Selected Topics in the Theory of Heavy Ion Collisions

Lecture 1

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‘Preface’

Starting point: Quantum Chromodynamics, QCD, the theory of strong interactions, is a mature theory with a precision frontier.

- background in search for new physics
- TH laboratory for non-abelian gauge theories

Open fundamental question: How do collective phenomena and macroscopic properties of matter emerge from the interactions of elementary particle physics?

Heavy Ion Physics: addresses this question in the regime of the highest temperatures and densities accessible in laboratories.

How? 1. Benchmark: establish baseline, in which collective phenomenon is absent.
2. Establish collectivity: by characterizing deviations from baseline
3. Seek dynamical explanation, ultimately in terms of QCD.

These lectures give examples of this ‘How?’

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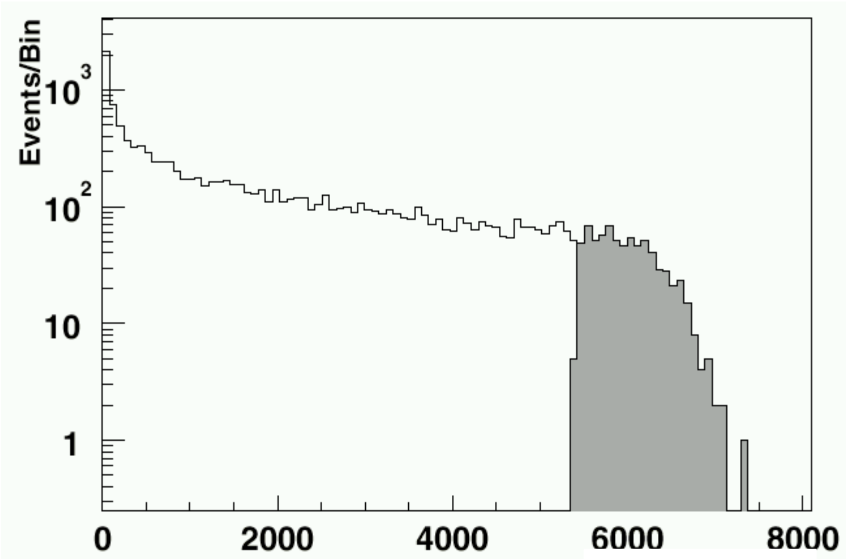
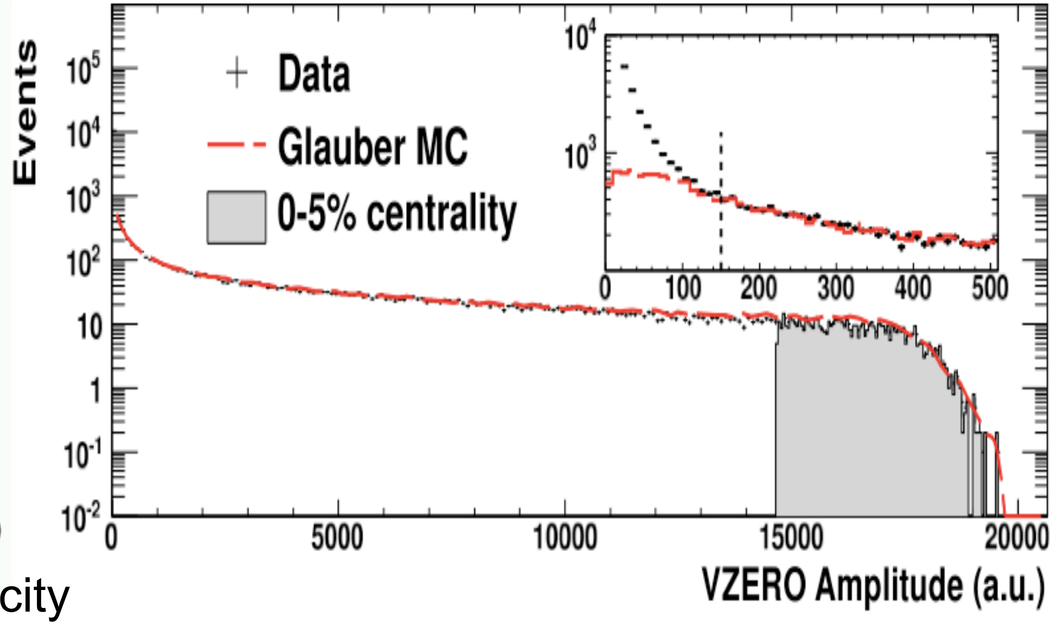
I.1. The very first measurement at an Heavy Ion Collider

PHOBOS, RHIC, 2000



ALICE, PRL 105 (2010) 252301, arXiv:1011.3916



Signal proportional to multiplicity

What is the benchmark for multiplicity distributions?

Multiplicity in inelastic A+A collisions is incoherent superposition of inelastic p+p collisions.

(i.e. extrapolate p+p -> p+A -> A+A without collective effects)

→ **Glauber theory**

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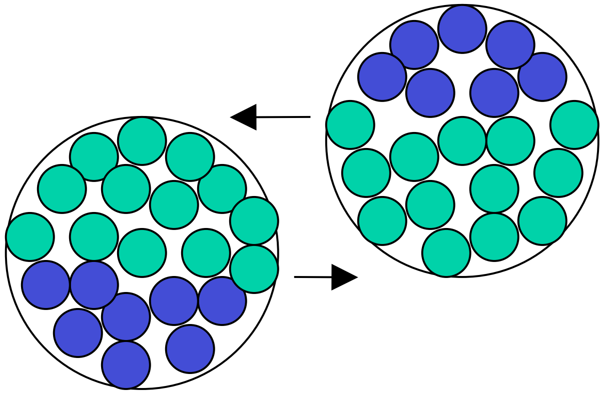
I.2. Glauber Theory

Assumption: inelastic collisions of two nuclei (A-B) can be described by incoherent superposition of the collision of "an equivalent number of nucleon-nucleon collisions".

How many?

Establish counting based on

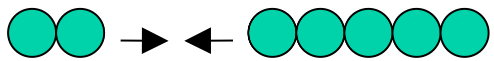
- Spectator nucleons
- Participating nucleons



To calculate N_{part} or N_{coll}, take

σ = inelastic n-n cross section

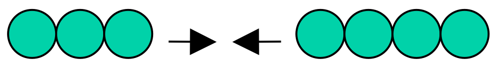
A priori, no reason for this choice other than that it gives a useful parameterization.



$N_{part} = 7$

$N_{coll.} = 10$

$N_{quarks+gluons} = ?$



$N_{inelastic} = 1$

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I.3. Glauber theory for n+A

We want to calculate:

N_{part} = number of participants = number of 'wounded nucleons',
which undergo at least one collision

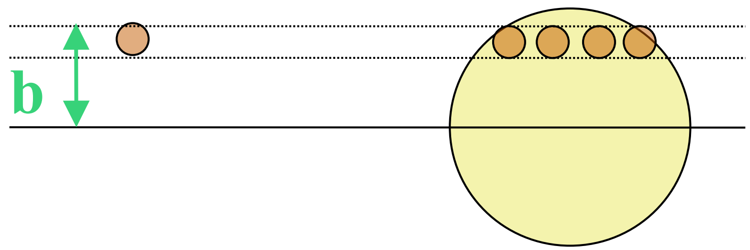
N_{coll} = number of n+n collisions,
taking place in an n+A or A+B collision

We know the single nucleon probability distribution within a nucleus A, the so-called nuclear density

(1.1)

$$\rho(b,z)$$

$$\int dz\, db\, \rho(b,z) = 1$$



Normally, we are only interested in the transverse density, the nuclear profile function

(1.2)

$$T_A(b) = \int_{-\infty}^{\infty} dz\; \rho(b,z)$$

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I.4. Glauber theory for n+A

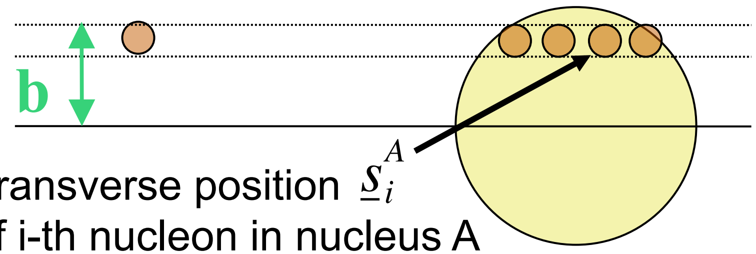
The probability that no interaction occurs at impact parameter b:

