Selected Topics in the Theory of Heavy Ion Collisions

Lecture 2

Urs Achim Wiedemann
CERN Physics Department
TH Division

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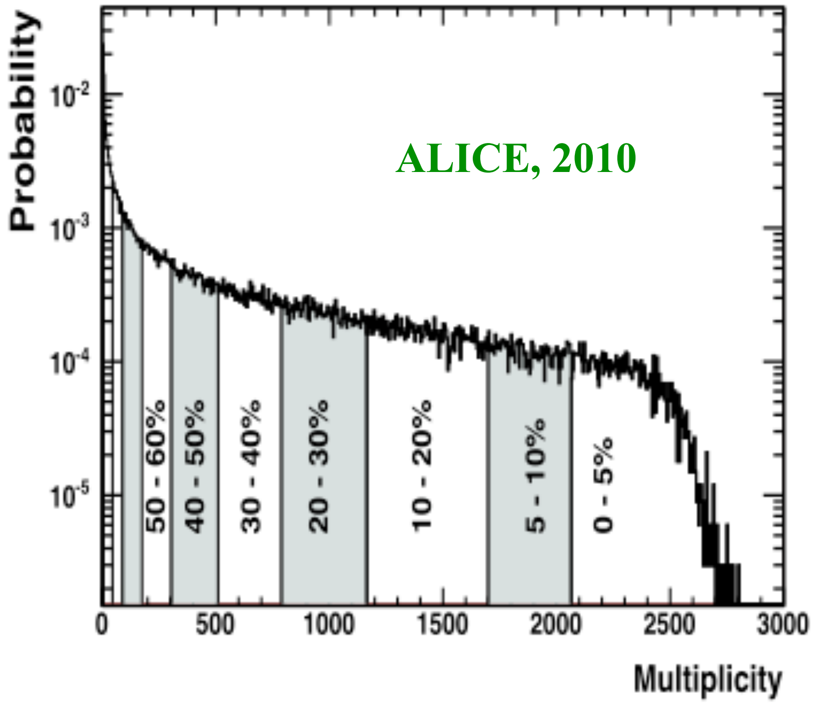
Recall of lecture 1: Azimuthal Anisotropies of Particle Production

We know how to associate an impact parameter range $b \in [b_{min}, b_{max}]$ to an event class in A+A, namely by selecting a multiplicity class.



What can we learn by characterizing not only the modulus b, but also the orientation $\underline{b}$?

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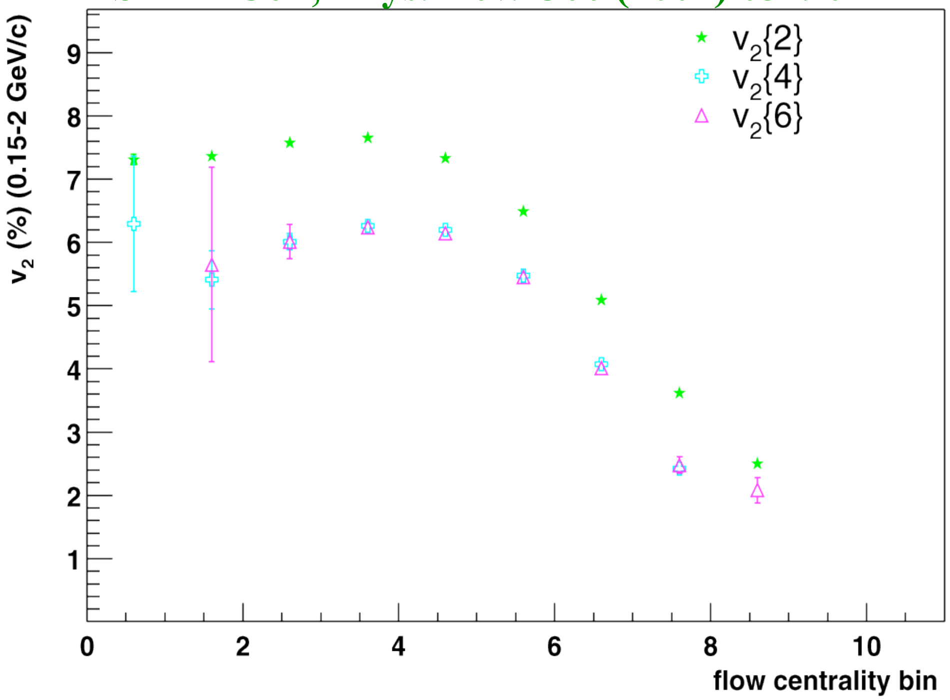
Recall: elliptic flow = hallmark of collectivity

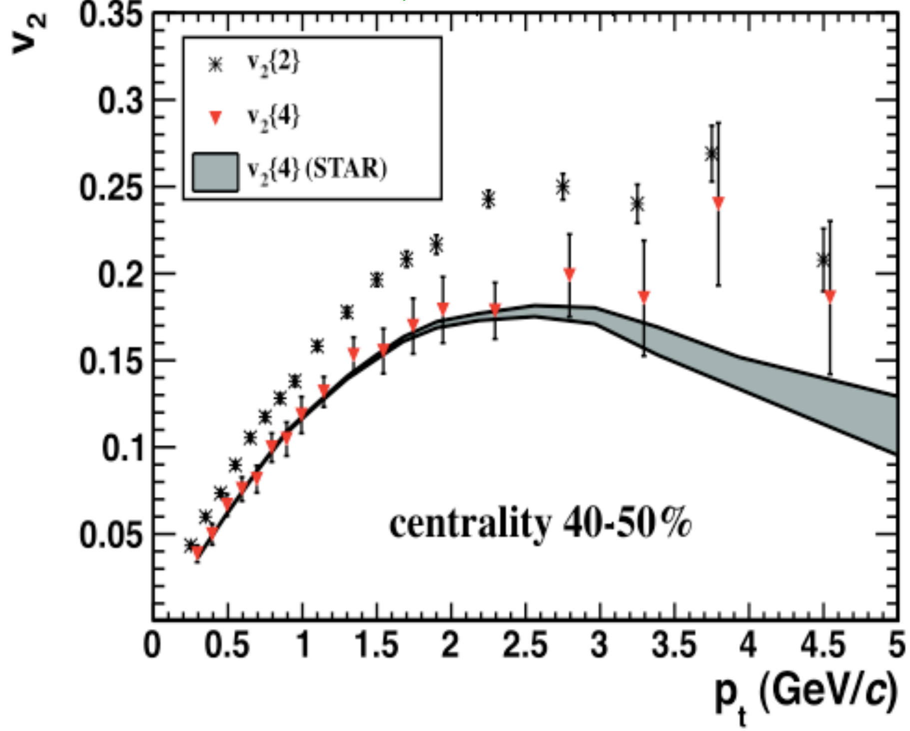
- pt-integrated v2 stabilizes at 4th order cumulants

STAR Coll, Phys. Rev. C66 (2002) 034904



- pt-differential v2 from 2nd and 4th order cumulants

ALICE Coll., arXiv:1011.3914



Elliptic flow signal is stable if reconstructed from higher order cumulants.

We have established a **strong collective effect**, which cannot be mimicked by multiplicity fluctuations in the reaction plane.

