Nuclear Structure in Muonic Atoms: μH and μD

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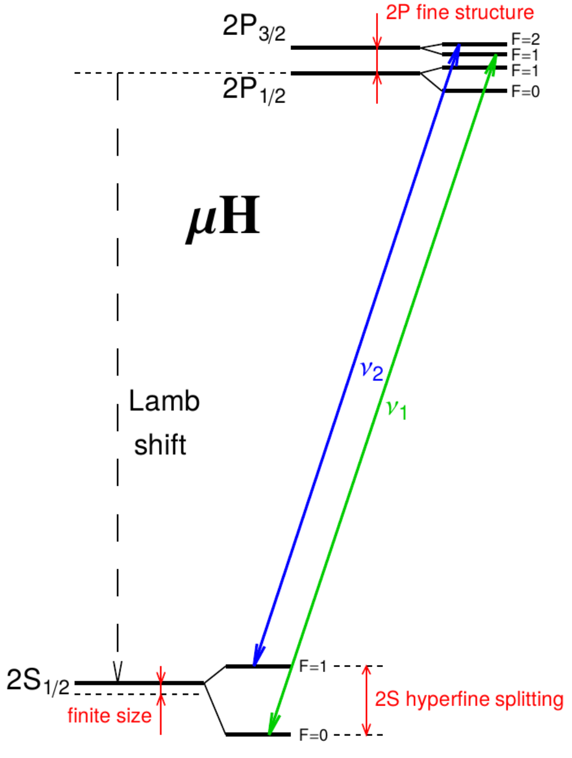
Lamb Shift in (Muonic) Atoms and Ions

- Atomic spectra are sensitive to nuclear properties:

Lamb Shift: $E_{\mathrm{LS}} = E_{\mathrm{QED}} + \mathcal{C}\, R_E^2 + E_{\mathrm{NS}}$

charge radius

- Expand in $\alpha,\ Z\alpha$
  - nuclear size $\mathcal{C} \propto \alpha^4 + \ldots$
  - nuclear structure $E_{\mathrm{NS}} \propto \alpha^5 + \ldots$



- Muonic atoms: greater sensitivity to charge radii

Bohr radius $a = (Z\alpha m_r)^{-1}$

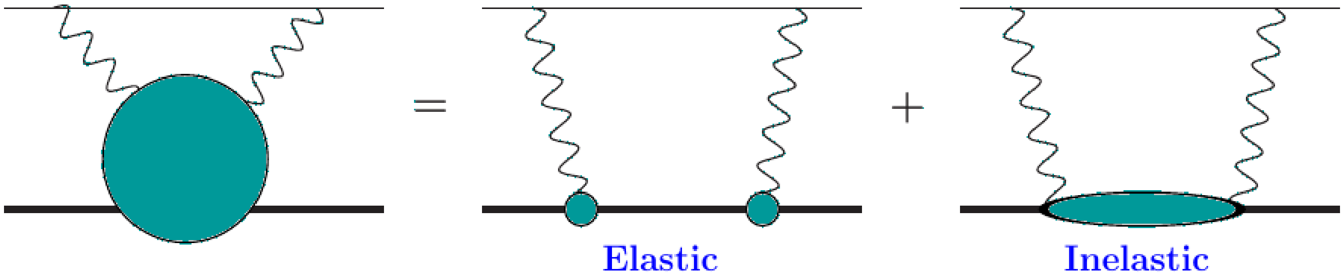
- But also greater sensitivity to subleading nuclear response

$$\Delta E = \frac{2\pi Z\alpha}{3} \frac{1}{\pi (an)^3} \left[R_E^2 - \frac{Z\alpha m_r}{2} R_F^3 \right] + \ldots$$

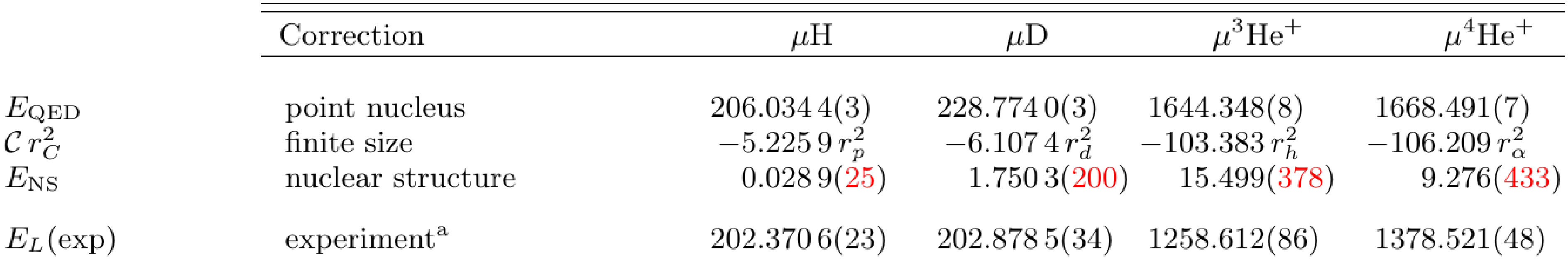
Friar radius (only a part of the subleading nuclear response)

Two-Photon Exchange (TPE)

- Described in terms of (doubly virtual forward) Compton scattering: VVCS
- Elastic ($\nu = \pm Q^2/2M_{\mathrm{target}}$, elastic e.m. form factors) and inelastic (~ nuclear generalised polarisabilities)



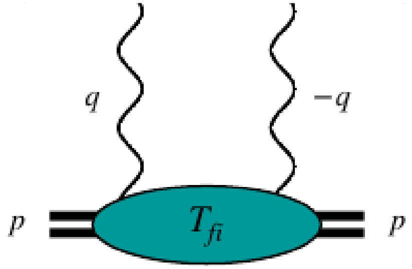
- Defines the theoretical uncertainty as of now



	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
E_{QED}	point nucleus	$206.034\,4(3)$	$228.774\,0(3)$	$1644.348(8)$	$1668.491(7)$
$\mathcal{C}\,r_C^2$	finite size	$-5.225\,9\,r_p^2$	$-6.107\,4\,r_d^2$	$-103.383\,r_h^2$	$-106.209\,r_\alpha^2$
E_{NS}	nuclear structure	$0.028\,9(25)$	$1.750\,3(200)$	$15.499(378)$	$9.276(433)$
$E_L(\mathrm{exp})$	experiment[a]	$202.370\,6(23)$	$202.878\,5(34)$	$1258.612(86)$	$1378.521(48)$

Pachucki, VL, Hagelstein, Li Muli, Bacca, Pohl – theory review (2022)
[a] experiment: CREMA (2013-2023)

VVCS and Structure Functions

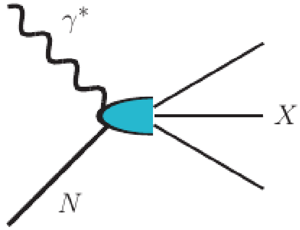


- Forward spin-1/2 VVCS amplitude

$$\alpha_{\rm em}\, M^{\mu\nu}(\nu,Q^2) = -\left\{\left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right) T_1(\nu,Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu-\frac{p\cdot q}{q^2}\,q^\nu\right) T_2(\nu,Q^2) \right.$$
$$\left. + \frac{i}{M}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\nu,Q^2) + \frac{i}{M^3}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\; s_\beta - s\cdot q\; p_\beta)\, S_2(\nu,Q^2)\right\}$$

Lamb Shift: $E^{2\gamma}_{nS} = -8i\pi\alpha m\, [\phi_n(0)]^2 \int \frac{d^4q}{(2\pi)^4}\, \frac{(Q^2-2\nu^2)\, T_1(\nu,Q^2) - (Q^2+\nu^2)\, T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$ Any spin!

