Measurement of the cross-section ratio $\sigma_{\psi(2S)}/\sigma_{J/\psi(1S)}$ in exclusive photoproduction at HERA

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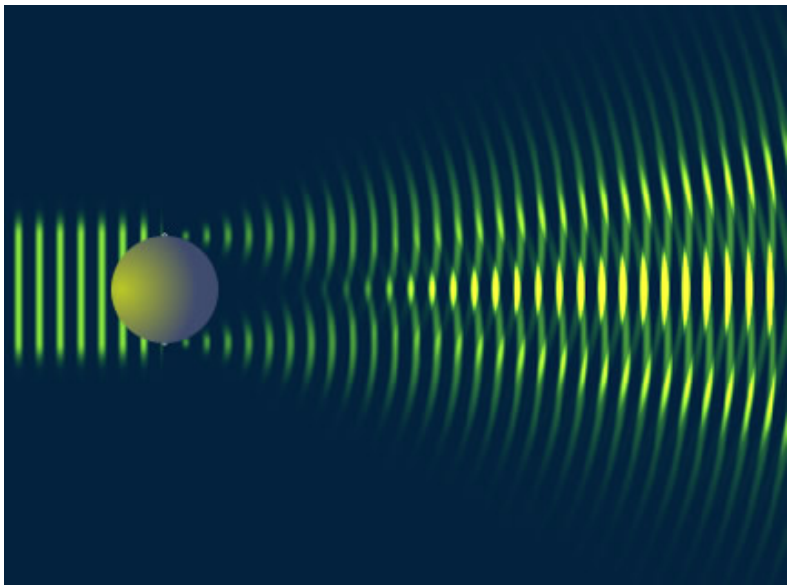
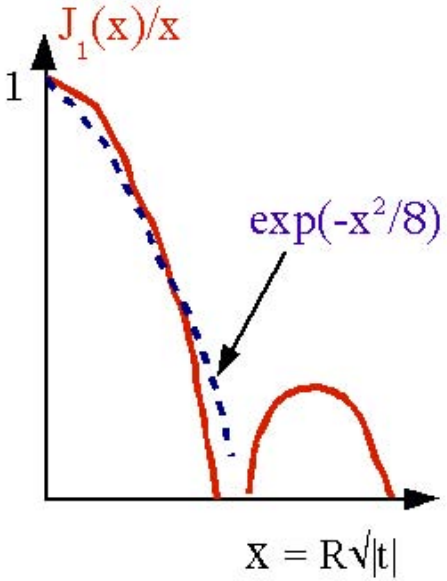
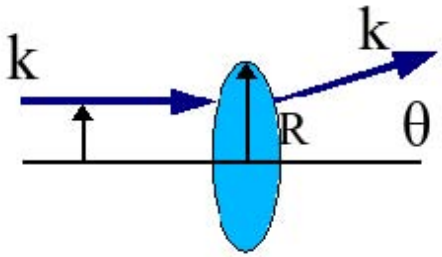
Diffraction of Light





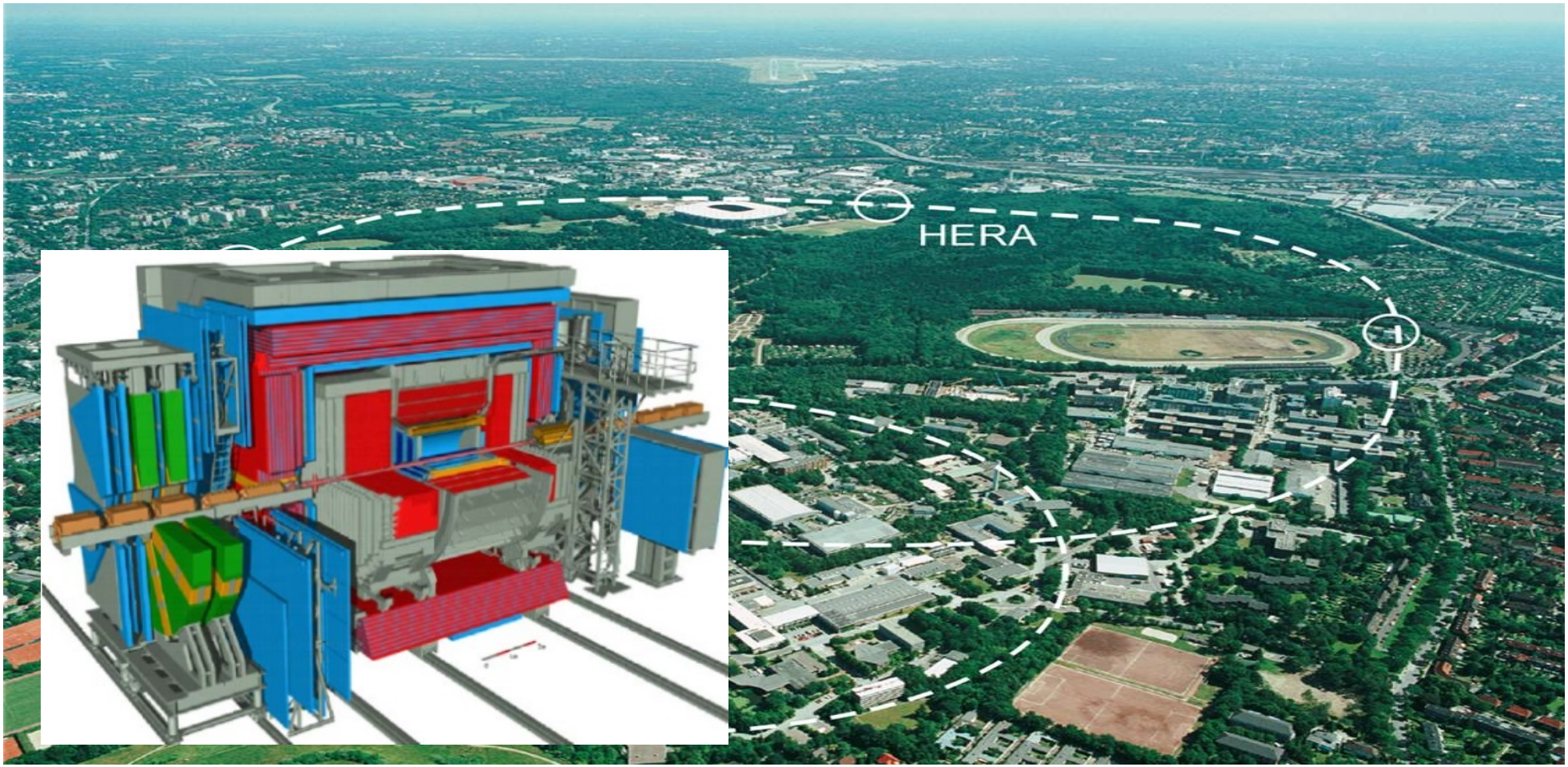


Light Scattering in Fraunhofer approximation (wavelength $\lambda \sim 1/k << R$)

- $|t| = 4k^2 sin^2(\theta/2)$
- $d\sigma/dt \sim e^{-b|t|}$ (first diffractive peak approximated from Bessel function)
- $b = (R/2)^2 \rightarrow$ transverse size of the target
- in the presented studies: target $\equiv$ proton and photon energy $\gg 1$ GeV

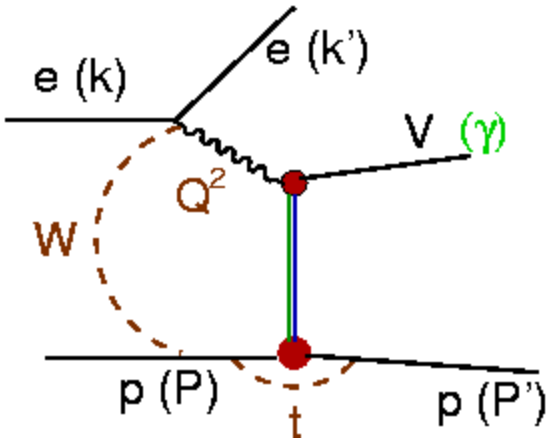
HERA and ZEUS: 1992 – 2007, DESY, Hamburg

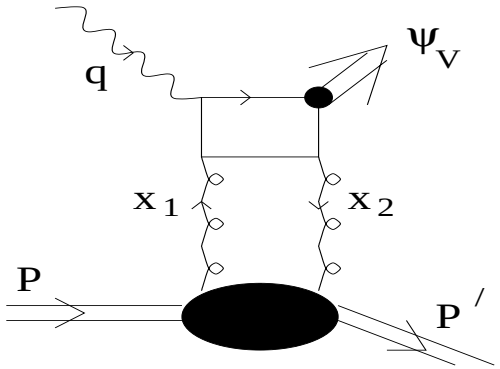
HERA: world's first and only $e^{\pm}p$ collider, $\mathbf{E_e = 27.5\ GeV}$, $\mathbf{E_p = 920\ GeV}$ ($\sqrt{s} = 318$ GeV)



ZEUS: multipurpose, hermetic detector (MVD, CTD, CAL, F/B/RMUON, BAC, ...)
Total luminosity: $\int \mathcal{L} \sim \mathbf{500\ pb^{-1}}$ collected during HERA I + II running periods

Production of Vector Mesons in Exclusive Diffraction in *ep* Scattering





Exclusive process: proton stays intact
Proton dissociation also possible → background

Kinematics: $\mathbf{M_V^2}, \mathbf{Q^2}, \mathbf{W}, \mathbf{|t|}$

M_V^2 - vector meson mass squared

Q^2 $(= -q^2 = -(k - k')^2)$ - the photon virtuality (emitted by the incoming electron):

- $Q^2 \approx 0$ GeV^2 PHP (*Photoproduction*)
- larger Q^2 for DIS (*Deep Inelastic Scattering*)

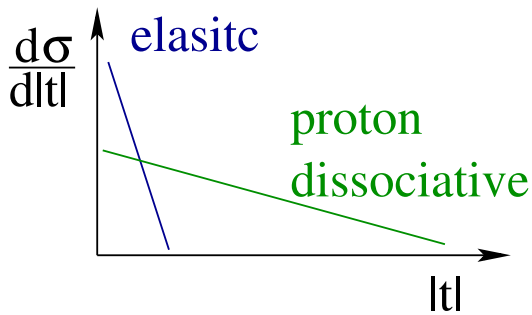
$W = (q + P)^2$ - invariant mass of the γp system

$W \approx \sqrt{2E_P(E - p_Z)_V}$

$|t|$ - 4-momentum transfer at the proton vertex

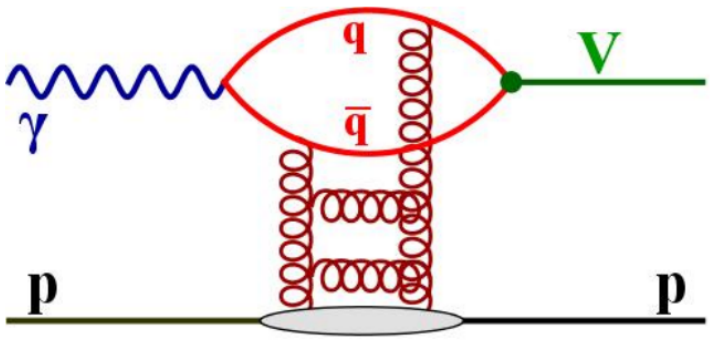
$t = (P - P')^2$

$t \approx -p_{T,V}^2$

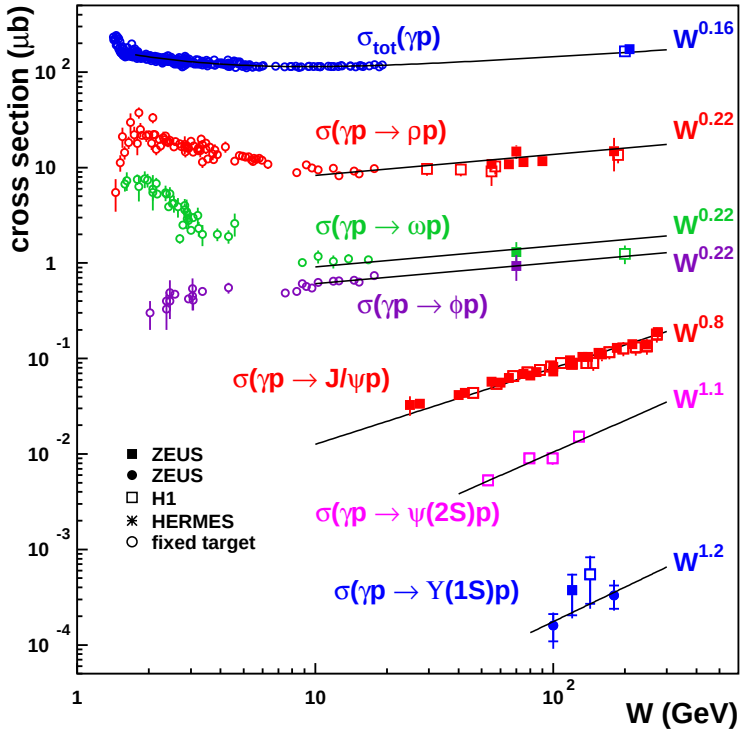


pQCD: M_V^2 and Q^2 - set the scale at which the W and $|t|$ are probed
Process sensitive to the **gluon density** in the proton

W-dependence of the VM exclusive cross section for PHP: $\sigma(\gamma p \to Vp)$

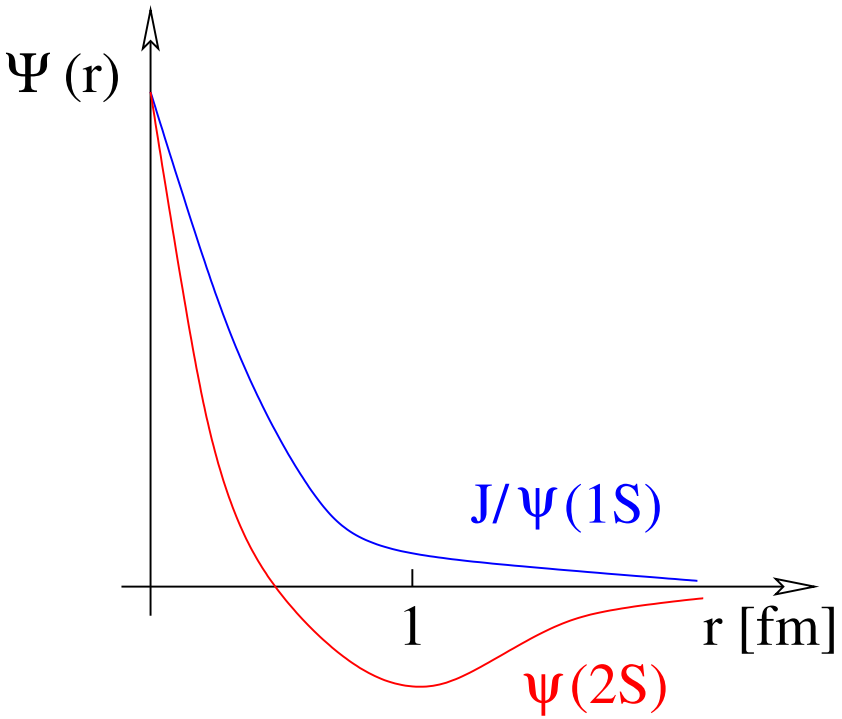


- process driven by gluons
- $\sigma \propto [xg(x,\mu^2)]^2$
- $\mathbf{x = \mu^2/W^2}$: high $W \to$ small x
- $\mu^2 \propto (Q^2 + M_V^2)$



- total cross sections σ_{VM} for VM photoproduction spans over 6 orders of magnitude ! (for higher VM masses $\to$ smaller transverse size of $q\bar{q}$ $\to$ "color screening")
- σ_{VM} **rises with** γp **c.m. energy** W **as** W^{δ} **(**$x \sim 1/W^2$ **: high** $W \to$ **small** x**)** (for heavy VMs expected from the gluon behaviour in the proton $\to$ probing small x)
- power δ rises with M_V^2 from "soft" ($\delta \approx 0.22$) to "hard" ($\delta \approx 1.2$) processes

Cross section ratio $\psi(2S)/J/\psi(1S)$

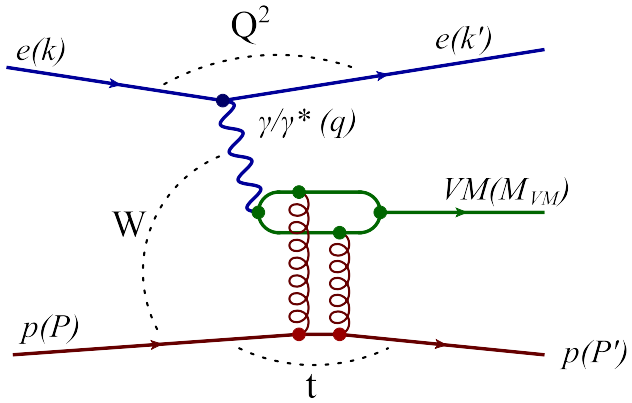


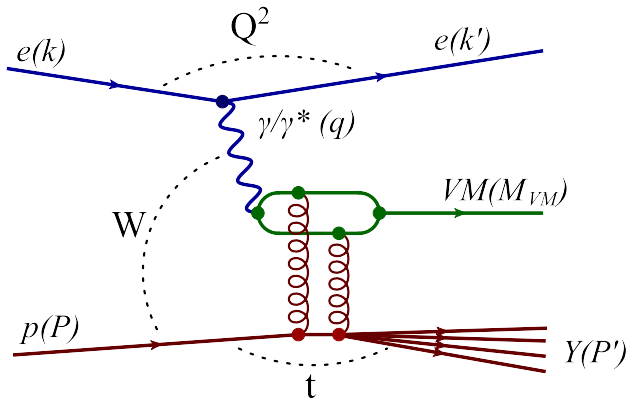
$$\text{Ratio } R = \frac{\sigma_{\gamma p \to \psi(2S) p}}{\sigma_{\gamma p \to J/\psi(1S) p}}$$

- sensitive to radial wave function of charmonium
- provides insight into the dynamics of the hard process
- (some systematics cancel out)

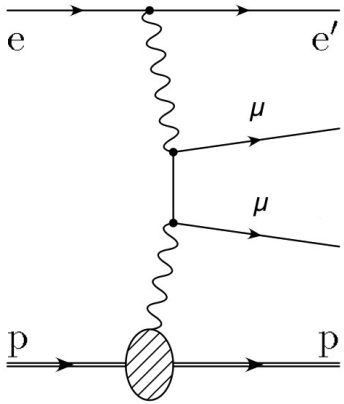
- $J/\psi(1S)$ and $\psi(2S)$ have the same quark composition but distinctive wave functions
- $\psi(2S)$ has a node at ≈ 0.4 fm
- $\langle r^2_{\psi(2S)} \rangle \approx 2 \langle r^2_{J/\psi(1S)} \rangle$
- pQCD models predict $R \sim 0.17$ in PHP and rise of R with Q^2 in DIS
- $\psi(2S)$ **cross section is expected to be suppressed w.r.t. the** J/ψ **production**
- (Both Vector Mesons masses are much smaller then the γp center-of-mass energy)

