Exploring the non-linear viscoelastic properties of a mass-rubber band oscillator

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Abstract. This contribution outlines an experimental procedure that can be executed via smartphones in order to analyse the oscillations of a mass-loaded damped oscillator composed of an elastic rubber loop. By utilising this system, data regarding the viscoelastic characteristics of the rubber material can be acquired. The analysis is conducted by varying the mass. A non-linear model for the elastic force was determined to be necessary in order to account for the experimental data. This experiment gives students accurate experimental data and challenges them to evaluate models' capacity to explain observed behaviour.

Introduction

One-dimensional mechanical oscillators are beneficial for teaching fundamental physical principles and demonstrating the application of differential equations. Damped oscillator models are beneficial for demonstrating the impact of friction in basic mechanical systems [1,2].

A simple oscillator with significant educational benefits can be constructed by attaching a mass to a rubber band. For instance, it can allow students to understand the meaning of some thermodynamics variable like entropy [3]. An oscillator with rubber extension in the longitudinal degree of freedom exhibits more damping compared to an identical oscillator with a metal spring. The damping is derived from the rubber band and poses a modelling problem related to the possibility to explain the phenomenon by a linear differential equation with viscous damping. While viscoelasticity is typically not covered in introductory physics courses, some aspects can be introduced at a basic level to complement introductory texts and serve as a stimulating topic for further exploration.

To investigate this problem, it is necessary conducting experiments, creating models, and comparing them with the data. Studying the behaviour of rubber in undergraduate courses is crucial as it connects theoretical concepts to practical applications and allows for the evaluation of basic models for a common material.

The main goal of the contribution is to familiarise students with the examination of viscoelastic properties in non-linear oscillating systems through a practical experimental arrangement. The measurements acquired are quite accurate to allow us to thoroughly analyse the system's properties, especially focusing on damping and non-linearity.

Experimental setup and data analysis

By the phyphox app [4] that controls the smartphone's built-in accelerometer, we record the periodic acceleration of a damped oscillator made of an elastic rubber band and a mass. The smartphone is suspended by a rubber loop with a cross-section of 1 mm by 3 mm, allowing it to move up and down. The oscillations occur along the vertical y-axis. The acceleration values along the x- and z-axis stay nearly zero. We can determine the angular frequency ω and damping coefficient γ as the mass changes from the acceleration as a function of the time. Newton's second law can be applied to analyse rubber-band oscillators, created by combining a spring with an elastic constant k and a damper with a viscous coefficient β in series, which produces a frictional force proportional to the velocity. We computed the motion equation for both linear and non-linear elastic force and then compared the outcomes, as depicted in Figure 1. The non-linear elastic force is expressed by a third-degree polynomial function of the deformation.

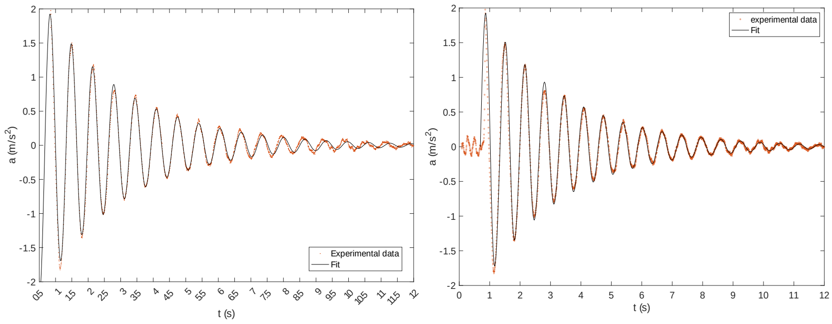

Figure 1. Acceleration as a function of the time for m = 440 g

The analysis was extended obtaining as the pulsation and the damping coefficient vary as a function of the mass (see Fig. 2).

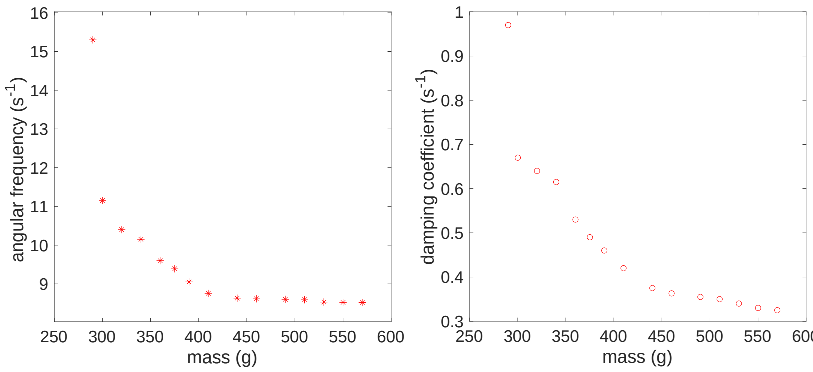

Figure 2. Angular frequency and damping coefficient as a function of mass

Discussion and conclusions

Our study of the results revealed that a linear model of the elastic force does not align with the experimental data Upon close examination, a little phase discrepancy is evident just from 6 seconds. We achieved improved agreement with the experimental results by using a non-linear model that relates force to deformation through a third-degree polynomial. Utilising a smartphone acceleration sensor in basic physics labs at the undergraduate level allows for the creation of affordable and adaptable laboratories, offering a novel approach to teaching and learning, particularly in remote education settings. This experiment gives students accurate experimental data and challenges them to evaluate models' capacity to explain observed behaviour.

References

[1] A. Ricchiuto and A. Tozzi, Motion of a harmonic oscillator with sliding and viscous friction, *Am. J. Phys.* **50** (1982) 176–179.

[2] A. Filipponi, L. Di Michele, C. Ferrante, Viscoelastic behavior of a mass- rubber band oscillator, *Am. J. Phys.* **78** (2010) 437–444.

[3] D. Roundy and M. Rogers, Exploring the thermodynamics of a rubber band, *American Journal of Physics*. **81** (2013) 20.

[4] T. Pierratos and H. M. Polatoglou, Utilizing the phyphox app for measuring kinematics variables with a smartphone, *Phys. Educ.* **55** (2020) 025019–6.