Selected Topics in the Theory of Heavy Ion Collisions

Lecture 3

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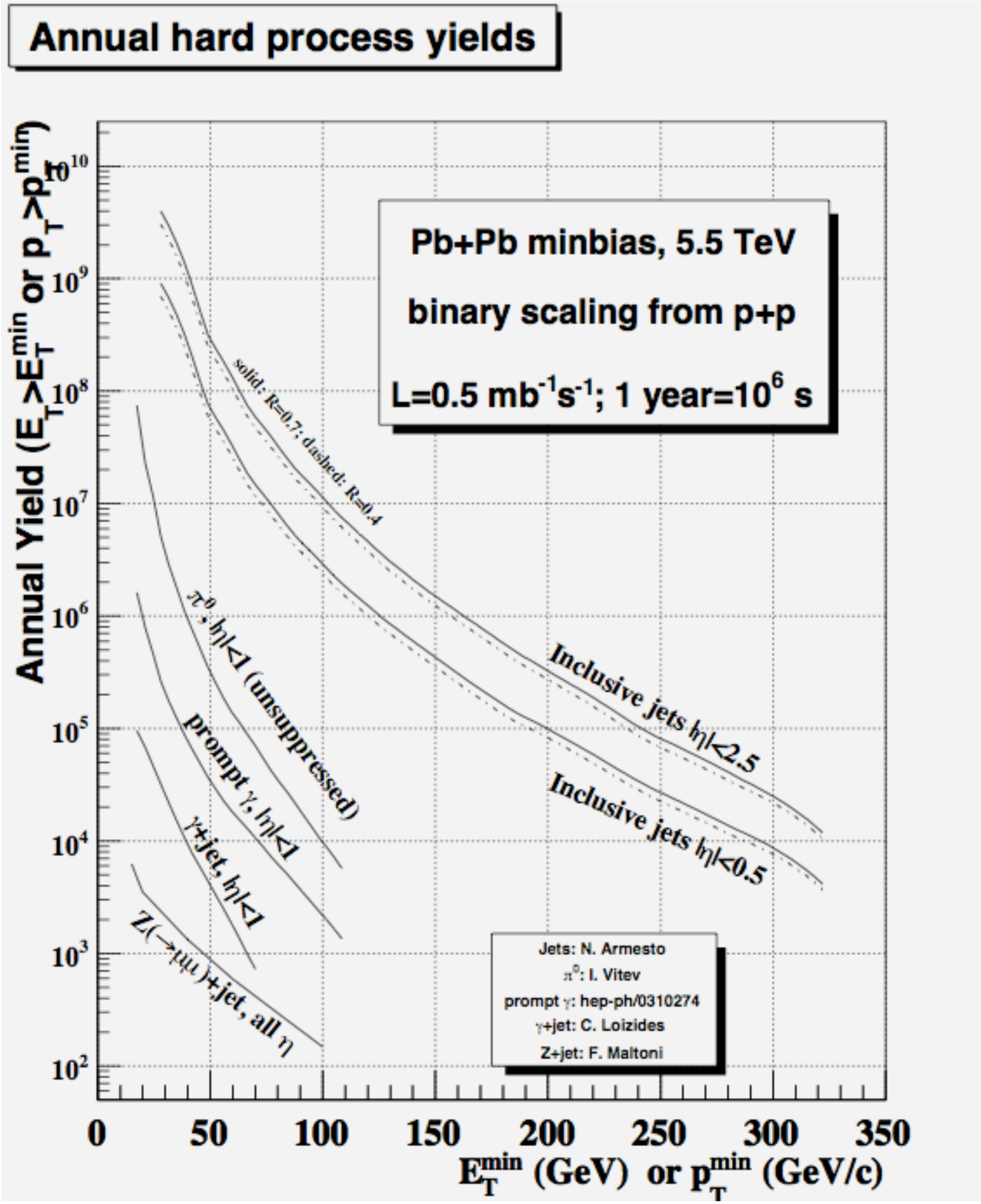
16 March 2011

'Hard Probes' at the LHC



Hard probes
= hard processes
embedded in dense nuclear
matter (and sensitive to its
'properties')

These are produced
abundantly at the LHC.

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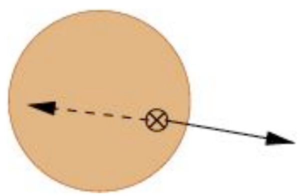
Bjorken's original estimate and its correction

Bjorken 1982: consider jet in p+p collision, hard parton interacts with underlying event ⇒ collisional energy loss

$$dE_{coll}/dL \approx 10\,GeV/fm$$ (error in estimate!)

Bjorken conjectured monojet phenomenon in proton-proton



But: radiative energy loss expected to dominate

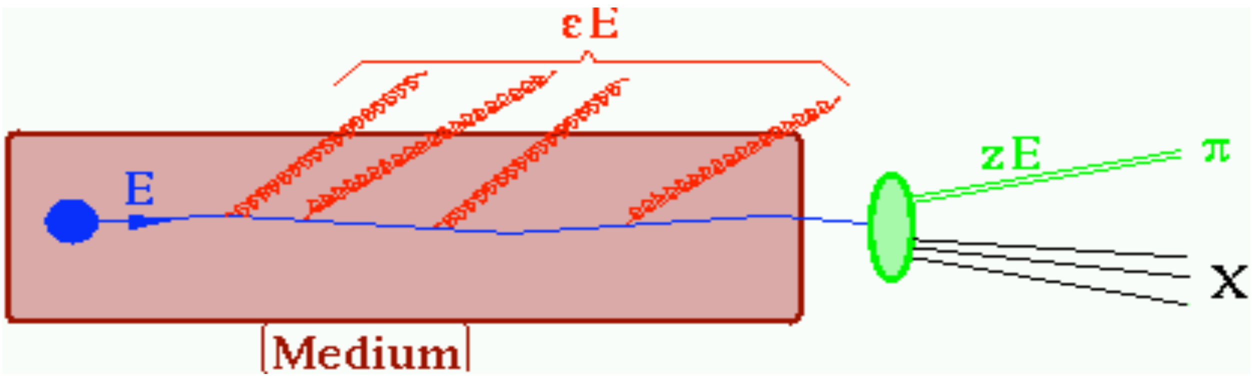
$$\Delta E_{rad} \approx \alpha_s \hat{q} L^2$$ Baier Dokshitzer Mueller Peigne Schiff 1995

- p+p: $L \approx 0.5\ fm,\quad \Delta E_{rad} \approx 100\ MeV$ Negligible !
- A+A: $L \approx 5\ fm,\quad \Delta E_{rad} \approx 10\ GeV$ Monojet phenomenon! Observed at RHIC

Today: we want to understand how these estimates arise and how energetic partons lose energy in dense matter.

IV.1 “Jet” Quenching at RHIC and LHC

So far, ‘jet quenching’ is mainly tested by suppressed leading hadron production:



Nuclear modification factor characterizes medium-effects:

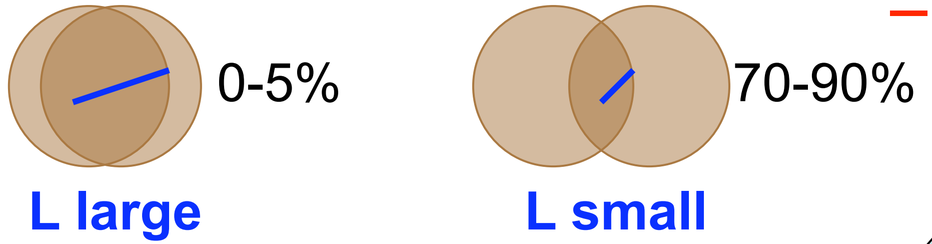
$$R_{AA}(p_T) = \frac{dN^{AA} / dp_T}{n_{coll}\; dN^{NN} / dp_T}$$

