JETSCAPE Online Summer School, Aug 1, 2022

Jet-medium excitations

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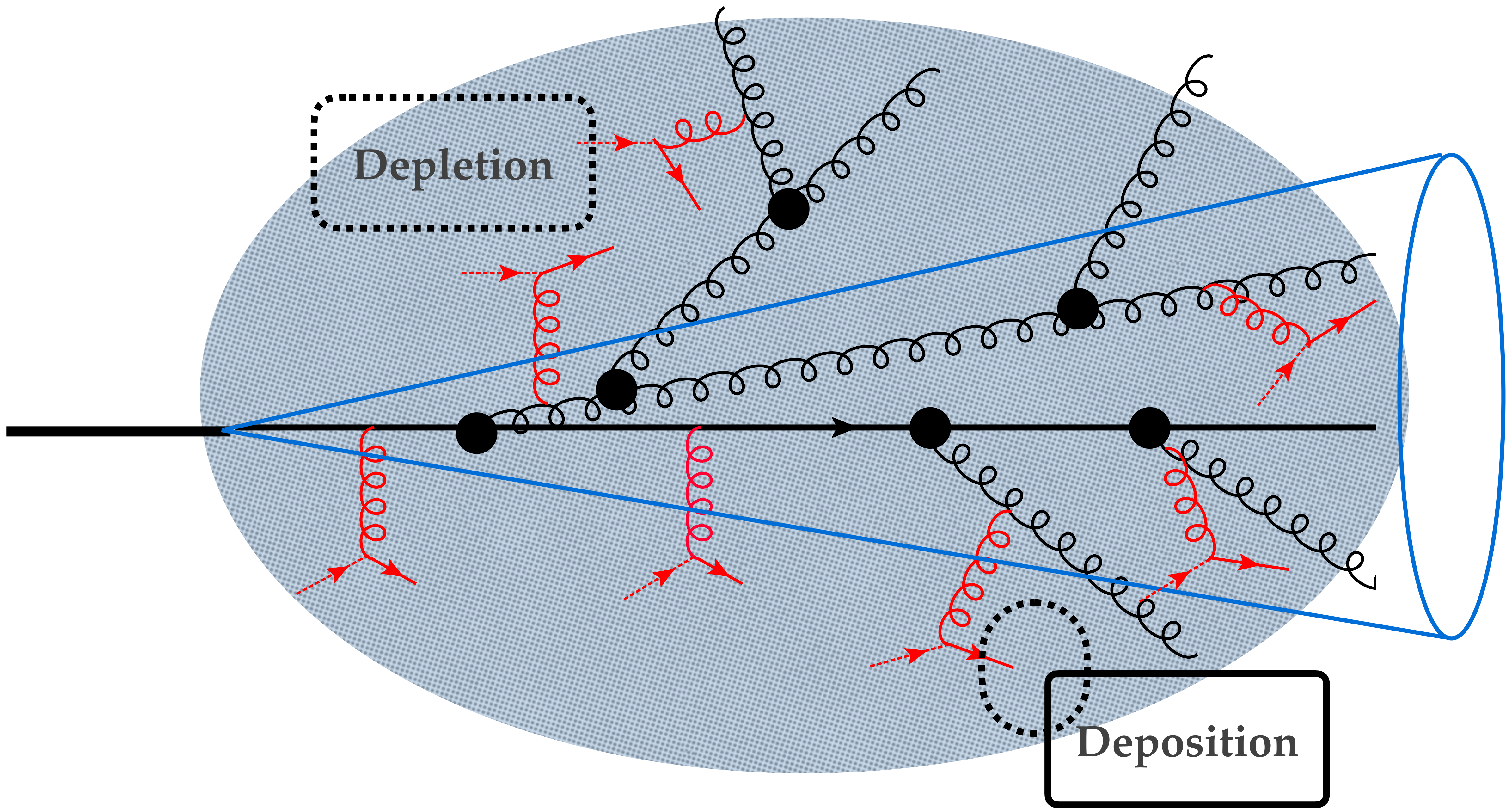
Outline

1. Introduction to medium response
2. Kinetic description of the parton/medium interaction
   - Collinear cascade
   - Thermalization/broadening to large angles
   - Sensitivity to the soft-sector
3. Medium response in JETSCAPE
   - Parton scatterings in the medium => Recoils/holes
   - Hydrodynamics with a Source term
   - Causal Diffusion => Medium response

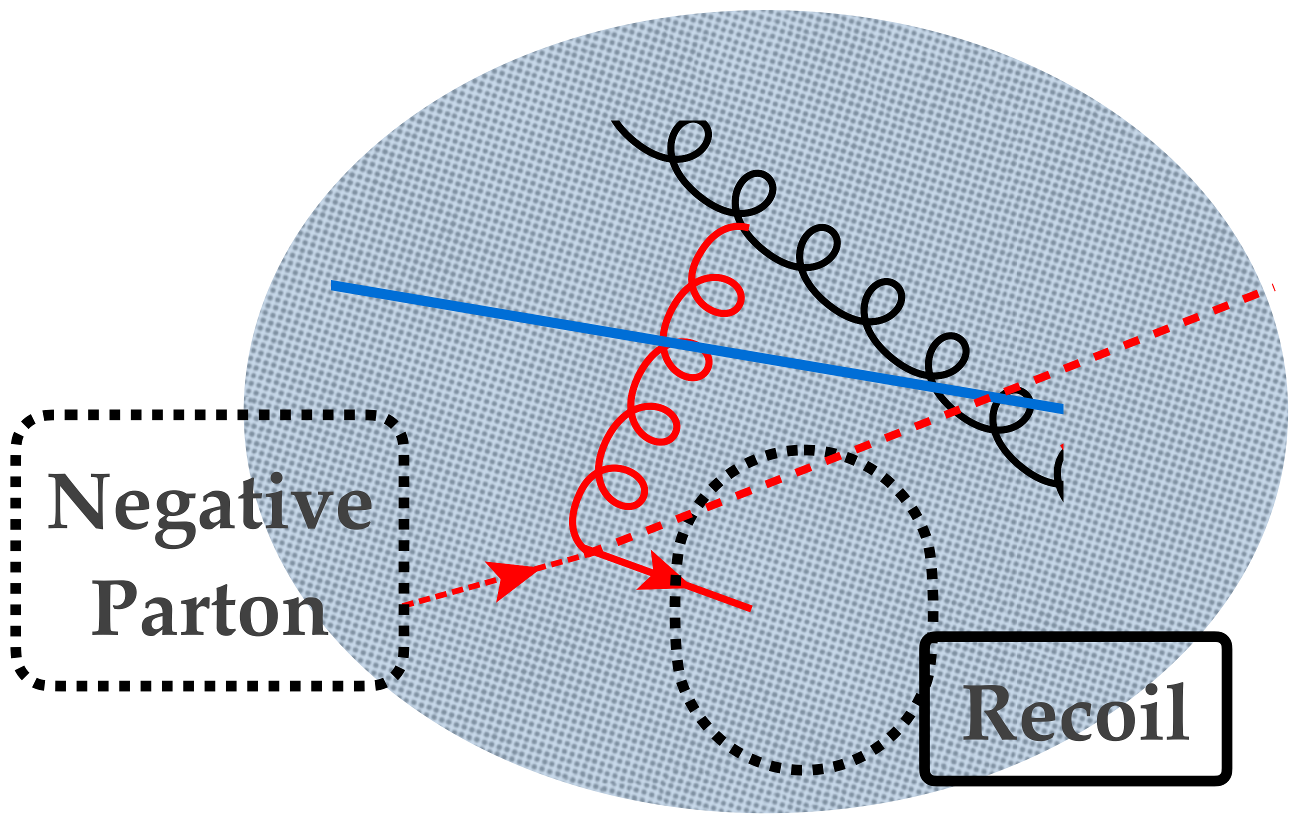
Introduction

- Hard parton - medium interactions:



- How about the energy deposited in the medium, where does it go?

Modeling Medium Response



- Perturbative approximation :
  - Parton sampled from the distribution are marked as negative partons and free stream until they are subtracted from the spectrum before computing observables
  - Recoil partons follow the hard parton evolution
- What if the energy of the recoil $\sim T$, does it really make sense to follow a perturbative cascade? Shouldn't it be thermalizing with the medium?

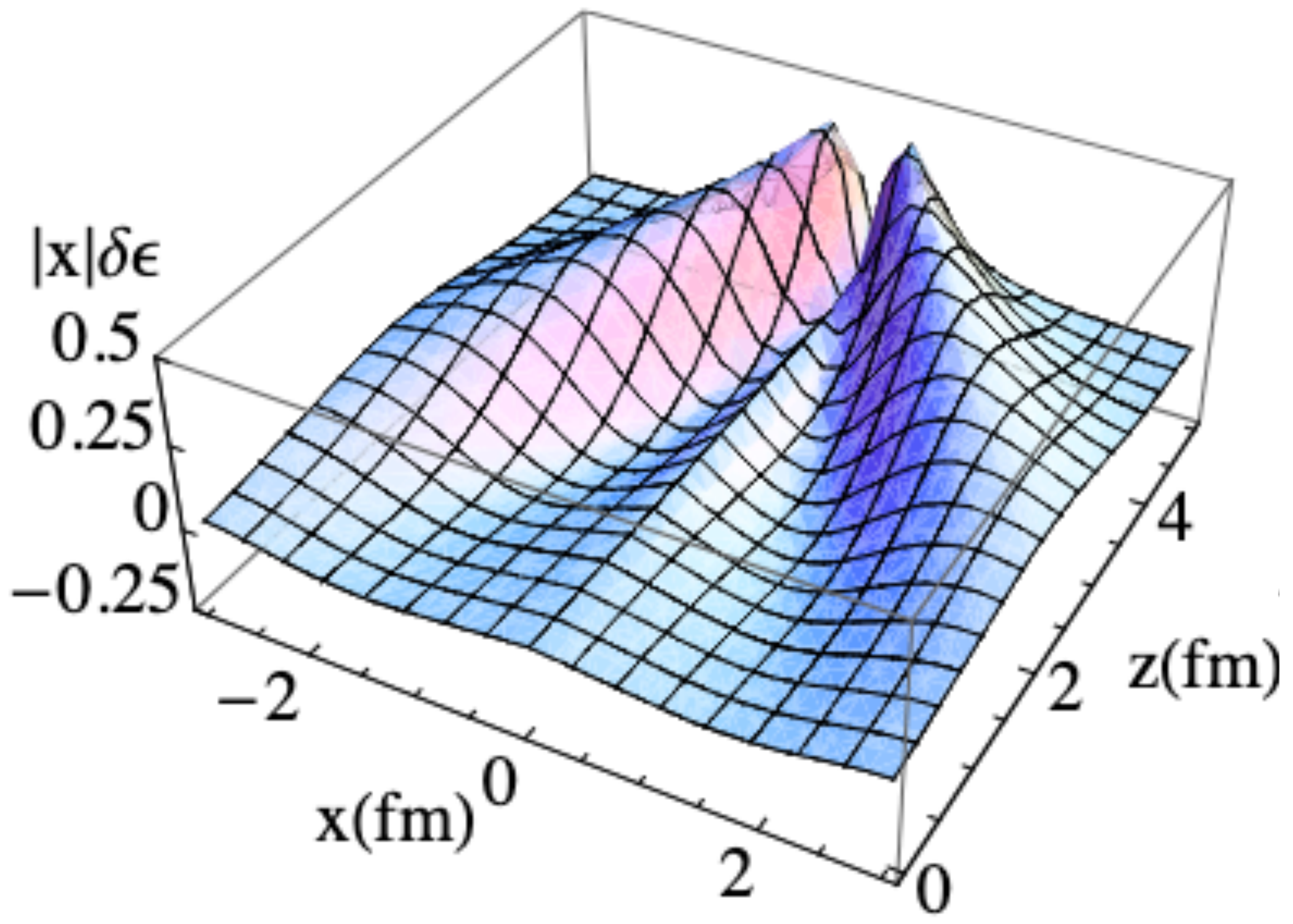
Modeling Medium Response

- Hydrodynamic response :
  - Linear approximation: Hard partons as linear excitation of the bulk medium arXiv: 0807.2996, 0802.2254, 0903.2255

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu},$$

$$\partial_\mu \delta T^{\mu\nu} = J^\nu,$$

$$\partial_\mu T_0^{\mu\nu} = 0$$

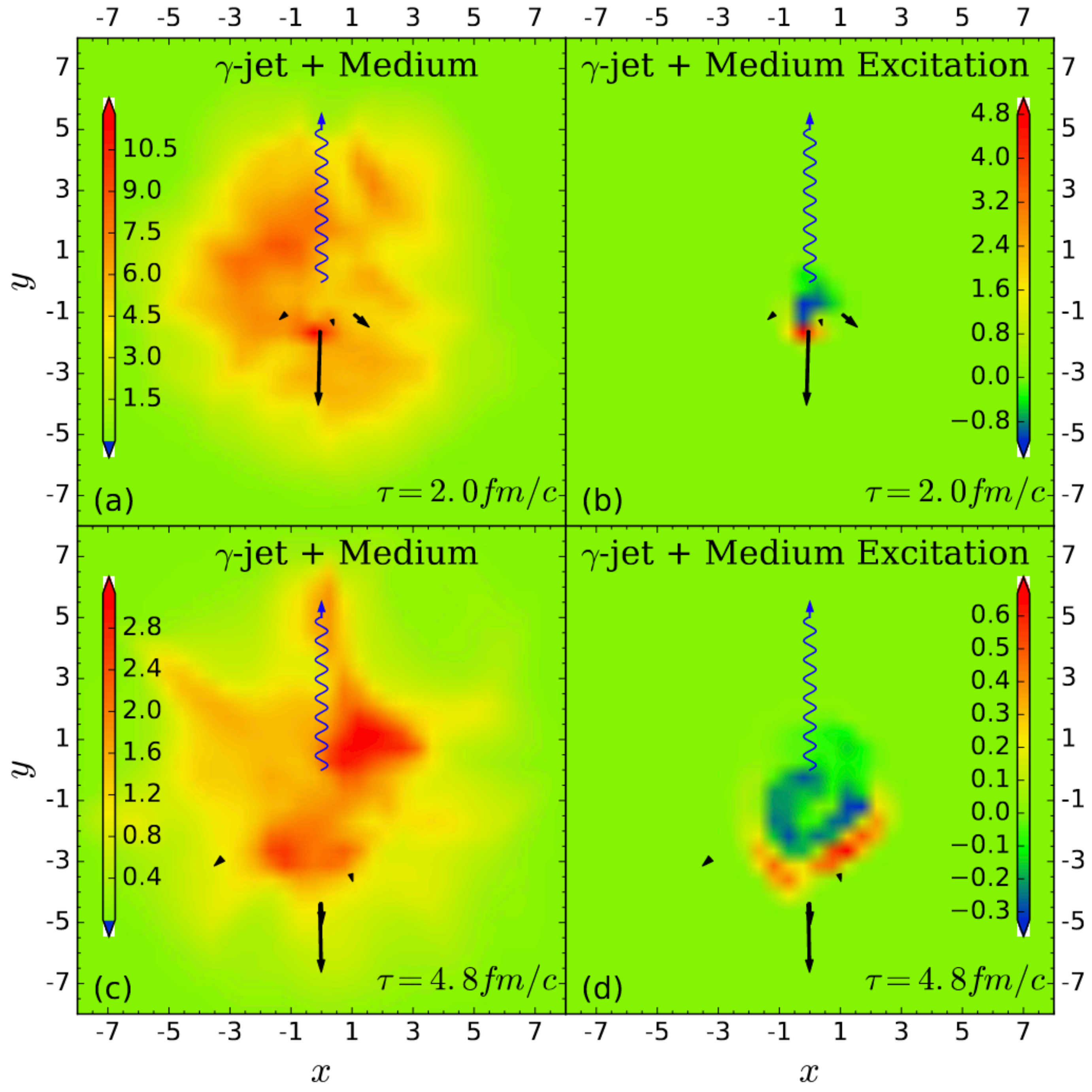


[G.-Y. Qin et Al. arXiv:0903.2255]

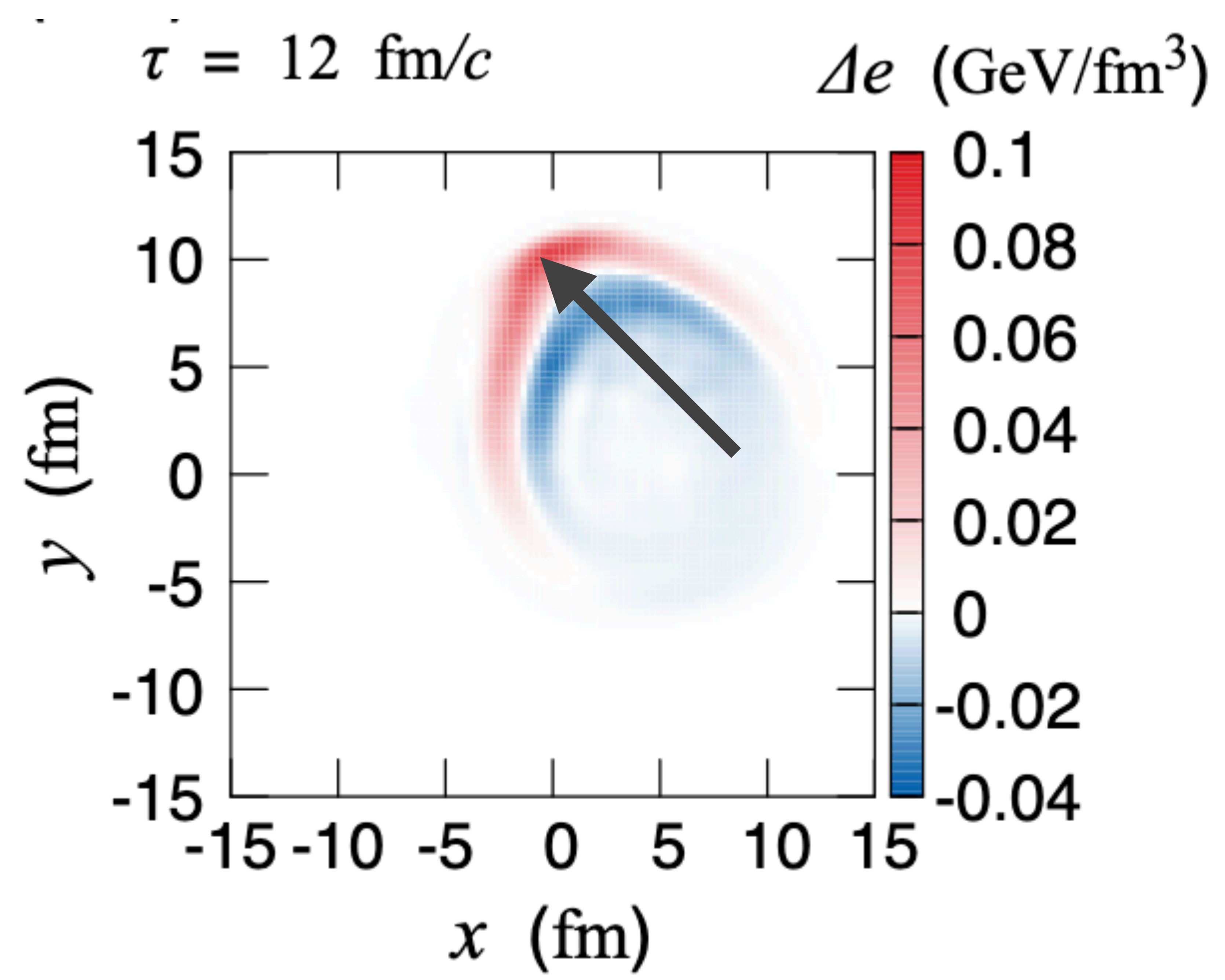
Modeling Medium Response

- Full Hydro:
  - CoLBT-hydro arXiv: 1704.03648
  - Hydro response: arXiv: 0503028,1407.1782,1701.07951

$$\partial_\mu T^{\mu\nu}(x) = J^\nu(x)\,.$$



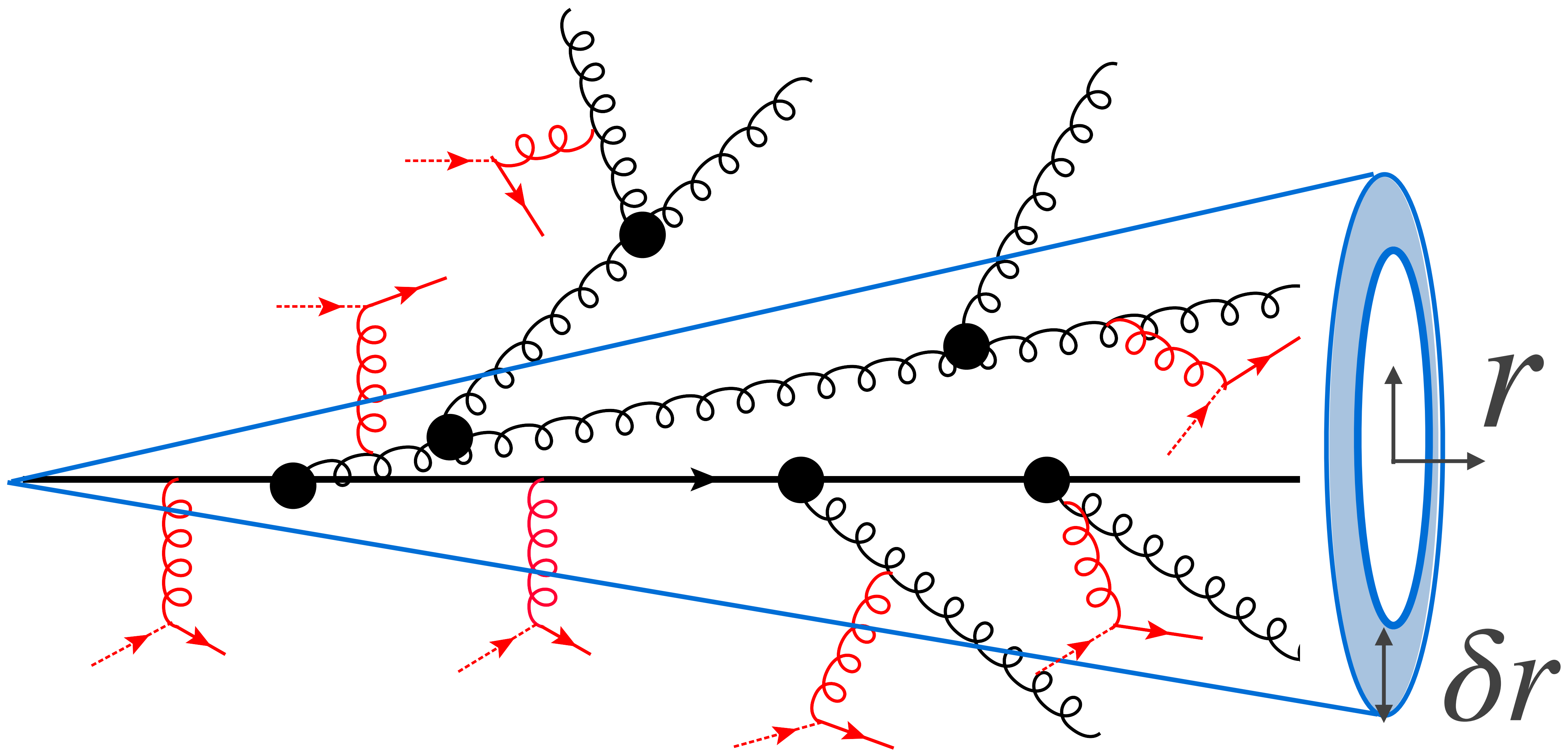
[W. Chen, et Al. arXiv:1704.03648]