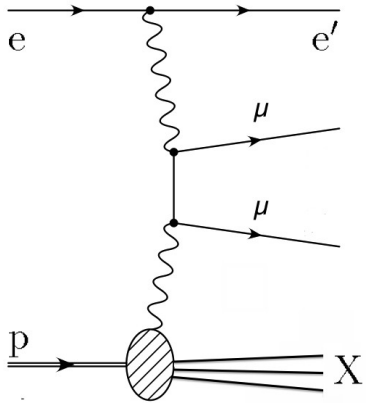
- **Signal MC:** DIFFVM (VM production in *ep* scattering)





- **Background MC:** GRAPE (Bethe-Heitler continuum $\mu^+\mu^-$)

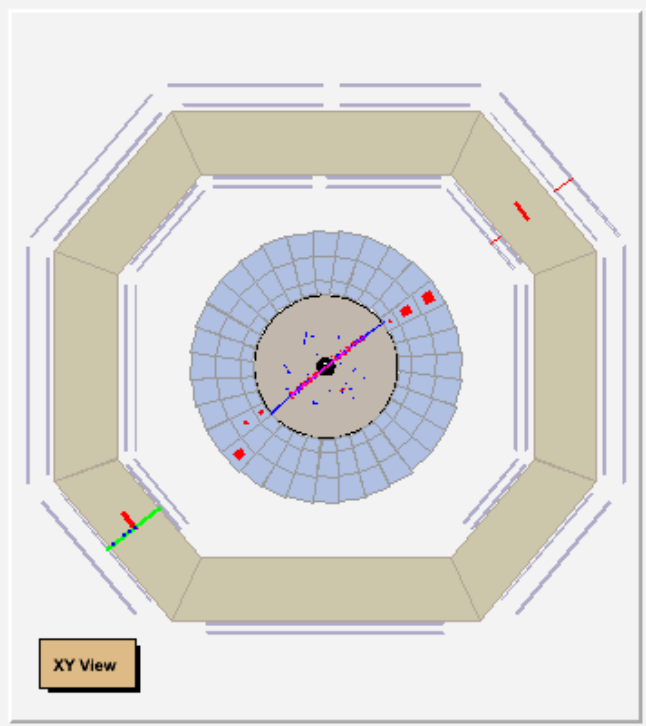


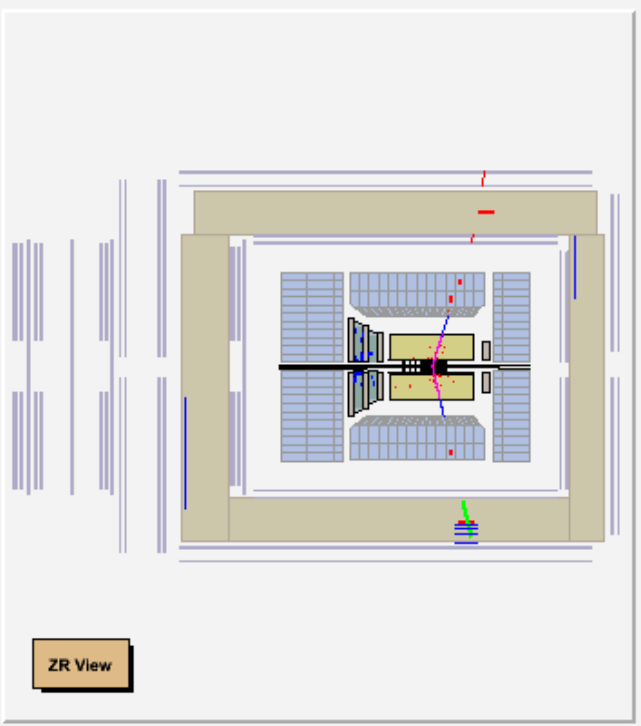


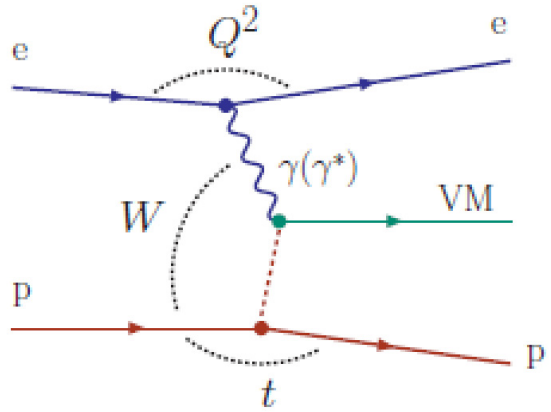
Data Sample and Phase Space

- **HERA II DATA:** $\mathcal{L} = 373$ pb^{-1} (2003 - 2007)
- **Investigated decay channels:**
  - $\psi(2S) \to \mu^+\mu^-, \quad \psi(2S) \to J/\psi + \pi^+\pi^-, \quad J/\psi(1S) \to \mu^+\mu^-$
- exclusive (elastic) photoproduction sample
- **kinematic range (analysis phase space):**
  - $30 < W < 180$ GeV
  - $|t| < 1.0$ GeV^2
  - $Q^2 < 1$ GeV^2 (median $Q^2 \approx 3 \times 10^{-5}$ GeV^2)

Example of Final State Topology for $ep \to J/\psi p$, $J/\psi \to \mu^+\mu^-$







Exclusive process, reaction mediated by exchange of colorless object;
proton stays intact.

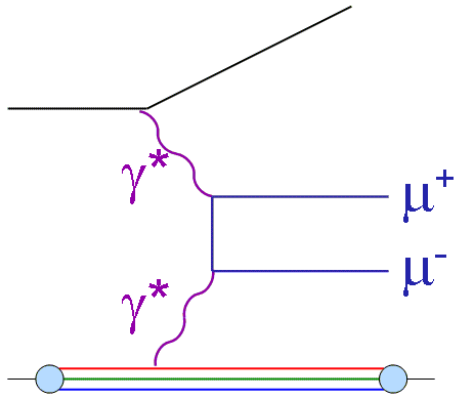
J/ψ and $\psi(2S)$ are detected in the 2- or 4-prong final states ($\mu^+\mu^-$ or $\mu^+\mu^-\pi^+\pi^-$)
very clean final state topology:

Photoproduction ($Q^2 < 1$ GeV^2): two or four charged particles and nothing else
$\Longrightarrow$ **experimental challenge: triggering on soft muons**
(Electroproduction ($Q^2 > 1$ GeV^2): scattered electron also visible in the detector)

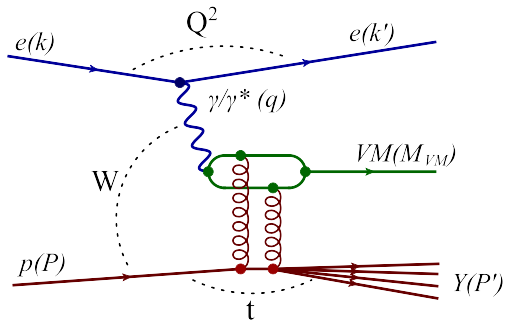
(NOTE THE OPENING ANGLE OF THE DECAY PRODUCTS !)

Background Sources

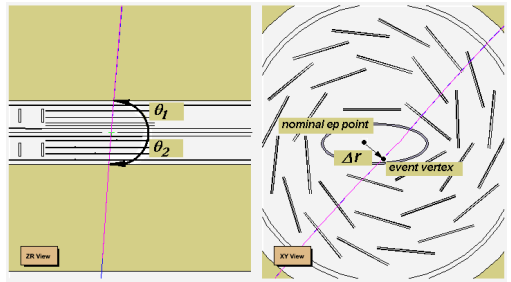
- QED di-muons (like $\gamma^*\gamma^* \to \mu^+\mu^-$) from the Bethe-Heitler process



- J/ψ and $\psi(2S)$ mesons production with the dissociation of the proton



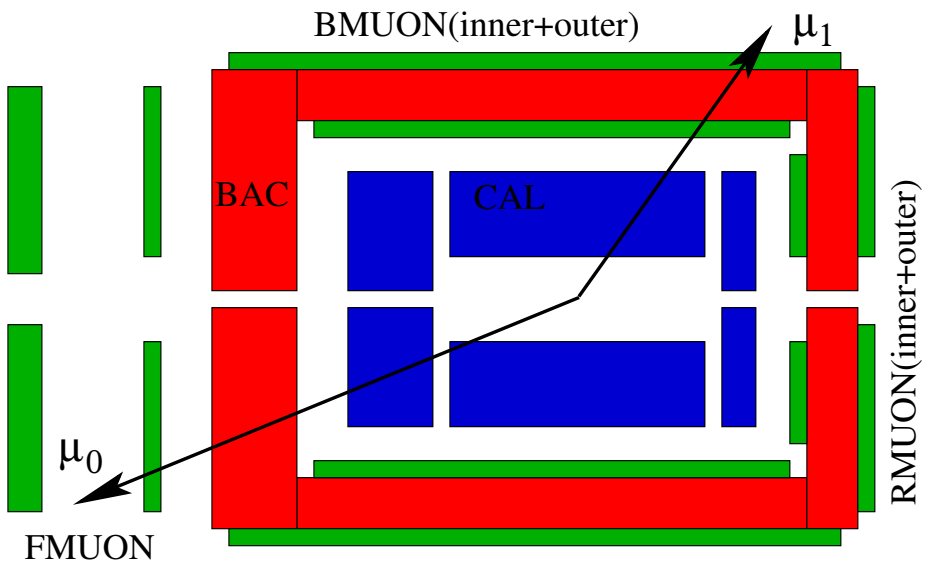
- Cosmic muons can mimic $\mu^+\mu^-$ pairs when passing close to the interaction point



- **Muon efficiency corrections**

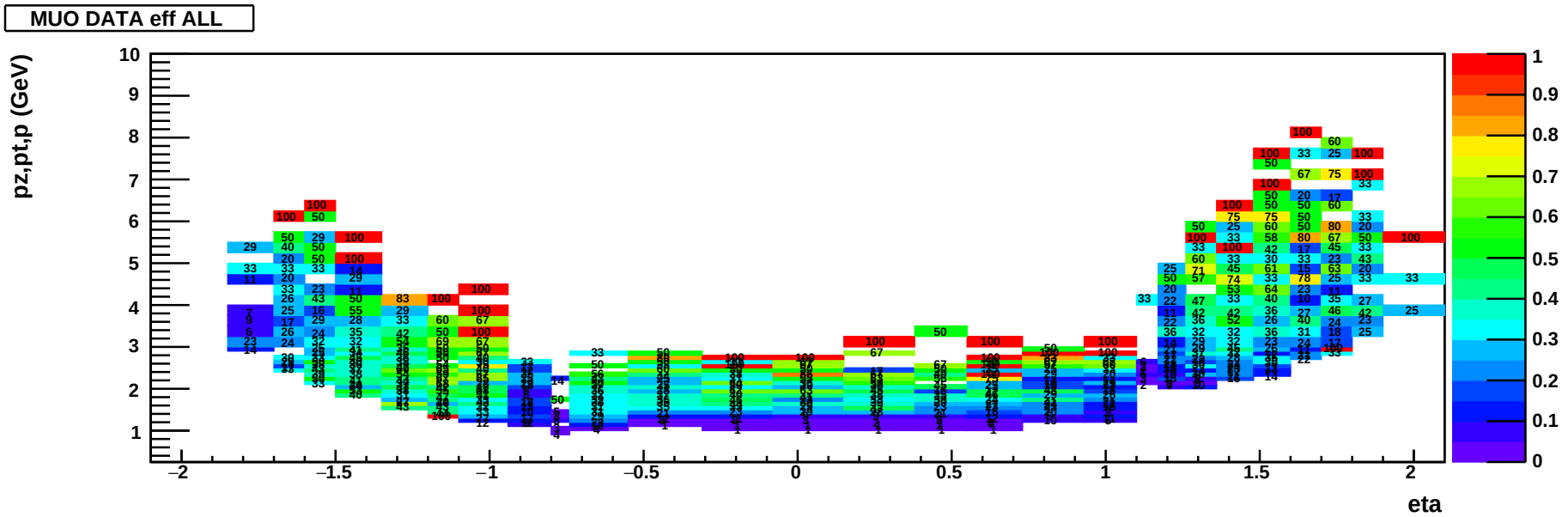
Single muon efficiency corrections: TAG and PROBE

- TAG: "the triggering" muon
- PROBE: "the tested" muon
- effic in given (pt^j_{eff}, η^j) bin:
  $\epsilon = N^j_{PROBE}/N^i_{TAG}$



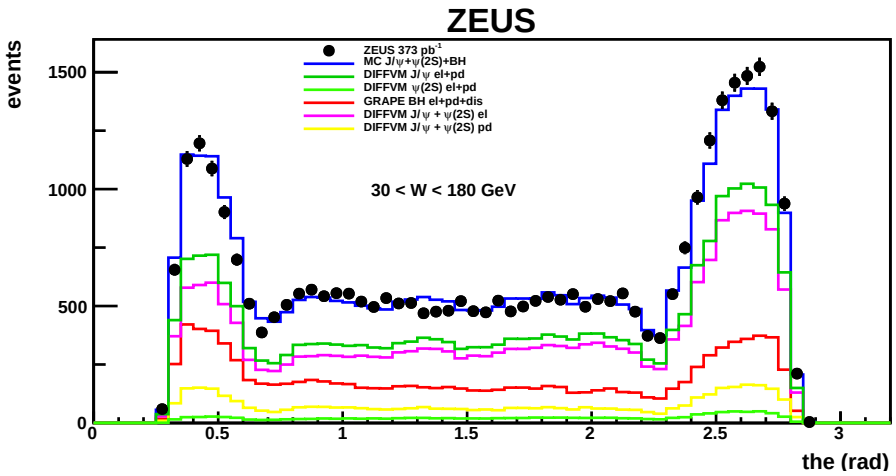
- one step correction for (FLT and SLT and TLT and off-line REC)
- separate maps for F/B/R/MUO, BAC and CAL (off-line only)
- evaluated for single muon in (pt_{eff}, η) bins, where as pt_{eff} is used:
  (motivated by the CAL/BAC geometry and scaling of the muon path length)
  - p in Forecap
  - p_t in Barrel
  - p_z in Rearcap
- proper identification of **the triggering muon** is crucial
- $\rightarrow$ the DATA/MC ratio delivers the correction weight: $\epsilon_x = \frac{\epsilon_{DATA}}{\epsilon_{MC}}$

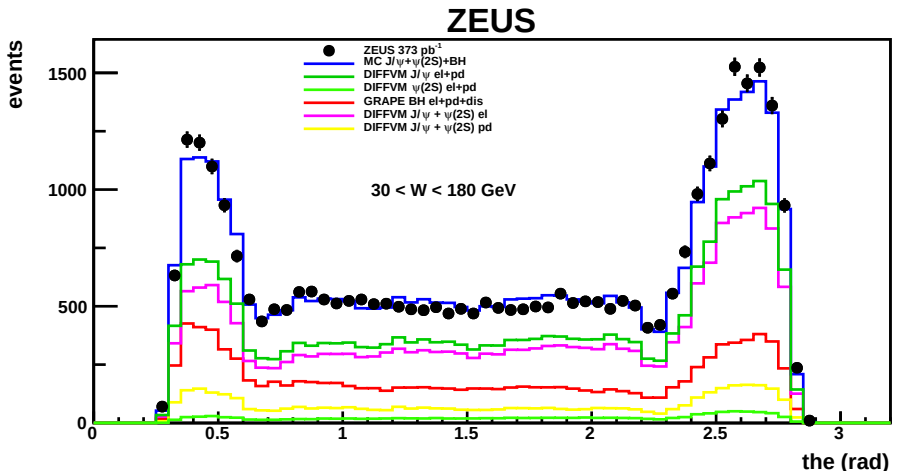
Muon correction maps: $(p_z, p_t, p$ *vs.* $\eta)$ - DATA

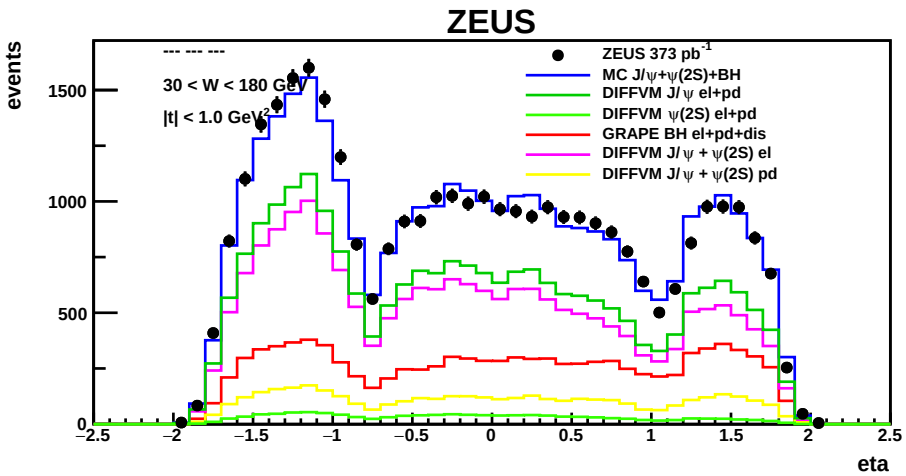


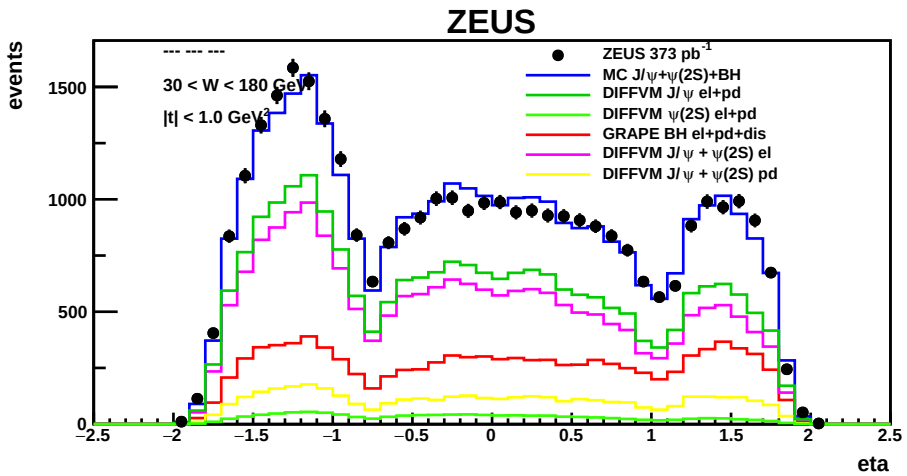
- **muon tomography (+ muon range)**
- probability (%) to fire FLT-SLT-TLT-REC chain by single muon on $(p_z, p_t, p; \eta)$ grid
- X-axis (along eta): Rear-MUO, Barrel-MUO, Forward-MUO detectors
- only events with $M(\mu^+, \mu^-) < 6$ GeV
  (ie. in the phase space range of di-muon mass fits)
- p_z, p_t, p grid: 100 MeV per bin ($p_{eff} < 3$ GeV), 250 MeV per bin ($p_{eff} > 3$ GeV)
- size of the grid is subject to systematics