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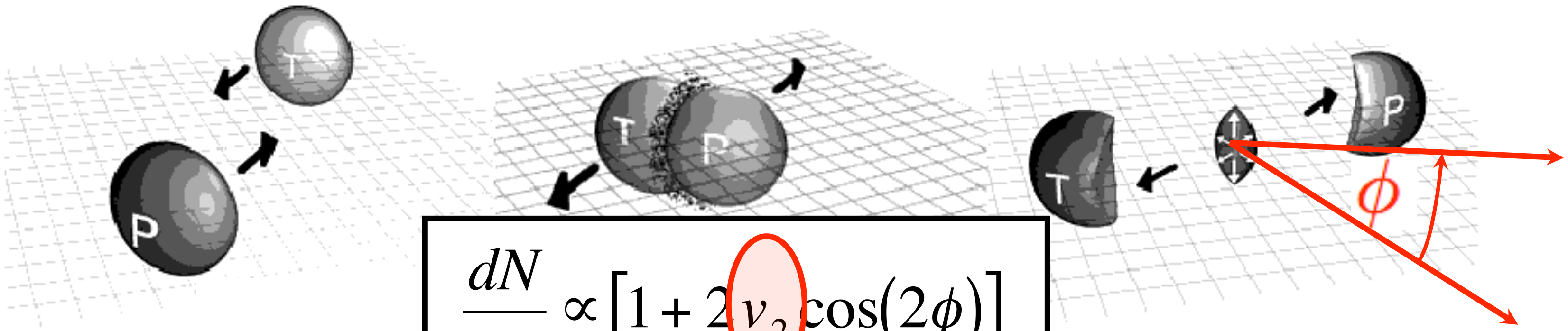
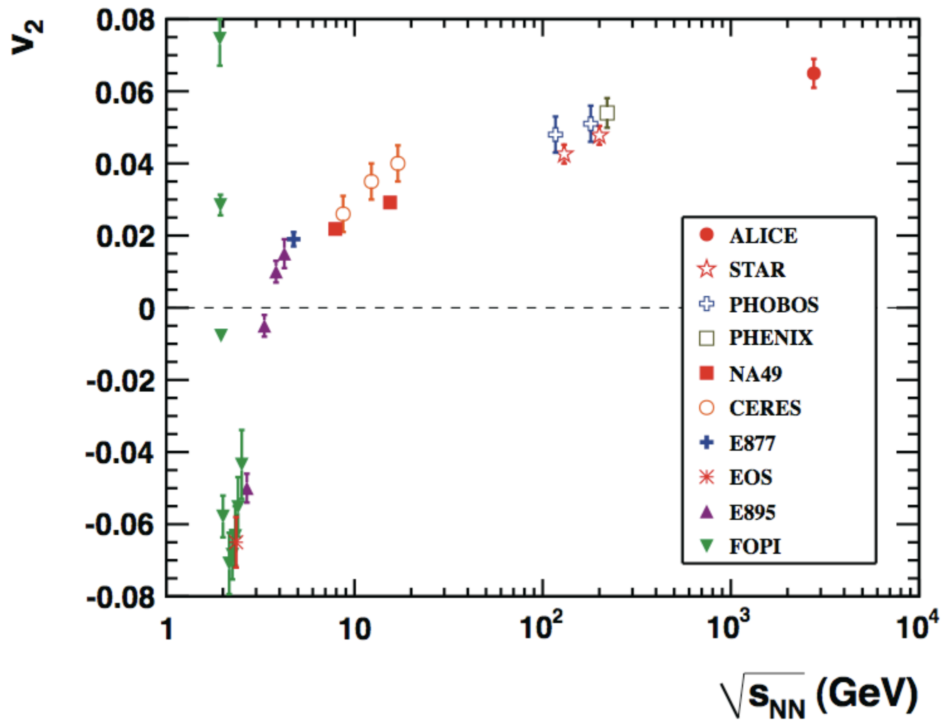
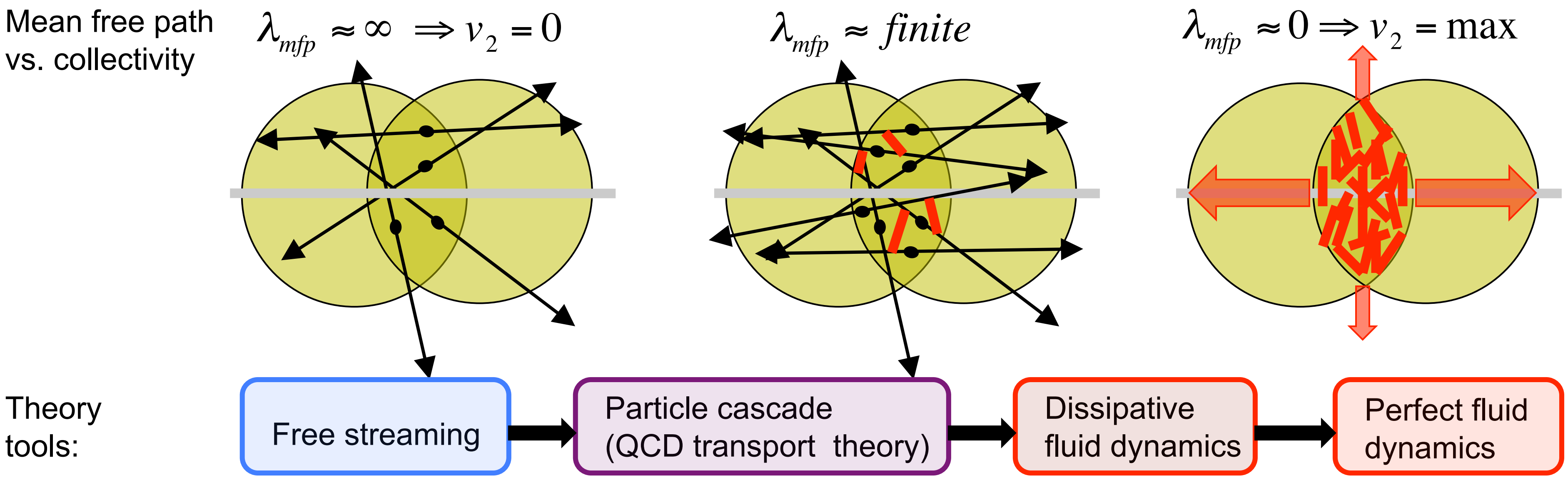
Elliptic Flow:
Hallmark of a collective phenomenon



$$\frac{dN}{d\phi} \propto \left[1 + 2 v_2 \cos(2\phi)\right]$$



Dynamical framework for modeling elliptic flow



To understand the size of v2, let us study a theoretical baseline:
the zero mean free path limit of final state interactions:

→ **Fluid dynamics**

II.1. Fluid dynamics - the basics

Consider matter in local equilibrium, characterized locally by its energy momentum tensor, the density of n charges, and a flow field:

- energy momentum tensor $T^{\mu\nu}$ ……. 10 indep. components
- conserved charges N_i^μ …………… 4n indep. components

Tensor decomposition w.r.t. flow field $u_\mu(x)$ projector $\Delta_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

(2.1) $$N_i^\mu = n_i\, u^\mu + \bar{n}_i$$

(2.2) $$T^{\mu\nu} = \varepsilon\, u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$$

(2.3) (1 comp.) $\varepsilon \equiv u_\mu T^{\mu\nu} u_\nu$ energy density

(2.4) (1 comp.) $p \equiv -T^{\mu\nu}\Delta_{\mu\nu}/3$ isotropic pressure

(2.5) (3 comp.) $q^\mu \equiv \Delta^{\mu\alpha} T_{\alpha\beta} u^\beta$ heat flow

(2.6) (5 comp.) $\Pi^{\mu\nu} \equiv \left[\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right] T^{\alpha\beta}$ shear viscosity

In Local Rest Frame (LRF) $u_\mu = (1,0,0,0)$

Convenient choice of frame: Landau frame: $u = u_L \Rightarrow q^\mu = 0$

Eckard frame: …

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II.2. Equations of motion for a perfect fluid

A fluid is perfect if it is locally isotropic at all space-time points. This implies

(2.7) $$N_i^{\mu} = n_i\, u^{\mu} + \cancel{\bar{n}_i}$$ (n comp.)

