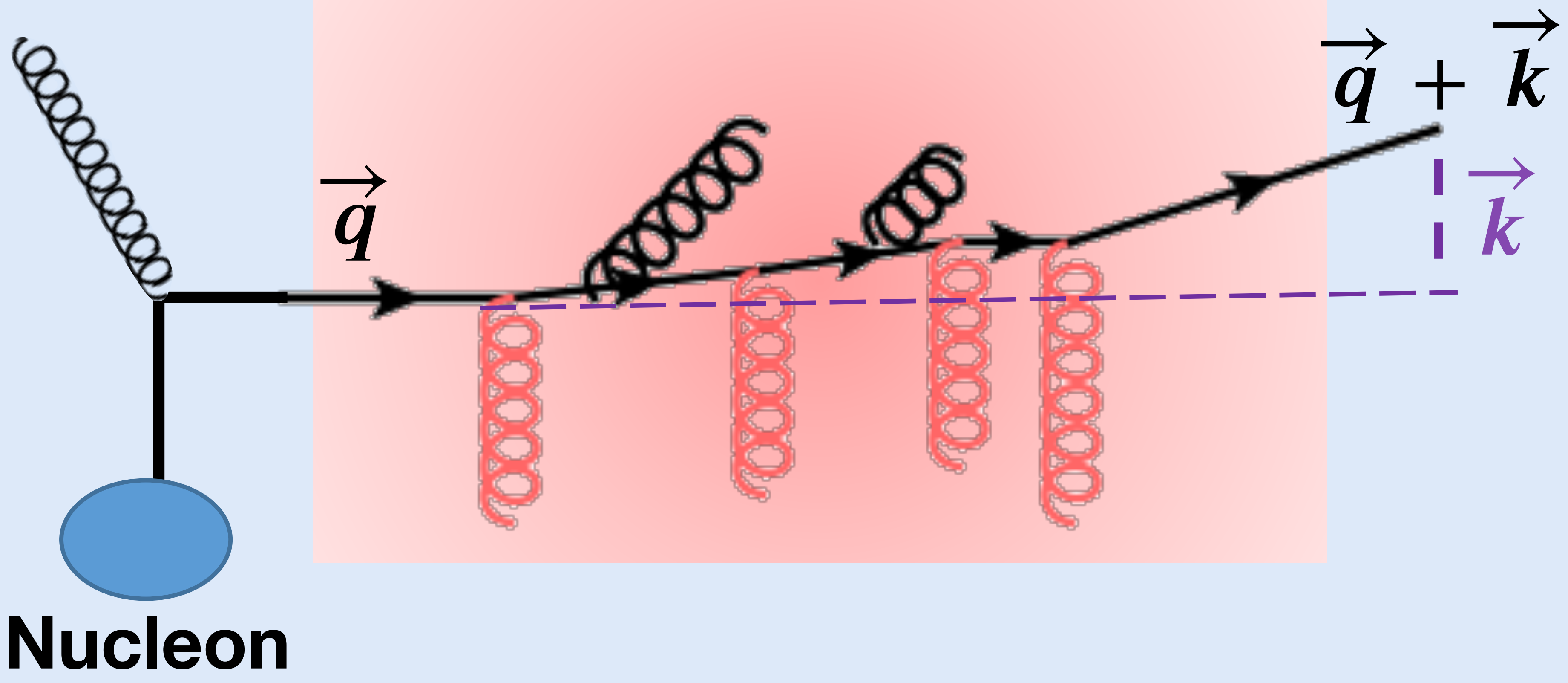
Heavy quark energy loss:
Talk by Wenkai Fan

Medium response to jets:
Talk by Ismail Saudi

High virtuality phase

Jet energy loss transport coefficients

❑ **Factorized approach to jet evolution**



❑ **Transport coefficient $\hat{q}$:**

Average transverse momentum squared per unit length

$$\hat{q}(\vec{r},t) = \frac{\langle \vec{k}_\perp^{\,2} \rangle}{L} \propto \langle M | F_\perp^{+}(y^-, y_\perp) F^{+\perp}(0) | M \rangle$$

❑ **Transport coefficient $\hat{e}$:**

$$\hat{e}(\vec{r},t) = \frac{\langle k_z \rangle}{L} \propto \langle M | \partial^- A^+(y^-, y_\perp) A^+(0) | M \rangle$$

❑ **Transport coefficient $\hat{e}_2$:**

$$\hat{e}_2(\vec{r},t) = \frac{\langle k_z^2 \rangle}{L} \propto \langle M | F^{+-}(y^-, y_\perp) F^{+-}(0) | M \rangle$$

Higher-twist formalism: (collinear expansion)

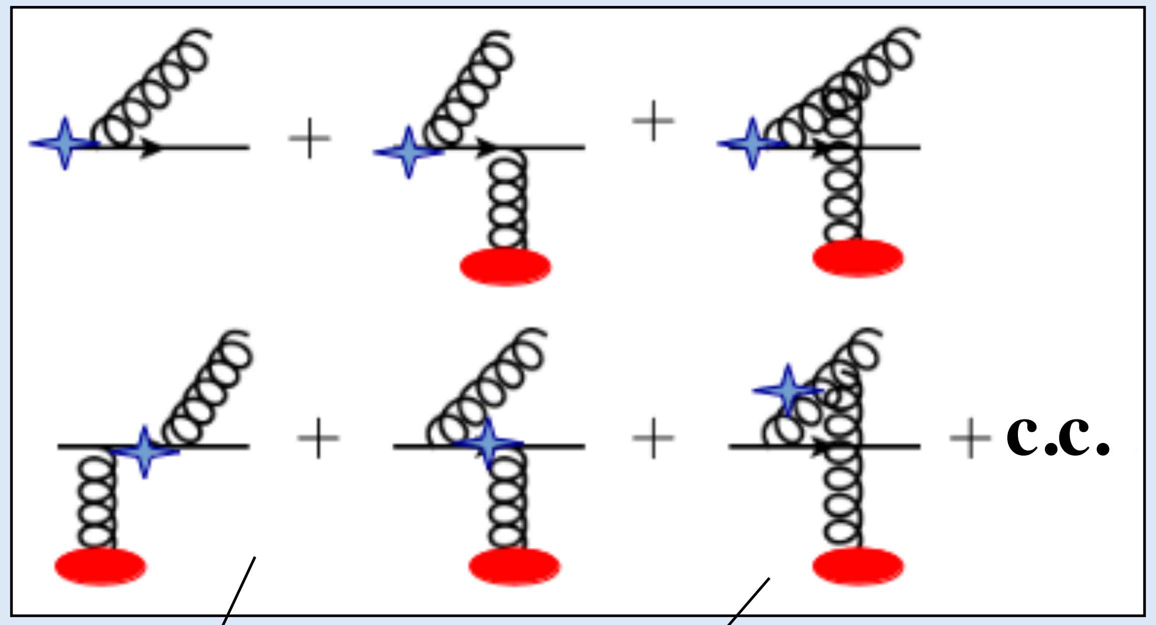
$$\frac{dN}{dy\,d\mu^2} = \frac{\alpha_s}{2\pi} \frac{P_{qg}(y)}{\mu^2} \left[1 + \int_{\xi_o^+}^{\xi_o^+ + \tau^+} d\xi^+ K(\xi^+, \xi_o^+, y, q^+, \mu^2) \right];$$

$$K(\xi^+, \xi_o^+, y, q^+, \mu^2) = \frac{1}{y(1-y)\mu^2(1+\chi)^2} \left\{ 2 - 2\cos\left(\frac{\xi^+ - \xi_o^+}{\tau^+} \right) \right\} \times \left\{ C_{qg}^{\hat{q}^-} \hat{q} + C_{qg}^{\hat{e}^-} \hat{e} + C_{qg}^{\hat{e}_2^-} \hat{e}_2 \right\}$$

MATTER jet energy loss

- ❑ **Modular All Twist Transverse-scattering Elastic-drag and Radiation**
- ❑ **Based on in-medium DGLAP evolution equation**

$$Q_1^2 \geq Q_2^2 \geq Q_3^2 \ldots\ldots$$



Repeating single emission single scattering kernel

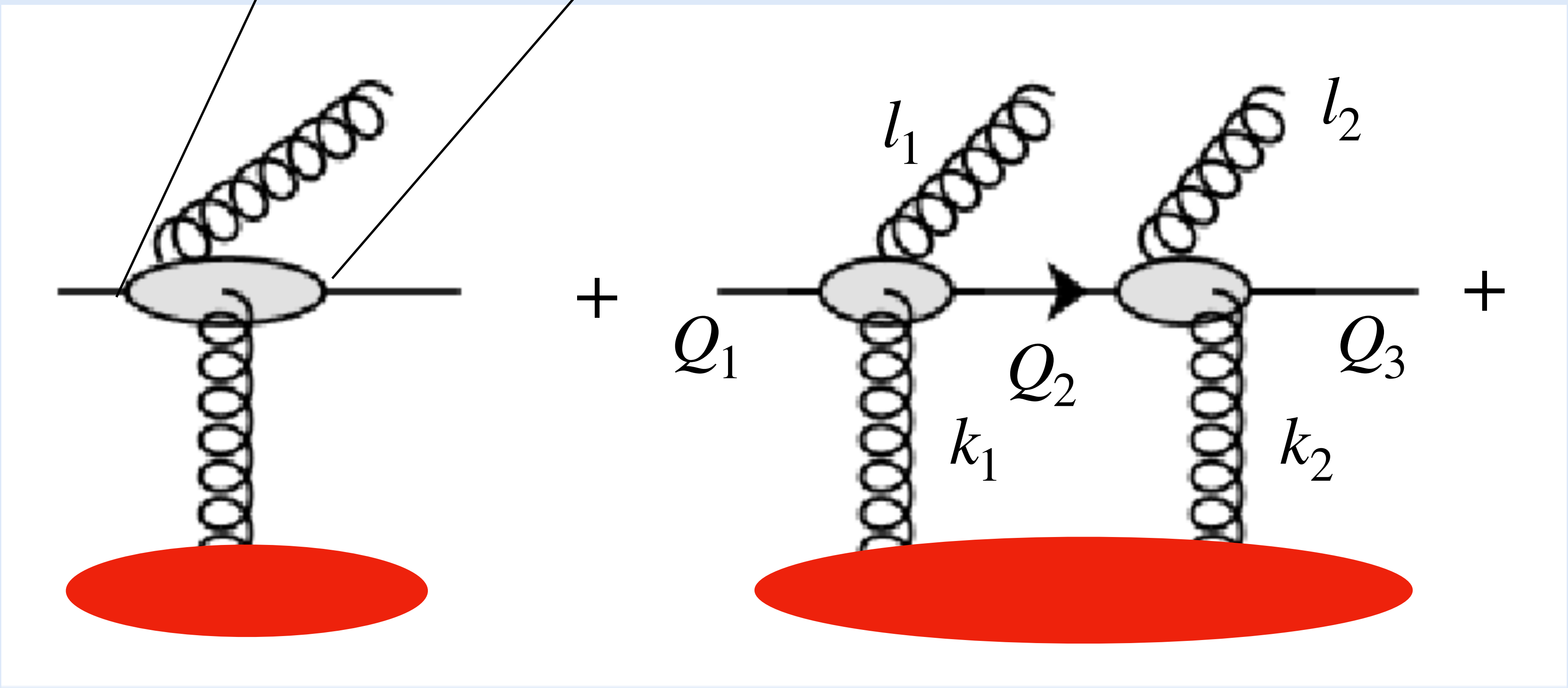
In limit: $< k_\perp^2 > \sim \hat{q}\tau^- << l_\perp^2 \sim Q^2$ Formation time: $\tau^- \sim q^-/Q^2$

$$\frac{\partial D\left(z,Q^2,\zeta_i^-\right)}{\partial \log Q^2} = \frac{\alpha_S}{2\pi}\int_z^1 \frac{dy}{y}\left[P_+\left(y\right) D\left(\frac{z}{y},Q^2,\zeta_i^-\right) + \right.$$

