

# Introduction to Neutrino Physics

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Michael A. Schmidt

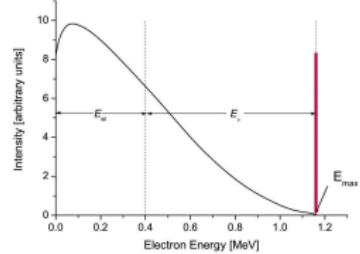
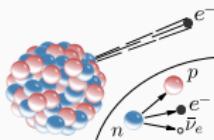
27 June 2022

Sydney-CPPC meeting

UNSW

# Neutrinos - a brief history

1920s Ellis: beta decay spectrum is continuous



1931 Pauli postulates neutrino  
to explain energy and (angular) momentum conservation

More than 25 years until detection

1959 Cowan, Reines: neutrino detection

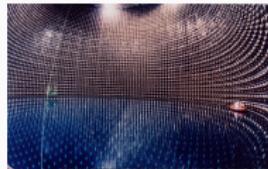


1962 Lederman, Schwartz, Steinberger:  $\nu_\mu$  detection

1989 Large Electron Positron collider:  $Z$  decay width  $\Rightarrow N_\nu = 2.984 \pm 0.008$

1998 Super-Kamiokande: atmospheric neutrino oscillations

2001 SNO: solar neutrino oscillations



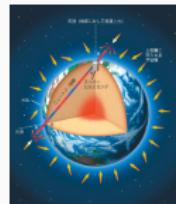
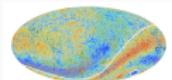
2012 Daya Bay, RENO, Double CHOOZ:  $\theta_{13}$

2017 COHERENT: CE $\nu$ NS

# Neutrinos in Nature

## Atmosphere

Cosmic neutrino background



Super-K



Sun

Super-novae



SN1987a

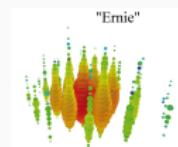


NASA

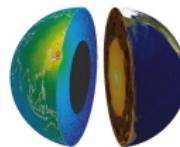


Daya Bay

Cosmic accelerators

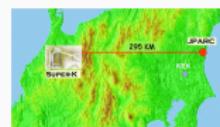


IceCube



KamLAND

Earth crust



Accelerators

T2K

# Outline

What do we know?

Neutrino Oscillations

Neutrino Mass Limits

Neutrino Mass Generation

Dirac vs. Majorana Mass

Seesaw Mechanisms

Beyond seesaw

Lepton mixing and symmetries

Leptogenesis

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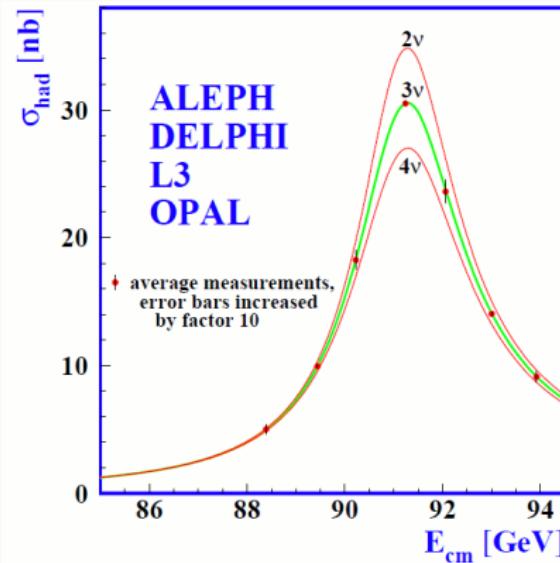
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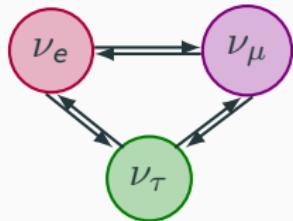
# How many Neutrinos?



- LEP measured invisible partial width  $\Gamma_{inv}$  of  $Z$  boson
- Used to determine number of neutrinos  $N_\nu$
- The combination of four LEP experiments yields

$$N_\nu = 2.984 \pm 0.008$$

# Neutrino oscillations



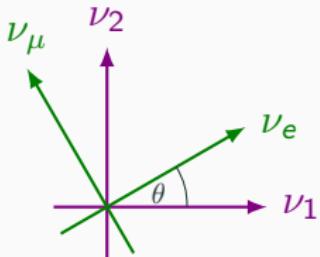
Neutrinos change flavour – they oscillate

Neutrino weak interaction eigenstates  $\nu_\alpha$   
are a superposition of mass eigenstates  $\nu_i$

interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

mass eigenstates  
with masses  
 $m_1, m_2, m_3$



The transition probability is

$$P_{e \leftarrow \mu} = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right)$$

[ $\Delta m^2 = m_2^2 - m_1^2$ ]

distance source to detector

energy



2015: T. Kajita, A. McDonald [for atmospheric and solar  $\nu$  oscillations] 7

# Neutrinos in SM

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

weak interactions

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} j_\rho^{CC} W^\rho - \frac{g}{2 \cos \theta_W} j_\rho^{NC} Z^\rho + c.c.$$

$$j_\rho^{CC} = 2 \sum_{\ell=e,\mu,\tau} \bar{\nu}_{\ell L} \gamma_\rho \ell_L \quad j_\rho^{NC} = \sum_{\ell=e,\mu,\tau} \bar{\nu}_{\ell L} \gamma_\rho \nu_{\ell L}$$

weak interaction eigenstate  $\neq$  mass eigenstate

$$\nu_{\alpha L} = \sum_i U_{\alpha i} \nu_{i L}$$

Thus the states are related by

$$|\nu_\alpha\rangle = \nu_\alpha^\dagger |0\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle$$

# Quantum Mechanical Description

Time evolution described by Schrödinger equation

$$i \frac{d}{dt} |\nu(t)\rangle = H |\nu(t)\rangle$$

Vacuum Hamiltonian diagonal and constant in mass basis

$$|\nu(t)\rangle = \mathcal{T} e^{-i \int_0^t H(t') dt'} |\nu_\alpha\rangle = \sum_i U_{\alpha i}^* e^{-i E_i t} |\nu_i\rangle$$

Thus the transition from flavour state  $\nu_\alpha \rightarrow \nu_\beta$

$$\mathcal{A}_{\beta\alpha}(t) \equiv \langle \nu_\beta | \nu(t) \rangle = \sum_i U_{\beta i} e^{-i E_i t} U_{\alpha i}^*$$

Transition probability

$$P_{\beta\alpha} = |\mathcal{A}_{\beta\alpha}|^2$$

## Quantum Mechanical Description 2

Ultra-relativistic neutrinos  $m_i \ll E_i$

$$E_i = \sqrt{p^2 + m_i^2} \simeq p + \frac{m_i^2}{2p}$$

Using distance travelled  $L \simeq t$  and  $E \simeq p$

$$P_{\beta\alpha} = \left| \sum_i U_{\beta i} e^{-i \frac{m_i^2}{2E} L} U_{\alpha i}^* \right|^2 = \sum_{i,j} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} e^{i \phi_{ij}}$$

with phase

$$\phi_{ij} = \frac{m_j^2 - m_i^2}{2E} L$$

What is the problem with this derivation?

- See 0905.1903 for an in depth discussion of issues
- Proper derivation using wave packet formalism or QFT [See e.g. 0905.1903 and references]

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## Quantum Mechanical Description 3

$$P_{\beta\alpha} = \sum_{i,j} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} e^{i\phi_{ij}} \quad \text{with} \quad \phi_{ij} = \frac{m_j^2 - m_i^2}{2E} L$$
$$= \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2 + 2 \operatorname{Re} \left( \sum_{i>j} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} e^{i\phi_{ij}} \right)$$

- First term classical part (no interference)
- Second term is interference term, averages to zero for  $\phi_{ij} \gg 1$  due to spread in energy  $E$  or baseline  $L$ .