- Unitarity and analyticity, data-driven: dispersive relations



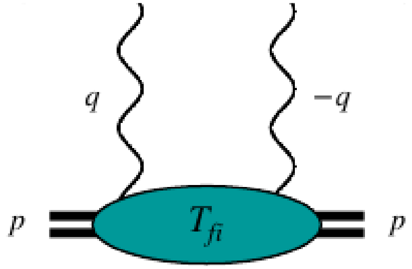
Structure functions $F_1(x,Q^2)$, $F_2(x,Q^2)$: inclusive electron scattering

$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi M\nu^2}{Q^4}\int_0^1 dx\, \frac{x\, F_1(x,Q^2)}{1-x^2(\nu/\nu_{\rm el})^2 - i0^+},$$
$$T_2(\nu,Q^2) = \frac{16\pi M}{Q^2}\int_0^1 dx\, \frac{F_2(x,Q^2)}{1-x^2(\nu/\nu_{\rm el})^2 - i0^+}$$

$x = Q^2/(2M\nu)$

- The subtraction function $T_1(0,Q^2)$ is not directly accessible in experiment
- Data on structure functions is deficient (for anything other than proton)

VVCS and Structure Functions



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$$\left. + \frac{i}{M}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\nu, Q^2) + \frac{i}{M^3}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\; s_\beta - s\cdot q\; p_\beta)\, S_2(\nu, Q^2)\right\}$$

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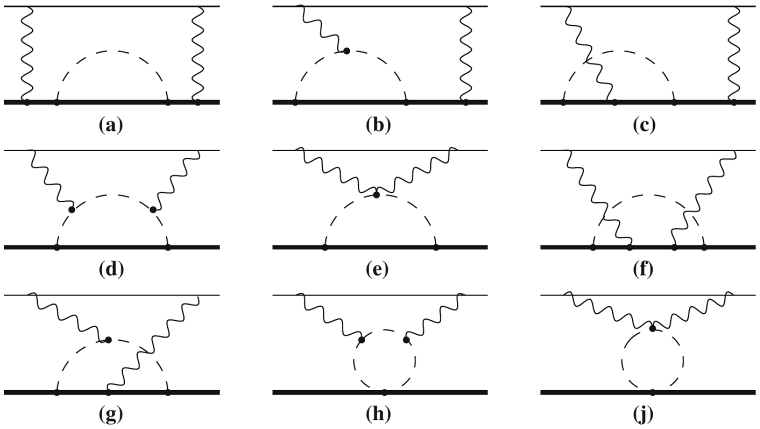
- Typical energies in (muonic) atoms are small: use effective field theories
  - chiral EFT (covariant, HB, ...)
  - or even pionless EFT for nuclear effects
  - expansion in powers of a small parameter
  - order-by-order uncertainty estimate
- Calculate VVCS or structure functions
- In nuclei heavier than proton: also calculate the elastic form factors



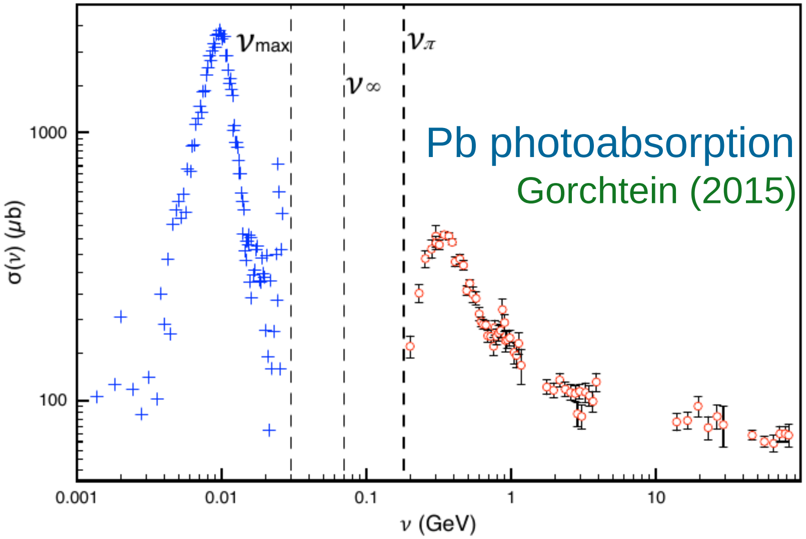
Nuclei Heavier than Proton

- Most of the TPE correction is nuclear (as in: no nucleon polarisation)
- Nuclear part of subtraction function converges (finite energy sum rule)

Gorchtein (2015)

  - TPE integrals with nuclear response functions from **χ**EFT will converge
  - „Most popular“ method

$$E_{\text{pol}} = -\frac{4\pi\alpha^2}{3}\,\phi^2(0)\int_{E_T} dE\,\sqrt{\frac{2\mu}{E}}\,|\langle\phi_N|\vec{d}|E\rangle|^2$$

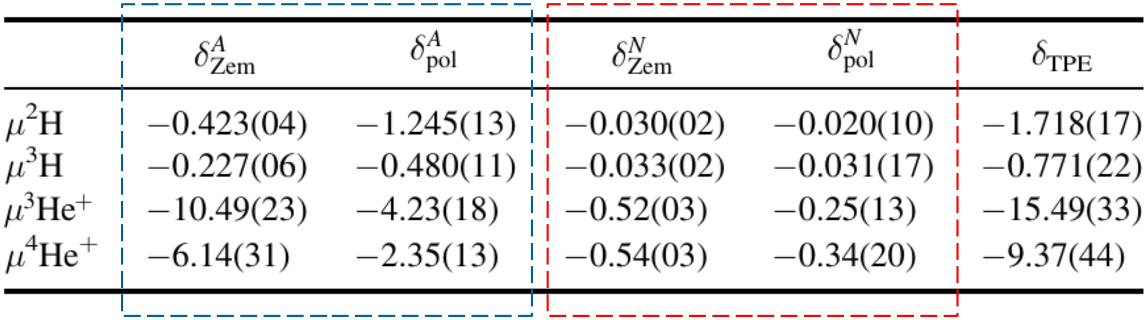


Pb photoabsorption

Gorchtein (2015)

Friar, Pachucki, Wienczek, Kalinowski, Rosenfelder, Leidemann, Bacca, Ji, Hernandez, Acharya, Li Muli, VL, ...