[Y. Tachibana, et Al. arXiv:1701.07951]

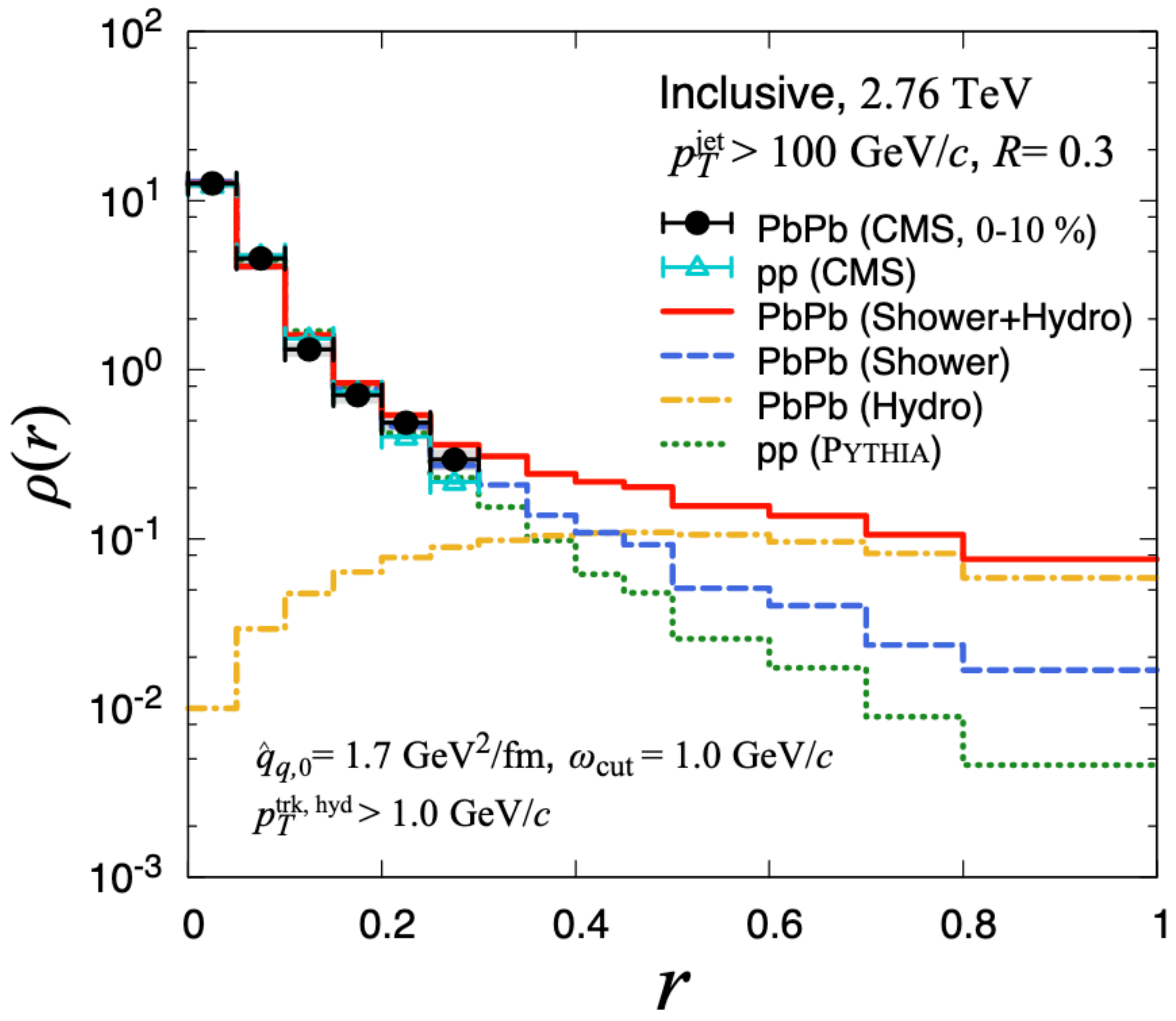
Jet Shape

- Jet Shape: Angular structure of the jet



$$\rho_{\rm jet}(r) = \frac{1}{N_{\rm jet}} \sum_{\rm jet} \left[\frac{1}{p_T^{\rm jet}} \frac{\sum_{\rm trk \in (r-\delta r/2, r+\delta r/2)} p_T^{\rm trk}}{\delta r} \right],$$

$$r = \sqrt{(\eta_p - \eta_{\rm jet})^2 + (\phi_p - \phi_{\rm jet})^2}$$



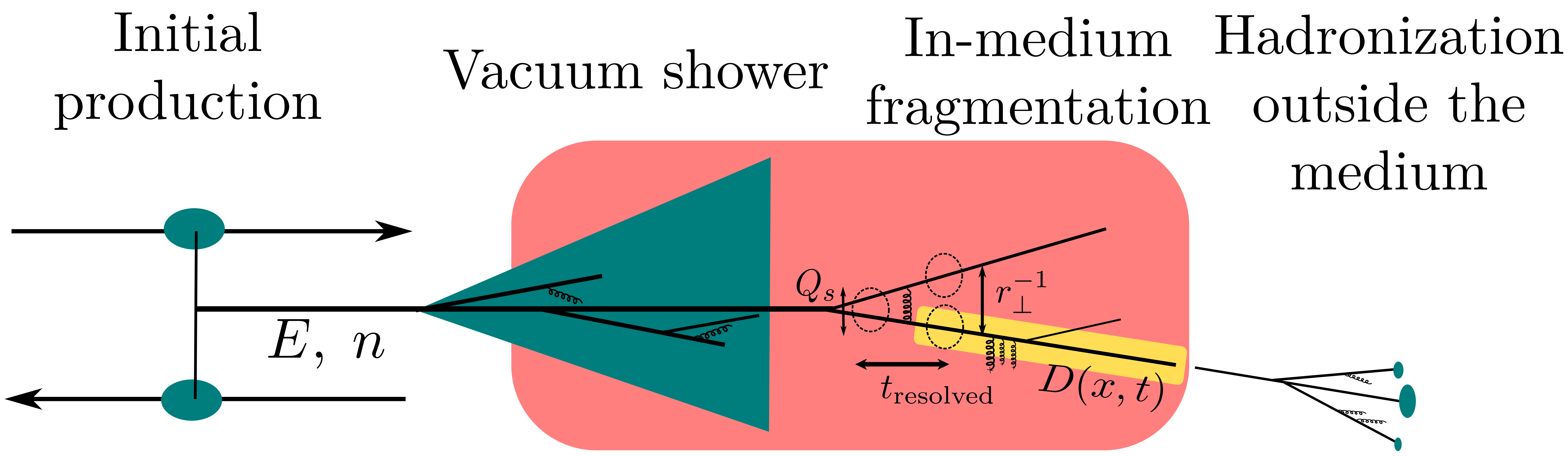
[Y. Tachibana, et Al. arXiv:1701.07951]

Kinetic Study

Based on: S. Schlichting, I.S. arXiv:2008.04928

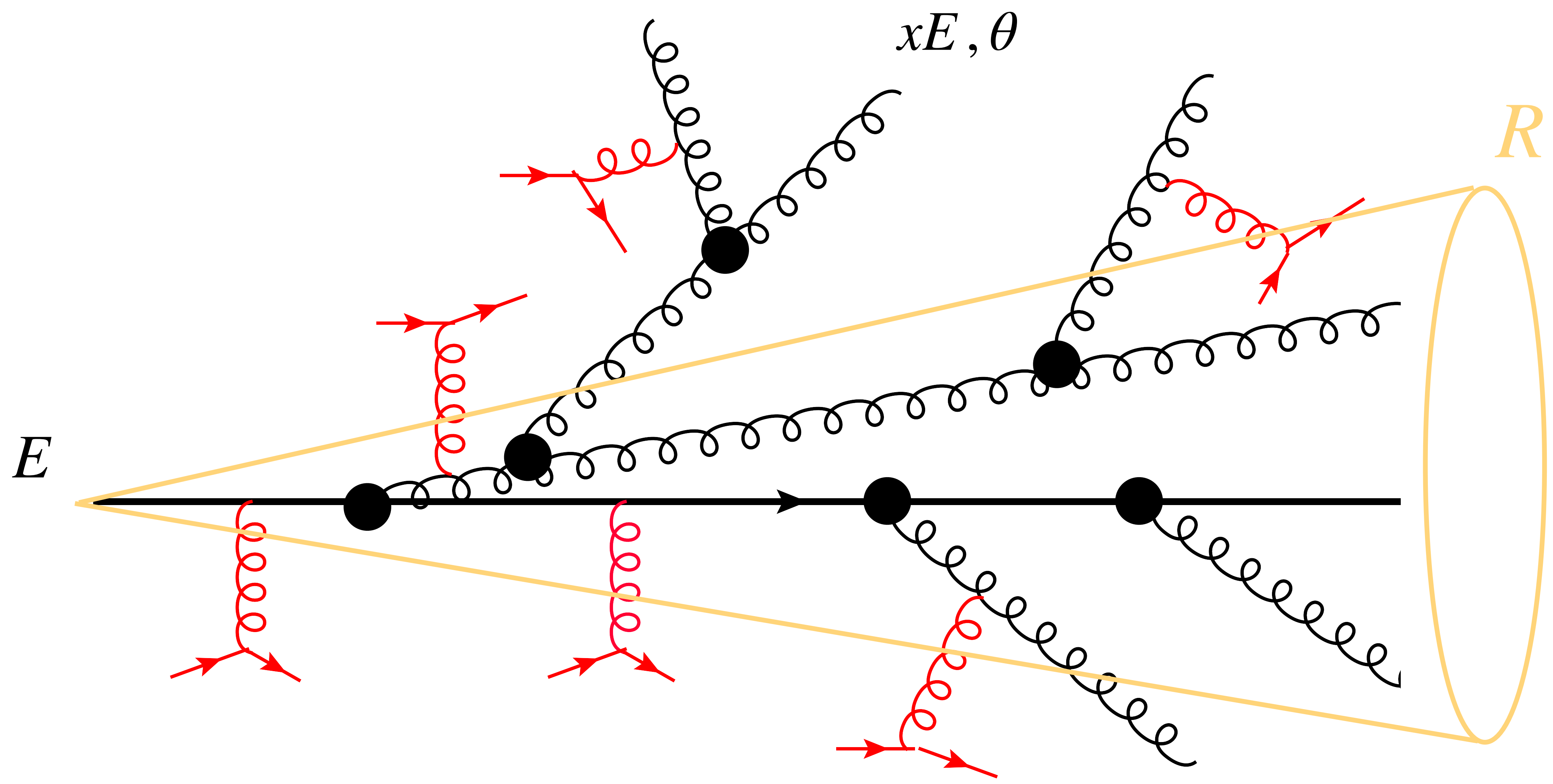
S. Schlichting, I.S., Y. Mehtar-Tani work in progress

QCD Jets



- Complete picture of jet evolution in HIC is a complex task
- **We focus mainly on energy loss and equilibration of hard partons in the medium**

Our Focus



- Main focus: Hard Parton traversing the medium
- Understand: energy cascade, out-of-cone energy loss, medium response and full thermalization of the initial hard parton => Important for low energy jets at RHIC (sPHENIX)

Effective Kinetic description

- Based on an effective kinetic theory at leading order:

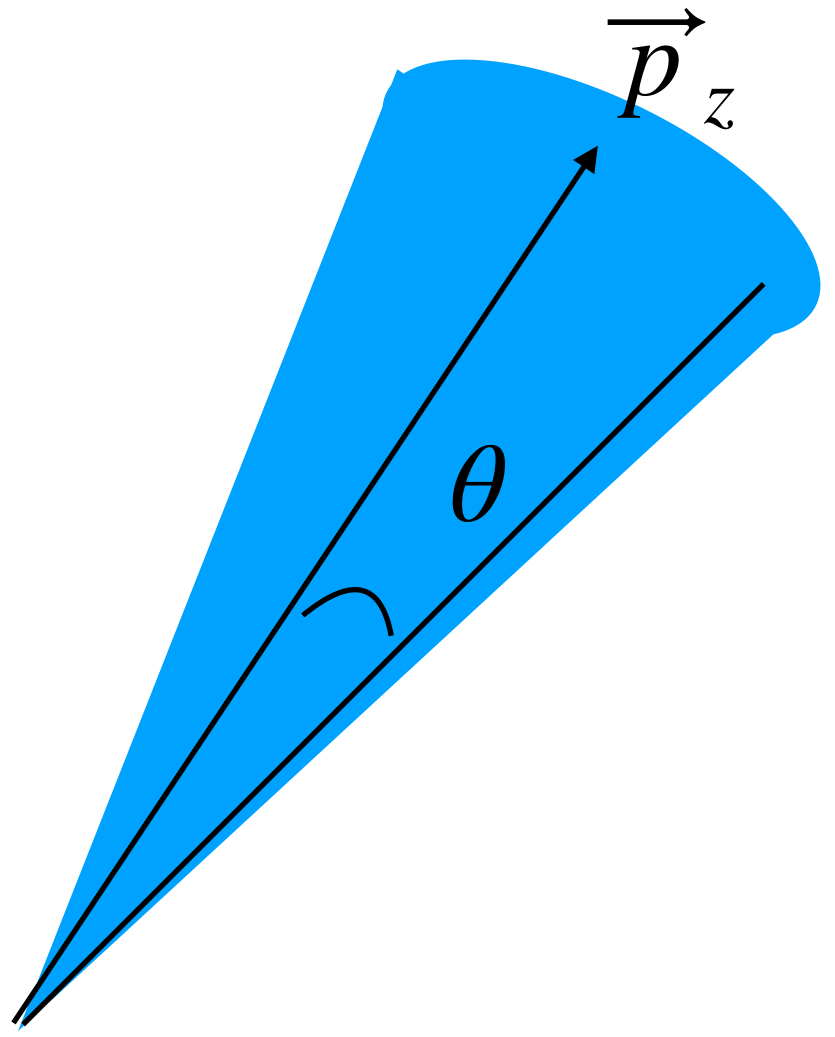
$$p^\mu \partial_\mu f_i(\vec{x}, \vec{p}, t) = C[\{f_i\}],$$

- We consider high energetic partons as linearized fluctuation over static background equilibrium

$$f(p,t) = n_{\rm eq}(p;T) + \delta f_{\rm jet}(p,t),$$

- Define energy distribution:

$$D_a(x,\theta,t) \equiv x\frac{dN_a}{dx\,d\cos\theta} \sim \frac{\nu_a(N_f)}{E_j} p^3 \delta f(p,\theta)\Big|_{p=xE_j},$$



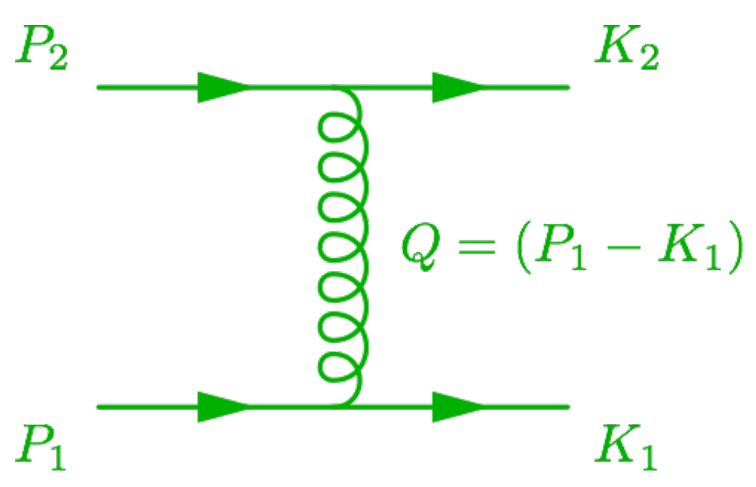
- $x = \dfrac{p}{E_j}$ is the parton momentum fraction

- θ : Polar angle of the momentum

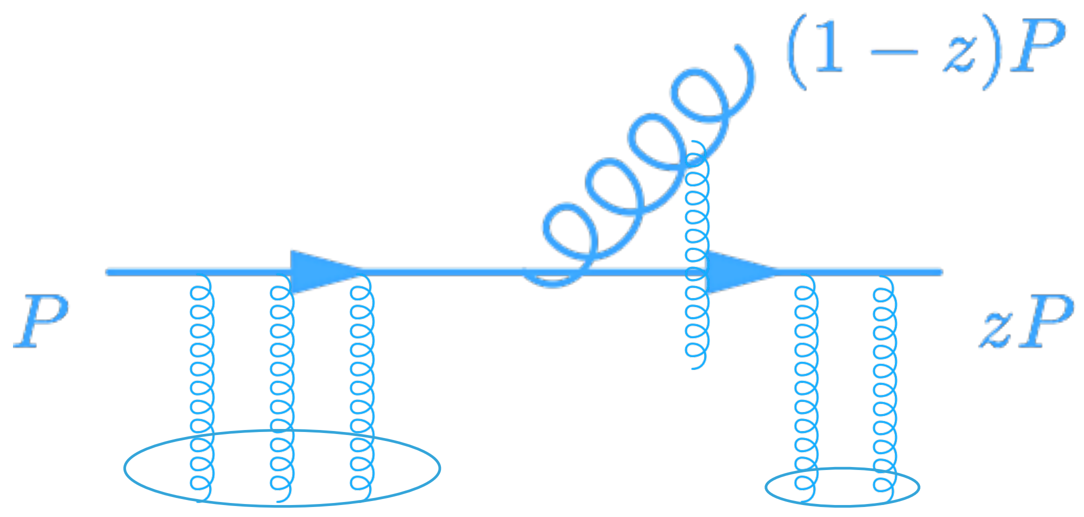
- Exact conservation of energy, momentum and valence charge → allows to study evolution from $\sim E$ to $\sim T$ including thermalization of the hard partons

Effective Kinetic description

Elastic scatterings



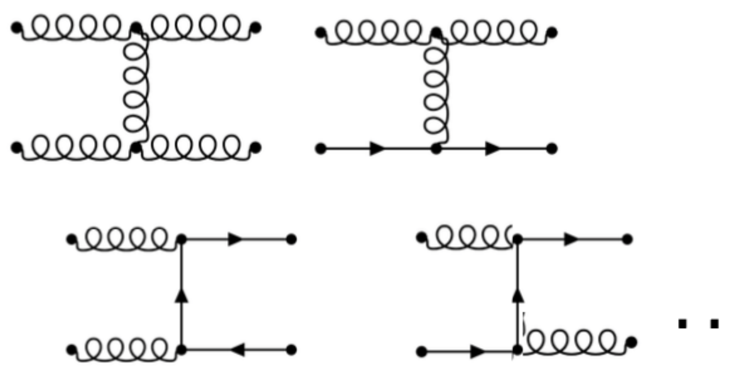
LPM resummed Rate.

