The SMASH transport approach

Introduction into transport and hybrid approaches and their usage within JETSCAPE

Jan Staudenmaier at the JETSCAPE Summer School 2022

July 27 2022

Separation of *hard* and *soft* physics

A fountain of water (*hard jet*) shooting through a dense fog (*soft bulk*)

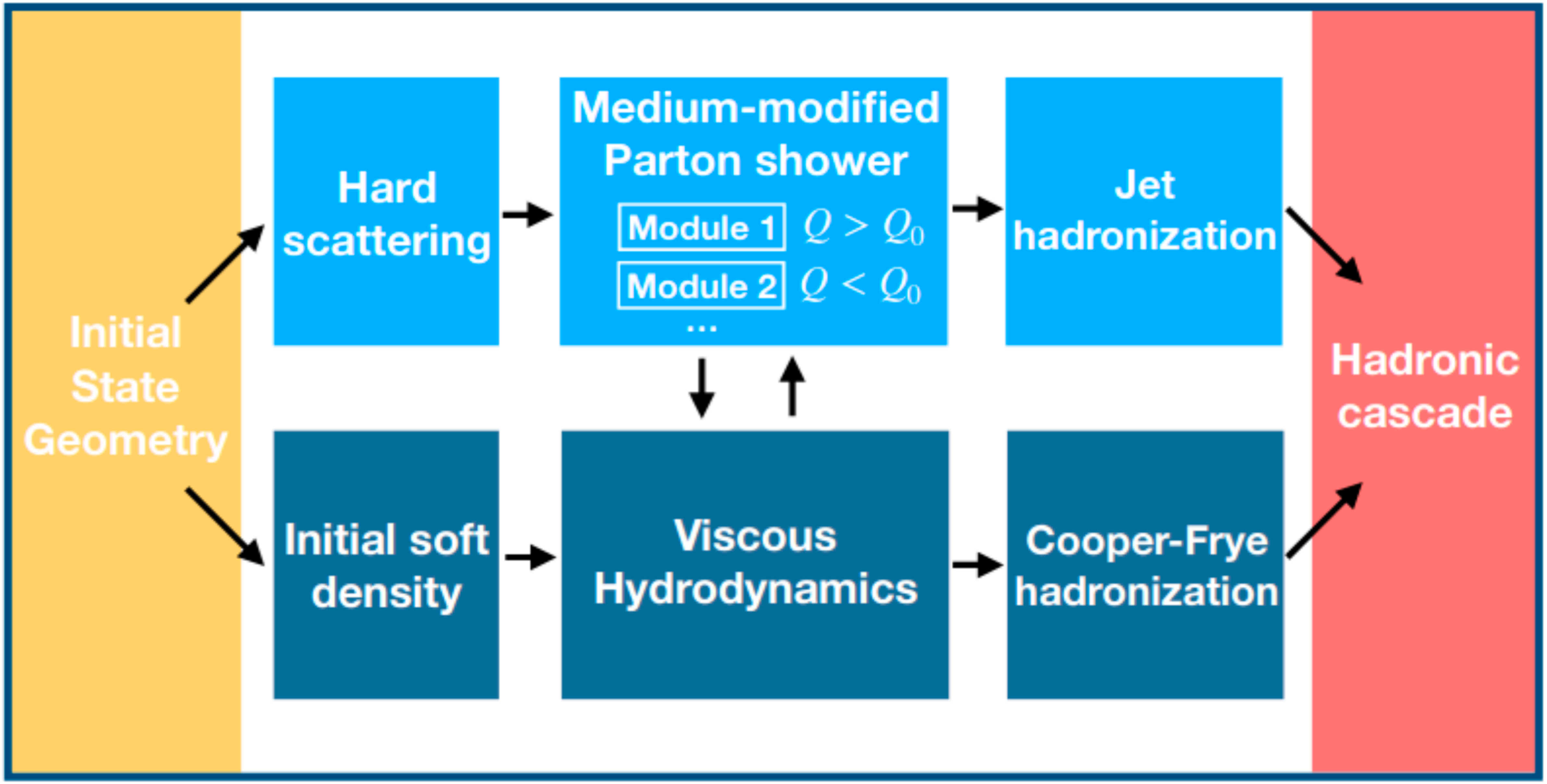
Separation of *hard* and *soft* physics

- In heavy-ion collisions …
  - *hard* physics about high momentum particles $\gtrapprox 5 - 10\,\mathrm{GeV}$
    ↳ Perturbative QCD, Jets, …
  - *soft* or *bulk* physics about low momentum particles $\lessapprox 5 - 10\,\mathrm{GeV}$
    ↳ Statistical Models, Hydrodynamics, …
- Separate theoretical description of the different regimes