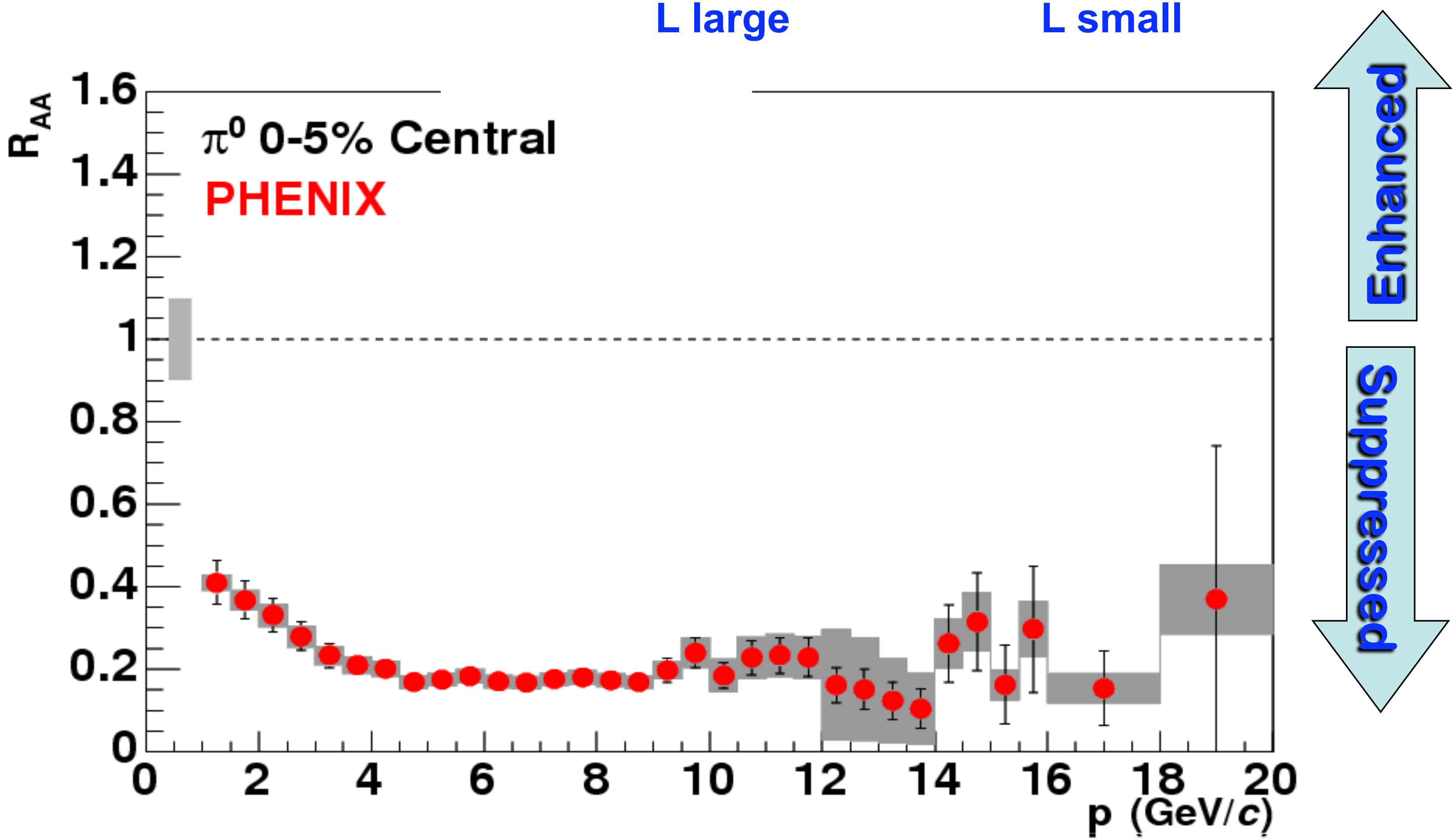
$R_{AA}(p_T) = 1.0$ no suppression

$R_{AA}(p_T) = 0.2$ factor 5 suppression

IV.2. Suppression persists to highest p_T

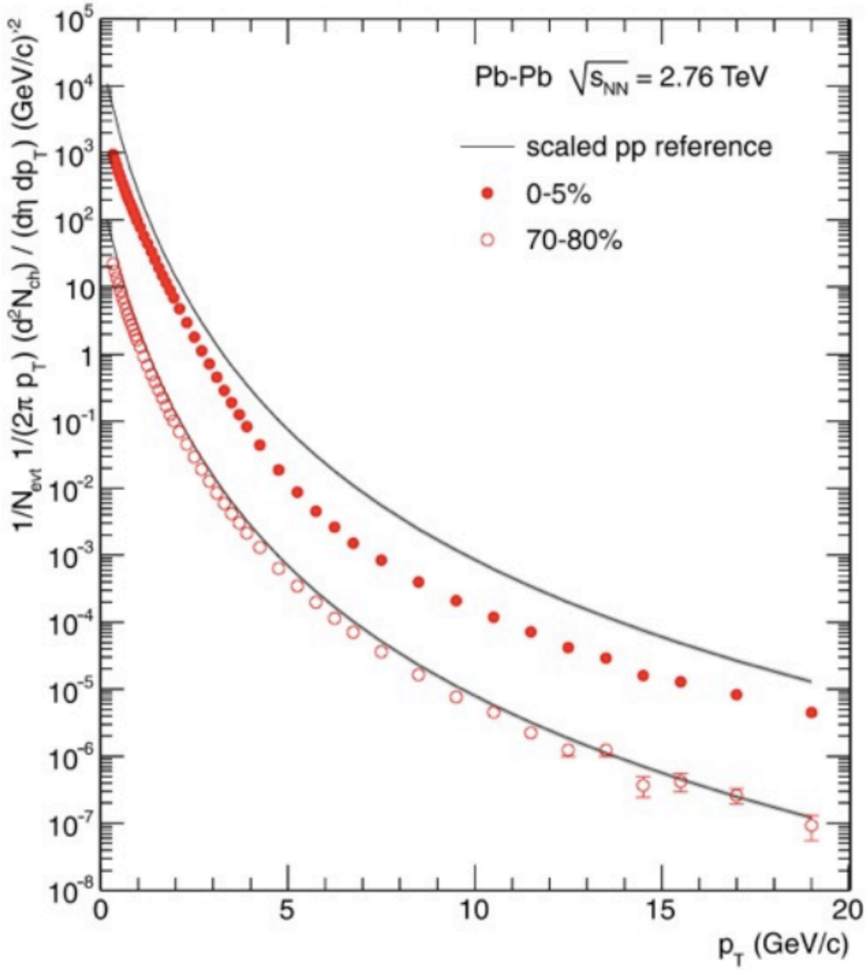
Centrality dependence:





IV.3. Suppression persists to highest p_T

- Spectra in AA and pp-reference



- Nuclear modification factor shows pt-dependence

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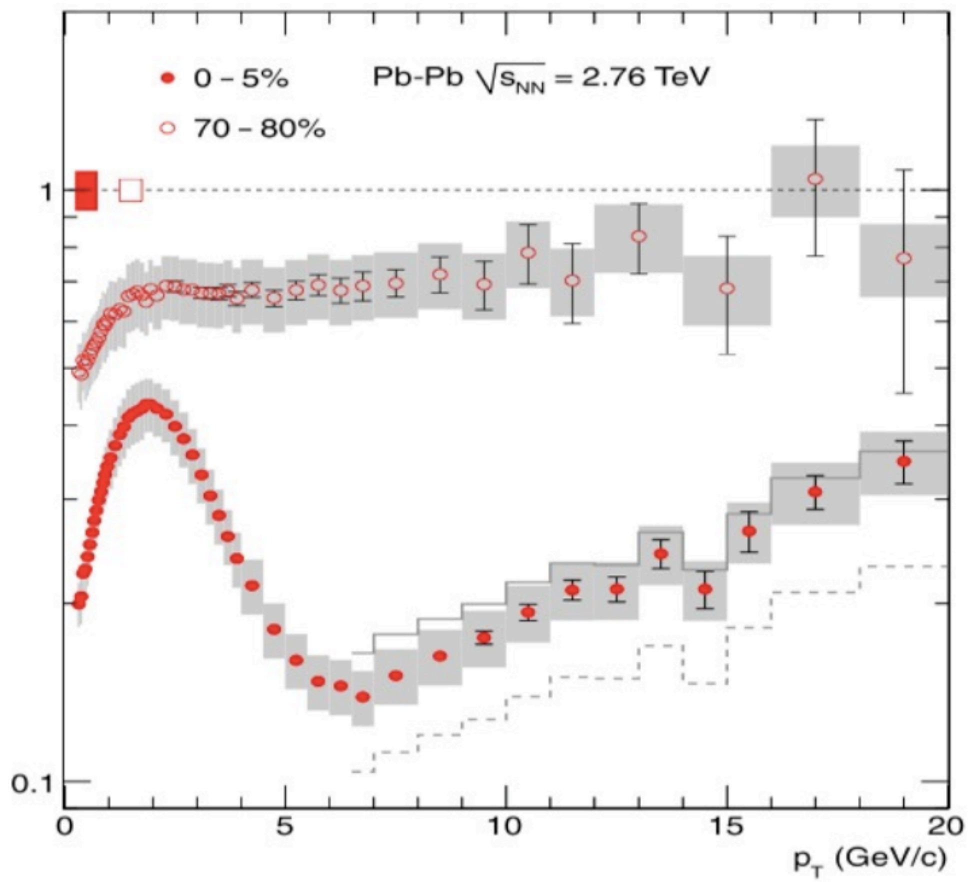
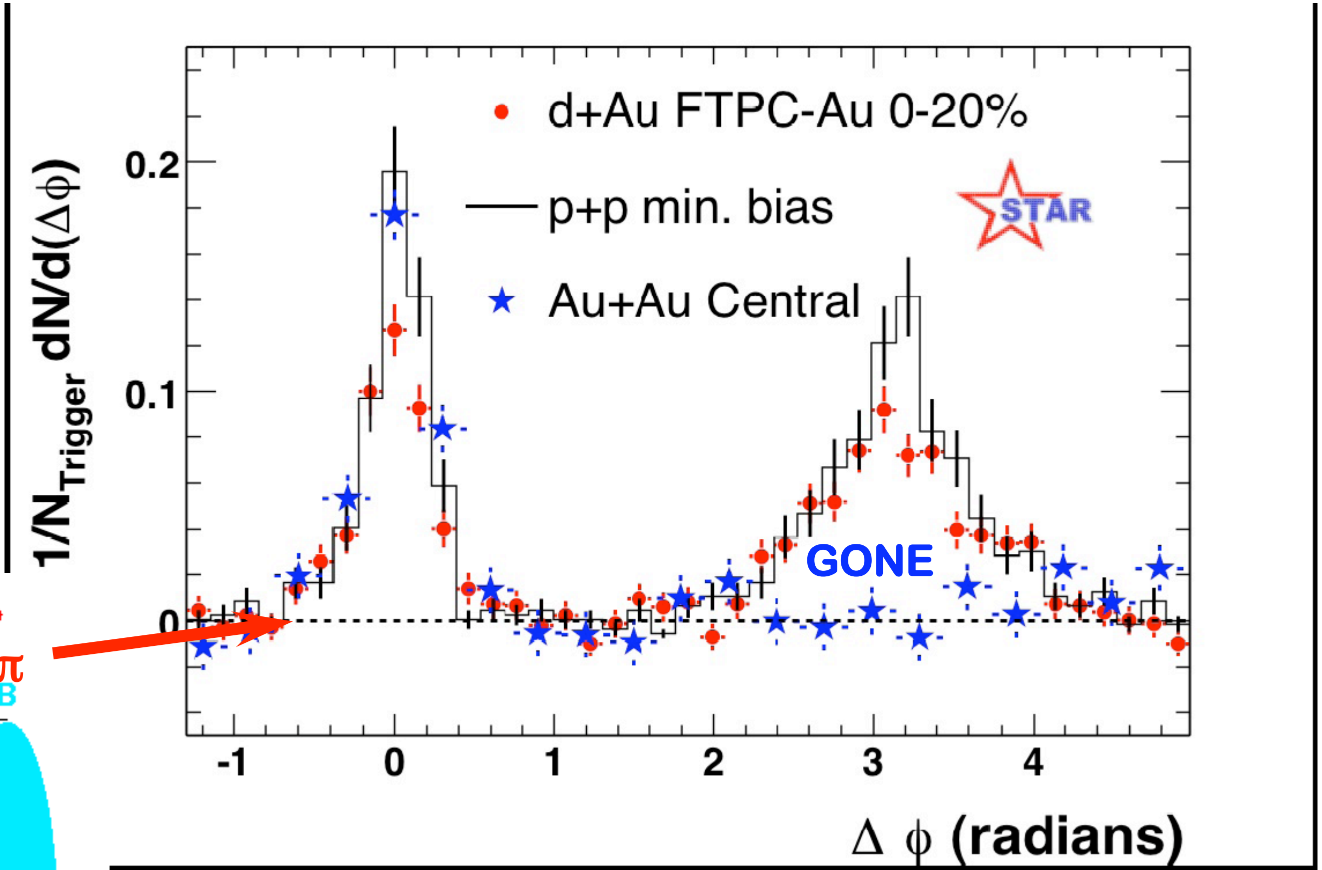
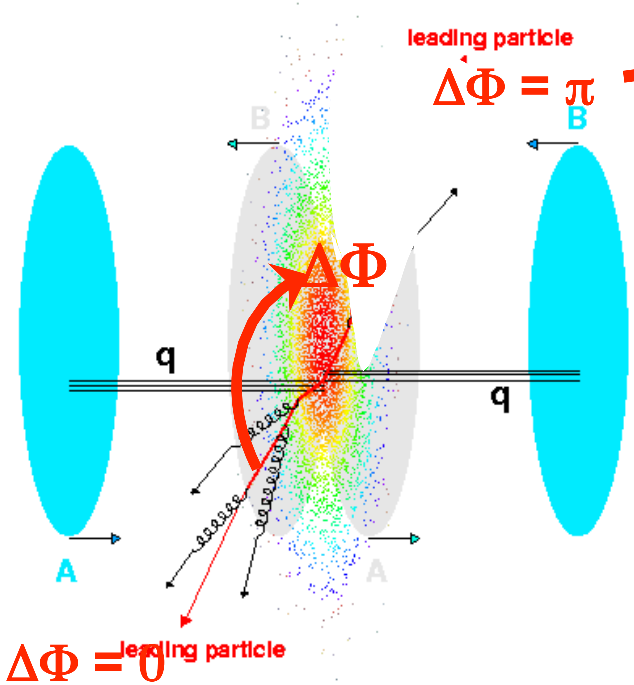