- Single-nucleon contributions need to be accounted for separately
  - relatively more important in heavier nuclei
  - sizeable uncertainty!
  - neutron not so well constrained empirically



	δ^A_{Zem}	δ^A_{pol}	δ^N_{Zem}	δ^N_{pol}	δ_{TPE}
μ^2H	−0.423(04)	−1.245(13)	−0.030(02)	−0.020(10)	−1.718(17)
μ^3H	−0.227(06)	−0.480(11)	−0.033(02)	−0.031(17)	−0.771(22)
μ^3He$^+$	−10.49(23)	−4.23(18)	−0.52(03)	−0.25(13)	−15.49(33)
μ^4He$^+$	−6.14(31)	−2.35(13)	−0.54(03)	−0.34(20)	−9.37(44)

nuclear

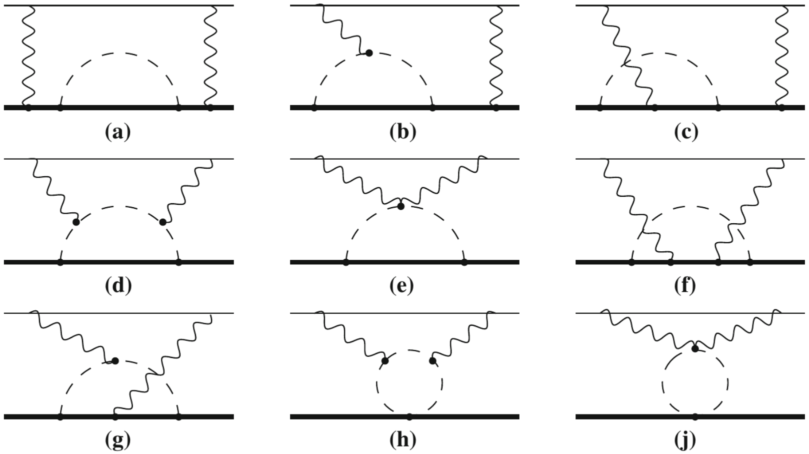
individual nucleons

Ji et al. (2018)

Lamb Shift of μH in Covariant BχPT

- Delta counting: $\Delta = M_\Delta - M \gg m_\pi$ Pascalutsa, Phillips (2003)
- The contributions of the Delta isobar are suppressed by powers of m_π / Δ
- Expansion in powers of

$$p/\Delta \sim m_\pi/\Delta \sim 0.5$$

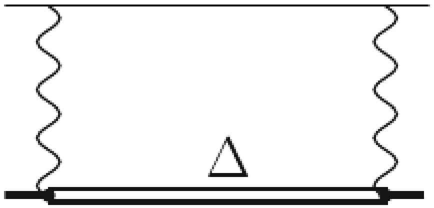


Alarcon, VL, Pascalutsa (2014)

- LO BχPT: pion-nucleon loops

$$\Delta E_{2S}^{\text{LO, pol}} = -9.6^{+1.4}_{-2.9}\ \mu\text{eV}$$

- Delta exchange:



  - suppressed in $\Delta E_{2S}^{\text{pol}}$ but affects the subtraction
  - insert transition form factors (Jones-Scadron)

$$\Delta E_{2S}^{\Delta-\text{pole}} = 0.95 \pm 0.95\ \mu\text{eV}$$

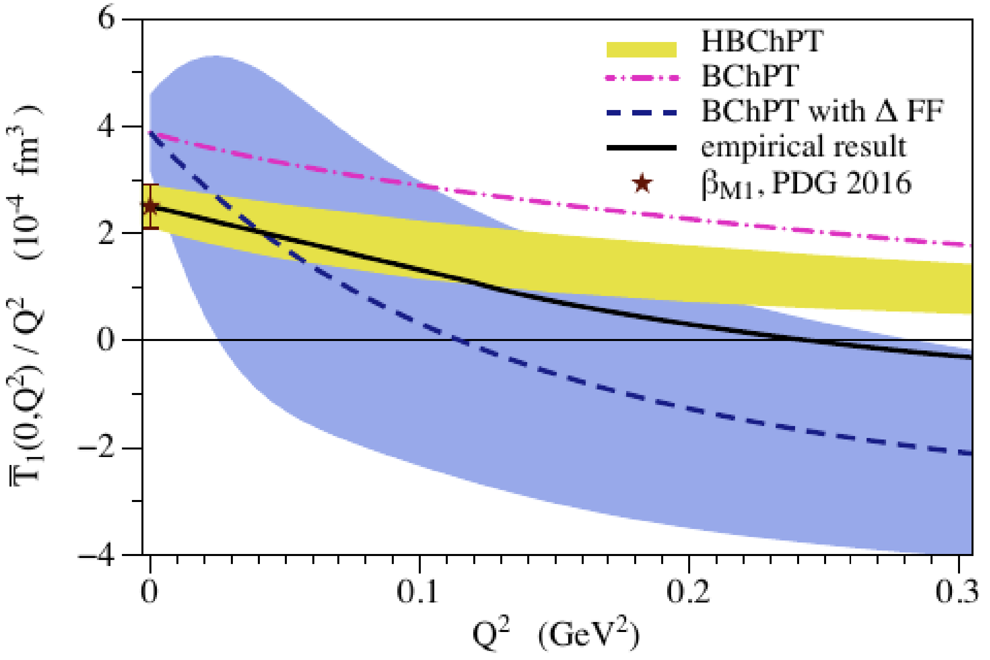
VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

Various Subtraction Functions

- The diversity of the results for the proton subtraction function $T_1(0, Q^2)$
  - HBChPT: dipole FF, matches β_{M1}[PDG] and the slope at 0

    modification of Birse, McGovern (2012)
  - BChPT: transition FFs change the subtraction function
  - Empirical: Regge asymptotic at high energy subtracted

    Tomalak, Vanderhaeghen (2015)



VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

- Zero crossing at low Q^2 – emerges in BChPT with FFs; established in the empirical derivation, but the position not well known (0.1..0.4 GeV^2)
- Big cancellations between different mechanisms (πN and πΔ loops vs. Δ pole), also cancellations in the LS integral because of the sign change
- Empirical derivation has sizeable errors towards $Q^2 = 0$ (not shown)

  attributed to mismatch between structure function fit in the resonance region (Christy-Bosted) and at high energies (Donnachie-Landshoff) =>