θ and η distribution for μ^+ and μ^-



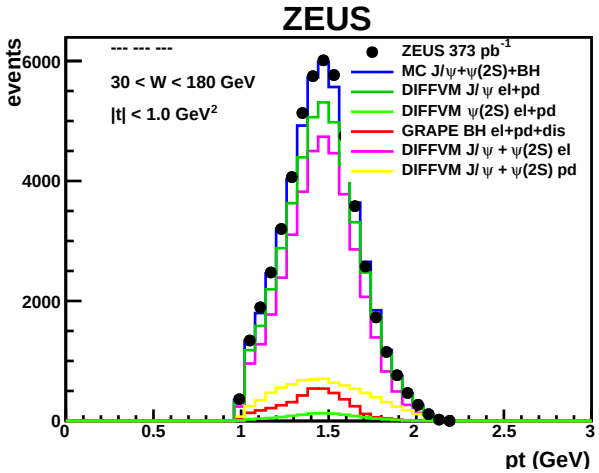


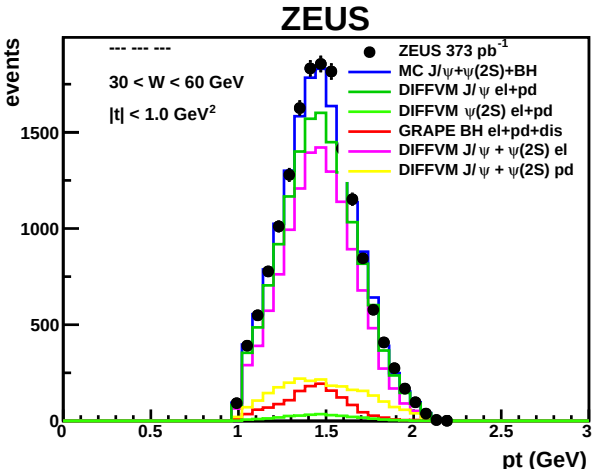


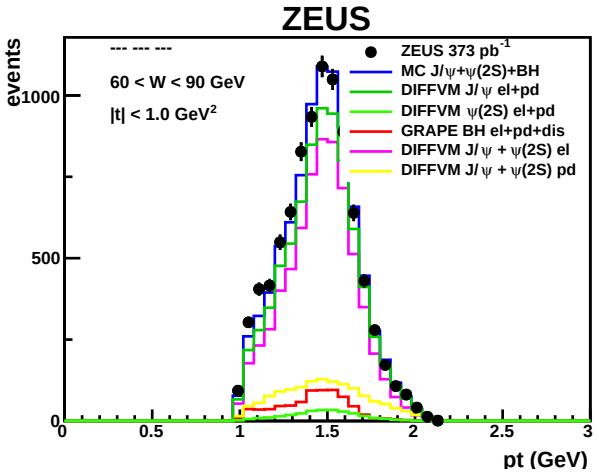


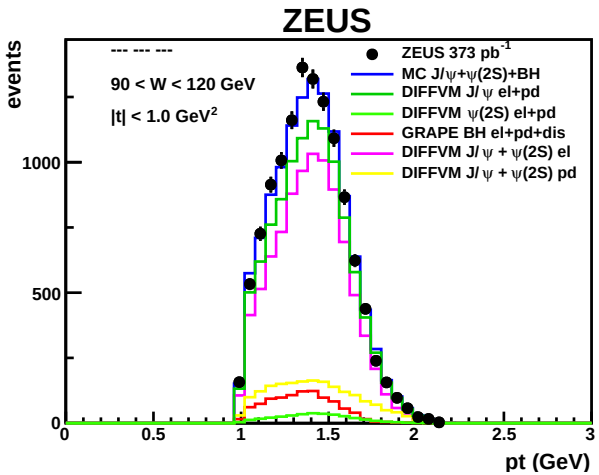
- θ and η distribution for muon tracks (μ^+ and μ^-, $\eta \in (-2, 2)$)
- **after MC tuning and muon corrections**
- well reproduced acceptance and efficiency of the detector

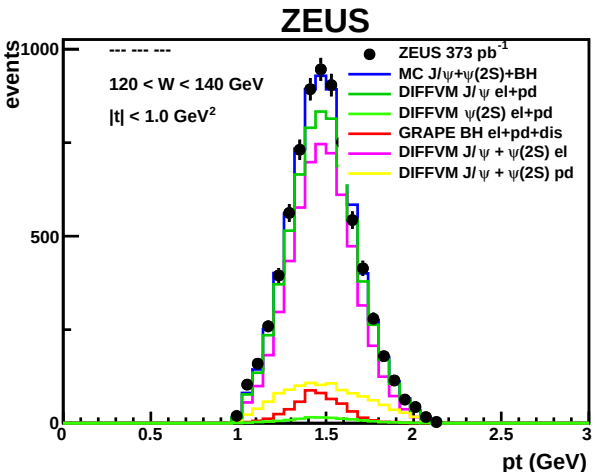
2-prongs: JPSI mass window: $p_t^{\mu^\pm}$ in W bins

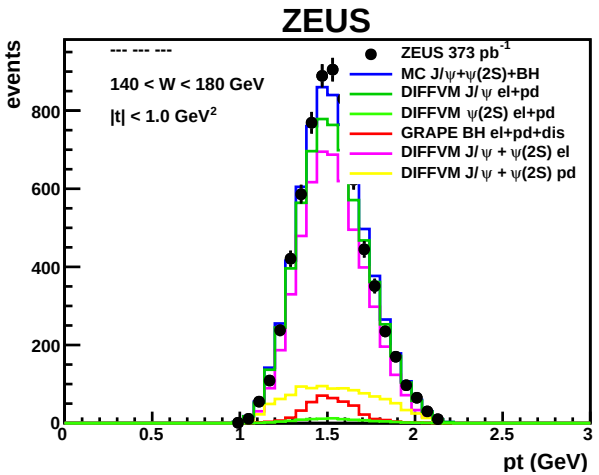












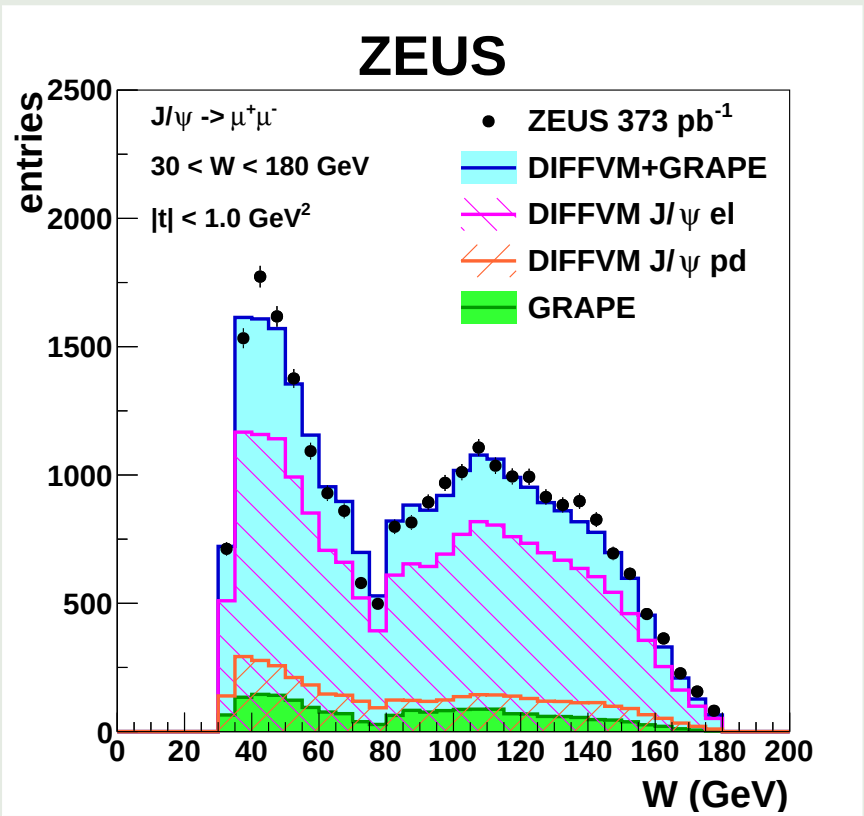
- ALL events and in W1, W2, W3, W4, W5 bins
- **Jacobian peak around** 1.5 **GeV** ($M_{JPSI}/2$)
- almost no JPSI (and PSI2S) event loss due to the $p_t^{\mu^\pm} > 1$ GeV cut

W and $|t|$ distributions

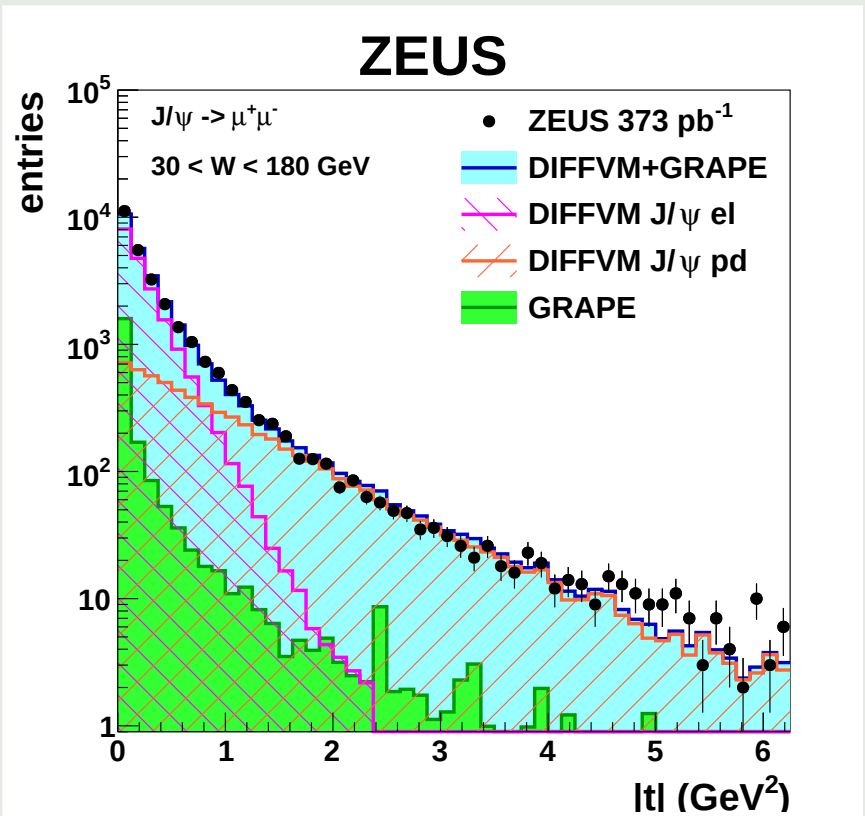
- **W and $|t|$ distributions: 2-prongs**

2-prongs: W and $|t|$ distributions: J/ψ mass window

W: $2.8 < M(\mu^+\mu^-) < 3.4$ GeV



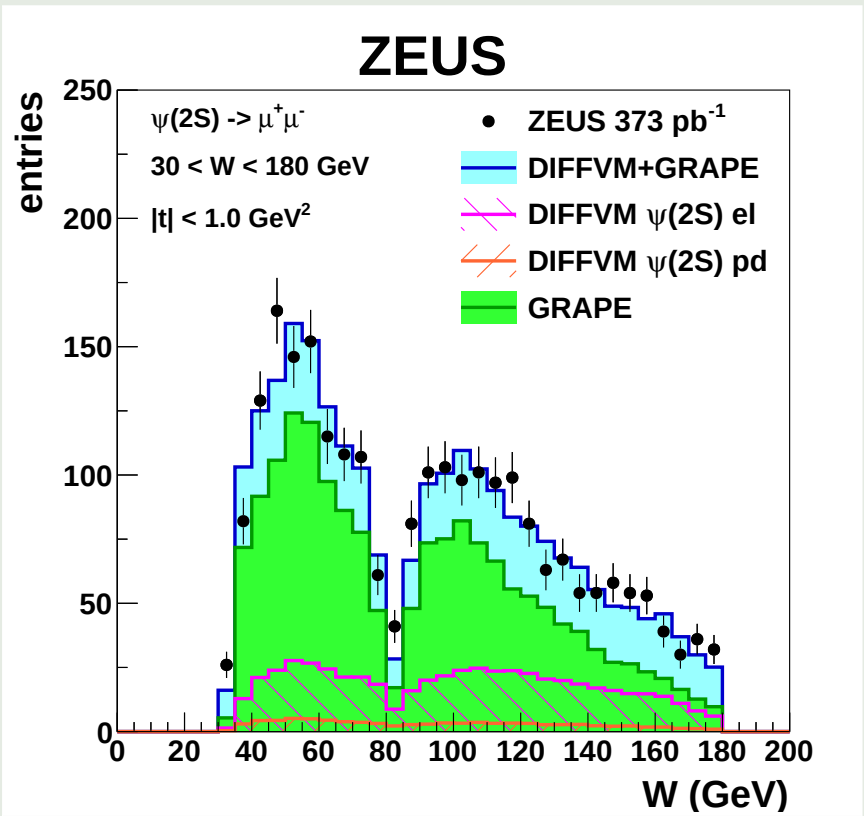
$|t|$: $2.8 < M(\mu^+\mu^-) < 3.4$ GeV



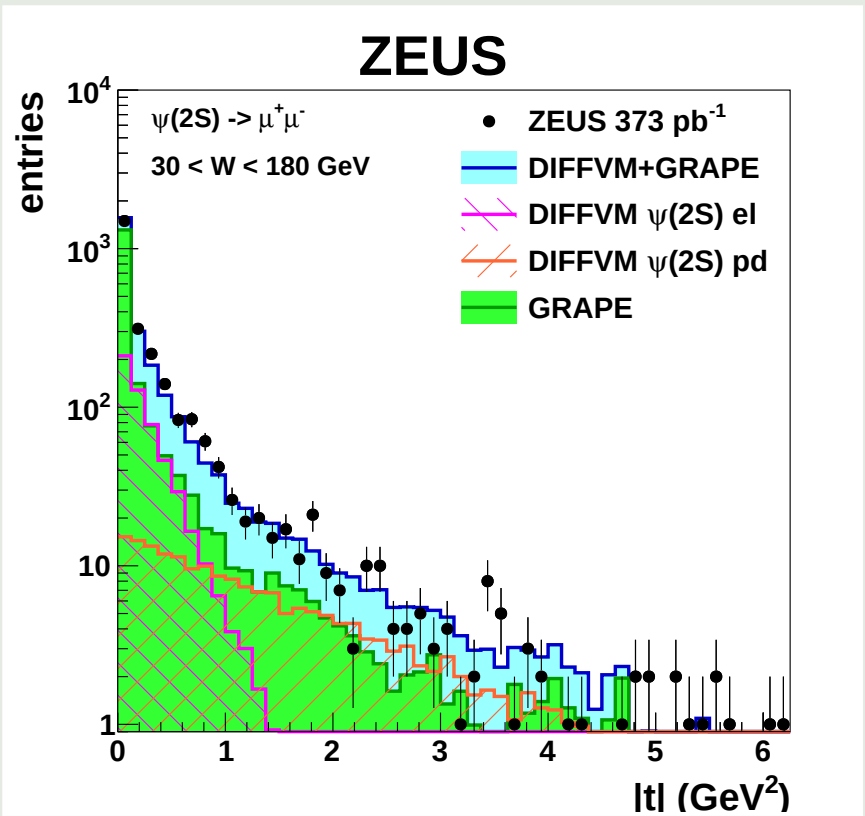
- dip in W distribution due to the anti-COSMIC cut: $\cos(\mu^+, \mu^-) < -0.985$
- proton dissociation dominates for $|t| > 1.0$ GeV^2
- proton dissociative fraction: $f_{p.diss} = 0.17 \pm 0.01$ ($|t| < 1.0$ GeV^2) from t-spectra fit

2-prongs: W and $|t|$ distributions: $\psi(2S)$ mass window

W: $3.4 < M(\mu^+\mu^-) < 4.0$ GeV



$|t|$: $3.4 < M(\mu^+\mu^-) < 4.0$ GeV



- dip in W distribution due to the anti-COSMIC cut: $\cos(\mu^+, \mu^-) < -0.985$
- proton dissociation dominates for $|t| > 1.0$ GeV^2
- channel dominated by Bethe-Heitler continuum $\mu^+\mu^-$ background

Signal Extraction

- **Signal Extraction, Mass spectra**

2-prongs: Signal extraction: fit parameterization

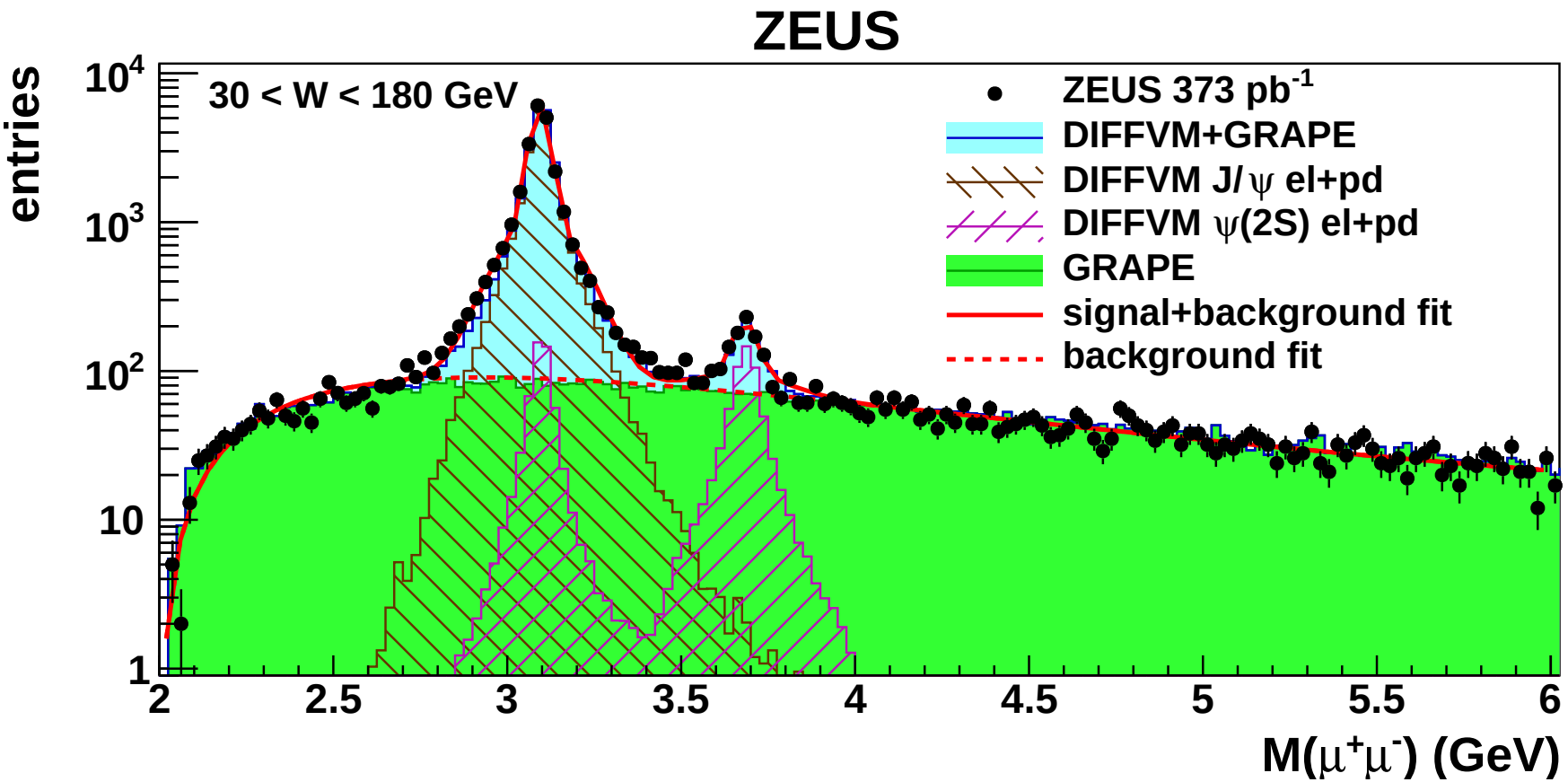
- Double Gaussian shape: $G(x)$ or $g(x) = N \cdot \Delta \cdot \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(x-m)^2}{2\sigma^2}\right)$
  where: N – number of events, Δ – mass bin width,
  m – mean value, σ – RMS
- for J/ψ: $N_1 \cdot G_1(x) + N_2 \cdot G_2(x)$
- for ψ': $N_1' \cdot g_1(x) + N_2' \cdot g_2(x)$
- introducing: $N = N_1 + N_2$, $N' = N_1' + N_2'$, $R = \frac{N'}{N}$
- with additional constrains: $m_1 = m_2$, $m_1' = m_2'$,
  $\frac{\sigma_1'}{\sigma_1} = \frac{\sigma_2'}{\sigma_2} = \alpha$, $\xi = \frac{N_1}{N} = \frac{N_1'}{N'}$ (**scaling of the mass resolution**)
- final formulae:

$$F(x) = N \cdot ((\xi \cdot G_1(x) + (1-\xi) \cdot G_2(x)) + R \cdot (\xi \cdot g_1(x) + (1-\xi) \cdot g_2(x))) + BG(x)$$