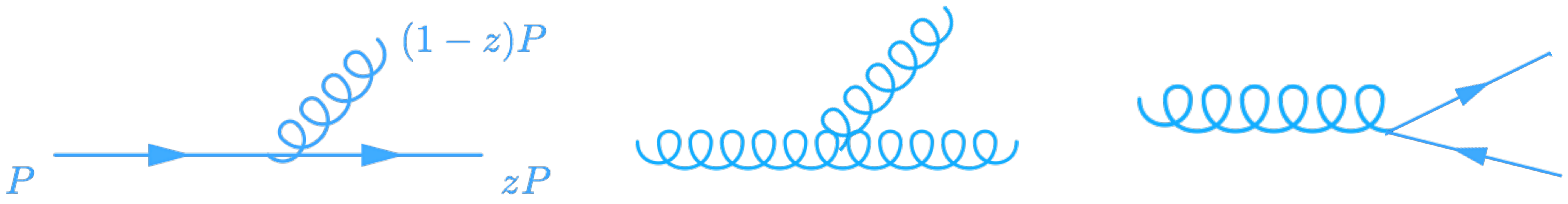


$$C[\{f_i\}] = C^{2\leftrightarrow 2}[\{f_i\}] + C^{1\leftrightarrow 2}[\{f_i\}] \quad ,$$

[J. Blaizot et al. arXiv:1402.5049]
[J. Ghiglieri et al. arXiv: 1509.07773]

[P. B. Arnold, G. D. Moore, and L. G. Yaffe (AMY) (2003)]

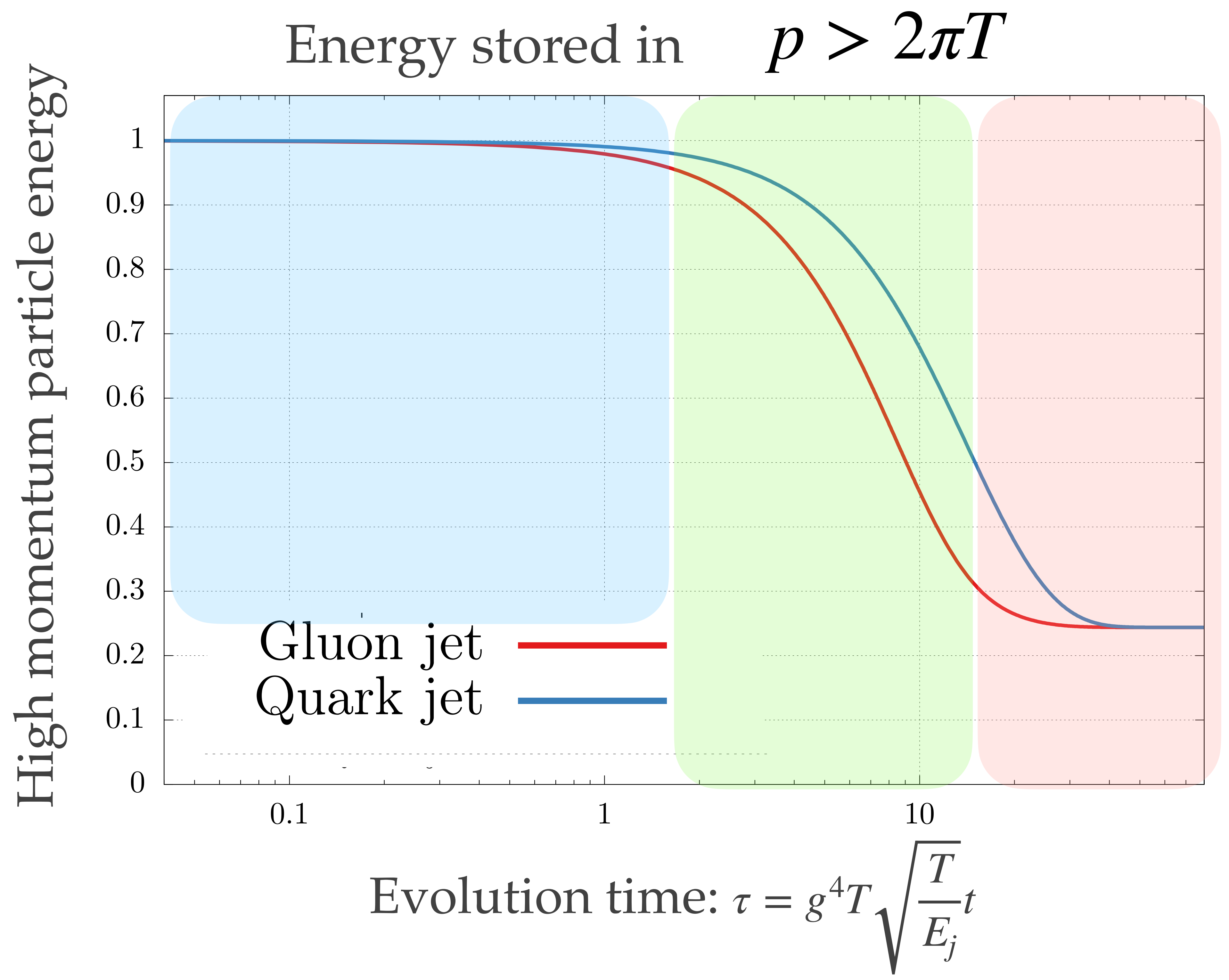




Energy Loss: Collinear cascade

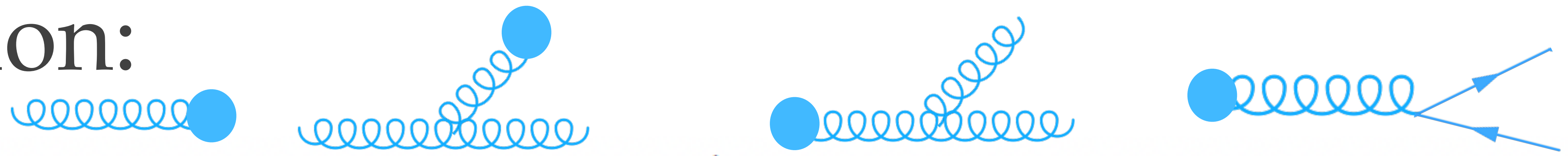
- Three regimes:
  - Initial energy loss: mediated by gluon radiation and re-coil terms.
  - Energy cascade: universality between gluon/quark Jet→ radiative break-up via successive splittings, reminiscent of turbulence
  - Equilibration: exponential decay, linear response.

Jet energy $E_j = 1000T$ and $g = 1$.



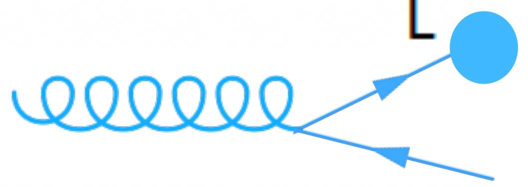
Early Time Behavior

- Consider single emission:



Gluon jet:

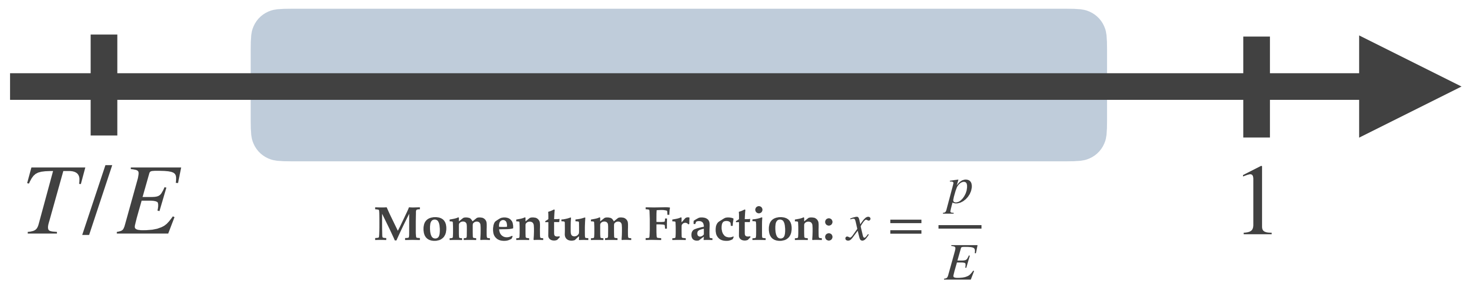
$$D_{\mathrm{g}}(x,\tau) \simeq \delta(1-x) + \left[x\mathcal{K}_{\mathrm{gg}}(x) - \int_0^1 \mathrm{d}z\, z\left(\mathcal{K}_{\mathrm{gg}}(z)+\mathcal{K}_{\mathrm{qg}}(z)\right)\delta(1-x)\right]\tau$$



$$D_{\mathrm{S}}(x,\tau) \simeq xK_{\mathrm{qg}}(x)\,\tau\,,$$

- The distribution follows the behavior of single splitting for $T/E \ll x \ll 1$:

$$D_g(x,t) \simeq \frac{G(t)}{\sqrt{x}}\,,\, D_S(x,t) \simeq S(t)\sqrt{x}\,,$$



with linear rising amplitudes

- Direct energy deposition:

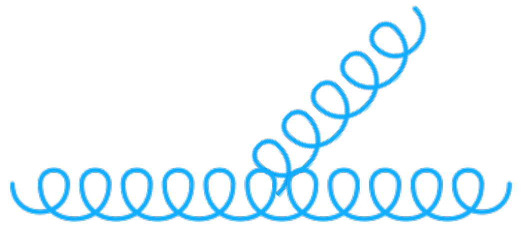
$$\left.\frac{dE}{d\tau}\right|_{\tau\ll 1} = \int_{2\pi T}^{\infty} dx\; \partial_\tau D(x,\tau) = \gamma^{\text{soft}-\text{radiation}} + \gamma^{\text{recoil}}\,,$$

Early Time Behavior

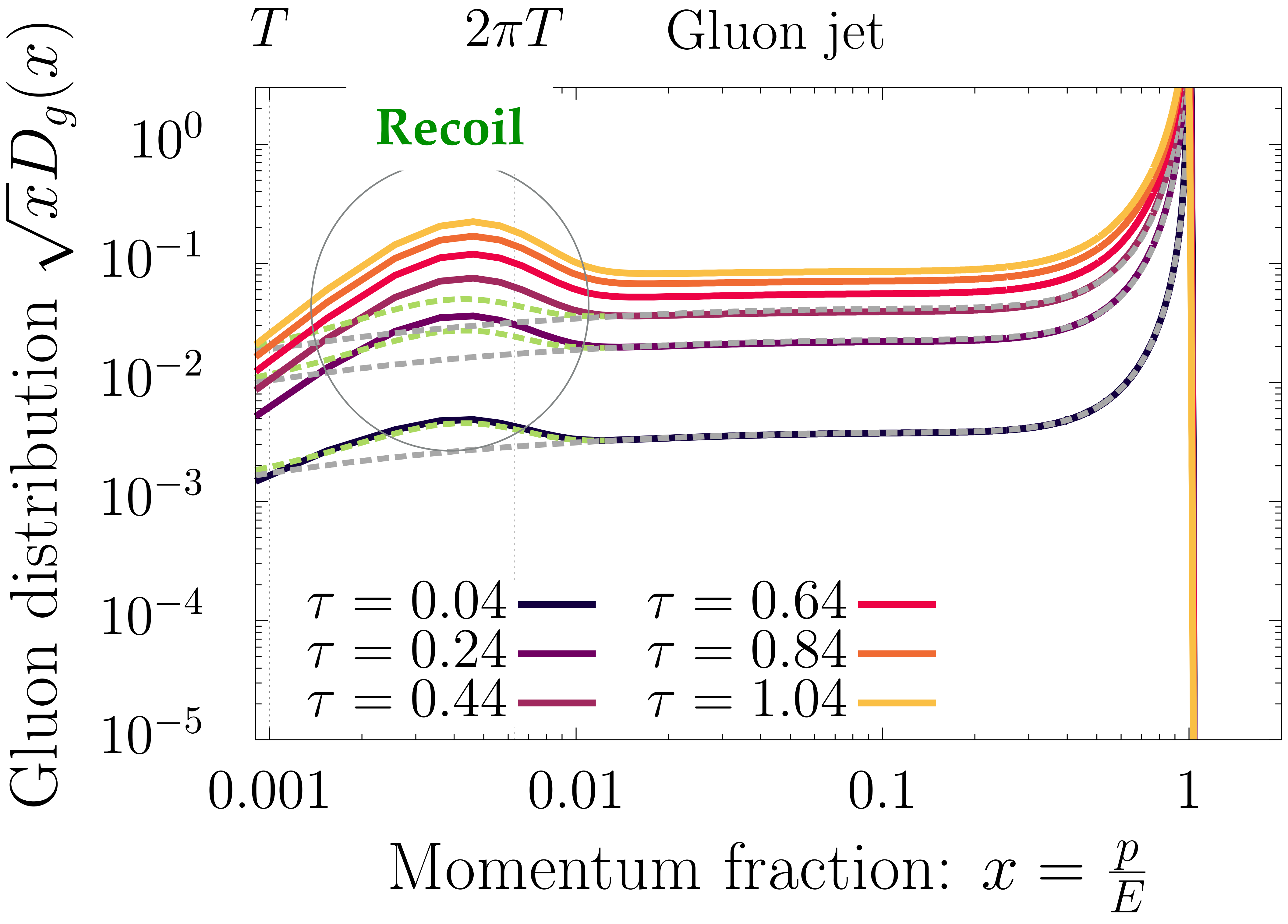
Initial Gluon Jet

$$\text{Singlet} = \frac{D_q(x) + D_{\bar{q}}(x)}{2}$$

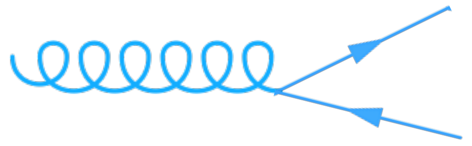
Driven by the rate $g \leftrightarrow gg$



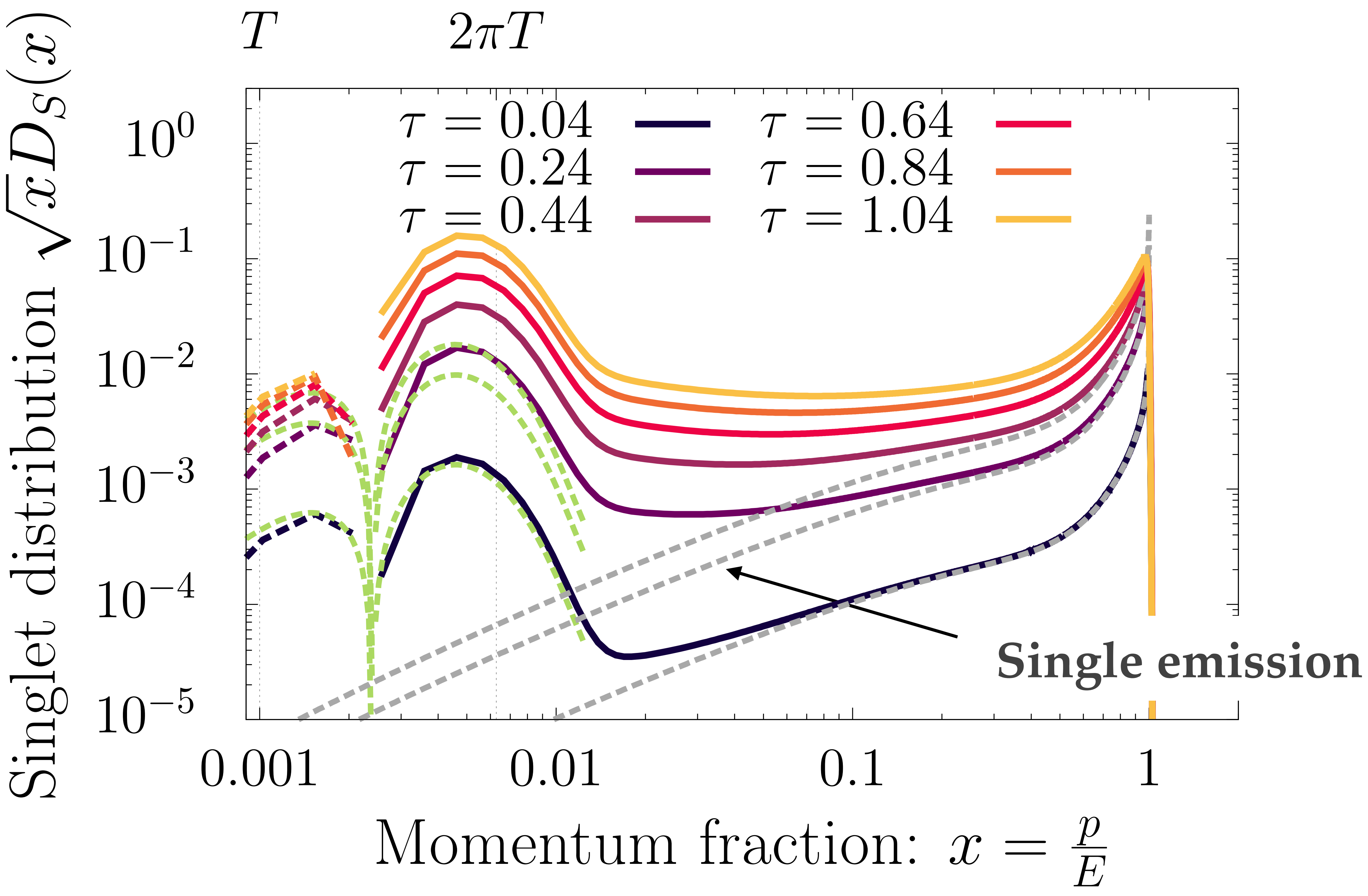
Gluon Energy distribution



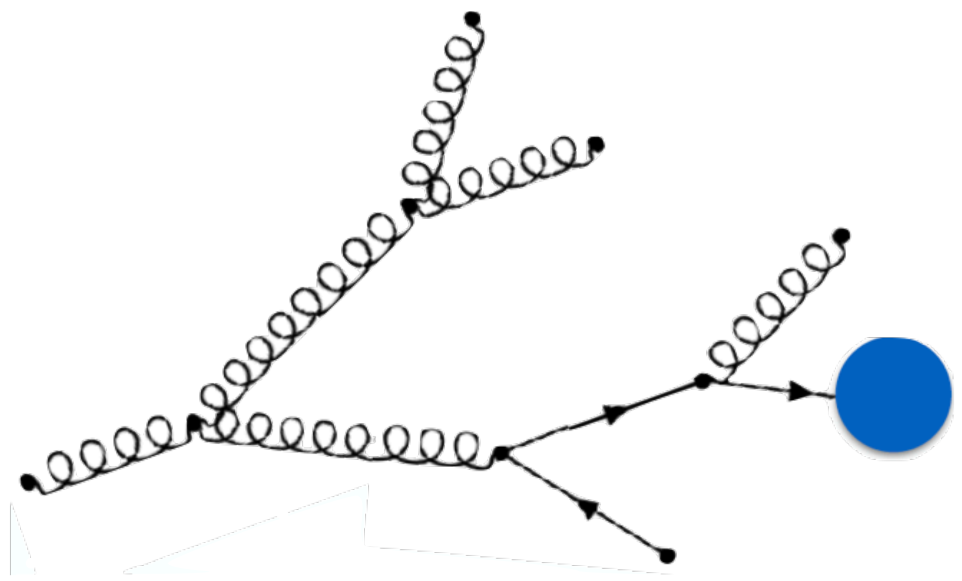
Driven by the rate $g \leftrightarrow q\bar{q}$