Vacuum term

$$+ \left(\frac{P(y)}{y\left(1-y\right)}\right)_+ D\left(\frac{z}{y},Q^2,\zeta_i^- + \tau^-\right) \times \int_{\zeta_i^-}^{\zeta_i^- + \tau^-} d\zeta^- \frac{\hat{q}\left(\zeta^-\right)}{Q^2}\left\{2 - 2\cos\left(\frac{\zeta^- - \zeta_i^-}{\tau^-}\right)\right\}\left.\right]$$

Medium term



Virtuality ordered emission approximation

Phys. Rev. C 88, 014909 (2013)
Phys. Rev. C 96, 024909 (2017)

Vacuum contribution are dominant, and medium-induced radiations are treated as correction

Low virtuality phase

LBT jet energy loss model

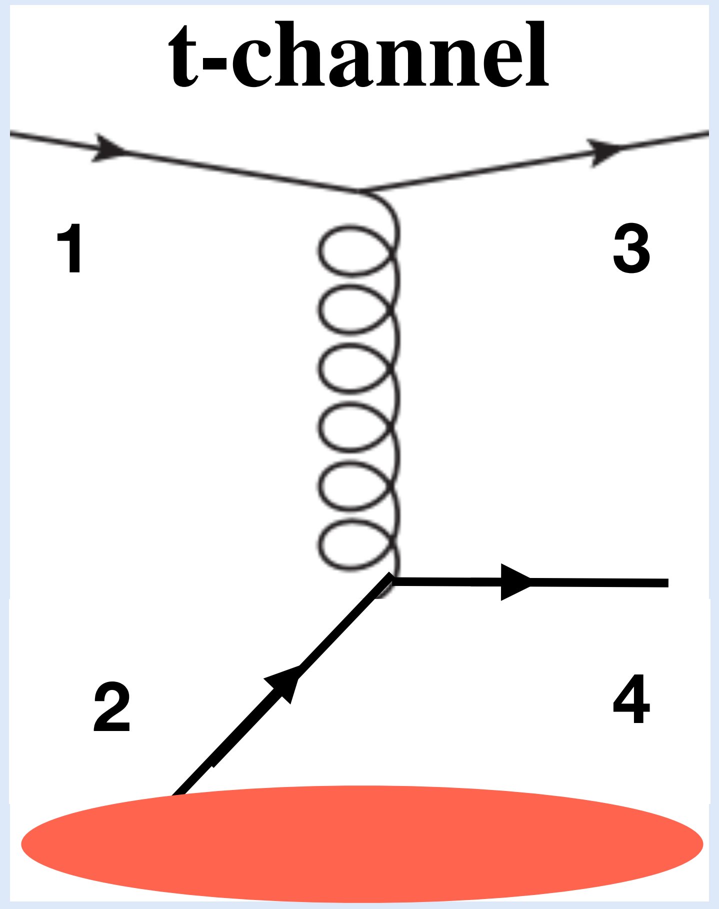
❑ **Based on linear Boltzmann transport equation**

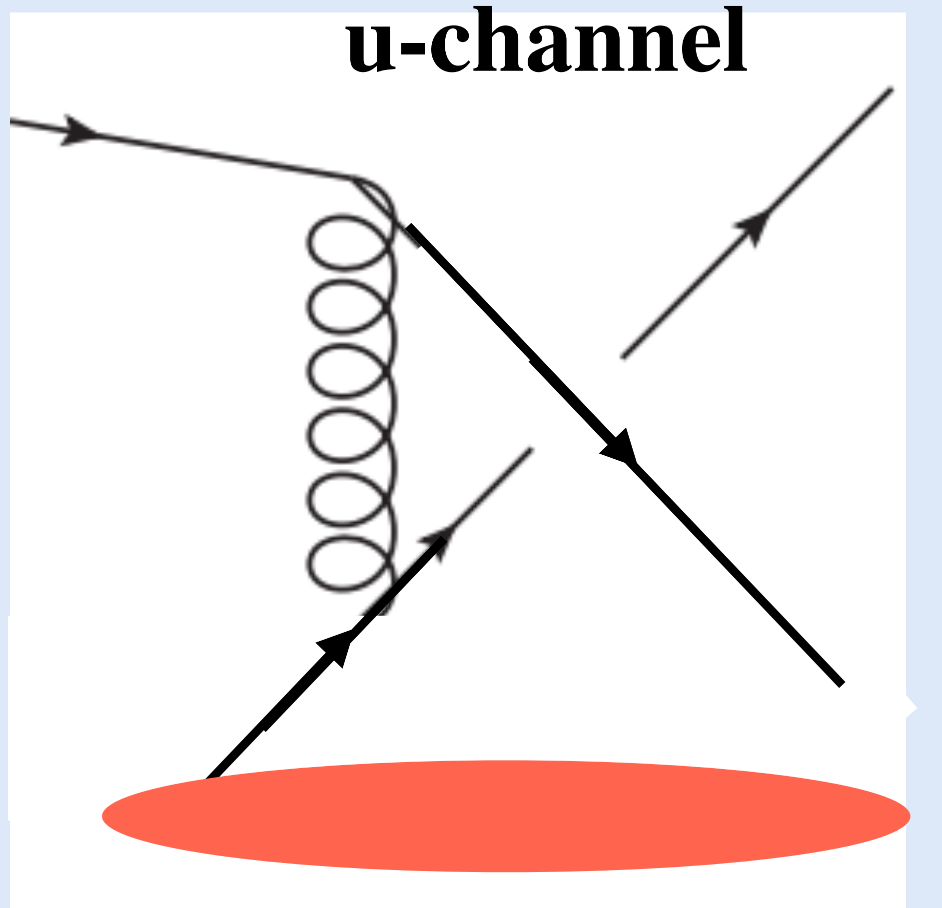
Evolution of phase-space distribution

$$p_i \cdot \partial f_i(x,p) = \Gamma_{el} + \Gamma_{inel}$$

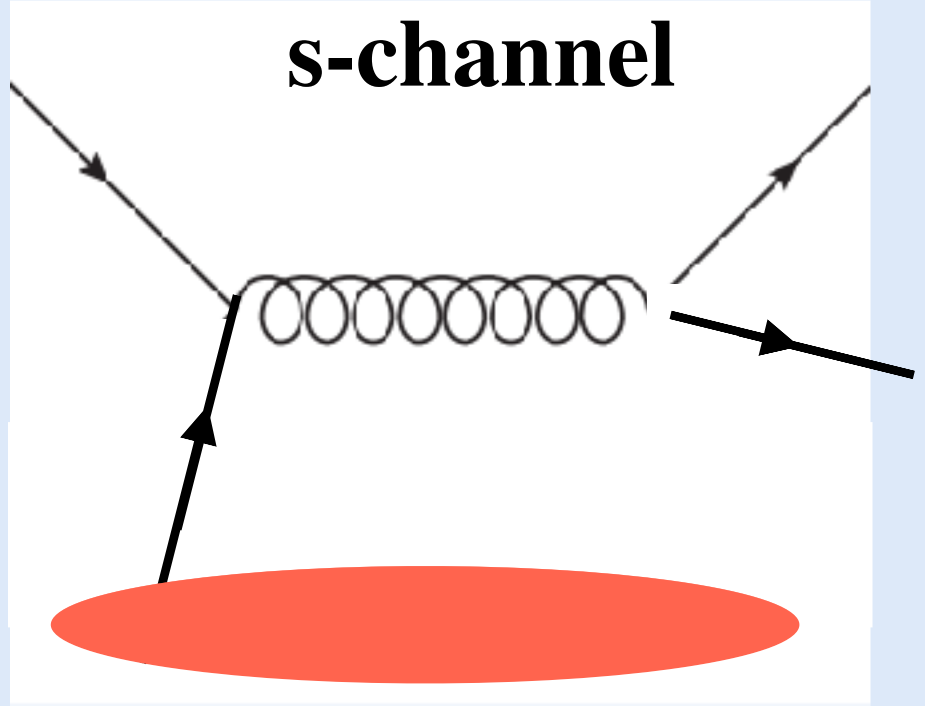
Elastic scattering: LO $2 \leftrightarrow 2$ proccess

Inelastic scattering: Single gluon emission rate using Higher Twist (depends on $\hat{q}$)


t-channel


u-channel

❑ **Elastic scattering kernel**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$$\Gamma_{12\rightarrow 34}(p_1) = \int \frac{d^3p_2}{(2\pi)^3 2E_2}\frac{d^3p_3}{(2\pi)^3 2E_3}\frac{d^3p_4}{(2\pi)^3 2E_4}(f_1 f_2 - f_3 f_4)(2\pi)^4\delta^4(p_1+p_2-p_3-p_4)\,|\mathscr{M}12 \rightarrow 34|^2$$

❑ **Total elastic scattering rate and probability**

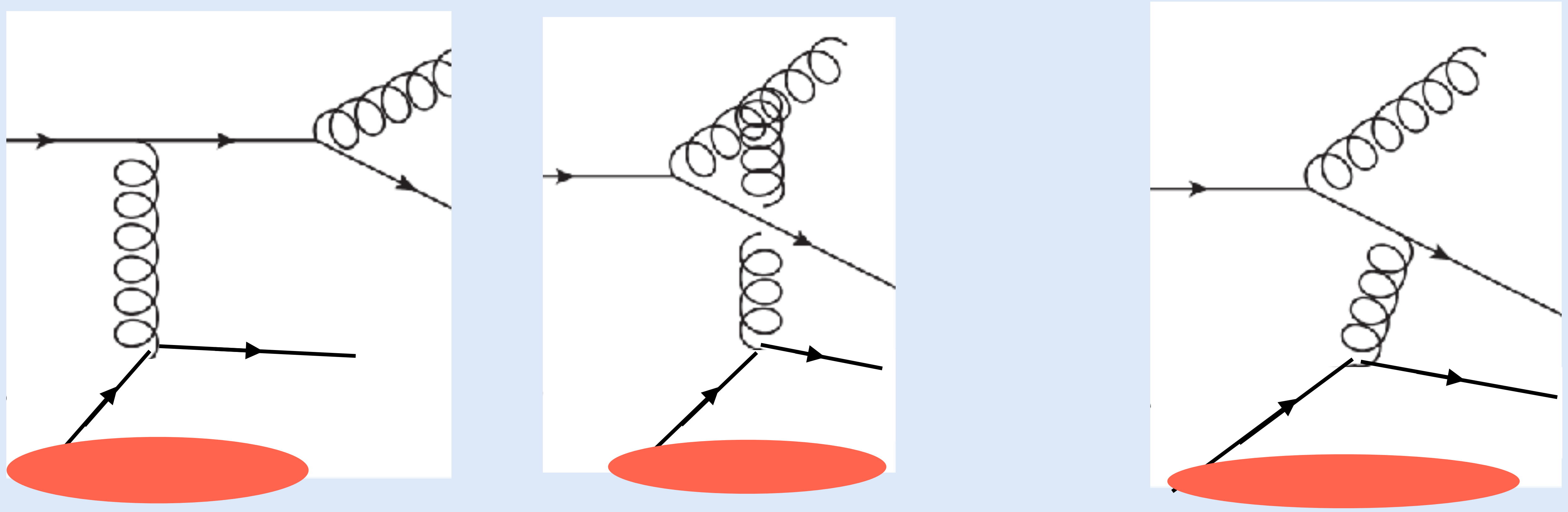
$$\Gamma_{total} = \sum_i \Gamma_i; \quad P_{el} = \Gamma_{total}\Delta t$$