  needs a better (combined) structure function parametrization

Lamb Shift of μH in Various Approaches

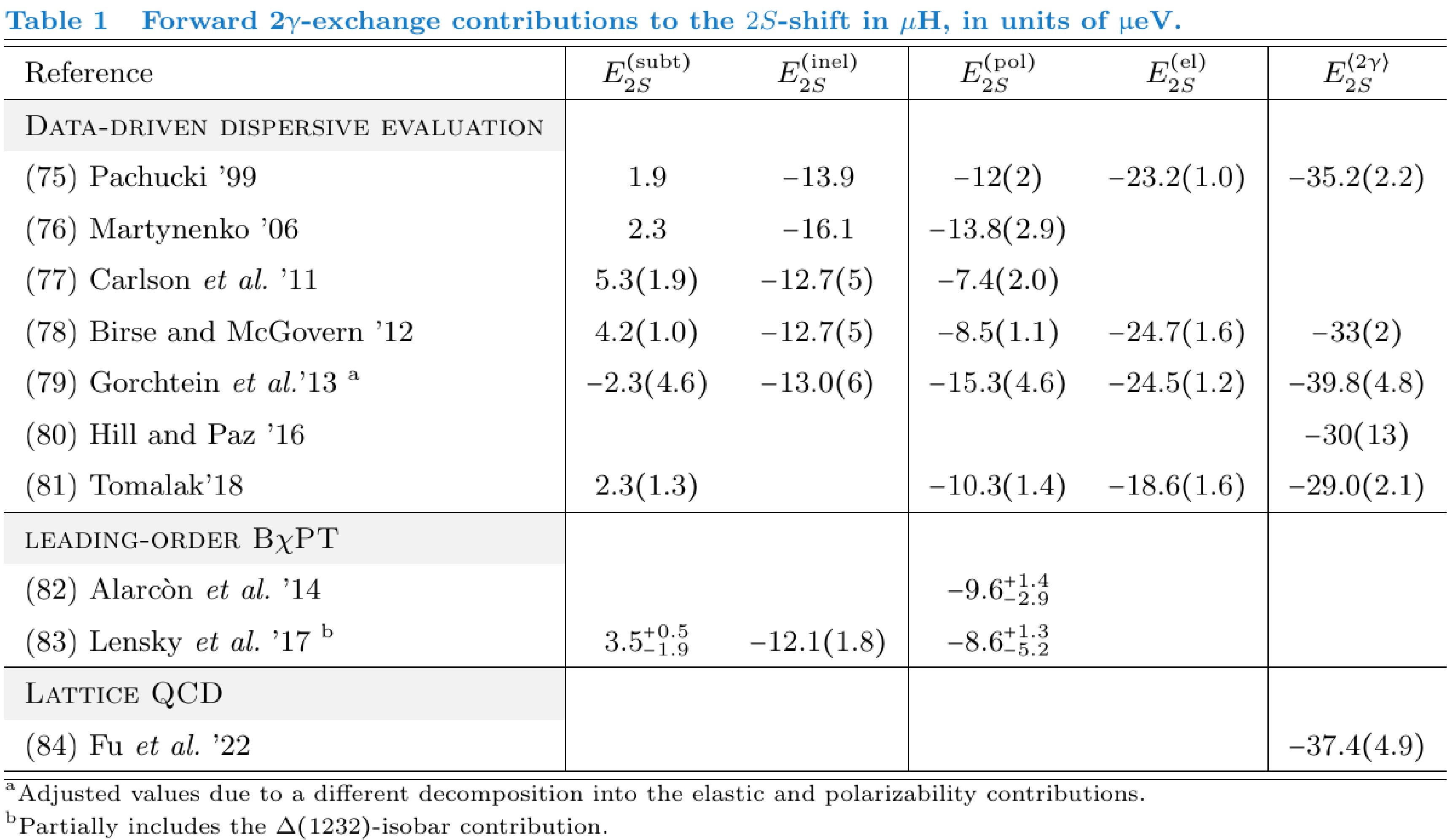


Table 1 **Forward 2γ-exchange contributions to the $2S$-shift in μH, in units of μeV.**

Reference	$E_{2S}^{(\mathrm{subt})}$	$E_{2S}^{(\mathrm{inel})}$	$E_{2S}^{(\mathrm{pol})}$	$E_{2S}^{(\mathrm{el})}$	$E_{2S}^{\langle 2\gamma \rangle}$
DATA-DRIVEN DISPERSIVE EVALUATION					
(75) Pachucki ’99	1.9	−13.9	−12(2)	−23.2(1.0)	−35.2(2.2)
(76) Martynenko ’06	2.3	−16.1	−13.8(2.9)		
(77) Carlson *et al.* ’11	5.3(1.9)	−12.7(5)	−7.4(2.0)		
(78) Birse and McGovern ’12	4.2(1.0)	−12.7(5)	−8.5(1.1)	−24.7(1.6)	−33(2)
(79) Gorchtein *et al.*’13 [a]	−2.3(4.6)	−13.0(6)	−15.3(4.6)	−24.5(1.2)	−39.8(4.8)
(80) Hill and Paz ’16					−30(13)
(81) Tomalak’18	2.3(1.3)		−10.3(1.4)	−18.6(1.6)	−29.0(2.1)
LEADING-ORDER BχPT					
(82) Alarcòn *et al.* ’14			$-9.6^{+1.4}_{-2.9}$		
(83) Lensky *et al.* ’17 [b]	$3.5^{+0.5}_{-1.9}$	−12.1(1.8)	$-8.6^{+1.3}_{-5.2}$		
LATTICE QCD					
(84) Fu *et al.* ’22					−37.4(4.9)

[a] Adjusted values due to a different decomposition into the elastic and polarizability contributions.

[b] Partially includes the Δ(1232)-isobar contribution.

Antognini, Hagelstein, Pascalutsa (2022)

- Agreement between different approaches, also on the size of the subtraction contribution separately – despite the variation in $T_1(0, Q^2)$

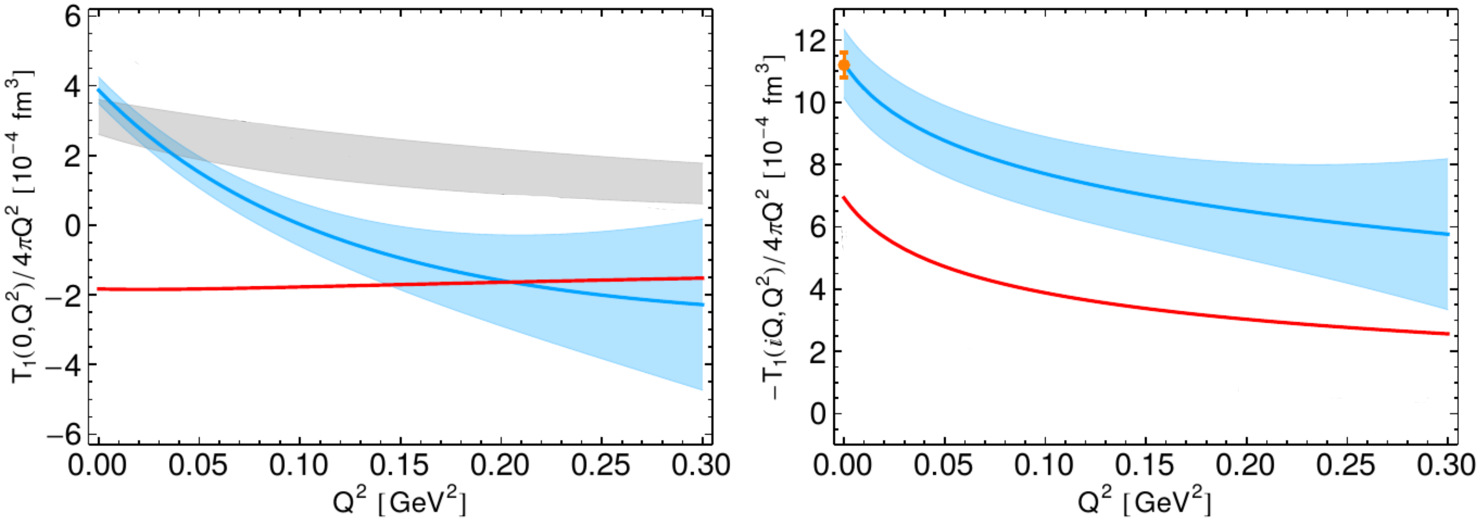
- Still, the subtraction contribution has the biggest uncertainty, and needs to be further constrained