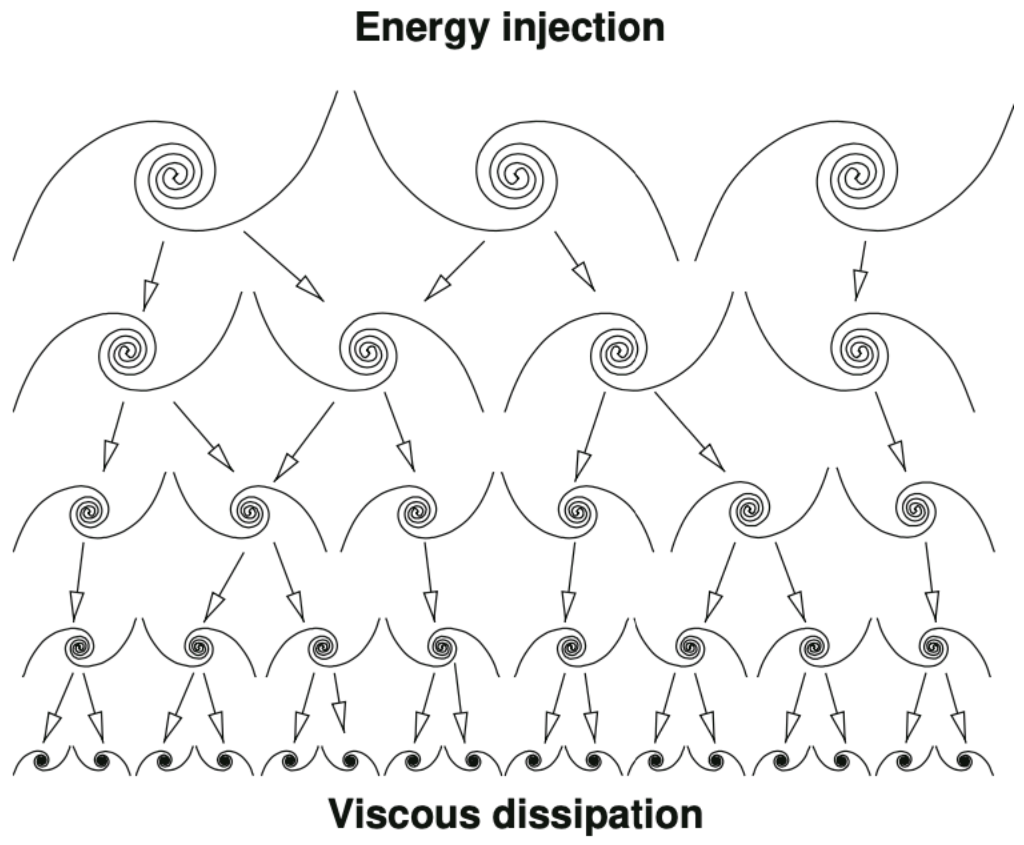


Singlet Energy distribution



Energy Cascade





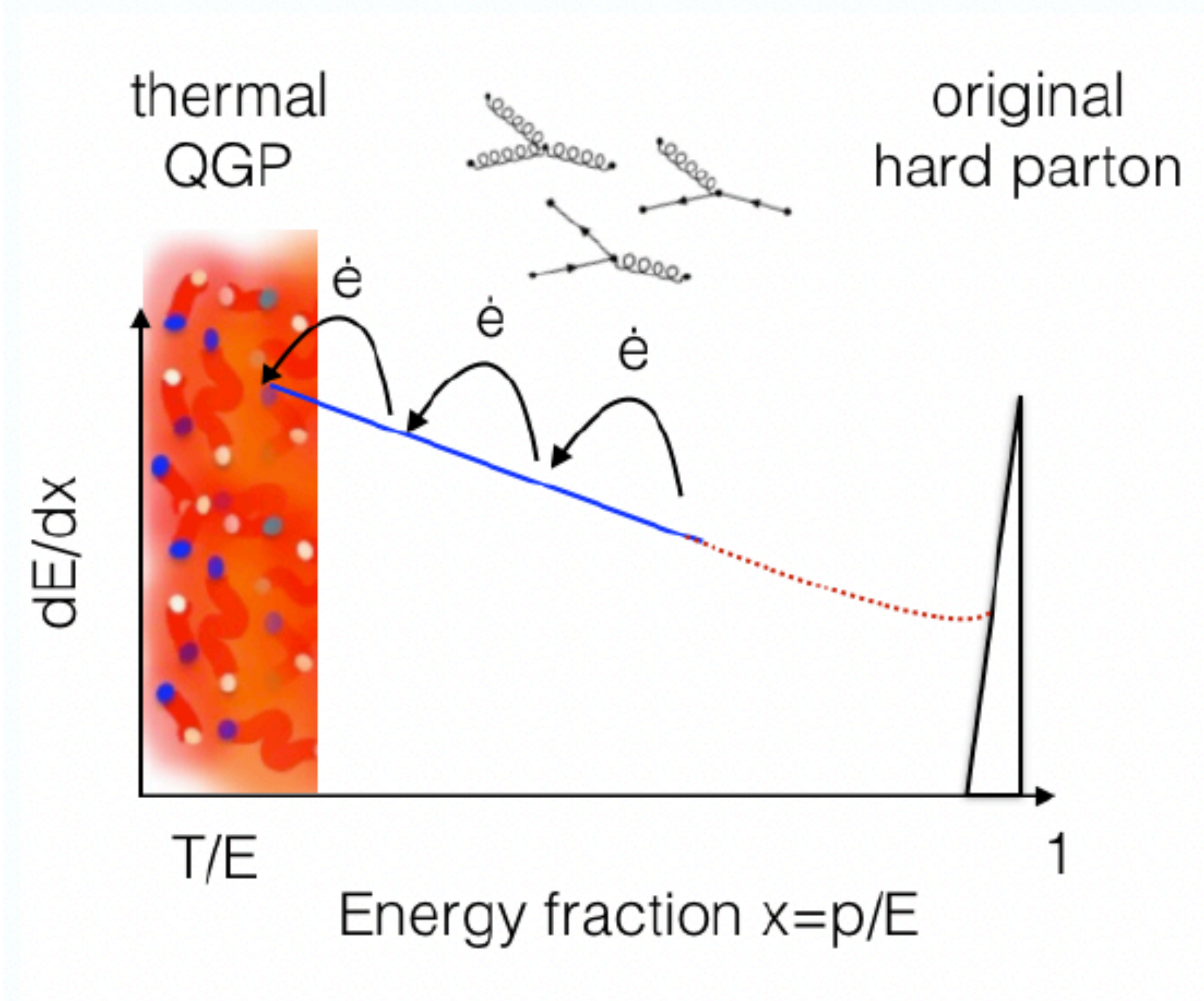
- Stationary turbulent solution in intermediate range $T/E \ll x \ll 1$

$$D_g(x) = \frac{G}{\sqrt{x}} \,, \quad D_S = \frac{S}{\sqrt{x}} \,,$$

- Scale invariant energy flux :

$$\frac{dE}{d\tau}(\Lambda) = \sum_i \int_{\Lambda/E}^{\infty} dx\ \partial_\tau D_i(x) = \left(\tilde{\gamma}_g + \frac{S}{G}\tilde{\gamma}_q\right) G(\tau) \,,$$

- Time dependent amplitude accounts for injection of energy due to radiation of hard particles $x \sim 1$:



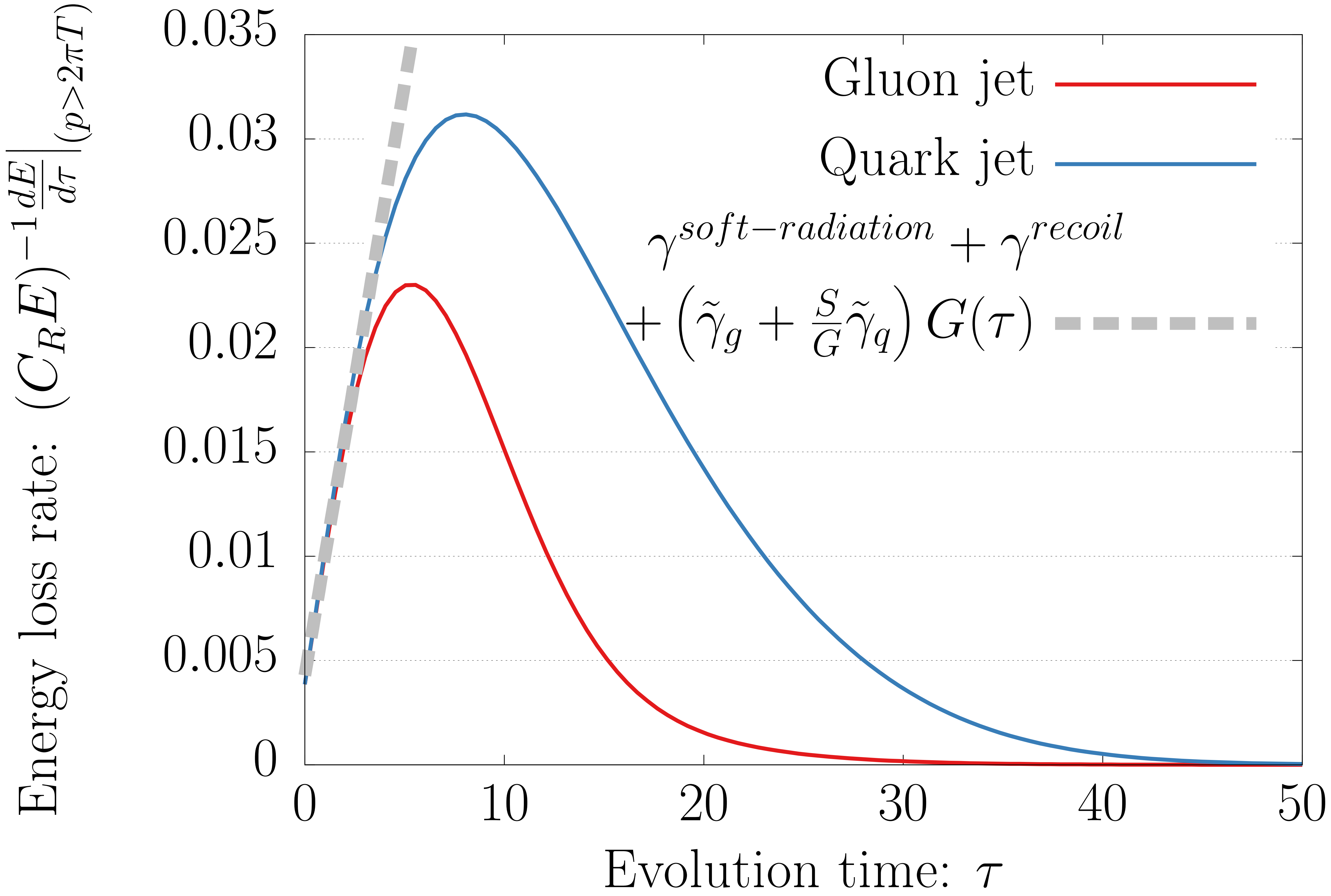
- Chemistry fixed by the Kolmogorov spectrum:

$$\frac{S}{G} = \frac{2N_f \int dz\ z\ \mathcal{K}_{qg}(z)}{\int dz\ z\ \mathcal{K}_{gq}(z)} \approx 0.07 \times 2N_f$$

Energy Cascade

- These estimates match with the early behavior of the energy loss rate

$$\frac{dE}{d\tau} = \gamma^{\text{soft}-\text{radiation}} + \gamma^{\text{recoil}} + \left(\tilde{\gamma}_g + \frac{S}{G}\tilde{\gamma}_q\right) G(\tau) \,,$$

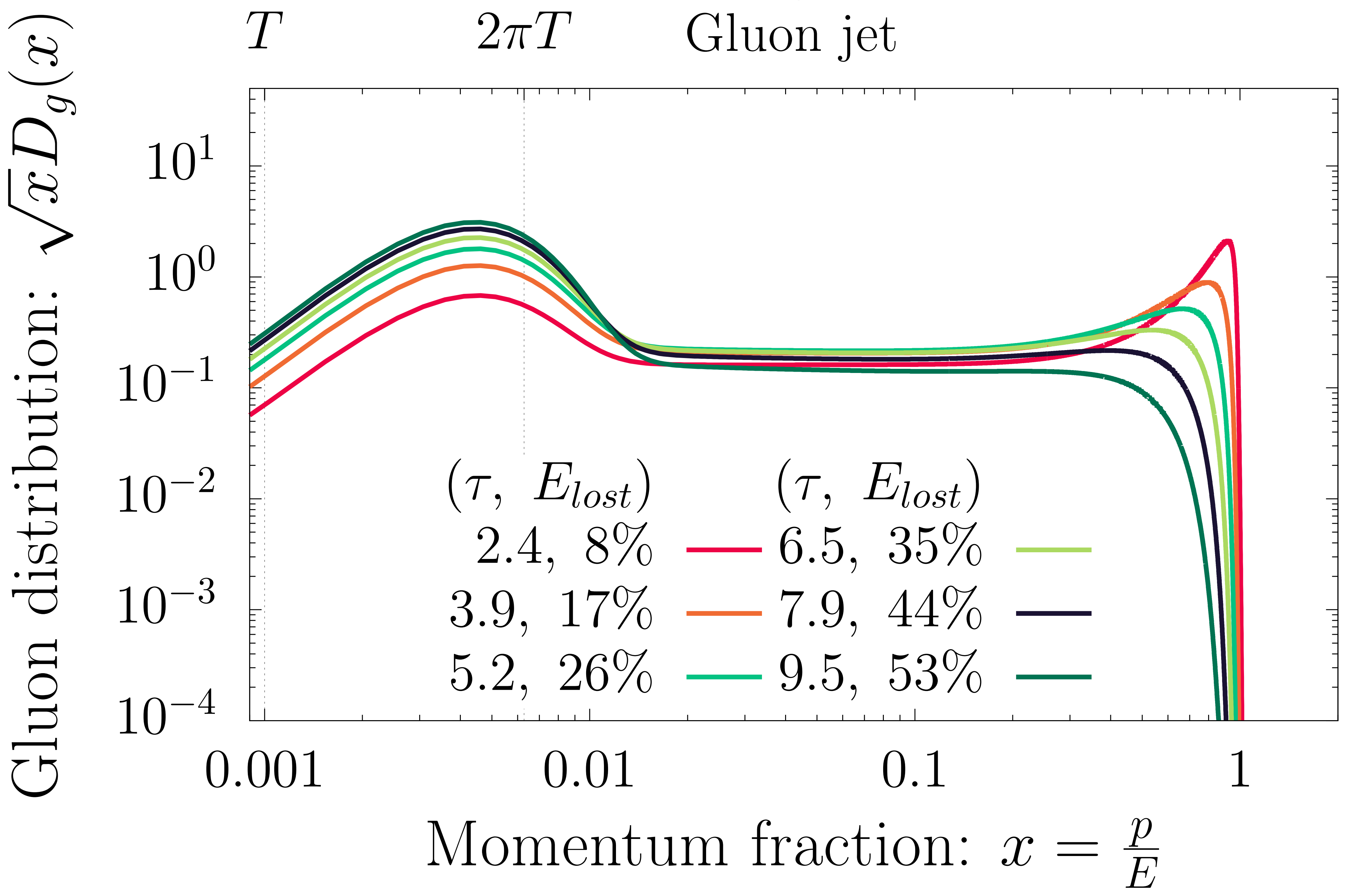


Turbulent behavior

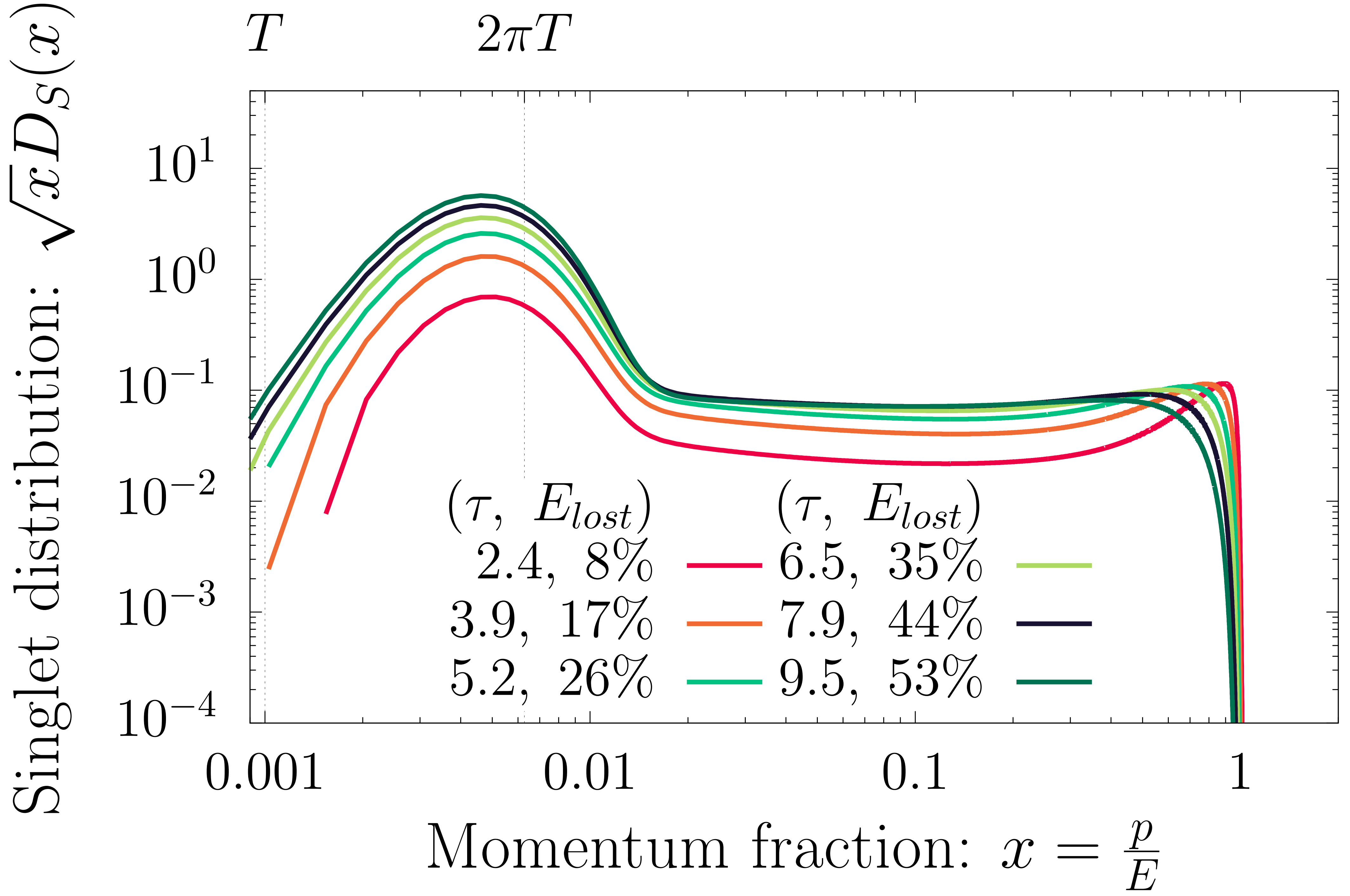
Initial Gluon Jet

$$\text{Singlet} = \frac{D_q(x) + D_{\bar{q}}(x)}{2}$$

Gluon Energy distribution



Singlet Energy distribution



- Characteristic $D(x) \sim \dfrac{1}{\sqrt{x}}$ behavior, associated with invariant energy flux.

[Mehtar-Tani et al. arXiv: 1807.06181]
[Blaizot et al. arXiv: 1301.6102]

Late Time Thermalization

Ultimately the jet equilibrate with the medium.