## Quantum Mechanical Description 4

Using unitarity of PMNS matrix  $\sum_i U_{\beta i} U_{\alpha i}^* = \delta_{\alpha\beta}$  and thus

$$\sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2 = \delta_{\alpha\beta} - 2 \operatorname{Re} \left( \sum_{i>j} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} \right)$$

we find

$$\begin{aligned} P_{\beta\alpha} &= \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2 + 2 \operatorname{Re} \left( \sum_{i>j} U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j} e^{i\phi_{ij}} \right) \\ &= \delta_{\alpha\beta} - 4 \sum_{i>j} \operatorname{Re} (U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j}) \sin^2 \frac{\phi_{ij}}{2} \\ &\quad - 2 \sum_{i>j} \operatorname{Im} (U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j}) \sin \phi_{ij} \end{aligned}$$

Third term describes CP violation (vanishes for  $\alpha = \beta$ )

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) - P(\nu_\alpha \rightarrow \nu_\beta) = 4 \sum_{i>j} \operatorname{Im} (U_{\beta i} U_{\alpha i}^* U_{\beta j}^* U_{\alpha j}) \sin \phi_{ij}$$

## Two Generations

$$P_{\beta\alpha} = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \quad P_{\alpha\alpha} = 1 - P_{\beta\alpha}$$

- Typical length scale  $L = \frac{4\pi E}{\Delta m^2} = 2.48 \frac{E[\text{MeV}]}{\Delta m^2[\text{eV}^2]} \text{m}$   
e.g. atmospheric neutrinos:  
 $E \sim 1 \text{ GeV}, \Delta m^2 \sim 2 \times 10^{-3} \text{ eV}^2 \Rightarrow L \sim 1000 \text{ km}$
- Oscillation probability invariant under

$$\Delta m^2 \rightarrow -\Delta m^2$$

$$\theta \rightarrow \frac{\pi}{2} - \theta$$

- Mass hierarchy and octant can not be determined for two neutrino oscillations in vacuum

# Matter Effect

- Hamiltonian in matter

$$H^{(f)} = H_{vac}^{(f)} + V^{(f)}$$

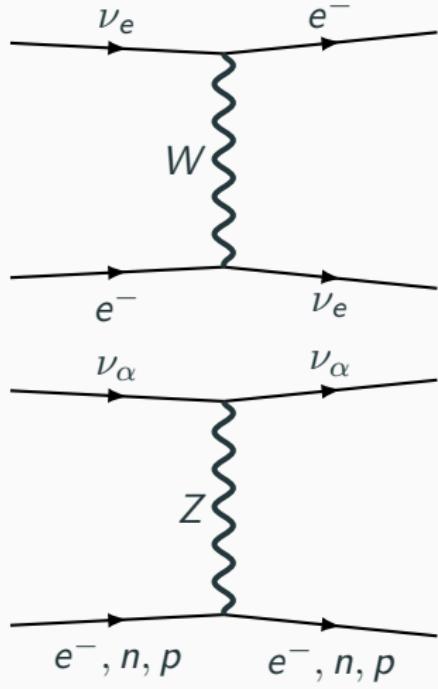
- NC interaction results in overall phase
- ✗ Does not apply in presence of sterile  $\nu$ s
- Matter potential of electron density  $n_e$

$$V_{ee}^{(f)} = \pm \sqrt{2} G_f n_e(x)$$

- Rediagonalisation of  $H^{(f)}$  required

$$\nu^{(f)}(x) = \tilde{U}(x) \nu^{(\tilde{m})}(x)$$

$$H^{(f)} = \frac{1}{2E} \tilde{U}(x) \tilde{M}^2(x) \tilde{U}^\dagger(x)$$



## Matter Effect 2

Rediagonalisation of  $H^{(f)}$

$$\nu^{(f)}(x) = \tilde{U}(x)\nu^{(\tilde{m})}(x)$$

$$H^{(f)} = \frac{1}{2E} \tilde{U}(x) \tilde{M}^2(x) \tilde{U}^\dagger(x)$$

Time-evolution equation

$$i \frac{d\nu^{(\tilde{m})}}{dx} = \left[ \frac{\tilde{M}^2}{2E} - i \tilde{U}^\dagger(x) \frac{d\tilde{U}(x)}{dx} \right] \nu^{(\tilde{m})}(x)$$

- The second term describes change of mass eigenstates
- Adiabatic approximation: It can be neglected if

$$L = \frac{4\pi E}{\Delta \tilde{M}^2} \ll \left( \frac{d \ln n_e(x)}{dx} \right)^{-1}$$

→ Unitary evolution, magnitude of mass eigenstates unchanged

## Two Generations: e.g. Solar Neutrino

Effective mixing angle and mass in matter

$$\sin(2\tilde{\theta}) = \frac{\sin(2\theta)}{\sqrt{\sin^2(2\theta) + C^2}}$$

$$\Delta\tilde{m}^2 = \Delta m^2 \sqrt{\sin^2(2\theta) + C^2}$$

with the parameter

$$C(x) = \cos(2\theta) - \frac{2\sqrt{2}G_F n_e(x)E}{\Delta m^2}$$

- Matter effect breaks degeneracies:  $\Delta m^2 \not\rightarrow -\Delta m^2$  and  $\theta \not\rightarrow \frac{\pi}{2} - \theta$
- Mikheyev-Smirnov-Wolfenstein (MSW) resonance  $C(x) = 0$

$$\Rightarrow \Delta m^2 \cos(2\theta) = 2\sqrt{2}G_F n_e(x)E$$

# Global fit to neutrino oscillation experiments



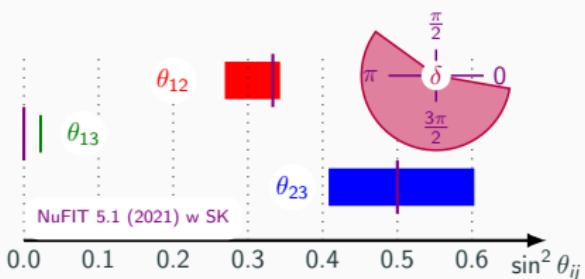
## Leptonic mixing (PMNS) matrix $U_{\alpha i}$

interaction eigenstates

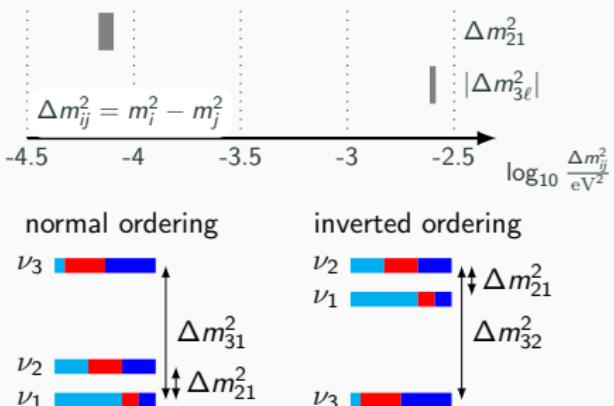
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mass eigenstates  
with masses  $m_1, m_2, m_3$

3 angles  $\theta_{ij}$ , 1 Dirac phase  $\delta$  and 2 Majorana phases  $\alpha_{ij}$