IV.4. The Matter is Opaque at RHIC

- STAR azimuthal correlation function shows ~ complete absence of “away-side” (high-pt) particle





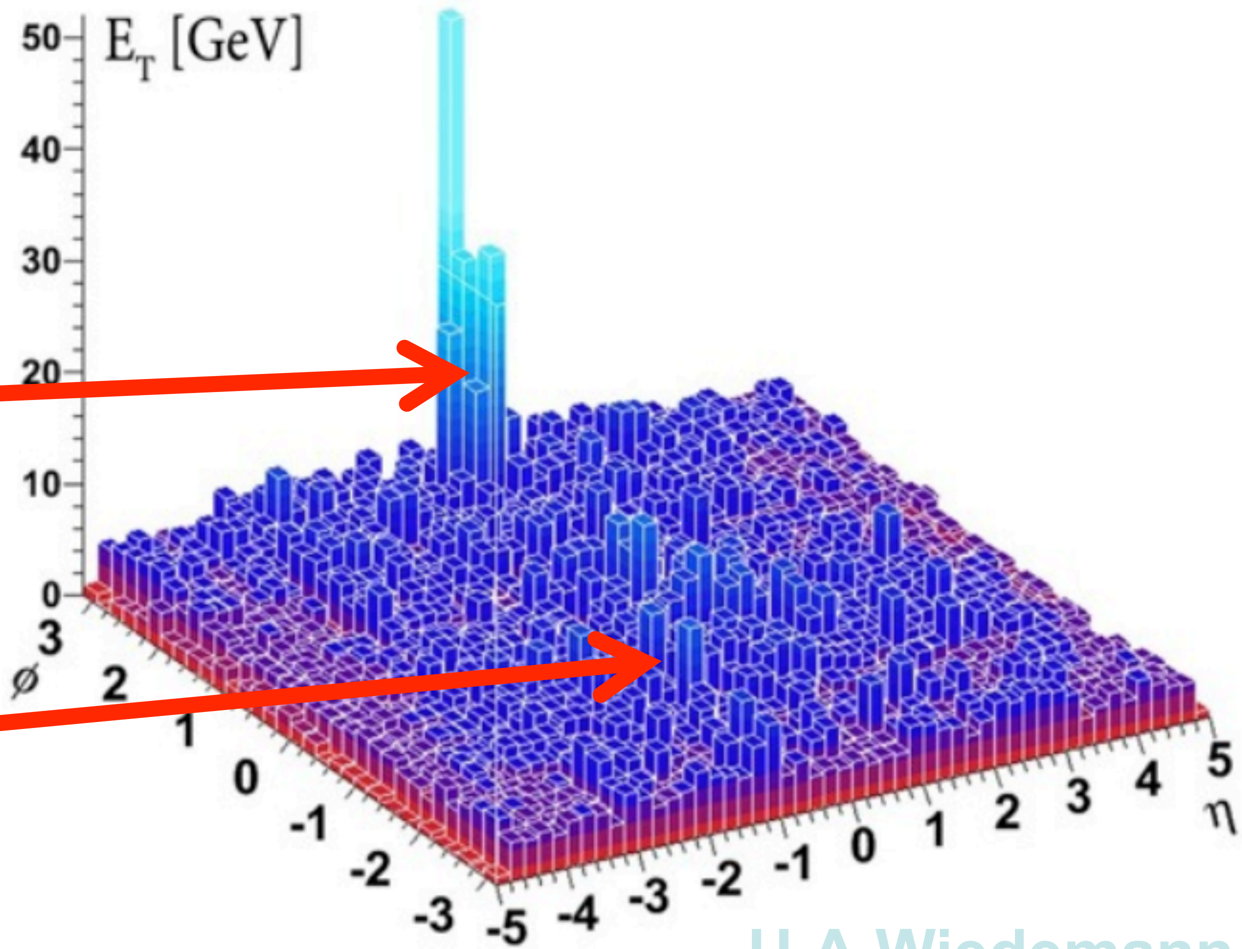
✦ Partner in hard scatter is *completely absorbed* in the dense medium

IV.5. Dijet asymmetries at LHC

First observation of absorption/modification of recoil in **reconstructed jets** (rather than leading hadrons)
=> new handle on understanding microscopic dynamics of jet quenching

ATLAS arXiv:1011.6182
CMS arXiv:1102.1957

Trigger jet
E_T ~ 100 GeV



Recoil
GONE

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IV.6. Issues

Problem: How does a parton propagate and fragment in spatially extended dense QCD matter? By studying its hadronic remnants, what can we learn about *properties of this matter*?

Physics: *Propagation/fragmentation* of highly energetic parton in the vacuum *is modified* by the interaction of the parton with spatially extended color field of the medium.

Purpose of this lectures: sketch current state of the art of the 'theory of jet quenching' and its testable consequences.

Warning: This theory is far from complete!
Our presentation is simplified.

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IV.7. Eikonal formalism

$$W(x_i) = P\exp\left[i\int dz^- T^a A_a^+(x_i, z_-)\right]$$

Here, A_a^+ is target gauge field, T^a is SU(3) generator in representation of the parton $|\alpha_i, x_i\rangle$, z_- is light cone coordinate.