- background function: $BG(x) = A \cdot (x-B)^C \cdot \exp(-D(x-B) - E(x-B)^2)$
  where A, B, C, D, E are fit parameters, B fixed $(= 2p_{t,min}^{\mu})$

2-prongs: dimuon mass spectra

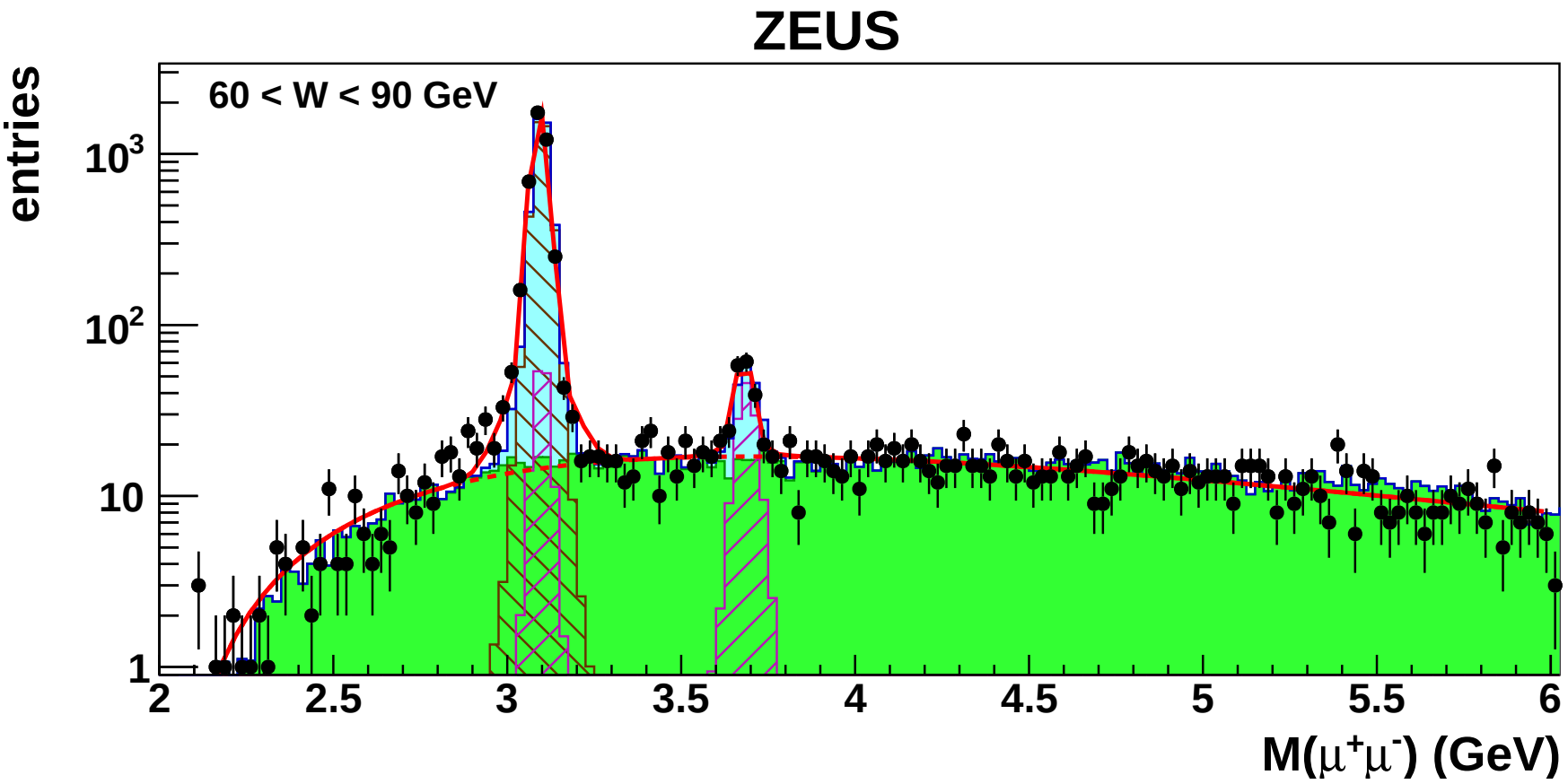
$M(\mu^+\mu^-)$



- **full phase space:** $30 < W < 180$ GeV, $|t| < 1.0$ GeV^2
- events yield: $\sim 23\,000$ J/ψ and ~ 700 $\psi(2S)$
- resonant background under J/ψ peak

2-prongs: dimuon mass spectra

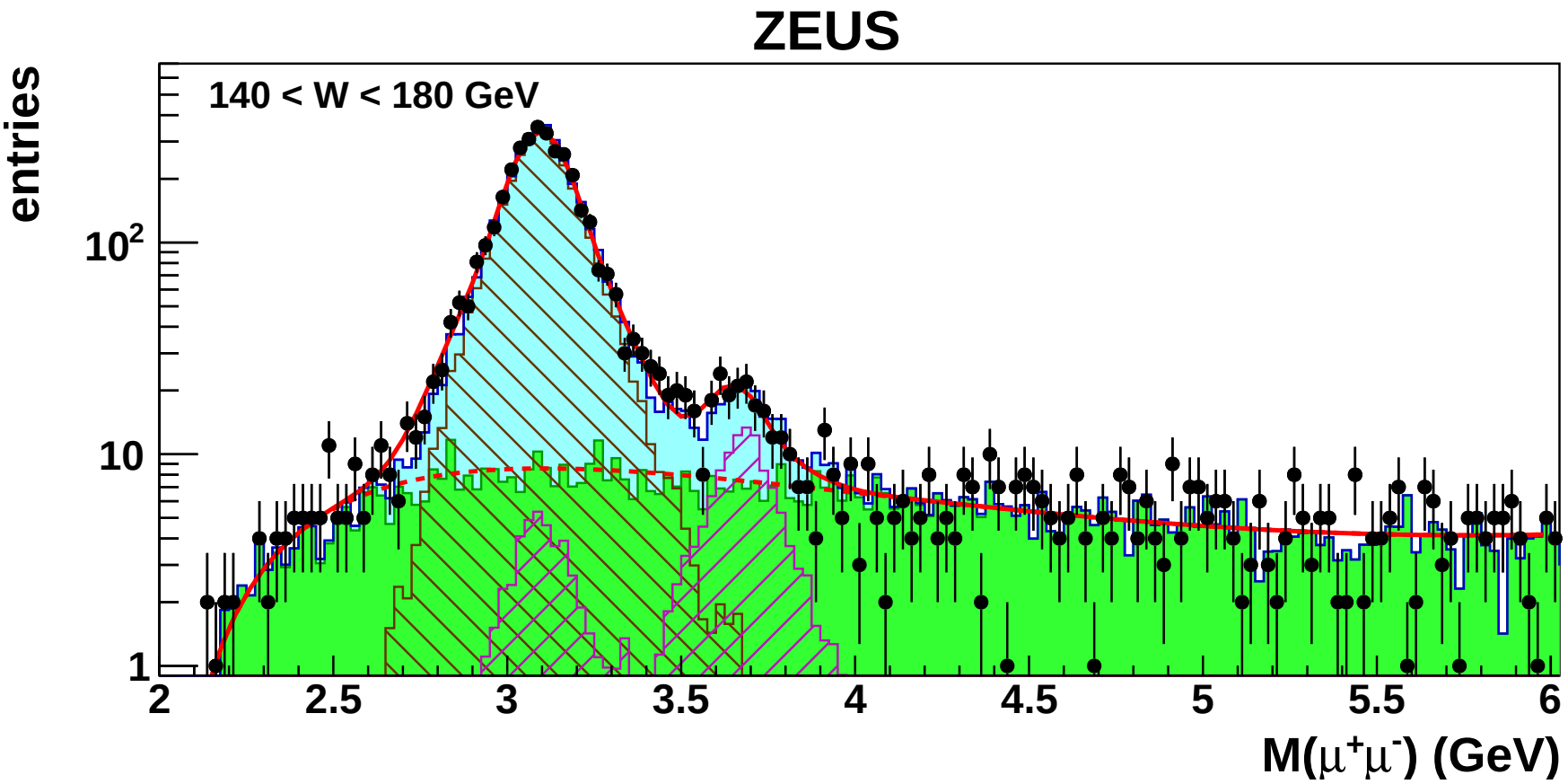
$M(\mu^+\mu^-)$



- **W2 bin:** $60 < W < 90$ GeV, $|t| < 1.0$ GeV^2
- central rapidity region, long tracks, mass resolution: $\sigma_M(\mu\mu) \sim 22$ MeV

2-prongs: dimuon mass spectra

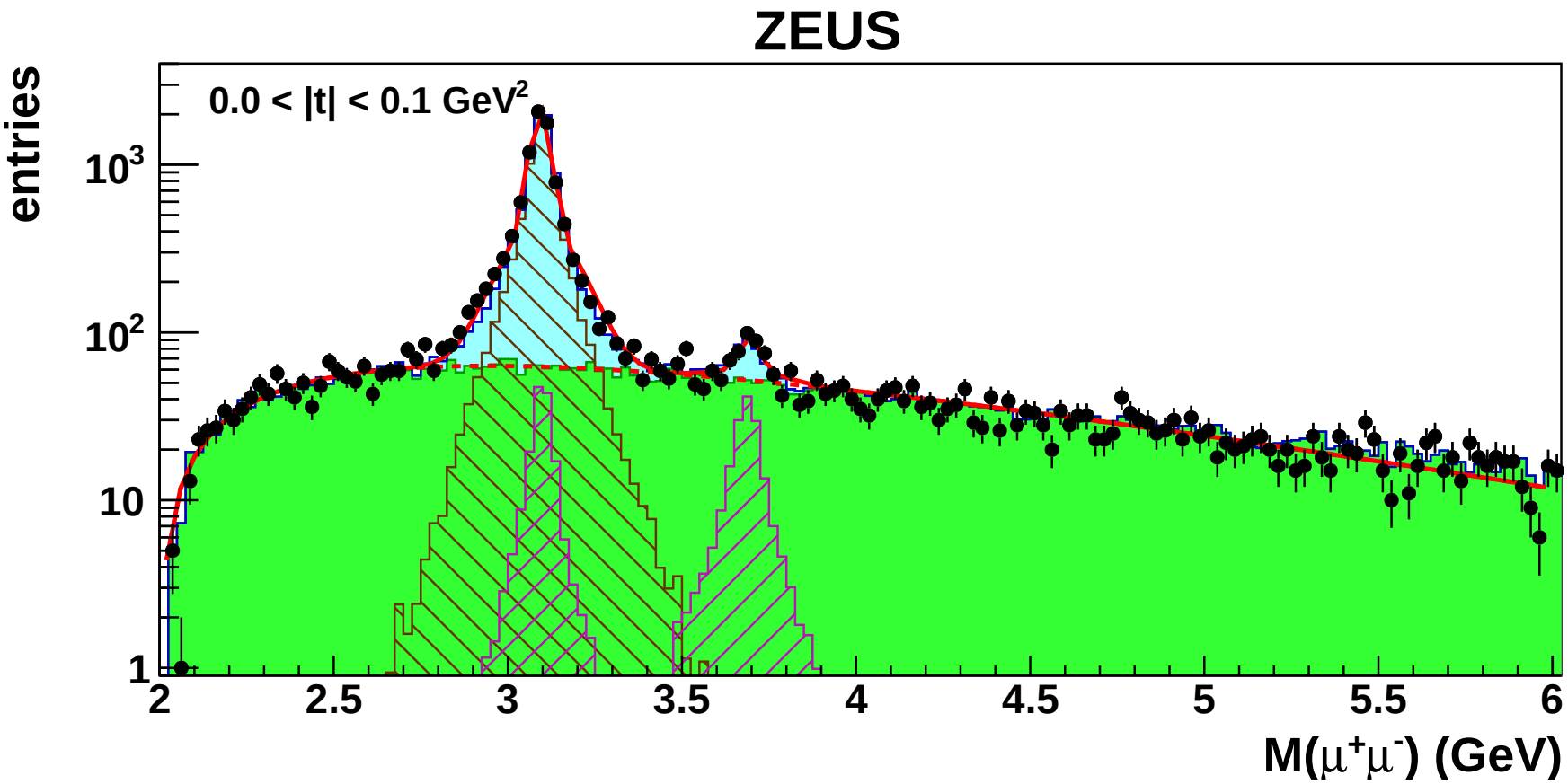
$M(\mu^+\mu^-)$



- **W5 bin:** $140 < W < 180$ GeV, $|t| < 1.0$ GeV^2
- high W, backward short tracks, mass resolution: $\sigma_M(\mu\mu) \sim 73$ MeV

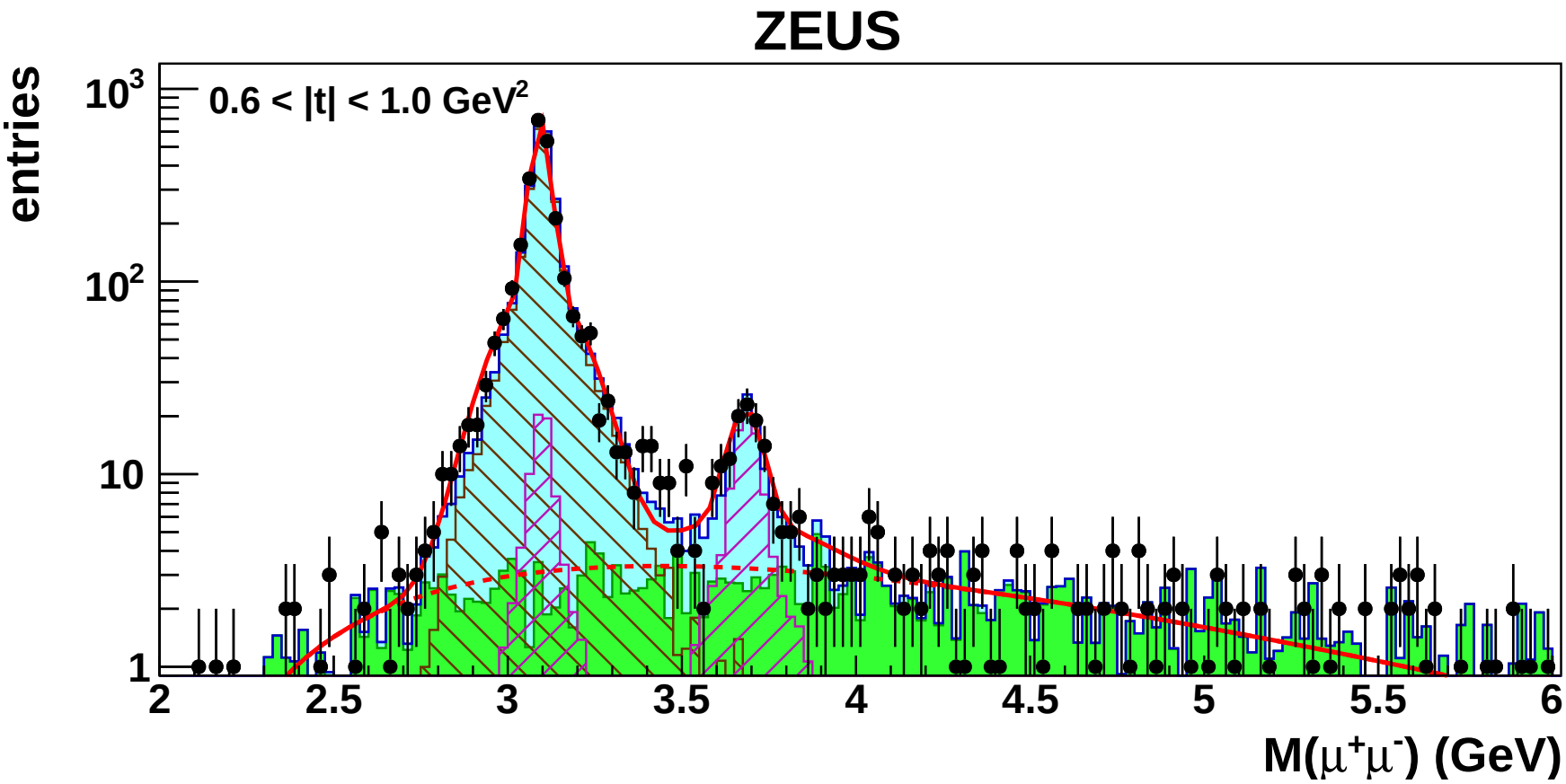
2-prongs: dimuon mass spectra

$M(\mu^+\mu^-)$



- **t1 bin:** $30 < W < 180$ GeV, $|t| < 0.1$ GeV^2
- low $|t|$, dominated by Bethe-Heitler continuum $\mu^+\mu^-$ background