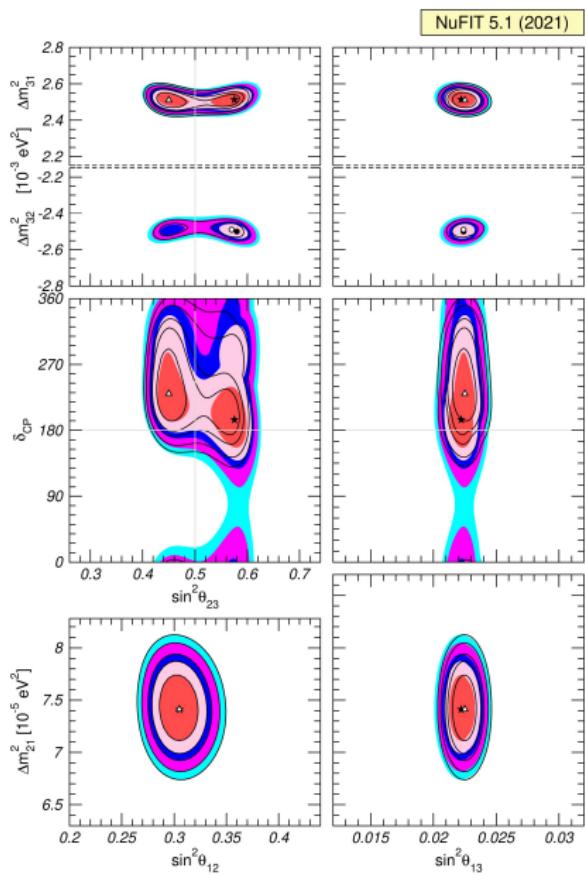


Possibly explained by symmetry



Neutrino oscillations independent of Majorana phases  $\alpha_{ij}$  and absolute mass scale

# Global Fit to Neutrino Oscillation Experiments

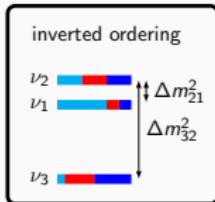
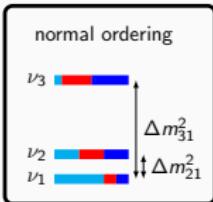
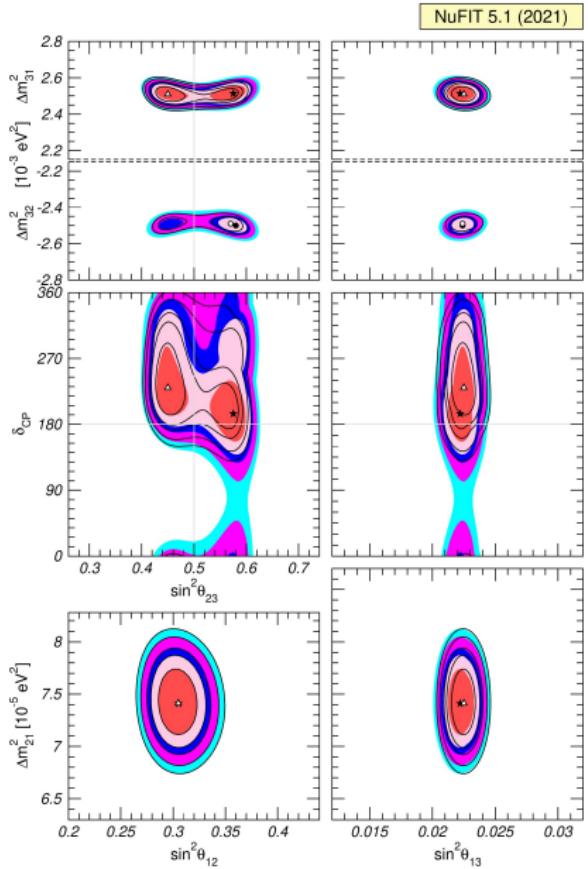


Leptonic mixing (PMNS) matrix

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

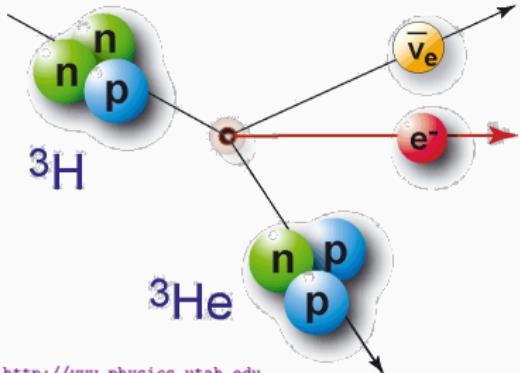
NuFIT 5.1 2007.14792	Normal Ordering (best fit)	
	bfp $\pm 1\sigma$	$3\sigma$ range
$\sin^2 \theta_{12}$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$
$\theta_{12}/^\circ$	$33.45^{+0.77}_{-0.75}$	$31.27 \rightarrow 35.87$
$\sin^2 \theta_{23}$	$0.450^{+0.019}_{-0.016}$	$0.408 \rightarrow 0.603$
$\theta_{23}/^\circ$	$42.1^{+1.1}_{-0.9}$	$39.7 \rightarrow 50.9$
$\sin^2 \theta_{13}$	$0.02246^{+0.00062}_{-0.00062}$	$0.02060 \rightarrow 0.02435$
$\theta_{13}/^\circ$	$8.62^{+0.12}_{-0.12}$	$8.25 \rightarrow 8.98$
$\delta_{CP}/^\circ$	$230^{+36}_{-25}$	$144 \rightarrow 350$
$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$
$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	$+2.510^{+0.027}_{-0.027}$	$+2.430 \rightarrow +2.593$

# Global Fit to Neutrino Oscillation Experiments



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# Mass limit: tritium endpoint measurement



<http://www.physics.utah.edu>

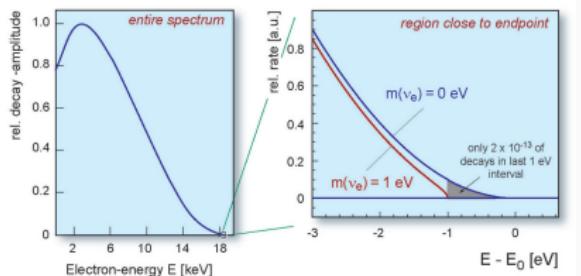
If energy resolution worse than mass splitting

$$S(E_e) \propto (Q - E_e) \sum_j U_{ej}^* U_{ej} \sqrt{(Q - E_e)^2 - m_j^2}$$
$$\simeq (Q - E_e) \sqrt{(Q - E_e)^2 - \sum_j U_{ej}^* U_{ej} m_j^2}$$

KATRIN experiment 2105.08533

$$\sqrt{\sum_i |U_{ei}|^2 m_i^2} \leq 0.8 \text{ eV}$$

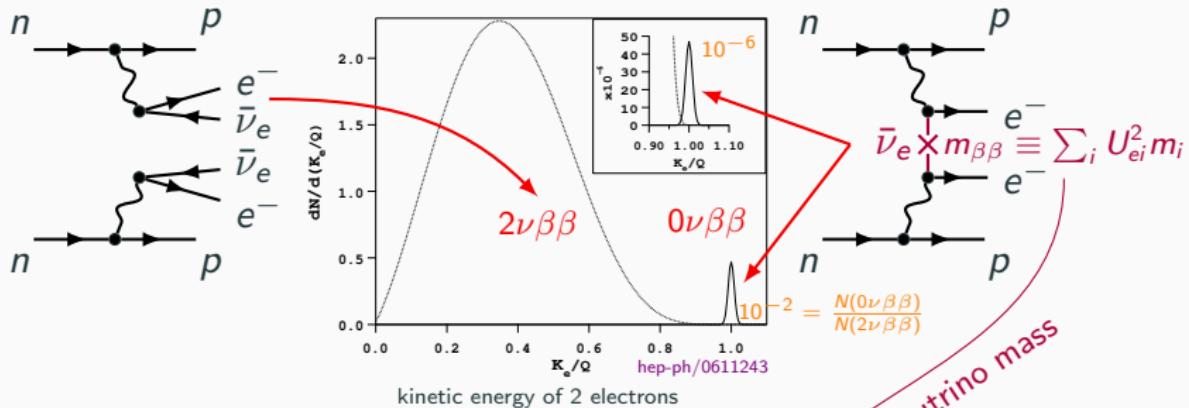
ultimate sensitivity 0.2 eV



<http://www.katrin.kit.edu>



# Mass limit: neutrinoless double beta decay



nuclear physics

lifetime

$$T_{1/2}^{0\nu} \propto \frac{1}{|M_{\text{nucl}}|^2} \quad \frac{1}{\langle m_{\beta\beta} \rangle^2}$$