Interpretation of

$$\Psi_{out} = \hat{S}\Psi_{in} = \sum_{\{\alpha_i, x_i\}} \psi(\alpha_i, x_i)\left(\Pi_i W^{r_i}_{\alpha_i\beta_i}(x_i)\right)|\beta_i, x_i\rangle$$

High-energy scattering ⟺ Rotation in color space

⟺ Components of Ψ_{in} decohere

⟺ Inelastic particle production

Measure of decoherence:

(4.4)

$$|\delta\Psi\rangle = \left[1 - |\Psi_{in}\rangle\langle\Psi_{in}|\right]|\Psi_{out}\rangle$$

E.g. probability of inelastic scattering of projectile given by $\langle\delta\Psi\|\delta\Psi\rangle$

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IV.8. Example: gluon production in q+A

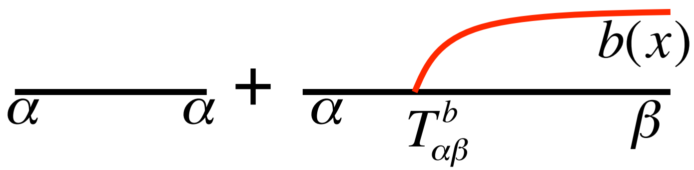
Consider high energy quark centered at x=0 and projectile rapidity y=0.
The wave packet to zeroth order in coupling is

(4.5) $$\left|\alpha(\underline{0},0)\right\rangle =$$

But to 1st order in coupling, the quark is not any more a bare quark, it has a gluon in its wavefunction

(4.6) $$\left|\Psi_{in}^{q}\right\rangle = \left|\alpha(\underline{0},0)\right\rangle + \int d\underline{x}\, d\xi\, \vec{f}\left(\underline{x}\right) T_{\alpha\beta}^{b} \left|\beta(\underline{0},0), b(\underline{x},\xi)\right\rangle + O(g^2)$$

=



This distribution of gluons is flat in rapidity. In transverse space, it follows a Coulomb-type Weizsäcker-Williams field

(4.7) $$\vec{f}(\underline{x}) \propto g \frac{\underline{x}}{\underline{x}^2}$$

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IV.9. Example: gluon production in q+A

Note that $\left|\Psi_{in}^{q}\right\rangle$ results from unitary free time evolution of bare quark from the very past $t = -\infty$ to the present $t = 0$

(4.8) $$\left|\Psi_{in}^{q}\right\rangle = U_{-}\left|\alpha\right\rangle = \exp\left[-\int d\underline{x}\left|\vec{f}(\underline{x})\right|^{2} + i\int dx\, d\xi\, \vec{f}(\underline{x})\left(a_{d}(x,\xi) + a_{d}^{+}(x,\xi)\right)T^{d}\right]\left|\alpha\right\rangle$$

i.e. U_{-} creates the cloud of gluons around the bare quark.

Now comes the scattering

(4.9) $$\left|\Psi_{out}^{q}\right\rangle = \hat{S}U_{-}\left|\alpha\right\rangle = W_{\alpha\gamma}^{F}(\underline{0})\left|\gamma\right\rangle + \int d\underline{x}\; \vec{f}\left(\underline{x}\right)T_{\alpha\beta}^{b}W_{\beta\gamma}^{F}(\underline{0})\,W_{bc}^{A}(\underline{x})\left|\gamma(\underline{0}),c(\underline{x})\right\rangle$$

Gluons are produced in those components of $\left|\Psi_{out}^{q}\right\rangle$, which lie in the subspace orthogonal to the incoming state with arbitrary color orientation γ

(4.10) $$\left|\delta\Psi_{\alpha}\right\rangle = U_{+}\left[\left|\Psi_{out}^{\alpha}\right\rangle - \sum_{\gamma}U_{-}\left|\gamma\right\rangle\left\langle\gamma U_{-}^{+}U_{+}^{+}\left|\Psi_{out}^{\alpha}\right\rangle\right.\right]$$

To calculate this, use $\left\langle\gamma U_{-}^{+}U_{+}^{+}\left|\Psi_{out}^{\alpha}\right\rangle\right. = \left\langle\gamma\left|W_{\alpha\delta}^{F}(\underline{0})\right|\delta\right\rangle + O(f^{2}) = W_{\alpha\gamma}^{F}(\underline{0})$

SUPPLEMENT

IV.10. Example: gluon production in q+A

(4.10) $$\left|\delta\Psi_\alpha\right\rangle = U_+\left[\left|\Psi^\alpha_{out}\right\rangle - \sum_\gamma U_-\left|\gamma\right\rangle\left\langle\gamma U_-^+ U_+^+\right|\Psi^\alpha_{out}\right\rangle\right]$$ Use **(4.6), (4.9)**

$$= U_+\begin{bmatrix} W^F_{\alpha\beta}(\underline{0})\left|\gamma\right\rangle + \int dx\,\vec{f}(\underline{x})T^b_{\alpha\beta}W^F_{\beta\gamma}(\underline{0})W^A_{bc}(\underline{x})\left|\gamma,c(\underline{x})\right\rangle \\ -W^F_{\alpha\beta}(\underline{0})\left|\gamma\right\rangle - \int dx\,\vec{f}(\underline{x})W^F_{\alpha\gamma}(\underline{0})T^b_{\gamma\beta}\left|\beta,b(\underline{x})\right\rangle \end{bmatrix}$$

$$= \int dx\,\vec{f}(\underline{x})\left[T^b_{\alpha\beta}W^F_{\beta\gamma}(\underline{0})W^A_{bc}(\underline{x}) - T^c_{\beta\gamma}W^F_{\alpha\beta}(\underline{0})\right]\left|\gamma,c(\underline{x})\right\rangle$$

The number spectrum of produced gluons reads then

(4.11) $$N^{qA}(k_T) = \frac{1}{N_c}\sum_\alpha\left\langle\delta\Psi_\alpha\right|a_d^+(k_T)a_d(k_T)\left|\delta\Psi_\alpha\right\rangle$$

$$= \int d\underline{x}\,d\underline{y}\,e^{i\underline{k}\cdot(\underline{x}-\underline{y})}\frac{1}{N_c}\sum_\alpha\left\langle\delta\Psi_\alpha\right|a_d^+(\underline{y})a_d(\underline{x})\left|\delta\Psi_\alpha\right\rangle$$

To calculate this expression, use

(4.12) $$a_d(\underline{x})\left|\delta\Psi_\alpha\right\rangle = \vec{f}(\underline{x})\left[\left(T^bW^F(\underline{0})\right)_{\alpha\gamma}W^A_{bd}(\underline{x}) - \left(W^F(\underline{0})T^d\right)_{\alpha\gamma}\right]\left|\gamma\right\rangle$$

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IV.11. Example: gluon production in q+A

Again:

(4.12)
$$a_d(\underline{x})\left|\delta\Psi_\alpha\right\rangle = \vec{f}(\underline{x})\left[\left(T^b W^F(\underline{0})\right)_{\alpha\gamma} W^A_{bd}(\underline{x}) - \left(W^F(\underline{0})T^d\right)_{\alpha\gamma}\right]\left|\gamma\right\rangle$$

$$\left\langle\delta\Psi_\alpha\right| a_d^+(\underline{y}) = \left\langle\gamma\right|\left[W^{A+}_{d\bar{b}}(\underline{y})\left(W^{F+}(\underline{0})T^{\bar{b}}\right)_{\gamma\alpha} - \left(T^d W^{F+}(\underline{0})\right)_{\gamma\alpha}\right]\vec{f}(\underline{y})$$

To calculate from this $\left\langle\delta\Psi_\alpha\left|a_d^+(\underline{y})a_d(\underline{x})\right|\delta\Psi_\alpha\right\rangle$, we use

$$Tr\left[T^a T^b\right] = \delta^{ab}/2 \qquad\qquad \vec{f}(\underline{x}).\vec{f}(\underline{y}) = \frac{\alpha_s}{2\pi}\frac{\underline{x}.\underline{y}}{x^2 y^2}$$

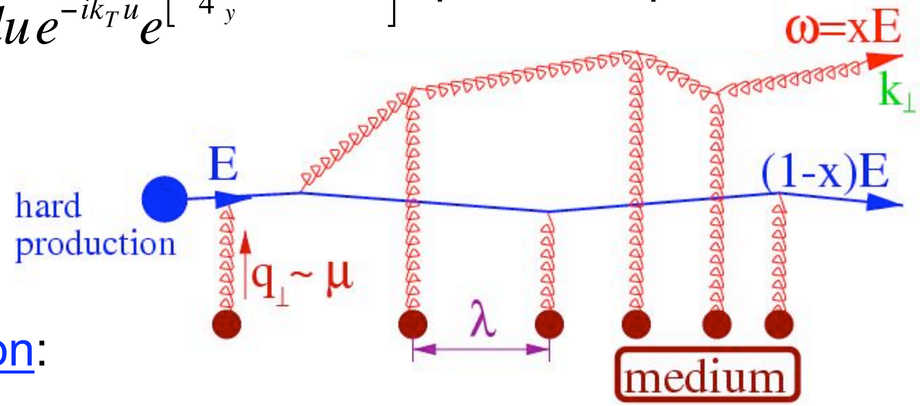
(4.13)
$$W^A_{ab}(\underline{x}) = 2Tr\left[T^a W^F(\underline{x}) T^b W^{F+}(\underline{x})\right]$$

