Different spins on the two-state paramagnet: Pedagogical advantages and considerations

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Abstract. The two-state paramagnet has previously been proposed as a valuable pedagogical tool for teaching statistical mechanics. In this study, we focused on the role that the two-state paramagnet example played in student discussions while solving problems in statistical mechanics. Students' use of the example was both advantageous and challenging. While we saw students using the two-state paramagnet example to reason analogically about similar systems, we also saw a general lack of in-depth reflection of what makes the model transferable to other contexts. We suggest some considerations for teachers to better leverage this example and facilitate student learning in statistical mechanics.

Introduction

Within physics education research (PER), studies on upper-division topics are still underrepresented compared to introductory courses. The importance of expanding such research has been emphasized in the PER community over the last two decades [1], motivated by gaining insights into students' reasoning in more advanced content as they progress from novice to expert physicists. Statistical mechanics is one such relatively under-researched area in PER. Previous studies on the topic [e.g. 1-4] have reported on many challenges faced by physics students. These are often related to various abstract and often subtle concepts, as well as the demanding use of mathematics. To explain concepts and mathematical approaches in statistical mechanics, teachers and textbooks often utilize simplified toy models, such as the two-state paramagnet, which has been suggested to be a valuable pedagogical tool in the subject [5]. In particular, it has been recommended to demonstrate some subtle aspects of the behaviour of systems with discrete and bounded energy levels, and promote students to reason in terms of the more fundamental concept of entropy instead of temperature. This is related to the conceptually subtle relationship between entropy, energy, and temperature, which has a precise mathematical definition in statistical mechanics:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V}. \tag{1}$$

In our recently published work [4], we explored what challenges students face when solving statistical mechanics problems. Our findings suggested that idealized models, like the two-state paramagnet or similar examples, can be beneficial, but can also cause various obstacles for students. We report here on a part of an ongoing study to analyse these factors further. Here we intend to answer the question: *As a pedagogical tool, what are the advantages and challenges related to the two-state paramagnet for students learning statistical mechanics?*

Method and findings

We briefly summarize the methods of our previous project [4], since the work presented here is a continued and deepened analysis of the same data. In our previous exploratory study, we adopted a grounded approach to investigate the challenges faced by upper-division physics students engaging in collaborative problem solving on the topic of statistical mechanics. This overarching methodology, similar to the starting points of e.g. grounded theory and phenomenography, was chosen since the topic of our research is still novel and complex. We video recorded nine groups of two to three student volunteers from a statistical mechanics course at a large Swedish university (1.5 h sessions). Then we performed a qualitative analysis of the data using iterative thematic coding. Our empirical findings included ten emerged categories of student challenges [4]. From this initial study we also found several interesting aspects to investigate in future work. Informed by this, we set out to analyse certain episodes of the same data in more detail. In particular, we focused on the students' reasoning and analogical use of the two-state paramagnet in relation to a task involving a similar three-state system.

Our findings suggest that students can productively use the two-state paramagnet, either spontaneously or after an interviewer prompt. When a conflict arose between the students' qualitative reasoning and their quantitative result, the two-state paramagnet example was in some cases used to reframe the task, activate other ideas and overcome the conflict. The students' communication and use of representations changed notably after introducing the two-state paramagnet into their discussion. However, most students still struggled to recognize all the relevant underlying similarities of their task and the two-state paramagnet model and could not fully transfer between them. Their problem solving seemed to depend on surface features and they often resorted to a 'plug and chug' approach. Additionally, the two-state paramagnet did not necessarily lead the students to reason about the problem in terms of the more fundamental quantity entropy rather than temperature, as it has been suggested by others [5]. The subtleties of key concepts and their connections in statistical mechanics, e.g. Eq. (1), appear to be very challenging for students. Our findings suggest that strong associations from other areas of physics and everyday experience seem to contribute to these difficulties.

Conclusions

Our findings revealed that the two-state paramagnet can be both beneficial and challenging for students engaging in collaborative problem-solving in statistical mechanics. Some students were able to leverage the example as an analogy to reframe their reasoning about a similar problem in a productive way. Others struggled to recognize the similarities in the underlying structure and could not resolve their conflict. Despite allowing simpler calculations and having the potential to serve as an illustration of important physics concepts, the subtle and "unintuitive" aspects of the two-state paramagnet seem to be challenging for students. Based on our findings, we suggest that teachers consider *how* they utilize the two-state paramagnet as an illustrative example. For example, teachers could spend time unpacking the hidden assumptions and subtleties of the idealized model and help students connect insights gained from studying the model to other contexts.

References

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