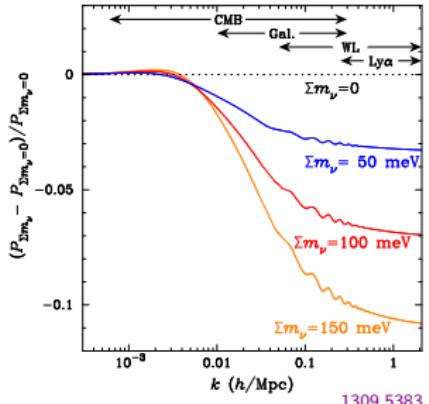
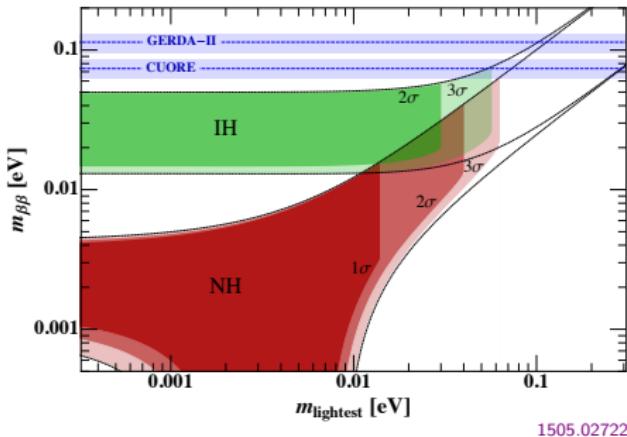
$^{76}\text{Ge}$ : Gerda

$^{136}\text{Xe}$ : KamLAND-Zen, EXO

$^{130}\text{Te}$ : CUORE

Limit:  $|m_{\beta\beta}| \lesssim 0.06 - 0.4 \text{ eV}$

# Mass Limits



$$\sqrt{\sum_i |U_{ei}|^2 m_i^2} \leq 0.8 \text{ eV}$$

$$m_{\beta\beta} \equiv \sum_i U_{ei}^2 m_i < (0.2 - 0.4) \text{ eV}$$

$$\sum_{\text{cosm}} m_i \equiv \sum_i m_i \lesssim (0.11 - 0.54) \text{ eV}$$

KATRIN 2105.08533

GERDA, EXO,  
KamLAND-Zen

Planck 1807.06209

## Open questions

- Are neutrinos their own antiparticles?
- What is the absolute neutrino mass scale?
- Normal [ $m_1 < m_3$ ] vs. inverted [ $m_3 < m_1$ ] mass ordering?
- What are the values of the remaining parameters?
- Is there anything beyond 3 neutrinos?



## Theoretical questions

- Why are neutrinos so much lighter than all other matter?
- Is there any explanation for the mixing angles?

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Dirac vs. Majorana Mass

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Beyond seesaw

Lepton mixing and symmetries

Leptogenesis

# Neutrino Mass



P. Dirac [Wikipedia]

## Dirac neutrinos

Three Generations of Matter (Fermions) spin $\frac{1}{2}$			
I	II	III	
mass $\rightarrow$ charge $\rightarrow$ name $\rightarrow$	2.4 MeV $\frac{2}{3}$ u up left	1.27 GeV $\frac{2}{3}$ c charm left	171.2 GeV $\frac{2}{3}$ t top right
Quarks	d $-\frac{1}{3}$ down right	s $-\frac{1}{3}$ strange right	b $-\frac{1}{3}$ bottom right
Leptons	$\nu_e$ 0 eV electron neutrino left	$\nu_\mu$ 0 eV muon neutrino left	$\nu_\tau$ 0 eV tau neutrino right
	0.511 MeV -1 e electron right	105.7 MeV -1 $\mu$ muon right	1.777 GeV -1 $\tau$ tau right
Bosons (Forces) spin 1			
	Z 0.2 GeV weak force right	W 80.4 GeV weak force right	
		Higgs 125 GeV spin 0 right	

arXiv:1301.5516 [hep-ph]

- Effective Dirac mass term

$$\mathcal{L}_\nu = -m_D \bar{\nu} N$$

- Dirac mass term in SM

$$\mathcal{L}_\nu = -Y_D \bar{L} H N$$

# Neutrino Mass



P. Dirac [Wikipedia]

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Quarks	d down Left $-\frac{1}{3}$	s strange Left $-\frac{1}{3}$	b bottom Left $-\frac{1}{3}$
Leptons	$\nu_e$ electron sterile neutrino neutrino Left $\frac{1}{2}$	$\nu_\mu$ muon sterile neutrino neutrino Left $\frac{1}{2}$	$\nu_\tau$ tau sterile neutrino neutrino Left $\frac{1}{2}$
	0.511 MeV e electron Right	105.7 MeV $\mu$ muon Right	1777 GeV $\tau$ tau Right
Bosons (Forces) spin 1	g gluon 0 0	$\gamma$ photon 0 0	$Z^0$ weak force 91.2 GeV Higgs boson 0 0
	125 GeV	arXiv:1301.5516 [hep-ph]	

- Effective Dirac mass term

$$\mathcal{L}_\nu = -m_D \nu N$$

- Dirac mass term in SM

$$\mathcal{L}_\nu = -Y_D L H N$$

# Neutrino Mass



P. Dirac [Wikipedia]

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mass → charge → name →	2.4 MeV $\frac{2}{3}$ u up Left	1.27 GeV $\frac{2}{3}$ c charm Left	171.2 GeV $\frac{2}{3}$ t top Left
Quarks	d $-\frac{1}{3}$ down Left	s $-\frac{1}{3}$ strange Left	b $-\frac{1}{3}$ bottom Left
Leptons	$e^-$ $N_1$ electron sterile neutrino neutrino neutrino Left	$\nu_\mu$ $N_2$ muon sterile neutrino neutrino neutrino Left	$\nu_\tau$ $N_3$ tau sterile neutrino neutrino neutrino Left
	0.511 MeV -1 e electron Right	105.7 MeV -1 $\mu$ muon Right	1777 GeV -1 $\tau$ tau Right
Bosons (Forces) spin 1	g gluon 0 0	$\gamma$ photon 0 0	$Z^0$ weak force 93.2 GeV 0 Higgs boson 125 GeV spin 0

arXiv:1301.5516 [hep-ph]

- Effective Dirac mass term

$$\mathcal{L}_\nu = -m_D \nu N$$

- Dirac mass term in SM

$$\mathcal{L}_\nu = -Y_D L H N$$

# Neutrino Mass

## Majorana neutrinos

- Neutrino is its own anti-particle

→ Majorana mass term

$$\mathcal{L}_\nu = -\frac{1}{2} m_M \bar{\nu} \nu$$

- Majorana mass term in SM

$$\mathcal{L}_\nu = -\frac{\kappa}{\Lambda} L H L H$$



E. Majorana [Wikipedia]

## Effective Operator

- It is an effective operator like in Fermi theory of beta decay

$$[G_F \propto \frac{1}{m_W^2}]$$

⇒ There is an underlying, more fundamental theory.