Phys. Rev. C 91, 054908 (2015)
Phys. Rev. C 94, 014909 (2016)

LBT jet energy loss model

❑ **Inelastic scattering: Single gluon emission**

Inelastic scattering: Single gluon emission rate using Higher Twist (depends on $\hat{q}$)



❑ **Medium-induced gluon radiation:**

$$\langle N_g \rangle = \Delta t \int dx dk_\perp^2 \frac{dN_g}{dx dk_\perp^2 dt}$$

$$\frac{dN_g}{dx dk_\perp^2 dt} = \frac{2\alpha_s C_A \hat{q} P(x) k_\perp^4}{\pi (k_\perp^2 + x^2 m^2)^4} \sin\left(\frac{t - t_i}{2\tau_f}\right)$$

❑ **Multiple scattering (n) during each time step are allowed (Poisson distribution):**

$$P(n) = \frac{\langle N_g \rangle^n}{n!} e^{-\langle N_g \rangle}$$

❑ **Inelastic probability for medium-induced gluon radiation**

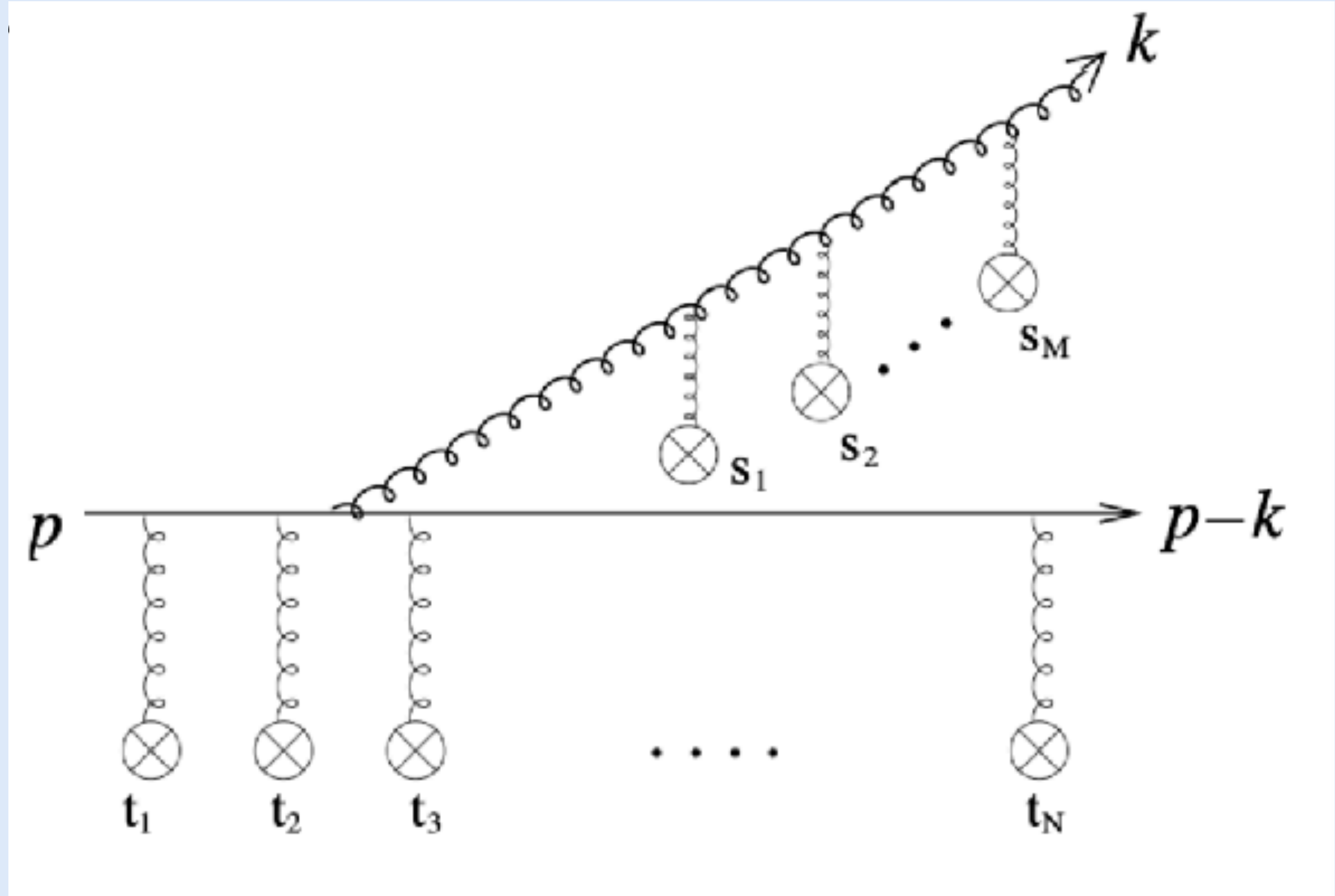
$$P_{inel} = 1 - e^{-\langle N_g \rangle}$$

Phys. Rev. C 91, 054908 (2015)
Phys. Rev. C 94, 014909 (2016)

MARTINI jet energy loss model

- **Modular Algorithm for Relativistic Treatment of heavy IoN Interactions**

Phys. Rev. C 80:054913 (2009)



Momentum distribution of the hard parton is given by
The following Fokker-Planck type rate equations

$$\frac{dP(p)}{dt} = \int_{-\infty}^{\infty} dk \left(P(p+k)\frac{d\Gamma(p+k,k)}{dk} - P(p)\frac{d\Gamma(p,k)}{dk} \right)$$

- **Based on Arnold-Moore-Yafffe (AMY) formalism**

  In limit of high temperature so QCD coupling is weak $g << 1$

JHEP 01, 030 (2003)
JHEP 06, 030 (2002)

- **Landau-Pomeranchuk-Migal (LPM) effect:**

  Scattering centers act coherently during formation time when $\tau_f > \lambda_{MFP}$

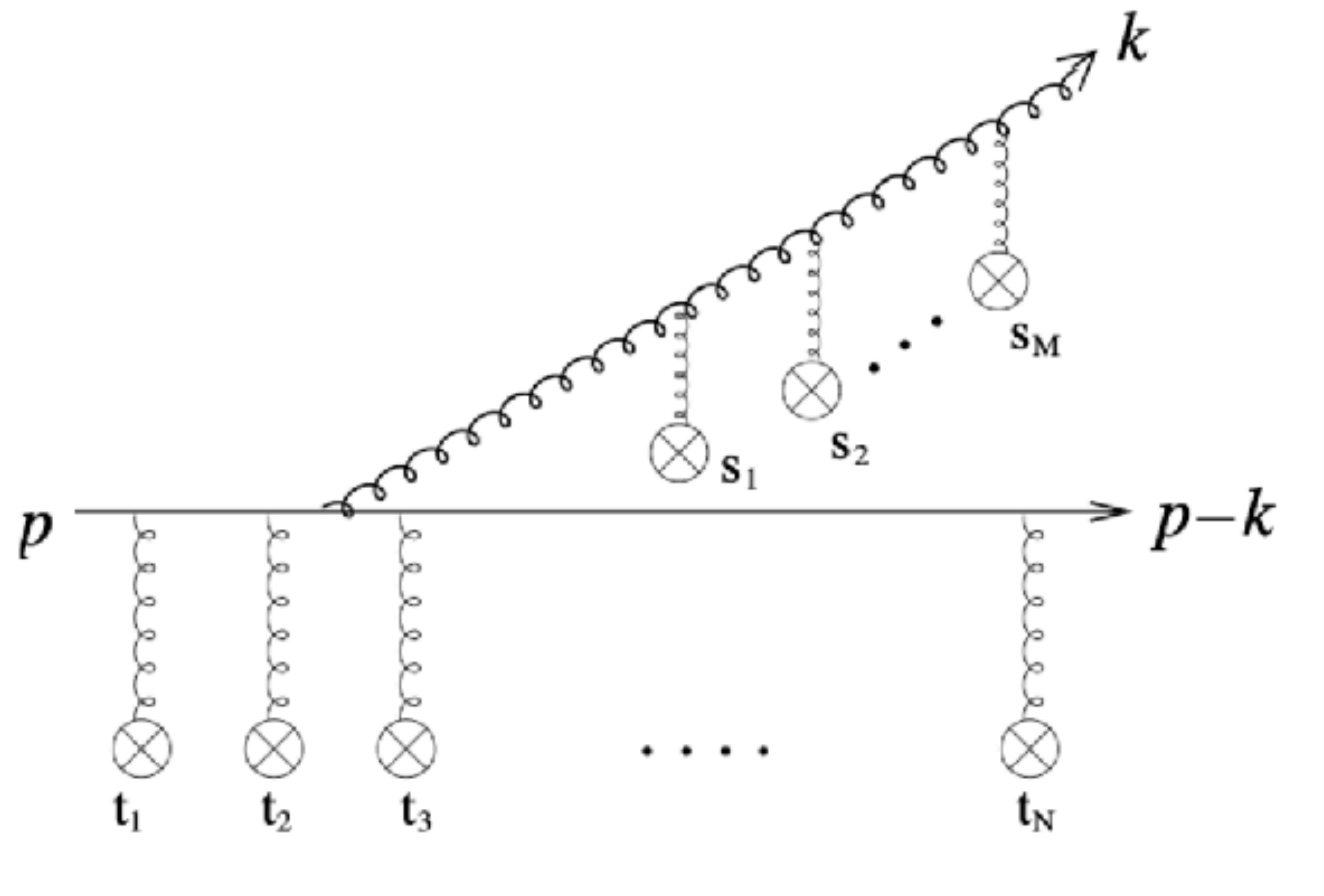
  Coherent scatterings leads to the suppression of emissions
  In AMY formalism LPM effect is calculated by resuming infinite ladder diagrams