JETSCAPE Flow

Separation of hard and soft sector

Topic of this lecture

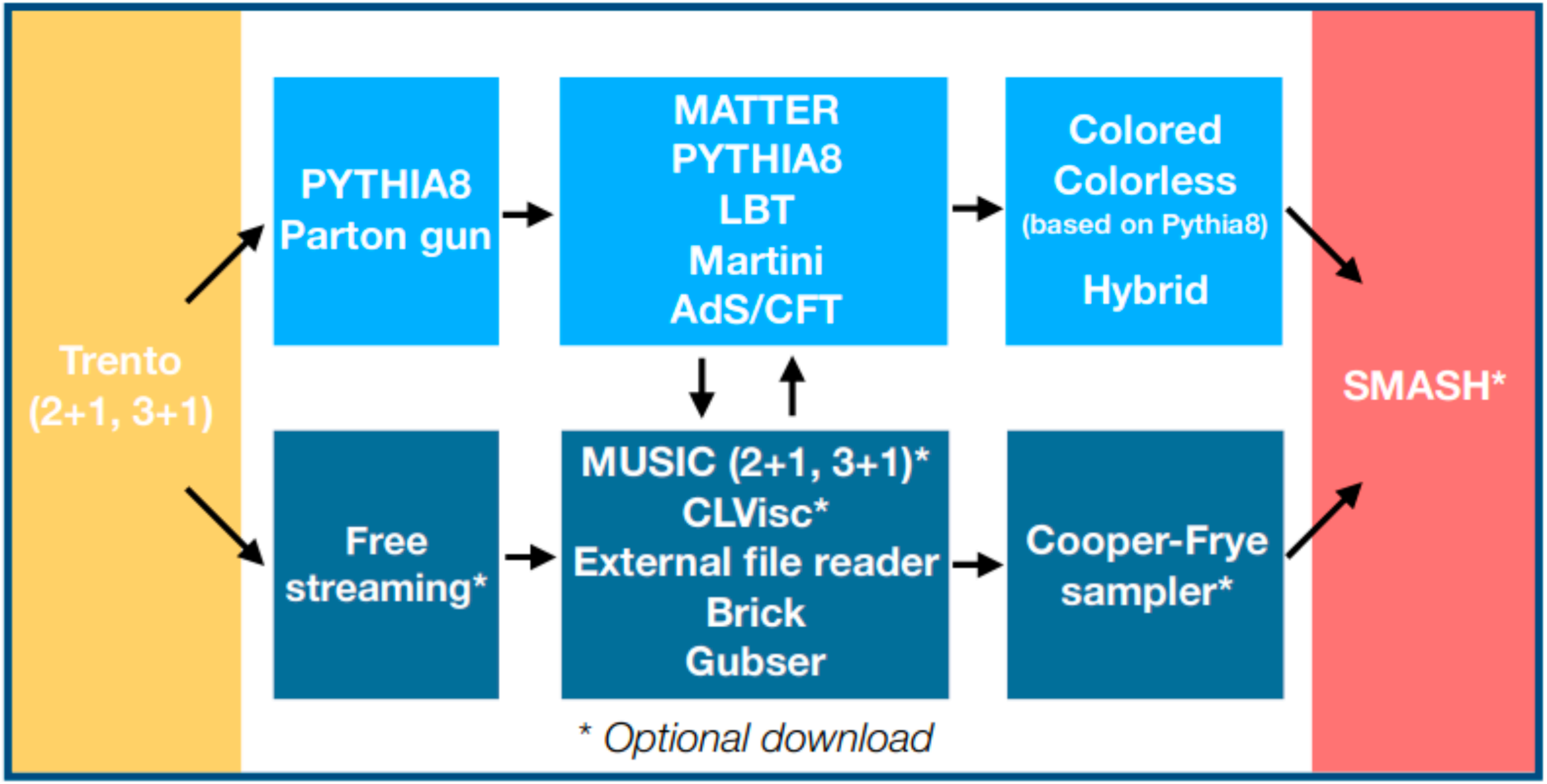


by J. Mulligan

JETSCAPE Modules

Separation of hard and soft sector

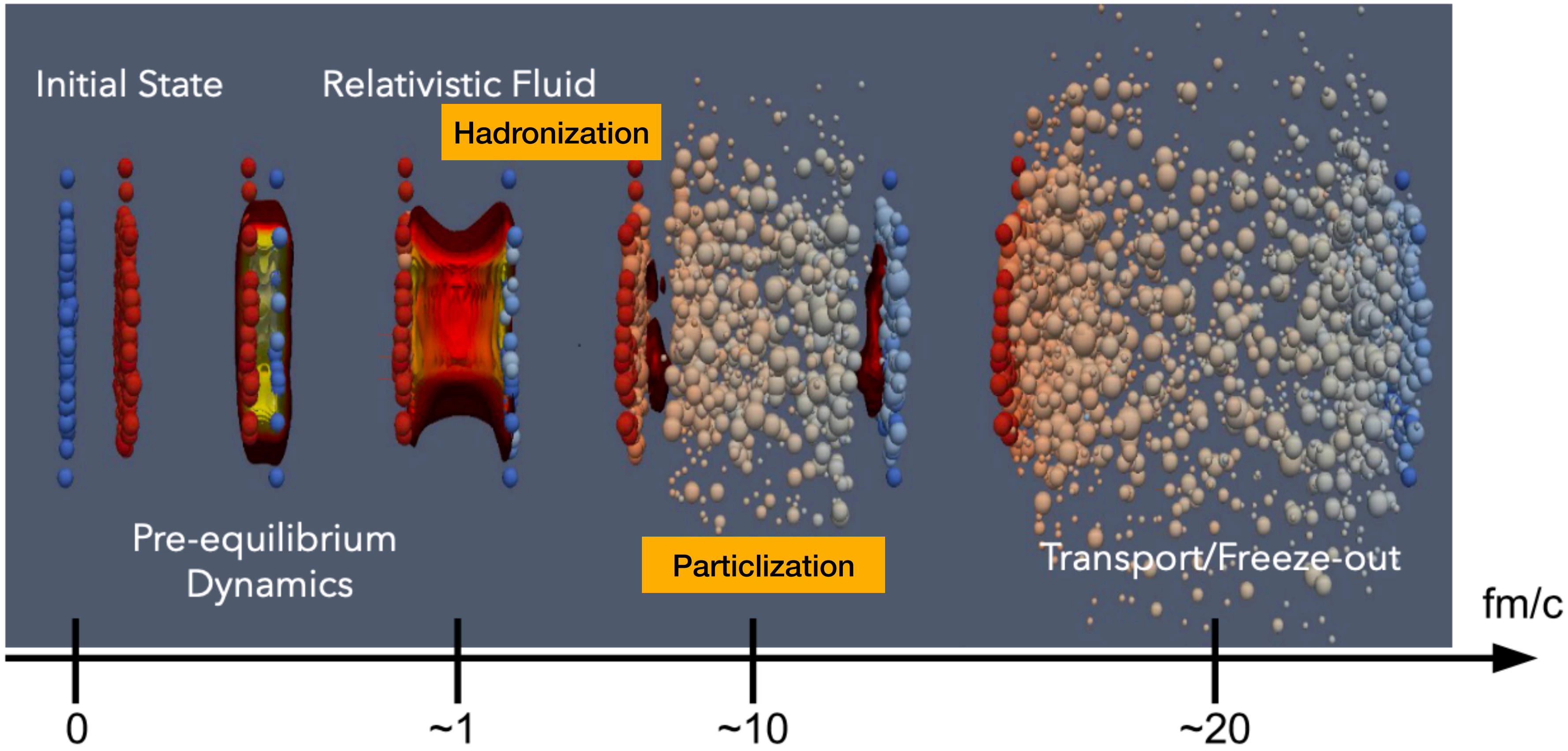
Topic of
this lecture

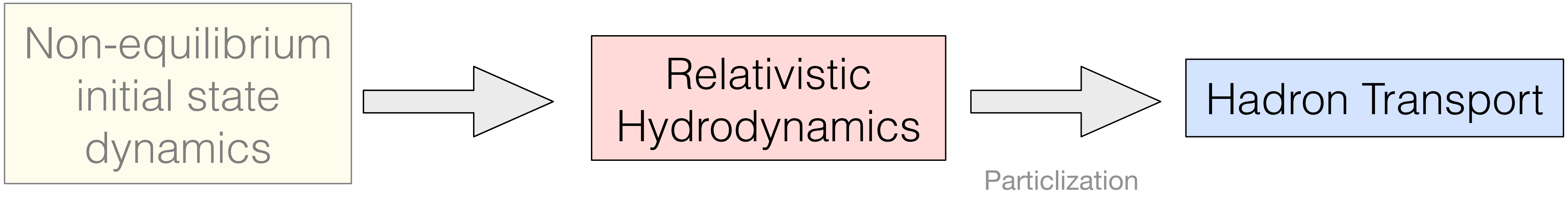


by J. Mulligan

The approach for *soft* physics

At high energies: Hybrid models





Non-equilibrium initial state dynamics

Relativistic Hydrodynamics

Hadron Transport

Particlization

Hybrid approaches

Relativistic Hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0,\ \partial_\mu j^\mu = 0$$

- Conservation laws
- Local thermal equilibrium
- Macroscopic
- Equation of state encodes phase transition
- Applicable at high densities
  mean free path $\ll$ system size

Hadron Transport

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\nabla f + \vec{F}\frac{\partial f}{\partial \vec{p}} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

- Boltzmann equation
- Non-equilibrium
- Microscopic
- Hadronic (or partonic) degrees of freedom
- Applicable at low/dilute system
  mean free path $\gg \lambda_{\mathrm{Compton}}$

Different regions of applicability ➡️ Hybrid approach

Hybrid approaches

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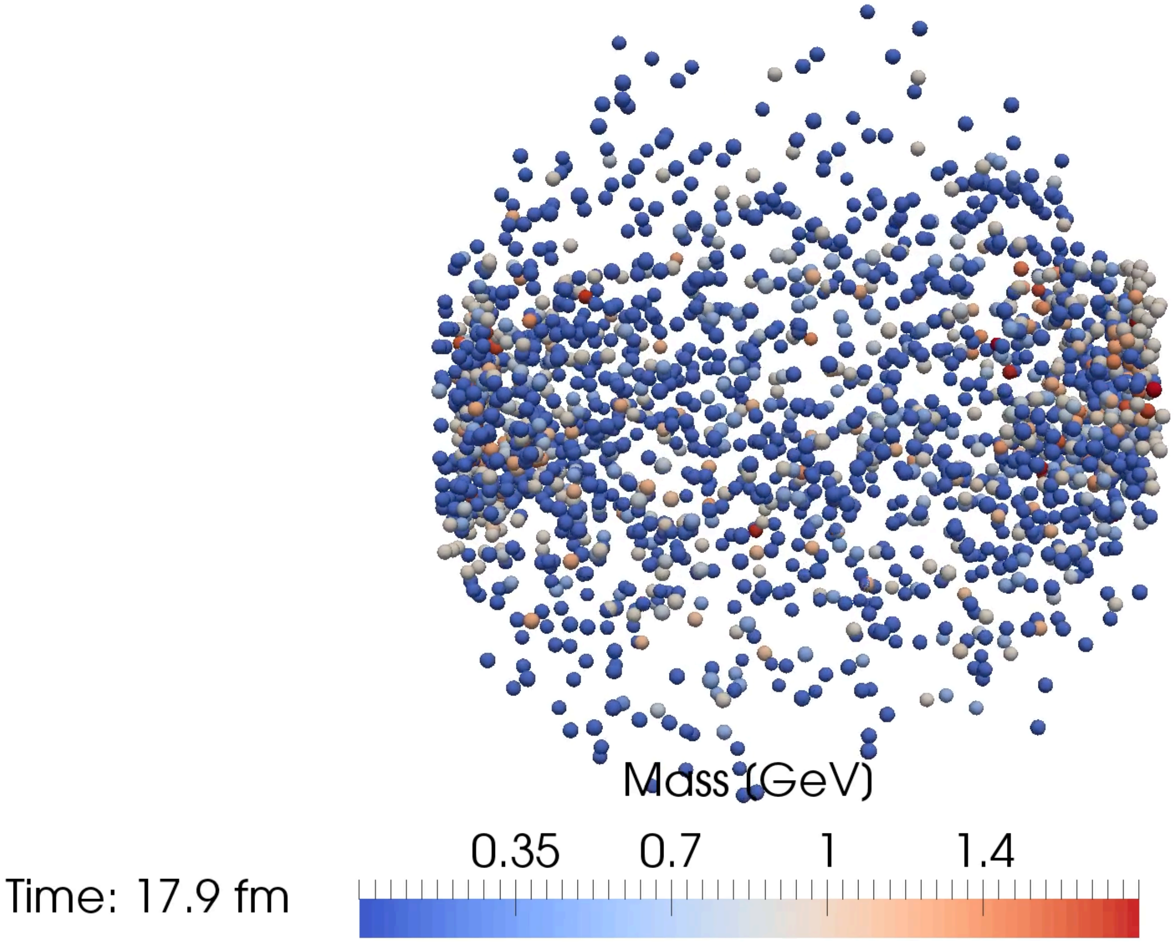
(Ideal) hydrodynamic equations can be derived from Boltzmann equation assuming equilibrium (distribution)

Visualization of transport approach

Pb-Pb at 17.3 GeV



Simple idea: Particles propagate, interact and decay

… but devil is in all the details

Theoretical Foundation

- Transport approaches are based on the **Boltzmann equation**

non-relativistic version

$$\boxed{\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\nabla f + \vec{F}\frac{\partial f}{\partial \vec{p}} = \left(\frac{\partial f}{\partial t}\right)_{\text{coll}}}$$

free streaming term

external force (potential) term

- Time evolution of particle density distribution function $f_i(\vec{r}, \vec{p}, t)$ for each species i

- f is the number of particles per phase space cell $dN_i = f_i(\vec{r}, \vec{p}, t)\, d^3r d^3p$

- Right-hand side: **Collision Integral** (next slide)

- Neglect quantum effects like interference and assume space and time span of collisions is small compared to mean free path