(1.3) $$P_0(\underline{b}) = \prod_{i=1}^{A}\left[1 - \int d\underline{s}_i^A\, T_A\left(\underline{s}_i^A\right)\sigma\left(\underline{b} - \underline{s}_i^A\right)\right]$$ $$\int d\underline{s}\,\sigma(\underline{s}) = \sigma_{nn}^{inel}$$

If nucleon much smaller than nucleus

(1.4) $$\sigma(\underline{b} - \underline{s}) \approx \sigma_{nn}^{inel}\,\delta(\underline{b} - \underline{s})$$

(1.5) $$P_0(\underline{b}) = \left[1 - T_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right]^A$$



Transverse position $\underline{s}_i^A$
of i-th nucleon in nucleus A

The resulting nucleon-nucleon cross section is:

(1.6) $$\sigma_{nA}^{inel} = \int d\underline{b}\left(1 - P_0\left(\underline{b}\right)\right) = \int d\underline{b}\left[1 - \left[1 - T_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right]^A\right]$$

$$\xrightarrow{A>>n} \int d\underline{b}\left[1 - \exp\left[-A T_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right]\right]$$ Optical limit

(1.7) $$= \int d\underline{b}\left[A T_A\left(\underline{b}\right)\sigma_{nn}^{inel} - \frac{1}{2}\left(A T_A\left(\underline{b}\right)\sigma_{nn}^{inel}\right)^2 + \ldots\right]$$

Double counting correction

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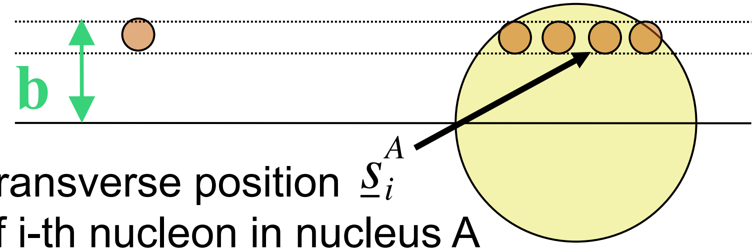
I.5. Glauber theory for n+A

To calculate number of collisions: probability of interacting with i-th nucleon in A is

(1.8) $$p(\underline{b},\underline{s}_i^A) = \int d\underline{s}_i^A T_A\left(\underline{s}_i^A\right)\sigma\left(\underline{b}-\underline{s}_i^A\right) = T_A(\underline{b})\,\sigma_{nn}^{inel}$$

Probability that projectile nucleon undergoes n collisions
= prob that n nucleons collide and A-n do not



Transverse position $\underline{s}_i^A$
of i-th nucleon in nucleus A

(1.9) $$P(\underline{b},n) = \binom{A}{n}(1-p)^{A-n}\,p^n$$

Average number of nucleon-nucleon collisions in n+A

(1.10) $$\overline{N}_{coll}^{nA}(\underline{b}) = \sum_{n=0}^{A} n\,P(\underline{b},n) = \sum_{n=0}^{A} n\binom{A}{n}(1-p)^{A-n}\,p^n = A\,p$$

$$= A\,T_A(\underline{b})\,\sigma_{nn}^{inel}$$

Average number of nucleon-nucleon collisions in n+A

(1.11) $$\overline{N}_{part}^{nA}(\underline{b}) = 1 + \overline{N}_{coll}^{nA}(\underline{b})$$

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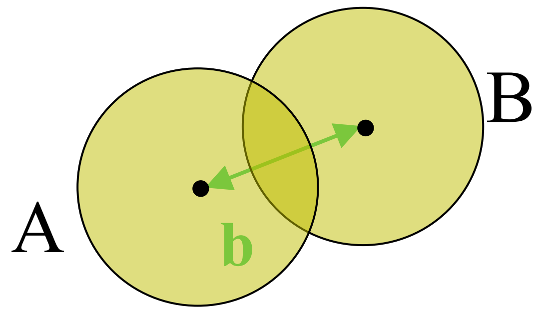
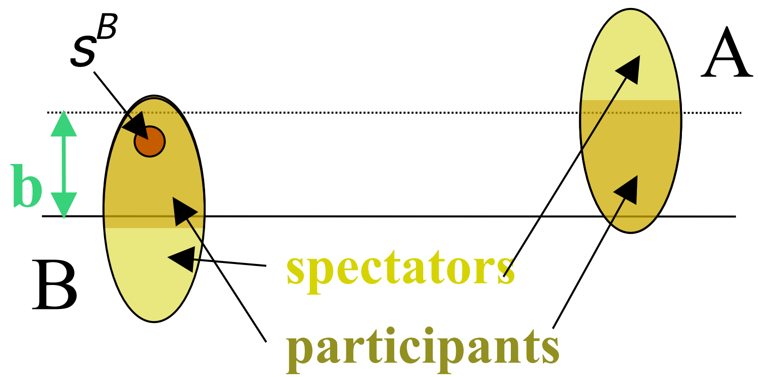
I.6. Glauber theory for A+B collisions

We define the nuclear overlap function



(1.12) $$T_{AB}(\vec{b}) = \int_{-\infty}^{\infty} d\vec{s}\; T_A(\vec{s})\, T_B(\vec{b}-\vec{s})$$

The average number of collisions of nucleon at s^B with nucleons in A is



(1.13) $$\overline{N}^{nA}_{coll}(\underline{b}-\underline{s}^B) = A\, T_A(\underline{b}-\underline{s}^B)\, \sigma^{inel}_{nn}$$

The number of nucleon-nucleon collisions in an A-B collision at impact parameter b is

(1.14) $$\overline{N}^{AB}_{coll}(\underline{b}) = B \int d\underline{s}^B\, T_B(\underline{s}^B)\, \overline{N}^{nA}_{coll}(\underline{b}-\underline{s}^B)$$

$$= AB \int d\underline{s}\, T_B(\underline{s})\, T_B(\underline{b}-\underline{s})\, \sigma^{inel}_{nn}$$

$$= A\,B\,T_{AB}(\underline{b})\, \sigma^{inel}_{nn}$$

determined in terms of nuclear overlap only

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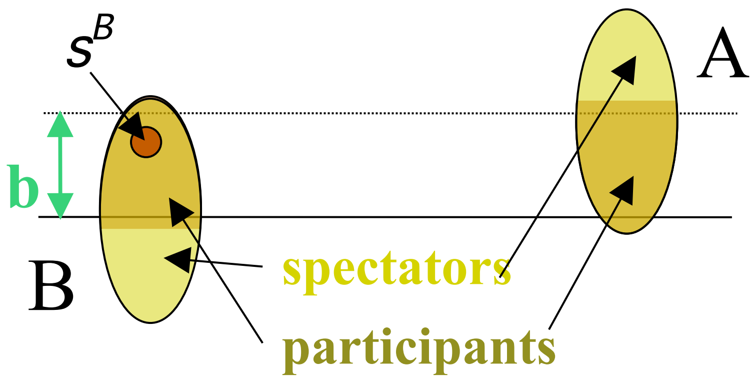
I.7. Glauber theory for A+B collisions

Probability that nucleon at s^B in B is wounded by A in configuration $\{s_i^A\}$



(1.15) $$p(\underline{s}^B, \{\underline{s}_i^A\}) = 1 - \prod_{i=1}^{A} \left[1 - \sigma(\underline{s}^B - \underline{s}_i^A)\right]$$

Probability of finding w_B wounded nucleons in nucleus B:

(1.16) $$P(w_b, \underline{b}) = \binom{B}{w_B} \left(\prod_{i=1}^{A} \prod_{j=1}^{B} \int d\underline{s}_i^A \, d\underline{s}_j^B \, T_A(\underline{s}_i^A) T_B(\underline{s}_j^B - \underline{b}) \right) p(\underline{s}_1^B, \{\underline{s}_i^A\}) \ldots$$

$$\ldots p(\underline{s}_{w_B}^B, \{\underline{s}_i^A\}) \left[1 - p(\underline{s}_{w_B+1}^B, \{\underline{s}_i^A\})\right] \ldots \left[1 - p(\underline{s}_B^B, \{\underline{s}_i^A\})\right]$$

Nuclear overlap function defines inelastic A+B cross section.