# Neutrino Mass

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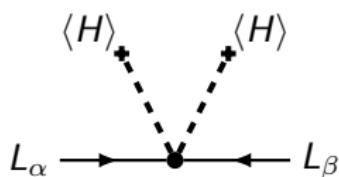
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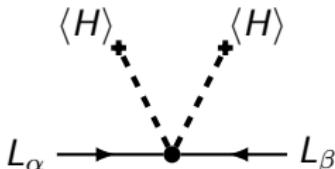
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## Dirac neutrinos

Three Generations of Matter (Fermions) spin $\frac{1}{2}$			Bosons (Force) spin 1		
I	II	III	0	0	125 GeV
mass → $\frac{2.4 \text{ MeV}}{\frac{2}{3}}$ charge → $\frac{2}{3}$ name → u left up flavor	mass → $\frac{1.27 \text{ GeV}}{\frac{2}{3}}$ charge → $\frac{2}{3}$ name → c left charm flavor	mass → $\frac{171.2 \text{ GeV}}{\frac{2}{3}}$ charge → $\frac{2}{3}$ name → t left top flavor	0 g gluon	0 0 $\gamma$ photon	$>114 \text{ GeV}$ $Z^0$ weak force
Quarks left down d	left strange s	left bottom b	0 $Z^0$ weak force	0 $H^0$ Higgs boson	spin 0
Leptons left electron neutrino sterile neutrino neutrino e	left muon neutrino sterile neutrino neutrino $\mu$	left tau neutrino sterile neutrino neutrino $\tau$	0.511 GeV $e^-$ electron	105.7 MeV $\mu^-$ muon	1.777 GeV $\tau^-$ tau
Bosons (Force) spin 0 $\text{W}^\pm$ weak force			80.4 GeV $\pm 1$		

arXiv:1301.5516 [hep-ph]

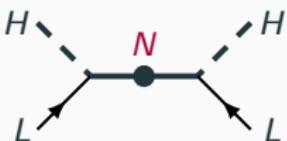
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$$\mathcal{L}_\nu = -m_D \bar{\nu} N$$

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# Seesaw Mechanism



Minkowski; Yanagida; Glashow;  
Gell-Mann, Ramond, Slansky; Mohapatra, Senjanovic.

$$N \sim (1, 1, 0)$$

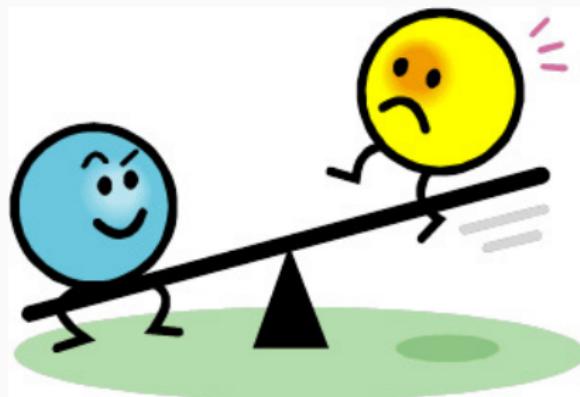
$$-\mathcal{L} = y_D L N H + \frac{1}{2} M_N N N$$

At low energy scales  $\mu \ll M$

$$\Rightarrow m_\nu = -y_D M^{-1} y_D^T \langle H \rangle^2$$

Three Generations of Matter (Fermions) spin $\frac{1}{2}$					
mass →	I	II	III		
charge →	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$		
name →	u up left right	c charm left right	t top left right		
Quarks	d down left right	s strange left right	b bottom left right	g gluon	$125 \text{ GeV}$
	$2.4 \text{ MeV}$	$1.27 \text{ GeV}$	$171.2 \text{ GeV}$	$\gamma$ photon	
	$-1/3$	$-1/3$	$-1/3$	$Z$ weak force	
	$4.8 \text{ MeV}$	$104 \text{ MeV}$	$4.2 \text{ GeV}$	$H$ Higgs boson	
Leptons	$e^-$ electron neutrino neutrino	$\nu_\mu$ muon neutrino neutrino	$\nu_\tau$ tau neutrino neutrino	$W^\pm$ weak force	
	$0.511 \text{ MeV}$	$105.7 \text{ MeV}$	$1.777 \text{ GeV}$		

Bosons (Forces) spin 1



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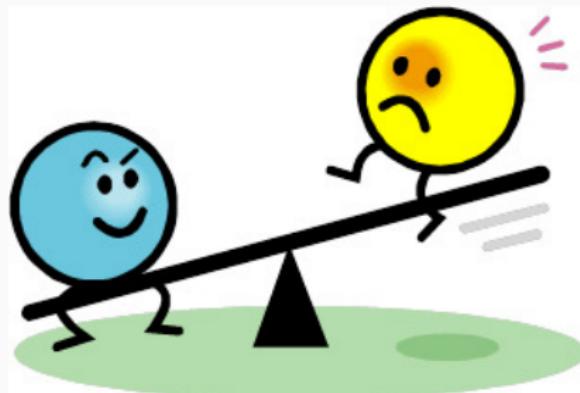
$$N \sim (1, 1, 0)$$

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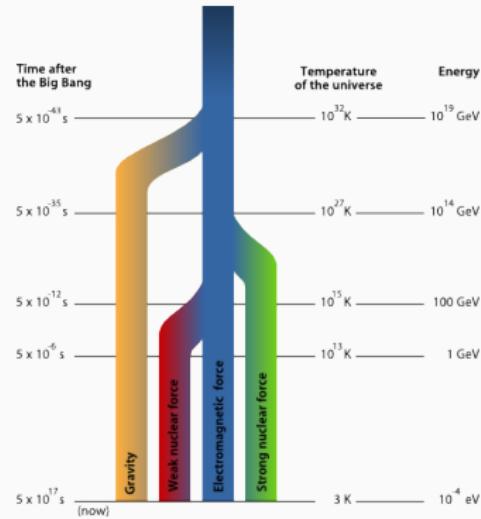
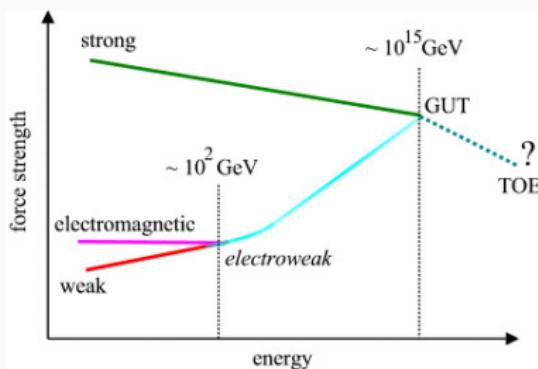
$$\Rightarrow m_\nu = -y_D M^{-1} y_D^T \langle H \rangle^2$$

- Two RH neutrinos  $N$  are enough to explain data
- Typically three  $N$
- $U(1)_{B-L}$  is anomaly-free
- $M_N \sim 10^{14} \text{ GeV}$ ,  $y_D \sim 1$   
 $\Downarrow$   
 $m_\nu \sim 0.1 \text{ eV}$



# Grand Unification

- e.g. SO(10) Fritsch, Minkowski; Glashow 1973
- In SO(10) all SM fermions unified in one 16 dimensional representation
- Including right-handed neutrinos
  - Seesaw mechanism naturally realised
  - Neutrinos have mass



- ⇒ Relations between fermion masses
- ⇒ Unification of Yukawa couplings more difficult

# Type I Seesaw: Flavour Physics

- LFV rare decay  $\mu \rightarrow e\gamma$

$$Br(\mu \rightarrow e\gamma) = \frac{3\alpha}{32\pi} \left| U_{\mu i}^* U_{ei} \frac{m_{\nu i}^2}{m_W^2} \right|^2 \lesssim 10^{-54}$$

- Constant term in  $m_\nu$  drops out due to GIM-mechanism (unitarity)

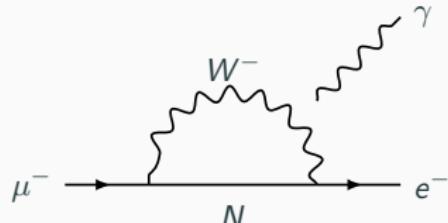
- Similarly for  $\mu \rightarrow 3e$  as well as  $\tau$  decays

- No mentionable contribution to quark flavour physics

⇒ Not much to see!

- ★ This changes in extensions, e.g.

- in SUSY due to slepton soft mass terms
  - in LR models due to RH gauge bosons
  - in inverse seesaw with TeV scale masses due to larger couplings
- LFV D6 operator can be large despite small LNV D5 operator
- ...



