Teaching and learning quantum entanglement

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Abstract. One of the central features of quantum mechanics is entanglement. It is crucial for understanding the theory, and its applications, especially in quantum technologies. At the 2023 GIREP meeting, an activity of the GTG "Teaching and learning in quantum physics" will address questions about this fundamental principle. The workshop organizers have invited interested parties to participate with their contribution to shed light on the specific challenges and solutions to address this aspect of quantum mechanics.

Motivation

The mathematical description of entanglement is clear and unambiguous: The superpositions of product states, which themselves cannot be represented as product states, are entangled states. In some cases, the entangled quantum entities are spatially separated, but appear to "communicate" between each other faster than the speed of light. While experts may be satisfied with the mathematical description, neuroscience tells us that a brain that has been newly introduced to the concept will inevitably search for interpretation, representation, and analogy among the things that it already knows. It is then the job of physics educators to provide the learners with a supporting learning environment by carefully and thoughtfully addressing each of the four elements mentioned above. These four elements are derived from a deeper analysis of current needs attached as the only predetermined contribution of the workshop.

Aims of the workshop

The discussion workshop will focus on three topics:

1) Theoretical approaches and conceptual reconstruction of entanglement including aspects concerning formalism and interpretation;
2) Role of representation and visualization in teaching entanglement and tools for teaching entanglement such as experiments, videos, simulations, games, models and analogical experiments;
3) Empirical research on teaching and learning entanglement: analysis of learning difficulties of students or evaluation of teaching proposals (teaching/learning sequence on entanglement) implemented with students.

Topic 1) will address conceptualization of entanglement. While experts might be comfortable with the formal definition of entanglement, novices likely search for interpretations beyond the formal. Especially in spatially separated entangled sates, the terms *locality*, *separability* and *holism* play a central role in the literature. Similarities and differences between them will be discussed and we will address questions of what kind of language should be used so that it encompasses non-locality, but can still efficiently describe the performed experiments and their results.

Representations are crucial for any physics topics, and especially challenging in the case of entanglement, because it is a phenomenon that does not occur in classical physics. Visual representations are a fundamental part of many tools, be it games, simulations, animations or simply figures. It is crucial that representations do not invoke harmful associations, so they should bypass formalism but map it appropriately. In topic 2), the participants will discuss experience with different tools for teaching: their representations, their suitability and potential inadequate conceptions fostered by them.

Topic 3) will address any known empirical research on the topic of entanglement. In the forefront will be teaching and learning activities already developed and tested in class, along with possible identified students' difficulties. A synthesis of empirical data will help identify relations to the other two topics.

Methods

The GTG has contacted members of the GTG, members of the Girep community on teaching and learning quantum mechanics and other interested researchers with an invitation to contribute to the topics described above. Based on the received contributions, the GTG will divide participants into groups with each group discussing one of the topics above.

The workshop will start with a 15-minutes introduction to the topic based on contributions and an extended literature review.

This will be followed by a 60-minutes discussion on specific questions as they will arise from the contributions. Participants will be expected to read all the contributions in advance. Only short, few-minutes reminder presentations will be allowed in the workshop.

The last 15 minutes will be reserved for reports from the groups, and the search for newly identified synergies between the topics.

Outcome

The outcome of the discussion workshop will be a position paper detailing the findings of the workshop. Where consensus has been reached, evidence supporting such consensus will be referenced. Where consensus could not be reached, points of contention or lack of evidence will be reported to motivate further research.

The outcome of the workshop will provide an important stepping stone in the teaching of entanglement as it will be the product of discussion among experts in the field and based on available evidence. Ideally, it may even set standards of language use for the topic.

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Abstract. Modern quantum technologies, especially quantum computing, are based on the fundamental principles of quantum physics. Herewith, entanglement is one of the central notions. At the 2023 GIREP meeting, an activity of the GTG "Teaching and learning in quantum physics" will address questions about this fundamental principle. There are different foci, some of which overlap: (1) formalism and interpretation of entanglement. (2) representation and visualization of entanglement. (3) model or analogy experiments. (4) prior understanding and conceptions of learners about entanglement. The GTG would like to discuss the topics and identify any existing evidence, consensus, and/or topics for future research.