Collision Integral

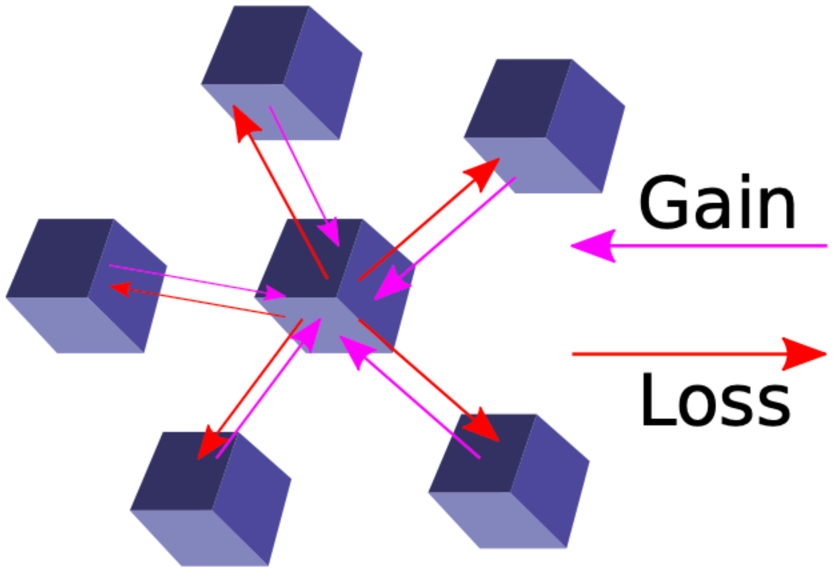
- Change of particle number dN in phase space cell $d^3r d^3p$

$$\frac{d}{dt}N(t,r,p) = dN_{coll}(p', \cdots \rightarrow p, \dots) - dN_{coll}(p, \cdots \rightarrow p', \dots)$$

Gain Term   Loss Term

- For 2-to-2 scatterings, one gets the integral:

$$\left(\frac{\partial f}{\partial t}\right)_{\text{coll}} = \int \frac{d^3p_2}{(2\pi)^3}(f_1' f_2' - f_1 f_2) v_{rel} \int d\sigma$$



Input from quantum field theory through matrix element

$$\sigma_{12\rightarrow 1'2'} = \frac{1}{2E_1}\frac{1}{2E_2}\frac{1}{v_{rel}}\frac{1}{S_{1'2'}}\int \frac{d^3p_1'}{(2\pi)^3 2E_1'}\frac{d^3p_2'}{(2\pi)^3 2E_2'} \times \overline{|\mathcal{M}_{12\rightarrow 1'2'}|^2}(2\pi)^4\delta^{(4)}(P_1 + P_2 - P_1' - P_2'),$$

Solving the Boltzmann equation

Let's solve the equation then, right?

- In a realistic scenarios want to describe interaction between more than 100 hadron species (π, ρ, K, a_2, f_1, ϕ, N, Δ, …)

- Need to solve the coupled system of integro-differential equations with collision terms for any interactions between the particles

$$
\begin{aligned}
Df_\pi &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\
Df_N &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\
Df_\Delta &= I_{coll}(f_\pi, f_N, f_\Delta, \dots) \\
\dots
\end{aligned}
\qquad (D = \frac{d}{dt})
$$

- Generally **impossible analytically** ➡️ Need numeric Monte-Carlo approach with an effective description of the different equations terms

Particle evolution

$$\frac{\partial f}{\partial t} + \frac{\vec{p}}{m}\nabla f + \vec{F}\frac{\partial f}{\partial \vec{p}} = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{coll}}$$

- **Propagation**: Particles are propagated according to their momenta along straight lines (assuming no potentials) *free streaming term*
- **Collisions**: When particles are close → perform earliest reaction *collision term*
- ***Geometric*** collision criterions (usually default in transport approaches)
  - $d_T \leq \sqrt{\sigma/\pi}$ (limited to binary collisions)
  - Time of closest approach → Time-sorting depends on frame (Issues with Lorentz-invariance)
  - Kodama criterion improves Lorentz-invariance
- ***Stochastic*** collision criterion
  - Lorentz-invariant collision probability for particles in same phase space cell
  - Allows for multi-particle reaction

Treatment of potentials *external force term*

BUU and QMD approaches

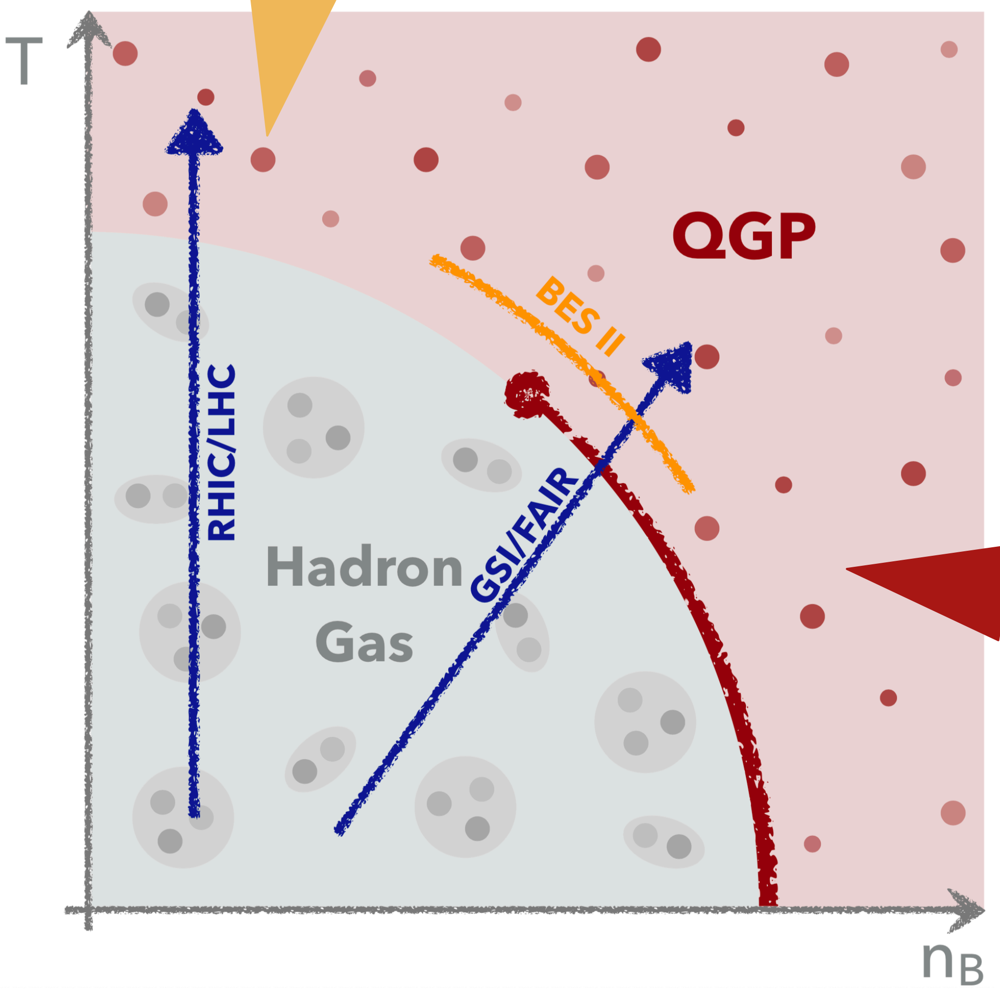
- **Boltzmann-Ühling-Uhlenbeck** (BUU) approaches
  - f represented by testparticles: $N \rightarrow NN_{\text{Test}}, \sigma \rightarrow \sigma/N_{\text{Test}}$
  - Density dependent mean-field potentials $U(\rho)$ $\boxed{\frac{d\vec{p}}{dt} = -\nabla_{\vec{r}} U}$
  - Solves Boltzmann equations in the limit of $N_{\text{Test}} \rightarrow \infty$
- **Quantum Molecular Dynamics** (QMD) approach
  - Particles are Gaussian wave packets
  - Potentials are sum of pairwise potentials
  - Solves many-body Hamiltonian (not based on an equation for f)

Application of transport approaches

Non-equilibrium systems of microscopic particles

Hybrid (JETSCAPE)

Standard approach **at high energies**
- Non-equilibrium initial evolution
- Viscous hydrodynamics
- Hadronic rescattering



- Two regimes with well-established approaches (split at $\sqrt{s} \approx 20$ GeV)

