Understanding how students recognize and connect mathematics ideas in physics contexts: A pilot study

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Abstract. A long-standing problem within physics education is the difficulty of transferring ideas learned in a mathematics context into a physics context. To address this, first and second year mathematics courses at the University of Edinburgh are taught to physics majors by physics faculty rather than mathematics faculty. To evaluate this, we have devised a problem categorization task using 12 items relating to vectors from the *Test of Calculus and Vectors in Mathematics and Physics* to pilot the viability of this task. Six students were given this and interviewed. We discuss the viability of this task and reflect on future research.

Introduction

Previous work has demonstrated that there is a profound difficulty for many physics students reconciling the mathematical skills that they have been taught with their application in a real-world context. Students tend to use different problem-solving techniques and strategies when presented with questions that exist in different contexts [1], and performance when approaching physics questions tends to be impaired in comparison to similar questions that are mathematical and without context [2]. Exposure to similar modes of reasoning in different contexts can help alleviate such difficulties, making mathematical preparation an important factor in success in future physics courses [3]. While card sorting tasks like the one presented here have been done before [4], and there have been efforts to understand student reasoning in solving physics problems [5 for example] as well as mathematical problems [6 for example], we believe that using a card sorting task in this way can potentially provide new insights into how and why students connect, or fail to connect, knowledge and skills gained from mathematics classes to a physics context.

Research Questions

We wish to understand how students connect ideas presented in a mathematics context to those presented in a physics context. To do this, we wished to design an instrument and associated task to give students to ultimately measure this. We wish to address the following questions:

1. How do students view the purpose of this task?
2. What groups can students form and why do they choose to form such groups?

Methods

We presented to student respondents a group of 12 items taken from the *Test of Calculus and Vectors in Mathematics and Physics* [7], six items relating to mathematics and six relating to physics. Questions on this test were intentionally designed in paired “isomorphs”, pairs of questions that involve the same underlying mathematical process to solve with one of the questions being presented with a physics context. Items chosen all came from the vectors portion of the exam. The set of cards given to students included five item isomorph pairs and two items that were not a pair. Students were then told that they had approximately 30 minutes to group items in ways that made sense to them. They chose the names of the groups as well as their description. After the students were done, they were interviewed about their experiences with the task and their reasoning for the groups they formed.

Results

An example of the groups with descriptions formed by one student is presented in Table 1. The number of groups students formed ranged from five groups of items to twelve. Some students connected the isomorph items to one another in their own groups. Common groups included grouping the mathematics and physics items separately Other common groupings related to various features of the problem (problems having boxes or a grid). Students also grouped items based on how they would solve the problem (as shown in Susan's groups in Figure 1). Student interviews provided insights into their approaches to forming their groups.

Conclusions

In this study, students presented various ways that they could organise the same set of mathematics and physics problems. Students presented a wide range of how they viewed the purpose of this task or how their approaches to it. Because we wish to distribute this task to a large sample of students, we must fully understand the possible groups students may form as well as how students interpret the task itself. Further study will be needed to address this if this card sorting task will ultimately be used as a means of measuring how students connect mathematics and physics ideas. We believe these results show promise of this possibility and mark the beginning of a long term study.

Table 1. Groups formed by "Susan". Group names and descriptions were written by her. Each item that is in a group is marked with an "X". Isomorphic problem pairs are coded with the same colour.

Participant Name	Group Name	Group Description	Item A	Item E	Item B	Item J	Item C	Item H	Item G	Item L	Item I	Item D	Item K	Item F
Susan	Dot Product	Problems asks explicitly to take dot product			X	X								
Susan	Component of dot product	Project asks for x-component of dot product									X	X		
Susan	Components	x & y components of vector are required					X	X						X
Susan	Vector subtraction	Problem asks to subtract 2 vectors							X	X				
Susan	Magnitude of sum	Magnitude of sum of vectors is asked for	X	X										
Susan	Dot Product required	Problem involves taking dot product in any way			X	X		X			X	X		X
Susan	Vector addition & subtraction	Vector addition requied in any way	X	X					X	X	X	X		

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