Formalism and interpretation of entanglement

The mathematical description of entanglement is clear and unambiguous: The superpositions of product states, which themselves cannot be represented as product states, are entangled states.

In some cases entanglement is independent of distance of the two photons. The question is how to incorporate these properties into a physical world view. In the literature the terms locality, separability and holism play a central role.

By *locality* we mean "that there is an influence between variables that are spatially located with respect to each other, such that the influence between them would have to propagate faster than the speed of light" [1].

Separability ultimately means that the common state of two spatially separated areas is determined by the individual descriptions of each area and spatio-temporal relations between them.

Often in the context of the discussion about entanglement also the term holism is mentioned. On the one hand, this can simply mean "non-separability", namely that separate spatio-temporal areas must be considered together and cannot be described as two individual objects and their spatio-temporal relations. This implies that in an entangled system it makes no sense to speak of subsystems or parts at all.

This leads to considerable difficulties in speaking about entanglement. What is the "object" in question: is it the whole system, or does the whole system consist of two parts (but without spatio-temporal relations in between)? A successful communication needs compromise: even if strictly speaking it does not really make sense to speak of "parts" of the whole system, one needs ways of speaking which allow to describe the performed experiments and their results.

GTG would like to discuss the questions:

- Which of the terms non-locality, non-separability, or holism should preferably be used in teaching quantum physics? In what ways should these terms be used?
- Are there differences between high school and college students or student teachers?
- What understanding of entanglement should learners achieve?

Representation and visualization of entanglement

While the mathematical description is clear and unambiguous, an accurate representation of entanglement is difficult because it is a phenomenon that does not occur in classical physics. Most importantly, it should be noted that interpreting and understanding a visualization can create undesirable associations or conceptions if the underlying formalism is not known. Therefore, a visualization must make the translation of its elements into the formalism as unambiguous as possible.

Three basic types can be distinguished among representations: (a) static or moving images, iconic representations that attempt to demonstrate the non-separability explained above; (b) XR techniques, which try to make entanglement in space tangible with the help of VR glasses; (c) Metaphors that verbally describe appropriate situations (not necessarily from physics, but from everyday life). In doing so, they bypass formalism but try to map it appropriately.

GTG would like to discuss the questions:

- What kind of representations are suitable, which ones can cause misunderstandings?
- What experiences have already been made with different visualizations in school or studies at university?
- Have certain student beliefs already been observed to be fostered by certain representations?

Model or analogy experiments/games

In teaching physics experiments play an important role. But quantum experiments as a rule cannot be done in school, not even in university. Therefore, people more often try to show entanglement using model experiments or games.

GTG would like to discuss:

- What suitable model experiments or games to introduce entanglement exist? What experiences have been made with their use?
- How suitable are model experiments or games to make the phenomenon of entanglement visible?
- Can formalism be translated into concrete actions without requiring students to learn the mathematics?

Learners' prior understanding and concepts of entanglement.

As clearly and unambiguously as entanglement can be described by the mathematical formalism, it defies purely conceptual representation. Therefore, two cases should be distinguished when investigating student conceptions:

1. the students or pupils know at least the mathematical basics of two-state systems and the notion of basis and basis change using spin states as an example. Furthermore, they can distinguish product and entangled states.
2. The students have received a conceptual introduction and know the most important terms such as superposition and the measurement process. However, they do not know a mathematical description

GTG would like to discuss:

- What student conceptions occur in each of these two cases?
- Are there student conceptions that are avoided or reduced by the mathematical representation?
- What learning difficulties are observed in each of the two cases?

References

[1] C. Friebe, M. Kuhlmann, H. Lyre, P. M. Näger, O. Passon, and M. Stöckler. *Philosophie der Quantenphysik: Zentrale Begriffe, Probleme, Positionen*. Springer (2018), page 149.