(2.8) $$T^{\mu\nu} = \varepsilon\, u^{\mu} u^{\nu} - p\Delta^{\mu\nu} + \cancel{q^{\mu} u^{\nu}} + \cancel{q^{\nu} u^{\mu}} + \cancel{\Pi^{\mu\nu}}$$ (5 comp.)

The equations of motion are then determined by conservation laws

(2.9) $$\partial_{\mu} N_i^{\mu} \equiv 0$$ (n constraints)

(2.10) $$\partial_{\mu} T^{\mu\nu} \equiv 0$$ (4 constraints)

and the equation of state

(2.11) $$p = p(\varepsilon, n)$$ (1 constraint)

Here, information from ab initio calculations (lattice) or models enters.

Hydrodynamic simulations are numerical solutions of (2.7),(2.8).
'Systematic' model uncertainties arise from
- specifying initial conditions
- specifying the decoupling of particles ('freeze-out')
- assuming that non-perfect terms in (2.7),(2.8) can be dropped
- specifying (2.11)

II.3. Two-dimensional Bjorken fluid dynamics

Main assumption: initial conditions for thermodynamic fields do not depend on space-time rapidity

(2.12) $$\eta = \frac{1}{2}\ln\left[\frac{t+z}{t-z}\right]$$

Longitudinal flow has 'Hubble form':

(2.13) $$v_z = z/t$$

Bjorken scaling means that hydrodynamic equations preserve Hubble form

(2.14) $$u^\mu = \cosh y_T \left(\cosh\eta, v_x, v_y, \sinh\eta\right)$$ **Longitudinally boost-invariant flow profile**

(2.15) at mid-rapidity $$v_r(\tau, r, \eta = 0) \equiv \tanh y_T(\tau, r)$$

(2.16) at forward rapidity $$v_r(\tau, r, \eta) \equiv \frac{v_r(\tau, r, \eta = 0)}{\cosh\eta}$$

Problem: show that e.o.m. (2.10) preserve longitudinal boost-invariance of initial conditions.
solution see e.g. Kolb+Heinz, PRC62 (2000) 054909

II.4. 2-dim ‘perfect’ Hydro Simulations: Input

Initialization: thermo-dynamic fields $\varepsilon(\tau, r, \eta = 0)$ have to be initialized, e.g. by

(2.17) $$\varepsilon_{init}(\underline{r}) = \varepsilon(\tau_0, \underline{r}, \eta = 0) \propto \left(\frac{1-x}{2} \overline{N}^{AB}_{part}(\underline{b}, \underline{r}) + x \overline{N}^{AB}_{coll}(\underline{b}, \underline{r}) \right)$$

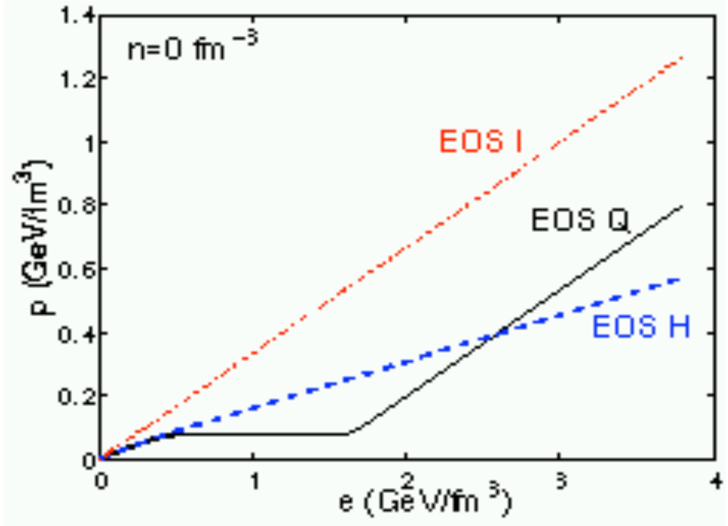
Equation of state: $p(\varepsilon, n)$

(2.18) Velocity of sound: $$c_s^2 = \frac{\partial p}{\partial \varepsilon}$$

(2.19) Expectations: $c_s^2 \approx 0.15$ **Soft EOS**

$c_s^2 = 1/3$ **Hard EOS**

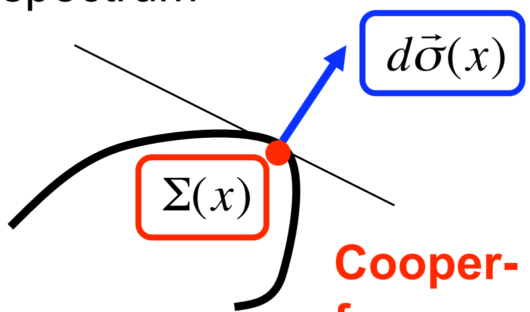
Input from (many) models and from lattice QCD.



Freeze-out: local temperature $T(x) = T_{fo}$ defines space-time hypersurface $\Sigma(x)$,
from which particles decouple with spectrum

(2.20) $$E \frac{dN_i}{d\vec{p}} = \frac{g_i}{(2\pi)^3} \int_\Sigma \vec{p}.d\vec{\sigma}(x)\, f_i(p.u(x), x)$$