(4.14)
$$\boxed{N^{qA}(k_T) = \frac{\alpha_s C_F}{2\pi}\int d\underline{x}\, d\underline{y}\, \frac{\underline{x}.\underline{y}}{x^2 y^2} e^{i\underline{k}.(\underline{x}-\underline{y})}\begin{bmatrix}1 - \left\langle Tr\left[W^{A+}(\underline{0})W^A(\underline{x})\right]\right\rangle \\ -\left\langle Tr\left[W^{A+}(\underline{y})W^A(\underline{0})\right]\right\rangle + \left\langle Tr\left[W^{A+}(\underline{y})W^A(\underline{x})\right]\right\rangle\end{bmatrix}}$$

V.1. BDMPS-Z gluon radiation spectrum

R. Baier et al. (BDMPS) 1996, Zakharov 1996
Wiedemann, 2000

Radiation off produced parton



(5.1)

$$\frac{dI}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2\omega^2}\, 2\mathrm{Re}\int_0^\infty dy \int_y^\infty d\bar{y} \int du\, e^{-ik_T u}\, e^{\left[-\frac{1}{4}\int_y^\infty d\xi\, \hat{q}(\xi) u^2\right]}$$

$$\times \frac{\partial}{\partial u}\cdot\frac{\partial}{\partial s} K(s=0,y;u,y\,|\,\omega)$$

Target average includes Brownian motion:

(5.2)

$$K\left(s,y;u,\bar{y}\,|\,\omega\right) = \int_{s=r(y)}^{u=r(\bar{y})} Dr \exp\left[\int_y^{\bar{y}} d\xi \left\{\boxed{\left(\frac{i\omega}{2}\dot{r}^2\right)} - \frac{1}{4}\boxed{\hat{q}(\xi)}\, r^2\right\}\right]$$

$$\xrightarrow{\omega\to\infty} \exp\left[-\frac{1}{4}\hat{q}\, L_{long}\, r^2\right]$$

$$\equiv \left\langle Tr\left[W^{A+}(0)\, W^A(r)\right]\right\rangle$$

BDMPS transport coefficient

Expectation value of light-like Wilson line

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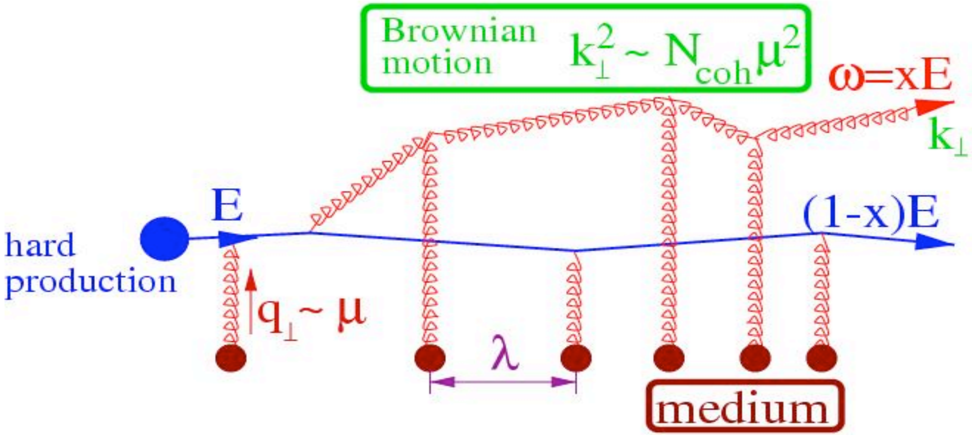
V.2. Parton energy loss - what to expect?



Medium characterized by BDMPS transport coefficient:

$$\hat{q} \cong \frac{\mu^2}{\lambda}$$

- How much energy is lost?

(5.3) Phase accumulated in medium: $\left\langle k_T^2 \, \Delta z \big/ 2\omega \right\rangle \approx \frac{\hat{q}L^2}{2\omega} = \frac{\omega_c}{\omega}$ Characteristic gluon energy

(5.4) Number of coherent scatterings: $N_{coh} \approx \frac{t_{coh}}{\lambda}$, where $t_{coh} \approx \frac{2\omega}{k_T^2} \approx \sqrt{\omega/\hat{q}}$

$k_T^2 \approx \hat{q}\, t_{coh}$

(5.5) Gluon energy distribution: $\omega \frac{dI_{med}}{d\omega\, dz} \approx \frac{1}{N_{coh}} \omega \frac{dI_1}{d\omega\, dz} \approx \alpha_s \sqrt{\frac{\hat{q}}{\omega}}$

(5.6) Average energy loss $\Delta E = \int_0^L dz \int_0^{\omega_c} d\omega\, \omega \frac{dI_{med}}{d\omega\, dz} \sim \alpha_s \omega_c \sim \alpha_s \hat{q} L^2$

Quadratic increase with L!

V.3. Opacity Expansion - zeroth order

To understand in more detail the physics contained in

$$\frac{dI}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{(2\pi)^2\omega^2} 2\,\mathrm{Re}\int_0^\infty dy \int_y^\infty d\bar{y} \int du\, e^{-ik_T u} e^{\left[-\int_y^\infty d\xi\, n(\xi)\sigma(u)\right]}$$

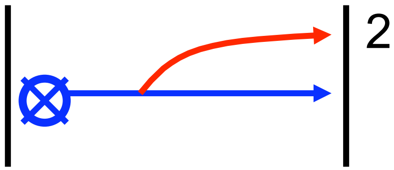
$$\times \frac{\partial}{\partial u} \cdot \frac{\partial}{\partial s} K(s=0,y;u,y\,|\,\omega)$$

We expand this expression in **'opacity'** (=density of scattering centers times dipole cross section)

(5.7) $$K(\underline{s},y;\underline{u},\bar{y}) = K_0(s;u) - \int d\underline{r}\, d\xi\, K_0(\underline{s},y;\underline{r},\xi) \boxed{n(\xi)\sigma(\underline{r})} K_0(\underline{r},\xi;\underline{u},\bar{y}) + \ldots.$$

To zeroth order, there is no medium (vaccum case), and one finds:

(5.8) $$\omega\frac{dI^{(0)}}{d\omega\, dk_T} = \frac{\alpha_s C_F}{\pi^2} H(k_T) =$$



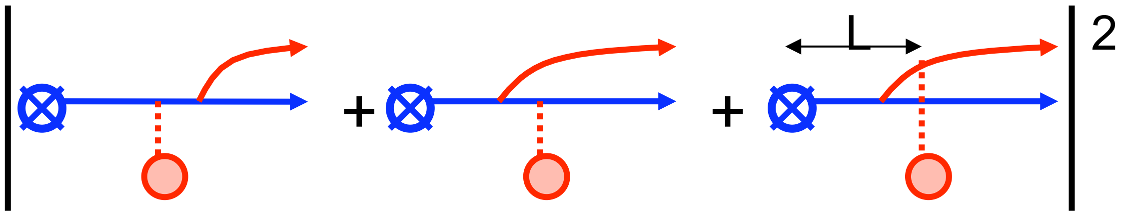
$$, \qquad H(k_T) = \frac{1}{k_T^2}$$

So, in the vacuum, the gluon energy distribution displays the dominant $1/\underline{k}^2$ piece of the DGLAP parton shower.
This is correct vacuum 'baseline', but with insufficient accuracy.

V.4. Opacity Expansion - up to 1st order

To first order in opacity, there is a generally complicate interference between vacuum radiation and medium-induced radiation.

$$\omega \frac{dI^{(1)}}{d\omega\, dk_T} =$$

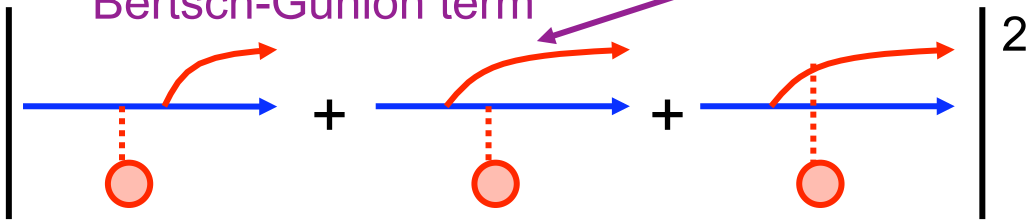


in the totally incoherent limit $L \to \infty$, we identify three contributions:

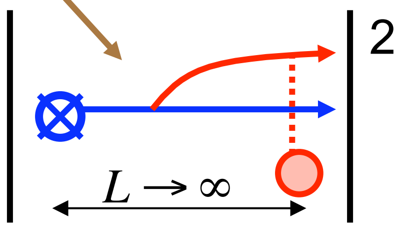
1. **Probability conservation** of medium-independent vacuum terms.
2. **Transverse phase space** redistribution of vacuum piece.
3. **Medium-induced gluon radiation** of quark coming from minus infinity

$$\lim_{L\to\infty}^{nL=const} \omega \frac{dI^{(1)}}{d\omega\, dk_T} = -\boxed{w_1} H(k_T) + nL \int_{q_T} dq_T \left[\boxed{R(q_T, k_T)} + \boxed{H(q_T + k_T)} \right]$$