(1.17) $$\sigma_{AB}^{inel} = \int d\underline{b} \, \sigma_{AB}(\underline{b}) = \int d\underline{b} \, P(w_B = 0, \underline{b})$$

$$= \int d\underline{b} \left[1 - \left(\prod_{i=1}^{A} \prod_{j=1}^{B} \int d\underline{s}_i^A \, d\underline{s}_j^B \, T_A(\underline{s}_i^A) T_B(\underline{s}_j^B - \underline{b}) \right) \prod_{j=1}^{B} \left[1 - p(\underline{s}_j^B, \{\underline{s}_i^A\})\right] \right]$$

$$\approx \int d\underline{b} \left[1 - \left[1 - T_{AB}(b) \sigma_{NN}^{inel}\right]^{AB} \right]$$

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I.8. Glauber theory for A+B collisions

It can be shown

Problem 1: derive the expressions (1.17), (1.19)
Use e.g. A. Bialas et al., Nucl. Phys. B111 (1976) 461

(1.18) Number of collisions:

$$\overline{N}_{coll}^{AB}(\underline{b}) = AB\, T_{AB}(\underline{b})\, \sigma_{NN}^{inel}$$

(1.19) Number of participants:

$$\overline{N}_{part}^{AB}(\underline{b}) = \frac{A\sigma_B^{inel}(\underline{b})}{\sigma_{AB}^{inel}(\underline{b})} + \frac{B\sigma_A^{inel}(\underline{b})}{\sigma_{AB}^{inel}(\underline{b})} \neq \overline{N}_{coll}^{AB}(\underline{b}) + 1$$

1. There is a difference between 'analytical' and 'Monte Carlo' Glauber theory: For 'MC Glauber, a random probability distribution is picked from T_A.

2. The nuclear density is commonly taken to follow a Wood-Saxon parametrization (e.g. for A > 16)

(1.20)

$$\rho(\vec{r}) = \rho_0 \Big/ \left(1 + \exp\left[-(r-R)/c\right]\right); \qquad R \equiv 1.07\, A^{1/3}\, fm, c = 0.545\, fm.$$

C.W. de Jager, H.DeVries, C.DeVries, Atom. Nucl. Data Table 14 (1974) 479

3. The inelastic Cross section is energy dependent, typically

(1.21)

$$\sigma_{nn}^{inel} \approx 40\ (65)\, mb \qquad at \qquad \sqrt{s_{nn}} = 100\ \ (2700)\ GeV.$$

But σ_{nn}^{inel} is sometimes used as fit parameter.

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I.9 Event Multiplicity in wounded nucleon model

Model assumption: If $\bar{n}_{nn}$ is the average multiplicity in an n-n collision, then

(1.22)

$$\bar{n}_{AB}(b) = \left(\frac{1-x}{2} \bar{N}^{AB}_{part}(b) + x\, \bar{N}^{AB}_{coll}(b) \right) \bar{n}_{NN}$$

is average multiplicity in A+B collision
(x=0 defines the wounded nucleon model).

The probability of having w_b wounded nucleons fluctuates around the mean,,
so does the multiplicity n per event (the dispersion d is a fit parameter, say d~1)

(1.23)

$$P(n,\underline{b}) = \frac{1}{\sqrt{2\pi\, d\, \bar{n}_{AB}(\underline{b})}} \exp\left(- \frac{\left[n - \bar{n}_{AB}(\underline{b}) \right]^2}{2d\, \bar{n}_{AB}(\underline{b})} \right)$$

How many events dN_{events} have event multiplicity dn?

(1.24)

$$\frac{dN_{events}}{dn} = \int db\, P(n,b) \underbrace{\left[1 - \left(1 - \sigma_{NN} T_{AB}(b) \right)^{AB} \right]}_{1-P_0(b)}$$

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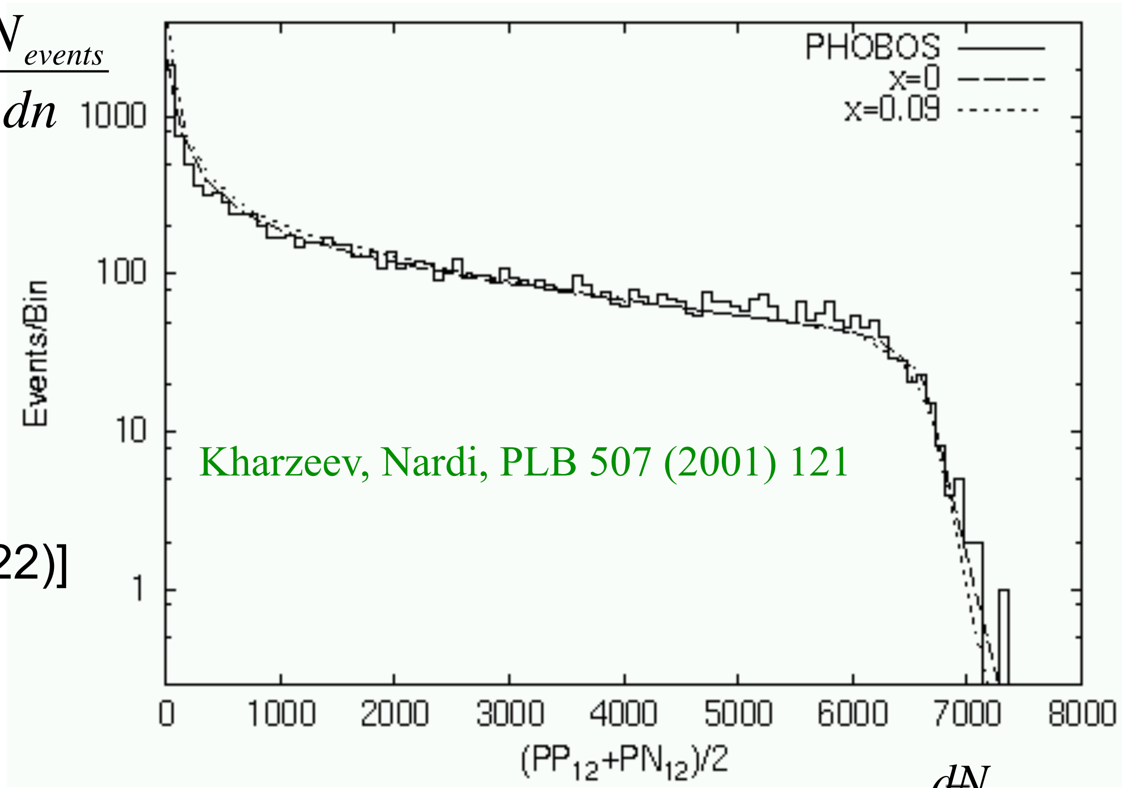
I.10 Wounded nucleon model vs. multiplicity

Compare data to multiplicity distribution (1.24): $\frac{dN_{events}}{dn} = \int d\underline{b}\, P(n,\underline{b})\left[1 - P_0(\underline{b})\right]$

$\frac{dN_{events}}{dn}$

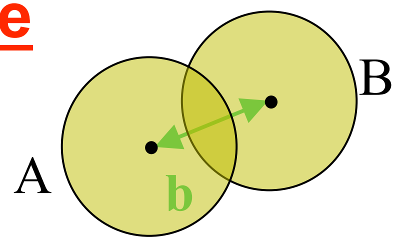


- determined by geometry only
- insensitive to details of particle production [there is only a weak dependence on parameter x in (1.22)]
- insensitive to collective effects

Sensitivity to geometry but insensitivity to model-dependent dynamics makes $\frac{dN_{events}}{dn}$

A well-suited centrality measure

(i.e. a measure of the impact parameter b)



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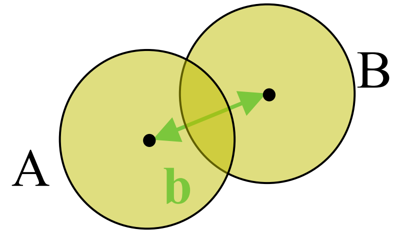
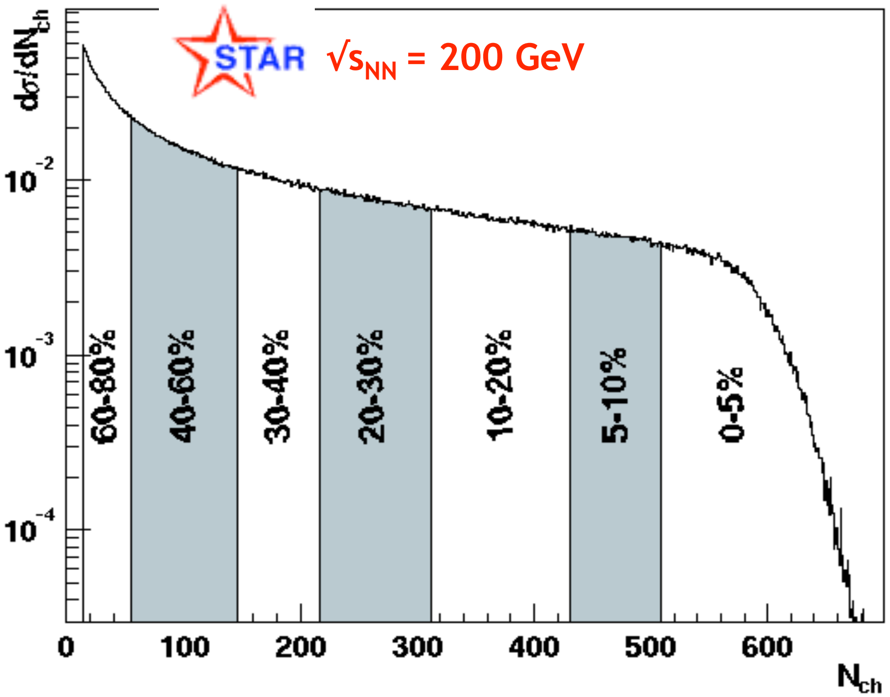
I.11. Multiplicity as a Centrality Measure