MARTINI jet energy loss model

- ❑ Transition rate for process 1->2 is given by

$$\frac{d\Gamma}{dk}(p,k,T) = \frac{C_s g^2}{16\pi p^7}\frac{1}{1 \pm e^{-k/T}}\frac{1}{1 \pm e^{-(p-k)/T}}\left\{\begin{array}{ll} \frac{1+(1-x)^2}{x^3(1-x)^2} & q \to qg \\ N_f\frac{x^2+(1-x)^2}{x^2(1-x)^2} & g \to q\bar{q} \\ \frac{1+x^4+(1-x)^4}{x^3(1-x)^3} & g \to gg \end{array}\right\}$$

$$\times \int \frac{d^2h}{(2\pi)^2} 2h \,.\, Re\, F(h,p,k)$$



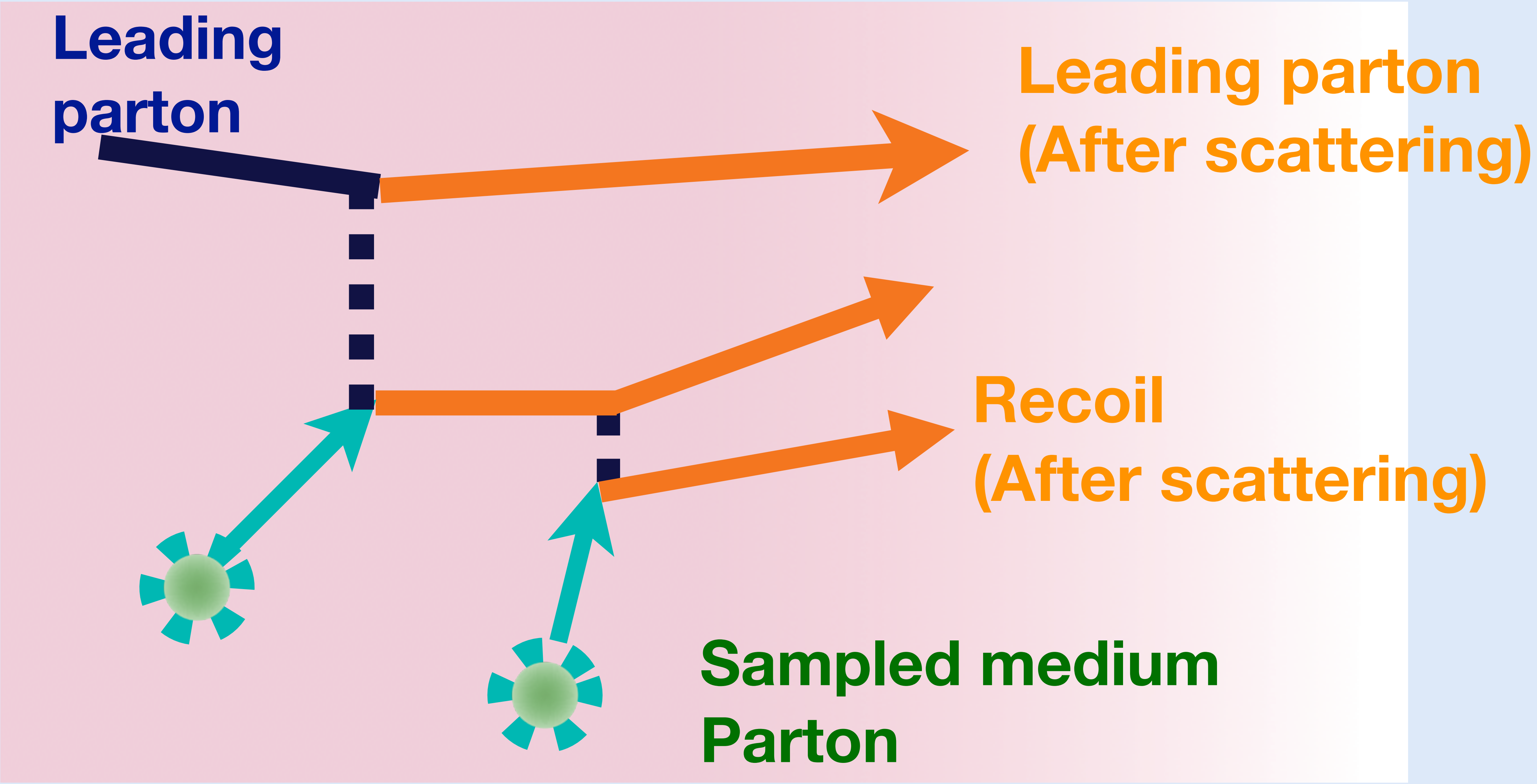
The function $F(h,p,k)$ is the solution of the integral equation that depends on Collision kernel

$$C(q_\perp) = \frac{m^2 D}{q_\perp^2(q_\perp^2 + m_D^2)}, \quad m_D^2 = \frac{g_s^2 T^2}{6}(2N_c + N_f)$$

- ❑ Elastic scattering rates are same as LBT
- ❑ Quark-gluon conversion channel is also included

Recoil-hole: Medium response

- ❑ It is a weakly coupled approach to medium response



- ❑ Sampled medium parton (Holes)

  Medium partons kicked out the jet parton
  Propagates as a parton shower in jet shower

- ❑ Recoil Parton

  Sampled from the thermal distribution
  Subtracted from the total signal

$$\left.\frac{dp^{\mu}}{d\eta d\phi}\right|_{\text{jet shower}} - \left.\frac{dp^{\mu}}{d\eta d\phi}\right|_{\text{picked-up}} = \left.\frac{dp^{\mu}}{d\eta d\phi}\right|_{\text{signal}}$$

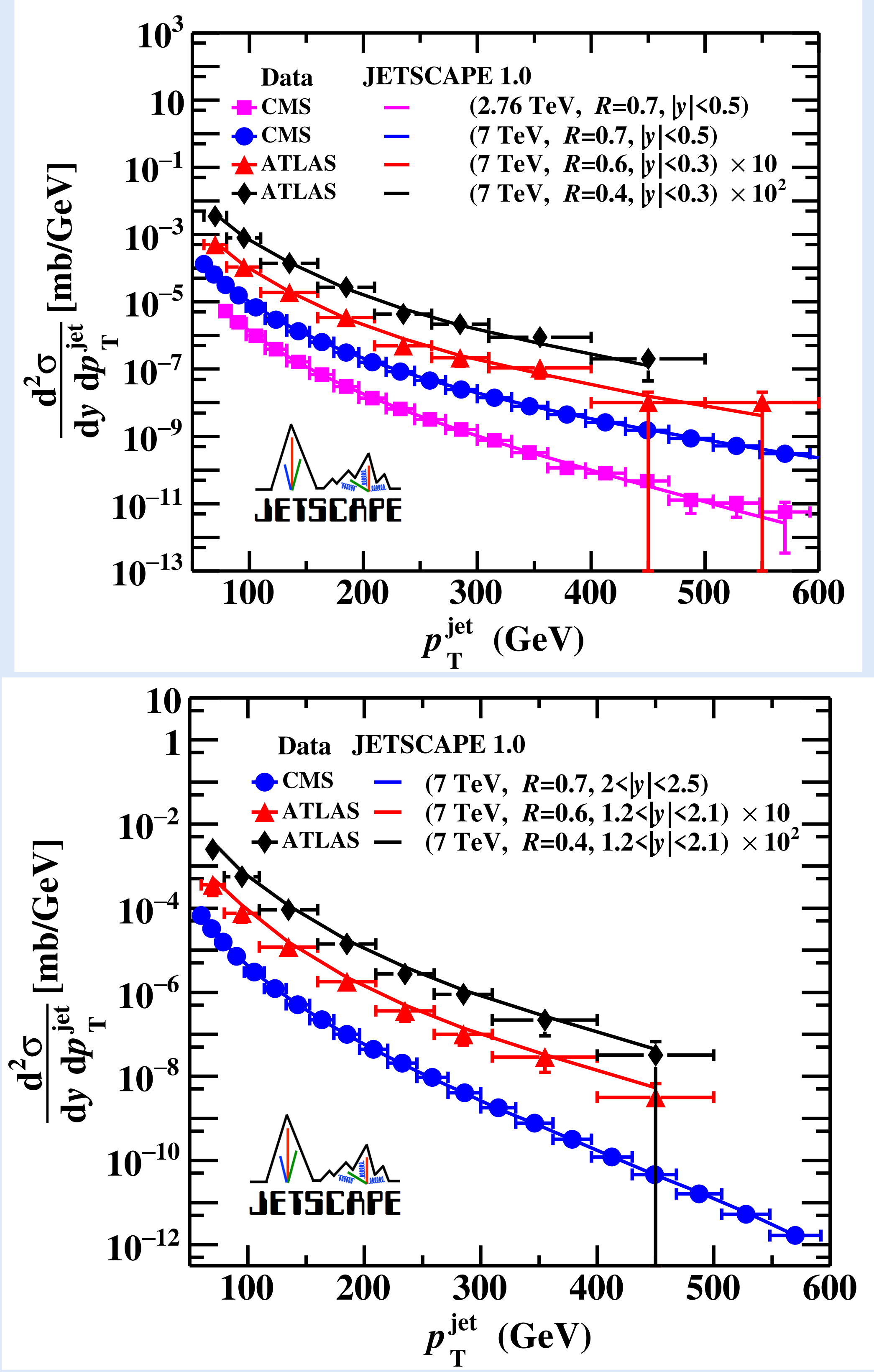
Jet parton and recoil are hadronized together to form total signal
Holes partons are hadronized separately and used to determine the correlated background to jet

JETSCAPE pp19 tune

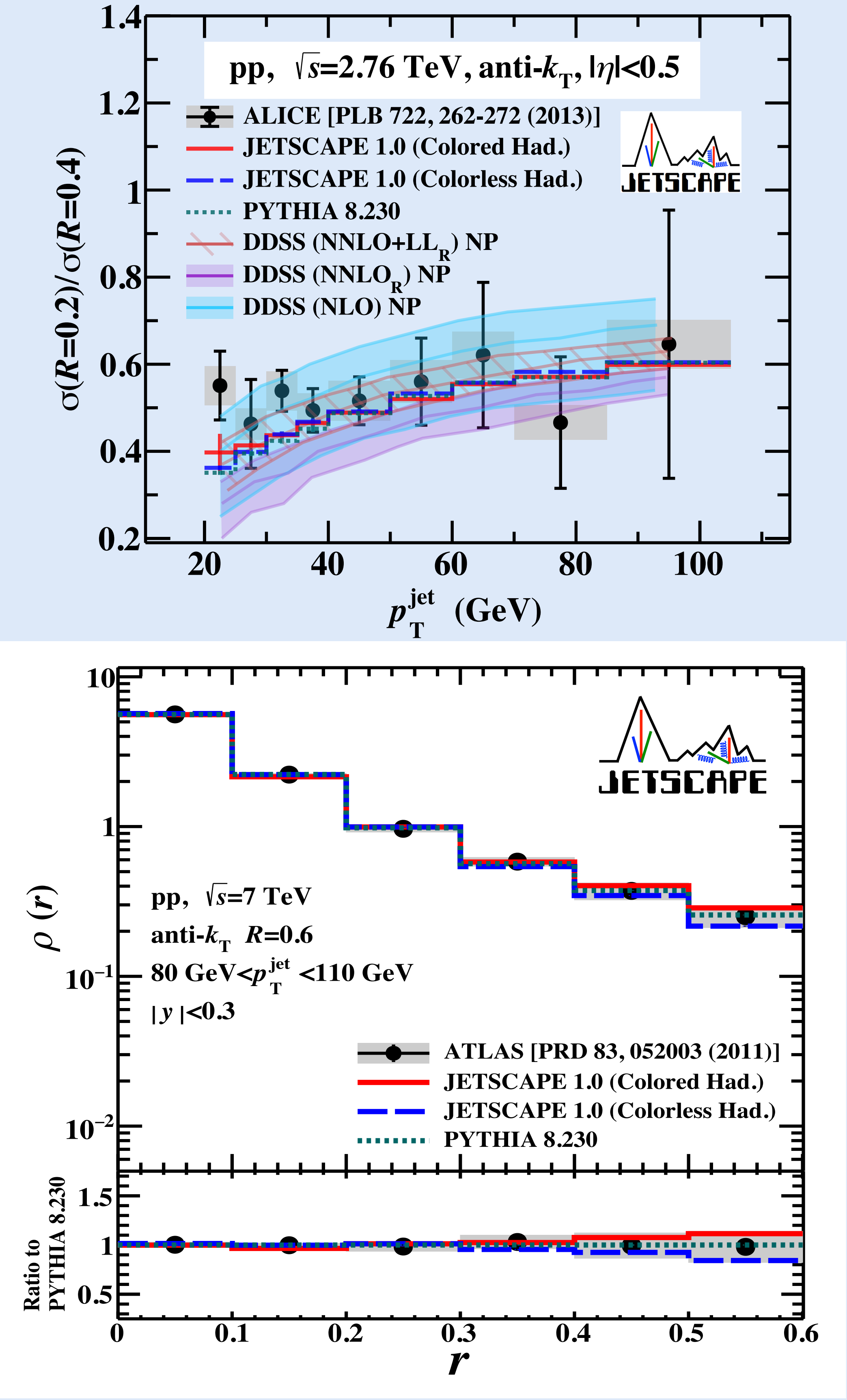
pp19 tune (arXiv:1910.05481)