Bertsch-Gunion term

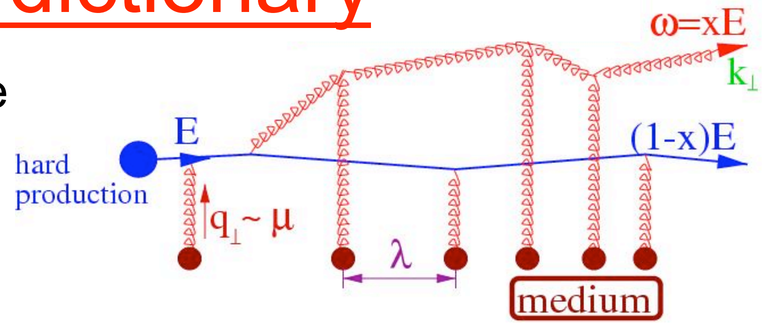


Rescattering of vacuum term



V.5. BDMPS – dictionary

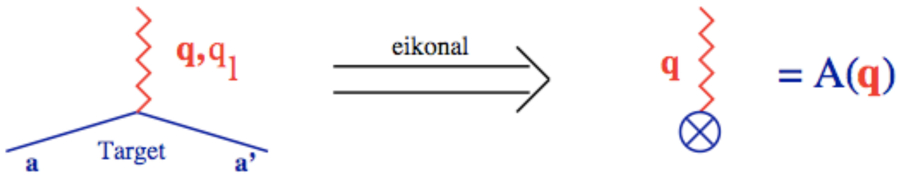
BDMPS-Z calculates in the kinematic regime



$$E >> \omega >> |k_T|,|q_T| >> \Lambda_{QCD}$$

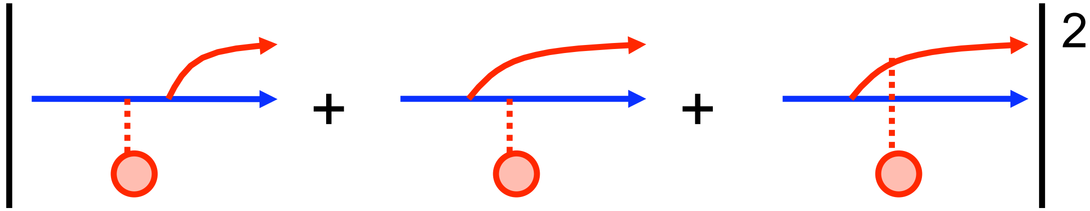
Elastic cross section in this limit

(5.9)



Inelastic cross section for one-fold incoherent scattering

(5.10) $$\frac{1}{\omega}|A(q)|^2 R(k,q) =$$



Inelastic cross section for multiple scattering

(5.11)

$$\propto \left(\prod_i |A(q_i)|^2\right) R\left(k + \sum_{i=2} q_i, q_1\right)$$

Incoherent limit

$$\propto \left(\prod_i |A(q_i)|^2\right) R\left(k, \sum_i q_i\right)$$

Coherent limit

V.6. Example: N=2 opacity

(5.12)
$$\frac{dI(N=2)}{d\ln\omega\, dk_T} = \frac{\alpha_s C_R}{\pi^2}\int dq_1\left(\left|A(q_1)\right|^2 - \sigma_{el}\delta(q_1)\right)\int dq_2\left(\left|A(q_2)\right|^2 - \sigma_{el}\delta(q_2)\right)$$
$$\times\left[\underbrace{\frac{(nL)^2}{2} R(k+q_1;q_2)}_{\textbf{Incoherent}} - n^2\frac{1-\cos LQ_1}{Q_1^2}\left\{\underbrace{R(k+q_1;q_2)}_{\textbf{Incoherent}} - \underbrace{R(k;q_1+q_2)}_{\textbf{Coherent}}\right\}\right]$$

Formation times

(5.13)
$$\tau_{f,n} = \frac{1}{Q_n} = \frac{2\omega}{\left(k_T + \sum_{i=1}^{n} q_i\right)^2}$$

define interpolation scale between totally coherent and incoherent limit

(5.14)
$$n^2\frac{1-\cos LQ_1}{Q_1^2} \longrightarrow \begin{cases} 0 & ,L > \tau_{f1} \\ n^2 L^2/2 & ,L < \tau_{f1} \end{cases}$$

Formally, determine totally coherent and incoherent limiting cases by taking $L \to 0$ or $L \to \infty$ for $nL = fix$

V.7. Main take-home message

- In high-energy limit, the medium-induced splitting a -> b+c, i.e., **medium-induced gluon radiation**) is regarded as the **most efficient mechanism to degrade energy of partonic projectile a**.
  It is more efficient than collisional mechanism a+b -> a'+b'

- Medium-induced gluon radiation has two 'classical' limiting cases:
  - incoherent limit: radiation = incoherent sum of radiation from all independent scattering centers
  - coherent limit: all scattering centers act coherently, as if radiation occurs from one scattering center with q = sum of the q_i

- The **interpolating scale** between coherent and incoherent limits is set by the **gluon formation time**

- Medium-induced quantum interference leads to characteristic parametric dependencies of medium-induced gluon radiation, in particular

$$\omega \frac{dI_{med}}{d\omega} \approx \alpha_s \sqrt{\frac{\omega_c}{\omega}}, \quad \omega_c = \hat{q} L^2 / 2 \qquad \left\langle k_T^2 \right\rangle \propto \hat{q}\, L \qquad \Delta E \propto \hat{q}\, L^2$$

- **Most important message**: these calculations are regarded as an important piece for a phenomenological understanding of jet quenching, but other pieces are missing: Multiple gluon radiation? Radiative vs. collisional energy loss? Evolution in virtuality? …

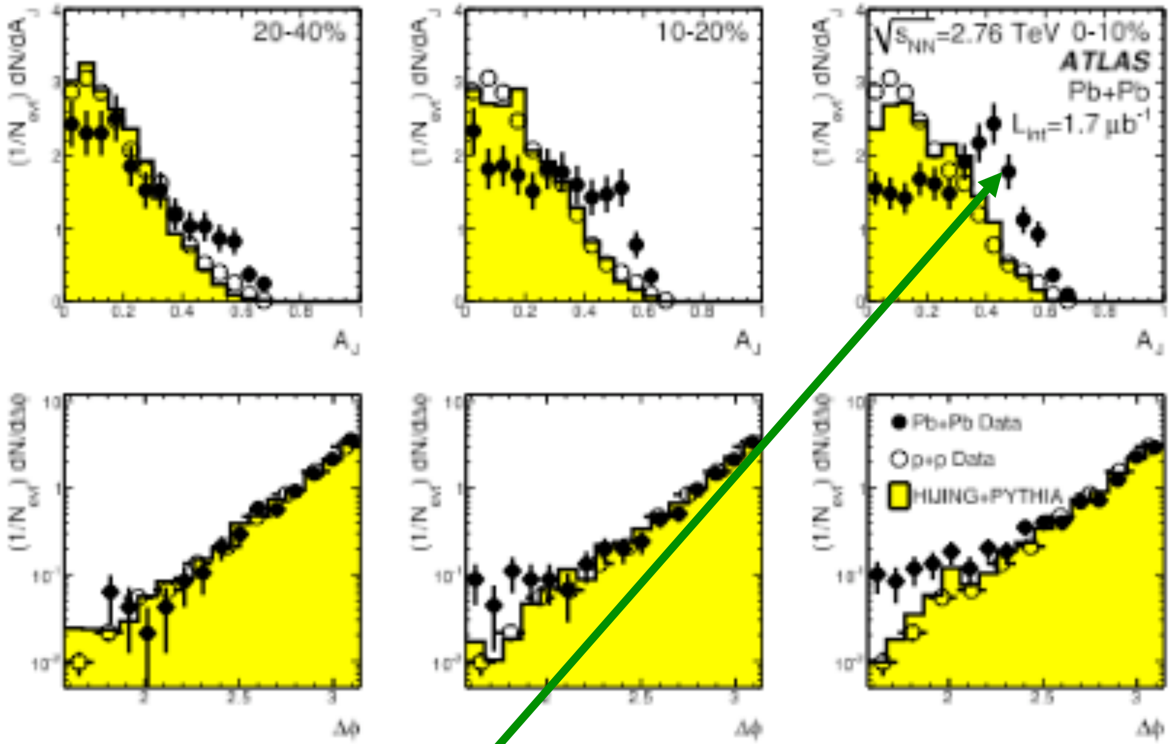
Back to pictures and questions…

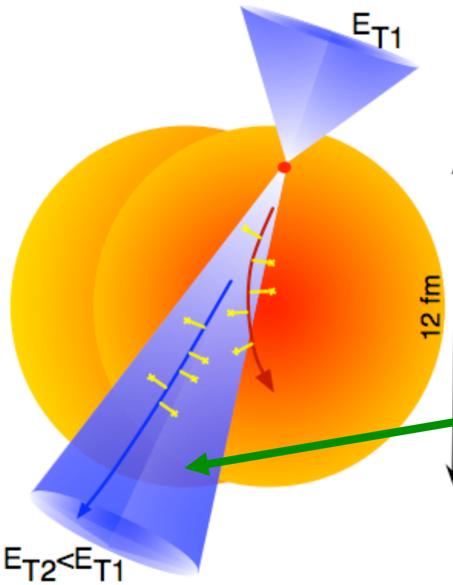
VI.1. Dijet asymmetry

Closing the lecture with one of the “latest news of the field”

$$A_J = \frac{E_{T1} - E_{T2}}{E_{T1} + E_{T2}}$$

- Asymmetry present in pp as result of 3-jet rates,
- but much enhanced in AA

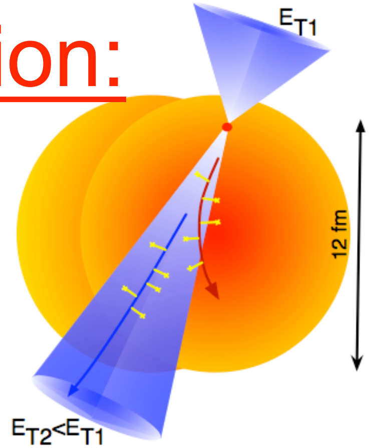




Problem: How can this jet broaden
(as suggested by A_j-dependence)
while $\Delta\Phi$ - dependence
is almost unaffected?