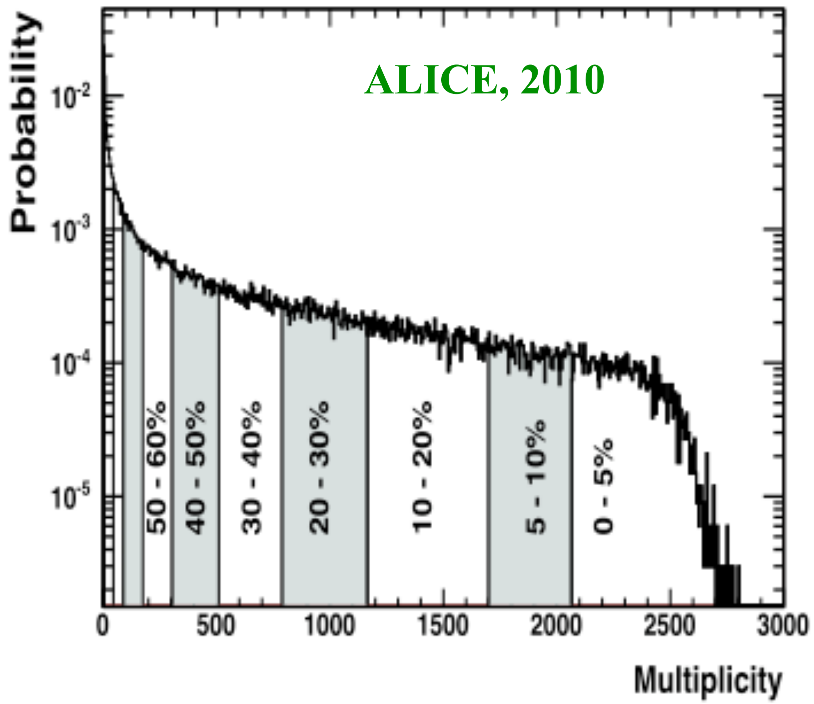
The connection between centrality and event multiplicity can be expressed in terms of



(1.25) $$\left\langle N_{part}^{A+A} \right\rangle_{n>n_0} = \frac{\int_{n_0} dn \int db\, P(n,\underline{b}) \left[1 - P_0(\underline{b})\right] N_{part}(\underline{b})}{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b}) \left[1 - P_0(\underline{b})\right]}$$

- Centrality class = percentage of the minimum bias cross section





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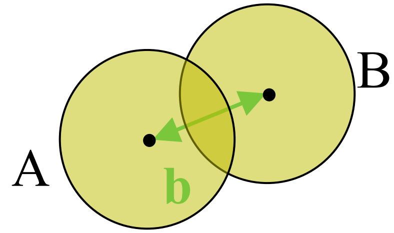
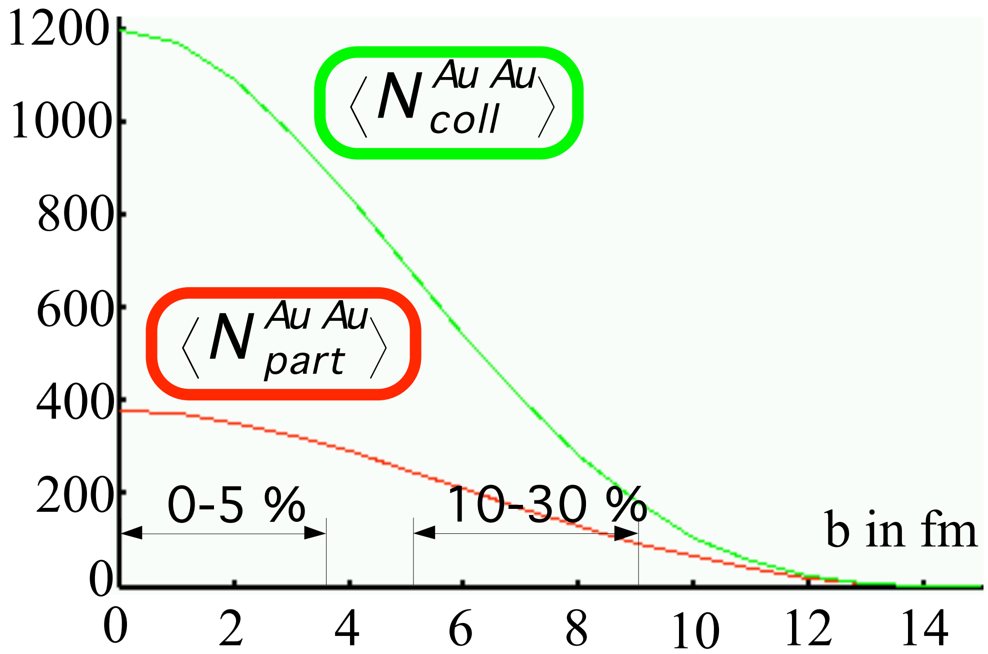
I.12. Centrality Class fixes Impact Parameter

The connection between centrality and event multiplicity can be expressed in terms of



(1.25) $$\left\langle N_{part}^{A+A} \right\rangle_{n>n_0} = \frac{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b})\left[1-P_0(\underline{b})\right] N_{part}(\underline{b})}{\int_{n_0} dn \int d\underline{b}\, P(n,\underline{b})\left[1-P_0(\underline{b})\right]}$$

- Centrality class specifies range of impact parameters



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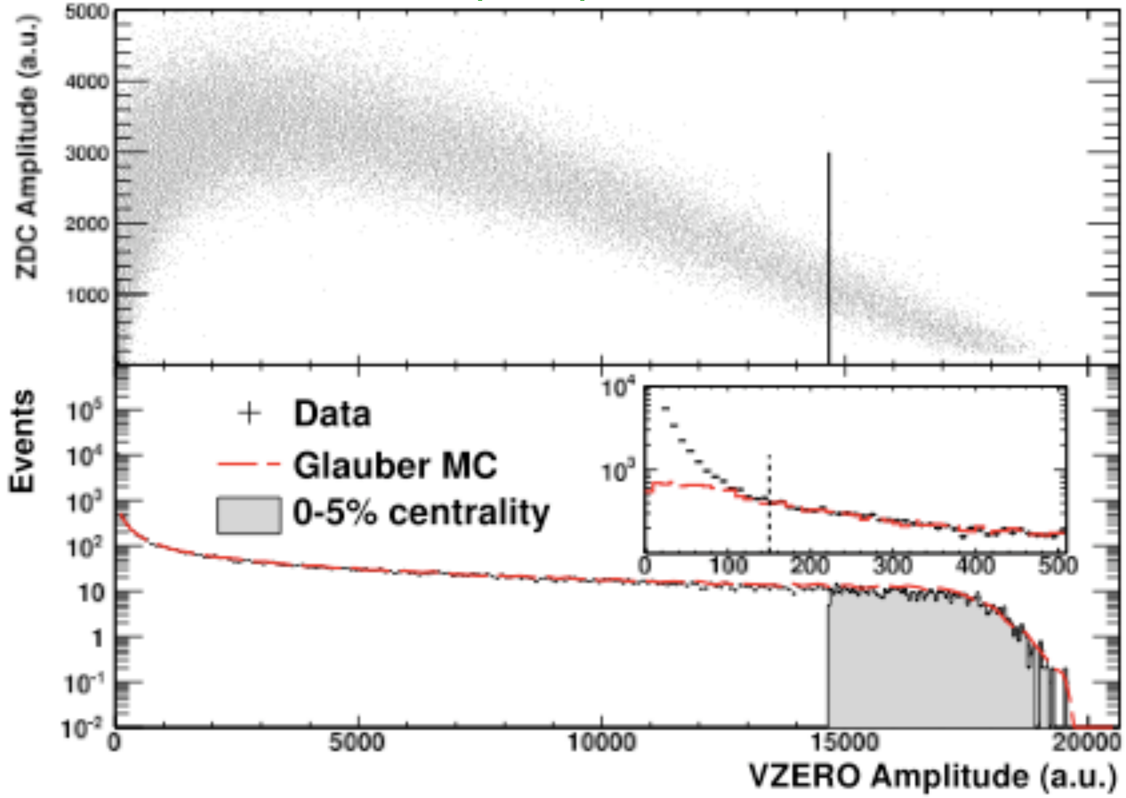
I.13. Cross-Checking Centrality Measurements

The interpretation of min. bias multiplicity distributions in terms of centrality measurements can be checked in multiple ways, e.g.