In Addition

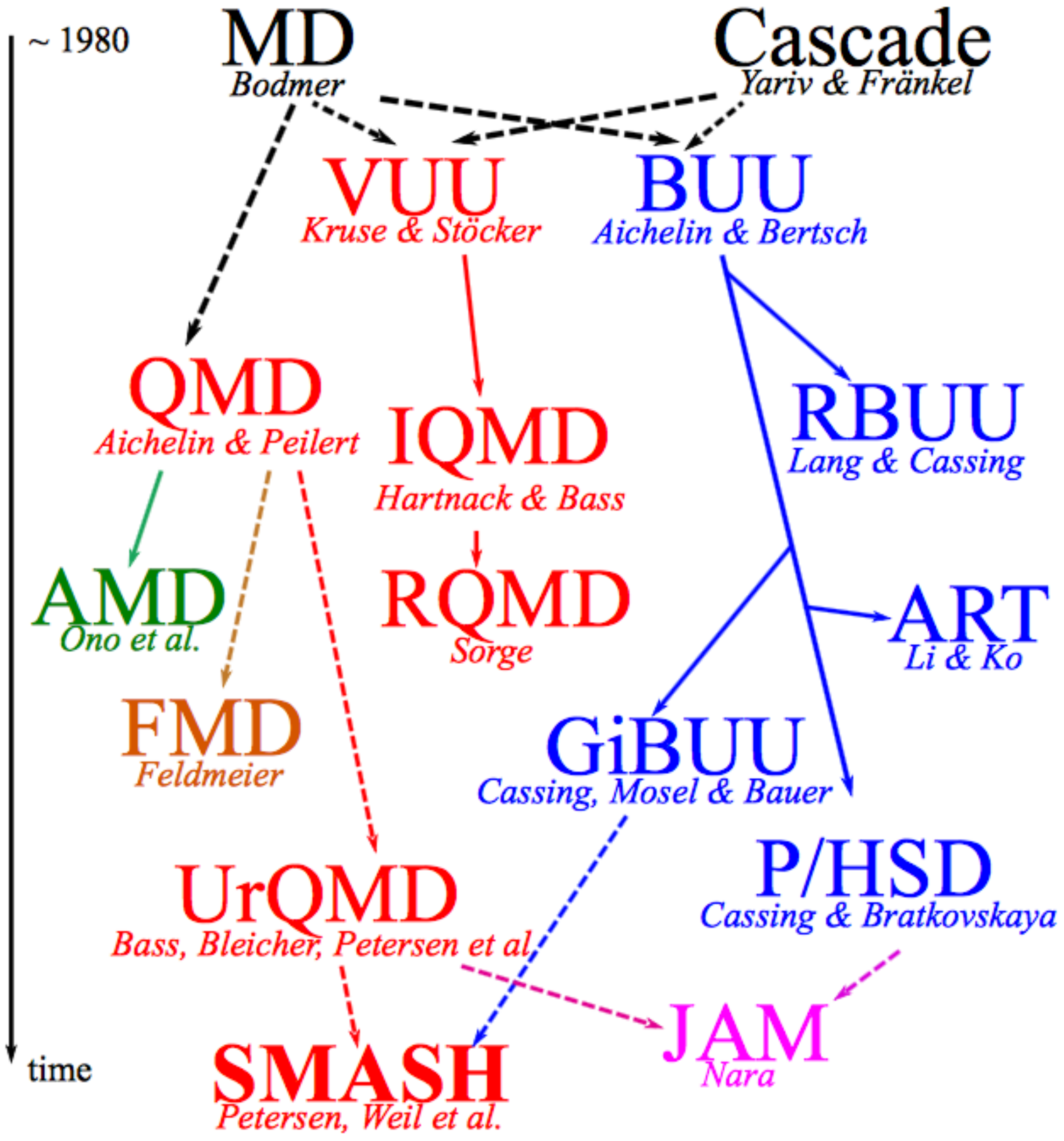
- Approaches with partons cover full energy range (e.g. AMPT, PHSD)
- Studies of neutrino collision with e.g. GiBUU
- Air-shower from cosmic rays (UrQMD, SMASH)

Standard approach **at low energies**
- Hadronic transport approaches
- Resonance dynamics
- Nuclear potentials

Pure Transport evolution

History of hadronic transport approaches

Successfully applied for decades



thanks to Steffen Bass

SMASH

- First C++ code in this historical chain written from scratch taking most successful aspects of existing approaches
- Goal: Reference for hadronic system with vacuum properties

SMASH

Simulating **M**any **A**ccelerated **S**trongly-interacting **H**adrons

- Newer hadronic transport approach for dilute non-equilibrium stages of HIC and low energy collisions
- BUU-type approach: Uses the Testparticle Method $N \rightarrow N N_{\mathrm{Test}}$
- Geometric collision criterion as default: $d_T \leq \sqrt{\sigma/\pi}$

  Recent addition: Stochastic collision criterion for multi-particle interactions

As in UrQMD

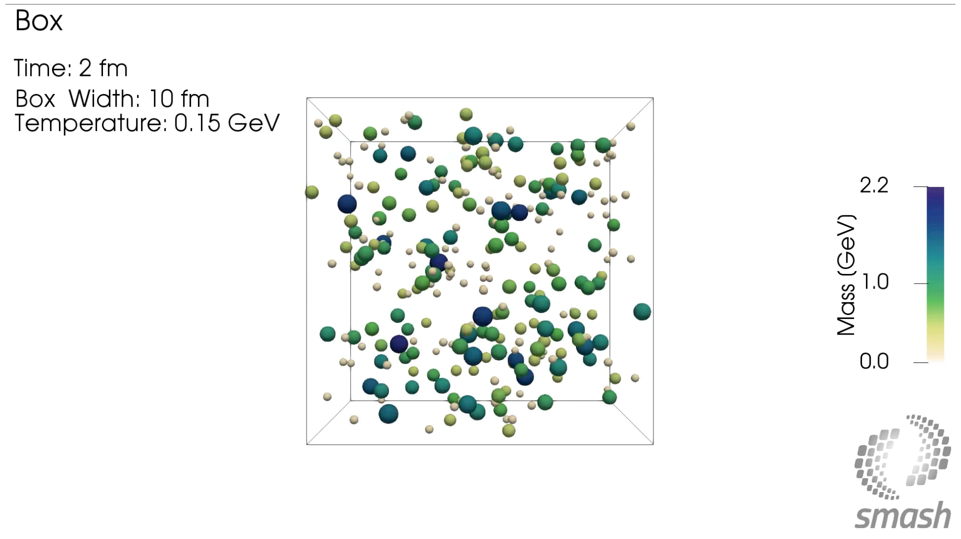
- Includes HepMC3 and ROOT output with newest release `SMASH-2.2`
- Open Source on Github

smash

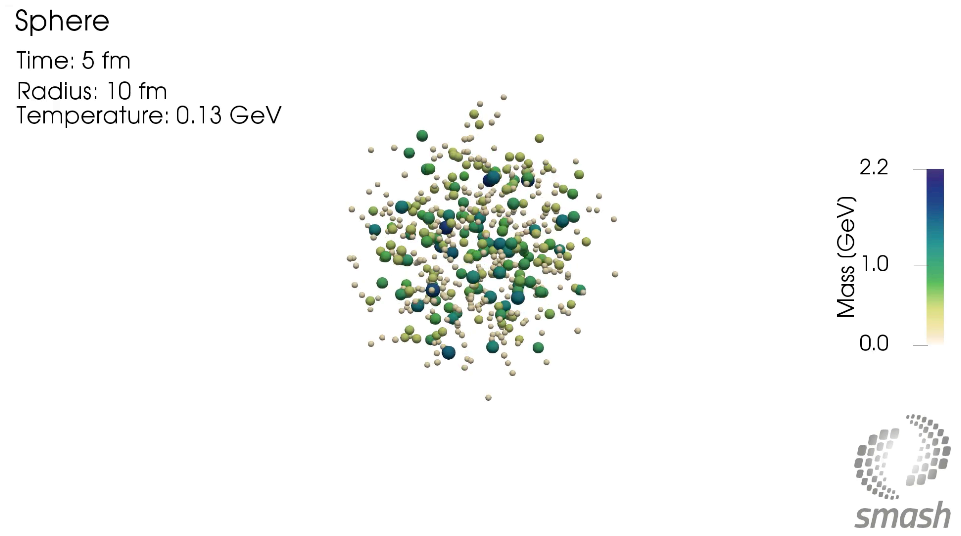
Initial conditions

Visualizations Webpage 🔗

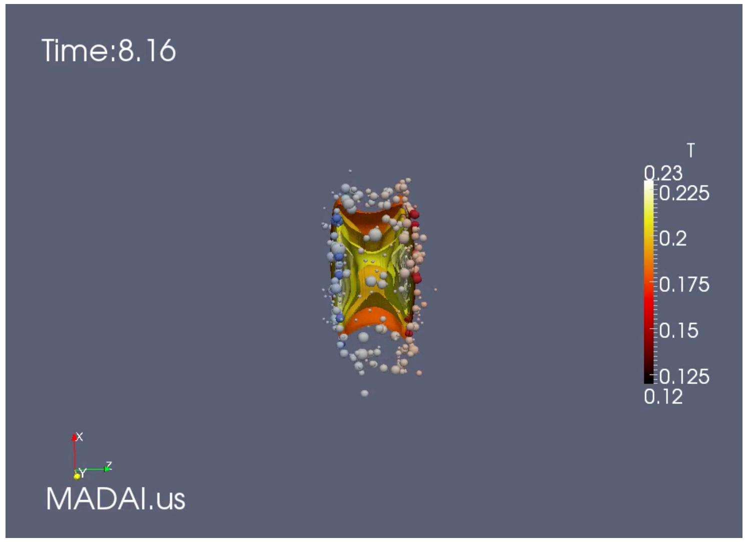
Collider - elementary or AA collisions



Box - infinite matter



Sphere - Expanding system



List - Afterburner of hydrodynamics

Degrees of freedom

Hadrons

N	**Δ**	**Λ**	**Σ**	**Ξ**	**Ω**
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^{-}_{1672}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^{-}_{2250}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1900}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1650}	Δ_{1905}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1910}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1680}	Δ_{1920}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1930}	Λ_{1810}	Σ_{1940}		
N_{1710}	Δ_{1950}	Λ_{1820}	Σ_{2030}		
N_{1720}		Λ_{1830}	Σ_{2250}		
N_{1875}		Λ_{1890}			
N_{1900}		Λ_{2100}			
N_{1990}		Λ_{2110}			
N_{2060}		Λ_{2350}			
N_{2080}					
N_{2100}					
N_{2120}					
N_{2190}					
N_{2220}					
N_{2250}					

