From probabilities to Bell's inequalities: a pathway for secondary school quantum literacy

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Abstract. Quantum Mechanics (QM) is a cornerstone of modern science and technology, yet its complexities have hindered its integration into school curricula. We propose a QM teaching/learning sequence for secondary schools that focuses on Bell's inequalities and their experimental verification, and outline a step-by-step approach, leveraging on students' existing knowledge of statistics and probability. Through hands-on activities, such as the demonstration of a simplified card game, and discussions of CHSH experiments, we clarify the main differences between classical and quantum probabilistic predictions. Preliminary results from informal and formal test sessions indicate promising results, suggesting the effectiveness of our approach.

Introduction

Quantum Mechanics (QM) lies at the core of our understanding of natural phenomena and is the basis of fundamental technological developments. Despite its importance, the complexity of mathematical tools and the degree of abstraction required to grasp QM concepts have so far substantially limited the way QM is presented at secondary school level. With the emergence of the second quantum revolution, there is a growing push to enhance quantum literacy at earlier education levels. A review of the current state of research on teaching QM in secondary education is discussed in [1], while an overview of QM-related activities proposed in secondary school curricula around the world is found in [2].

Within this framework, we considered the transition from classical correlation to Bell's inequality violation and its consequences, and examined whether it could be a suitable topic for the secondary school level. We started considering the difference between probabilistic predictions in classical and quantum physics. In classical physics, a probabilistic description is necessary only when information about the system is incomplete, namely, in the presence of unknown (hidden) variables. Conversely, QM states that the outcomes of an experiment probabilistically distribute even in the presence of complete information about the system state. This profound discrepancy with classical predictions is a key element to understand the foundations of QM and can be elucidated through the example of Bell's inequalities and their experimental falsification. This, together with the supposed 'innocence' of the assumptions (realism and locality) and the simplicity of the algebra involved, makes Bell's inequalities particularly apt to introduce QM fundamental aspects.

A route to Bell's inequalities: Description and preliminary results

In the present work, we explore this idea, devising an educational teaching/learning sequence which builds on students' curricular knowledge of the basic concepts of statistics to explore the full meaning and consequences of Bell's inequalities. The sequence develops according to the following phases:

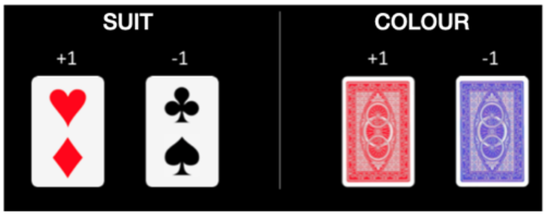
1. After revising basic probabilistic knowledge, the concept of correlations is introduced, insisting on the demarcation between correlations and causation.

2. Bell’s inequalities for classical systems are described; we devise a simple “card game” illustrating CHSH inequality, which only requires calculating mean values.

3. The axioms of QM and fundamental concepts as state superposition, non-compatible observables and entanglement are qualitatively introduced exploiting analogies and referring to key experiments (Feynman’s double slit, Stern-Gerlach) [3]. EPR argument is discussed and the validity of Bell's Theorem, even when hidden variables is stated.

4. The fundamental steps of the CHSH experiments of 2022 Nobel Prize winners [4] are reproduced involving the students in a sort of “live-crib”, using cards prepared as “entangled photons” in Charlie’s lab and measured independently by Alice and Bob.

The basic ideas of this educational path have been first tried in a few informal situations, as in particular the public exhibit “Speaking the Unspeakable, held in several Italian cities in 2023 [3]. A more formal and revised version has been proposed to 26 fourth-year high-school students in a three-days stage. The effectiveness of our sequence has been probed with pre- and post-tests.



$$S_i = B_A B_B + B_A S_B + S_A B_B - S_A S_B$$

N	ALICE’s SUIT S_A	BOB’s SUIT S_B	ALICE ’s BACK B_A	BOB’s BACK B_B	$S_A S_B$	$B_A S_B$	$S_A B_B$	$B_A B_B$	S_i
1	+1	+1	1	1	1	1	1	1	2
2	+1	-1	+1	-1	1	-1	-1	1	-2
3									
4									
5									
6									
7									
8									
9									
10									
TOT									

$$S = (S_1 + S_2 + \dots + S_{10})/10$$

Fig. 1. The card game illustrating the CHSH inequality. By compiling the table on the right it becomes apparent that, whatever combinations of suits and back colors, each S_i could assume only ±2 values, so that $|S| \leq 2$.

In the Pre-test, we probed any preliminary knowledge on entanglement and Bell’s inequalities through oral open questions; only two pupils had an inkling of the concept of entanglement. On the other hand, both open questions and simple problems probed pupils understanding of probability and statistics; the positive results assure us that this is indeed a viable starting point for our sequence. *In the Post-test,* open questions on entanglement, hidden variables and Bell’s inequalities were proposed. For instance, pupils were asked to choose, among different statements, those that correctly describe the CHSH experiment: 17 pupils chose the correct statements, 5 made only one mistake, while the remaining 3 gave no answers or made more than two mistakes.

Conclusions

The idea of starting from basic concepts of probability theory to build an educational path which explores some of the key ideas of QM – that is, the meaning and consequences of its intrinsic probabilistic nature – have been shown feasible and successful. We believe this is a promising route to introduce QM; as it builds on well-known and important topics in math curriculum, it is apt to an interdisciplinary approach which may be particularly appreciated by teachers.

References

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