1. Energy E_F of spectators is deposited in Zero Degree Calorimeter (ZDC)

$$E_F = \left(A - N_{part}(b)/2\right)\sqrt{s}/2$$

ALICE, PRL 105 (2010) 252301, arXiv:1011.3916



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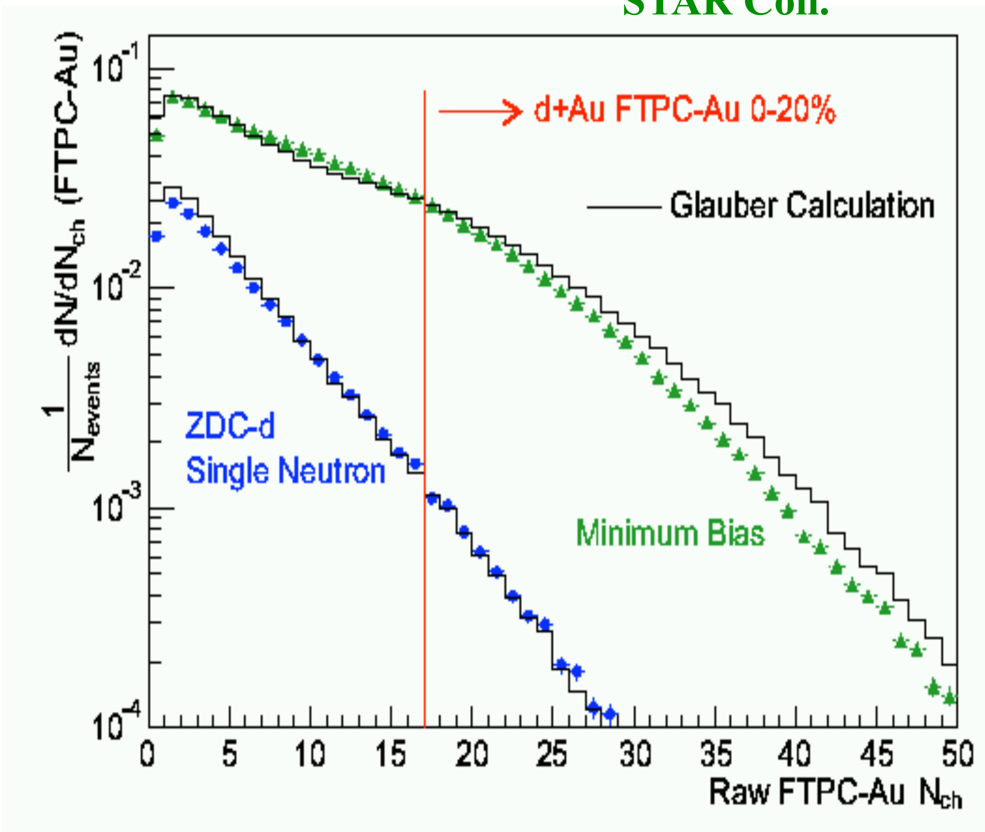
I.14. Cross-Checking Centrality Measurements

The interpretation of min. bias multiplicity distributions in terms of centrality measurements can be checked in multiple ways, e.g.

2. Testing Glauber in d+Au and in p+Au(+ n forward)

STAR Coll.



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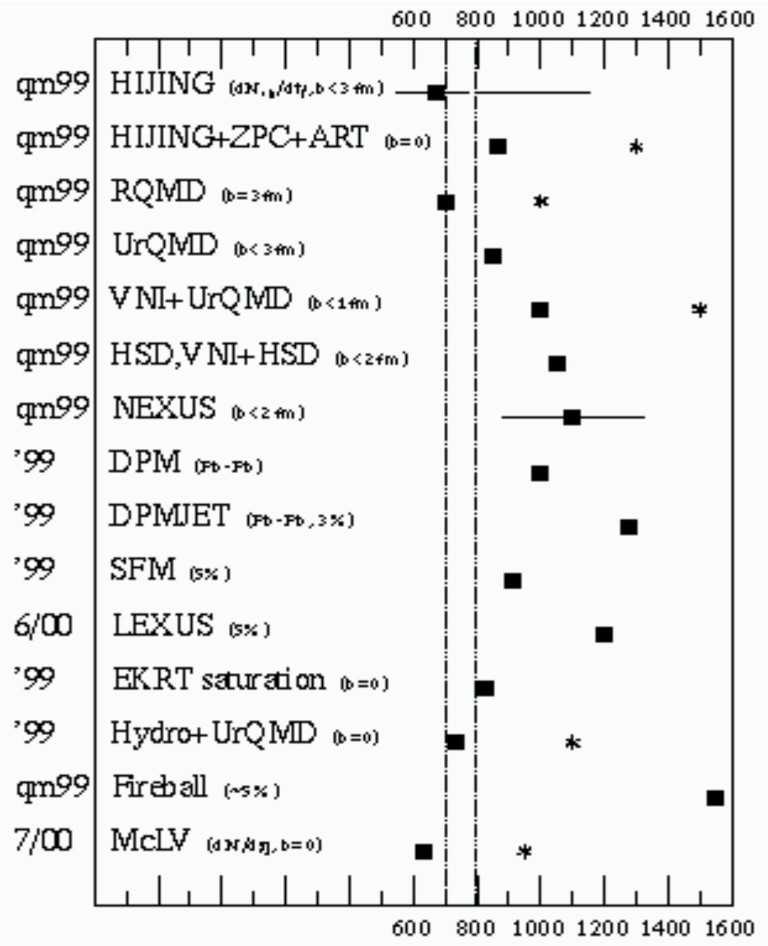
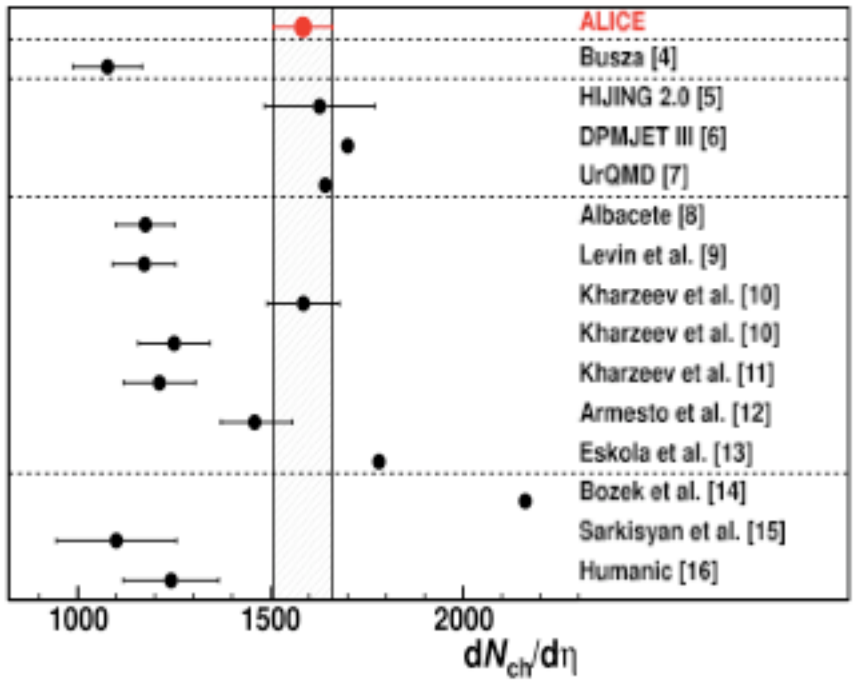
I.15. Final remarks on event multiplicity in A+B

There is **no 1st principle QCD calculation** of event multiplicity, neither in p+p nor in A+B