- **Optimized value of parameters:**
  - ✦ Lambda QCD: $\Lambda_{\mathrm{QCD}} = 200\mathrm{MeV}$
  - ✦ Initial virtuality (off-shellness) of the parton after hard scattering: $Q_{\mathrm{in}} = \dfrac{p_{\mathrm{T}}}{2}$

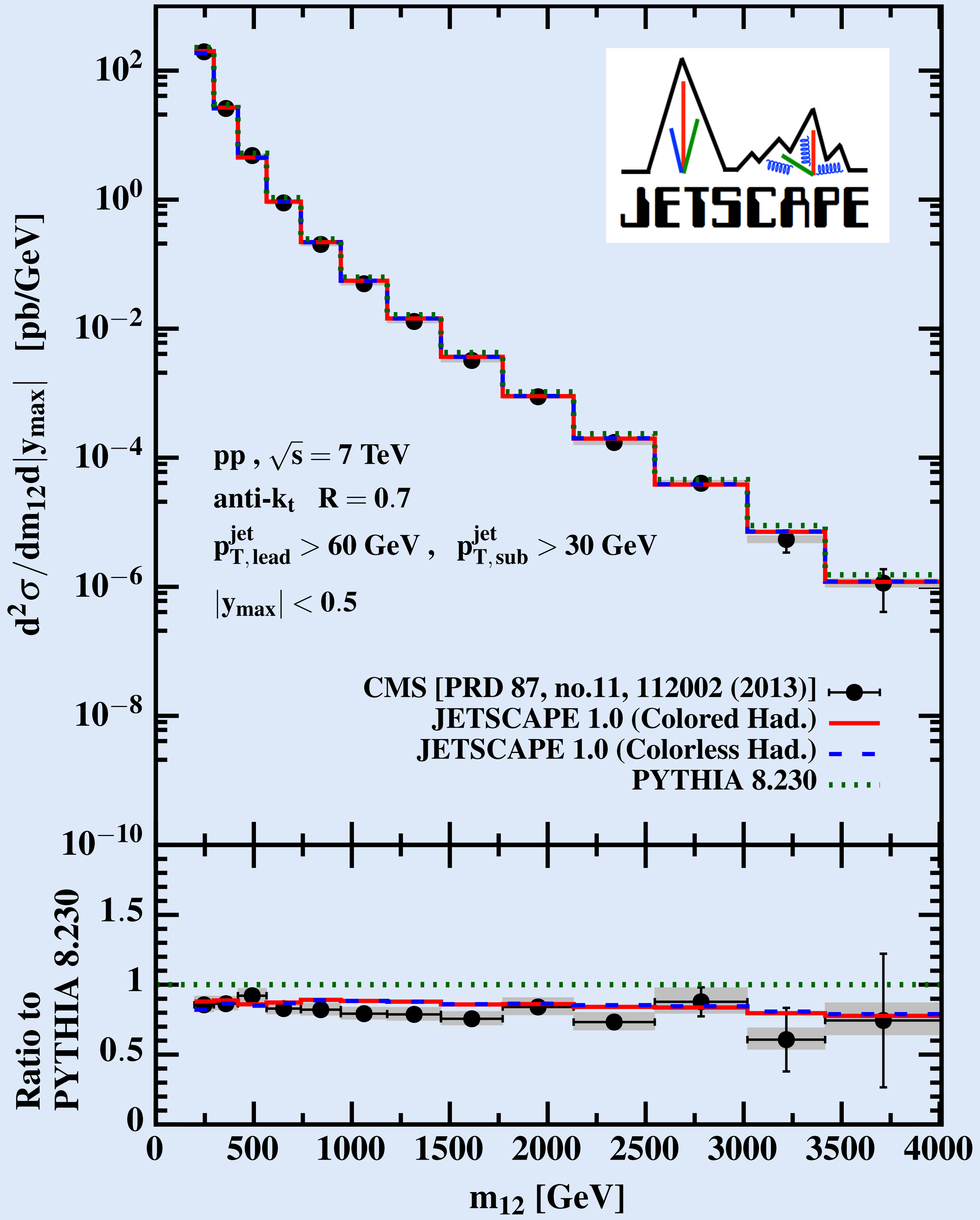
Inclusive jet cross section



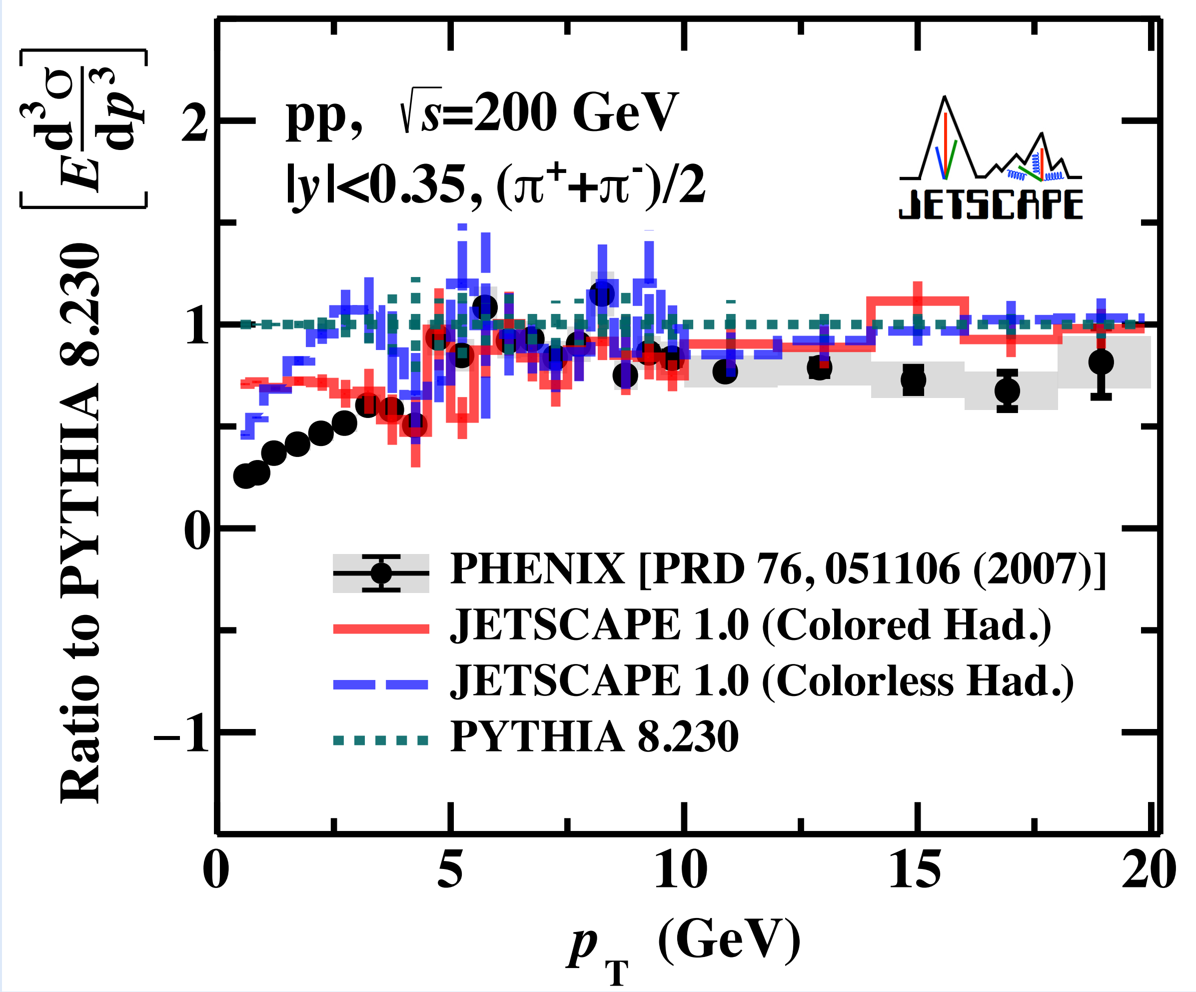
Jet shape



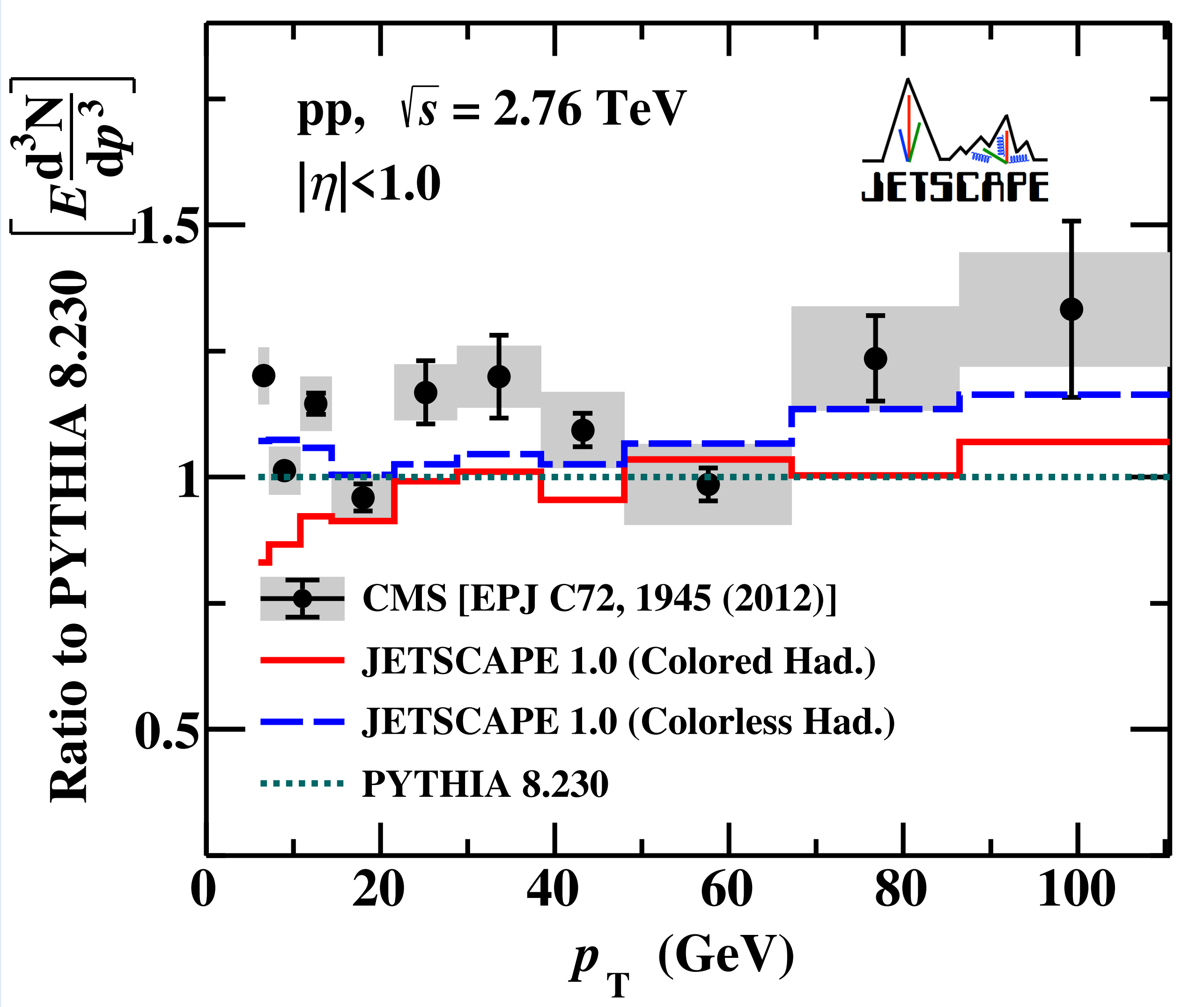
Jet Mass



Charged hadron yield



Charged hadron yield

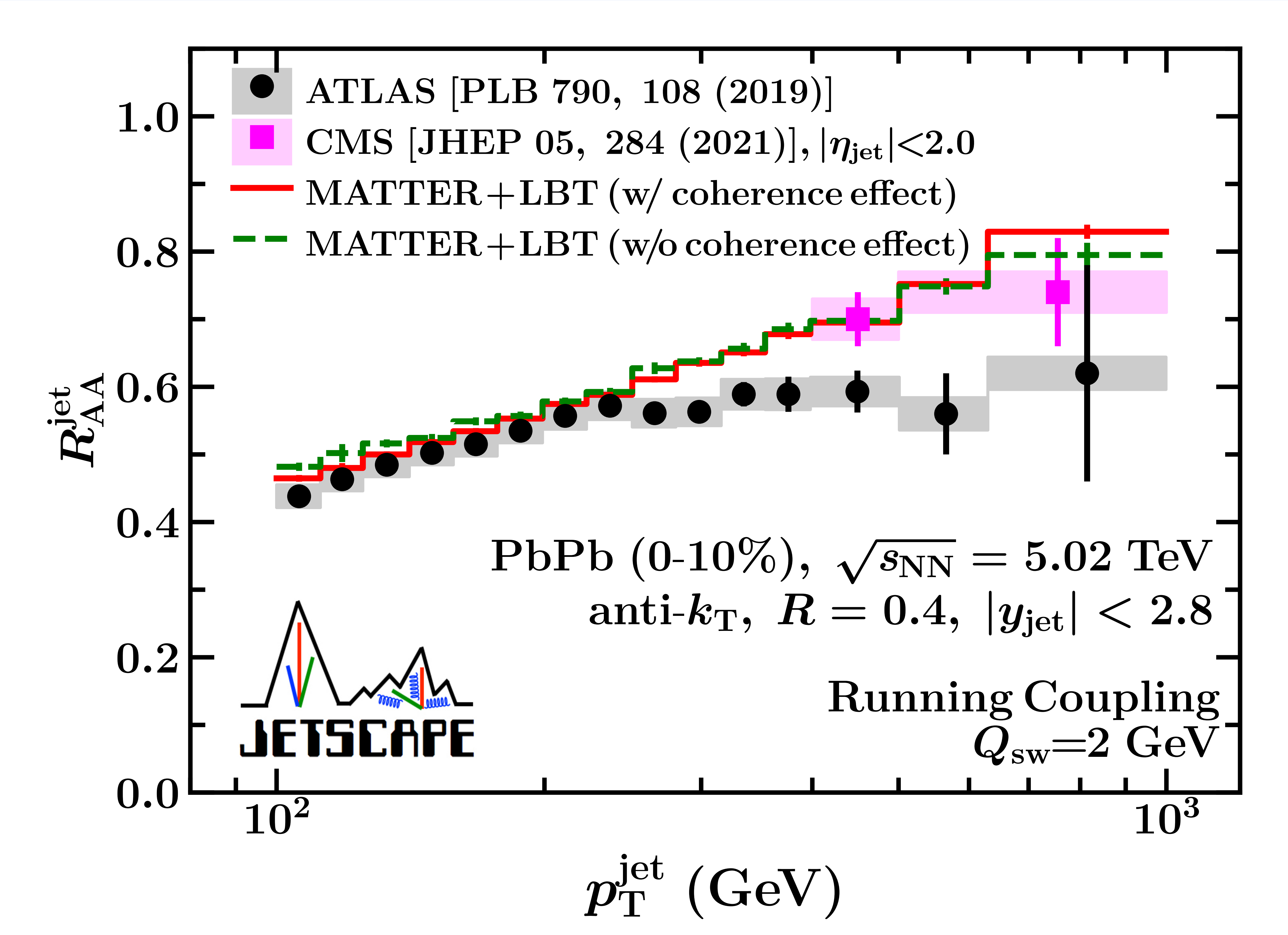


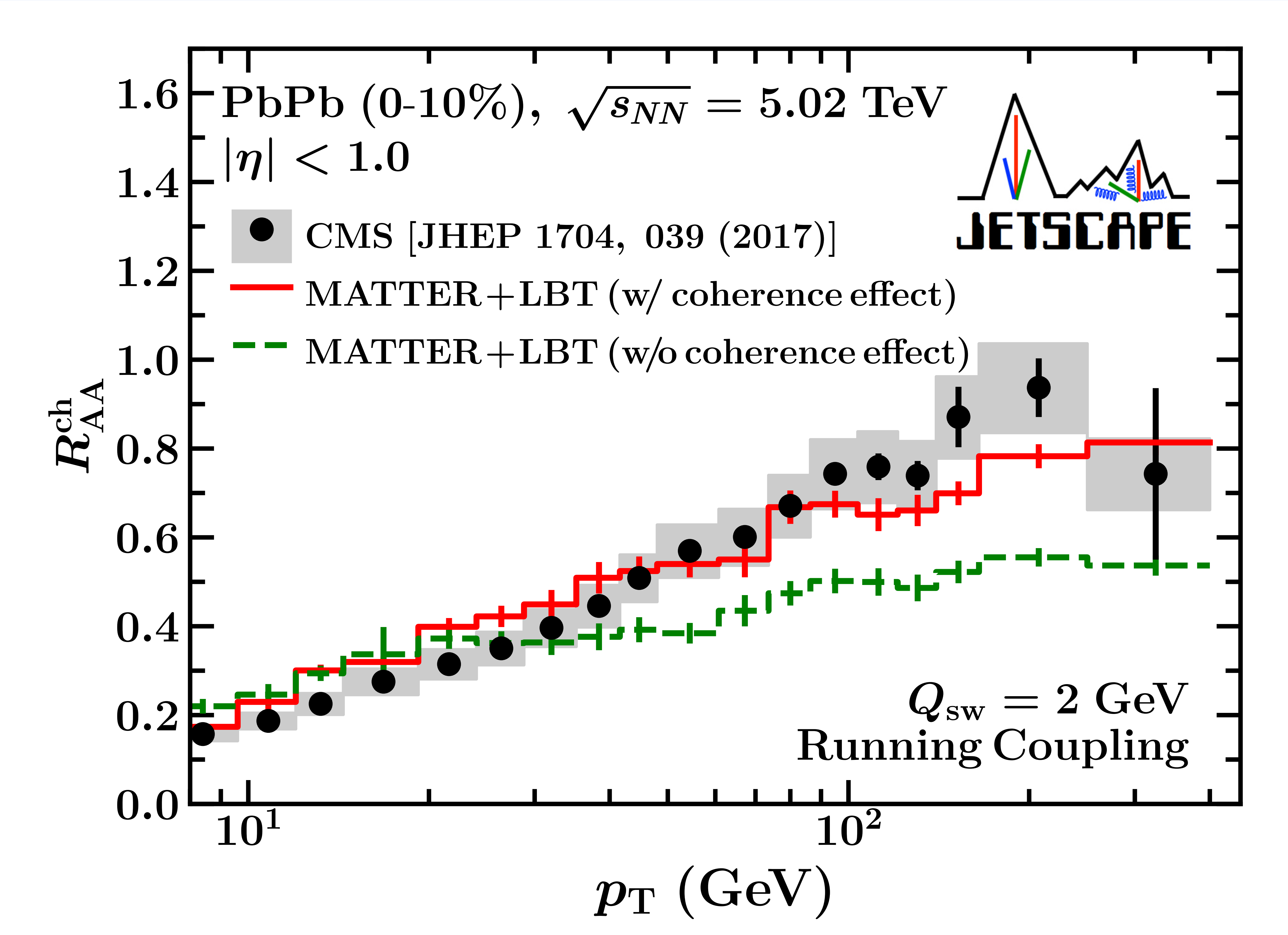
Jets and Leading hadron suppression at $\sqrt{s_{NN}} = 5.02$ TeV