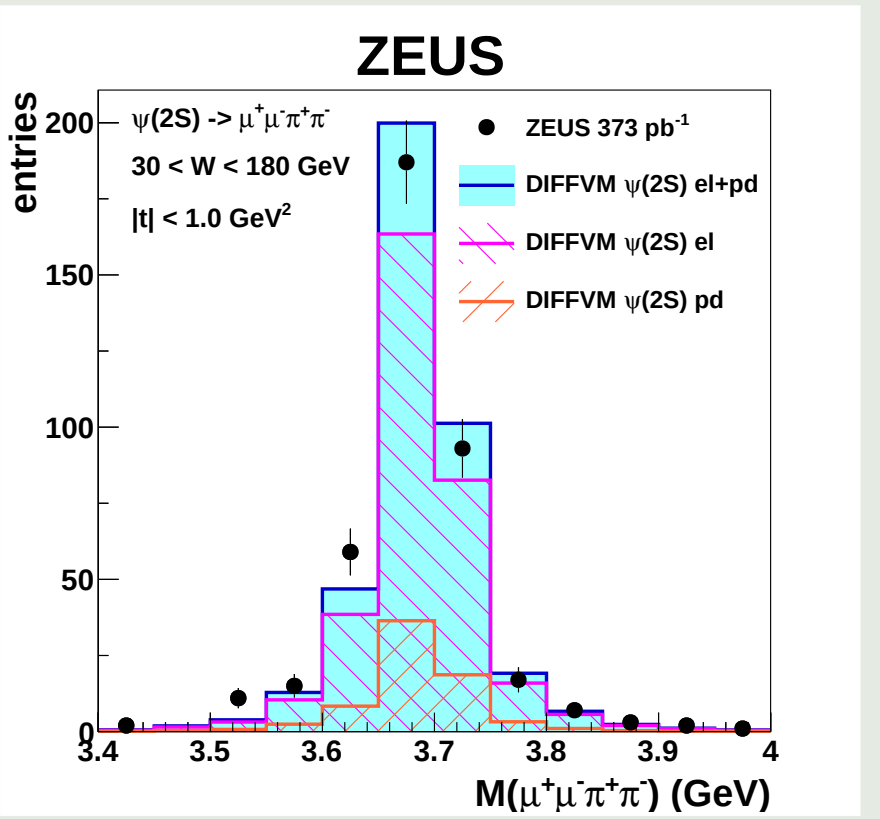
$M(\mu^+\mu^-)$



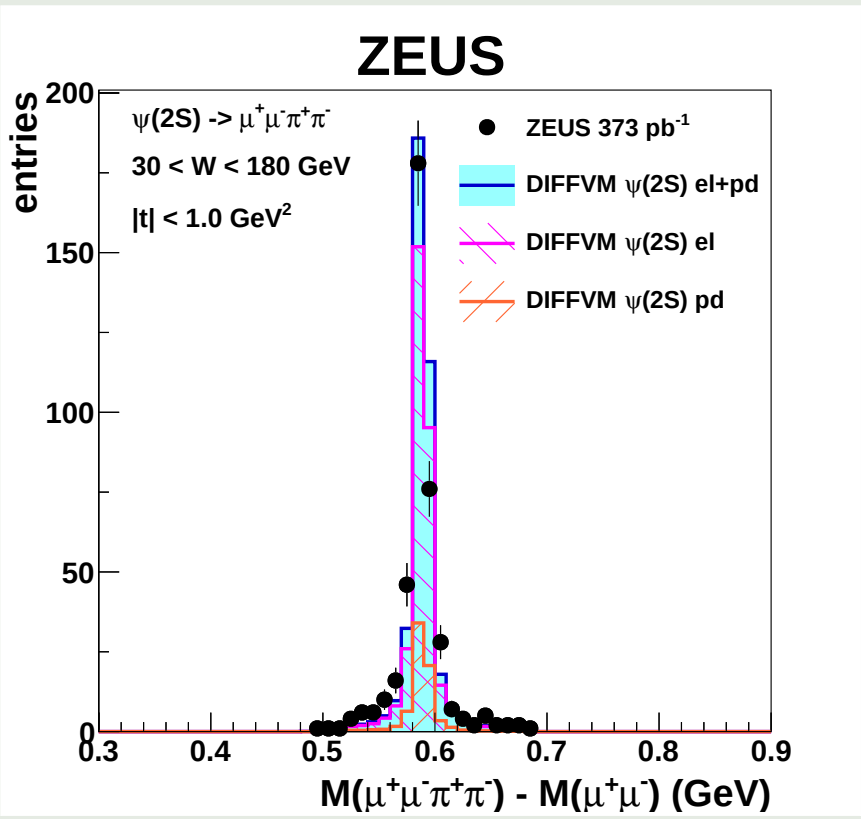
- **t5 bin:** $30 < W < 180$ GeV, $0.6 < |t| < 1.0$ GeV^2
- higher $|t|$, small Bethe-Heitler continuum $\mu^+\mu^-$ contribution
- **BUT: high contamination proton dissotiatoive events** $\rightarrow t-$spectra

4-prongs: mass spectra: $\psi(2S) \to \mu^+\mu^-\pi^+\pi^-$

$M(\mu^+\mu^-\pi^+\pi^-)$



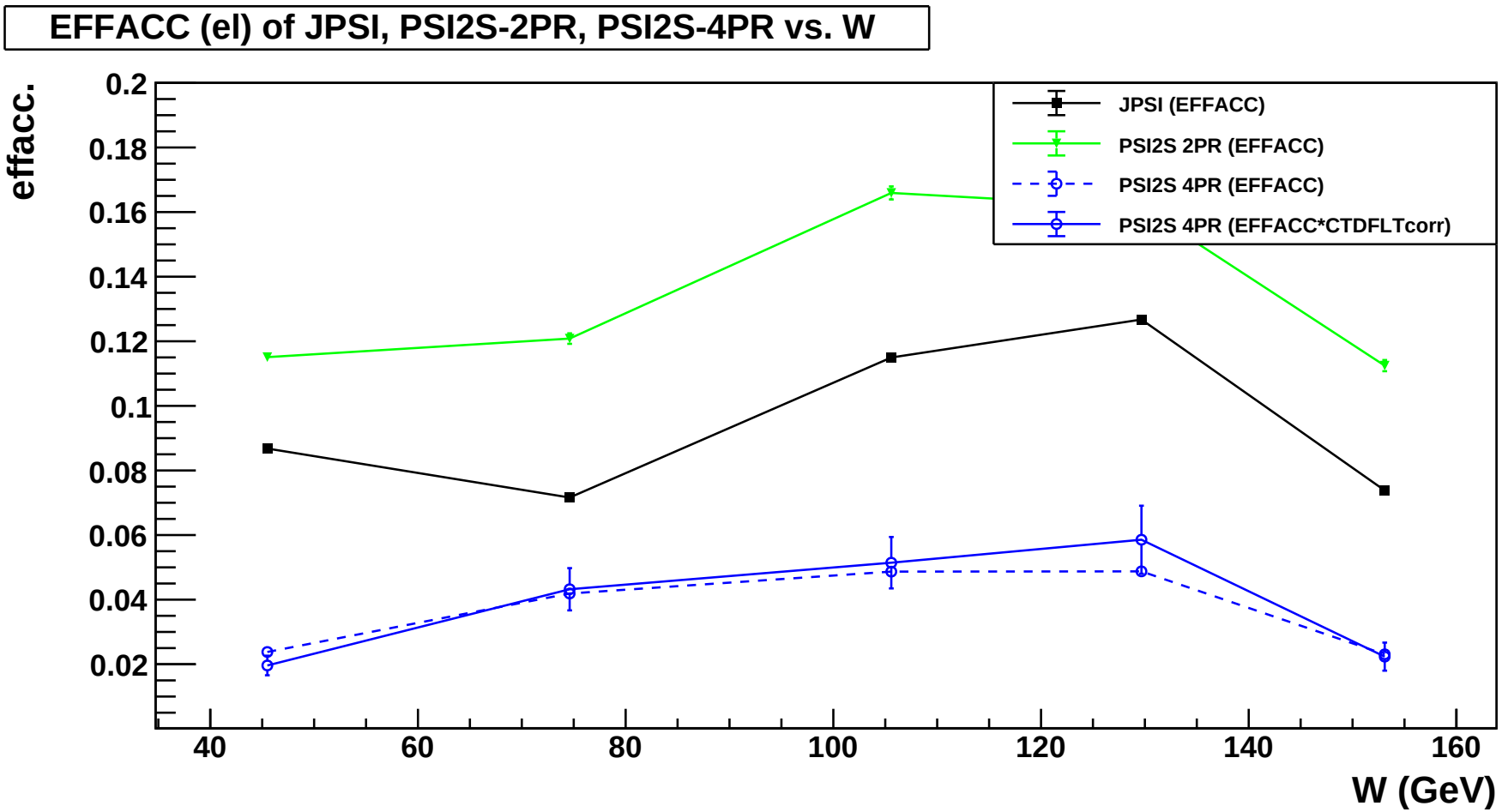
$M(\mu^+\mu^-\pi^+\pi^-) - M(\mu^+\mu^-)$



- events yield: ~ 400 $\psi(2S)$
- better resolution on mass difference $\to$ cascade decay of $\psi(2S)$
- proton dissociative fraction: $f_{p.diss} = 0.16 \pm 0.01$ from t-spectra fit
- **almost background free channel**

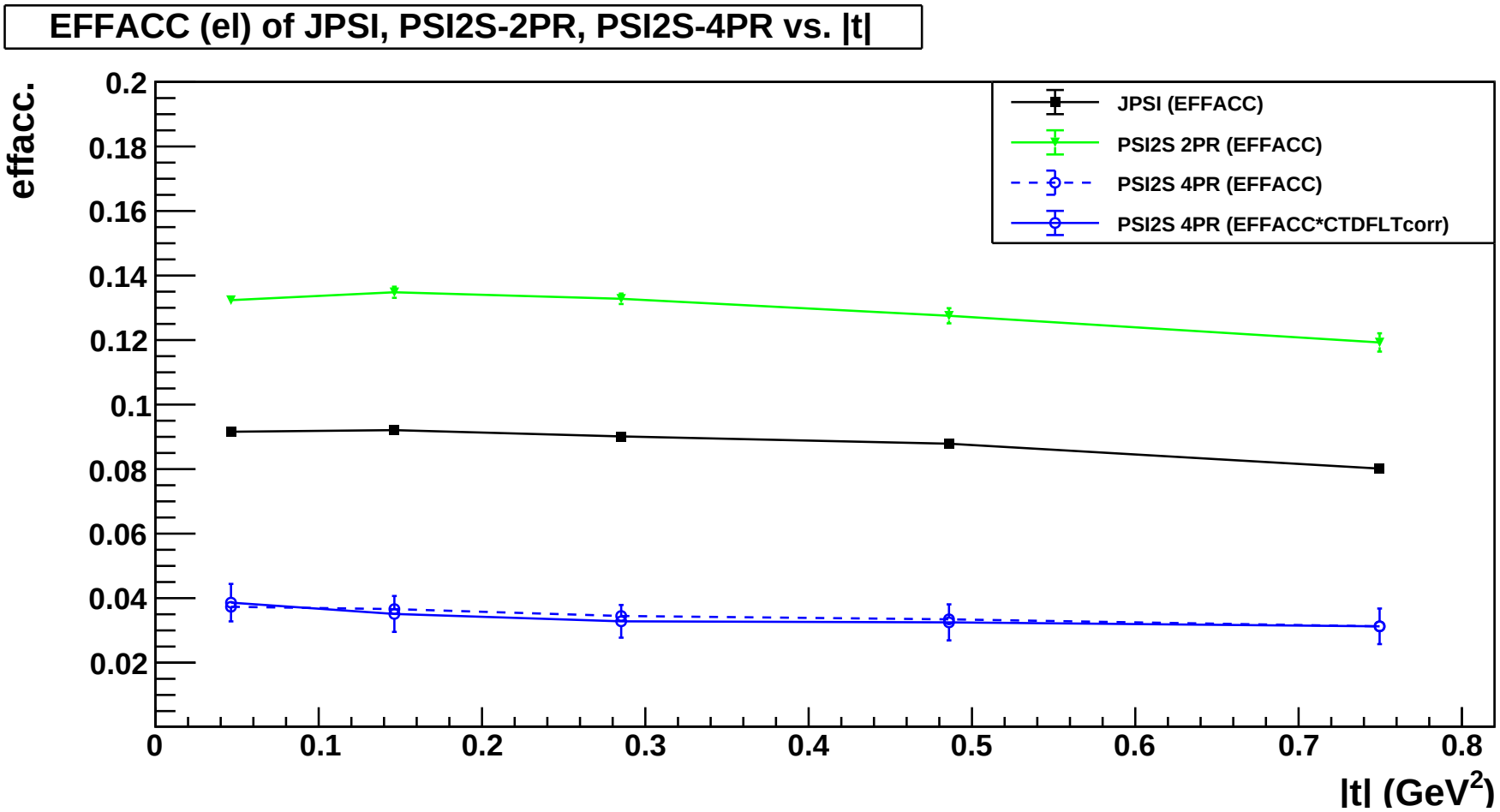
- **selection efficiency**

Acceptance*efficiency in W bins: elastic



- JPSI, PSI2S 2- and 4-prong (**2 ÷ 16%**)
- Higher di-muon acceptance for higher mass state (PSI2S)
- similar angular coverage for final state di-muons
- second W bin (W2) is the "dip" for di-muon acceptance

Acceptance*efficiency in $|t|$ bins: elastic



- JPSI, PSI2S 2- and 4-prong (**4 ÷ 14%**)
- Higher di-muon acceptance for higher mass state (PSI2S)
- flat in $|t|$ (no angular correlations to $|t|$)
- dashed line after CTD FLT corrections

- **Cross section ratio**

Cross section ratio $R = \frac{\sigma(\psi(2S))}{\sigma(J/\psi(1S))}$, full kinematic range

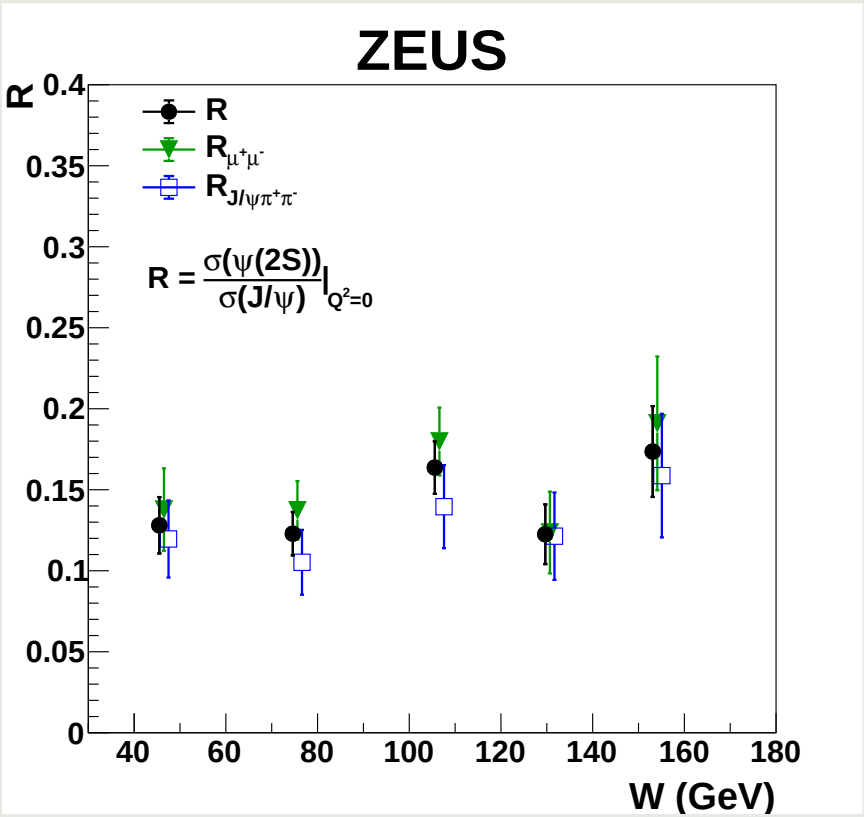
$30 < W < 180$ GeV, $|t| < 1.0$ GeV^2, $Q^2 < 1.0$ GeV^2

$\psi(2S)$ decay mode	$R = \frac{\sigma(\psi(2S))}{\sigma(J/\psi(1S))}$
$\mu^+\mu^-$	0.154 ± 0.012
$J/\psi(\to \mu^+\mu^-)\pi^+\pi^-$	0.125 ± 0.019
combined	$0.146 \pm 0.010^{+0.016}_{-0.020}$

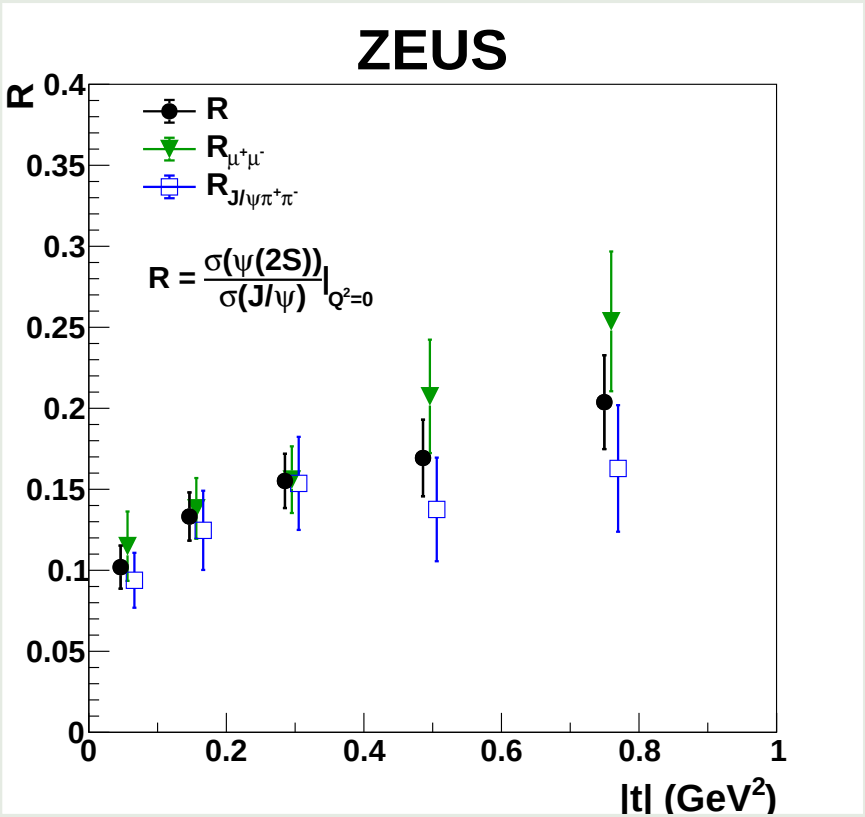
- $R_{J/\psi\pi\pi} = \frac{N_{\psi(2S)}}{N_{J/\psi(1S)}} \cdot \frac{Acc_{J/\psi(1S)\to\mu^+\mu^-}}{Acc_{\psi(2S)\to J/\psi\pi^+\pi^-}} \cdot \frac{1}{BR_{\psi(2S)\to J/\psi\pi^+\pi^-}} \cdot \frac{1-f_{pdiss}^{\psi(2S)}}{1-f_{pdiss}^{J/\psi(1S)}}$
- $R_{\mu\mu} = \frac{N_{\psi(2S)}}{N_{J/\psi(1S)}} \cdot \frac{Acc_{J/\psi(1S)\to\mu^+\mu^-}}{Acc_{\psi(2S)\to\mu^+\mu^-}} \cdot \frac{BR_{J/\psi(1S)\to\mu^+\mu^-}}{BR_{\psi(2S)\to\mu^+\mu^-}} \cdot \frac{1-f_{pdiss}^{\psi(2S)}}{1-f_{pdiss}^{J/\psi(1S)}}$
- $Acc_i = \frac{N_i^{reco}}{N_i^{true}}$, $f_{p.diss}^i$ - fraction of proton dissociative events
- $BR(\psi(2S) \to J/\psi\pi^+\pi^-) = (34.68 \pm 0.3)\%$, $BR(\psi(2S) \to \mu^+\mu^-) = (0.80 \pm 0.06)\%$, $BR(J/\psi \to \mu^+\mu^-) = (5.961 \pm 0.033)\%$, $BR(\psi(2S) \to \mu^+\mu^-\pi^+\pi^-) = (2.07 \pm 0.02)\%$ (PDG 2020)
- both channels have similar precision and provide consistent results

cross section ratio $R = \sigma_{\psi(2S)}/\sigma_{J/\psi(1S)}$: 2- and 4-prongs comparison