(2.21) $$f_i(E, x) = \frac{1}{\exp[(E - \mu_i(x))/T(x)] \pm 1}$$

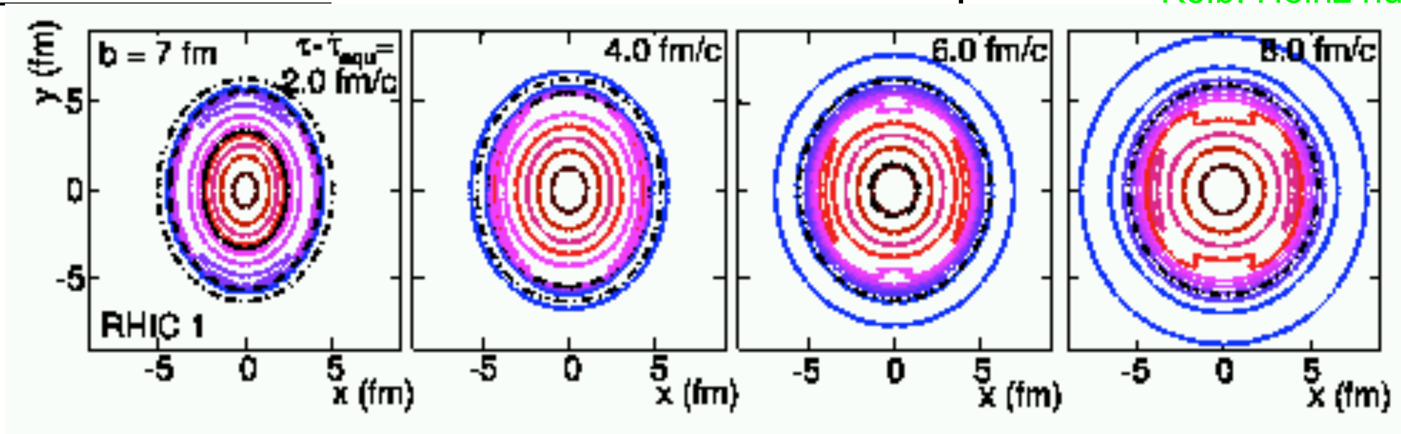


Cooper- Frye freeze-out

II.5. Elliptic flow vs. hydrodynamic simulations

Results of simulations: time evolution in transverse plane

Kolb, Heinz nucl-th/0305084

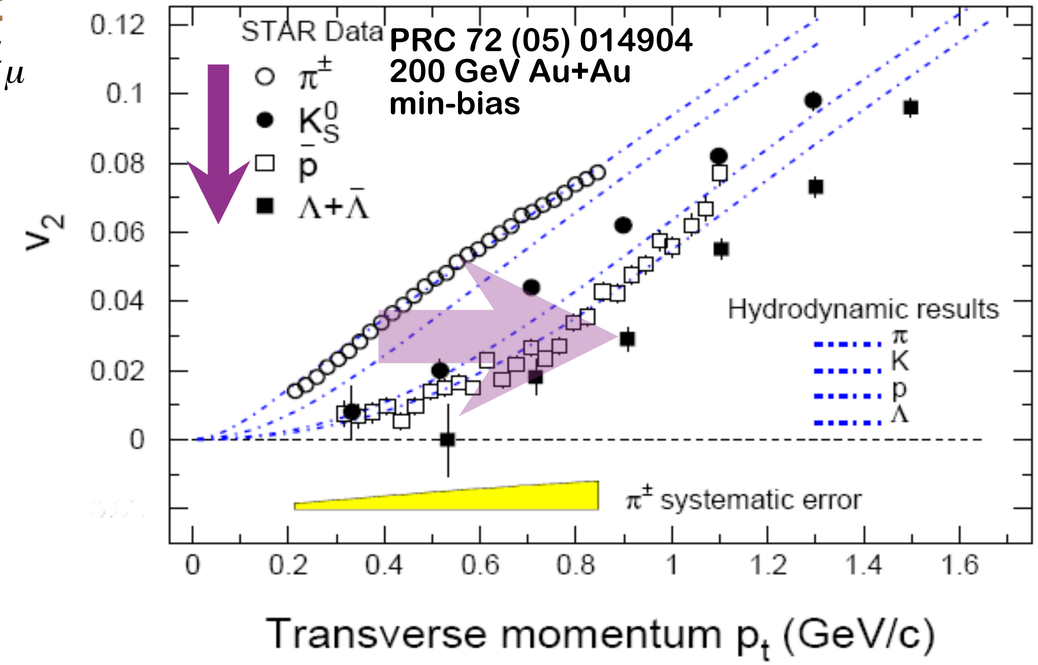


Conclusions from such studies:

- initial **transverse pressure gradient**
  - ⇒ ϕ - dependence of flow field u_μ
  - ⇒ elliptic flow $v_2(p_T)$

- size and pt-dependence of v_2 data accounted for by hydro ('maximal')

- characteristic **mass dependence**, since all particle species emerge from common flow field u_μ



II.6. Dissipative corrections to a perfect fluid

Small deviations from a locally isotropic fluid can be accounted for by restoring

(2.7) $$N_i^\mu = n_i u^\mu + \bar{n}_i$$ (4n comp.)

(2.8) $$T^{\mu\nu} = \varepsilon u^\mu u^\nu - p\Delta^{\mu\nu} + q^\mu u^\nu + q^\nu u^\mu + \Pi^{\mu\nu}$$ (10 comp.)

When does perfect fluid assumption fail? Consider conserved current:

(2.22) $$\partial_\mu j^\mu = \partial_\mu \left(\rho\, u^\mu\right) = \rho \underbrace{\partial_\mu u^\mu}_{\text{expansion scalar}} + \underbrace{u^\mu \partial_\mu}_{\text{comoving } t-\text{derivative}} \rho = 0$$

Spatio-temporal variations of macroscopic fluid should be small if compared to microscopic reaction rates

(2.23) $$\Gamma \cong n\sigma >> \theta = \partial_\mu u^\mu$$

Dissipative corrections characterized by gradient expansion!

Now, the conservation laws and equation of state

$\partial_\mu N_i^\mu \equiv 0$ (n constraints)

$\partial_\mu T^{\mu\nu} \equiv 0$ (4 constraints)

$p = p(\varepsilon, n)$ (1 constraint)

are not sufficient to constrain all independent thermo-dynamic fields in (2.7),(2.8).

How do we obtain additional constraints?

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II.7. 1st order dissipative fluid dynamics

Since conservation laws + eos do not close equations of motion, one seeks additional constraints from expanding 2nd law of thdyn to 1st order

(2.24) $$S^\mu = s\,u^\mu + \beta\, q^\mu$$ Entropy to first order

Use $\varepsilon + p = \mu n + Ts$ and $u_\nu \partial_\mu T^{\mu\nu} \equiv 0$ to write:

(2.25) $$T\partial_\mu S^\mu = (T\beta - 1)\partial.q + q.(\dot{u} + T\partial.\beta) + \Pi^{\mu\nu}\partial_\nu u_\mu + \Pi\theta \geq 0$$

To warrant that entropy increases, require:

$$\beta \equiv 1/T$$

Navier-Stokes
1st order hydro

(2.26) bulk viscosity $$\Pi \equiv \varsigma\,\theta$$

(2.27) heat conductivity $$q^\mu \equiv \kappa T \Delta^{\mu\nu}\left(\partial_\nu \ln T - \dot{u}_\nu\right)$$

(2.28) shear viscosity $$\Pi^{\mu\nu} \equiv 2\eta\left[\left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha\right)/2 - \Delta^{\mu\nu}\Delta_{\alpha\beta}/3\right]\partial^\alpha u^\beta$$

Determines $\Pi, q^\mu, \Pi^{\mu\nu}$ in terms of flow, energy density and dissipative coeff.

(2.29) $$\partial_\mu S^\mu = \frac{\Pi^2}{\varsigma T} - \frac{q.q}{\kappa T^2} + \frac{\Pi^{\mu\nu}\Pi_{\mu\nu}}{2\eta T} \geq 0$$

Problem: instantaneous acausal propagation.

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II.8. 2nd order viscous hydro – entropy derivation

Expand entropy to 2nd order in dissipative gradients

(2.30) $$S^\mu = s u^\mu + \beta q^\mu + \alpha_0 \Pi q^\mu + \alpha_1 \Pi^{\mu\nu} q_\nu + u^\mu \left(\beta_0 \Pi^2 + \beta_1 q q + \beta_2 \Pi^{\mu\nu} \Pi_{\mu\nu} \right)$$

Now, need 9 eqs. to determine $\Pi, q^\mu, \Pi^{\mu\nu}$

$\partial_\mu S^\mu \geq 0$ leads to differential equations for $\Pi, q^\mu, \Pi^{\mu\nu}$

which involve $\alpha_0, \alpha_1, \beta, \beta_0, \beta_1, \beta_2, \varsigma, \kappa, \eta$

Focus on shear viscosity only: (neglect vorticity)

(2.31) $$T \partial_\mu S^\mu = \Pi_{\mu\nu} \left[-\beta_2 D \Pi^{\mu\nu} + \tfrac{1}{2} \left\langle \nabla^\mu u^\nu \right\rangle \right] \equiv \tfrac{1}{2\eta} \Pi_{\mu\nu} \Pi^{\mu\nu}$$ $$\beta_2 = \tau_\Pi / 2\eta$$

Problem: does not specify shear tensor fully

Notations: covariant derivative $d_\mu u^\nu \equiv \partial_\mu u^\nu + \Gamma^\nu_{\alpha\mu} u^\alpha$

Convective derivative $D \equiv u^\mu d_\mu$

Nabla operator $\nabla^\mu \equiv \Delta^{\mu\nu} d_\nu = d^\mu - u^\mu D$

Angular bracket $\left\langle A^{\mu\nu} \right\rangle \equiv \left[\tfrac{1}{2} \left(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha \right) - \tfrac{1}{3} \Delta^{\mu\nu} \Delta_{\alpha\beta} \right] A^{\alpha\beta}$

II.9. Fluid dynamics from relativistic transport eq.