Effective jet-quenching strength $\implies \hat{q}_{\mathrm{HTL}} \cdot f(Q^2)$

$$f(Q^2) = \frac{1 + c_1 \ln^2(Q^2_{\mathrm{sw}}) + c_2 \ln^4(Q^2_{\mathrm{sw}})}{1 + c_1 \ln^2(Q^2) + c_2 \ln^4(Q^2)},$$

where $f(Q^2) \to 1$ in low virtuality phase

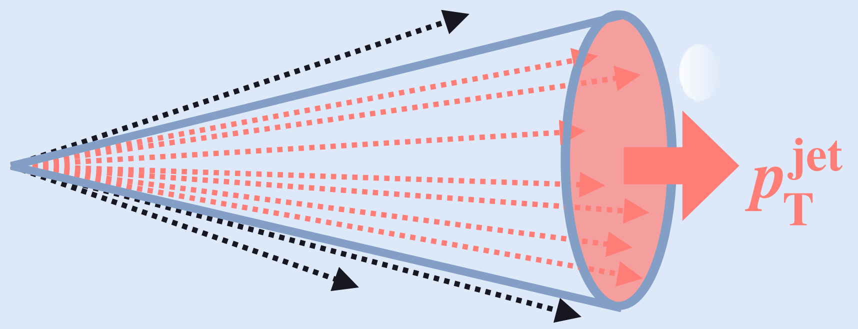




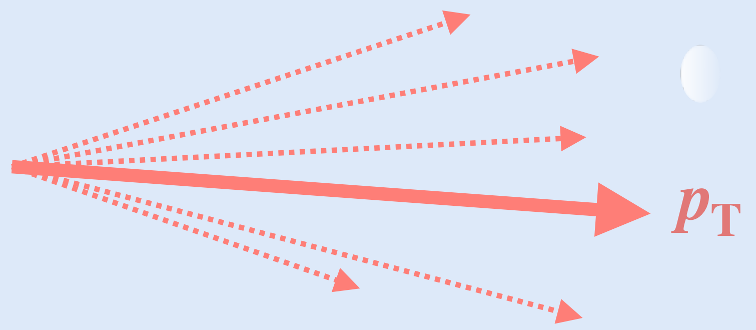
Strong coherence effects are observed for high-p_T hadrons