- Total charged event multiplicity: models failed to predict RHIC



- and failed to predict LHC



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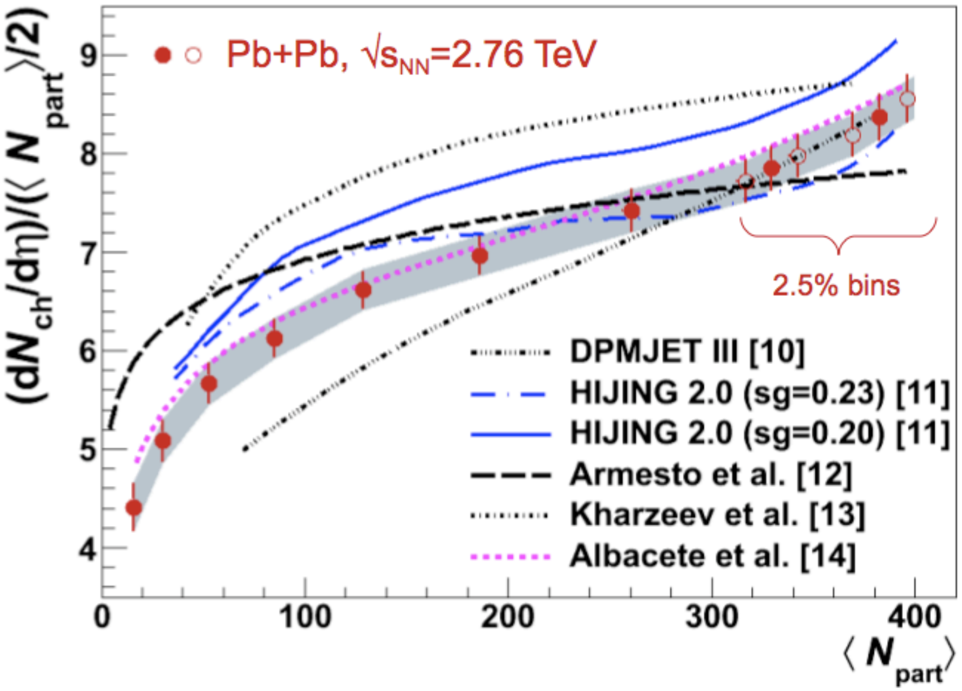
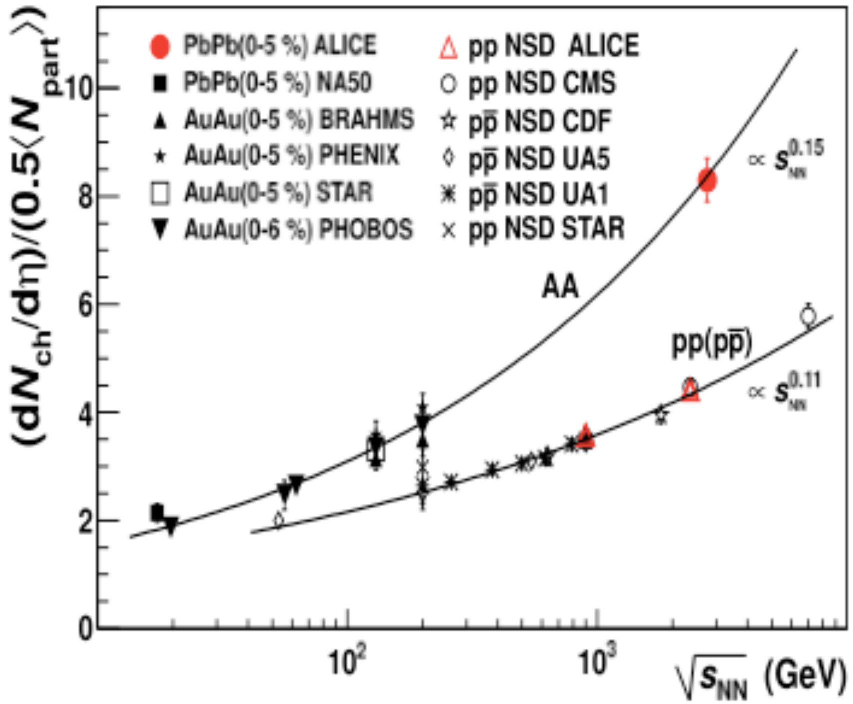
I.16. Final remarks on event multiplicity in A+B

There is **no 1st principle QCD calculation** of event multiplicity, neither in p+p nor in A+B

- Clear deviations from multiplicity of wounded nucleon model
- $\sqrt{s}$ - dependence of event multiplicity not understood in pp and AA

ALICE Coll., PRL 106, 032301 (2001) arXiv:1012.1657





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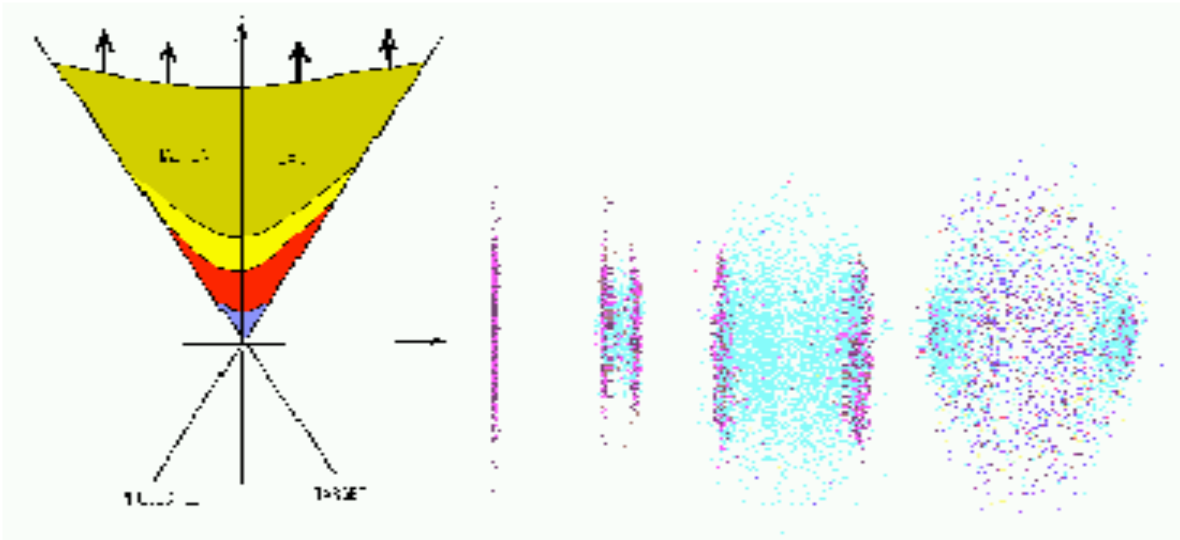
I.17. Final remarks on event multiplicity

Multiplicity distribution is not only used as centrality measure but:



Multiplicity (or transverse energy) thought to determine properties of produced matter

Bjorken estimate

$$\varepsilon(\tau_0) = \frac{1}{\pi R^2}\frac{1}{\tau_0}\frac{dE_T}{dy}$$

$$\frac{dE_T}{dy} \approx \frac{dN}{dy}\langle E_T \rangle$$

This estimate is based on geometry, thermalization is not assumed, numerically:

$$\varepsilon^{SPS}(\tau_0 \cong 1\, fm/c) = 3-4\, GeV/fm^3$$

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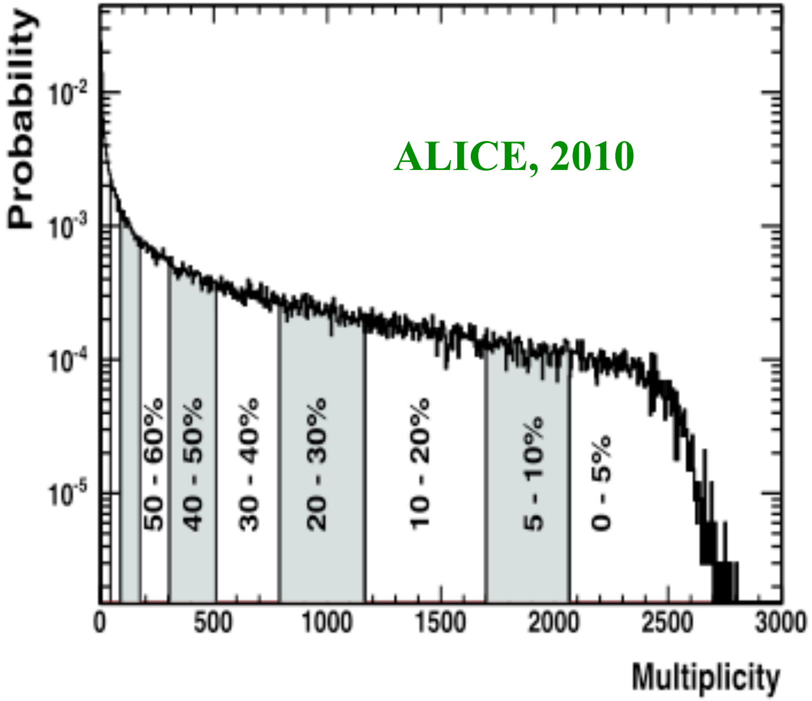
II.1. Azimuthal Anisotropies of Particle Production

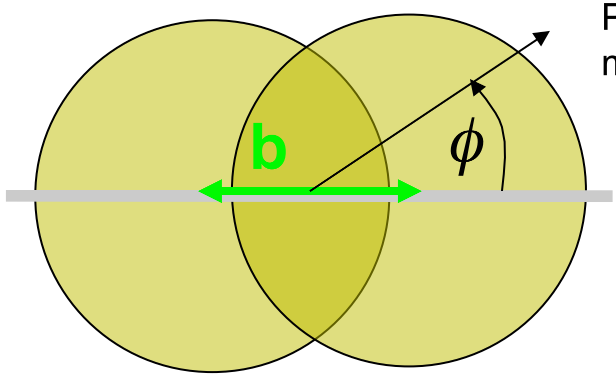
We know how to associate an impact parameter range $b \in [b_{\min}, b_{\max}]$ to an event class in A+A, namely by selecting a multiplicity class.