Consider Boltzmann equation with relaxation time approximation

(2.32)
$$p^\mu d_\mu f(x,p) = C \approx -\left(u^\mu p_\mu\right)\frac{f - f_{eq}}{\tau_\pi}$$

Consider small departures from local thermal equilibrium, quadratic ansatz

(2.33)
$$f = f_{eq}\left[1 + \varepsilon_{\mu\nu}(x,p)p^\mu p^\nu\right] \qquad \varepsilon_{\mu\nu} = \frac{1}{2T^2\left(\varepsilon + p\right)}\Pi_{\mu\nu}$$

With this ansatz, we write momentum moments from the Boltzmann eq.

… long journey …

(2.34)
$$\left(\varepsilon + p\right)Du^\mu = \nabla^\mu p - \Delta^\mu_\nu \nabla^\sigma \Pi^{\nu\sigma} + \Pi^{\mu\nu} Du_\nu$$
$$D\varepsilon = -(\varepsilon + p)\nabla_\mu u^\mu + \tfrac{1}{2}\Pi^{\mu\nu}\left\langle \nabla_\nu u_\mu \right\rangle$$
$$\tau_\pi \Delta^\mu_\alpha \Delta^\nu_\beta D\Pi^{\alpha\beta} + \Pi^{\mu\nu} = \eta\left\langle \nabla^\mu u^\nu \right\rangle - 2\tau_\pi \Pi^{\alpha(\mu}\omega^{\nu)}_\alpha$$

2nd order Israel-Stewart fluid dynamic equations of motion.

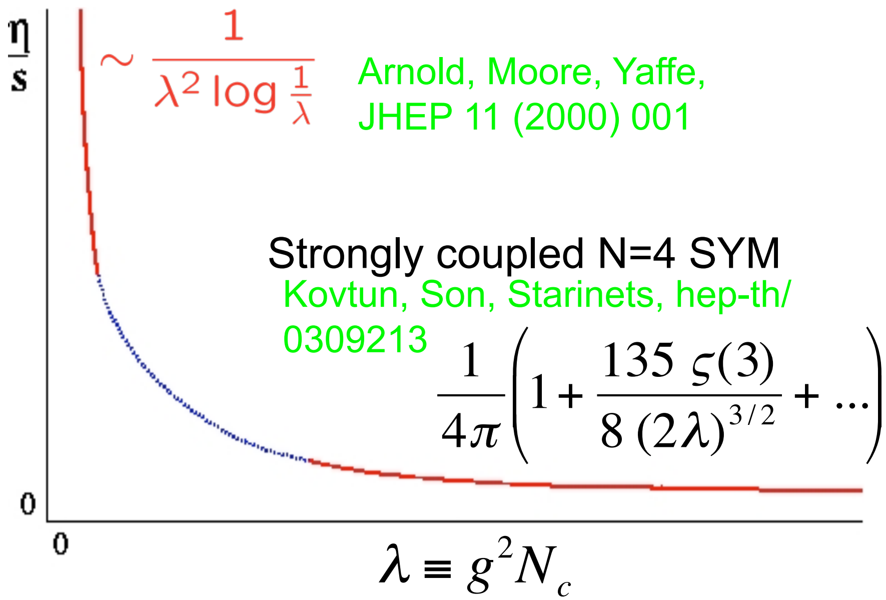
II.10. Transport coefficients are fundamental properties of hot QCD matter

The Green-Kubo formula defines transport coefficient as long wave-length limit of retarded Green's function of energy-momentum tensor

(3.30)
$$G^R_{xy,xy}(\omega,0) \equiv \int dt\, dx\, e^{i\omega t}\, \Theta(t) \left\langle \left[T_{xy}(t,x), T_{xy}(0,0) \right] \right\rangle_{eq}$$

$$\eta \equiv -\lim_{\omega \to 0} \frac{1}{\omega} \mathrm{Im}\, G^R_{xy,xy}(\omega,0)$$

Calculable from first principles in quantum field theory (QCD)



First attempts in finite temperature lattice QCD: H. Meyer, PRD76 (2007) 101701

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II.11. Results of fluid dynamic simulations

- Fluid dynamic simulations support very small dissipative corrections

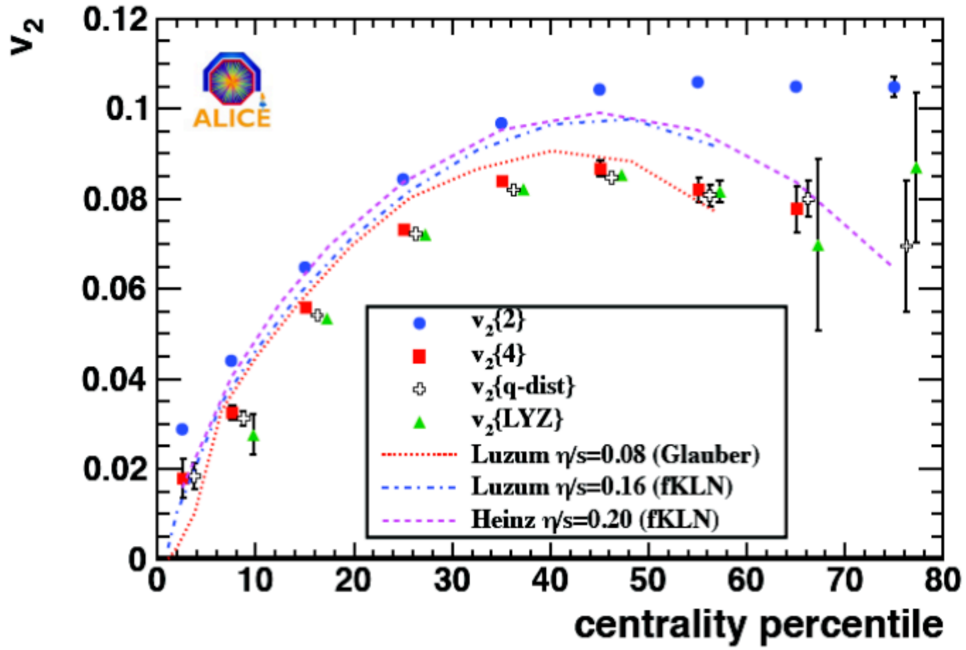
- Results depend on initial state

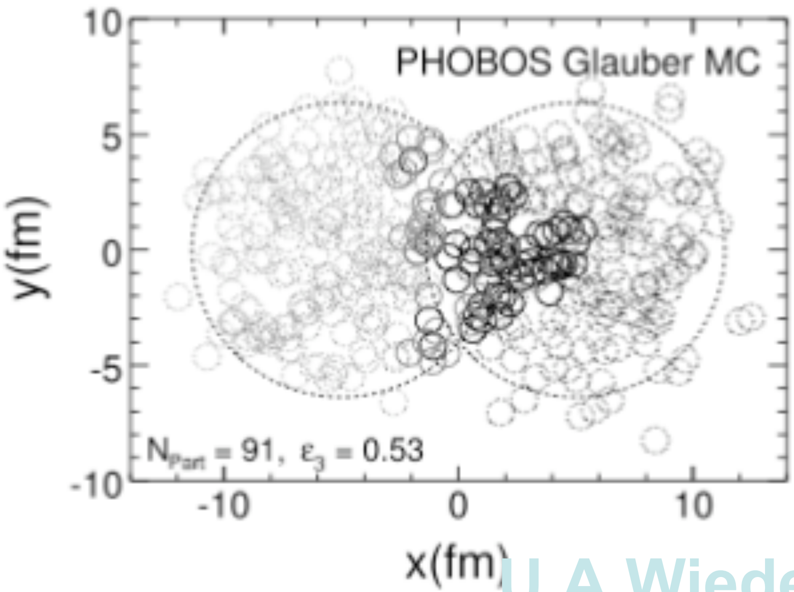
  - transverse profile: (e.g. “KLN”, “Glauber”, …)

  - initial state fluctuations (could be constrained by v_3 ...)

