Derivations as Mathematical Model Building: A Scaffold for Solving Novel Problems

KK MASCHOOD (1), Sanjay CHANDRASEKHARAN (2)

(1) Modeling and Conceptual Change Group,
(2) Learning Sciences Research Group Homi Bhabha Centre for Science Education, Tata Institute of Fundamental Research, Mumbai, 400 088, India.

Abstract. Traditional lecture-based approaches are used to teach the derivation of canonical models, and their connections to solving problems. We present an alternative approach to teaching derivations, by recasting them as an activity in mathematical model-building, particularly as a process of 'loading' the real-world into mathematics. A digital learning system - Interactive Derivations - was developed based on this approach, and was used as an intervention to study the impact of this loading approach in solving novel real-world problems. We present a case study that shows indicative evidence of the positive impact of the loading approach in solving novel real-world problems.

Introduction

Learning to build models, and using them to solve novel real-world problems, is an agreed-upon ideal goal of physics education [1]. Textbook problems embedded within the chapters, or those appearing at the end, are considered to provide students the necessary training for problem-solving. However, a review of the physics education research literature on problem-solving patterns among students reveals that the ideal goal of solving novel real-world problems is far from being met [1]. In this paper we discuss a possible solution, focussing in particular on the connections between derivations and problem-solving.

Derivations and problem-solving: The traditional instruction approach

Learning derivations, followed by solving textbook problems, constitute the routine in physics classrooms, particularly in countries like India [2, 3]. In traditional lecture-based classrooms, derivations are often perceived as an exercise in mathematical manipulation, resulting in an important equation at the end. These final equations are often presented in a box, and students are encouraged to memorize them, as they serve as important templates that help structure the problems that follow. One of the most important skills during problem-solving is the identification and recall of the 'correct' equation, relevant to the problem at hand. The problem statement is then parsed, to map to the variables in the equation. This is followed by mathematical manipulations, in accordance with the demands of the problem.

We maintain that the above described demarcation between derivations and problem-solving is not pedagogically optimal. Such an approach does not train students to solve novel, real-world problems, as the primary emphasis is on symbol-based manipulations. To address this issue, we present an alternative approach, where we recast the pedagogy of derivations - as an activity in mathematical model building [2]. In this approach, derivation is considered the process of loading the real-world into mathematics. This view is synergistic with some of the key steps involved in solving real-world problems.

Derivations as mathematical model building: Intervention, study design, and method

To help develop a pedagogical approach to solve novel real-world problems, we identified a key underlying conceptual structure involved in multiple physics derivations. This generic conceptual structure involves the following 5 steps: Reality —> Idealization —> Discretization —> Geometric stage —> Algebraic stage [2, 4]. This structure was then implemented as an interactive learning system, and its effectiveness in helping students solve novel problems was tested. Students at the undergraduate level were introduced to the new 5-step conceptual structure of the derivation, in a one to one session. The example used was the derivation of the equation of motion of a simple pendulum. Following this, students were introduced to the Interactive Derivation system, for the derivation of the wave equation [4]. This derivation was not part of the students' syllabus, and it was thus unfamiliar to them. Students interacted and engaged with the software-based derivation , with the researcher providing support and an overall narrative. After this session, we gave them problems that they had not done before, and interviewed them in a semi-structured format. The objective was to see whether the system had any impact on their problem-solving approach.

Results and Concluding remarks

Here we discuss the case of a student who was given 2 problems. The student managed to solve problem 1, but struggled with problem 2. Problem 1 was to model the motion of a massive metallic ball oscillating in an oil tank. Though the student had not solved the exact problem before, he had learned the derivation of a damped spring oscillator. He immediately identified the similarity, and recognized that both cases involve a damped oscillator. The damped oscillator model acted as a template, into which he could fit the given problem. The representational machinery was recalled, and appropriated to fit the given problem. The student then said that the experience with our system did not play any significant role in his thinking, for solving this specific problem.

Problem 2 required modeling the variation of atmospheric pressure with height. This was posed as a real-world problem faced by a mountaineer. The student had not encountered this problem before. Unlike problem 1, he had no template model to fit it. The student struggled with the problem. However, indicative evidence shows that the conceptual structure provided by our system [4] was helpful in this case. The student realized that the first step was to idealize. He generated a structural diagram, thereby moving from the real-world to the world of physics. However, he said that though he knows that certain things need to be stripped, and certain things need to be focused on, he could not decide exactly on what to omit, or what to focus on. The following excerpt from the interview illustrates the nature of student difficulty, his uptake from the interactive derivation and how he thought it could help in solving the problem at hand:

You know, you don't need to think about the Young's modulus or whether, you know, there is a stress or strain…. because we are dealing with the wave equation. So, given a sufficiently complex system like this…. and given a sufficiently simple task of only obtaining the wave equation, I needed to identify correctly what I needed to strip down and what I needed to keep. In this scenario [wave equation]… what I needed to keep, was that, if this kind of swayed, there will be forces that will just pull this back to, you know…. forces that want to pull this back to, you know, wherever it had been. That is the only thing I needed to know.

I know, I have to learn to realize what to strip away and what not. And the system is teaching me through examples. So, I do think that maybe if I see, you know, two or three more examples I will, you know, kind of get an intuition as to, you know, what I should consider and what not.

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