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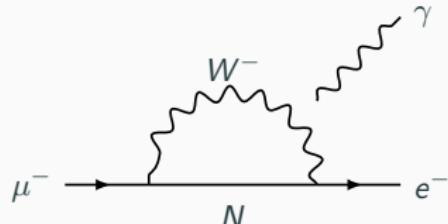
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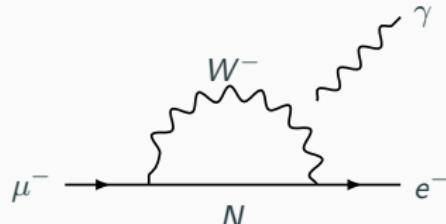


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- $\rightarrow$  LFV D6 operator can be large despite small LNV D5 operator
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# Inverse Seesaw

- Add three SM singlets  $S$  and no right-handed neutrino mass term

$$\begin{pmatrix} \nu & N & S \end{pmatrix} \begin{pmatrix} 0 & m_D & m_{\nu S} \\ . & 0 & M_{NS} \\ . & . & M_{SS} \end{pmatrix} \begin{pmatrix} \nu \\ N \\ S \end{pmatrix}$$

- Light neutrino mass  $m_\nu = m_\nu^{DS} + m_\nu^{LS}$

Double seesaw [Mohapatra, Mohapatra, Valle \(1986\)](#)

$$m_\nu^{DS} = m_D (M_{NS}^{-1T} M_{SS} M_{NS}^{-1}) m_D^T$$

and linear seesaw [Barr \(2004\)](#)

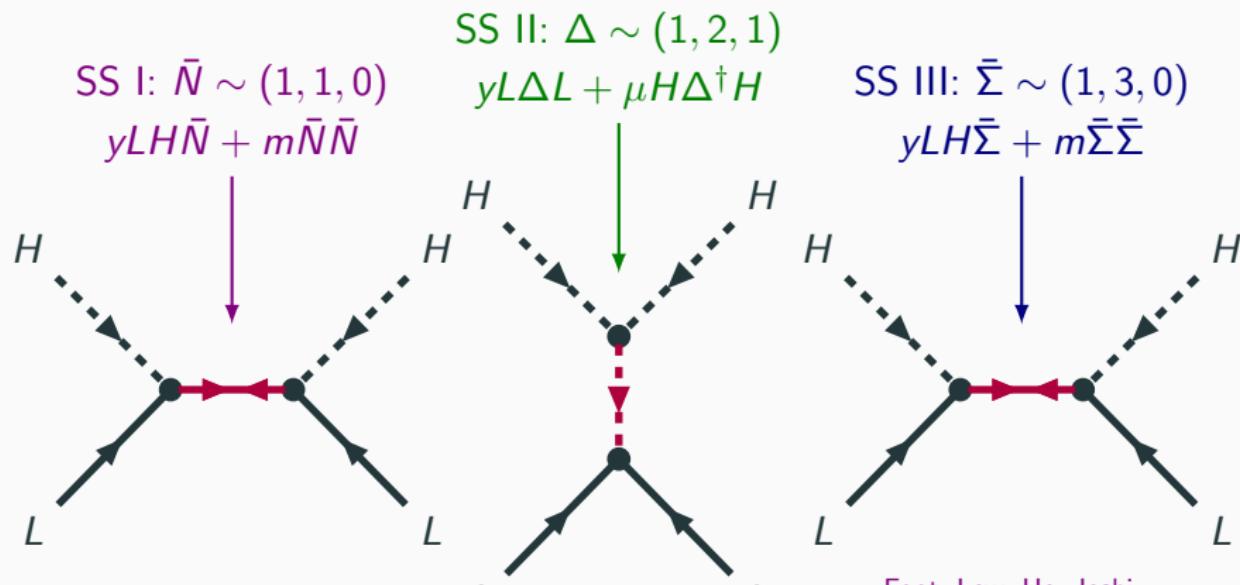
$$m_\nu^{LS} = - \left[ m_D \left( m_{\nu S} M_{NS}^{-1} \right)^T + \left( m_{\nu S} M_{NS}^{-1} \right) m_D^T \right]$$

- If  $m_\nu^{DS} \gg m_\nu^{LS}$  two interesting regimes:

1. Inverse seesaw:  $M_{SS} \ll m_D, M_{NS}$ : Lepton number only broken by  $M_{SS}$  and thus naturally small, but large Yukawa couplings
2.  $M_{SS} \sim M_{Pl}, M_{NS} \sim M_{GUT}, m_D \sim M_Z$   
 $\Rightarrow m_N \sim 10^{13} \text{ GeV}, m_\nu \sim 1 \text{ eV}$

Note, if  $M_{NS} \propto m_D^T$ , Dirac screening [Smirnov \(1993\), Lindner, MS, Smirnov \(2005\)](#)

## Tree level: seesaws



Minkowski; Yanagida; Glashow;

Gell-Mann, Ramond, Slansky;

Mohapatra, Senjanovic.

Mohapatra, Senjanovic;

Magg, Wetterich;

Lazarides, Shafi, Wetterich;

Schechter, Valle.

Foot, Lew, He, Joshi.

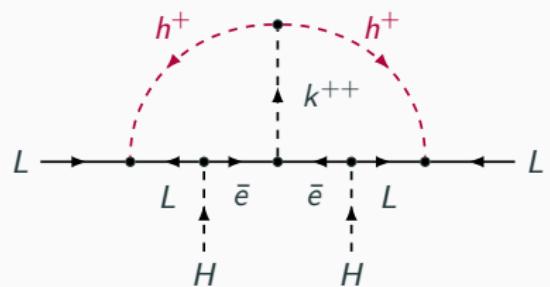
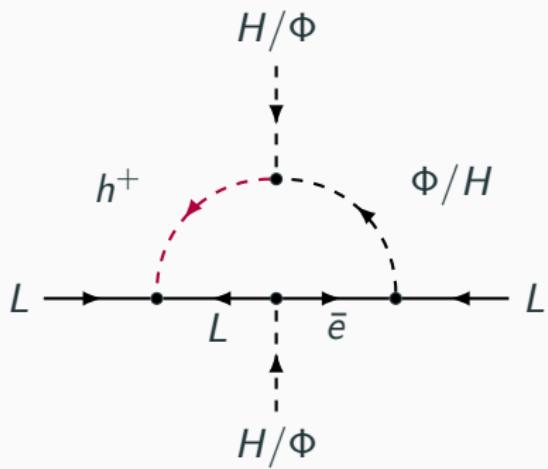
Singly-charged scalar:  $fLLh^+$

Zee model

$$y\bar{e}\phi^\dagger L + \mu h^{+*} H\phi$$

Zee-Babu model

$$g\bar{e}^\dagger \bar{e}^\dagger k^{++} + \mu h^+ h^+ k^{++*}$$



# $\Delta L = 2$ EFT operators

[Babu, Leung, De Gouvea, Jenkins]

Zee model    Zee-Babu model



$$O_2 = L^i L^j L^k \bar{e} H^l \epsilon_{ij} \epsilon_{kl}$$

$$O_{4a} = L^i L^j Q_i^\dagger \bar{u}^\dagger H^k \epsilon_{jk}$$

$$O_9 = L^i L^j L^k \bar{e} L^l \bar{e} \epsilon_{ij} \epsilon_{kl}$$

$$O_{11a} = L^i L^j Q^k \bar{d} Q^l \bar{d} \epsilon_{ij} \epsilon_{kl}$$

$$O_{12a} = L^i L^j Q_i^\dagger \bar{u}^\dagger Q_j^\dagger \bar{u}^\dagger$$

...

$$O_{59} = L^i Q^j \bar{d} \bar{d} \bar{e}^\dagger \bar{u}^\dagger H^k H_i^\dagger \epsilon_{jk}$$

$$O_{3a} = L^i L^j Q^k \bar{d} H^l \epsilon_{ij} \epsilon_{kl}$$

$$O_{4b} = L^i L^j Q_k^\dagger \bar{u}^\dagger H^k \epsilon_{ij}$$

$$O_{10} = L^i L^j L^k \bar{e} Q^l \bar{d} \epsilon_{ij} \epsilon_{kl}$$

$$O_{11b} = L^i L^j Q^k \bar{d} Q^l \bar{d} \epsilon_{ik} \epsilon_{jl}$$

$$O_{12b} = L^i L^j Q_k^\dagger \bar{u}^\dagger Q_l^\dagger \bar{u}^\dagger \epsilon_{ij} \epsilon^{kl}$$

operators up to dimension 11 classified



For details see review

[Cai, Herrero-Garcia, MS, Vicente, Volkas 1706.08524]

# Outline

What do we know?