Subtraction Function: How to Constrain it?

$$T_1(0,Q^2) \;=\; \beta_{M1}\,Q^2 + \left[\frac{1}{6}\beta_{M2} - \alpha_{\text{em}}\sqrt{\frac{3}{2}}P'^{(M1,M1)0}(0) + \frac{1}{(2M)^2}\beta_{M1} + \alpha_{\text{em}}\,b_{3,0}\right] Q^4 + \mathcal{O}(Q^6)$$

VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

- The knowledge of $b_{3,0}$ constrains the slope
- Get from dilepton electroproduction, $ep \to ep\ell^+\ell^-$ Pauk, Carlson, Vanderhaeghen (2020)
- A different subtraction point: $\nu_s = iQ$ instead of $\nu_s = 0$ Hagelstein, Pascalutsa (2021)
    - might be advantageous to use [no zero crossing at low Q^2, less affected by cancellations, smaller Δ contribution, inelastic contribution becomes small]



- An improvement in empirical extraction of $T_1(0,Q^2)$ [or $T_1(iQ,Q^2)$] is possible, needs better parametrizations of proton structure functions!

Theory Framework for μD: Pionless EFT

- pionless EFT for nuclear effects
  - expansion in powers of a small parameter $p/m_\pi \simeq \gamma/m_\pi \simeq 1/3$
  - order-by-order Bayesian uncertainty estimate
  - easier to solve than χEFT (analytic results for *NN*)
  - easier to analyse
  - explicit gauge invariance and renormalisability
  - slower convergence (~larger uncertainty) and (potentially) a narrower range of applicability than χEFT
    - the latter two issues do not seem to affect deuteron VVCS
- We in fact do go beyond strict pionless and use χEFT/data driven DR to estimate higher-order individual nucleon contributions

Setup for Deuteron VVCS and TPE

- Longitudinal and Transverse amplitudes in pionless EFT

$$f_L(\nu, Q^2) = -T_1(\nu, Q^2) + \left(1 + \frac{\nu^2}{Q^2}\right) T_2(\nu, Q^2), \qquad f_T(\nu, Q^2) = T_1(\nu, Q^2)$$

Lamb Shift:

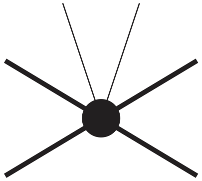
$$\Delta E_{nl} = -8i\pi m \, [\phi_{nl}(0)]^2 \int \frac{d^4 q}{(2\pi)^4} \frac{f_L(\nu, Q^2) + 2(\nu^2/Q^2) f_T(\nu, Q^2)}{Q^2(Q^4 - 4m^2\nu^2)}$$

$f_L = O(p^{-2}), \qquad f_T = O(p^0)$ in the VVCS amplitude

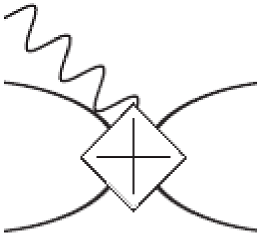
$\alpha_{E1} = 0.64 \text{ fm}^3$

$\beta_{M1} = 0.07 \text{ fm}^3$

$\text{longitudinal} = O(p^{-2}), \qquad \text{transverse} = O(p^2)$ in TPE

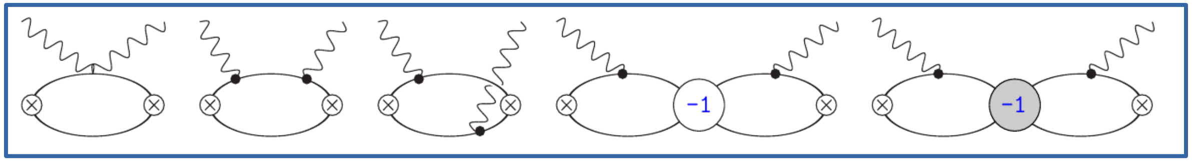


- Transverse contribution to TPE starts only at N4LO
- N4LO: ΔE_{nl} needs to be regularised, an unknown lepton-NN LEC
- We go up to N3LO in f_L, and up to (relative) NLO in f_T [cross check]
- One unknown LEC at N3LO in f_L
  - important for the charge form factor
  - extracted from the H-D isotope shift and proton R_E