- We write the EoM as an eigenvalue problem

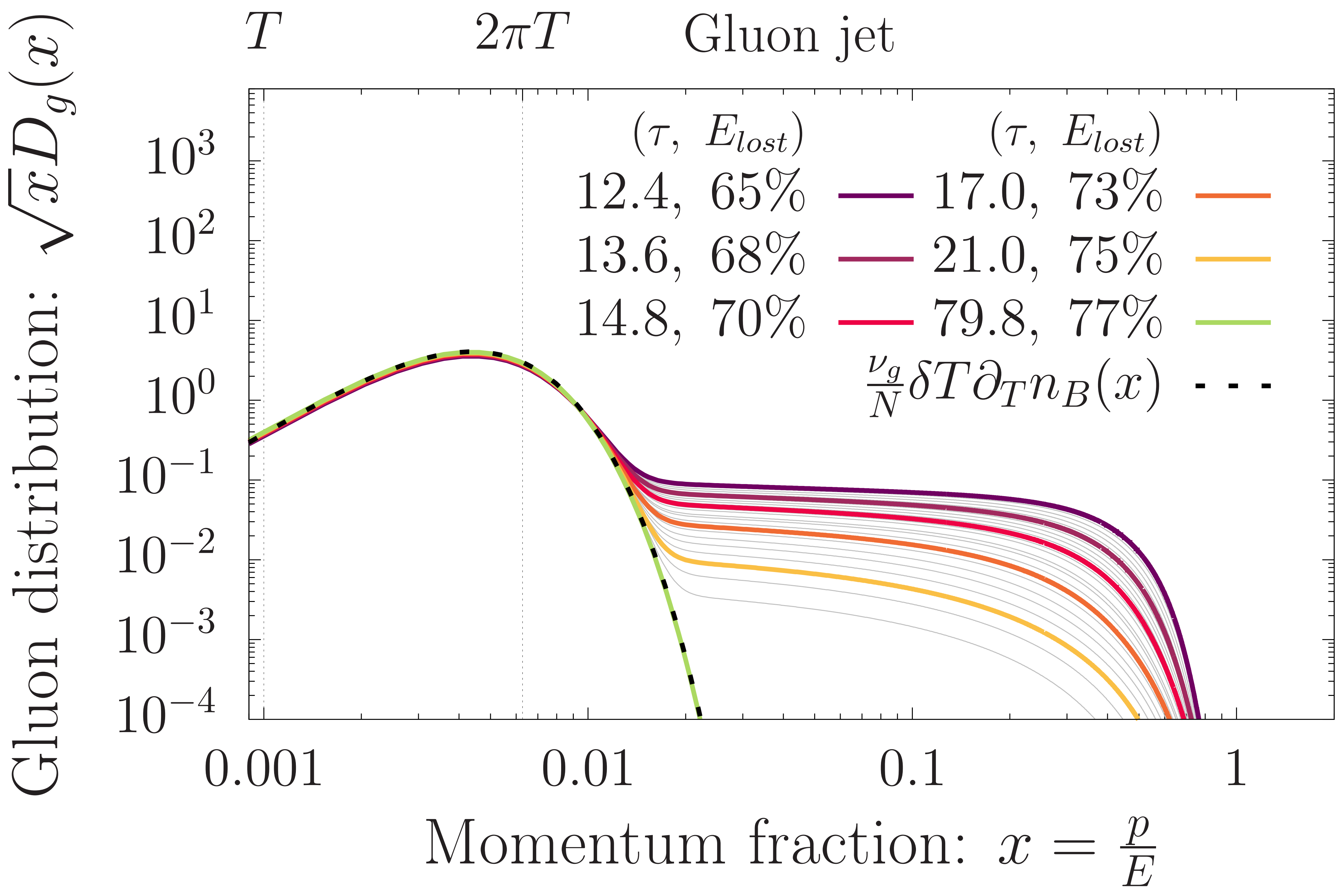
$$\partial_\tau D_i(x,\tau) = C[\{D_i\}] = \lambda_i D_i \,.$$

- Zero modes ($\lambda_0 = 0$) stems from conserved quantities (Energy / Valence charge) and its eigenvectors are the asymptotic behavior / stationary solution.

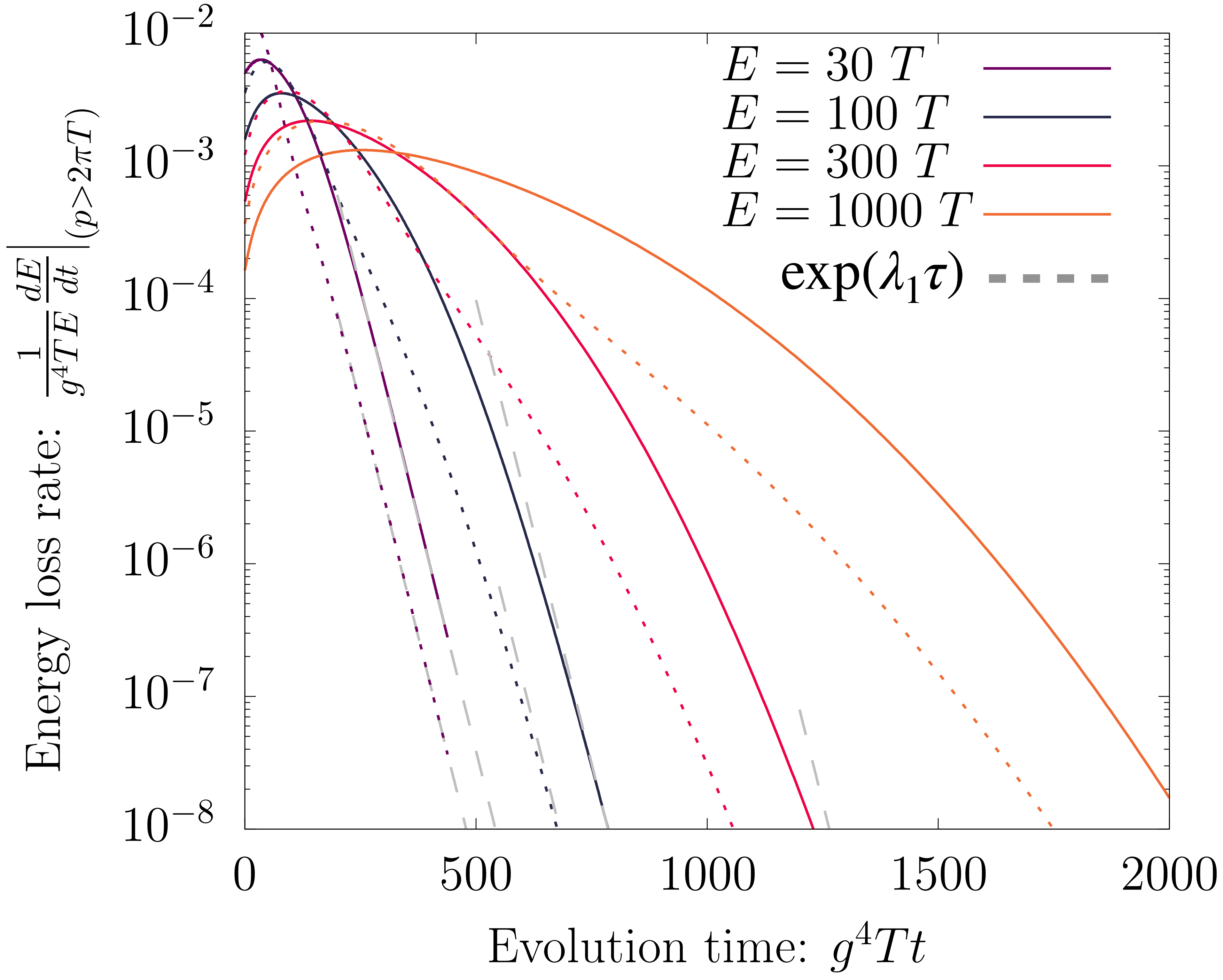
$$D(x,+\infty) = \delta T \partial_T n_{(Bose\ /\ Fermi)}(p;T)\,|_{p=xE_j}, \quad \text{and} \quad \delta\mu\partial_\mu n_{(Bose\ /\ Fermi)}(p;T)\,|_{p=xE_j}\,.$$

Late Time Thermalization

- The low-lying eigenvalues describe the equilibration at late times.



Energy Loss Rate



Angular Cascade

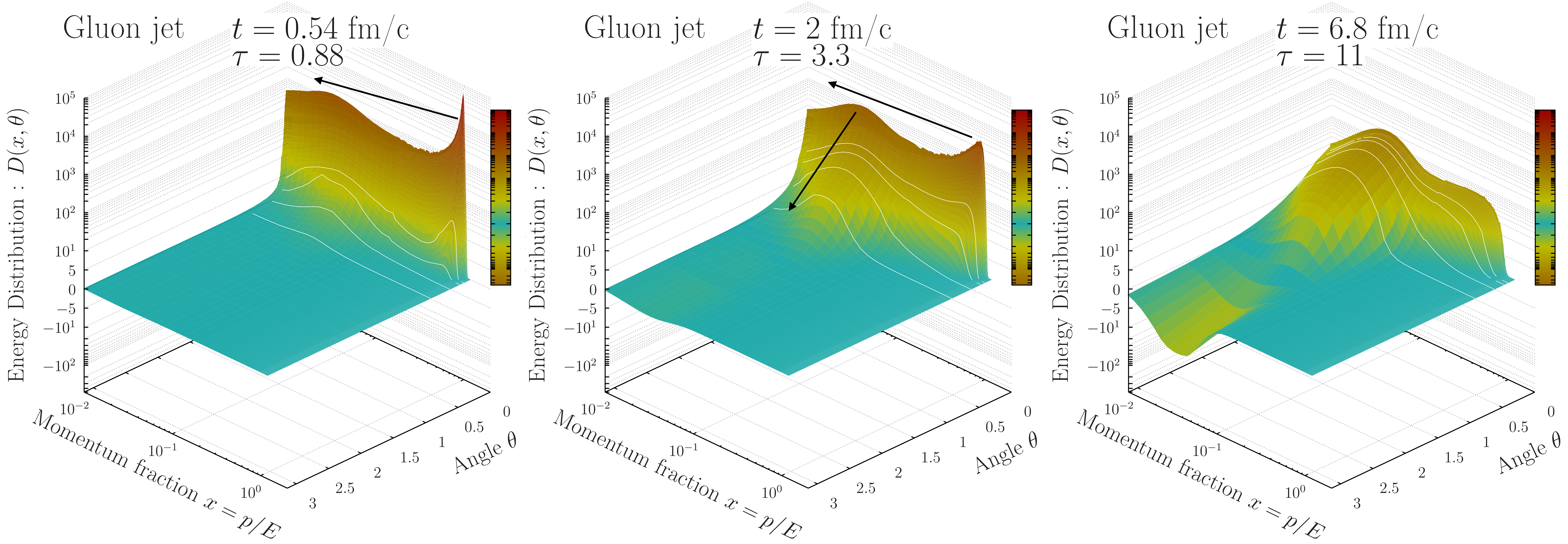
Based on: S. Schlichting, I.S. arXiv:2008.04928
S. Schlichting, I.S., Y. Mehtar-Tani work in progress

Energy Loss & Thermalization

- Collinear cascade

Jet energy $E_j = 100T$ and $g = 2$.

- Broadening of the soft fragments (p~T)

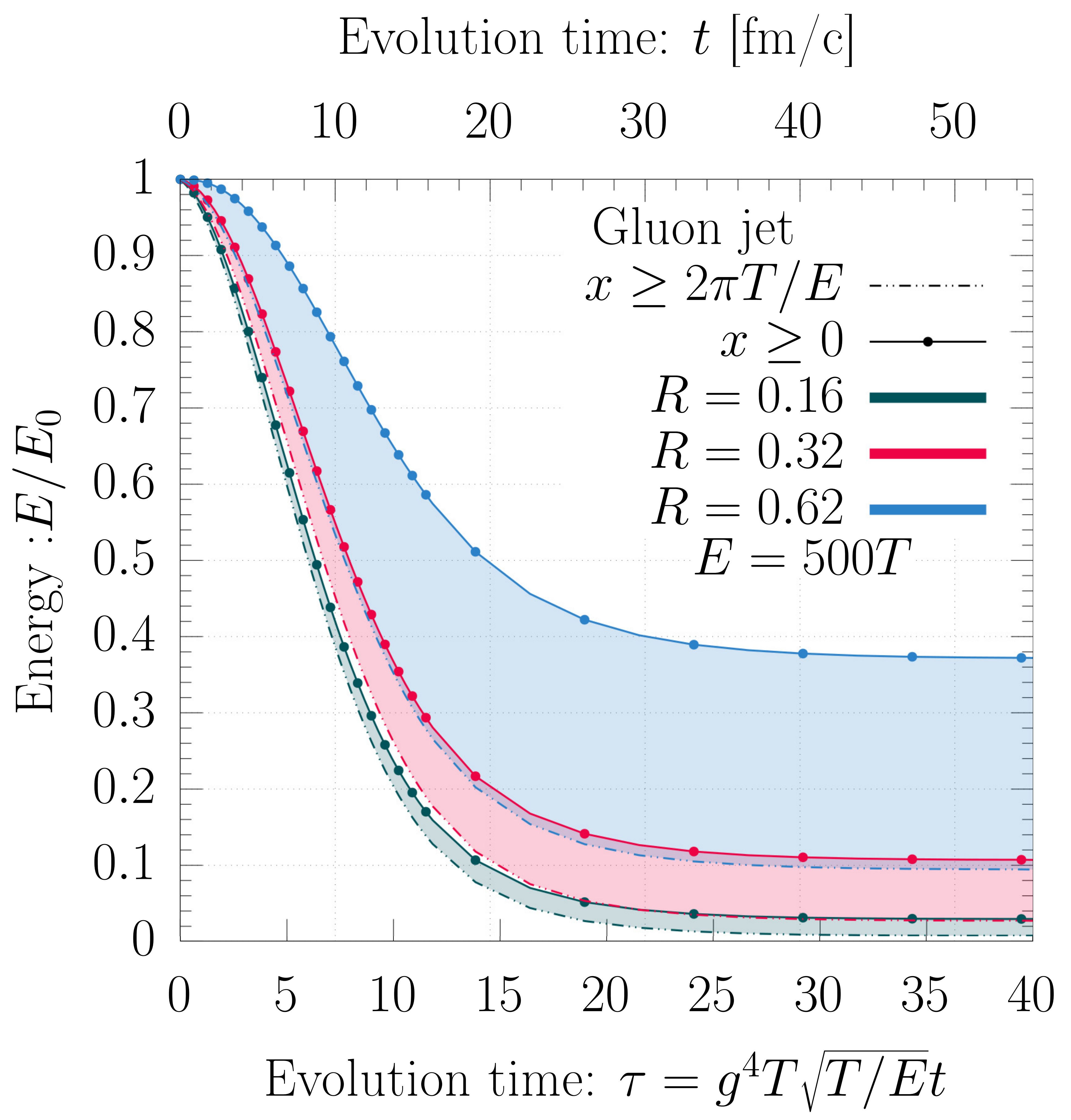


- Energy loss dominated by collinear branchings followed by thermalization of the soft sector
- Negligible broadening of hard particles; Energy loss out-of-cone mainly due to energy deposition in the soft sector

Energy Loss & Thermalization

$$E(R,\tau) = \int dx \int_{\cos R}^{1} D(x, \cos\theta, \tau)\ .$$

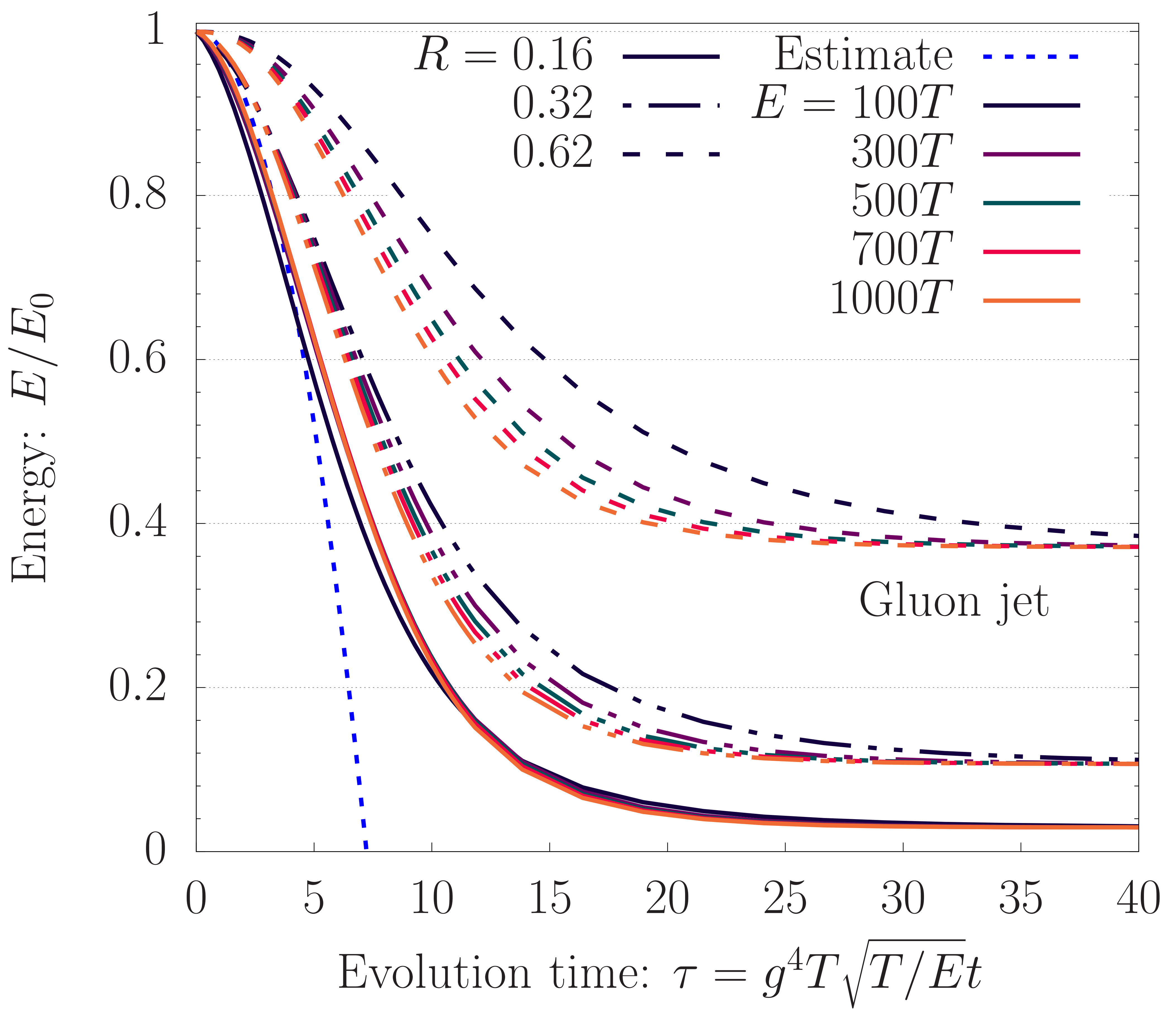
- Small cone-size: soft sector does not play a major role → similar energy loss in both momentum regions
- Larger cone-size: soft sector carries substantial fraction of the equilibrated energy at late times + early time energy loss diverges.



$$E_{2\pi}(R,\tau) = \int_{2\pi T/E}^{\infty} dx \int_{\cos R}^{1} D(x, \cos\theta, \tau)\ .$$

Sensitivity To The Initial Parton

- Characteristic time of the turbulent cascade is $t_{th} = \frac{1}{\alpha_s}\sqrt{\frac{E}{\hat{q}}}$ (time it takes a parton to thermalize)
- Small cone-sizes show a scaling between partons of different energies.
- W/ deviations for larger cone-sizes.