R vs. W



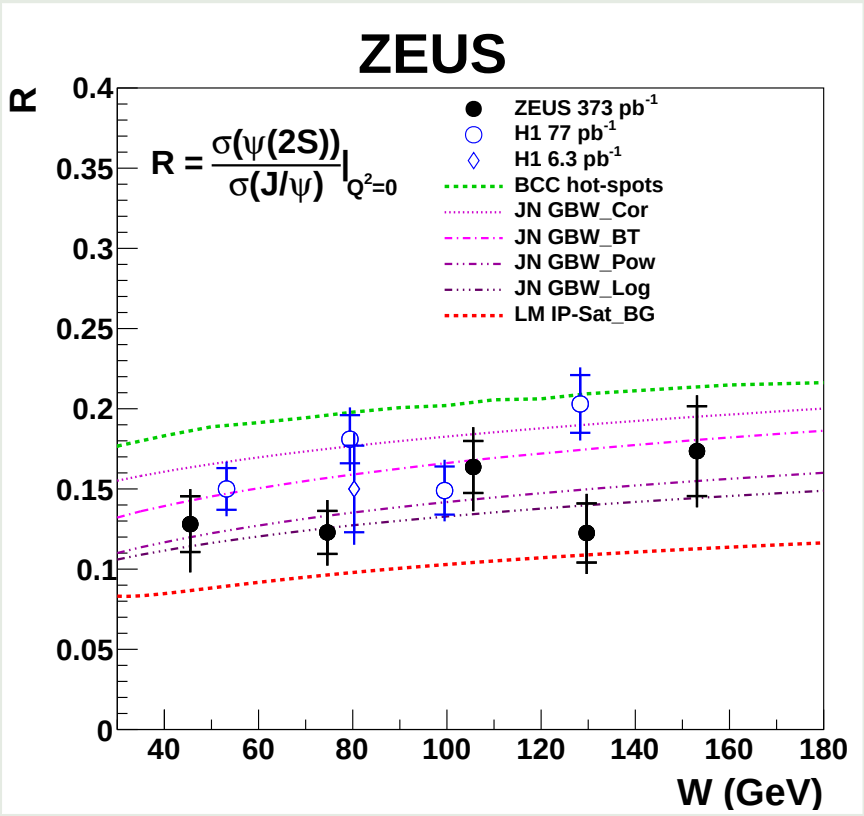
R vs. $|t|$



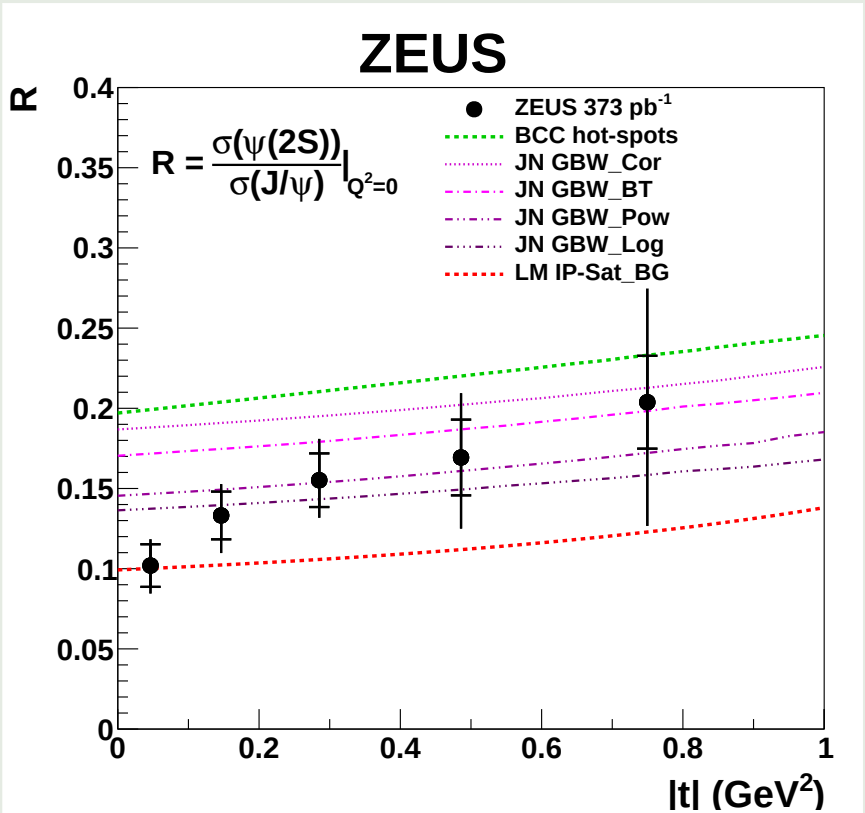
- $R_{\mu\mu}$ (2-prongs channel), $R_{J/\psi\pi\pi}$ (4-prongs channel) and combined R (full dots)
- statistical errors only
- **good agreement between two channels**

cross section ratio $R = \sigma_{\psi(2S)}/\sigma_{J/\psi(1S)}$: Final Results

R vs. W



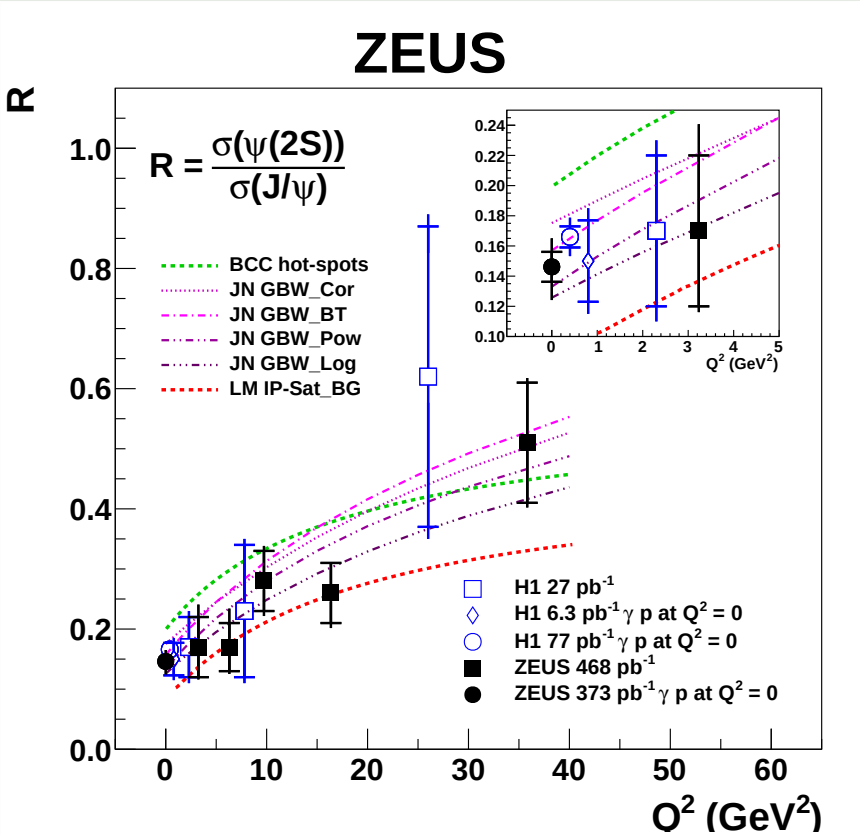
R vs. $|t|$



- for R vs. W ZEUS (full dots) and H1 (open markers) results are compared
- **no W dependence observed, moderate increase with $|t|$**
- good agreement between data and theoretical models (see next page)
- errors at high-$|t|$ points dominated by systematics ($\rightarrow$ proton dissociative fraction)

cross section ratio $R = \sigma_{\psi(2S)}/\sigma_{J/\psi(1S)}$: Final Results

R vs. Q^2

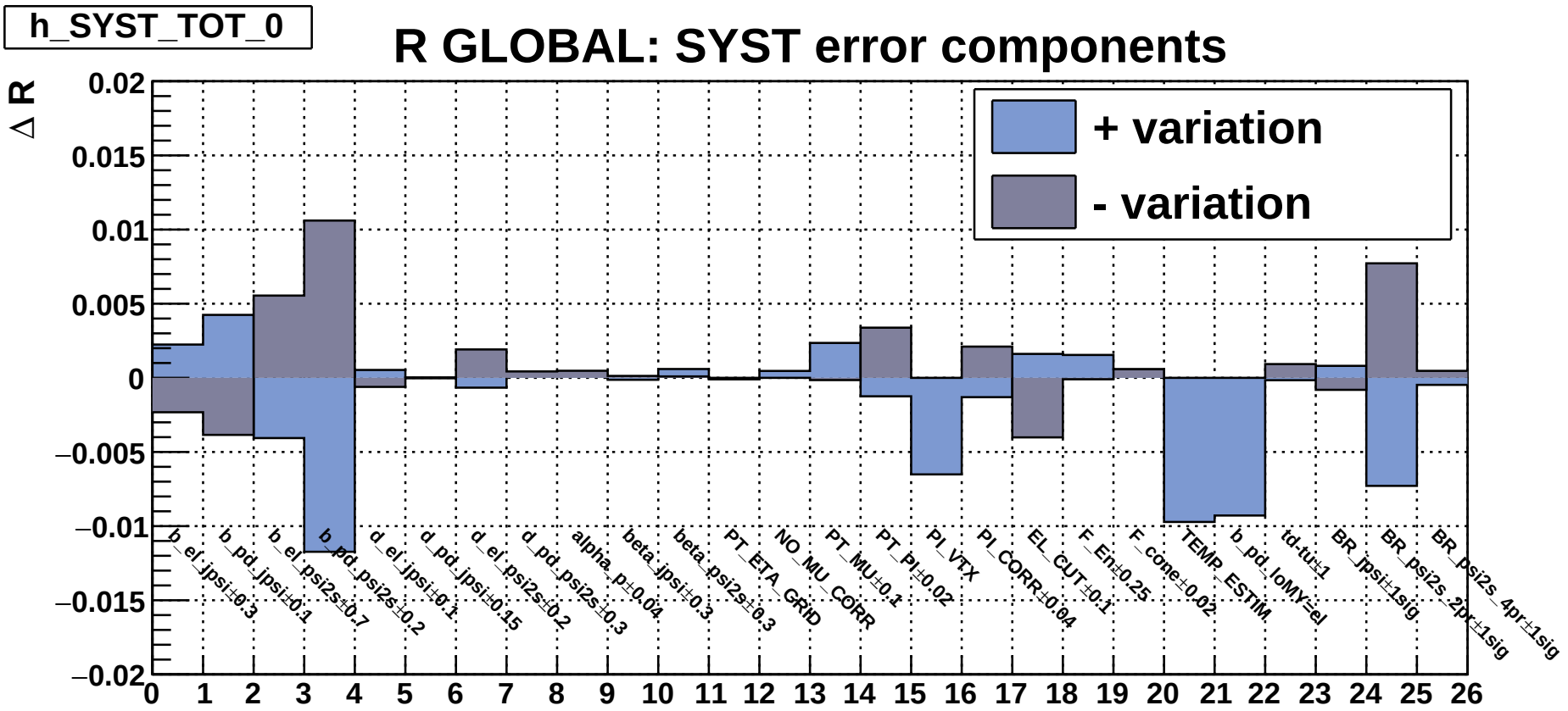


theoretical models:

- Bendova, Cepila and Contreras (BCC hot-spots):
  - Phys. Rev. **D 99**, 034025 (2019).
- Jan Nemchik et al. (JN):
  - Eur. Phys. J. **C 79**, 154 (2019).
  - Eur. Phys. J. **C 79**, 495 (2019).
  - Phys. Rev. **D 103**, 094027 (2021).
- Lappi and Mäntysaari (LM IP-Sat):
  - Phys. Rev. **C 83**, 065202 (2011).
  - Phys. Rev. **D 87**, 034002 (2013).
  - PoS (DIS2014), 069 (2014).

- ZEUS (full dot) and H1 (open markers) photoproduction results plotted at $Q^2 \sim 0$
- DIS results are also presented vs. Q^2: ZEUS (full squares) and H1 (open squares)
- good agreement between data and theoretical models (→ backup plots, page 52)
- **better precision of photoproduction points**

R: components of global systematic error



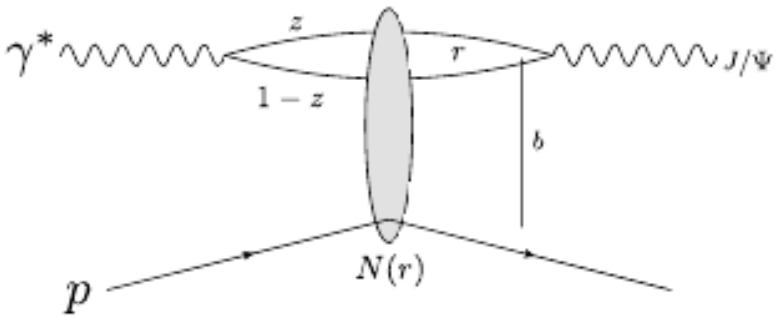
- biggest contributions from:
- b-slope variation of t-dependence (esp. for b_{pd} of $\psi(2S)$) $\rightarrow$ **p.diss fraction**
- event number estimator (MC templates fit instead of Gaussian fit) $\rightarrow$ **BH bckgr.**
- slow pions vertexing $\rightarrow$ **very soft tracks reconstruction**
- $BR(\psi(2S) \rightarrow \mu^+\mu^-) \rightarrow$ **PDG world average...**

- **Theoretical Models: Color Dipole**

Fluctuations and diffractive vector meson production

High energy factorization:

1. $\gamma^* \to q\bar{q}$ splitting, wave function $\Psi^{\gamma}(r, Q^2, z)$
2. $q\bar{q}$ dipole scatters elastically: $N(r, x, b)$
3. $q\bar{q} \to J/\Psi$, wave function $\Psi^{V}(r, Q^2, z)$



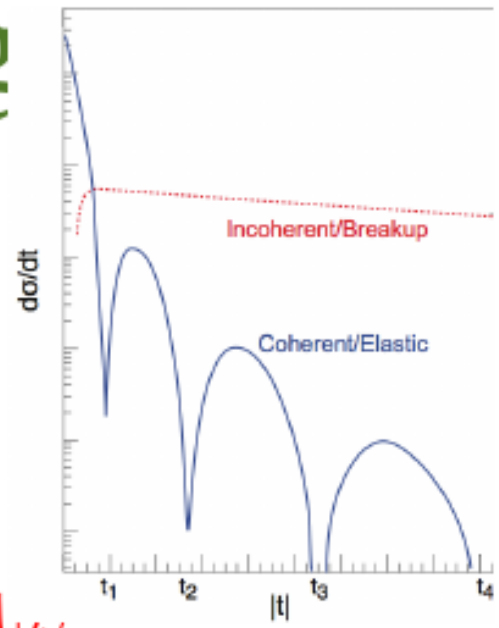
Diffractive scattering amplitude

$$\mathcal{A} \sim \int \mathrm{d}^2 b \mathrm{d} z \mathrm{d}^2 r \Psi^{\gamma *} \Psi^{V}(r, Q^2, z) \mathbf{e}^{-\mathbf{ib}\cdot\mathbf{\Delta}} N(r, x, b)$$

- Impact parameter is the Fourier conjugate to the momentum transfer $(-t \approx \Delta^2) \to$ **access to the spatial structure**

This is LO, see Jani's talk for NLO corrections!

Incoherent Scattering



Good, Walker (Phys. Rev. 120 (1960) 1857–1860):

Proton dissociates ($f \neq i$):

$$\sigma_{\text{incoherent}} \propto \sum_{f \neq i} \langle i|\mathcal{A}|f\rangle^{\dagger} \langle f|\mathcal{A}|i\rangle$$

complete set

$$= \sum_{f} \langle i|\mathcal{A}|f\rangle^{\dagger} \langle f|\mathcal{A}|i\rangle - \langle i|\mathcal{A}|i\rangle^{\dagger} \langle i|\mathcal{A}|i\rangle$$

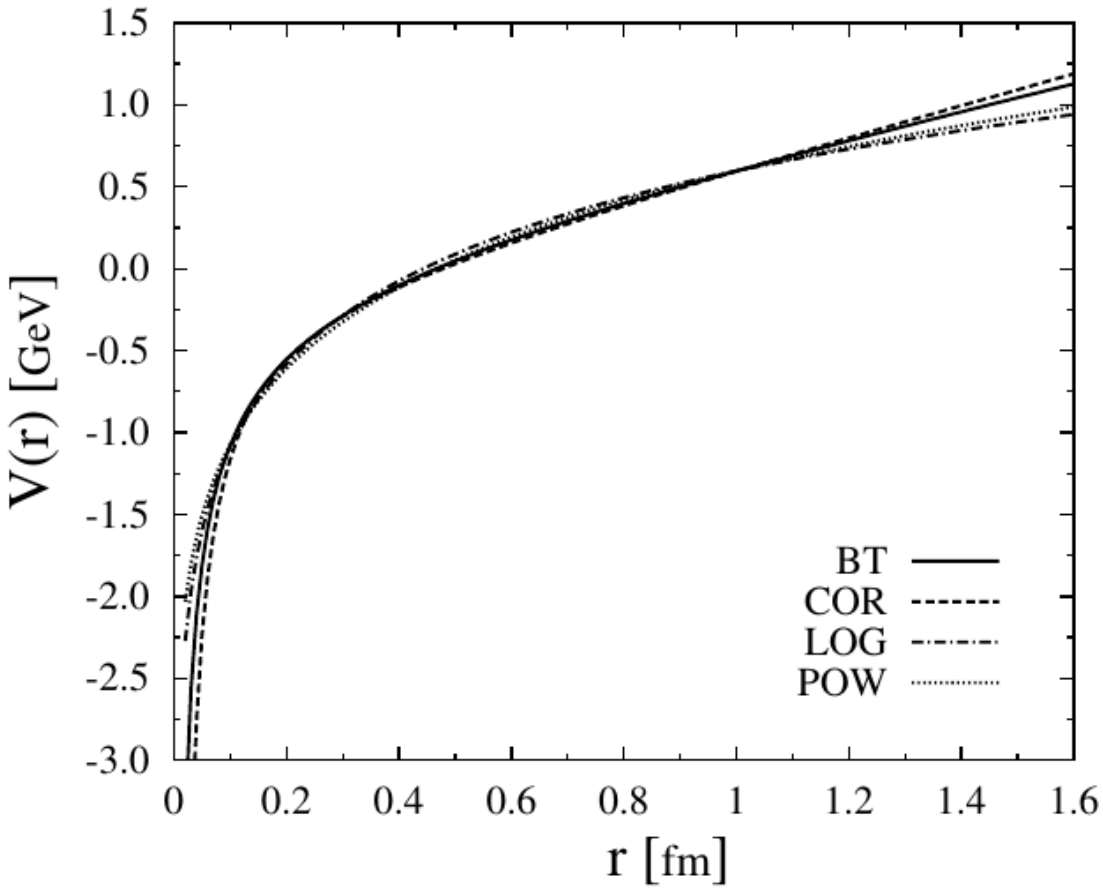
$$= \langle i\,||\mathcal{A}|^2|\,i\rangle - |\langle i|\mathcal{A}|i\rangle|^2 = \langle |\mathcal{A}|^2\rangle - |\langle \mathcal{A}\rangle|^2$$