What can we learn by characterizing not only the modulus b, but also the orientation $\underline{b}$?

II.2. Particle production w.r.t. reaction plane

Consider single inclusive particle momentum spectrum



(2.1) $$f(\vec{p}) \equiv dN/d\vec{p}$$

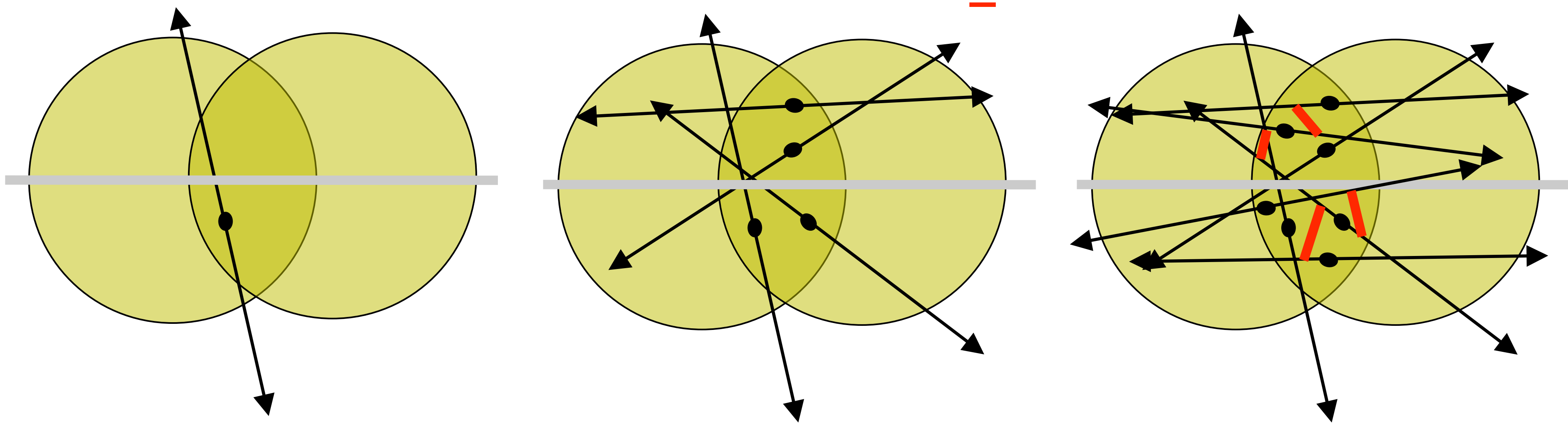
(2.2) $$\vec{p} = \begin{pmatrix} p_x = p_T \cos\phi \\ p_y = p_T \sin\phi \\ p_z = \sqrt{p_T^2 + m^2} \sinh Y \end{pmatrix}$$

To characterize azimuthal asymmetry, measure n-th harmonic moment of (2.1) in some detector acceptance *D* [phase space window in (p_T,Y)-plane].

(2.3) $$v_n(D) \equiv \left\langle e^{i n \phi} \right\rangle_D = \frac{\int_D d\vec{p}\, e^{i n \phi} f(\vec{p})}{\int_D d\vec{p}\, f(\vec{p})}$$ **n-th order flow**

<u>Problem</u>: Eq. (2.3) cannot be used for data analysis, since the orientation of the reaction plane is not known a priori.

II.3. Why is the study of v_n interesting?



- Single 2->2 process
- Maximal asymmetry
- NOT correlated to the reaction plane

- Many 2->2 or 2-> n processes
- Reduced asymmetry $\sim 1/\sqrt{N}$
- NOT correlated to the reaction plane

- final state interactions
- asymmetry caused not only by multiplicity fluctuations
- collective component is correlated to the reaction plane

The azimuthal asymmetry of particle production has a collective and a random component. Disentangling the two requires a statistical analysis of finite multiplicity fluctuations.

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II.4. Cumulant Method

If reaction plane is unknown, consider particle correlations

(2.4) $$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{D_1\wedge D_2} = \frac{\int_{D_1\wedge D_2} d\vec{p}_1 d\vec{p}_2\, e^{in(\phi_1-\phi_2)} f(\vec{p}_1,\vec{p}_2)}{\int_{D_1\wedge D_2} d\vec{p}_1 d\vec{p}_2\, f(\vec{p}_1,\vec{p}_2)}$$

A two-particle distribution has an uncorrelated and a correlated part

(2.5) $$f(\vec{p}_1,\vec{p}_2) = f(\vec{p}_1)f(\vec{p}_2) + f_c(\vec{p}_1,\vec{p}_2)$$

(2.6) Short hand $$(1,2) = (1)(2) + (1,2)_c$$

Correlated part

<u>Assumption</u>: Event multiplicity N>>1

→ <u>correlated</u> part is O(1/N)-correction to $f(\vec{p}_1)f(\vec{p}_2)$

(2.7) $$\left\langle e^{in(\phi_1-\phi_2)}\right\rangle_{D_1\wedge D_2} = v_n(D_1)\,v_n(D_2) + \underbrace{\left\langle e^{in(\phi_1-\phi_2)}\right\rangle^{corr}_{D_1\wedge D_2}}_{O(1/N)}$$

"Non-flow effects"

(2.8) If $v_n(D) >> \frac{1}{\sqrt{N}}$,then non-flow corrections are negligible.

What, if this is not the case?

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II.5. 4-th order Cumulants

2nd order cumulants allow to characterize v_n, if $v_n \gg 1/\sqrt{N}$.
Consider now 4-th order cumulants:

(2.9)
$$
\begin{aligned}
(1,2,3,4) = {} & (1)(2)(3)(4) + (1,2)_c(3)(4) + \ldots \\
& + (1,2)_c(3,4)_c + (1,3)_c(2,4)_c + (1,4)_c(2,3)_c \\
& + (1,2,3)_c(4) + \ldots \\
& + (1,2,3,4)_c
\end{aligned}
$$

If the system is isotropic, i.e. $v_n(D)=0$, then k-particle correlations are unchanged by rotation $\phi_i \to \phi_i + \phi$ for all i, and only labeled terms survive. This defines

(2.9)
$$
\left\langle\left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle
$$
$$
\equiv \left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle - \left\langle e^{i\,n(\phi_1-\phi_3)}\right\rangle\left\langle e^{i\,n(\phi_2-\phi_4)}\right\rangle - \left\langle e^{i\,n(\phi_1-\phi_4)}\right\rangle\left\langle e^{i\,n(\phi_2-\phi_3)}\right\rangle
$$

For small, non-vanishing v_n, one finds

Borghini, Dinh, Ollitrault, PRC (2001)

(2.10)
$$
\left\langle\left\langle e^{i\,n(\phi_1+\phi_2-\phi_3-\phi_4)}\right\rangle\right\rangle = -v_n^4 + O\left(\frac{1}{N^3}, \frac{v_{2n}^2}{N^2}\right)
$$

<u>Improvement</u>: signal can be separated from fluctuating background, if

$$
v_N \gg \frac{1}{N^{3/4}}
$$

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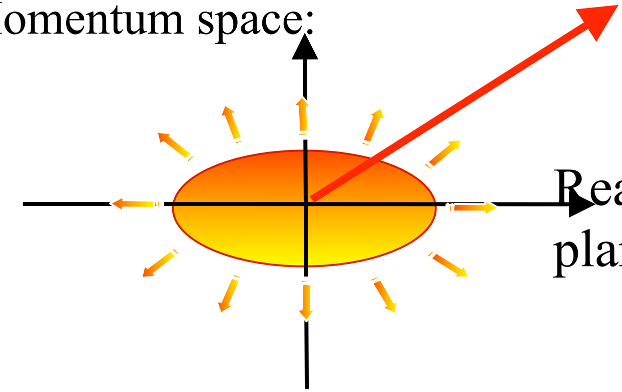
II.6. LHC and RHIC Data on Elliptic Flow: v_2