Leading Parton Quenching Factors

$$R^X_{\rm AA}(p_T, y, \phi) \equiv \frac{1}{N_{AA}} \frac{\frac{d^2N^X_{\rm AA}}{dp_T^2dy}}{\frac{d^2N^X_{\rm pp}}{dp_T^2dy}} ,$$

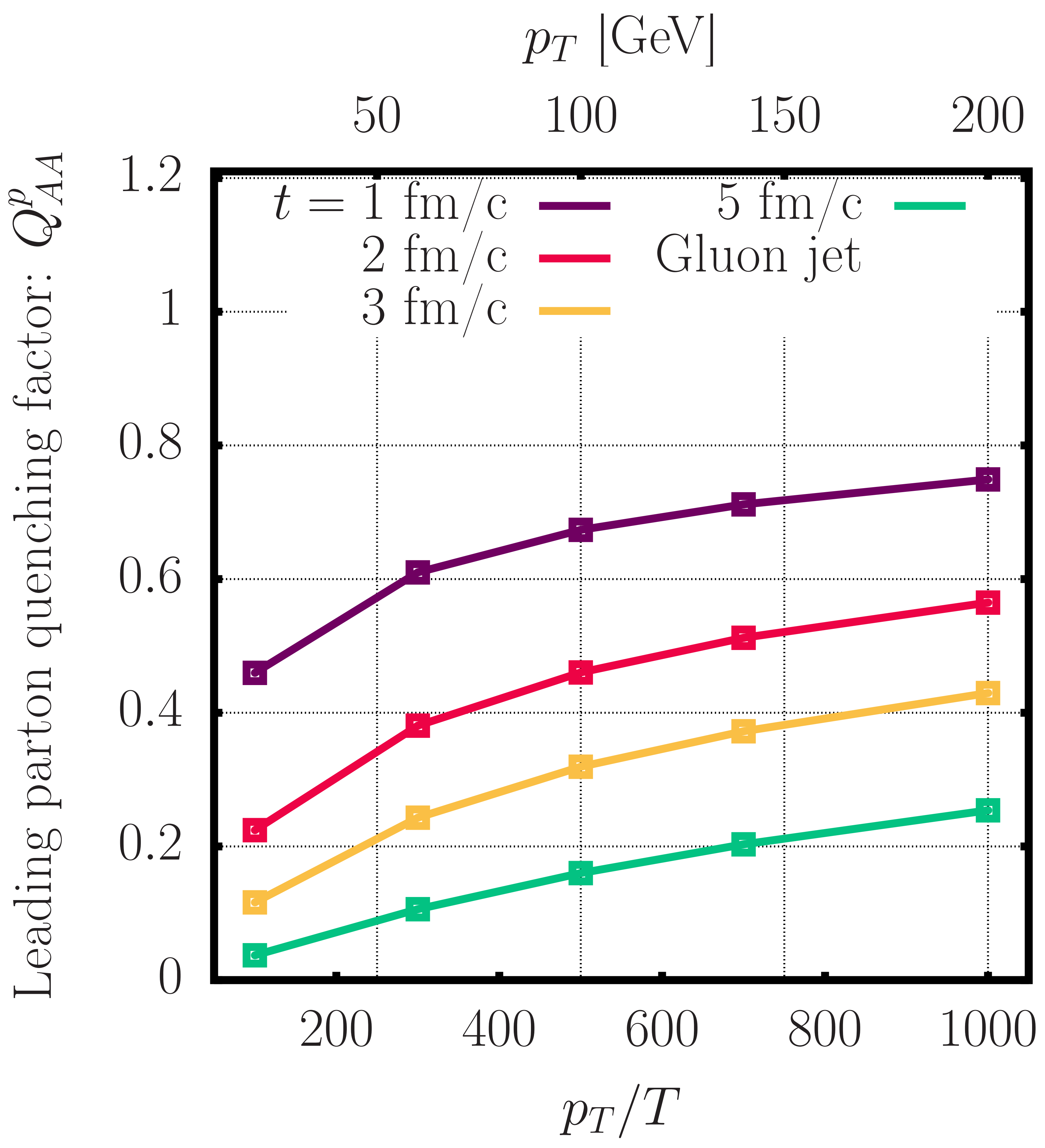
- Leading parton quenching can be modeled as a moment of the distribution

$$\frac{d^2\sigma_{AA}}{dp_T^2}(p_T) = \int_0^\infty \mathrm{d}^2 p_T^{in} \int_0^1 \frac{\mathrm{d}x}{x} \int_{-1}^1 \mathrm{d}\cos\theta \; \delta^2(p_T - x p_T^{in})$$
$$D\left(x, \theta, \tau \equiv g^4 T \sqrt{T/p_T^{in} t}\right) \frac{\mathrm{d}^2\sigma_0}{\mathrm{d}^2 p_T^{in}}(p_T^{in}) ,$$

[R. Baier et al. arXiv:0106347]

$$Q^{\rm p}_{AA}(p_T) = \frac{\frac{d^2\sigma_{AA}}{\mathrm{d}p_T^2}}{\frac{\mathrm{d}^2\sigma_0}{\mathrm{d}p_T^2}} = \int_0^1 \mathrm{d}x \int_{-1}^1 \mathrm{d}\cos\theta \; D\left(x, \theta, g^4 T\sqrt{xT/p_T t}\right) \left(\frac{1}{x}\right)^{2-n} .$$

- Hadron quenching only sensitive to hard constituents, i.e. collinear cascade => in-medium splittings



Modeling Jet Quenching

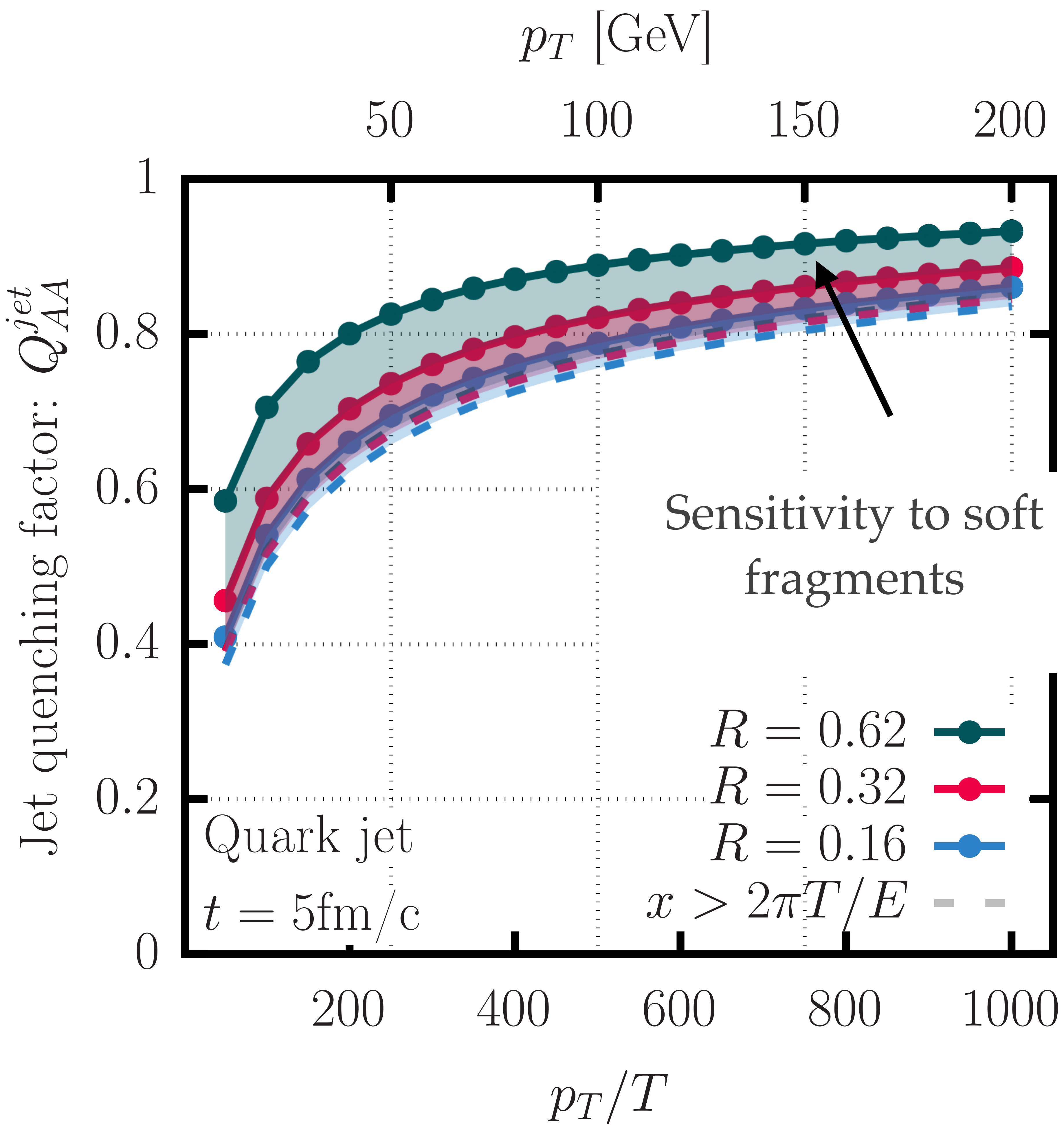
- We capture the first emission using the BDMPS finite medium rate $\frac{d\Gamma}{d\omega}(P,\omega,t)$
- Model medium energy loss by computing the energy remaining inside the cone $E(\omega,R,L-t)$ after a time $(L-t)$

$$Q(p_T) = \exp\left[\int_0^L \mathrm{d}t \int \mathrm{d}\omega \frac{\mathrm{d}\Gamma}{\mathrm{d}\omega}\left(1-\mathrm{e}^{-n\frac{\omega}{p_T}\left[1-E\left(\omega,R,\tau=\frac{L-t}{t_{\mathrm{th}}}\right)\right]}\right)\right].$$

[Y. Mehtar-Tani, & K. Tywoniuk arXiv: 1707.07361]

- Jet quenching recovers energy from the soft sector for large cone size => medium response
- Energy loss currently over-estimated due to neglecting finite size effects on medium-induced emission rates (work in progress)

$$\frac{\mathrm{d}\sigma_{\mathrm{vac}}(p_T+\epsilon)}{\mathrm{d}p_T^2\,\mathrm{d}y} \approx \frac{\mathrm{d}\sigma_{\mathrm{vac}}}{\mathrm{d}p_T^2\,\mathrm{d}y}\exp\left(-\frac{n\epsilon}{p_T}\right),$$



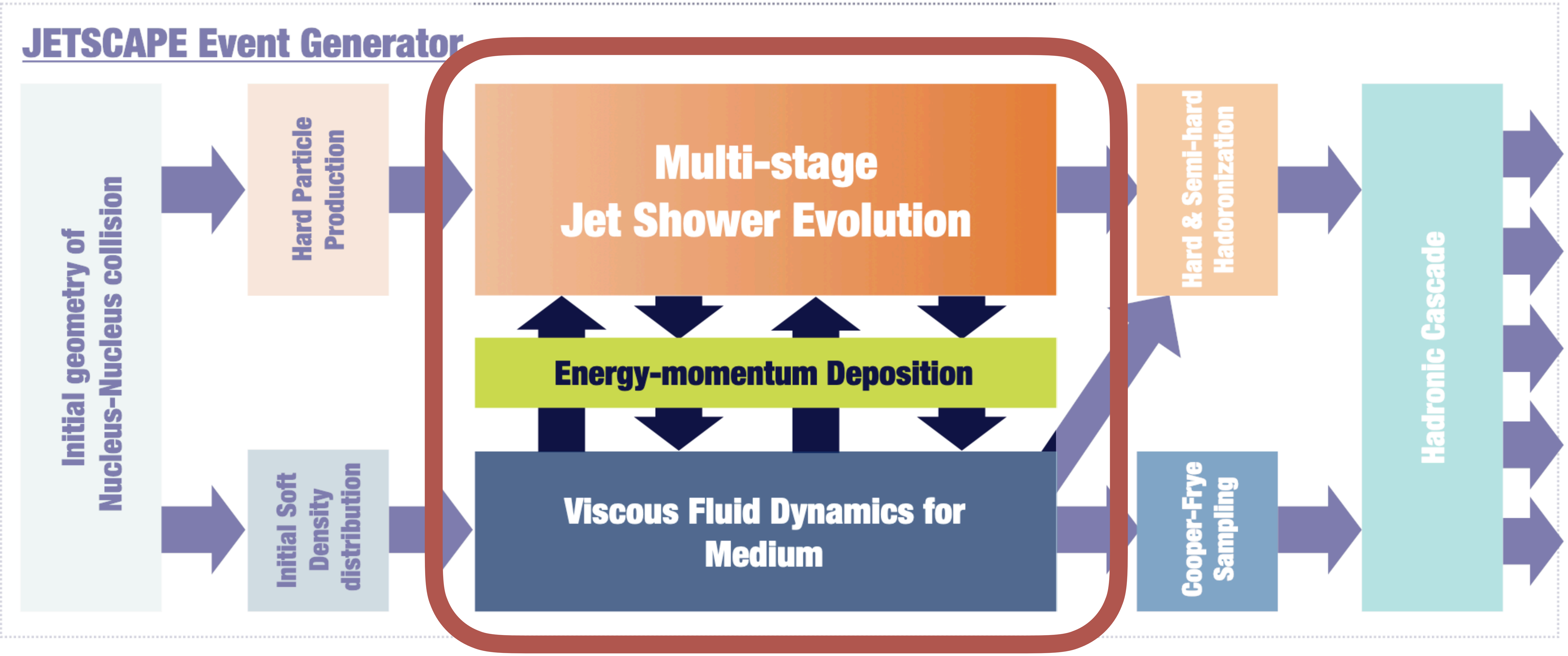
Conclusion I

- Mechanisms underlying energy loss similar to QGP thermalization → low energetic partons ($E \lesssim 30T$) more sensitive to the medium scale
- High energy distribution stays collinear → energy at large angles ($\theta > 0.2$) is mainly sensitive to soft scales
- Observables sensitive to large angle effects => Require a good understanding of medium response
- Energy deposition in the medium will need a non-perturbative approach

Multi-stage Framework

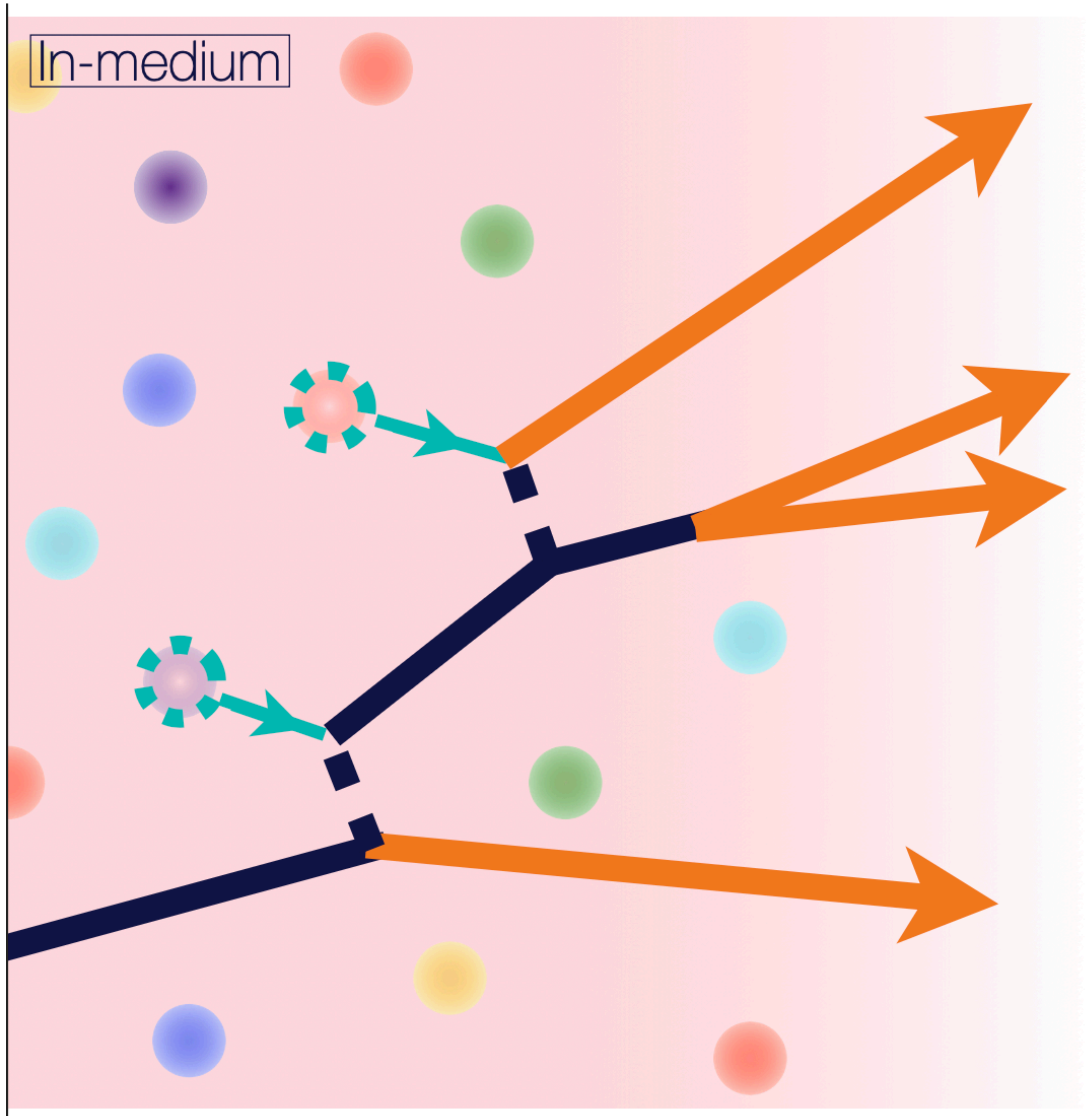
Based on slides from Y. Tachibana

JETSCAPE Framework



- How to connect the Multi-stage jet shower to the fluid dynamics of the medium

Weakly-Coupled Jet-Medium Interactions



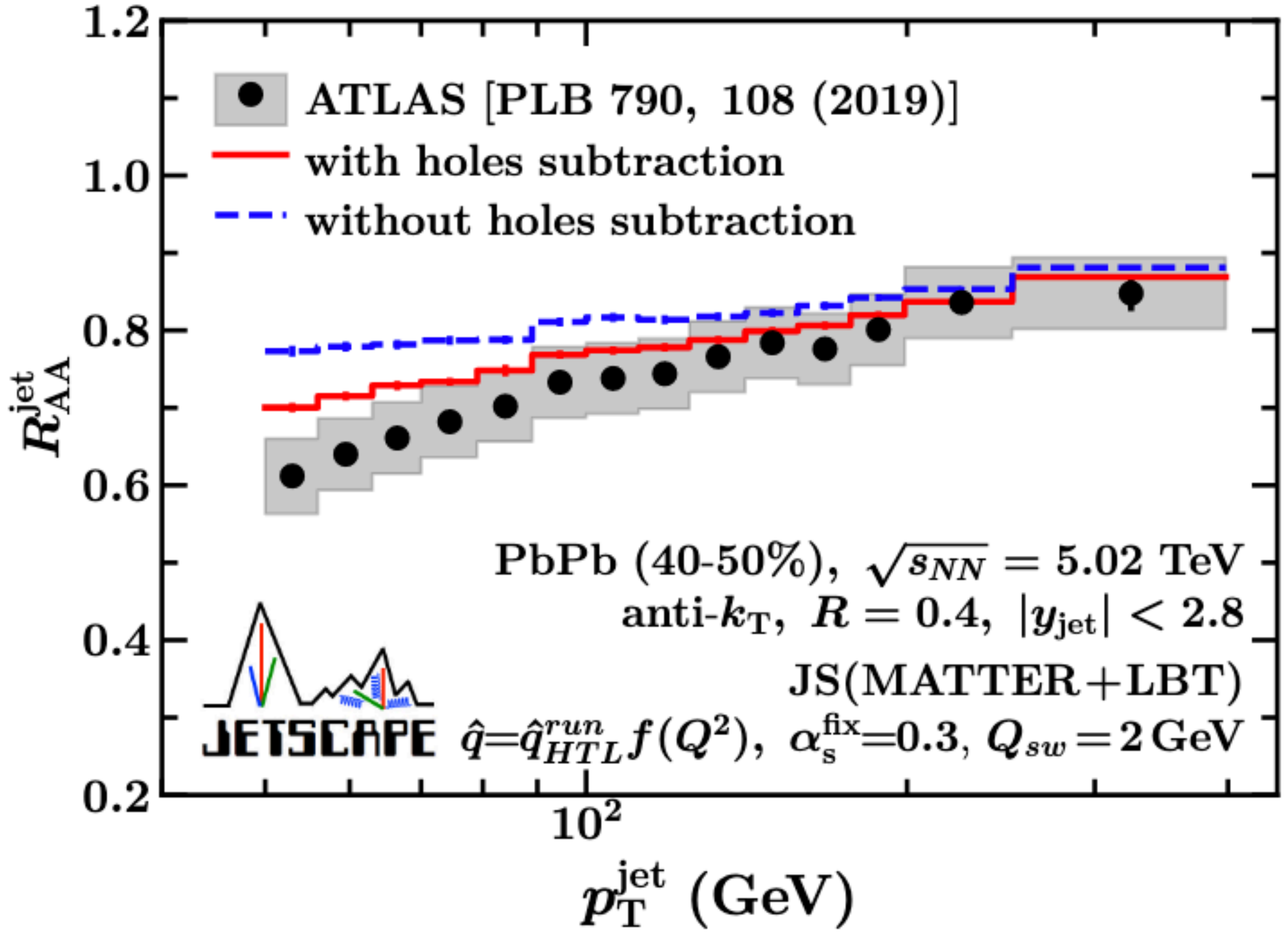
- In 2<->2 scatterings :
- A parton sampled from the medium => Hole (negative parton)
- Recoiling parton + holes
- What if the energy of the parton is $\sim E_{\mathrm{med}}$

$$\left.\frac{dp^{\mu}}{d\eta d\phi}\right|_{\mathrm{signal}} = \left.\frac{dp^{\mu}}{d\eta d\phi}\right|_{\mathrm{shower}} - \left.\frac{dp^{\mu}}{d\eta d\phi}\right|_{hole}$$

JETSCAPE Results: Weak-Coupling Method

- So far JETSCAPE has results for the perturbative method of recoil-hole formalism
- Hole subtraction describes the suppression

$$p^{\mu}_{\text{jet}} = p^{\mu}_{\text{shower}} - \sum_{\substack{i \in \text{holes} \\ \Delta r_i < R}} p^{\mu}_i .$$



[JETSCAPE arXiv: 2204.01163]

Hydrodynamics

- Bulk dynamics are described by Hydro Eq.