The incoherent CS is the variance of the amplitude!!

$$\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi}\left\langle |\mathcal{A}|^2 \right\rangle$$

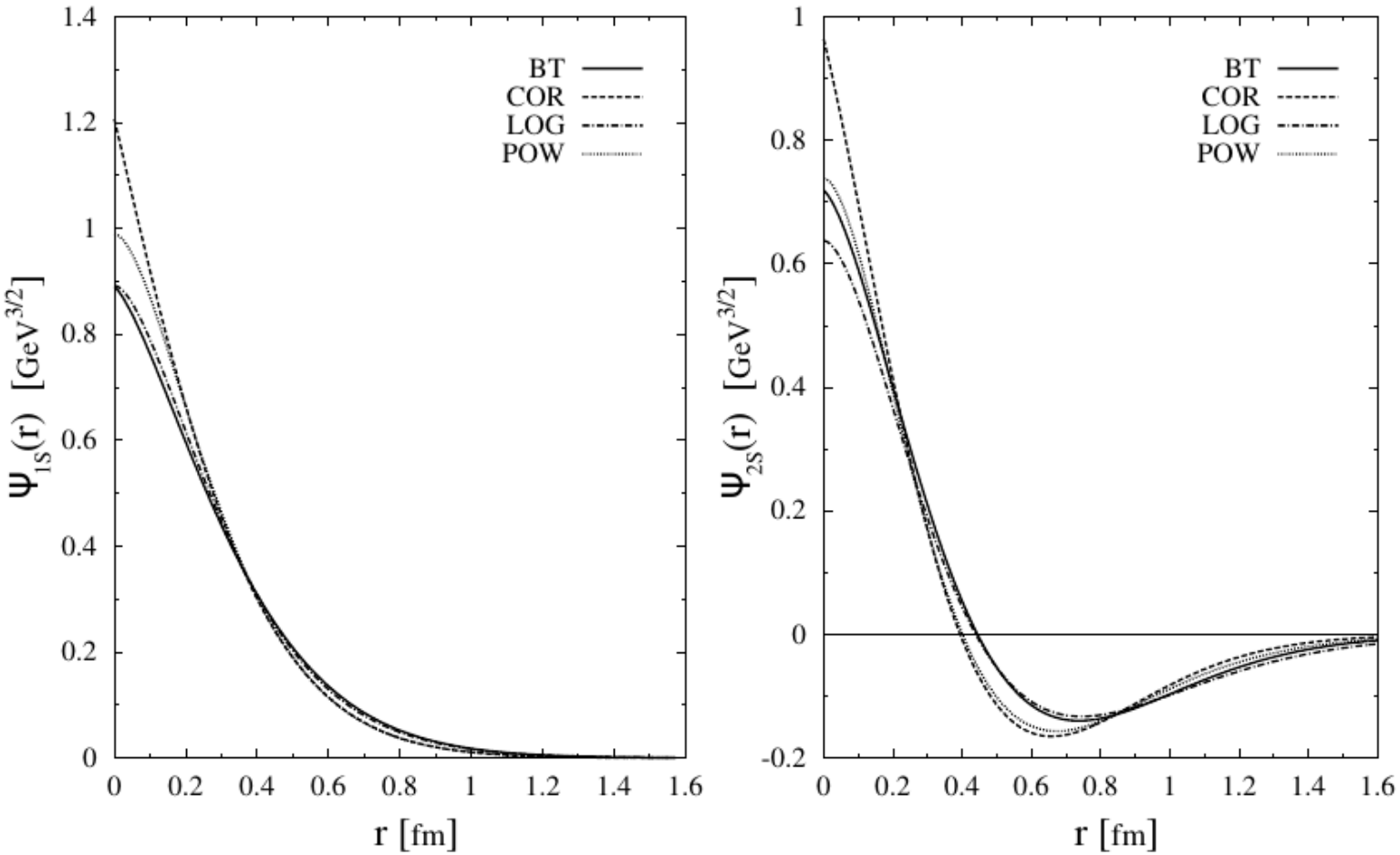
$$\frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi}|\langle \mathcal{A}\rangle|^2$$

$q\bar{q}$ potentials: modeling of quarks confinement



- **COR:** Cornell potential and **BT:** Buchmüller and Tye : $V(r) = -\frac{k}{r} + \frac{r}{a^2}$, $k = 0.52$, $a = 2.34$ GeV^{-1}, $m_c = 1.84$ GeV (**BT:** $m_c = 1.48$ GeV)
- **LOG:** Logarithmic potential: $V(r) = 0.6635\,\mathrm{GeV} + (0.733\,\mathrm{GeV})log(r \cdot 1\,\mathrm{GeV})$
- **POW:** Power-law potential: $V(r) = 8.064\,\mathrm{GeV} + (6.898\,\mathrm{GeV})(r \cdot 1\,\mathrm{GeV})^{0.1}$, $m_c = 1.8$ GeV

radial wave functions for $J/\psi(1S)$ and $\psi(2S)$



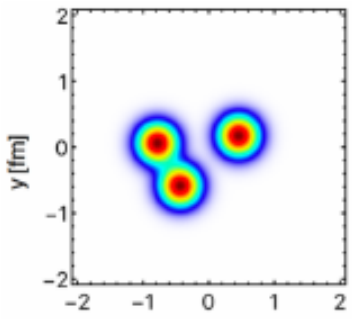
- **radial part of the wave functions** $\Psi(r)$ for $c\bar{c}$ $1S$ and $2S$ states
- $(\frac{\Delta}{m_c} + V(r))\Psi_{nlm}(\vec{r}) = E_{nl}\Psi_{nlm}(\vec{r})$, $\Psi_{nlm}(\vec{r}) = \Psi_{nl}(r) \cdot Y_{lm}(\theta, \phi)$
- **BT, COR, LOG, POW potentials**

- **Theoretical Models: Gluon Hot Spots**

Incoherent Scattering in *ep*

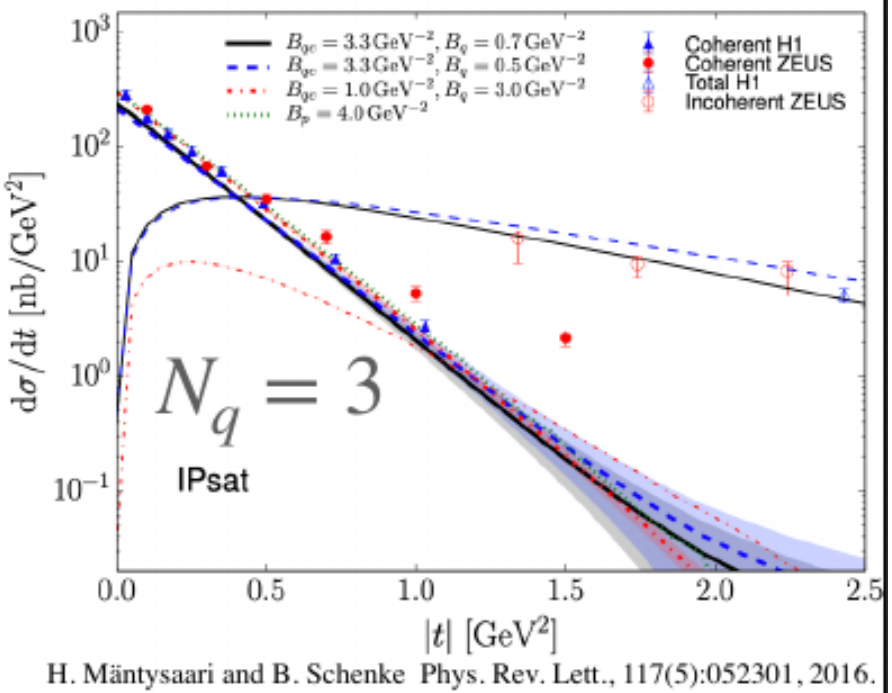
$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$

$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$



$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b} - \vec{b}_i)^2}{2B_q}}$$

$\vec{b}_i$ with a Gaussian distribution of width B_{qc}



H. Mäntysaari and B. Schenke Phys. Rev. Lett., 117(5):052301, 2016.

Also: large scale (small $|t|$) saturation scale fluctuations.

- Arjun Kumar, Tobias Toll : arXiv:2202.06631

Summary

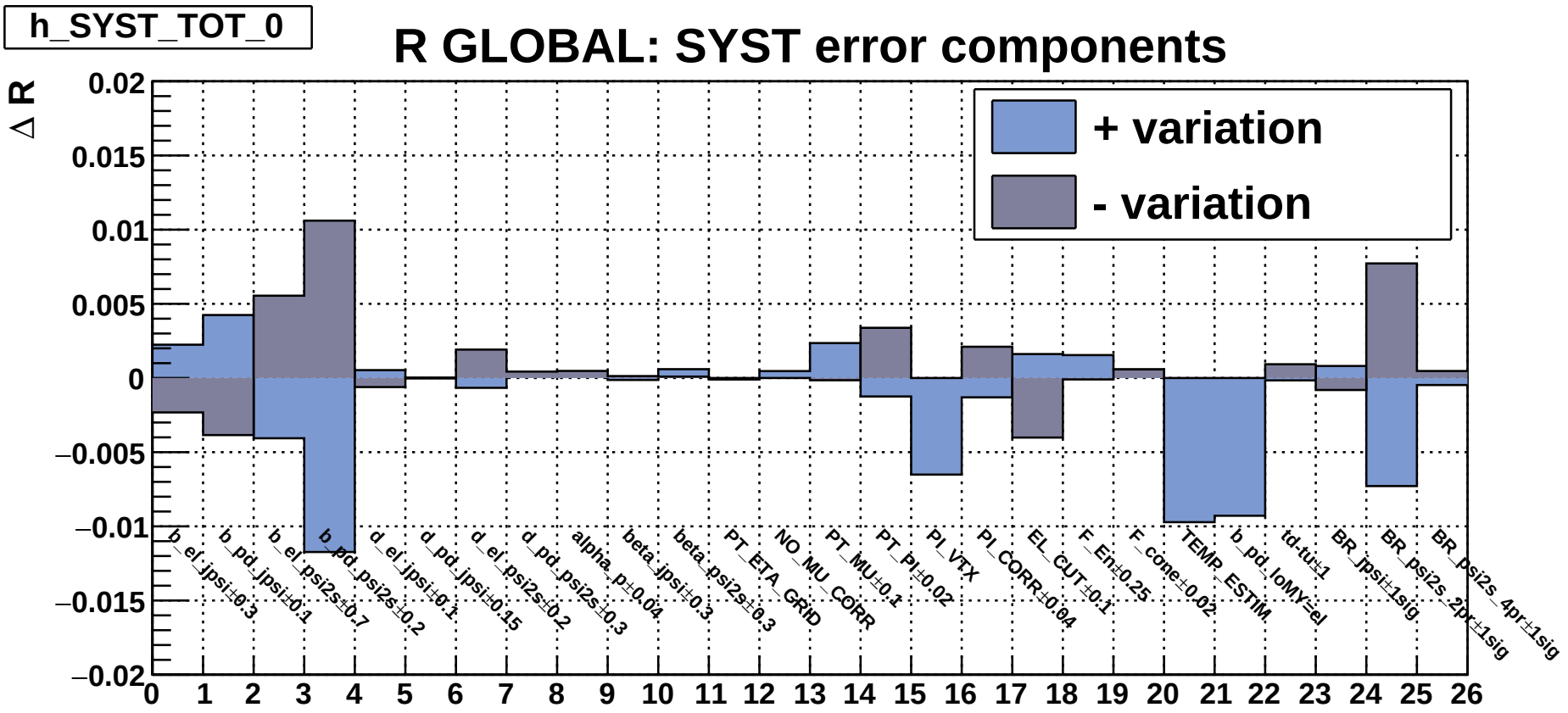
- **Cross section ratio R = $\frac{\sigma(\psi(2S))}{\sigma(J/\psi(1S))}$** in photoproduction using HERA II data was measured by ZEUS in the kinematic range: $30 < W < 180$ GeV, $|t| < 1.0$ GeV^2
- first ZEUS measurement of R in photoproduction (at $Q^2 = 0$): $R = 0.146 \pm 0.01(stat.)^{+0.016}_{-0.022}(syst.)$
- first HERA result for R vs. $|t|$ in photoproduction
- **moderate rise of cross section ratio as a function of $|t|$**
- **no W dependence observed within experimental errors**
- consistent results for 2- and 4-prongs decay channels
- comparable precision in both decay channels
- **theoretical calculations of the ratio $\frac{\sigma(\psi(2S))}{\sigma(J/\psi(1S))}$ for exclusive vector-meson production has been compared to the experimental data**
- $\rightarrow$ majority of the predictions are consistent with the data
- **data start to exhibit constraining power**

Thank You For Your Attention

BACKUP PLOTS FOLLOWS...

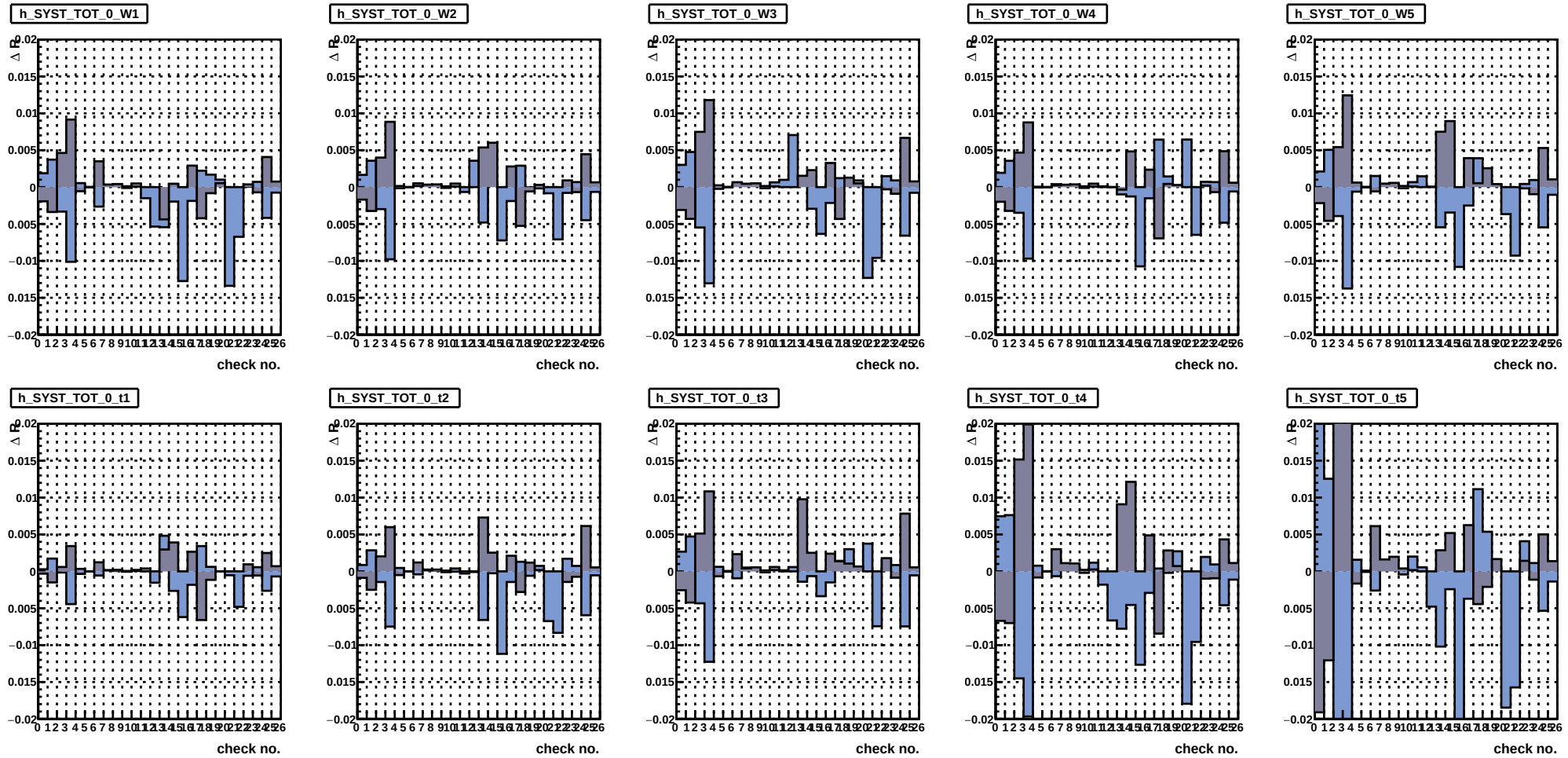
- **Systematics**

R: components of global SYSTEMATICS error



- biggest contributions from:
- b-slope variation of t-dependence (esp. for b_{pd} of $\psi(2S)$) $\rightarrow$ **p.diss fraction**
- event number estimator (MC templates fit instead of Gaussian fit) $\rightarrow$ **BH bckgr.**
- slow pions vertexing $\rightarrow$ **very soft tracks reconstruction**
- $BR(\psi(2S) \rightarrow \mu^+\mu^-) \rightarrow$ **PDG world average...**

R: components of SYST. error in W and $|t|$ bins



- upper row: contributions in 5 W bins
- bottom row: contributions in 5 $|t|$ bins
- bin order as on previous page

Selection cuts

- **Selection cuts**

Event Selection: 2-prongs sample: $J/\psi, \psi(2S) \to \mu^+\mu^-$

- Exclusive Muon Triggers (F/B/R/MUON or BAC)
- Tracking and Vertex
  - $N_{track} = 2$, oppositely charged tracks matched to the primary vertex ($\eta \in (-1.9, 1.9)$)
  - **both tracks identified as a muon in CAL, at least one in F/B/RMUON or BAC**
  - $p_T > 1.0$ GeV of each track
  - anti-COSMIC cuts (CAL timing, acolinearity: $\cos(\mu^+, \mu^-) < -0.985$)
- Elasticity/Exclusivity and Photoproduction cuts (on CAL Energy)
  - no scattered electron found in CAL
  - $E_{\mathrm{clu}} < 0.5$ GeV for clusters not matched to muons (or pions)
    (corresponds to an effective cut on $Q^2 < 1$ GeV^2)
  - $E(\theta < 0.12 rad) < 1$ GeV
    the sum of the energy in the FCAL cone around the beam-pipe;
    to suppress proton-dissociative events, $ep \to e + VM + Y$
    (corresponds to a requirement for $M_Y \lesssim 5$ GeV)
- Kinematic range (analysis phase space):
  - $30 < W < 180$ GeV
  - $|t| < 1.0$ GeV^2
  - $Q^2 < 1$ GeV^2 (median $Q^2 \approx 3 \times 10^{-5}$ GeV^2)

Event Selection: 4-prongs sample: $\psi(2S) \to \mu^+\mu^-\pi^+\pi^-$

- (only differences w.r.t. the 2-prong channel)
- $N_{track} = 4$, (two oppositely charged pairs, sorted by p_T)
- highest momentum pair: muon candidates
  lowest momentum pair: pion candidates
- no anti-COSMIC cuts
- transverse momentum of pion candidates: $p_T^{\pi} > 0.12$ GeV;
- $2.8 < M(\mu^+\mu^-) < 3.4$ GeV (J/ψ window)
- $M(\mu^+\mu^-\pi^+\pi^-) - M(\mu^+\mu^-)$ in $(0.5 - 0.7)$ GeV window
  (**cascade decay of** $\psi(2S)$)