- Results depend on freeze-out

  Do all particle species flow with the same flow field?





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II.12. A model illustrating viscosity

Model: fluid with Bjorken scaling and no transverse gradients

1. Zeroth order ideal fluid dynamics

(3.30) $$\partial_\tau \varepsilon = -\frac{\varepsilon + p}{\tau}$$

This e.o.m. implies that entropy s is conserved

(3.31) $$\frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T}$$

2. First order Navier-Stokes dissipative hydrodynamics

(3.32) $$\partial_\tau \varepsilon = -\frac{\varepsilon + p}{\tau} + \frac{4\eta}{3\tau^2} \qquad \frac{d(\tau s)}{d\tau} = \frac{\frac{4}{3}\eta}{\tau T}$$

A 'perfect liquid' description is applicable, if the change of entropy is small compared to its absolute size

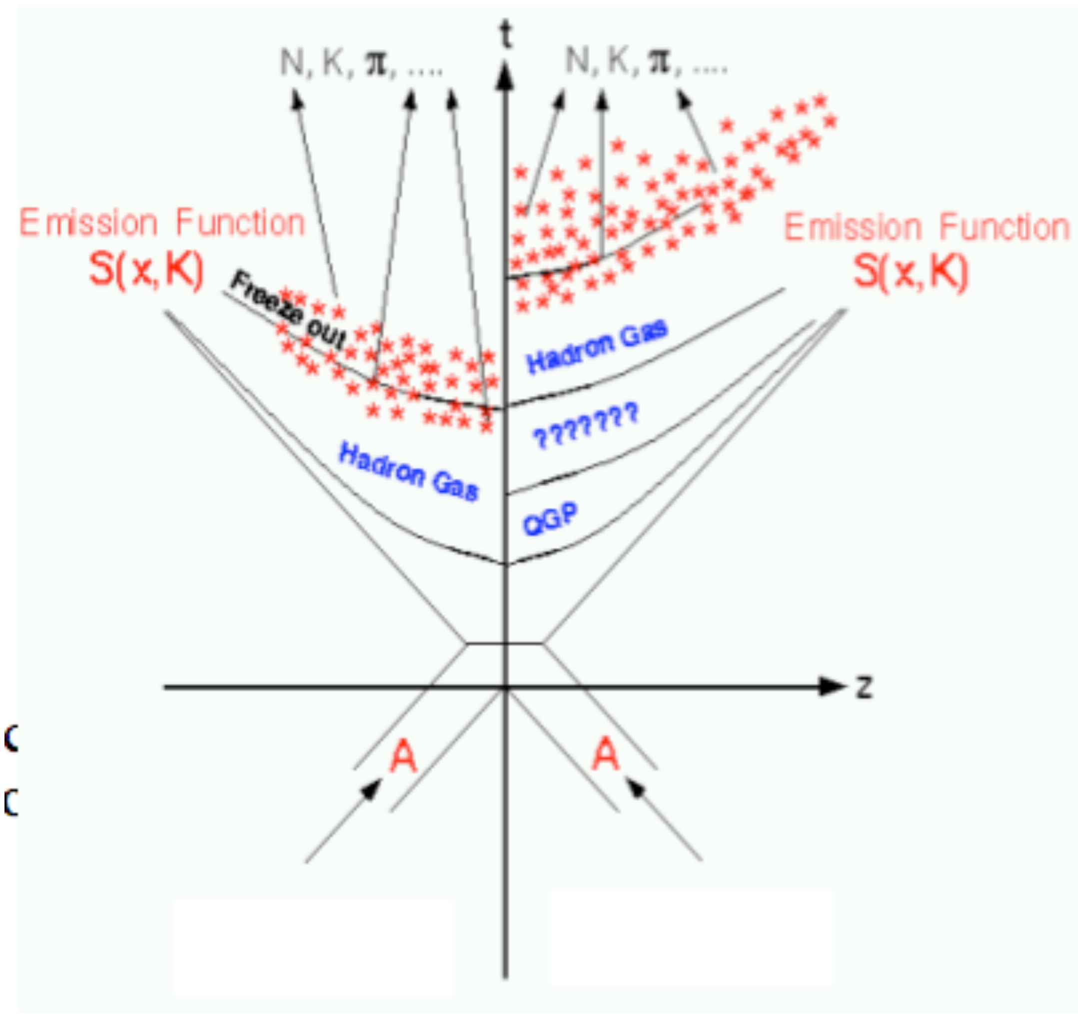
(3.33) $$\frac{\eta}{\tau T}\frac{1}{s} << 1$$

Put in numbers $\tau \sim 1\, fm/c, \quad T \sim 200\, MeV$ ⟹ $\frac{\eta}{s} << 1$

III.1. Measuring spatio-temporal extensions

- Can on characterize the space-time geometry of source from which particles are emitted in heavy ion collisions?

- Determine S(x,K) ?



model 1

model 2

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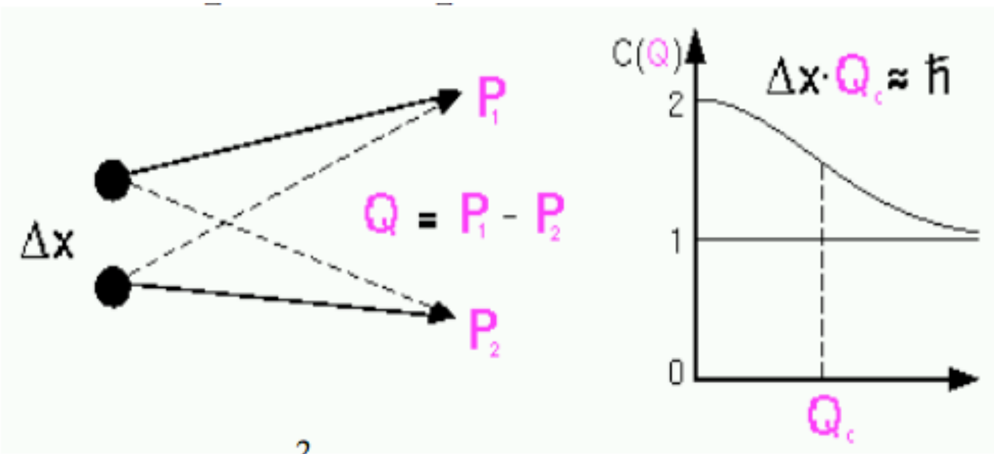
III.2. The use of idential two-particle correlations

- One-particle spectra are sensitive to **momentum information** only

$$E\frac{dN}{d^3p} = \int d^4x\, S(x,p) = \frac{1}{2\pi}\frac{dN}{p_T dp_T d\eta}\left[1+2v_2(p_T)\cos\left(2(\phi-\psi_r)\right)\right]$$