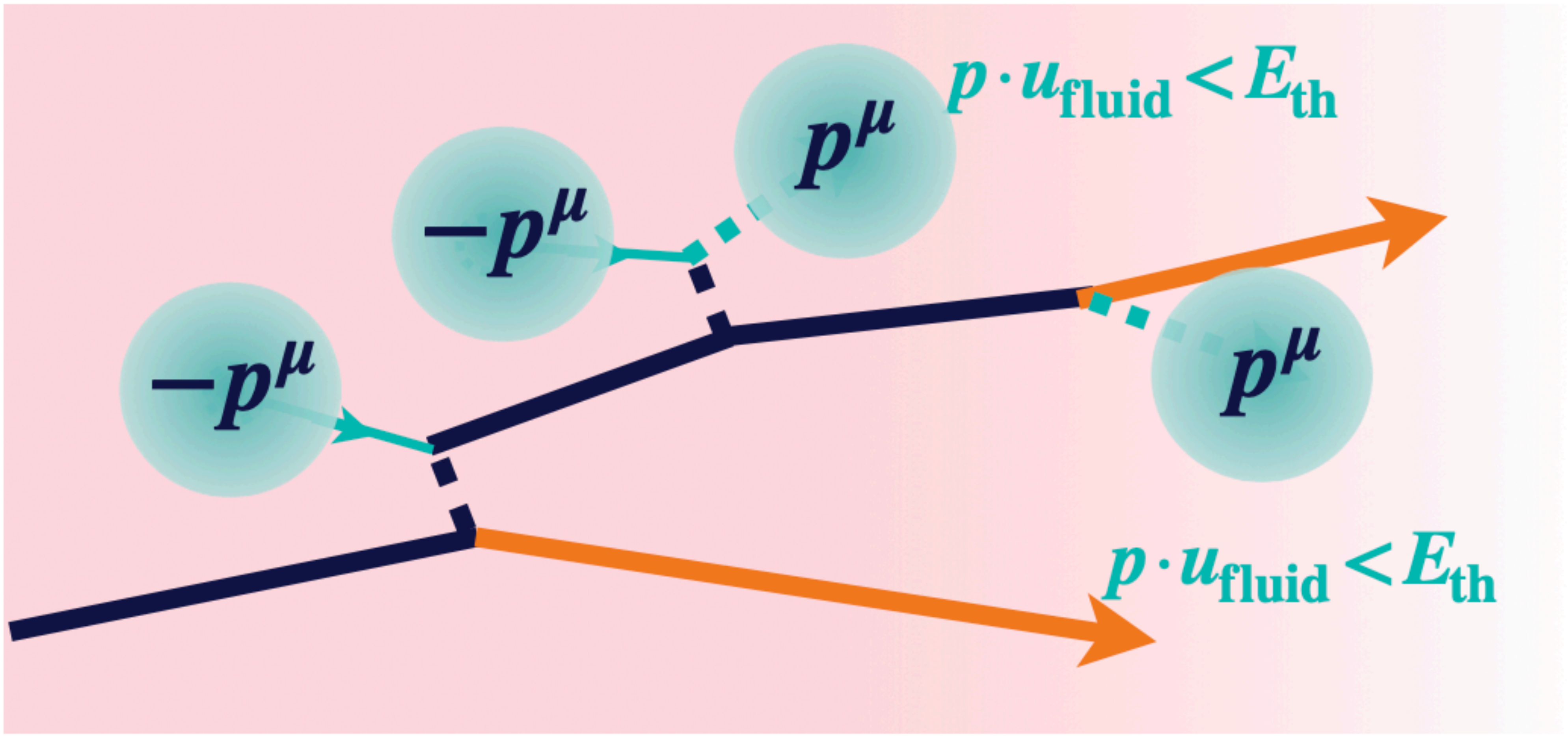
$$\nabla_\mu T^{\mu\nu}_{\text{med}}(x) = 0 \ ,$$

- Jet shower can included as a source term :

$$\nabla_\mu T^{\mu\nu}_{\text{med}}(x) = J^{\nu}_{\text{jet}}(x) \ ,$$

- However, direct deposition will be too narrow to be studied by Hydro
- Energy deposition into the medium modeled using Causal Diffusion (Liquefier) in JETSCAPE)

Causal Diffusion

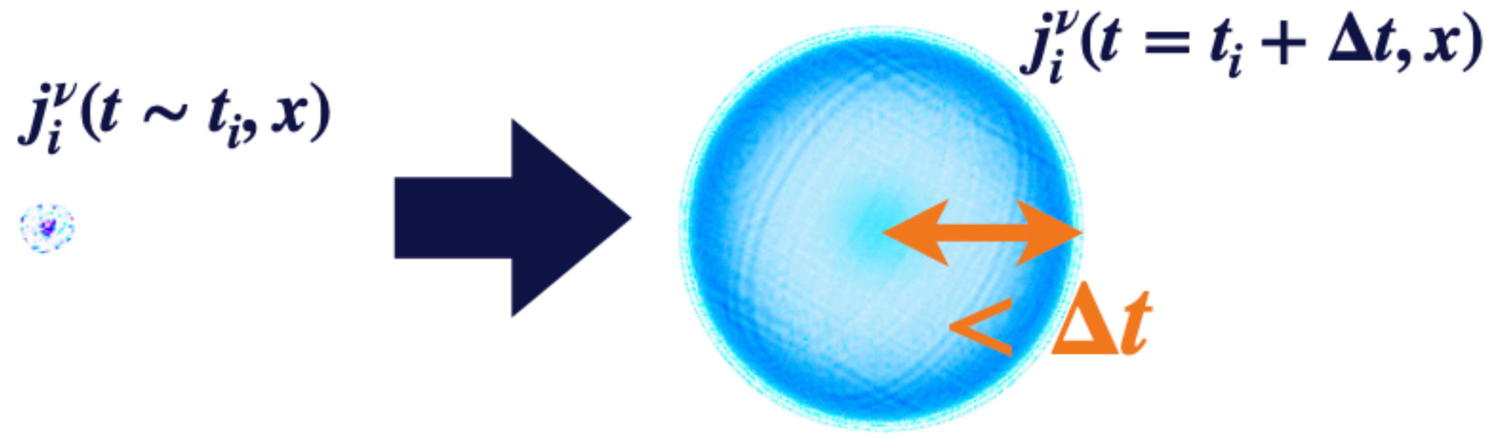


- Partons with energy $p \cdot u_{\text{fluid}} < E_{\text{th}}$ are sent to the "Liquefier" and diffused in position space

$$\left[\frac{\partial}{\partial t} + \tau_{\text{relax}} \frac{\partial^2}{\partial t^2} - D_{\text{diff}} \nabla^2\right] j_i^\nu(x) = 0,$$

- With initial condition:

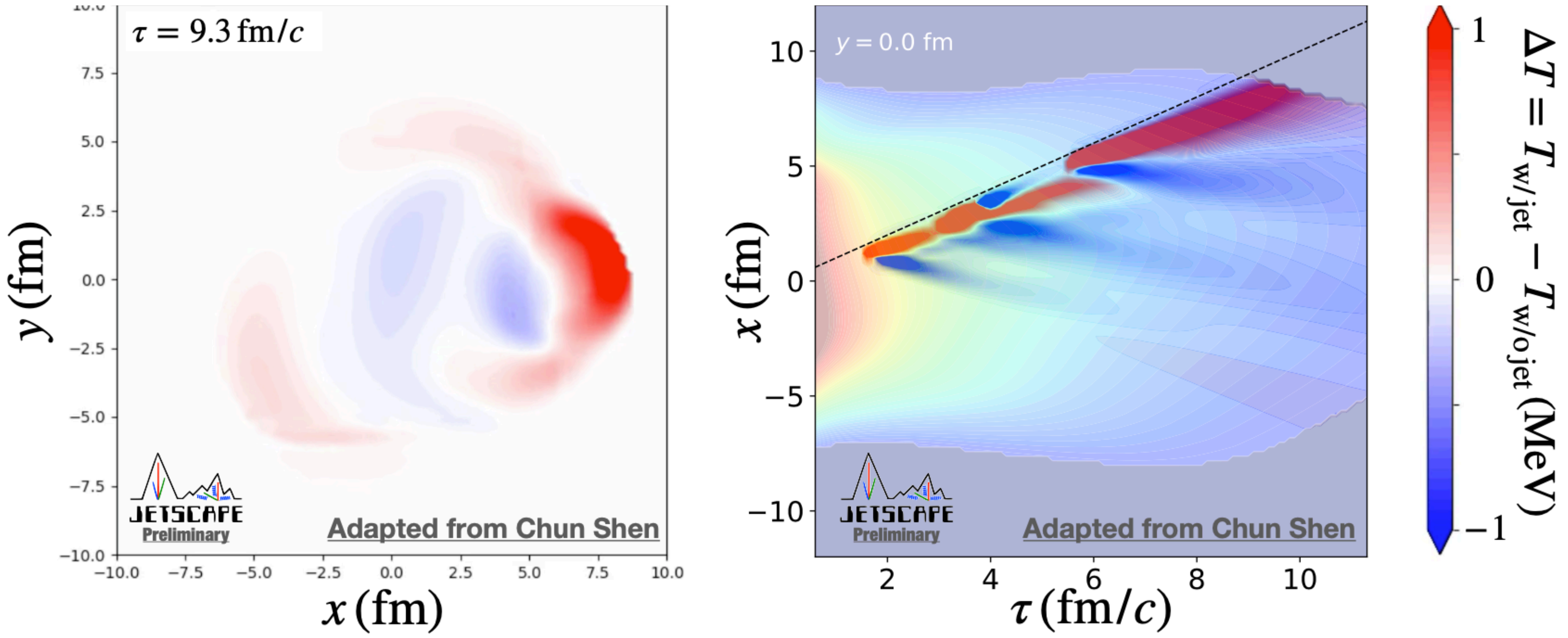
$$j_i^\nu = \pm_i \, p_i^\nu \delta^{(3)}(\vec{x} - \vec{x}_i^{\text{dep}})$$



Causal Diffusion

- Modification of the temperature profile due to the Jet propagation
- => Diffusion wake and loss of momentum due to partons kicked in the jet direction

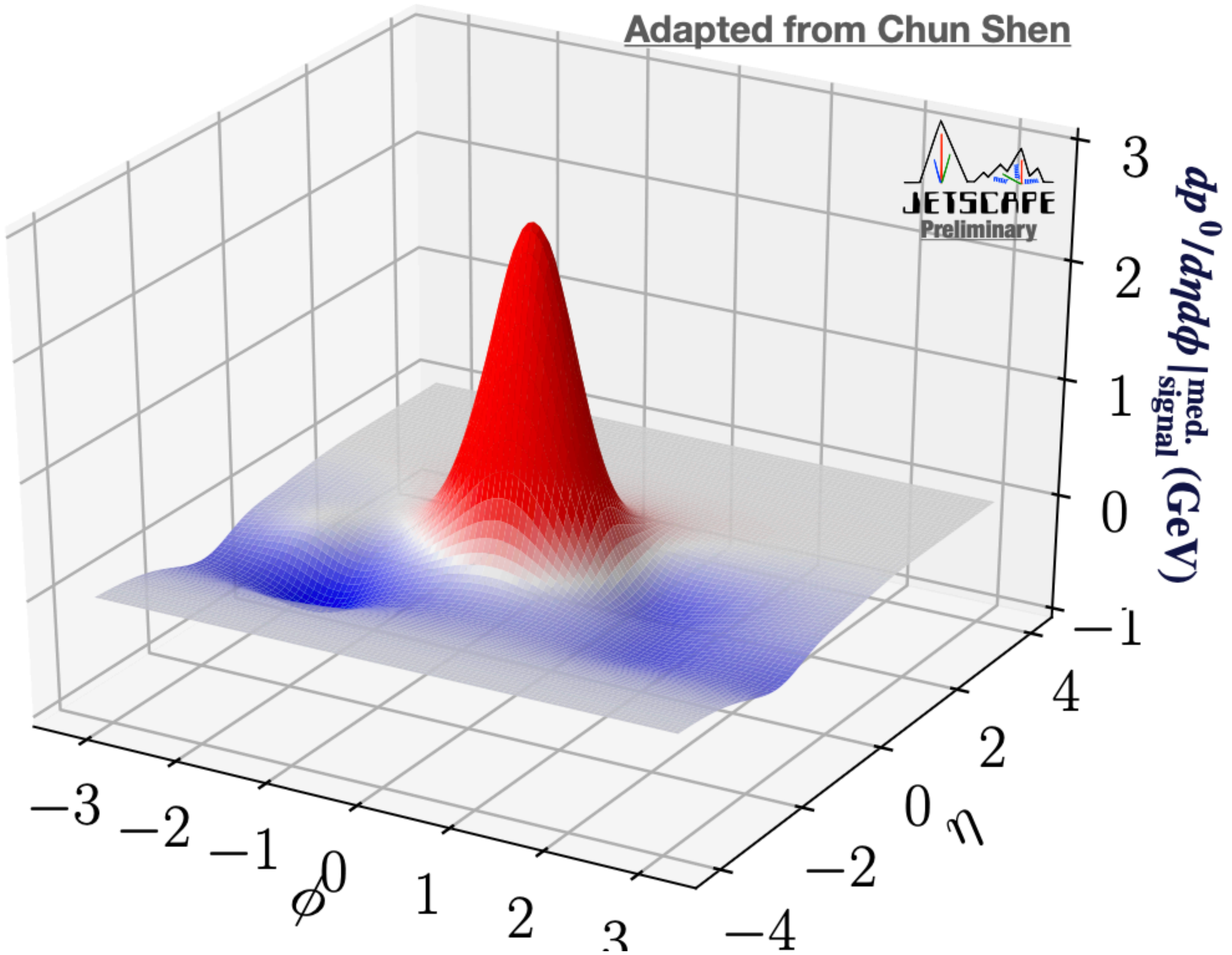
Matter+LBT + Causal Diffusion + Viscous Hydyo



Causal Diffusion

- Modification of the momentum due to the hard partons

Matter+LBT + Causal Diffusion + Viscous Hydyo



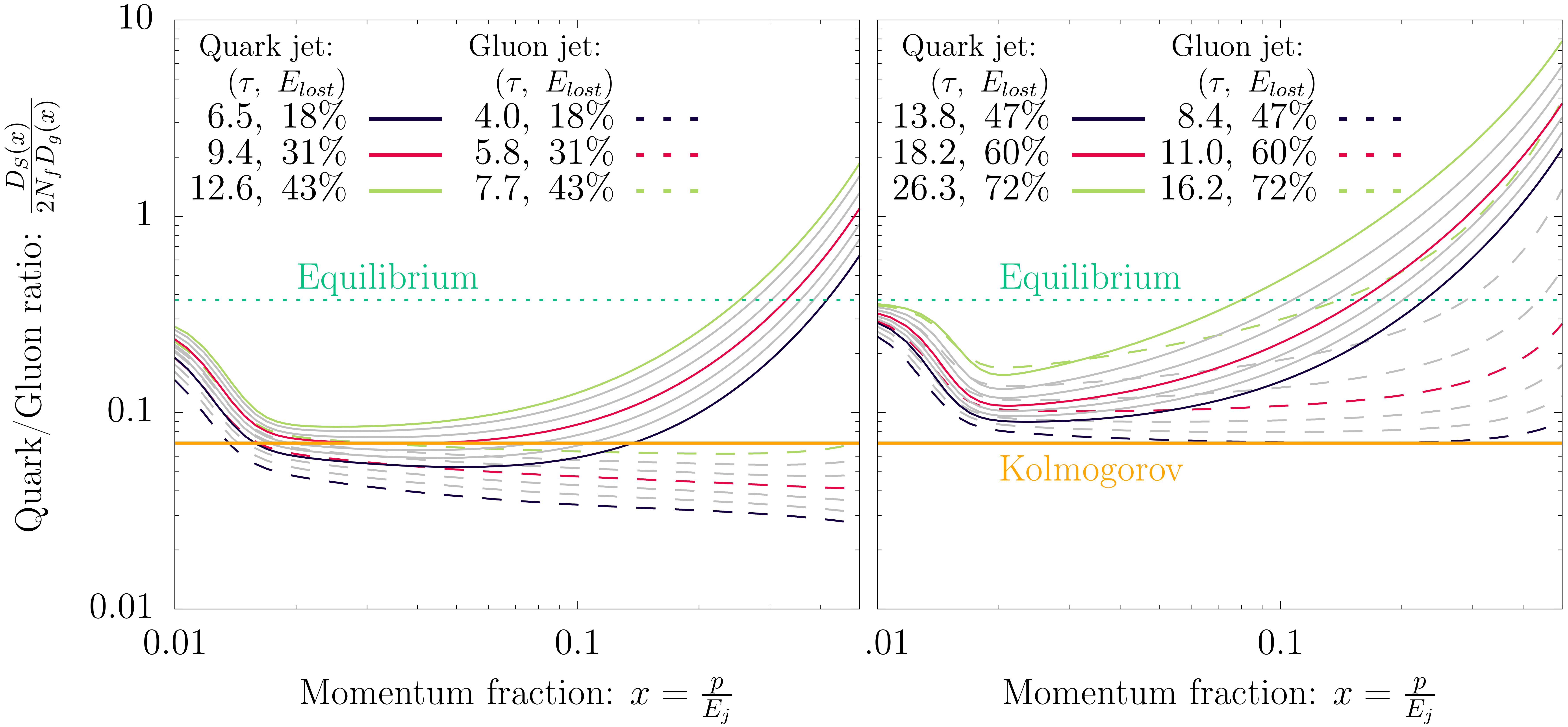
Conclusion II

- Medium response is important for the study of low pT partons -> something to look for at sPHENIX
- Different methods to study medium response:
  - Kinetic method is interesting theoretically but not realistic yet, needs improvements
  - Weakly coupled method is pushing perturbative method out of range of validity
  - 2-stage Hydro is demanding numerically to obtain large statistics

Backup

Jet Chemistry

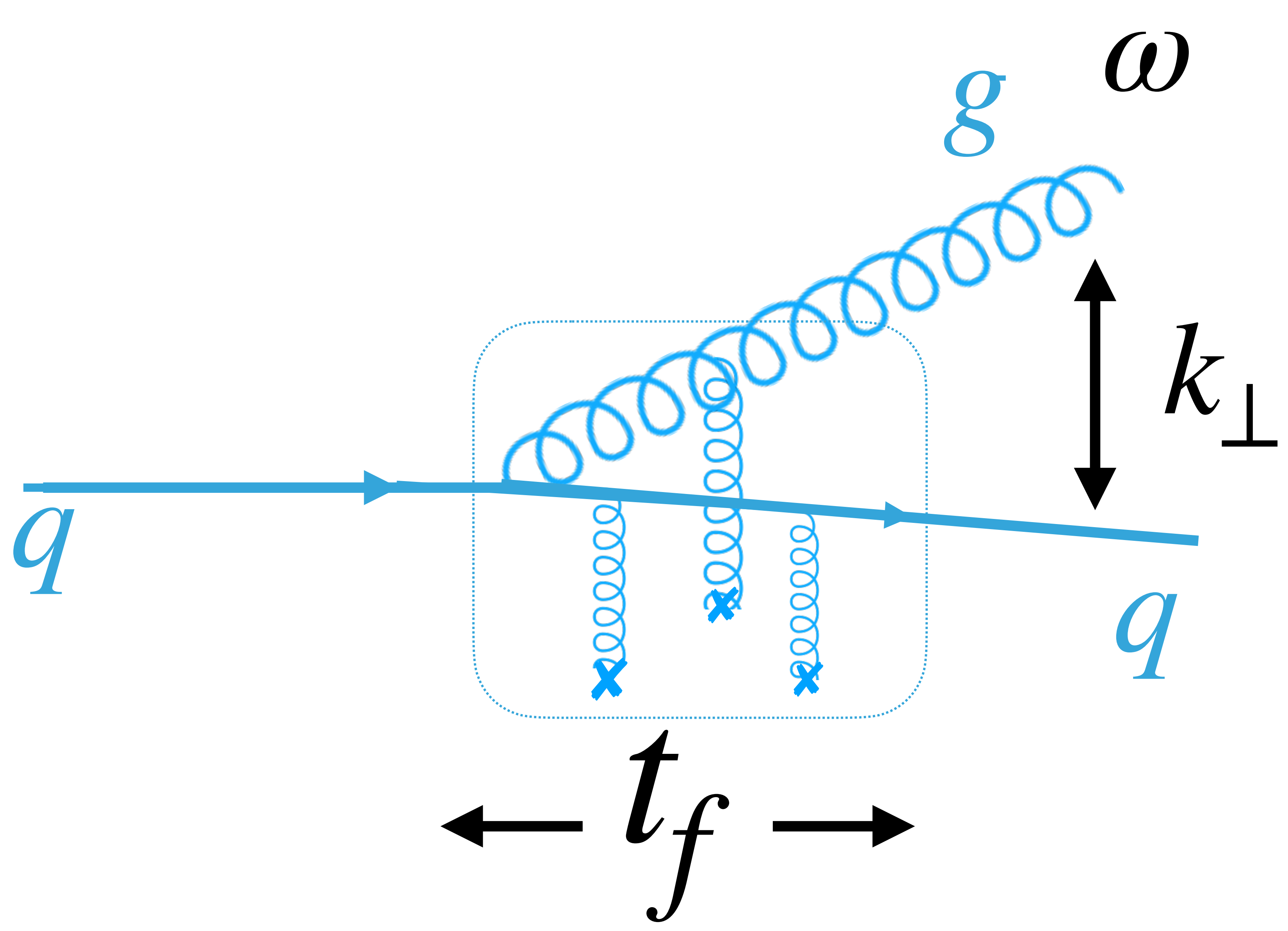
- Strongly quenched jets are quark rich



$x \sim T/E$ Thermal | $T/E \ll x \ll 1$ non-thermal (Kolmogorov) | $x \sim 1$ Jet core

