'Seeing quantum in a water droplet: On the theoretical development of an experimentally-backed educational analogy

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Abstract. Modeling the hydrogen atom is often considered an essential milestone in an introductory quantum mechanics course. Mathematically, the path is clear: from the square potential well, through the harmonic potential, and culminating with the hydrogen atom. However, a working conceptual path is lacking. Here, we present results from a real experiment where the light scattering of an evaporating, optically levitating water droplet can be explained with a quantum analogy, serving as an 'optical atom.' We have distilled two important concepts in this analogy, angular momentum and tunneling, to explain properties of the hydrogen atom conceptually in an undergraduate teaching context.

Introduction

An important goal in an introductory quantum mechanics (QM) course is to describe the hydrogen atom. Students solve the Shrödinger equation with a Coulomb potential and get the electron's wave function (e.g. [1]). This is argued to be crucial for understanding what QM does. However, this path does not provide students with conceptual models and helpful analogies that connect mathematical results to their previous, mainly classical, experiences with physics and the world.

When learning QM, students face both mathematical and conceptual challenges. Regarding the mathematical challenges, students can build upon previously acquired knowledge, such as numerical analysis and wave mechanics. When it comes to conceptual challenges, however, students tend to struggle to develop subject-correct interpretations of concepts in QM [2]. Using the case of tunneling as an example, previous research shows that students are hesitant to move away from the macroscopic, classical notion of describing it as overcoming a barrier [3]. This exemplifies the act of students unsuccessfully trying to make sense of a new, abstract concept by relating it to previous experiences or some real-world application. To better aid students in the process of creating and revising their internal models, it is helpful for teachers to involve them in their model construction and refinement process [4].

In the current study, we build on recent work where a quantum analogy can explain the scattering spectrum from an evaporating, twinkling water droplet [5]. Each twinkle is assigned to an energy level in a potential well. The rotation of the photon around the droplet is analogous to the electron's angular momentum, and the evanescent wave in and out is analogous to tunneling in QM. From this, it is possible to consider the water droplet as an 'optical atom.' the current work theoretically investigates the following question: *How can an optically levitated twinkling droplet be used as an analogy to aid students in understanding the hydrogen atom?*

Method

To theoretically investigate the potential usefulness of the analogy, it is necessary to identify similarities and differences between the analogy (photon in a droplet) and the target system (electron in a hydrogen atom). To do this, we follow the idea of how to map an analogy from [6]. By identifying analogous features, we categorize the nature of mapping as being either positive or negative, in the sense that a positive mapping would enable one to draw inferences from the analogous system about the target system. Next, we analyzed introductory QM textbooks to identify how and where the analogy could be utilized to qualitatively highlight QM concepts.

Findings and conclusions

We have extracted two main concepts the 'optical atom' could help illustrate in an undergraduate QM teaching course. We have distilled the relevant theoretical interpretations and the experimental backing to be useful in a teaching setting. These are presented together with a video of the experiment where the twinkles, which are analogous to energy levels, are visible to the naked eye.

Preliminary analysis of our findings shows several positive mappings between the analogy and the target system. Notably, we identify positive mappings related to the concept of tunneling and quantized angular momentum; which previous research identifies as problematic for students to comprehend conceptually. Further, by reviewing current teaching materials, we identify a conceptual gap when learning about potentials. Among the commonly taught potentials on the path to the hydrogen atom, i.e., square well, harmonic, and quantum rotor, only the quantum rotor has quantized angular momentum; none are in spherical coordinates, and the understanding of tunneling is entirely mathematical. Our proposed analogy could be a bridge for students to better comprehend the hydrogen atom conceptually. We argue that having access to a demonstration of an actual experiment, where students can see quantization (by identifying that the optically levitating droplet twinkles as it shrinks), is an important step to help students develop and refine internal models to make sense of what quantum mechanics does.

The experiment will be further presented and discussed at the conference, and mappings between the analogy and the target system will be related to previous research to provide initial answers to our research question. In the current study, we focus on a theoretical study of mappings between the analogy and the target system, as it provides essential building blocks for constructing teaching sequences. As such, our results do not include data on students' learning but lay the foundation for future studies involving students and developing teaching sequences.

References

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