

# Theoretical hyperfine splittings of ${}^7,{}^9\text{Be}^{2+}$ ions for future studies of nuclear properties

Xiao-Qiu Qi<sup>a,b</sup>, Pei-Pei Zhang<sup>b,d,†</sup>, Zong-Chao Yan<sup>c,b</sup>, Ting-Yun Shi<sup>b</sup>, G. W. F. Drake<sup>d</sup>, Ai-Xi Chen<sup>a</sup>,  
Zhen-Xiang Zhong<sup>e,b</sup>

<sup>a</sup> Key Laboratory of Optical Field Manipulation of Zhejiang Province and Physics Department of Zhejiang Sci-Tech University, Hangzhou 310018, China

<sup>b</sup> State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, Innovation Academy for Precision Measurement Science and Technology, Chinese Academy of Sciences, Wuhan 430071, China

<sup>c</sup> Department of Physics, University of New Brunswick, Fredericton, New Brunswick, Canada E3B 5A3

<sup>d</sup> Department of Physics, University of Windsor, Windsor, Ontario, Canada N9B 3P4

<sup>e</sup> Center for Theoretical Physics, Hainan University, Haikou 570228, China

The hyperfine structures of the  $2^3S_1$  and  $2^3P_J$  states of  ${}^7\text{Be}^{2+}$  and  ${}^9\text{Be}^{2+}$  are investigated within the framework of the nonrelativistic quantum electrodynamics (NRQED) [1], including relativistic and radiative corrections up to order  $m\alpha^6$ . Our results [2] are shown in Tables 1 and 2. The uncertainties of the calculated hyperfine splittings are on the order of tens of ppm, and for  ${}^9\text{Be}^{2+}$  our results improve the previous theoretical and experimental values by at least two orders of magnitude. The improved sensitivity of the hyperfine splittings of  ${}^7,{}^9\text{Be}^{2+}$  to the nuclear Zemach radius and electric quadrupole moment opens the way to future measurements to extract the atomic physics values of these two nuclear properties to an accuracy of 5% or better.

Table 1: Theoretical hyperfine intervals in the  $2^3S_1$  state of  ${}^7\text{Be}^{2+}$  and  ${}^9\text{Be}^{2+}$  with the Zemach radius  $R_{\text{em}} = 3.45(11)$  fm and  $R_{\text{em}} = 4.07(5)$  fm, respectively. The last column is the predicted accuracy of  $R_{\text{em}}$ .

|                      | $(J, F) - (J', F')$ | $\nu$ (This work)<br>cm <sup>-1</sup> | Scholl <i>et al.</i> [3]<br>cm <sup>-1</sup> | $ \delta R_{\text{em}}/R_{\text{em}} $<br>% |
|----------------------|---------------------|---------------------------------------|--|---|
| ${}^7\text{Be}^{2+}$ | (1, 1/2) – (1, 3/2) | 0.40952(1) at 24 ppm                  |  | 5   |
|                      | (1, 3/2) – (1, 5/2) | 0.68250(1) at 15 ppm                  |  | 3   |
| ${}^9\text{Be}^{2+}$ | (1, 1/2) – (1, 3/2) | 0.344574(9) at 26 ppm                 | 0.3448(10)                                   | 4   |
|                      | (1, 3/2) – (1, 5/2) | 0.574275(6) at 10 ppm                 | 0.5740(11)                                   | 2   |

Table 2: Theoretical hyperfine intervals in the  $2^3P_J$  state of  ${}^7\text{Be}^{2+}$  and  ${}^9\text{Be}^{2+}$  with the nuclear quadrupole moments  $Q_d = -6.11$  fm<sup>2</sup> and  $Q_d = 5.350(14)$  fm<sup>2</sup>, respectively. The last column is the predicted accuracy of  $Q_d$ .

|                      | $(J, F) - (J', F')$ | $\nu(Q_d)$ (This work)<br>cm <sup>-1</sup> | Johnson <i>et al.</i> [4]<br>cm <sup>-1</sup> | Scholl <i>et al.</i> [3]<br>cm <sup>-1</sup> | $ \delta Q_d/Q_d $<br>% |
|----------------------|---------------------|--|---|--|-------------------------|
| ${}^7\text{Be}^{2+}$ | (2, 1/2) – (2, 3/2) | 0.18726(1) at 53 ppm                       |   |  | 4                       |
|                      | (2, 3/2) – (2, 5/2) | 0.31574(1) at 32 ppm                       |   |  | 5                       |
|                      | (2, 5/2) – (2, 7/2) | 0.44953(1) at 22 ppm                       |   |  | 4                       |
|                      | (1, 1/2) – (1, 3/2) | 0.21130(1) at 47 ppm                       |   |  | 3                       |
| ${}^9\text{Be}^{2+}$ | (1, 3/2) – (1, 5/2) | 0.31346(1) at 32 ppm                       |   |  | 5                       |
|                      | (2, 1/2) – (2, 3/2) | 0.158371(7) at 44 ppm                      | 0.1581  | 0.1585(10)                                   | 3                       |
|                      | (2, 3/2) – (2, 5/2) | 0.266123(4) at 15 ppm                      | 0.2659  | 0.2659(11)                                   | 3                       |
|                      | (2, 5/2) – (2, 7/2) | 0.377128(4) at 11 ppm                      | 0.3773  | 0.3768(14)                                   | 2                       |
|                      | (1, 1/2) – (1, 3/2) | 0.175126(4) at 23 ppm                      | 0.1754  | 0.1751(10)                                   | 1                       |
|                      | (1, 3/2) – (1, 5/2) | 0.265662(3) at 11 ppm                      | 0.2654  | 0.2654(10)                                   | 2                       |

- [1] K. Pachucki *et al.*, Phys. Rev. A 85, 042517 (2012).  
[2] X.-Q. Qi *et al.*, Phys. Rev. A 107, L010802 (2023).  
[3] T. J. Scholl *et al.*, Phys. Rev. Lett. 71, 2188 (1993).  
[4] W. R. Johnson *et al.*, Phys. Rev. A 55, 2728 (1997).