As of SMASH-1.7

Unflavored				**Strange**
π_{138}	$f_{0\,980}$	$f_{2\,1275}$	$\pi_{2\,1670}$	K_{494}
π_{1300}	$f_{0\,1370}$	$f_{2}'{}_{1525}$		K^{*}_{892}
π_{1800}	$f_{0\,1500}$	$f_{2\,1950}$	$\rho_{3\,1690}$	$K_{1\,1270}$
	$f_{0\,1710}$	$f_{2\,2010}$		$K_{1\,1400}$
η_{548}		$f_{2\,2300}$	$\phi_{3\,1850}$	K^{*}_{1410}
η'_{958}	$a_{0\,980}$	$f_{2\,2340}$		$K_{0}^{*}{}_{1430}$
η_{1295}	$a_{0\,1450}$		$a_{4\,2040}$	$K_{2}^{*}{}_{1430}$
η_{1405}		$f_{1\,1285}$		K^{*}_{1680}
η_{1475}	ϕ_{1019}	$f_{1\,1420}$	$f_{4\,2050}$	$K_{2\,1770}$
	ϕ_{1680}			$K_{3}^{*}{}_{1780}$
σ_{800}		$a_{2\,1320}$		$K_{2\,1820}$
	$h_{1\,1170}$			$K_{4}^{*}{}_{2045}$
ρ_{776}		$\pi_{1\,1400}$		
ρ_{1450}	$b_{1\,1235}$	$\pi_{1\,1600}$		
ρ_{1700}				
	$a_{1\,1260}$	$\eta_{2\,1645}$		
ω_{783}				
ω_{1420}		$\omega_{3\,1670}$		
ω_{1650}				

see also `particles.txt`

- + corresponding antiparticles
- Perturbative treatment of photons and dileptons
- Isospin symmetry

- Mesons and baryon properties according to PDG
- Isospin multiplets and anti-particles are included

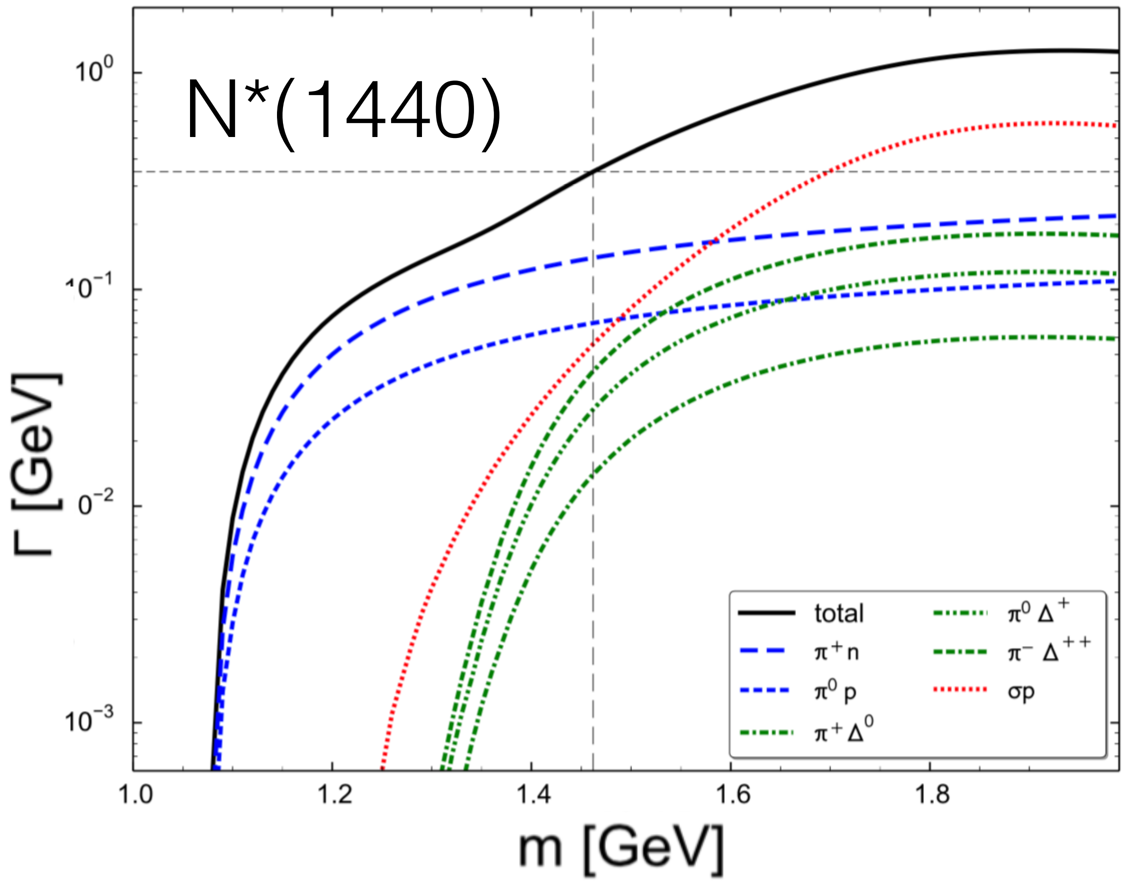
Resonances

- Spectral function
  - All unstable particles („resonances“) have relativistic Breit-Wigner spectral functions

$$\mathcal{A}(m) = \frac{2\mathcal{N}}{\pi}\frac{m^2\Gamma(m)}{(m^2 - M_0^2)^2 + m^2\Gamma(m)^2}$$

- Decay widths
  - Particles stable, if width < 10 keV $(\pi,\, \eta,\, K, \ldots)$
  - Treatment of Manley et al

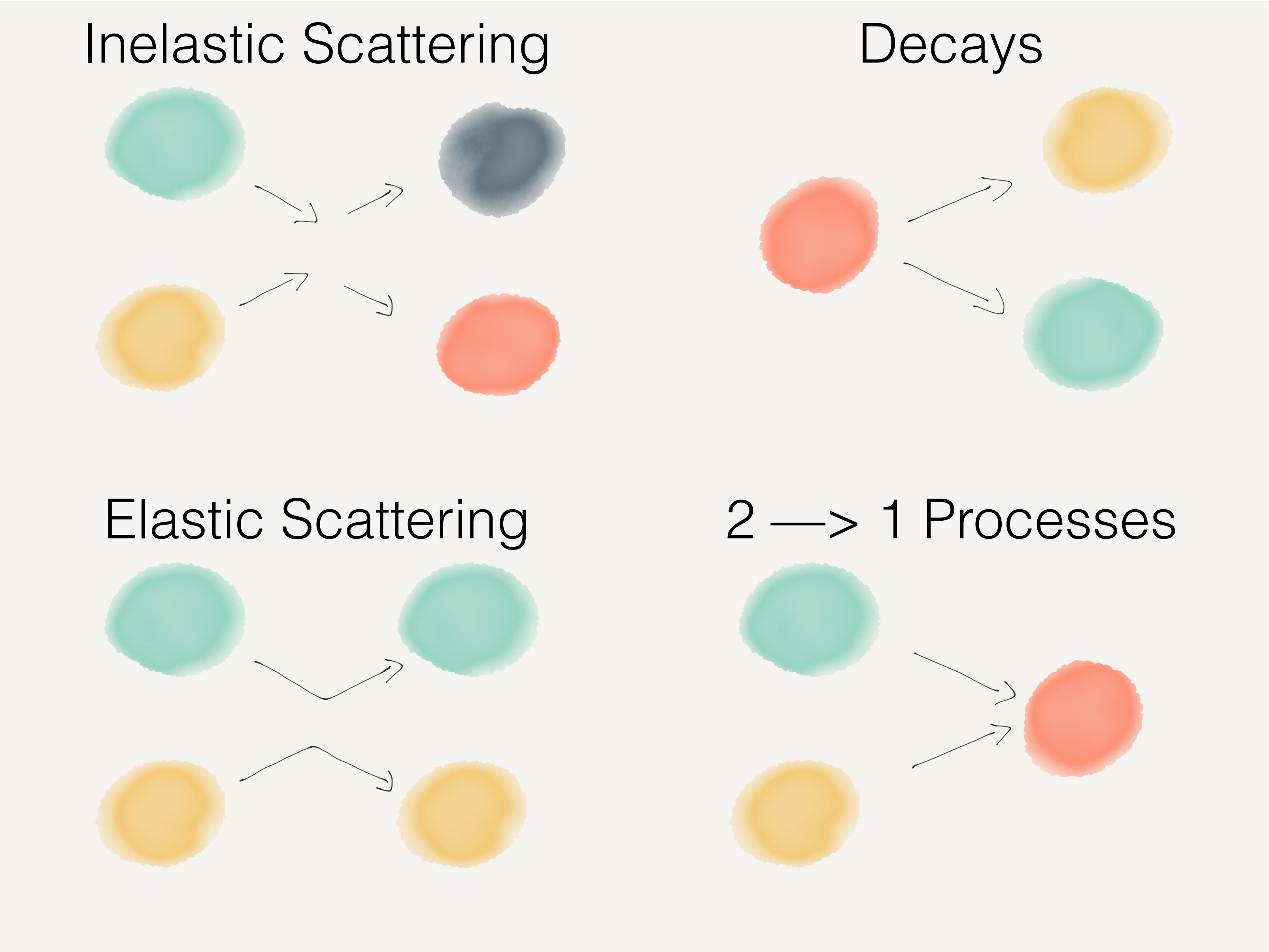
$$\Gamma_{R\to ab} = \Gamma^0_{R\to ab}\frac{\rho_{ab}(m)}{\rho_{ab}(M_0)}$$