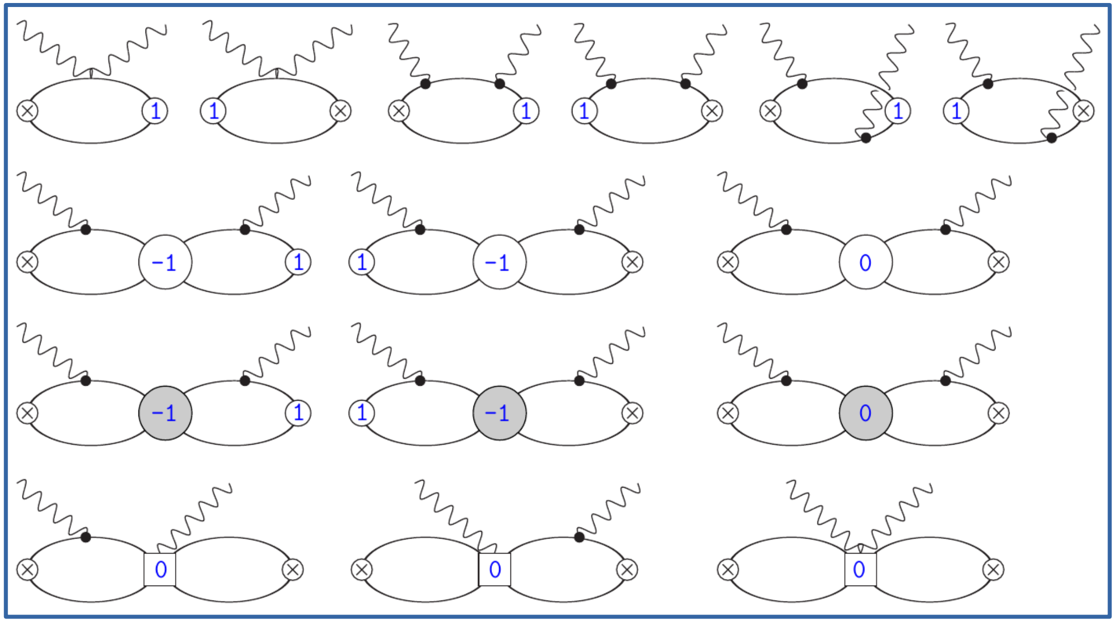


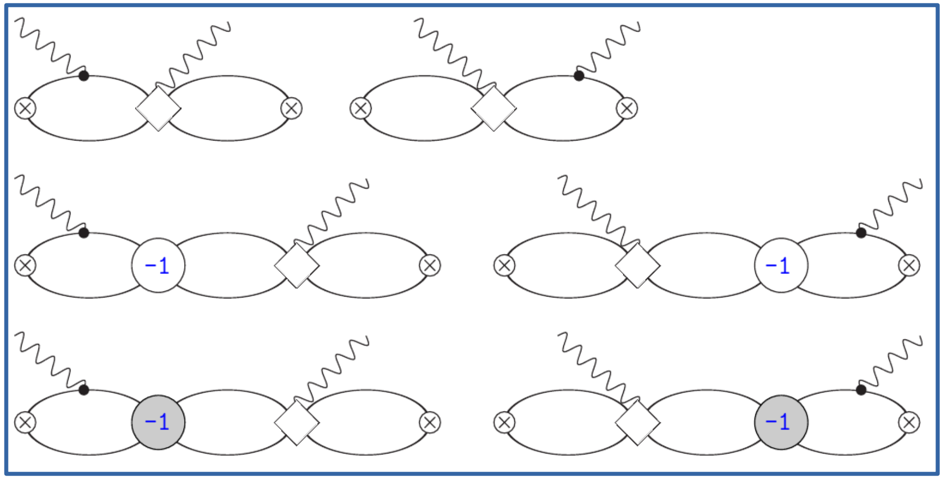
Deuteron VVCS: Feynman Graphs

LO



NLO

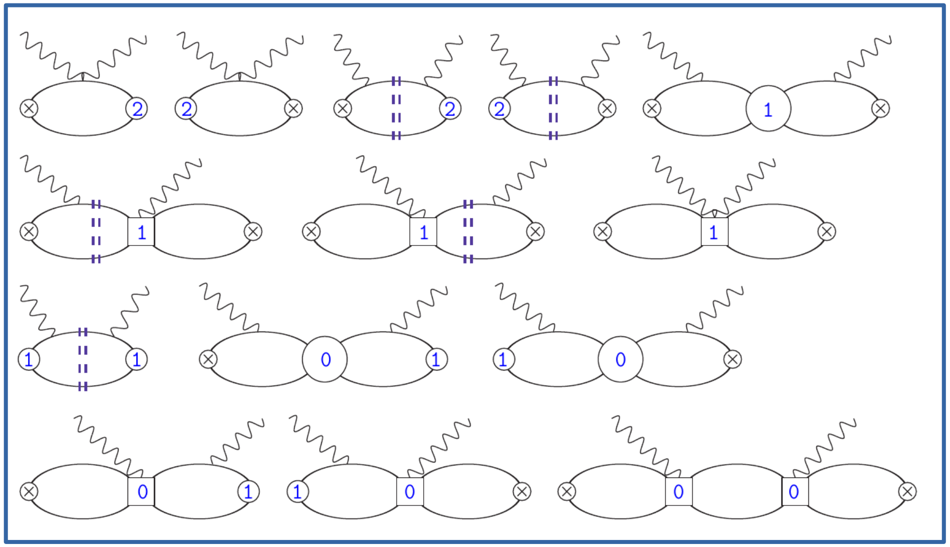


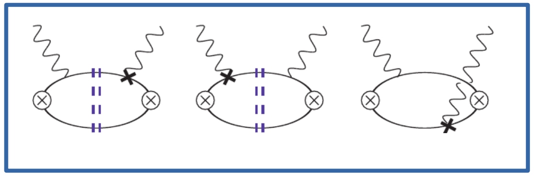


- Amplitudes are calculated analytically
- Checks:
  - ➔ the sum of each subgroup (+ respective crossed graphs) is gauge invariant
  - ➔ regularisation scale dependence has to vanish

