Investigating the Role of Mathematics for Learning Quantum Physics

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Abstract. Quantum physics is inherently described by mathematical structures. Hence, it seems plausible that mathematical knowledge might support its conceptual understanding already in upper-level secondary school. It is therefore desirable to develop and evaluate corresponding teaching-learning sequences. However, for a successful implementation, it is crucial that teachers accept and understand such sequences. In a first step, by an explorative interview study we investigate in-service teachers' and prospective teachers' acceptance and understanding of a mathematical description of fundamental principles of quantum physics. We present the analysis of the interview data with respect to attitudes about the teaching-learning sequence and difficulties in understanding.

Introduction

The teaching of quantum physics via two-state systems has been the focus of numerous educational research projects over the last years [e.g., 1]. Such approaches seem suitable mainly for two reasons. Firstly, learning about two-state systems enables insights into fundamental quantum physics principles such as superposition, the measurement process, uncertainty and entanglement. Secondly, since the mathematical description of such systems is comparatively simple, it seems possible to introduce a mathematical description already in secondary school. As the mathematical formalism can be regarded as part of conceptual knowledge, especially in quantum physics, mathematical abilities might even be necessary to develop a deeper understanding of the subject, already in upper-level secondary school level, as discussed in [2]. The authors argue that mathematics can support the development of adequate mental models as well as a consistent interpretation of experiments, phenomena and concepts. To make this interplay of mathematical structure and conceptual understanding happen, a careful selection of mathematical tools, visualization and explicit connection to the concepts is necessary.

While there are some investigations on students' difficulties with learning mathematics in quantum physics at an undergraduate level, to our knowledge there is only little research at a secondary school level [e.g., 1, 3]. Building on that, prior to developing new teaching-learning sequences for upper-level secondary school students it is necessary to investigate teachers' and prospective teacher' perspectives as they should be familiar with the mathematical formalism to gain a deeper understanding and to teach their students adequately.

Subject of study and research questions

Our research project investigates the role of a reduced mathematical formalism in the understanding of fundamental quantum physics principles, such as superposition, the measurement process, uncertainty and entanglement. In the mathematical description we focus on the combination of algebraic representations (Dirac notation) with a graphical representation (Bloch sphere). The target group is teachers and prospective teachers who do not yet have much knowledge of quantum theory. Hence, our research questions are:

1. Do secondary school teachers and prospective teachers accept a reduced mathematical formalism for describing fundamental principles of quantum physics?

2. Which elements of a mathematical formalism contribute to developing an adequate understanding of fundamental principles of quantum physics?

Research design and methods

In a first step, a new teaching-learning sequence on quantum physics was developed, focusing on the polarisation of single photons as a quantum physical two-state system and on a reduced Dirac-formalism in combination with the Bloch sphere for visualisation [4]. The mathematical formalism is intended for an undergraduate students' level and could also be used in secondary school in a slightly adapted form.

In order to gain insight into learning processes and attitudes, and to approach a clarification of our research questions we conduct exploratory material-based interviews in which we interview teachers and prospective teachers after we provide them with an instruction, following [5]. After explanations are provided, the participants are asked to express their opinion on comprehensibility, to paraphrase the content, to give own explanations and to work on tasks. This procedure is repeated several times and allows us to gain insights into teachers' assessments, difficulties as well as learning processes and resources used in their explanations.

In the study, a random sample of $N=15$ prospective teachers and $N=9$ teachers took part. The interviews were transcribed. The data were analysed by an evaluative qualitative content analysis to get an overview of the acceptance of the formalism and the solutions of the different tasks as well as by a structuring qualitative content analysis to investigate specific difficulties, strategies used in the solutions and to get deeper insights into opinions about the teaching-learning sequence.

Our results indicate that teachers and prospective teachers generally show good acceptance and understanding. However, in detail, some difficulties arise, for example challenges in the transition between an algebraic (Dirac notation) and the graphic (Bloch sphere) representation. The findings will be presented in the contribution.

Conclusion and outlook

As the research indicates a good understanding of the explanations, it seems conceivable that success in improving the understanding of secondary school students could be achieved through a mathematical approach as well. Due to that, in the second part of our study, the survey instruments are adapted for upper-level secondary school students and analogous research questions are addressed.

References

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