(2.11) $$E\frac{dN}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_T dp_T d\eta}\left[1 + 2v_2(p_T)\cos\left(2(\phi - \psi_{reaction\ plane})\right)\right]$$

- Momentum space:



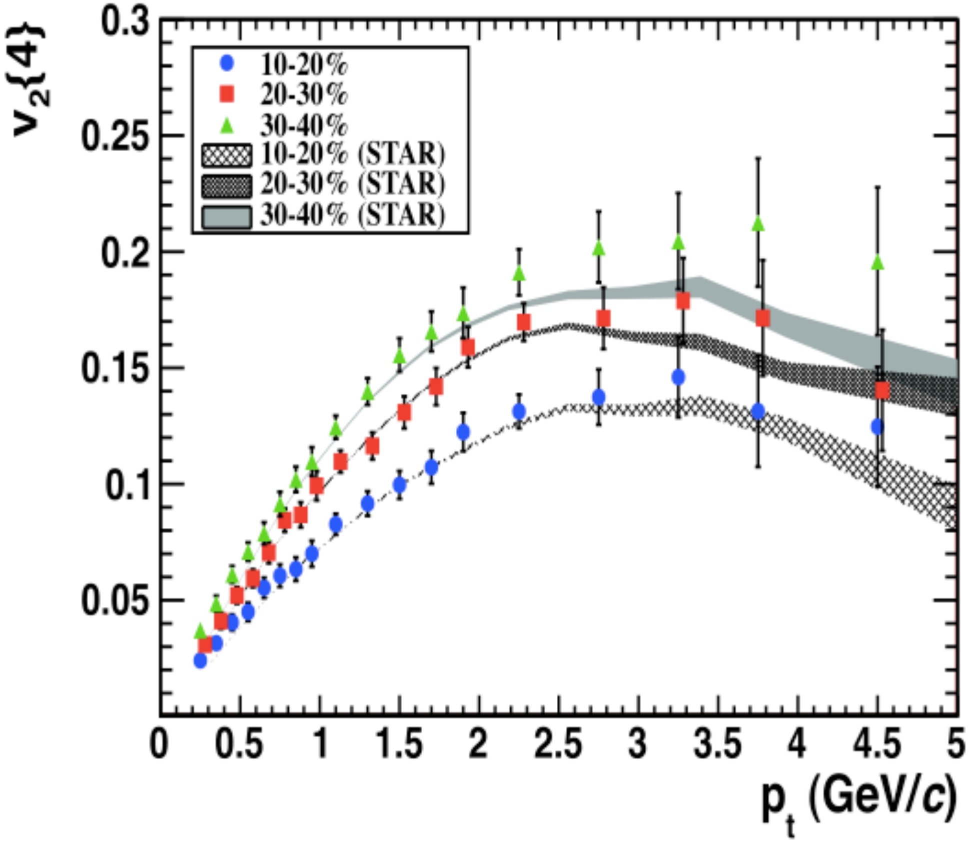
- Signal $v_2 \approx 0.2$ implies 2-1 asymmetry of particles production w.r.t. reaction plane.

- 'Non-flow' effect for 2nd order cumulants

(2.12) $$N \sim 100 \Rightarrow 1/\sqrt{N} \sim O(v_2)$$

2nd order cumulants do not characterize solely collectivity.

(2.13) $$1/N^{3/4} \sim 0.03 \ll v_2$$



Non-flow effects should disappear if we go from 2nd to 4th order cumulants.

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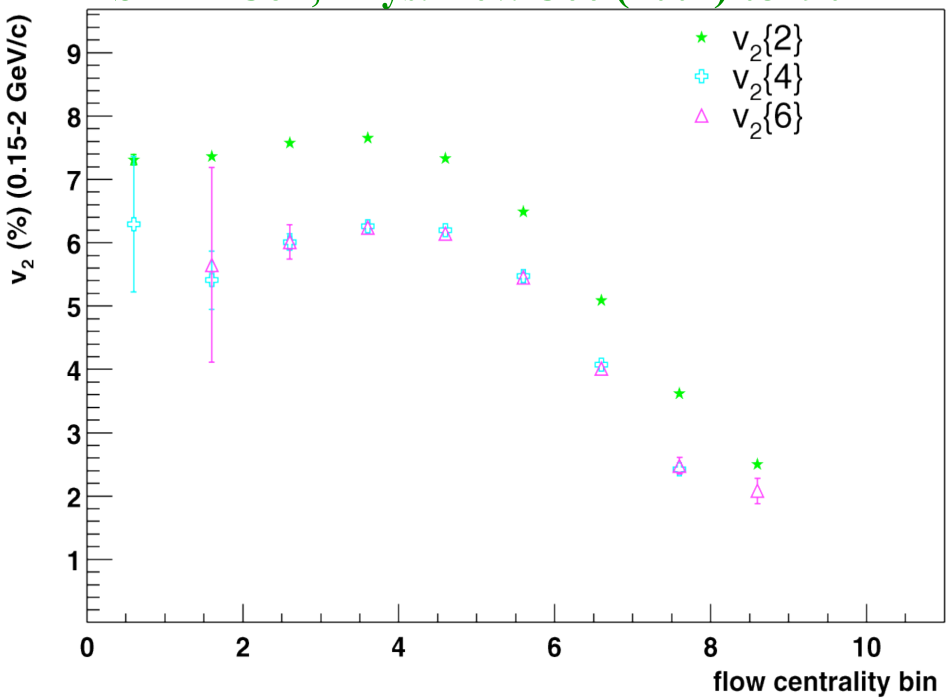
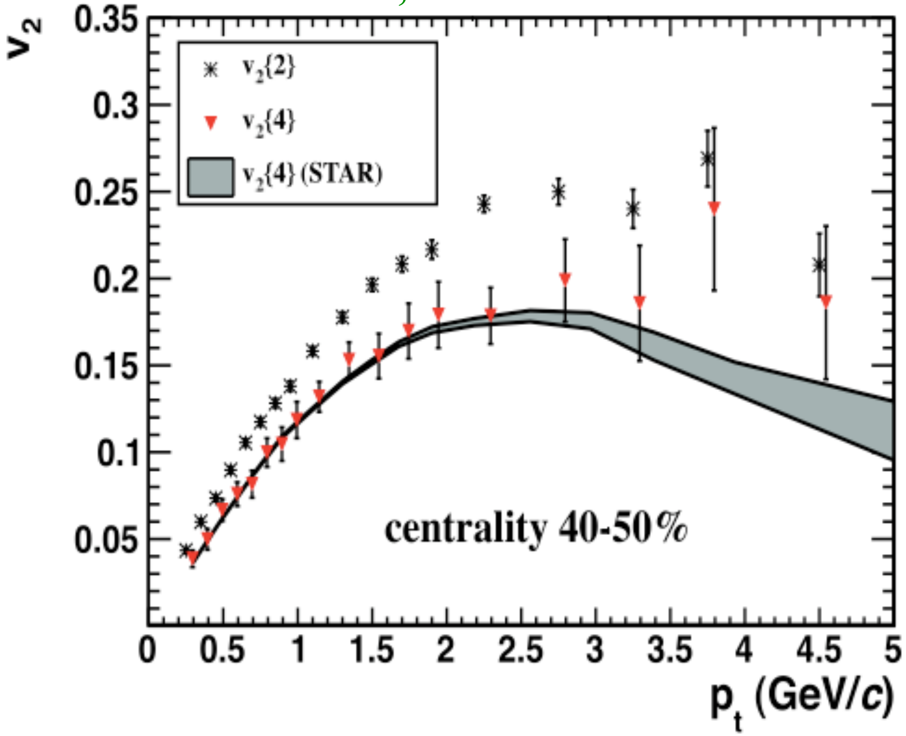
II.7. Establishing collectivity in v_2

- pt-integrated v2 stabilizes at 4th order cumulants
- pt-differential v2 from 2nd and 4th order cumulants

STAR Coll, Phys. Rev. C66 (2002) 034904



ALICE Coll., arXiv:1011.3914



Elliptic flow signal is stable if reconstructed from higher order cumulants.

We have established a **strong collective effect**, which cannot be mimicked by multiplicity fluctuations in the reaction plane.

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