Multiple representations for Quantum Mechanics

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Abstract. In a cultural, history-based, sense-making approach to Quantum Mechanics multi-representations are fundamental to improve conceptual learning of abstract, mathematical aspects. In fact, they can be integrated among each other allowing students to actively manipulate concepts and instruments which would otherwise be difficult and little accessible. In this presentation we will show different (and interconnected) representations of vectors in complex Hilbert spaces and of eigenvalues and eigenvectors of Hermitian matrices together with an evaluation of their efficacy for students conceptual understanding.

Introduction: cultural pedagogical reconstruction of subject matters – epistemological and educational framework

In most modern educational reconstructions of Quantum Mechanics (QM), historical context is often set aside to focus primarily on the conceptual aspects of the theory [1]. While we fully agree on the necessity for conceptual and formal foundations to be adequately addressed – otherwise, we would be discussing approaches built on entirely uncertain grounds – we believe that motivations for a paradigm shift (whether from the intuitive view of motion to classical mechanics or from classical mechanics to QM) must be well-founded and meaningful for students. These motivations should stem from interpretational difficulties arising from perceived anomalies in experimental results and/or attempts at theoretical description.

Therefore, motivations for introducing a new theory and a new way of viewing the world cannot be merely suggested by fragmented and circumstantial observations, even if these are chosen with particular care for their significance. Reflections on the Nature of Science appear indispensable for achieving a mature understanding of physics, particularly for those whose professional future may not lie in scientific research, and thus, especially for students in scientific degree programs other than physics and in the education of high school students [2,3]. However, these motivations must be logically well-sounded and meaningful, rooted in, and not merely suggested by, a few experimental pieces of "evidence". The educational and cultural reconstruction of contents should highlight why our knowledge has precisely the particular structure it does, within an active learning approach where the cultural and social context is seen as a resource rather than an obstacle. Physics should be perceived as a significant cultural stimulus for individuals and their way of positioning themselves in the world, as well as for society.

These considerations certainly prompt the inclusion of significant portions of the history of physics in the didactic reconstruction of QM contents. It is within this context that the proposal to actively engage in discussing and utilizing a possible formal formulation of QM, starting from high school education, should be considered [4]. Particular attention, moreover, must be paid to mathematical aspects, which, due to their importance in constructing the conceptual aspects of the theory, must be approached with great care [5].

Among the central mathematical aspects of QM are the concepts of state as a vector in a complex Hilbert space and the notions of eigenvector and eigenvalue of self-adjoint operators. Given the abstract nature of these concepts, it is particularly useful to provide students with various representations that can be integrated into the necessary sense-making process necessary for operating with these mathematical structures [6].

A multi-representation approach to mathematics for Quantum Mechanics

In this study, we will explore various methods of representing vectors in real and complex n-dimensional Hilbert spaces, utilizing algebraic, geometric (via GeoGebra), and web-based applications to introduce eigenvalue and eigenvector concepts [7]. We will evaluate their strengths and weaknesses, emphasizing the synergies achieved through their combined utilization. Furthermore, we will present the outcomes of their application in three distinct contexts, all related to the introduction, starting from historical perspectives, of the structural and formal elements of QM: 1) a 22-hour online experiment involving high school students, 2) a 30-hour experiment conducted by a high school teacher with his students [8], and 3) a discussion session with mathematics and physics master's degree students held in the "Preparation of Didactical Experiences" courses.

Results and comments

The analysis of the experimentation is based on the results coming from surveys (assigned both online and in classroom) and direct observation of the lessons. Classroom observation allowed us to keep track of students' physical conceptions and questions that arose during the lessons, including the discussions with the teacher that resulted from them. As part of the analysis an interviews session with the students has also been conducted aimed at assessing students' understanding of core conceptual contents and to collect opinions about the methods and technological tools proposed.

Although analysis is still ongoing, preliminary results show that students' performance on mathematical tasks are similar to those on other topics of physics that are already commonly included in high-school curricula, meaning that mathematics used during the course is not an issue by itself. Moreover, despite the fact that handling vectors on complex spaces with dimensions bigger than three is neither perceived as a conceptual nor practical issue, the possibility to give graphical representation of such vectors seems to be intrinsically valuable to students, allowing them to build intuitive knowledge around abstract topics (such as complex eigenvectors, infinite-dimensional vectors…) and improving their understanding of quantum issues.

References

[1] M. Michelini and A. Stefanel, Approaches on T/L Quantum Physics from PER Literature. In B. Jarosievitz and C. Sükösd (eds), *Teaching-Learning Contemporary Physics* (2021), Springer, Cham, 3-17.

[2] E. F. Redish, Who needs to learn physics in the 21st century and why?, *Plenary lecture, GIREP Conference Physics Teacher Education beyond 2000* (2000).

[3] U. Besson, *Didattica della fisica*, Carocci Ed., Rome, 2017.

[4] VV.AA., *Amer. Journ. of Phys.* **70** (2002) 199-367.

[5] G. Pospiech *et al.*, The Role of Mathematics in Teaching Quantum Physics at High School. In B. Jarosievitz and C. Sükösd (eds), *Teaching-Learning Contemporary Physics* (2021), Springer, Cham, 47-70.

[6] M. A. Geyer and W. Kuske-Janßen, Mathematical representations in physics lessons. In G. Pospiech, *et al.* (eds), *Mathematics in physics education* (2019), Springer, Dordrecht, 75-102.

[7] S. Ainsworth, The educational value of multiple-representations when learning complex scientific concepts. In: J. K. Gilbert, *et al.* (eds) *Visualization: theory and practice in science education* (2008), Springer, Dordrecht, 191-208.

[8] L. Lovisetti, G. Organtini and M. Giliberti, *Il Nuovo Cimento C* **46**(6) (2023) 200.