D. M. Manley and E. M. Saleski,
Phys. Rev. D 45, 4002 (1992)

As in GiBUU

Interactions *collision term*



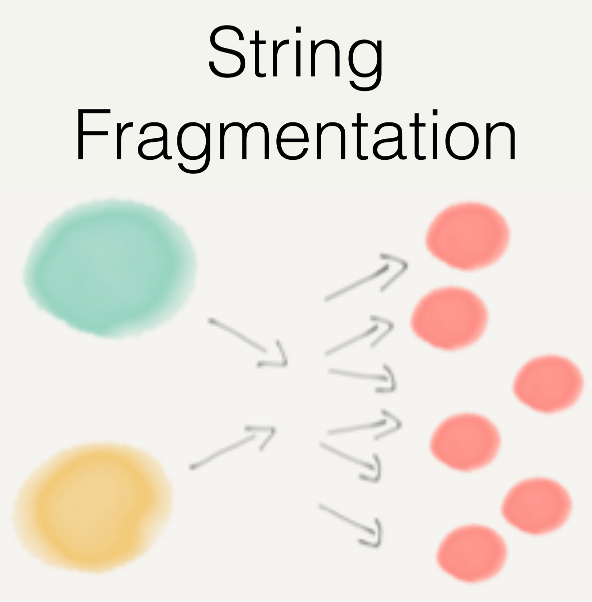
Inelastic Scattering

Decays

Elastic Scattering

2 —> 1 Processes

\+



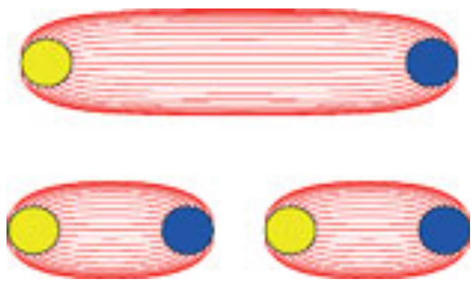
String Fragmentation

for high-energy scatterings

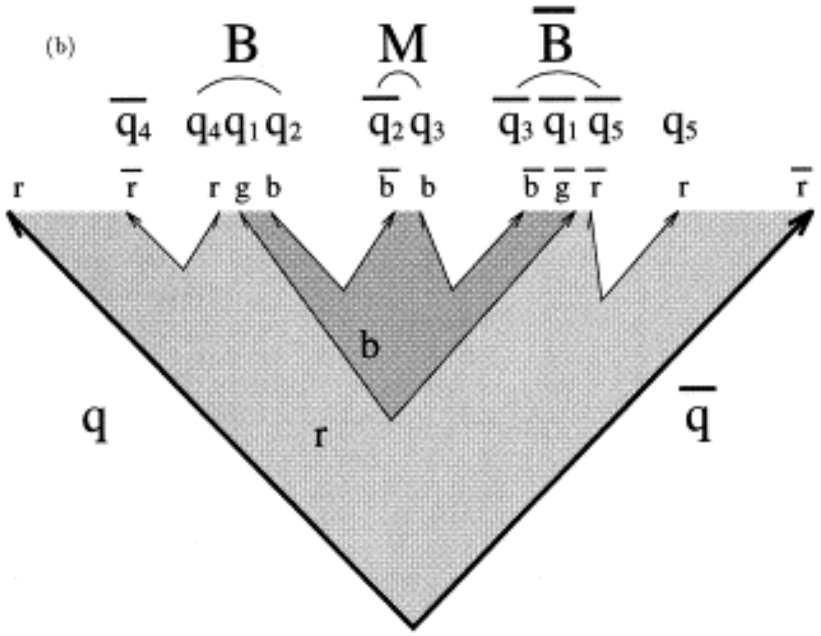
In the few-GeV energy regime decay and excitation of resonances dominate hadronic cross section

String fragmentation

- For high-energy ($\sqrt{s} > 3 - 4$ GeV) hadron-hadron collisions
- *Confinement*: Color flux tube produces new quark pairs if quarks are separated

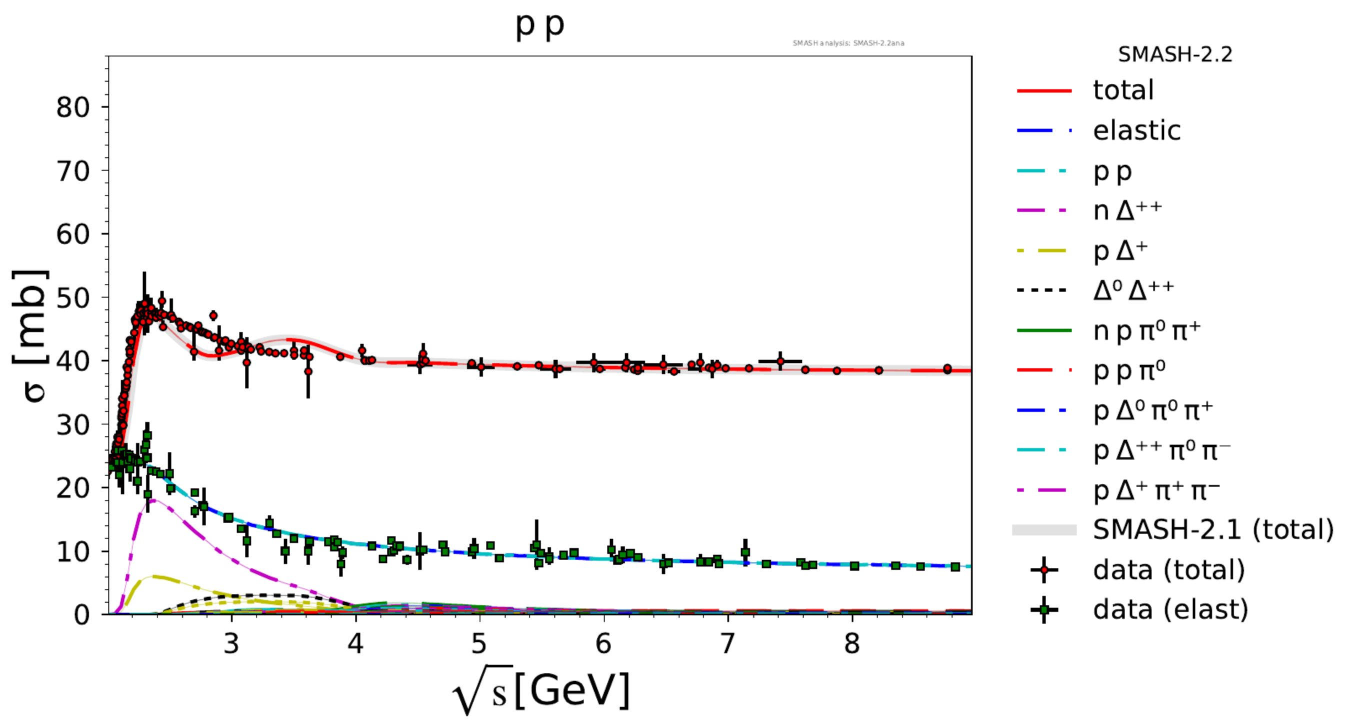


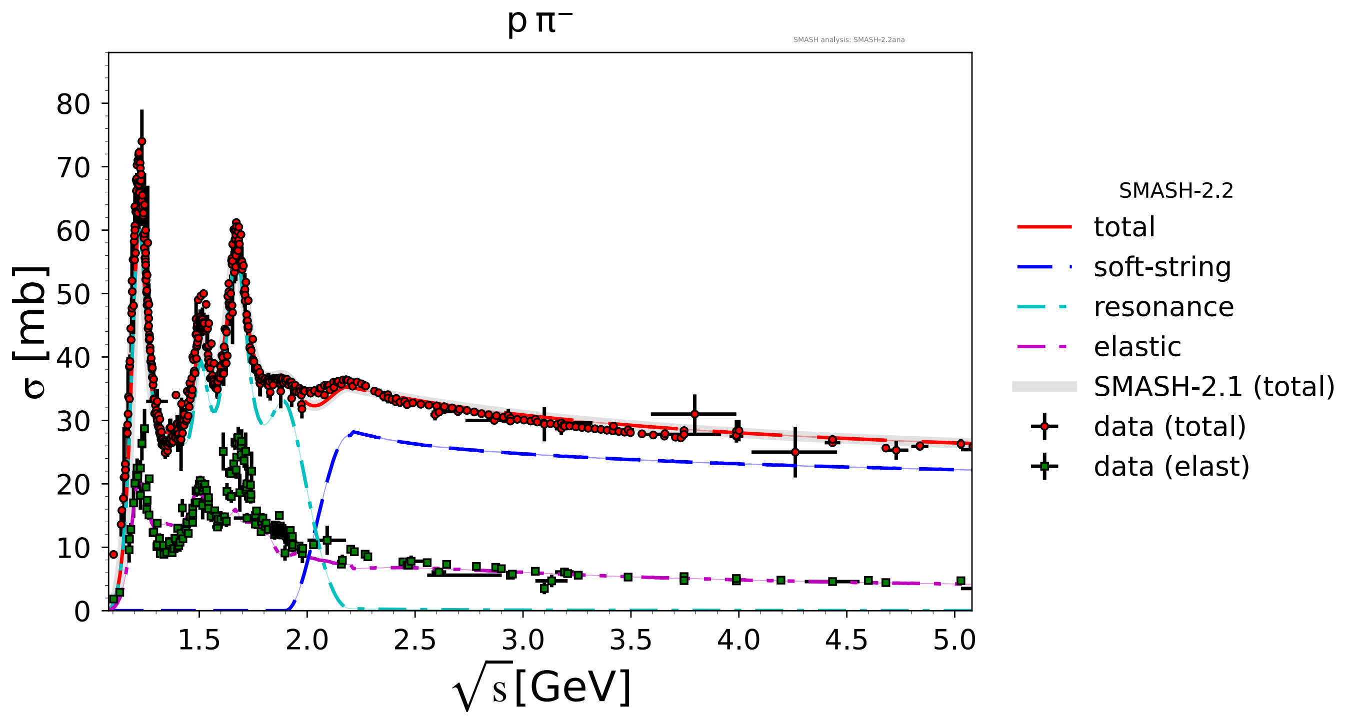
- Quark-antiquark or quark-diquark pairs from colliding hadrons form string that fragments into hadrons
- SMASH uses PYTHIA to perform hard scatterings and string fragmentation



Elementary Cross Sections

- Total cross sections of pp and $p\pi^-$
- One of the major constraints of transport approaches
- Different contributions at different energies
    - Low energies: Resonances
    - High energies: Strings
- Well described by SMASH