Neutrino Oscillations

Neutrino Mass Limits

Neutrino Mass Generation

Dirac vs. Majorana Mass

Seesaw Mechanisms

Beyond seesaw

Lepton mixing and symmetries

Leptogenesis

# A Closer Look at Flavour Structure

- Is there any pattern in the leptonic mixing (PMNS) matrix?
- Experimentally measured lepton mixing angles [NuFIT 5.1 \(2021\) 2007.14792](#)
- Tribimaximal mixing is a good starting point Harrison,Perkins,Scott (2002)

$$\sin^2 \theta_{12} = 0.304^{+0.012}_{-0.012}$$

$$\sin^2 \theta_{12} \equiv \frac{1}{3}$$

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$$\sin^2 \theta_{13} \equiv 0$$

→ However corrections necessary, particularly for  $\theta_{13}$

Smaller than starting with no mixing in quark sector

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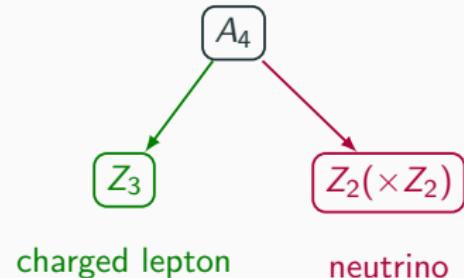
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# Flavour Symmetry: Basic Picture

- Discrete symmetries work well e.g.  $A_4$
- Angles predicted by symmetry breaking

Altarelli, Feruglio hep-ph/0512103; He, Keum, Volkas hep-ph/0601001



Breaking of symmetry  $G_f$  crucial

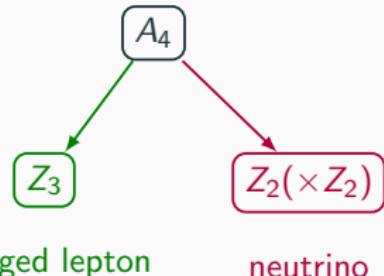
M.Holthausen,MS [JHEP 1201 (2012) 126]

- Breaking non-trivial: zero or random mixing
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- ⇒ Need mechanism to achieve alignment

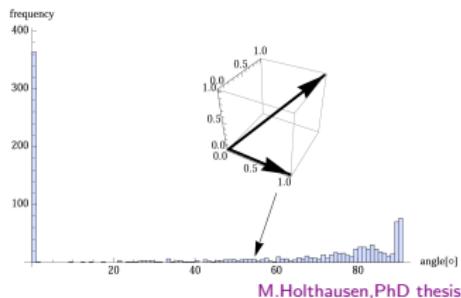
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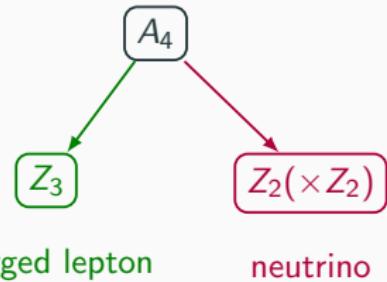


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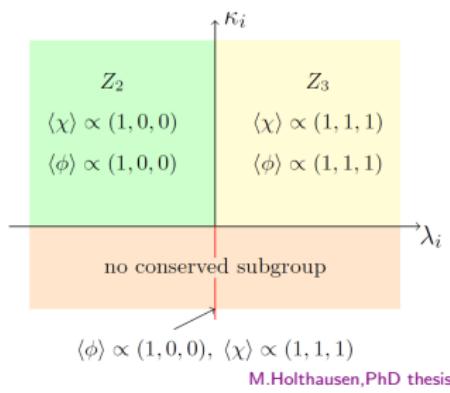
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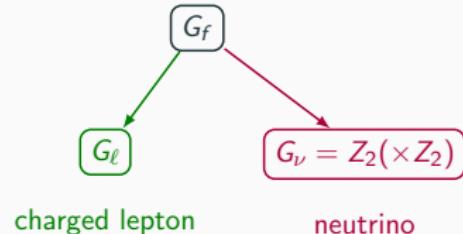
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Precise predictions [ $G_\nu = \mathbb{Z}_2 \times \mathbb{Z}_2$ ]

- Precise values of mixing parameters:  
→ For example tribimaximal mixing in models based on  $A_4$ ,  $S_4$

$$s_{12}^2 \equiv \frac{1}{3}, \quad s_{23}^2 \equiv \frac{1}{2}, \quad s_{13}^2 \equiv 0$$



Sum rules [ $G_\nu = \mathbb{Z}_2$ ]

- Relations between parameters:  
→ One example is an *atmospheric mixing sum rule*:

$$\sin \theta_{23} - \frac{1}{\sqrt{2}} = \lambda \sin \theta_{13} \cos \delta + \dots$$

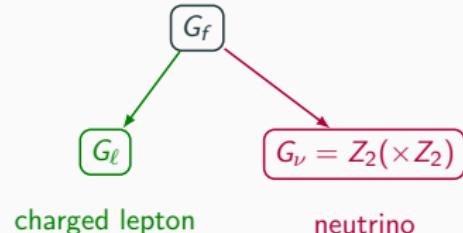
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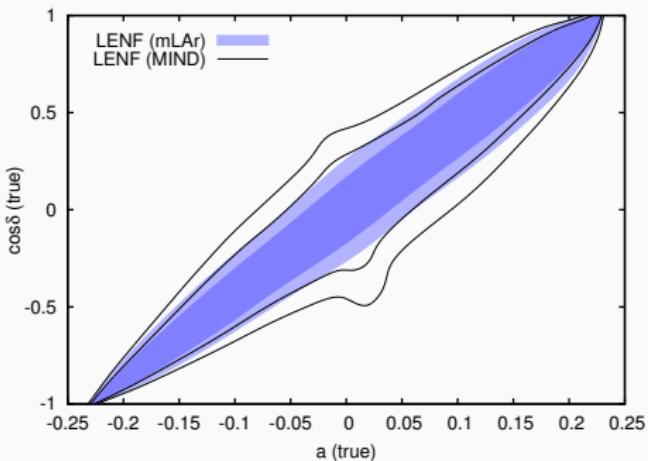
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# Testing Sum Rules

## Atmospheric Sum Rule

Ballett, King, Luhn, Pascoli, MS [Phys.Rev. D89 (2014) 016016]

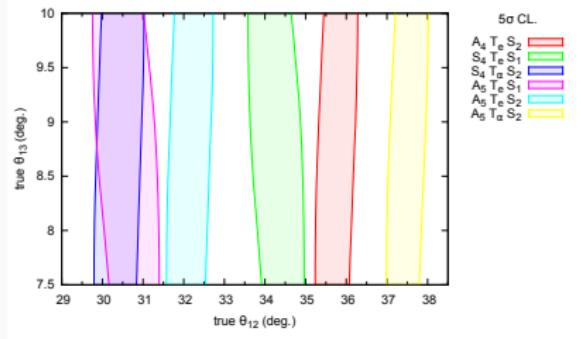
$$\frac{a}{\sqrt{2}} \equiv \sin \theta_{23} - \frac{1}{\sqrt{2}} \approx 1 \sin \theta_{13} \cos \delta$$



- $2\sigma$  and  $3\sigma$  allowed regions

## Solar Sum Rule

Ballett, King, Luhn, Pascoli, MS [J.Phys.Conf.Ser. 598 (2015) 012014]