VI.2. Jet quenching via jet collimation:

J. Casalderrey-Solana, G. Milhano, U.A. Wiedemann, arXiv:1012.0745

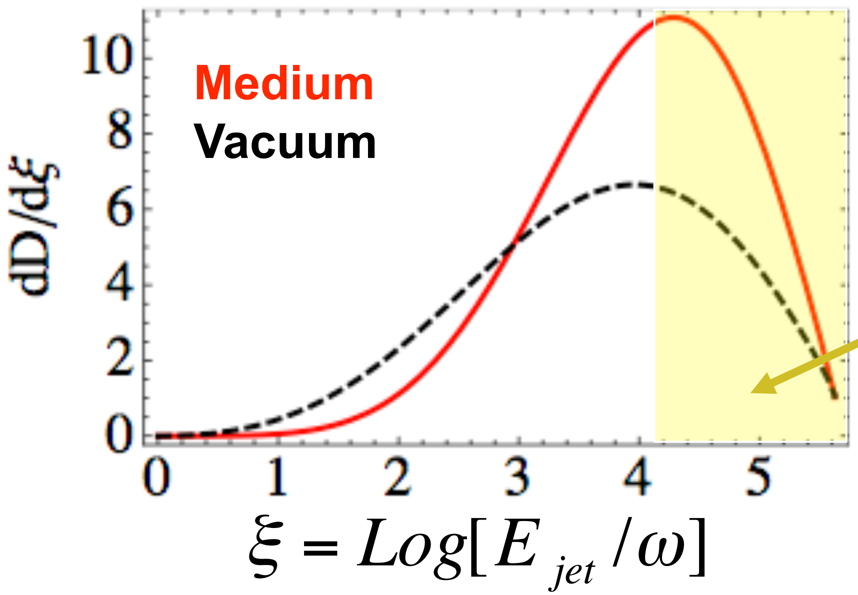


In medium, formation times of soft partons are shorter

$$\tau_f^{vac} \cong \frac{\omega}{k_T^2} = \frac{1}{\theta^2 \omega}, \qquad \tau_f^{med} \cong \frac{\omega}{k_T^2} = \sqrt{\frac{\omega}{\hat{q}}}$$

So **soft gluons** are there early in the shower and they are radiated at larger angle

$$\langle \theta^2 \rangle = \frac{\langle k_T^2 \rangle}{\omega^2} = \frac{\hat{q} L}{\omega^2}$$



A significant fraction of the total jet energy is soft modes

$$\omega \leq \sqrt{\hat{q} L}$$

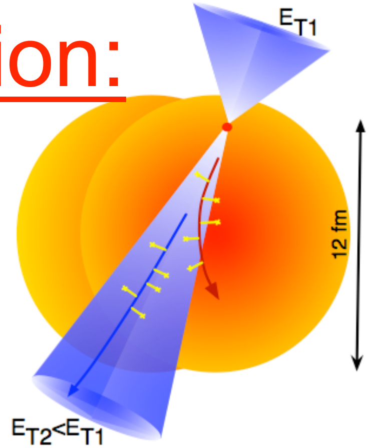
And can be radiated at angles

$$\langle \theta \rangle \geq 1$$

Complete decorrelation from jet axis!

VI.2. Jet quenching via jet collimation:

J. Casalderrey-Solana, G. Milhano, U.A. Wiedemann, arXiv:1012.0745



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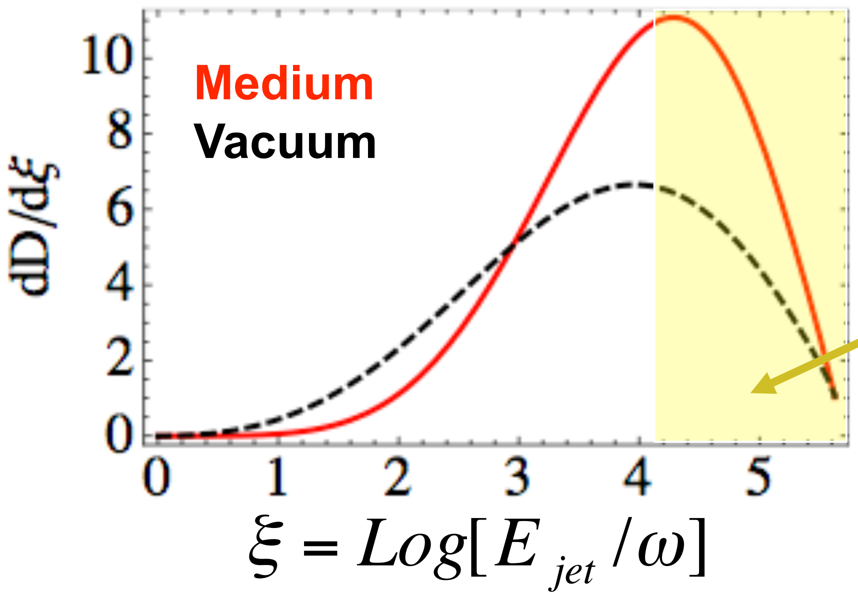
$$\tau_f^{vac} \cong \frac{\omega}{k_T^2} = \frac{1}{\theta^2 \omega}, \qquad \tau_f^{med} \cong \frac{\omega}{k_T^2} = \sqrt{\frac{\omega}{\hat{q}}}$$

So **soft gluons** are there early in the shower and they are radiated at larger angle

IS THIS TRUE?

$$\langle \theta^2 \rangle = \frac{\langle k_T^2 \rangle}{\omega^2} = \frac{\hat{q} L}{\omega^2}$$

CAN IT BE SUPPORTED BY DETAILED CALCULATIONS?



A significant fraction of the total jet energy is soft modes

$$\omega \leq \sqrt{\hat{q} L}$$

And can be radiated at angles

$$\langle \theta \rangle \geq 1$$

Complete decorrelation from jet axis!

VI.4. LHC: high-pt opportunities in coming years

The probes:

- Jets
- identified hadron specta
- D-,B-mesons
- Quarkonia
- Photons
- Z-boson tags

The range:

Q^2 ,x, A, luminosity

Abundant yield

of hard probes

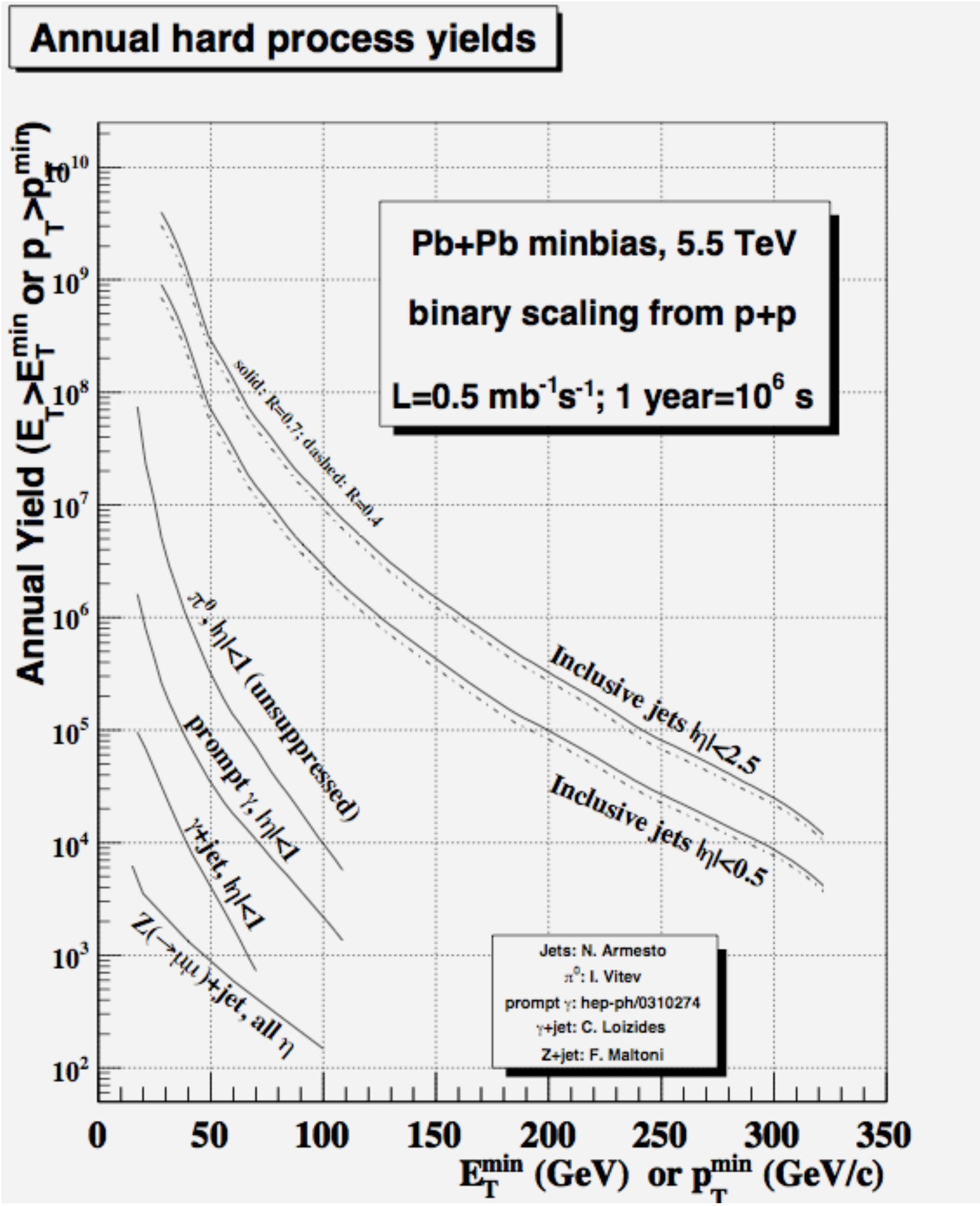
+ robust signal

(medium sensitivity

>> uncertainties)