Landau-Pomeranchuck-Migdal (LPM) effect

- Multiple soft scatterings with the medium kick the parton slightly off-shell → leading to radiation of a gluon (ω,**k**)
- $t_f \ll \lambda_{\mathrm{mfp}}$: the medium cannot resolve the quanta until it's formed
- $t_f \gg \lambda_{\mathrm{mfp}}$: multiple soft scatterings with the medium act coherently leading to interference effects that has to be resummed



$$t_f \sim \frac{2\omega}{k_\perp^2}$$

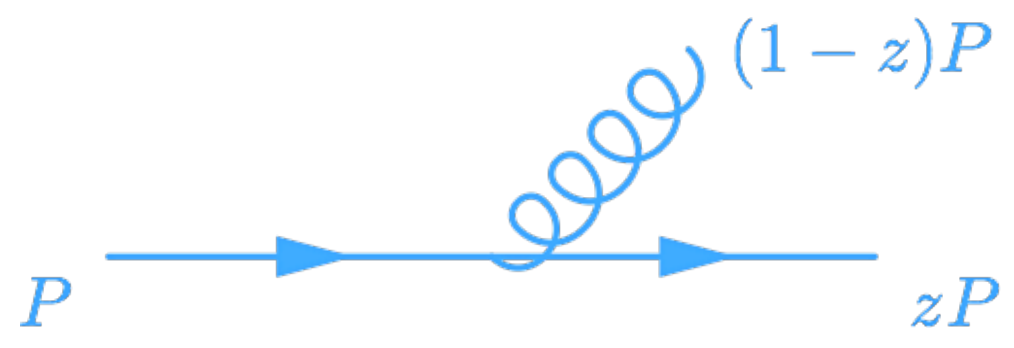
$$k_\perp \sim \hat{q} t_f$$

$$\rightarrow \quad t_f(\omega) = \sqrt{\frac{2\omega}{\hat{q}}}$$

Collinear Radiation

- In-medium radiation rates given by

$$\frac{d\Gamma^a_{bc}(p,z)}{dz} = \frac{\alpha_s P_{bc}(z)}{[2Pz(1-z)]^2} \int \frac{d^2\boldsymbol{p}_b}{(2\pi)^2} \,\mathrm{Re}\left[2\mathbf{p}_b \cdot \mathbf{g}_{(z,P)}(\mathbf{p}_b)\right] \;,$$



- where the g fct solves

$$2\mathbf{p}_b = i\delta E(z,P,\mathbf{p}_b)\mathbf{g}_{(z,P)}(\mathbf{p}_b) + \int \frac{d^2q}{(2\pi)^2}\, \bar{C}(\boldsymbol{q})\; \Big\{C_1\left[\mathbf{g}_{(z,P)}(\mathbf{p}_b) - \mathbf{g}_{(z,P)}(\mathbf{p}_b - \mathbf{q})\right] + C_z\left[\mathbf{g}_{(z,P)}(\mathbf{p}_b) - \mathbf{g}_{(z,P)}(\mathbf{p}_b - z\mathbf{q})\right] + C_{1-z}\left[\mathbf{g}_{(z,P)}(\mathbf{p}_b) - \mathbf{g}_{(z,P)}(\mathbf{p}_b - (1-z)\mathbf{q})\right]\Big\} \;,$$

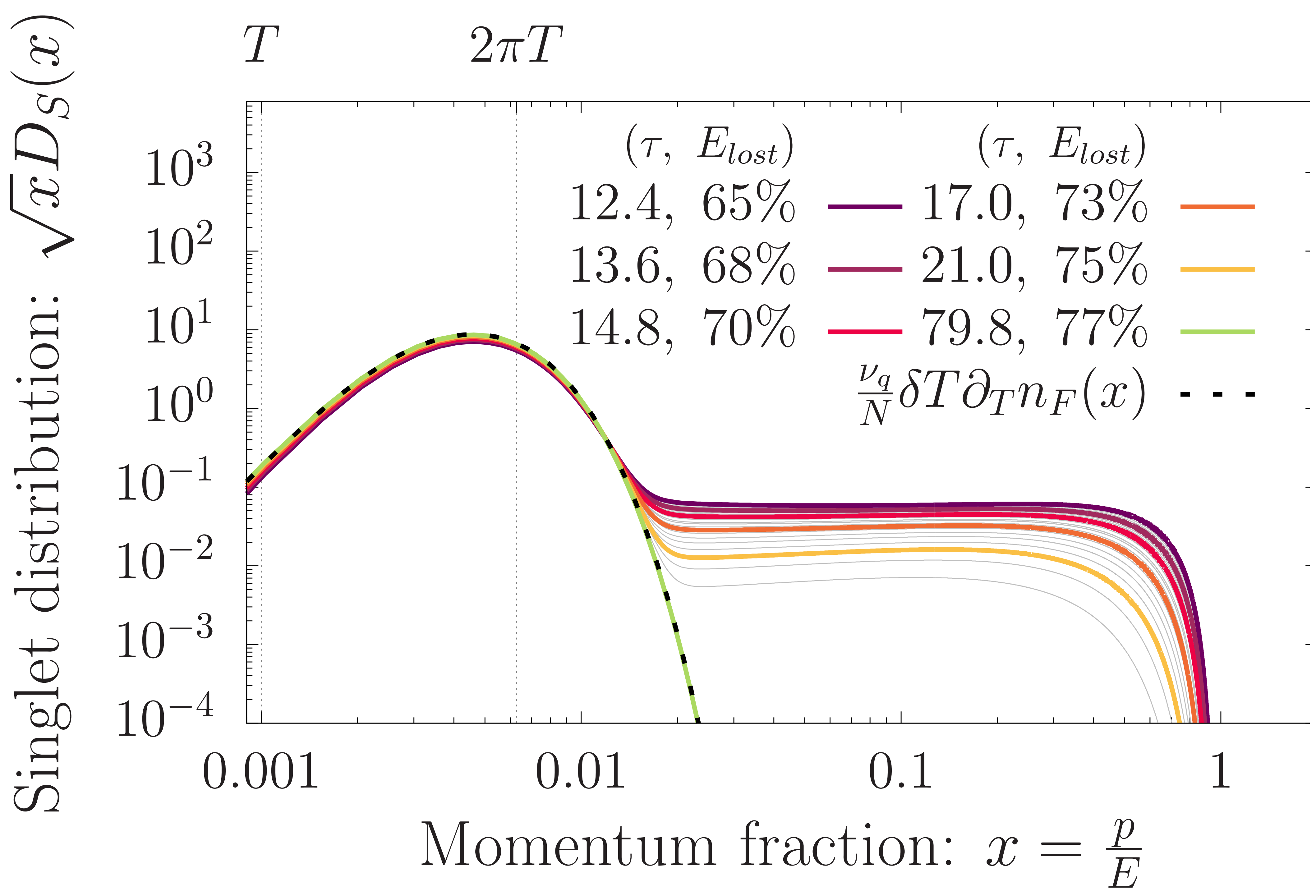
- Elastic scatterings are described using the broadening kernel

$$\bar{C}(\boldsymbol{q}) = \frac{g^2 T m_D^2}{q^2(q^2+m_D^2)} \;.$$

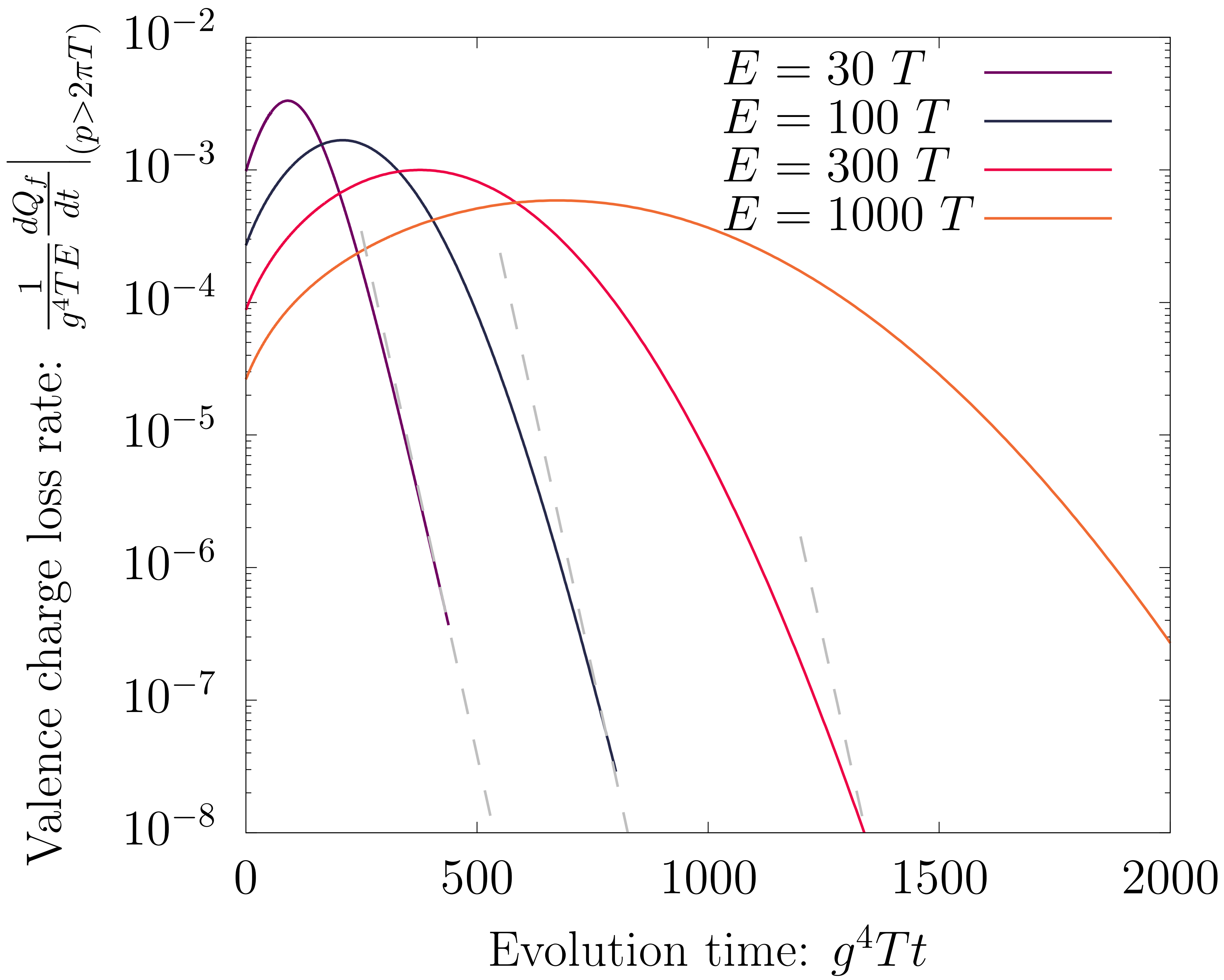
$$C_a^{1\leftrightarrow 2}[\{f_i\}] = \sum_{bc}\Bigg\{ -\frac{1}{2}\int_0^1 dz \frac{d\Gamma^a_{bc}(\boldsymbol{p},z)}{dz}\Big[f_a(\boldsymbol{p})(1\pm f_b(z\boldsymbol{p}))(1\pm f_c(\bar{z}\boldsymbol{p})) - f_b(z\boldsymbol{p})f_c(\bar{z}p)(1\pm f_a(\boldsymbol{p}))\Big] + \frac{\nu_b}{\nu_a}\int_0^1 \frac{dz}{z^3}\frac{d\Gamma^b_{ac}(\frac{\boldsymbol{p}}{z},z)}{dz}\Big[f_b\left(\frac{\boldsymbol{p}}{z}\right)(1\pm f_a(\boldsymbol{p}))\left(1\pm f_c\left(\frac{\bar{z}}{z}\boldsymbol{p}\right)\right) - f_a(\boldsymbol{p})f_c\left(\frac{\bar{z}}{z}\boldsymbol{p}\right)\left(1\pm f_b\left(\frac{\boldsymbol{p}}{z}\right)\right)\Big]\Bigg\} \;,$$

Late Time Thermalization

Energy Loss Rate



Valence Charge Loss Rate

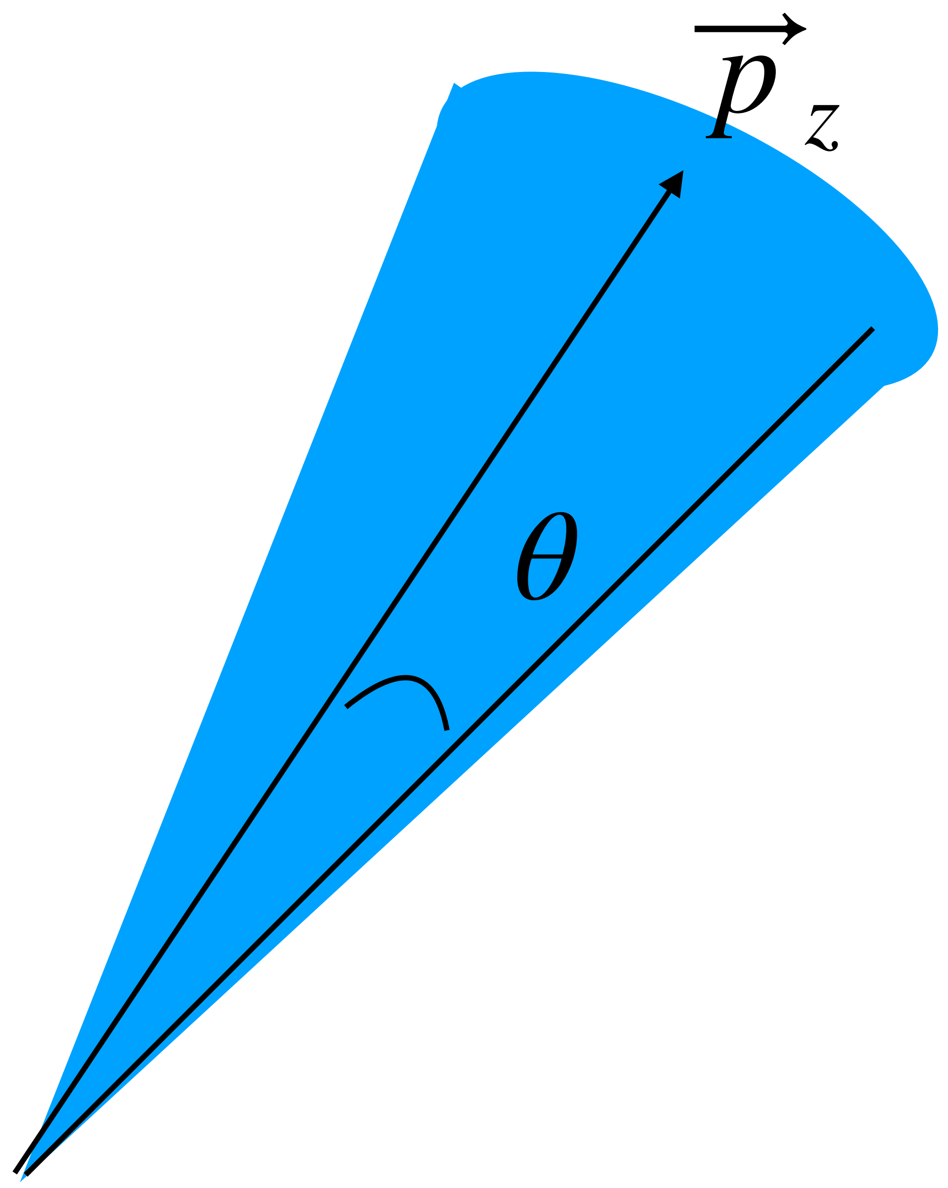


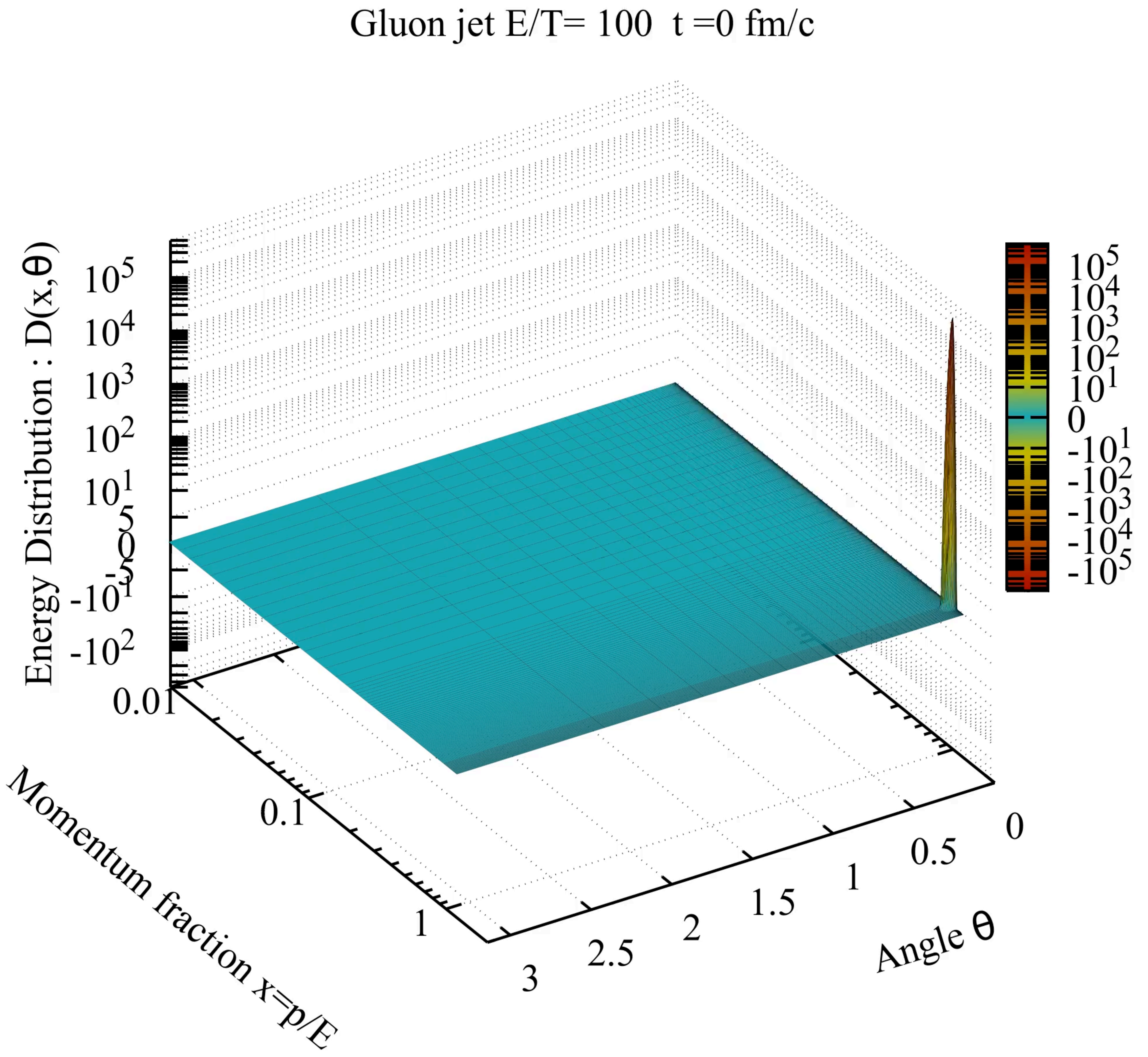
- The jet has lost most energy by the time near equilibrium physics sets in

  —> Not relevant for jet physics.

Angular Cascade

Jet energy $E_j = 100T$ and $g = 2$.





Quenching Factors

[R. Baier et al. In: JHEP 09 (2001), p. 033.]

Leading Parton Quenching

- The spectrum is computed using a convolution with particle distribution

$$\frac{d^2\sigma_{AA}}{dp_T^2}(p_T) = \int_0^\infty d^2p_T^{in} \int_0^1 \frac{dx}{x} \int_{-1}^1 d\cos\theta \; \delta^2(p_T - xp_T^{in}) \; D\left(x, \theta, \tau \equiv g^4 T \sqrt{T/p_T^{in} t}\right) \frac{d^2\sigma_0}{d^2p_T^{in}}(p_T^{in}) \,,$$

$$Q^{\mathrm{h}}_{AA}(p_T) = \frac{\frac{d^2\sigma_{AA}}{dp_T^2}}{\frac{d^2\sigma_0}{dp_T^2}} = \int_0^1 dx \int_{-1}^1 d\cos\theta \; D\left(x, \theta, \sqrt{x\hat{q}/p_T t}\right) \left(\frac{1}{x}\right)^{2-n} .$$

Jet Quenching

- The convolution is computed using the energy remaining inside the cone