- Dependence on relative pair momentum is sensitive to **space-time information**



$$C(K,Q) = 1 + \frac{\left|\int d^4x\, S(x,K)\exp[iK.x]\right|^2}{\int d^4x\, S(x,P_1)\int d^4y\, S(y,P_2)} = 1+\lambda\exp\left[-\sum_{ij} R_{ij}^2(K)\, q_i q_j\right]$$

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III.3. The out-side-long system

$$C(K,Q) = 1 + \lambda \exp\left[-R_o^2(K)q_o^2 - R_s^2(K)q_s^2 - R_l^2(K)q_l^2 - 2R_{ol}^2(K)q_o q_l\right]$$

• Different HBT radii measure different combinations of spatial and temporal information. This is characterized by the Gaussian widths (space-time variances) of S(x,K)

$$\langle f \rangle(K) = \frac{\int d^4x\, f(x) S(x,K)}{\int d^4x\, S(x,K)}$$

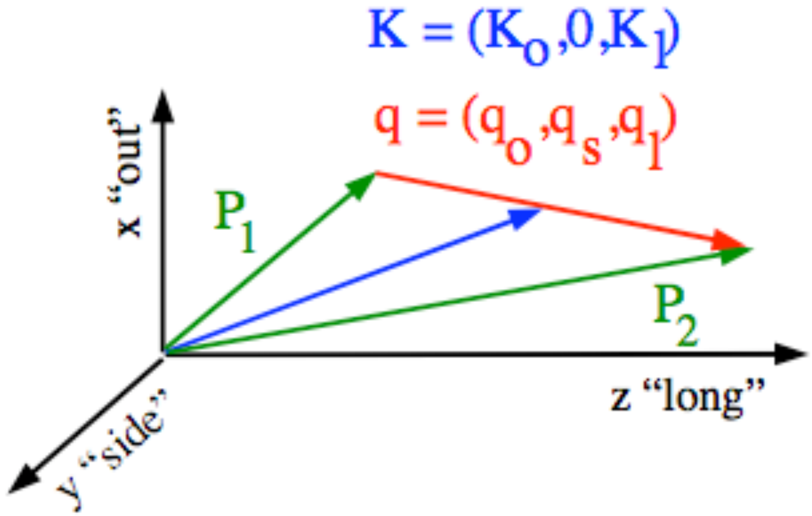
$$R_o^2(K) = \left\langle (\tilde{x} - \beta_t \tilde{t})^2 \right\rangle$$

$$R_s^2(K) = \left\langle \tilde{y}^2 \right\rangle$$

$$R_l^2(K) = \left\langle (\tilde{z} - \beta_l \tilde{t})^2 \right\rangle$$

$$R_{ol}^2(K) = \left\langle (\tilde{x} - \beta_t \tilde{t})(\tilde{z} - \beta_l \tilde{t}) \right\rangle$$

$$\vec{\beta} \equiv \frac{\vec{K}}{K_0}$$



• Distances measured w.r.t. source center,

$$\tilde{x} \equiv x - \langle x \rangle$$

• Momentum dependence of HBT radii can be exploited to extract space-time information, (over)simplified example:

$$R_o^2(K) - R_s^2(K) \approx \beta_t^2 \langle \tilde{t}^2 \rangle$$

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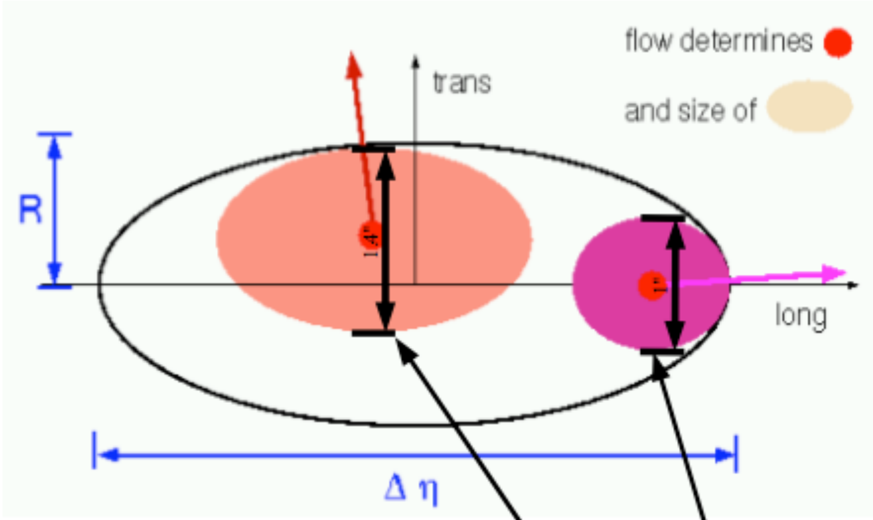
III.4. HBT-radii measure homogeneity regions

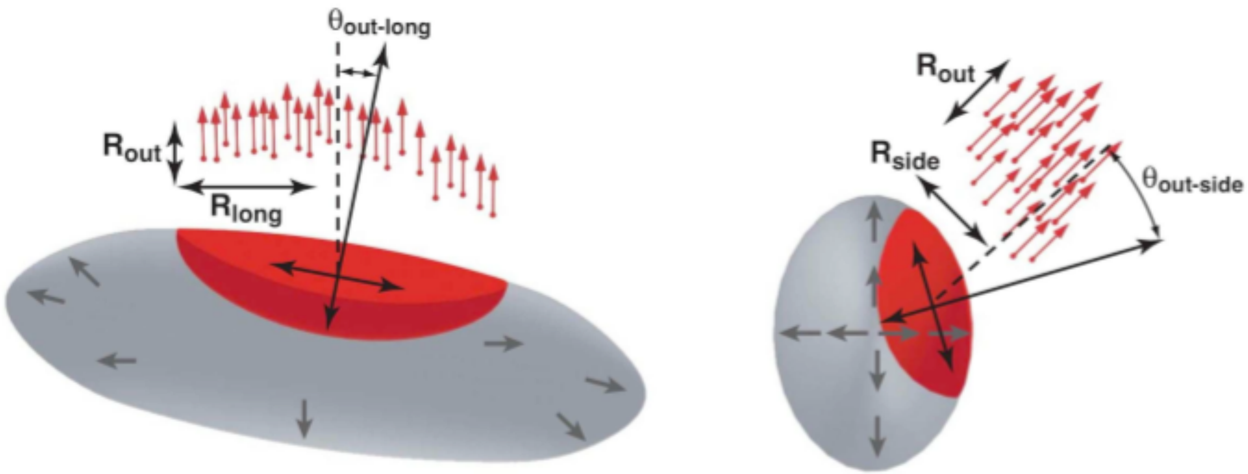
- They do NOT measure the entire source size
  K_T-dependence of HBT radii defines
  wave-length filter for observed region
  and thus contains dynamical information



$$V_{LHC} \approx 300\, fm^3 \approx 2 V_{RHIC}$$

$$R_s^2(K_{t1}) \neq R_s^2(K_{t2})$$

- In non-central collisions, out-long and side-long HBT radii are non-zero and provide direct access to spatial excentricity



