Student Understanding of Divergence and Curl

Zeynep TOPDEMIR (1), Michael LOVERUDE (2), John R. THOMPSON (3)

(1) Johannes Kepler University, School of Education, Altenberger Straße 69, 4040 Linz Austria
(2) California State University Fullerton, Fullerton, California 92834, USA
(3) Department of Physics and Astronomy, University of Maine, Orono, Maine 04469, USA

Abstract. As a part of a broader project that aims to investigate students' mathematical understanding in physics, this study explores how students understand the partial derivatives of divergence and curl of vector field diagrams. Student difficulties finding partial derivatives of divergence and curl of vector field diagrams will be reported.

Introduction

Several previous studies in physics education research have examined student understanding of divergence and curl in post-introductory and graduate courses [1-4]. These studies involved two-dimensional representations of a field as an array of vectors and asked students to determine the divergence and/or curl from these representations. Baily & Astolfi [1] and Bollen et al. [2] performed similar studies with different diagrams and reported that around 50% of their students could correctly determine whether the divergence and curl of the vector field diagrams are zero or not. In these previous studies, students determined the sign or value of the divergence and/or curl for a given field diagram. There has not been as much focus on the partial derivatives that constitute these operations, e.g., $\frac{\partial V_x}{\partial x}$ and $\frac{\partial V_y}{\partial y}$ for divergence or $\frac{\partial V_x}{\partial y}$ and $\frac{\partial V_y}{\partial x}$ for curl in Cartesian coordinates. This study explores student understanding of constituent derivatives of divergence and curl with vector field representations.

Research questions and methods

The research question of this study is "How do students reason with constituent derivatives of divergence and curl given a vector field diagram?" Data was collected in the Mathematical Methods for Physics course, an intermediate course intended to prepare students for the advanced mathematics they will encounter in upper-level physics courses. All students had completed introductory sequences in both physics and calculus. Written data were collected in the course after instruction on vector calculus. In the tasks, students were shown a 2-d field representation (see Figure 1) and asked to determine the signs first of the divergence and curl, then of the constituent derivatives. Field 1 has only V_x components; students were asked to determine $\frac{\partial V_x}{\partial x}$ and $\frac{\partial V_x}{\partial y}$. For Field 2 all constituent derivatives of divergence and curl were asked. Field 1 (N=14) and Field 2 (N=32) were asked in different semesters at two public universities; due to small N, data are combined. Only the results for constituent derivatives will be discussed here.

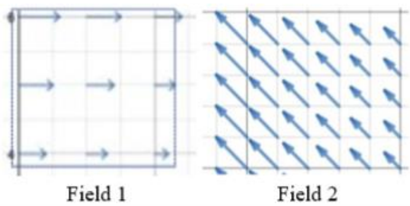

Fig. 1. Vector field diagrams used in the tasks.

Findings

For Field 1, 9 of 14 students were able to identify the sign of $\frac{\partial V_x}{\partial x}$. Determining $\frac{\partial V_x}{\partial x}$ for Field 2 was more challenging: 34% of the students (N=32) answered correctly. We have suggested that some students recognize that the vector magnitude is decreasing, but do not account for the negative direction of the vector and thus find the sign of $\frac{\partial V_x}{\partial x}$ to be negative [5]. More students correctly determined $\frac{\partial V_y}{\partial y}$ to be zero for Field 2 (78%).

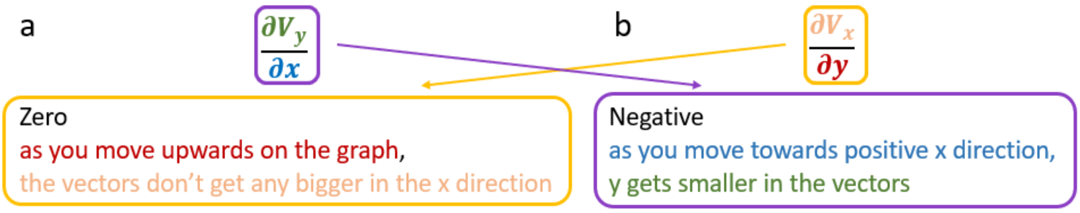

Fig. 2. Student responses showing incorrect notation mapping to constituent derivatives of the curl for Field 2 (a, b). Colored text in response corresponds to similarly colored elements of derivative

For Field 1, only 2 of 14 students were able to identify the change in V_x with respect to the *y*-direction. For Field 2, 72% of the students (N=32) answered each of $\frac{\partial V_x}{\partial y}$ and $\frac{\partial V_y}{\partial x}$ correctly, but only 50% answered both derivatives correctly. Some students incorrectly mapped notations to derivatives. Figures 2a and 2b show responses from a student for $\frac{\partial V_y}{\partial x}$ and $\frac{\partial V_x}{\partial y}$, respectively. In Figure 2a, the response explains how V_x changes along the *y*-axis even though the question asked about $\frac{\partial V_y}{\partial x}$. Similarly, the response in Figure 2b explains how V_y changes along the *x*-axis instead of $\frac{\partial V_x}{\partial y}$. Determining the signs of the constituent derivatives of divergence and curl was a challenging task for students: only 5 of 32 students (16%) correctly determined all four partial derivative signs.

Discussion and conclusions

Some challenges were dependent on the properties of the specific vector fields, e.g., when the vector field had a single component or when a vector field component was negative. Incorrect student responses suggested confusion between the change in a component and the change in a coordinate, confirming the findings of previous research [3]. How these challenges and difficulties may relate to understanding of the divergence and curl will be discussed as well.

References

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