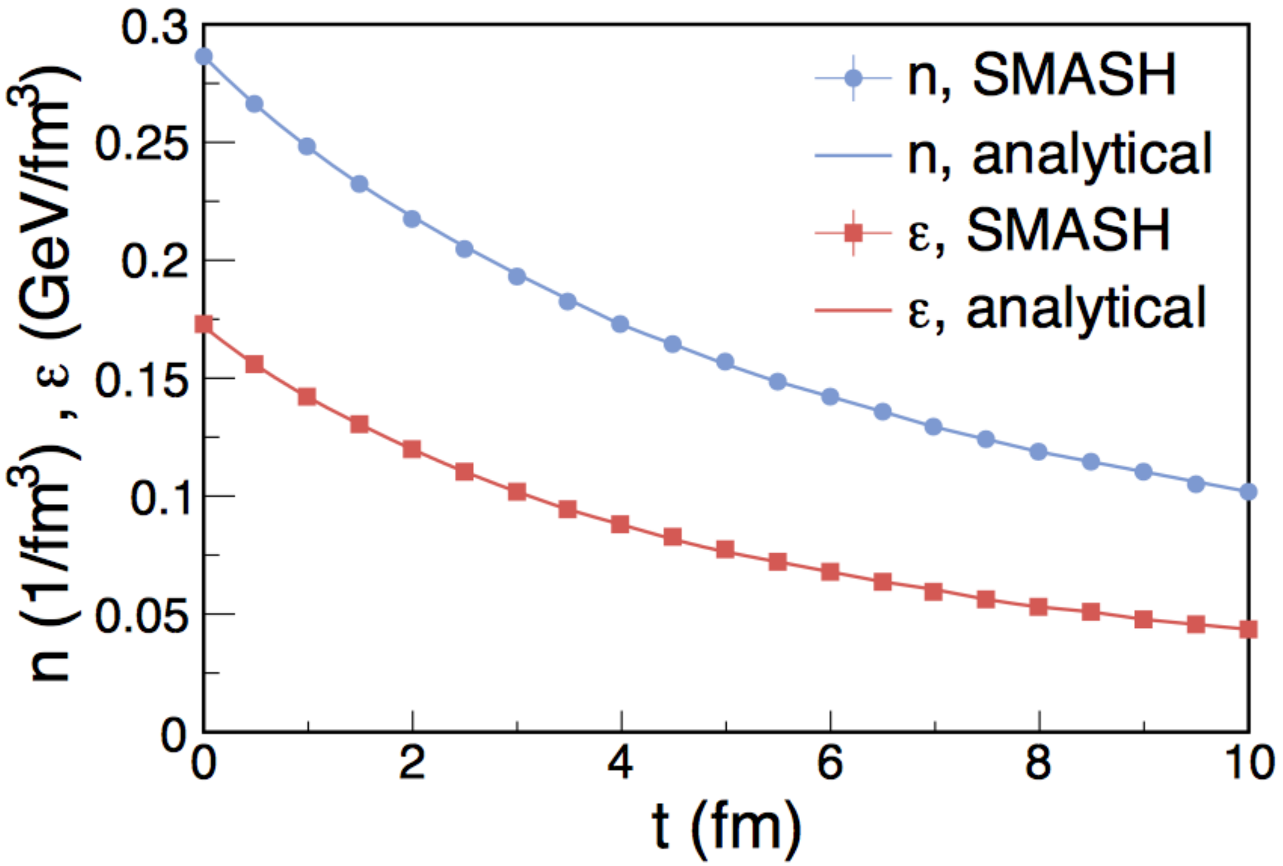


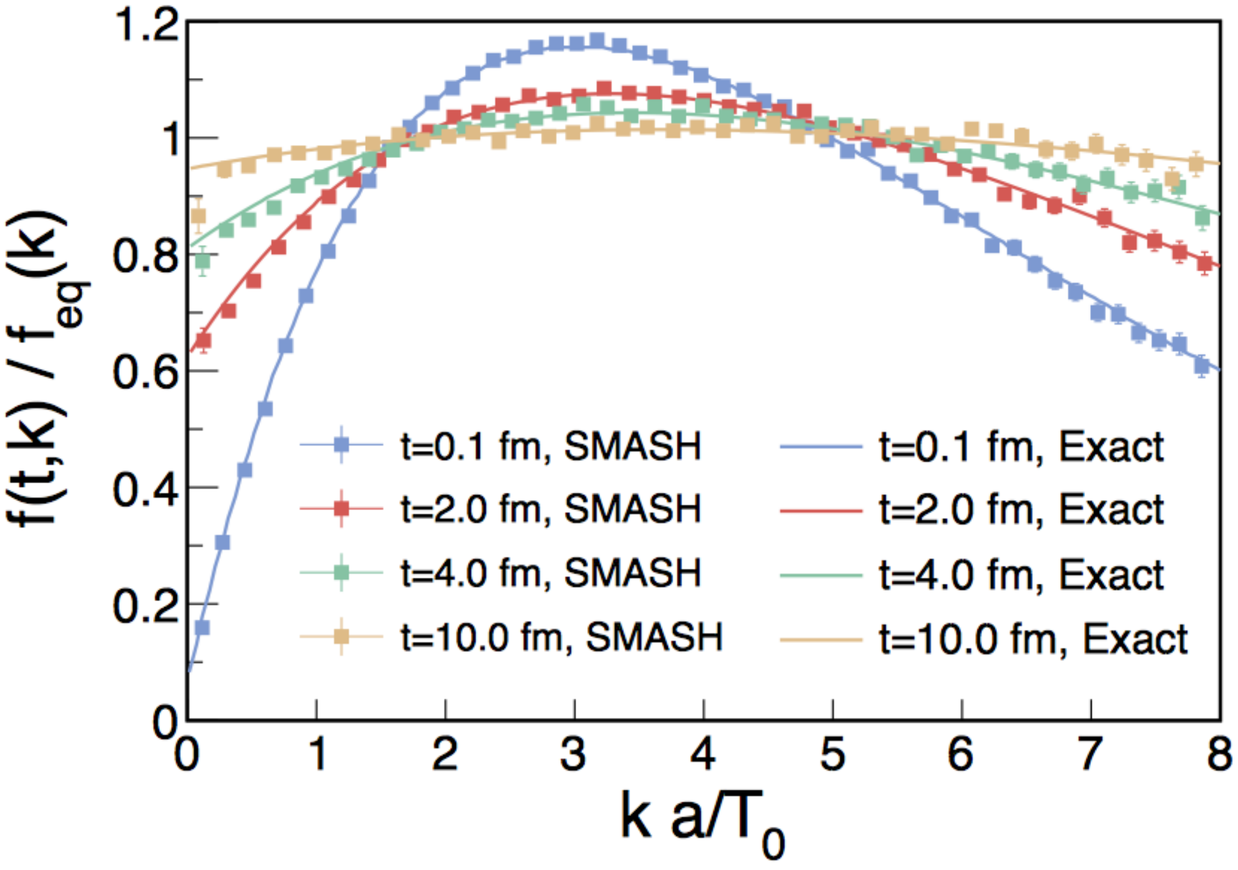


Results (I)

Comparison to analytic solution of Boltzmann eq.