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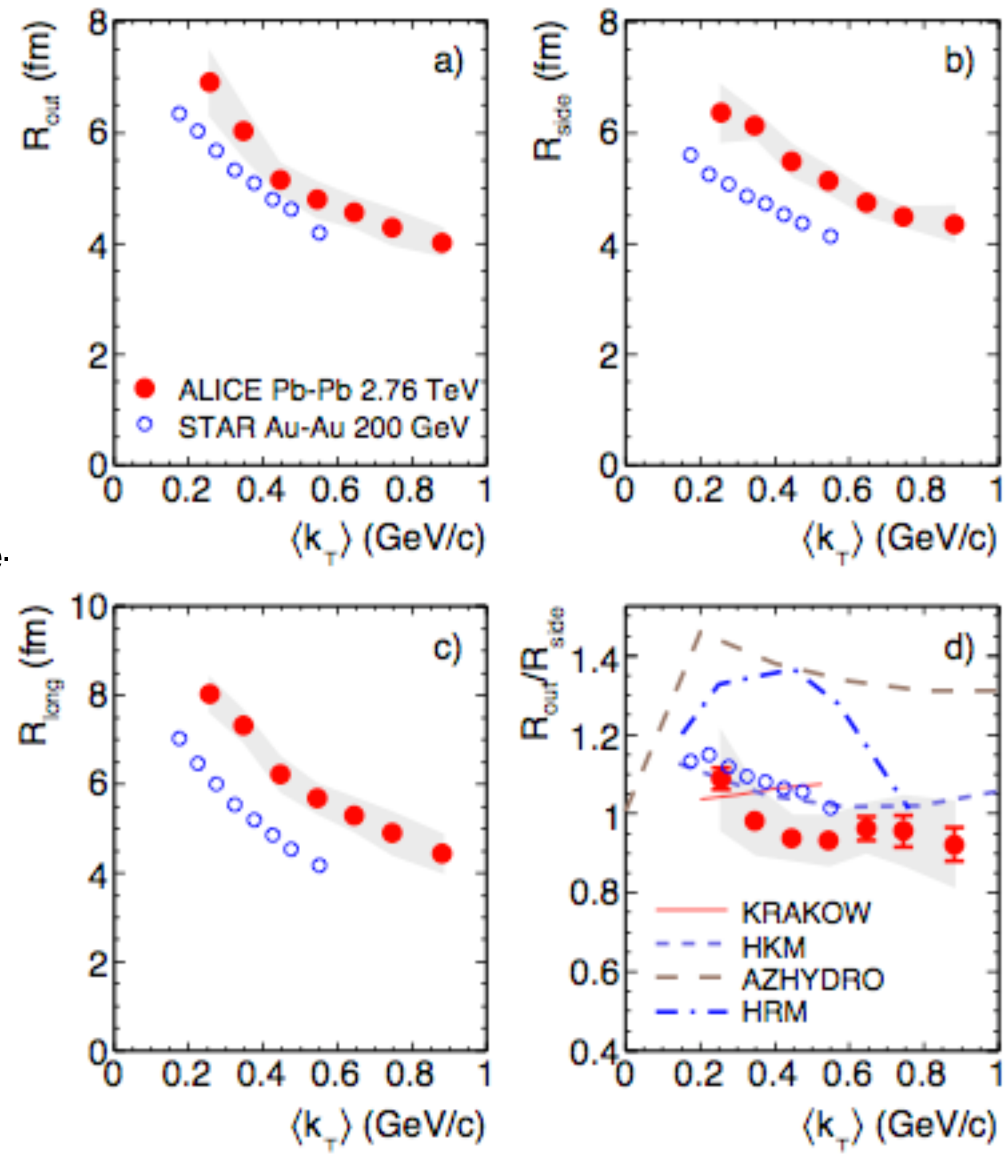
III.5. Measurements of HBT

- Characteristic K_T-dependence has origin in
  - position-momentum gradients induced by flow
  - resonance decay contributions that increase source size for soft particles

- In non-central collisions, out-long and side-HBT radii are non-zero and provide direct access to spatial excentricity



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III.6. Measurement of homogeneity volume

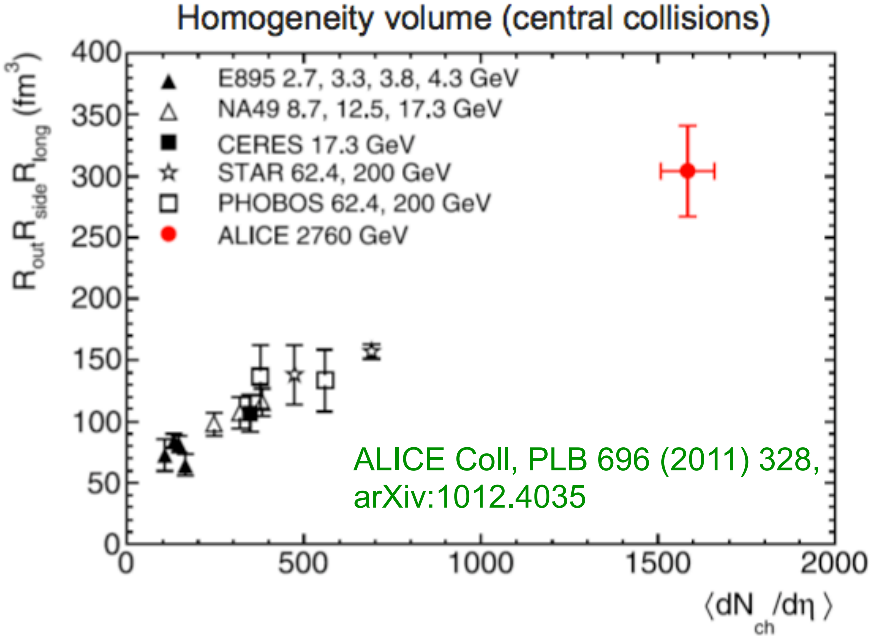
- Volume at freeze-out depends linearly on

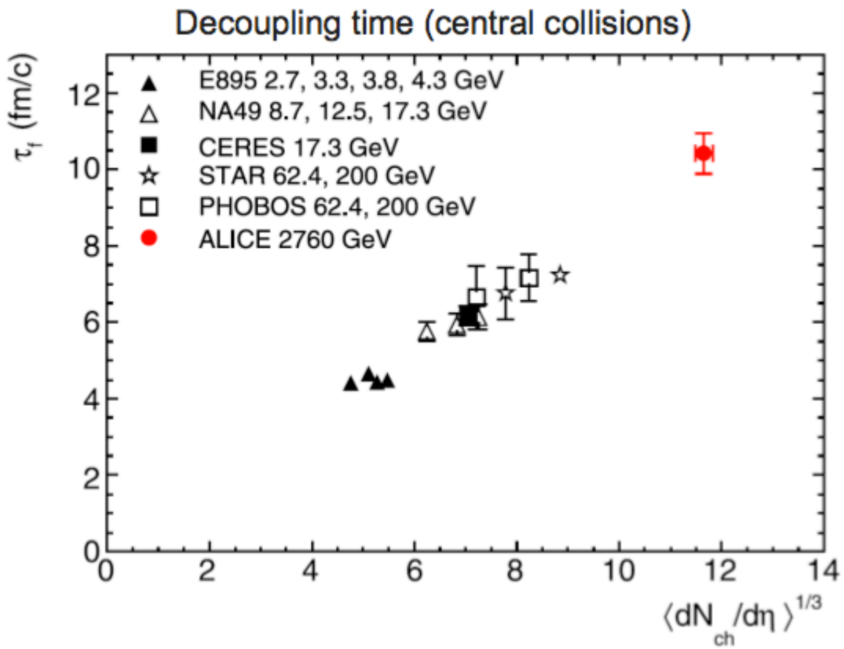
$$dN_{ch}/d\eta$$

$$V_{LHC} \approx 300\, fm^3 \approx 2V_{RHIC}$$

- From R_{long}, one gets access to freeze-out time

$$\tau_{f,LHC} \approx 10\, fm/c$$





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