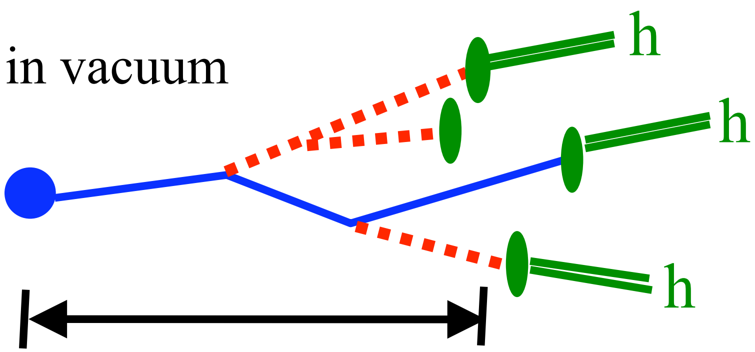
= detailed understanding

of dense QCD matter

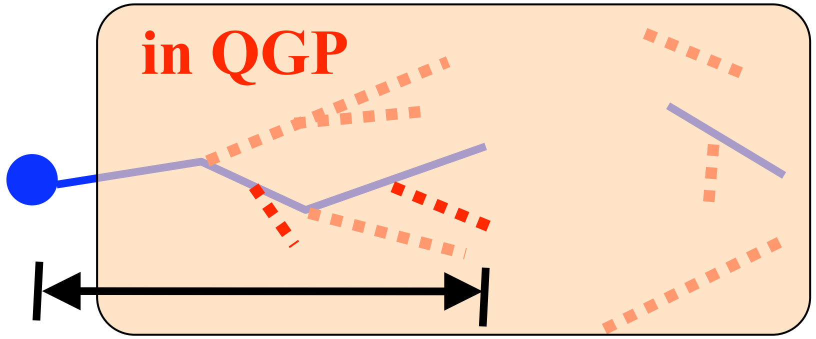


Back-up

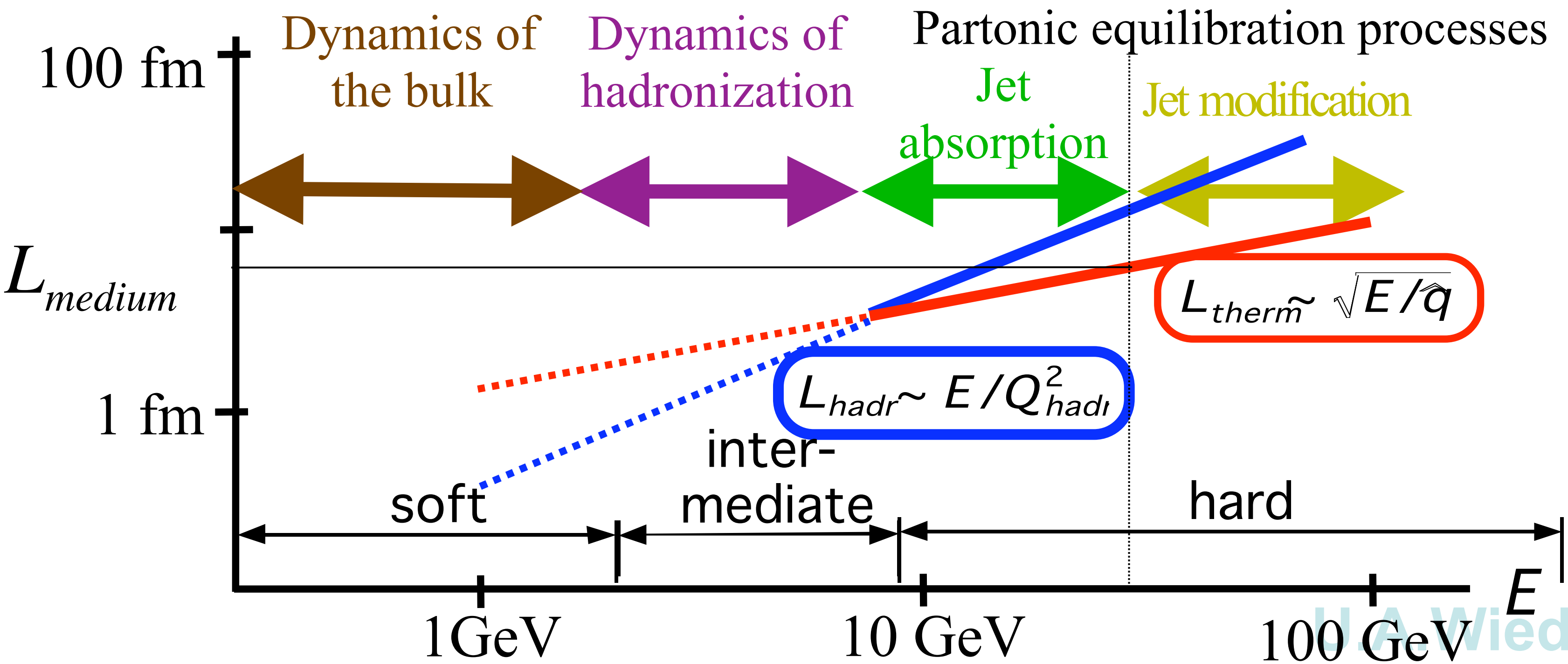
VI.1. Estimating Time scales for parton E-loss



$$L_{hadr} = const \frac{E}{Q^2}$$



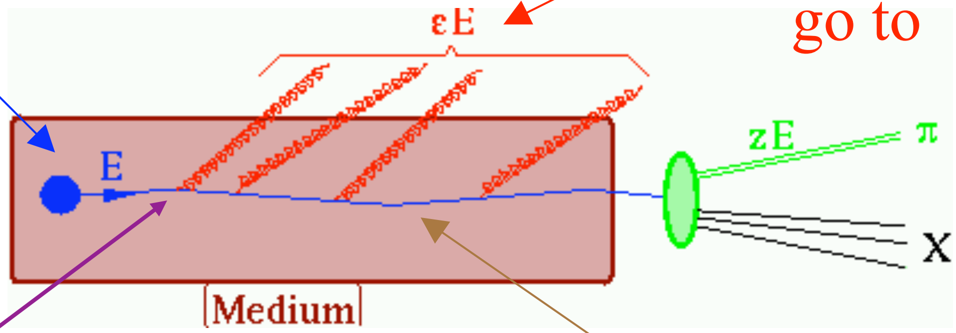
$$\Delta E \approx E \approx \alpha_s \hat{q} L^2_{therm}$$



Improved Characterization of jet quenching with hadron spectra and hadron correlations

How does this parton thermalize ?

Where does this associated radiation go to ?



What is the dependence on parton identity ?

How do we detect recoil in case of collissional energy loss ?

$$\Delta E_{gluon} > \Delta E_{quark,m=0} > \Delta E_{quark,m>0}$$

U.A.Wiedemann

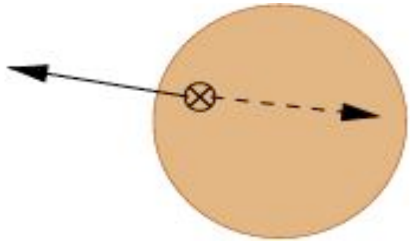
Trigger Bias in high-pt hadron production

Triggering on high-pt hadrons, one selects a biased parton fragmentation pattern:

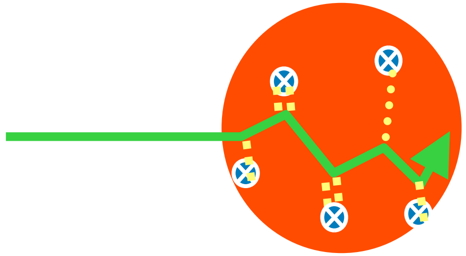
- A bias favoring hard fragmentation, determined by steepness of spectrum:

$$\frac{z^{n-1}}{(p_T^{hadron})^n} D_{h/q}^{(med)}(z,Q^2)$$

- A bias favoring small in-medium path length (surface emission)



- A bias favoring initial-state pt-broadening in the direction of the trigger



U.A.Wiedemann