Collision energy dependence of Jet and Hadron R_{AA}

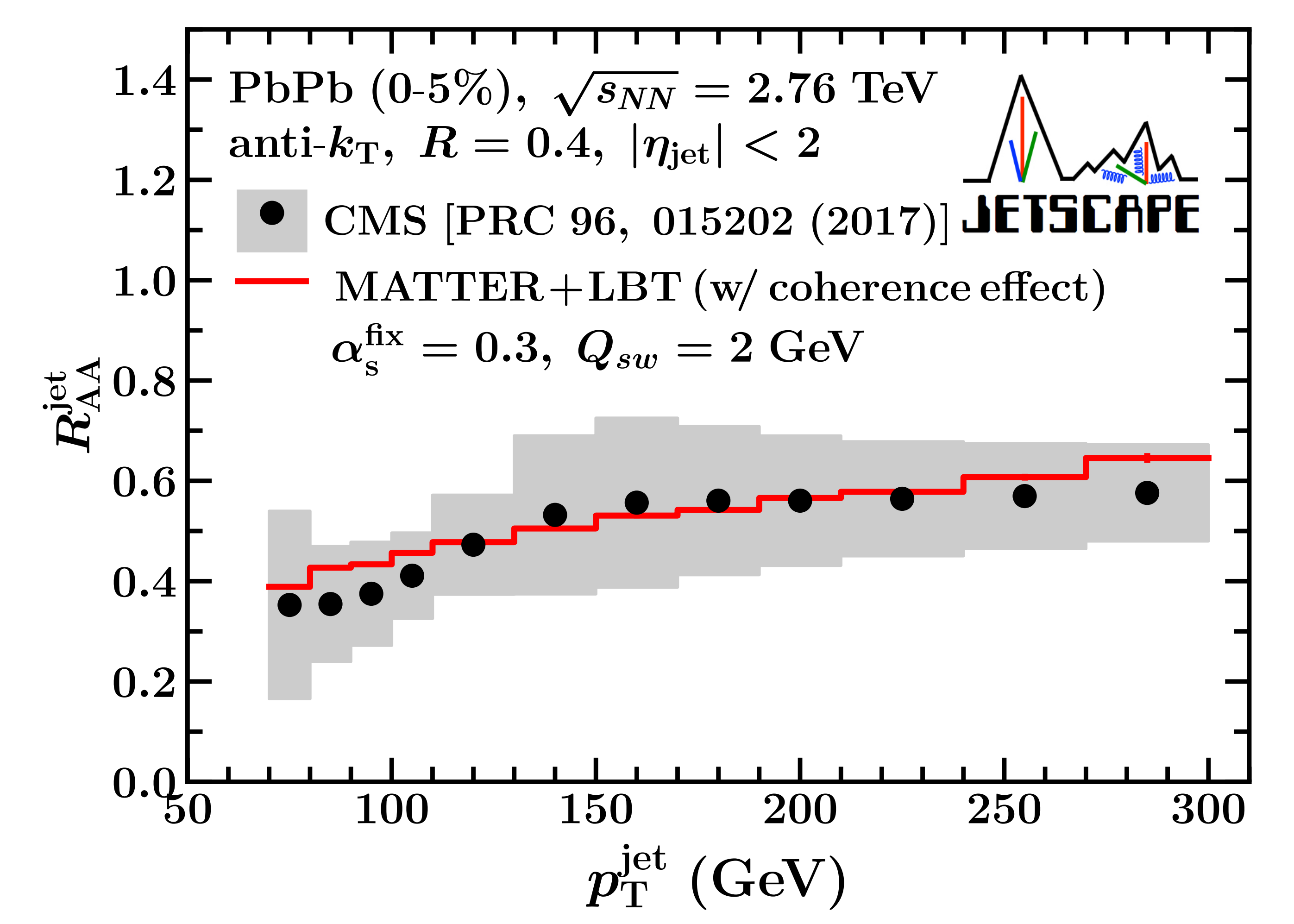
Jet R_{AA}

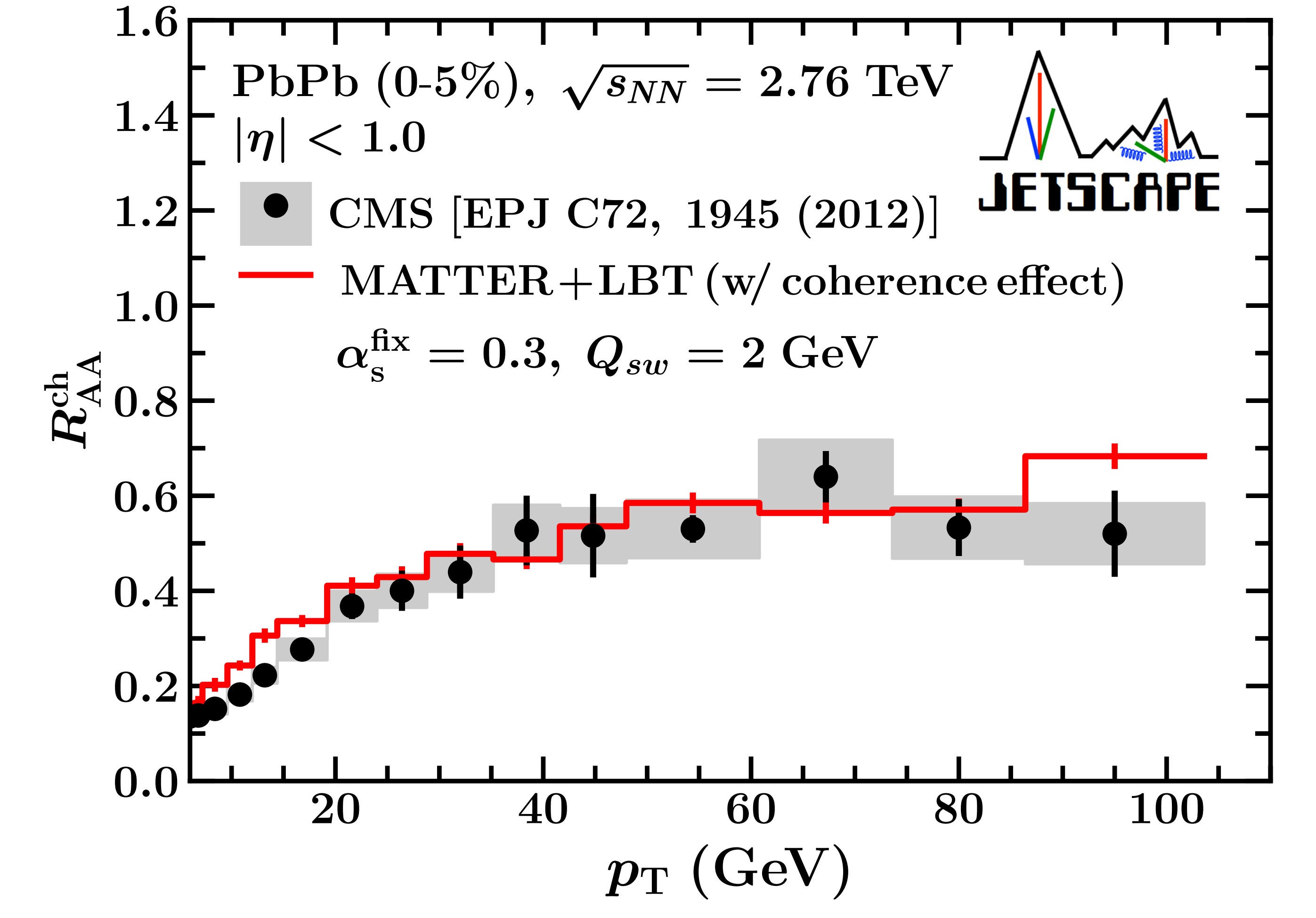


Particle R_{AA}

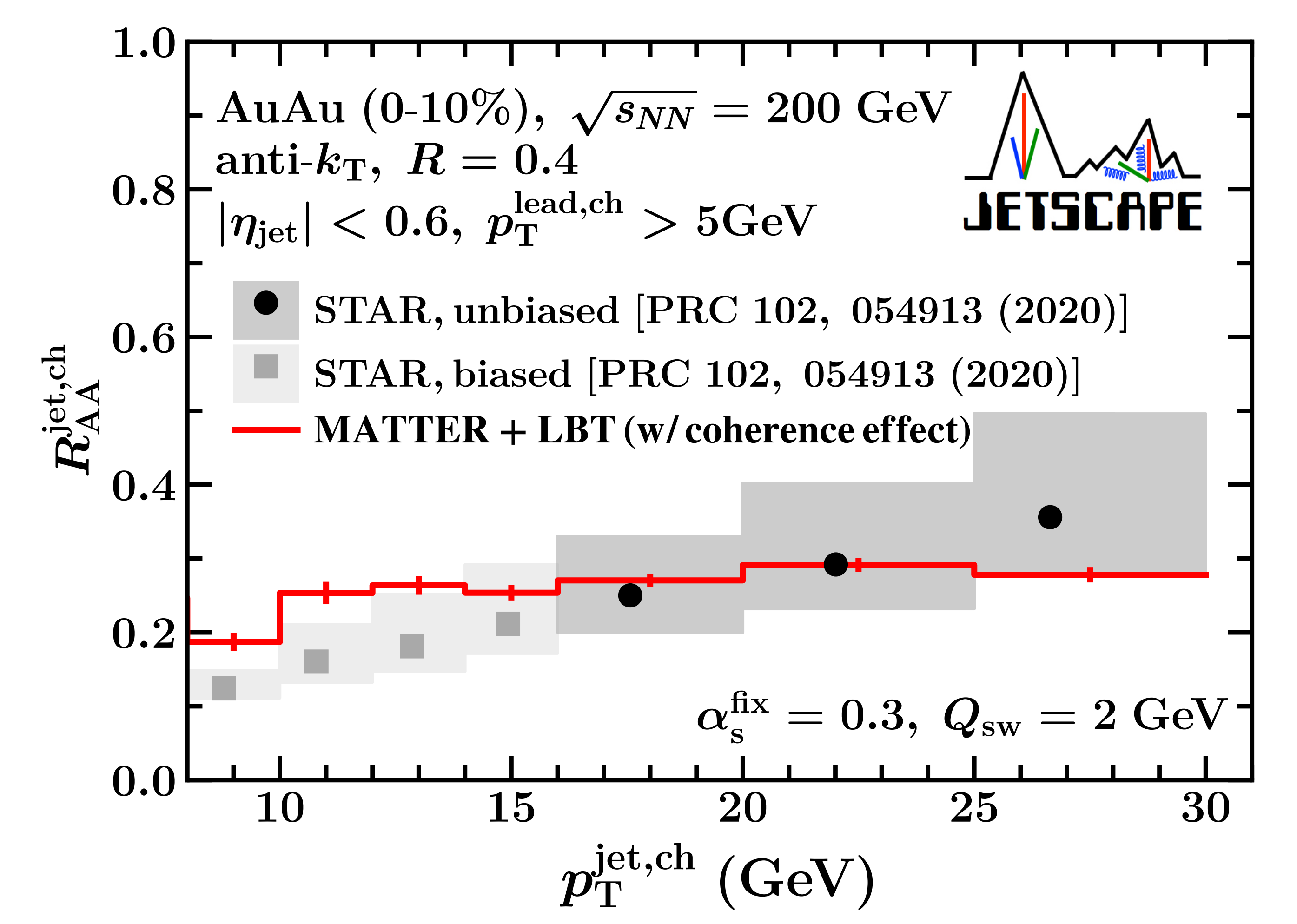


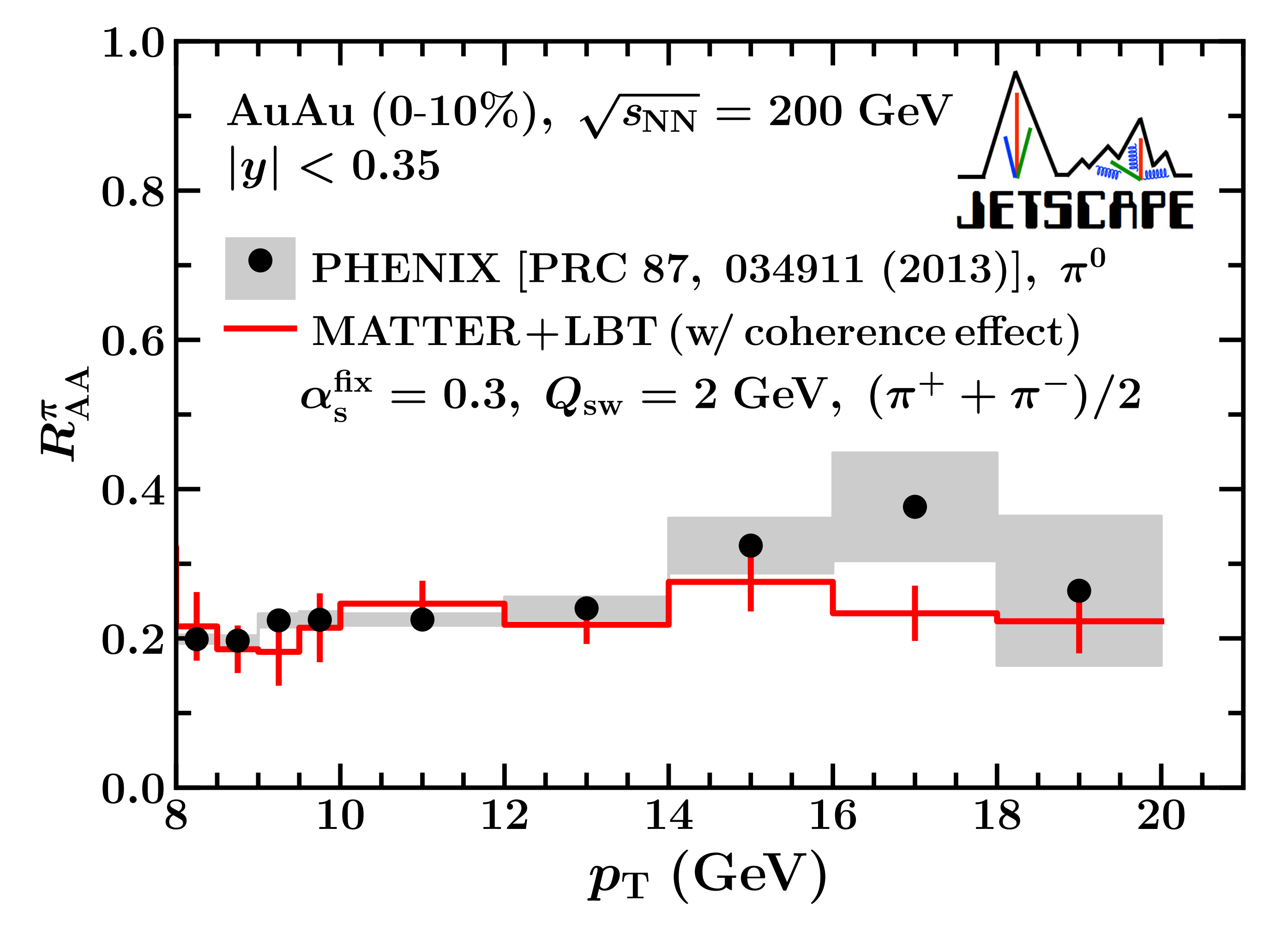
- **Pb+Pb at 2.76 TeV**





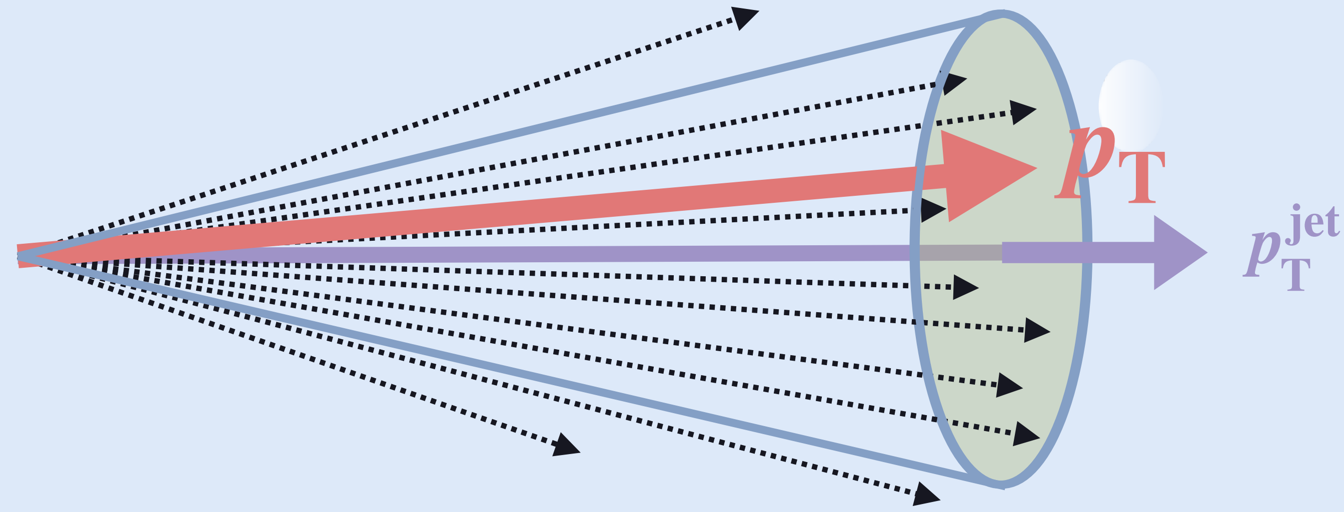
- **Au+Au at 200 GeV**



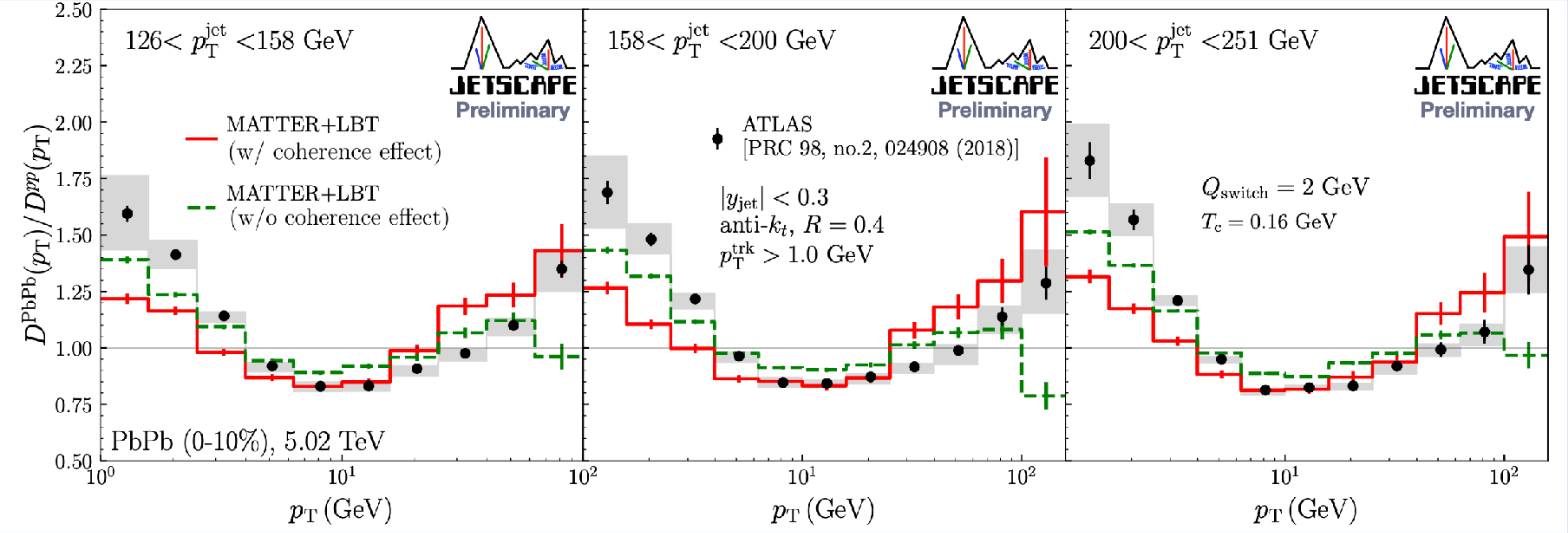


Jet Fragmentation function

$$D(p_T) = \frac{1}{N_{\mathrm{jet}}} \sum_{\mathrm{jets}} \frac{dN_{trk}}{dp_T^{ch}}$$



Shows sensitivity to coherence effects



Summary

- ❑Factorization of soft and hard scales
- ❑Parton distribution function and Fragmentation function
- ❑ Vacuum DGLAP equation and medium modified DGLAP equation
- ❑ Basic review of jet energy loss in high virtuality and low virtuality phase
- ❑ MATTER, LBT and MARTINI energy loss module

Next talks in jet session:

- ❑Wenkai Fan : Overview of heavy quark energy loss

- ❑ Ismail Soudi: Weakly-coupled and strongly-coupled approach of medium response

Thanks to all TA’s and Chairs