Phys.Lett.B 770 (2017) 532-538



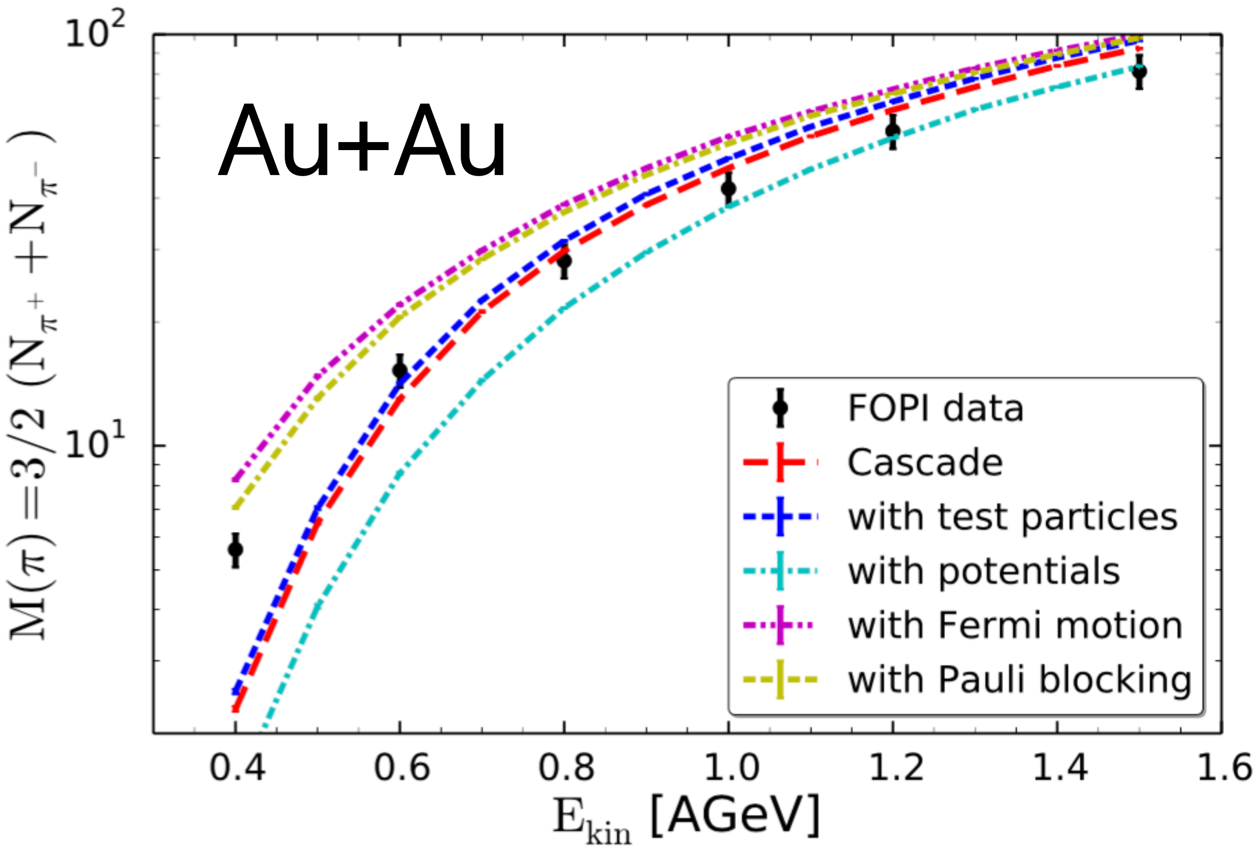


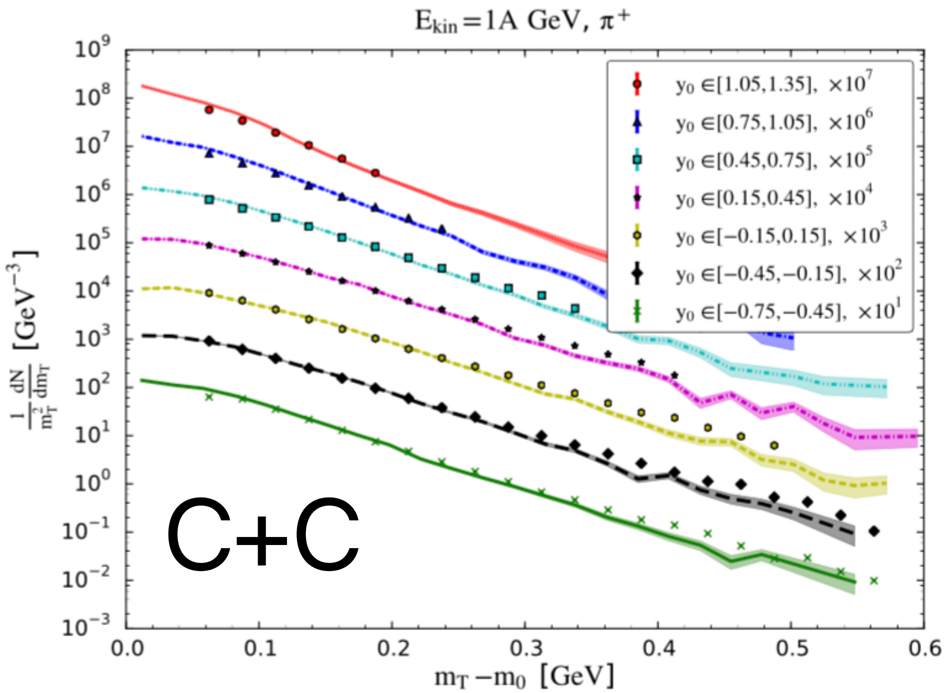
Perfect agreement with analytic solution for expanding metric shows correct numerical implementation of collision algorithm

Results (II)

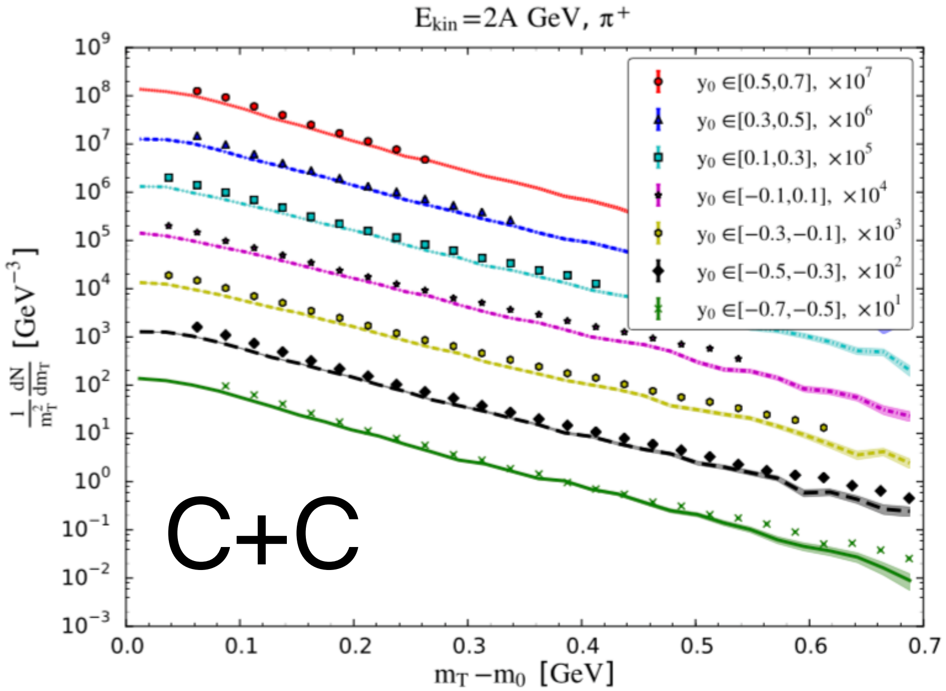
Pion production for collisions at low energies

Phys.Rev.C 94 (2016) 5, 054905





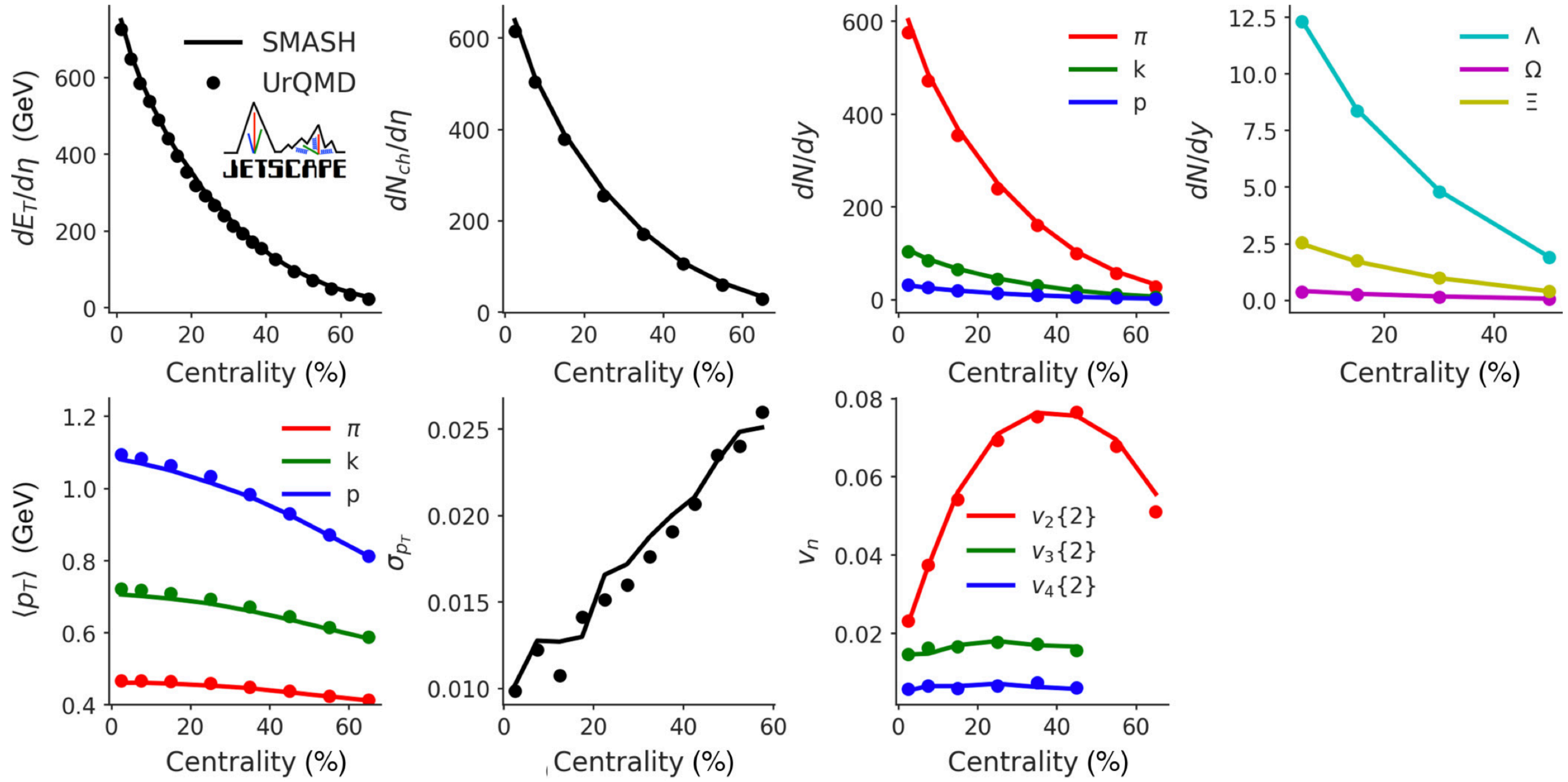
Nice agreement with experimental data for pions at low energies



Results (III)

Comparison of SMASH and UrQMD Afterburner Stage

Phys.Rev.C 103 (2021) 5, 054904



Good **agreement between both approaches** for different bulk observables

More information on SMASH

smash

Website 🔗

Github 🔗

User Guide 🔗

Documentation 🔗

Analysis Suite 🔗

Visualizations 🔗

Thanks for your attention

and
thanks to everyone involved in the school’s organization