- Solar sum rules, e.g.  $A_4 T_\alpha - S_2$

$$\sin \theta_{12} - \frac{1}{\sqrt{3}} = \sqrt{\frac{2}{2 - \sin^2 \theta_{13}}} - 1$$

- $5\sigma$  allowed regions
- Clear separation of predictions  
Overlapping solar sum rules have different atmospheric sum rule

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# Baryogenesis

Goal: Explain matter-antimatter asymmetry dynamically

$$\eta_B = \left. \frac{n_B - n_{\bar{B}}}{n_\gamma} \right|_0 = (6.21 \pm 0.16) \times 10^{-10}$$

$$Y_{\Delta B} = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.75 \pm 0.23) \times 10^{-11}$$

## Sakharov Conditions

1.  $\cancel{B}$ : Baryon number violation

SM: Electroweak sphalerons (non-perturbative field configuration)

2.  $\cancel{C}, \cancel{CP}$ : C and CP violation

- $\cancel{C}$ : production of baryons as efficient as anti-baryons
- $\cancel{CP}$ : production of left-handed baryons as efficient as right-handed anti-baryons

SM: CP violation in quark sector (CKM phase) → too small

3. Out-of-equilibrium: Otherwise reverse process immediately destroys asymmetry

SM: Electroweak phase transition → too weak

# Baryogenesis

Goal: Explain matter-antimatter asymmetry dynamically

$$\eta_B = \left. \frac{n_B - n_{\bar{B}}}{n_\gamma} \right|_0 = (6.21 \pm 0.16) \times 10^{-10}$$

$$Y_{\Delta B} = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.75 \pm 0.23) \times 10^{-11}$$

Sakharov Conditions

1.  $\cancel{B}$ : Baryon number violation

SM: Electroweak sphalerons (non-perturbative field configuration)

2.  $\cancel{C}, \cancel{CP}$ : C and CP violation

- $\cancel{C}$ : production of baryons as efficient as anti-baryons
- $\cancel{CP}$ : production of left-handed baryons as efficient as right-handed anti-baryons

SM: CP violation in quark sector (CKM phase) → too small

3. Out-of-equilibrium: Otherwise reverse process immediately destroys asymmetry

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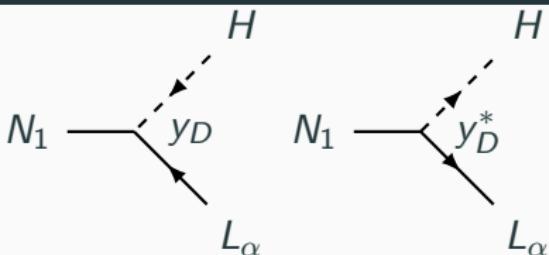
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# Thermal Leptogenesis

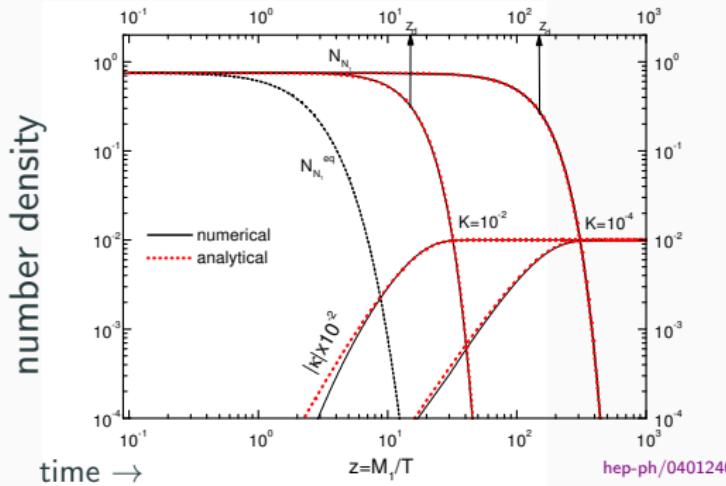
[Fukugita, Yanagida (1986)]

RH neutrinos are Majorana particles and can decay to both leptons and anti-leptons



$\Rightarrow \cancel{L}$ : Violation of lepton number  $L$  and hence  $B - L$

$\Rightarrow$  Sphalerons violate  $B + \cancel{L}$  and thus no-zero  $B$  produced

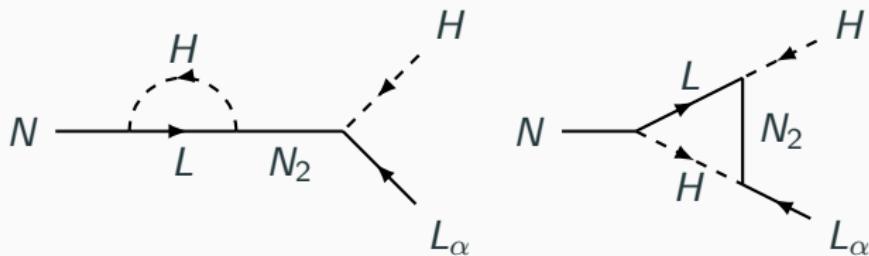


If  $y_D \ll 1$   $N$  long-lived,  
then out-of-equilibrium  
decay

$$K \equiv \frac{\Gamma_{N_1}(z = \infty)}{H(z = 1)}$$

## CP violation

- No C and CP violation at tree-level, because  $|\mathcal{M}|^2 \propto |y_D|^2$
- Interference of tree-level diagram with loop diagrams  $\Rightarrow \cancel{CP}$



- CP asymmetry:  $\epsilon = \frac{\Gamma(N_1 \rightarrow HL) - \Gamma(N_1 \rightarrow \tilde{H}\bar{L})}{\Gamma(N_1 \rightarrow HL) + \Gamma(N_1 \rightarrow \tilde{H}\bar{L})}$
- Efficiency factor:  $0 < \eta < 1$
- Baryon asymmetry  $Y_{\Delta B} \simeq \frac{135\zeta(3)}{4\pi^2 g_*} \times \epsilon \times \eta \times C$

First factor is equilibrium number density of  $N_1/s$  for  $T \gg M_1$

$g_*$  relativistic degrees of freedom

$C$  redistribution of asymmetry due to fast processes

Simplified discussion neglected other processes, washout effects, flavour effects

**Questions?**

# Mixing in the Lepton Sector



Бруно Понтекорво  
pontecorvo.jinr.ru

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix describes mixing in the lepton sector:

$$U = V \cdot \text{diag}(e^{-i\varphi_1/2}, e^{-i\varphi_2/2}, 1)$$

with ( $s_{ij} = \sin \theta_{ij}$ ,  $c_{ij} = \cos \theta_{ij}$ )

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

- Three mixing angles  $\theta_{12}$ ,  $\theta_{13}$ , and  $\theta_{23}$  [Euler angles]
- One Dirac CP phase  $\delta$
- Possibly two Majorana CP phases  $\varphi_{1,2}$ , if neutrinos are their own antiparticles

## Casas-Ibarra Parameterisation

Casas, Ibarra (2001)

It is possible to invert the type-I seesaw formula

$$m_\nu = -m_D M^{-1} m_D^T$$

In the mass basis of charged leptons and right-handed neutrinos, the PMNS matrix  $U$  diagonalises  $m_\nu$ ,

$$D_{m_\nu} = U^T m_\nu U = -U^T m_D D_{\sqrt{M}}^{-1} D_{\sqrt{M}}^{-1} m_D^T U$$

and thus

$$\mathbf{1} = \left[ i D_{\sqrt{M}}^{-1} m_D U D_\nu^{-1/2} \right]^T \left[ i D_{\sqrt{M}}^{-1} m_D U D_\nu^{-1/2} \right]$$

The combination in brackets is a complex symmetric matrix

$R^T R = \mathbf{1}$  and therefore

$$m_D = y_D \langle H \rangle = -i D_{\sqrt{M}} R D_{\sqrt{m_\nu}} U^\dagger$$