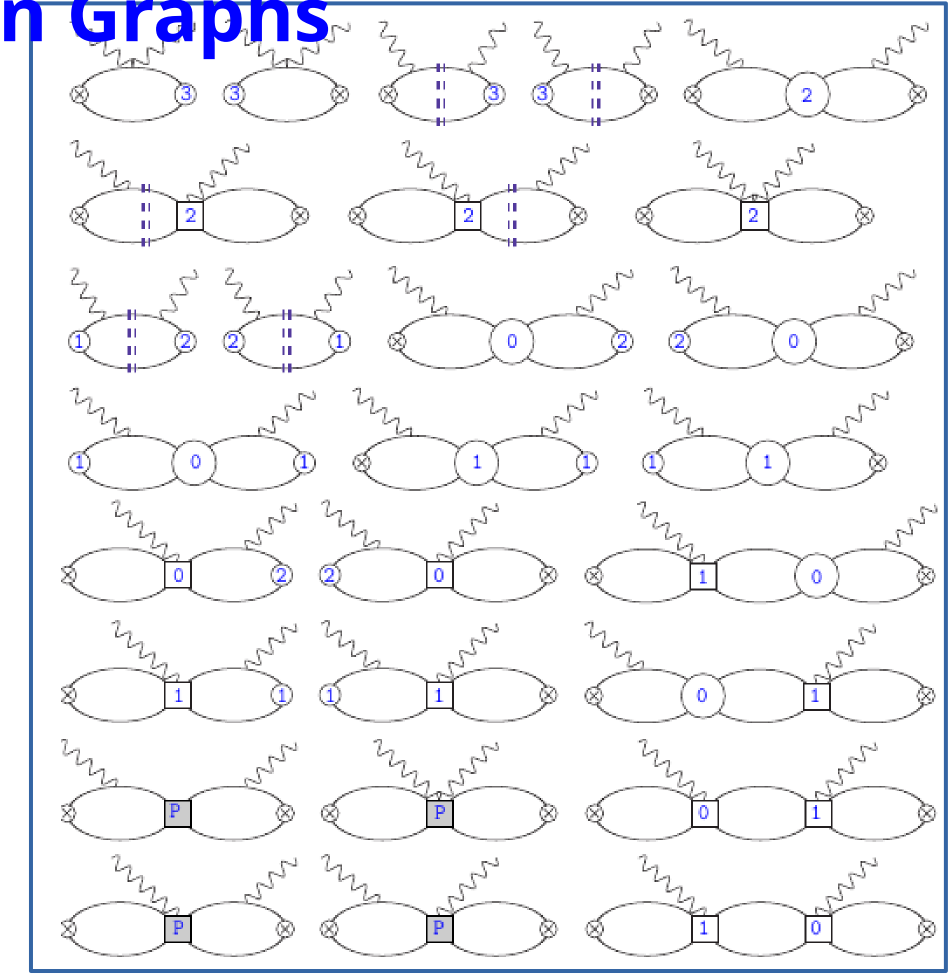
Deuteron VVCS: Feynman Graphs

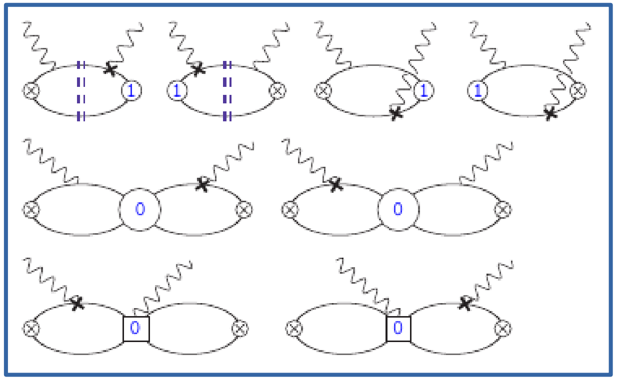
NNLO

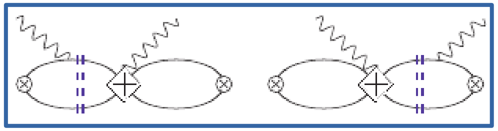




N3LO





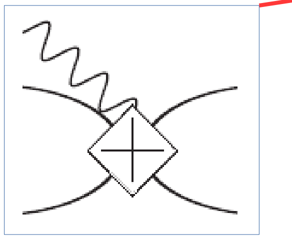


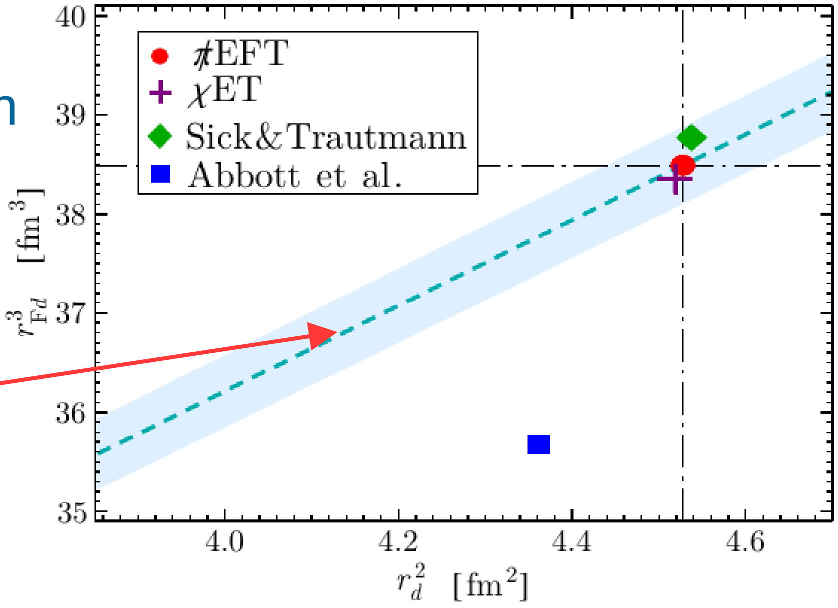
Deuteron Charge Form Factor and TPE in μD

- Correlation between the charge and Friar radii; can be used to test FF parametrisation

$$R_F^3 = \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^4} \left[G_C^2(Q^2) - 1 - 2G_C'(0)\,Q^2\right]$$

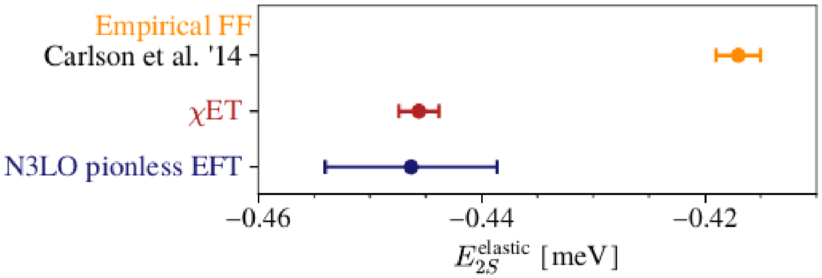
- The correlation is generated by the N3LO LEC

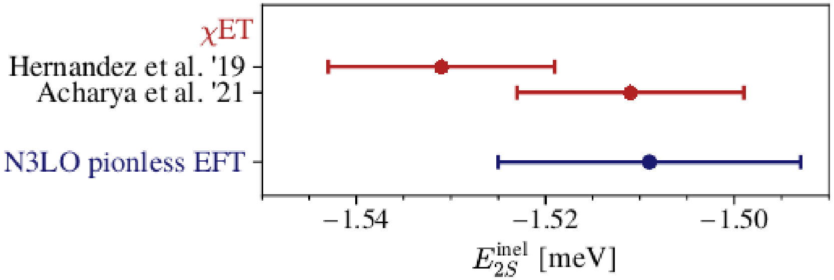




VL, Hagelstein, Pascalutsa (2022)

$$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$$





- Abbott et al. charge FF is not suitable for studying the low-Q properties
- Agreement with χEFT vindicates both EFTs

TPE in μD: Higher-Order Corrections

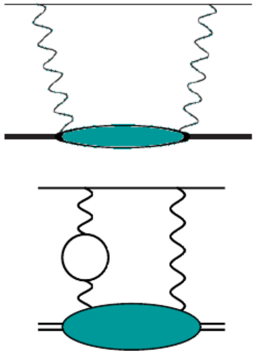
$$\Delta E_{2S} = \Delta E_{2S}^{\mathrm{elastic}} + \Delta E_{2S}^{\mathrm{inel}} = -1.955(19)\mathrm{meV}$$

- Higher-order in α terms are important in D
  - Coulomb $\left[\mathcal{O}(\alpha^6 \log \alpha)\right]$

    taken from elsewhere $\Delta E_{2S}^{\mathrm{Coulomb}} = 0.2625(15)\,\mathrm{meV}$
  - eVP $\left[\mathcal{O}(\alpha^6)\right]$ Kalinowski (2019)

    reproduced in pionless EFT $\Delta E_{2S}^{\mathrm{eVP}} = -0.027\ \mathrm{meV}$

non-forward



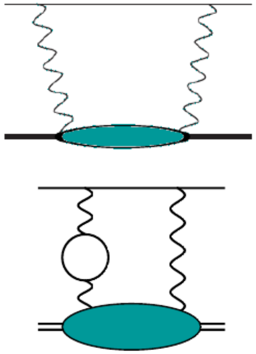
- Single-nucleon terms at N4LO in pionless EFT and higher
  - insert empirical FFs in the LO+NLO VVCS amplitude
  - polarisability contribution (inelastic+subtraction)
    - inelastic: *ed* scattering data above π threshold

      Carlson, Gorchtein, Vanderhaeghen (2013)
    - subtraction: nucleon subtraction function from **χ**EFT

      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\mathrm{hadr}} = -0.032(6)\ \mathrm{meV}$

TPE in μD: Higher-Order Corrections

$\Delta E_{2S} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} = -1.955(19)\text{meV}$



- Higher-order in α terms are important in D
  - Coulomb $\left[\mathcal{O}(\alpha^6 \log \alpha)\right]$

    taken from elsewhere $\Delta E_{2S}^{\text{Coulomb}} = 0.2625(15)\ \text{meV}$
  - eVP $\left[\mathcal{O}(\alpha^6)\right]$ Kalinowski (2019)

    reproduced in pionless EFT $\Delta E_{2S}^{\text{eVP}} = -0.027\ \text{meV}$

- Single-nucleon terms at N4LO in pionless EFT and higher
  - insert empirical FFs in the LO+NLO VVCS amplitude
  - polarisability contribution (inelastic+subtraction)
    - inelastic: *ed* scattering data Carlson, Gorchtein, Vanderhaeghen (2013)
    - subtraction: nucleon subtraction function from **χ**EFT

      VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)
  - in total: small but sizeable: $\Delta E_{2S}^{\text{hadr}} = -0.032(6)\ \text{meV}$

$$\Delta E_{2S}^{2\gamma} = \Delta E_{2S}^{\text{elastic}} + \Delta E_{2S}^{\text{inel}} + \Delta E_{2S}^{\text{hadr}} + \Delta E_{2S}^{\text{eVP}} + \Delta E_{2S}^{\text{Coulomb}} = -1.752(20)\ \text{meV}$$

TPE Corrections: A Challenge for Theory

- The uncertainties show that TPE corrections are a challenge:

	Correction	μH	μD	μ^3He$^+$	μ^4He$^+$
E_{QED}	point nucleus	206.034 4(3)	228.774 0(3)	1644.348(8)	1668.491(7)
$\mathcal{C}\, r_C^2$	finite size	$-5.225\,9\, r_p^2$	$-6.107\,4\, r_d^2$	$-103.383\, r_h^2$	$-106.209\, r_\alpha^2$
E_{NS}	nuclear structure	0.028 9(25)	1.750 3(200)	15.499(378)	9.276(433)
$E_L(\mathrm{exp})$	experiment^a	202.370 6(23)	202.878 5(34)	1258.612(86)	1378.521(48)

Pachucki, VL, Hagelstein, Li Muli, Bacca, Pohl (2022)

- Both progress in understanding nucleon structure and more precise nuclear interactions are needed to match experimental precision
- Analysing, e.g., isotope shift (H-D, ^{3}He-^{4}He) allows one to extract the TPE corrections

talk by Y. van der Werf on Wednesday

H-D: $r_d^2 - r_p^2 = 3.820\,61(31)\,\mathrm{fm}^2$

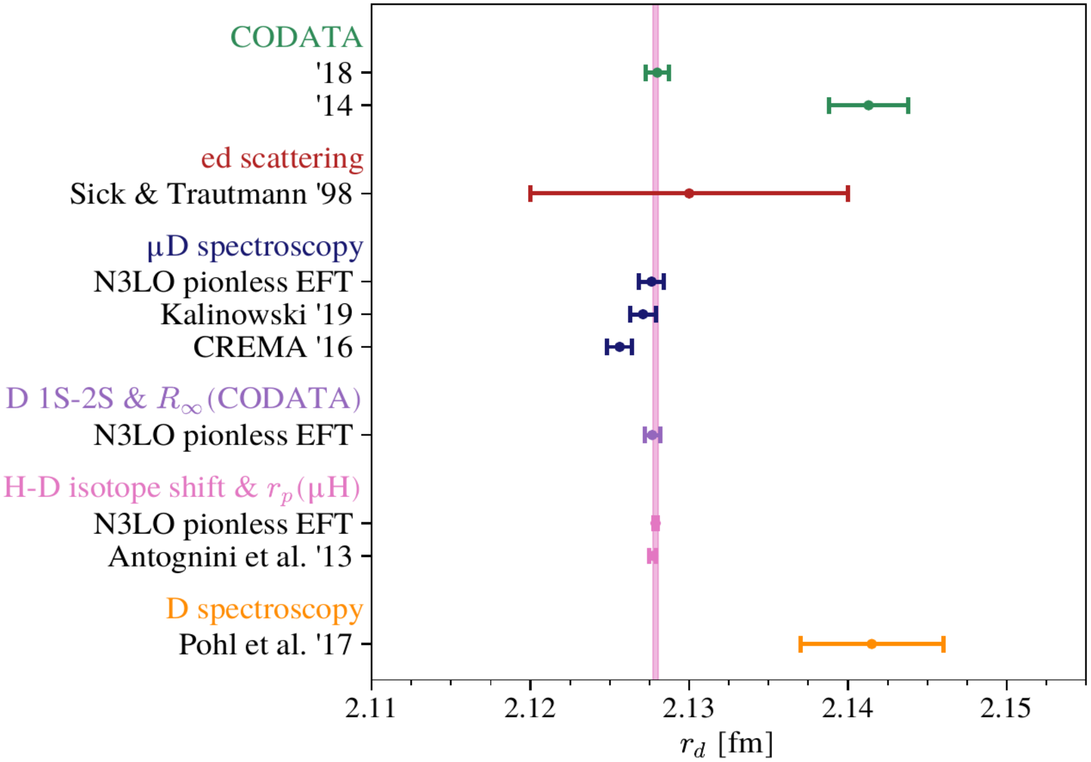
very precise due to partial cancellation of uncertainties

=> $E_{\mathrm{TPE}}(\mu\mathrm{D}) = -1.7585(56)$ meV

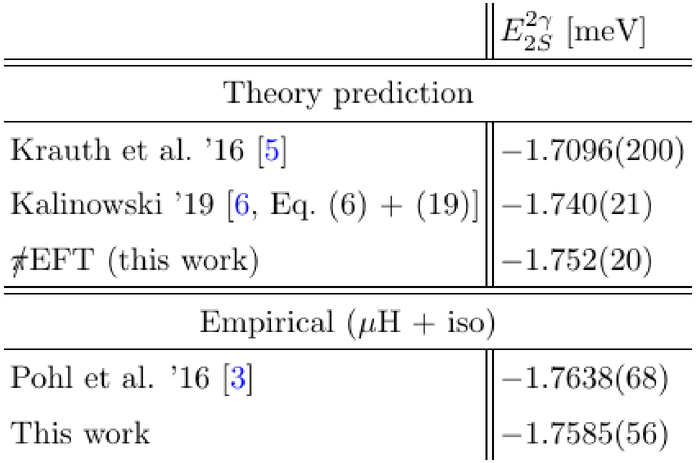
→ Benchmark for nuclear theory calculations

Deuteron Charge Radius and TPE in μD

- Reassessed with pionless EFT
- **μ**D, D, and H-D isotope shift all consistent with one another
- Agreement with the very precise empirical value of 2γ exchange



VL, Hagelstein, Pascalutsa (2022)



	$E_{2S}^{2\gamma}$ [meV]
Theory prediction	
Krauth et al. '16 [5]	−1.7096(200)
Kalinowski '19 [6, Eq. (6) + (19)]	−1.740(21)
$\not\pi$EFT (this work)	−1.752(20)
Empirical (μH + iso)	
Pohl et al. '16 [3]	−1.7638(68)
This work	−1.7585(56)

- Nuclear-level response well under control
- Single-nucleon structure starts to be important at this level of precision
  - even more important in heavier nuclei
- Experimental precision presents a challenge for theory

Outlook

- Experimental precision presents a challenge for theory
- Further progress in studying the proton structure is important for matching the precision in μH but also as input for heavier nuclei
  - more reliable structure function fits/parametrisations to further constrain proton subtraction function
  - alternative subtraction: $T_1(iQ, Q^2)$ instead of $T_1(0, Q^2)$ may work better in μH
- With more advanced nuclear potentials (χET) the uncertainty in μD and heavier atoms/ions will decrease
  - a naïve option is to go to higher orders (N4LO…)
  - one can also try an alternative way to fix the parameters of the potential, e.g., constraining the few-body charge radius
- More precise spectroscopy of normal atoms (isotope shift and so on): extract the TPE corrections to constrain the nuclear structure

Thank You for Your Attention!