- **Theoretical Models**

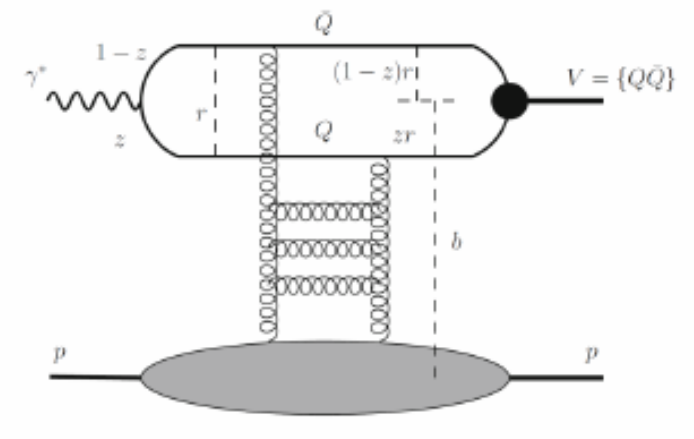
- predictions of all models were calculated within the phase space of this analysis:
  - $30 < W < 180$ GeV
  - $|t| < 1.0$ GeV^2
  - $Q^2 \sim 0$ (photoproduction points)

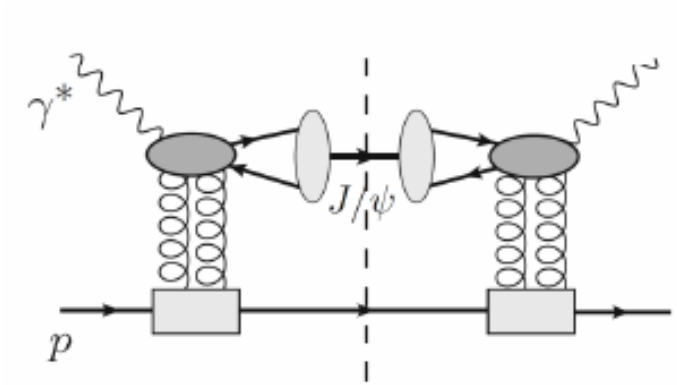
Theory predictions (1): Color Dipole Models

- **Jan Nemchik (JN) et al. :**
- Eur. Phys. J. C **79**, no.6, 495 (2019).
- Eur. Phys. J. C **79**, no.2, 154 (2019).
- calculations have been performed for various combinations of quarkonium wave functions:
  - **Cor** (Cornell potential)
  - **BT** (Buchmüller-Tye)
  - **Pow** (Power-law potential)
  - **Log** (Logarithmic potential)

  and models for the dipole cross sections:
  - BGBK, **GBW** $\leftarrow$ **used on the plots**
  - for each combinations calculations are performed with and w/o skewness in the gluon density
- the same b-slope parameters for both quarkonium states

Color dipole formalism for VM





Amplitude:

$$\mathcal{A}^{\gamma^* p \to V p}(x, Q^2, \vec{q}) = \langle V|\tilde{\mathcal{A}}|\gamma^*\rangle = \int d^2r \int_0^1 d\alpha\, \Psi_V^*(\vec{r},\alpha)\, \mathcal{A}_{Q\bar{Q}}(\vec{r}, x, \alpha, \vec{q})\, \Psi_{\gamma^*}(\vec{r},\alpha,Q^2)$$

t-dependent differential cross section:

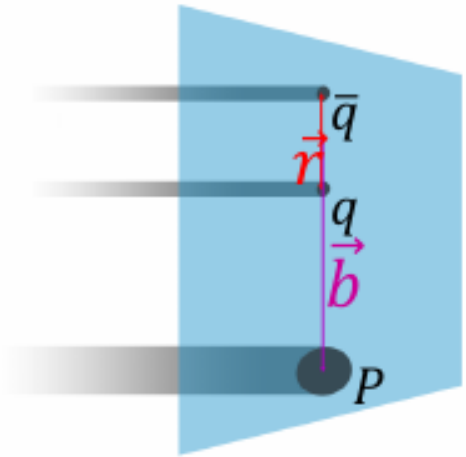
$$\frac{d\sigma^{\gamma^* p \to V p}(s, Q^2, t=-q^2)}{dt} = \frac{1}{16\,\pi} \left|\mathcal{A}^{\gamma^* p \to V p}(s, Q^2, \vec{q})\right|^2$$

Dipole cross section

- Describes the interaction of $Q\bar{Q}$ with a proton
- Nonperturbative effects, **no theoretically calculable**
- Just models
  - Nevertheless, qualitatively, we have ideas, what is going inside
  - In the perturbative area described by the gluon distribution function
- Various models on the market: GBW, KST, IP-Sat, BGBK, BK, ...
- Usually, they are fitted from DIS data, mostly from HERA
- However, such a fit is integrated over impact parameter $\vec{b}$
- For t-dependence, **we need a b-dependent dipole cross section (amplitude)**

More in B.Kopeliovich, MK, J.Nemchik, Phys.Rev.D 103 (2021) 9, 094027

$\vec{b}$-$\vec{r}$ correlation



This is the case of maximal contribution.

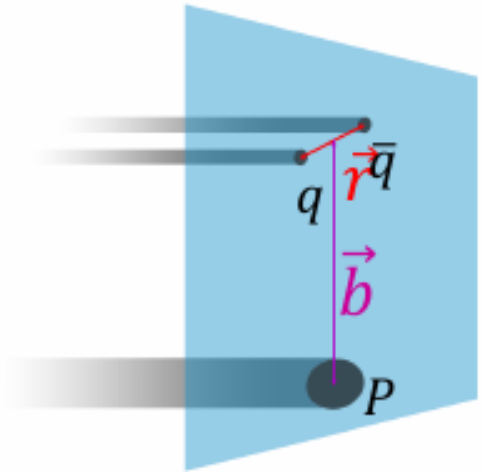
$\vec{b} \parallel \vec{r}$ is simpler to calculate, no angle dependence.

Solution: Inspired by the Born approximation with two-gluon exchange

In reality, the angle between $\vec{b}$-$\vec{r}$ can be arbitrary.

One should integrate over all possibilities.

This is a challenge e.g. for BK. So far, only the $\vec{b} \parallel \vec{r}$ approximation was used.

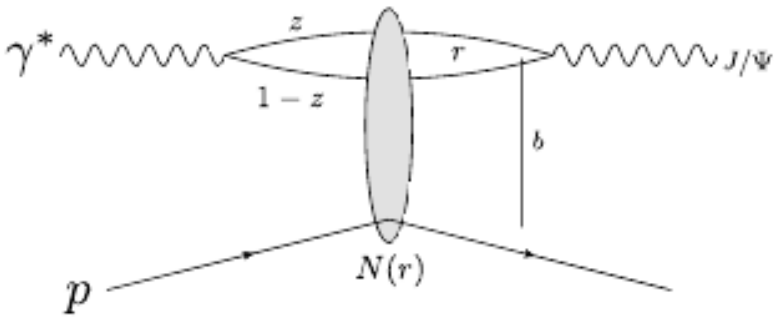


More in B.Kopeliovich, MK, J.Nemchik, Phys.Rev.D 103 (2021) 9, 094027 & DIS2021

Fluctuations and diffractive vector meson production

High energy factorization:

1. $\gamma^* \to q\bar{q}$ splitting, wave function $\Psi^{\gamma}(r, Q^2, z)$
2. $q\bar{q}$ dipole scatters elastically: $N(r, x, b)$
3. $q\bar{q} \to J/\Psi$, wave function $\Psi^{V}(r, Q^2, z)$



Diffractive scattering amplitude

$$\mathcal{A} \sim \int \mathrm{d}^2 b \mathrm{d} z \mathrm{d}^2 r \Psi^{\gamma *} \Psi^{V}(r, Q^2, z) \mathbf{e}^{-\mathbf{ib}\cdot\mathbf{\Delta}} N(r, x, b)$$

- Impact parameter is the Fourier conjugate to the momentum transfer $(-t \approx \Delta^2) \to$ **access to the spatial structure**

This is LO, see Jani's talk for NLO corrections!

Diffractive vector meson production

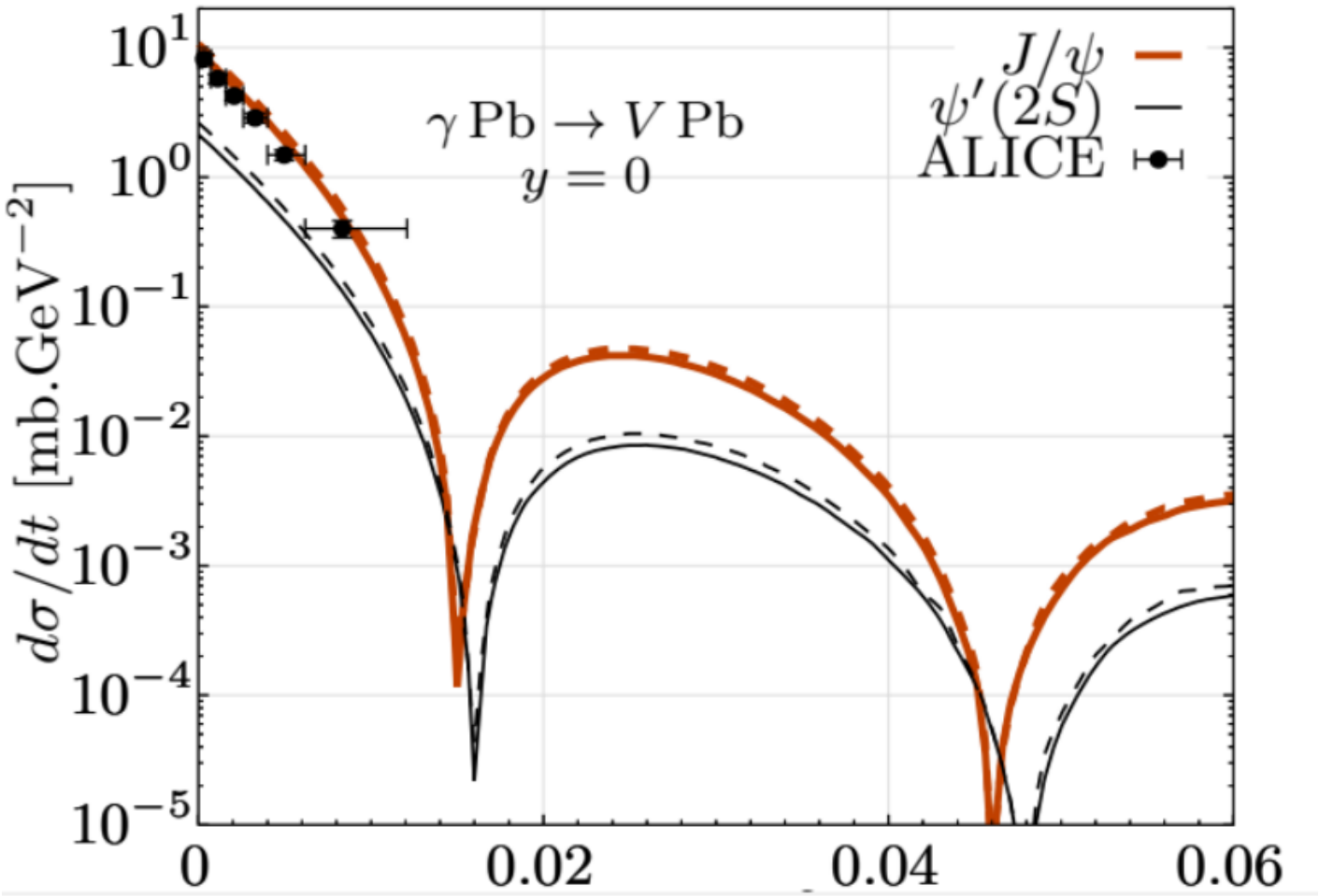
Coherent	Incoherent
$\sigma_{\text{coherent}} \sim \lvert\langle\mathcal{A}\rangle_\Omega\rvert^2$	$\sigma_{\text{incoherent}} \sim \langle\lvert\mathcal{A}\rvert^2\rangle_\Omega - \lvert\langle\mathcal{A}\rangle_\Omega\rvert^2$
• Proton stays intact	• Proton dissociates
Probes the average interaction $\Rightarrow$ average shape	Event-by-event fluctuations in the proton structure

- Experimental signature: rapidity gap
- Theoretically: no net color transfer
- Average over target configurations Ω at amplitude/cross section level

$$\mathcal{A}^{\gamma^* p \to Vp} \sim \int \mathrm{d}^2\mathbf{b}\mathrm{d}z\mathrm{d}^2\mathbf{r}\Psi^{\gamma^*}\Psi^{V}(|\mathbf{r}|, z, Q^2)\mathrm{e}^{-\mathrm{i}\mathbf{b}\cdot\mathbf{\Delta}}N(|\mathbf{r}|, x, \mathbf{b}, \Omega)$$

Miettinen, Pumplin, PRD 18, 1978; Caldwell, Kowalski, 0909.1254; H.M, Schenke, 1603.04349; H.M, 2001.10705

$d\sigma/dt$: example from Alice



- $\sim$ Fourier transform of the spatial gluon density distribution
- higher maxima hidden by proton-dissotiative background
- excellent resoution for t reconstruction needed !

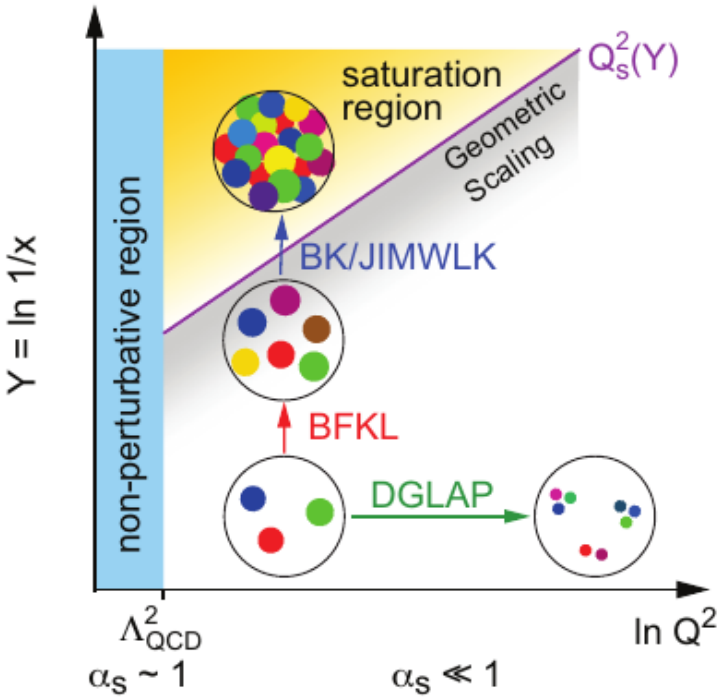
Theory predictions (2): Color Dipole Models

- **Lappi and Mntysaari (LM IP-Sat) :**
- the BFKL evolution plus the IP-Sat model to predict vector-meson production in ep and electronion collisions in the dipole picture
- 2S parameters from arXiv:1406.2877 (PoS DIS2014 (2014) 069)
- 1S parameters from hep-ph/0606272 (Phys.Rev. **D74** (2006) 074016)
- Calculation described in (Phys.Rev. **C83** (2011) 065202)
- **IP-Sat** dipole from fit (Phys.Rev. **D87** (2013) no.3, 034002)
- Wave function: Boosted Gaussian (**BG**), $Q^2 = 0$ GeV^2
- Skewedness and real part corrections included

Theory predictions (3): Gluon Hot-Spots Model(s)

- **Bendova, Cepila and Contreras (BCC hot-spots) :**
- Phys. Rev. D **99**, 034025 (2019).
- model with hot spots randomly sampled in the transverse plane bound by the size of the proton
- The slope parameter b is 4.72 GeV^{-2} and it is fixed by the combined H1 and ZEUS data from 2013 for JPsi photoproduction t-distribution.
- the same b-slope for both JPsi and Psi2s

Goal: confront linear vs. non-linear QCD evolution



kernel calculated in pQCD

BK evolution for dipole amplitude $N(x,r) \in [0,1]$

[related to gluon distribution]

$$\frac{dN(x,r)}{d\ln\frac{1}{x}} = \int d^2\boldsymbol{r}_1 K(\boldsymbol{r},\boldsymbol{r}_1)\left[N(x,r_1)+N(x,r_2)-N(x,r)-N(x,r_1)N(x,r_2)\right]$$

linear BFKL evolution = subset of complete BK

non-linear term relevant for N~1 (=high density)

- Nuclear Amplitude N(x,r)

Modelling x-dependence: 1. The Proton's Size

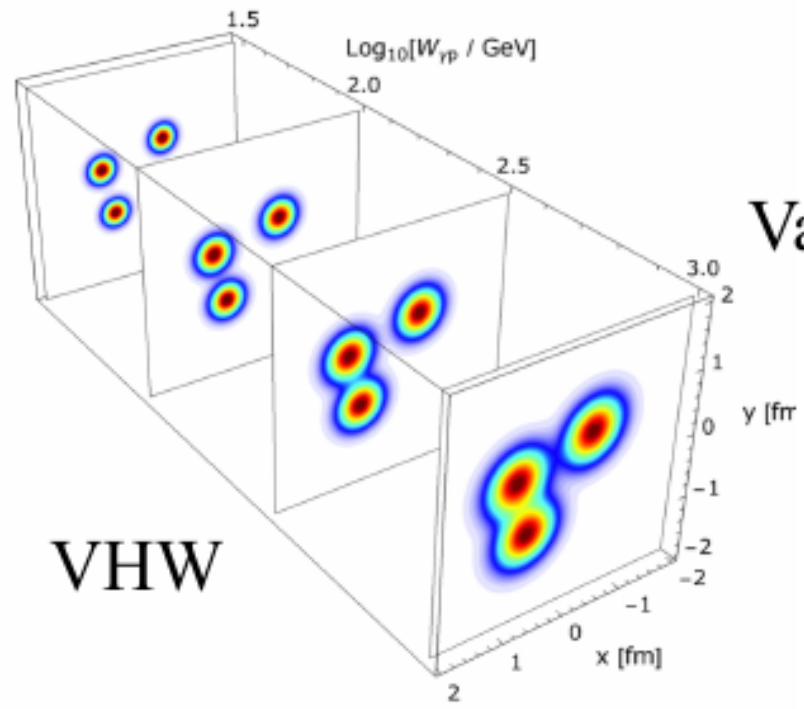
$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$B_G(x_{I\!P}) = B_p x_{I\!P}^{\lambda_p}$$

$$r_{\rm rms} = \sqrt{2B_G(x_{I\!P})}$$

- Arjun Kumar, Tobias Toll : arXiv:2202.06631

Modelling x-dependence: 2. The Hotspot Size



$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q}}$$

Variable Hotspot Width (VHW):

$$B_q(x_{I\!P}) = B_{hs} x_{I\!P}^{\lambda_{hs}}$$

Logarithmic model:

$$B_q(x_{I\!P}) = b_0 \ln^2 \frac{x_0}{x_{I\!P}}$$

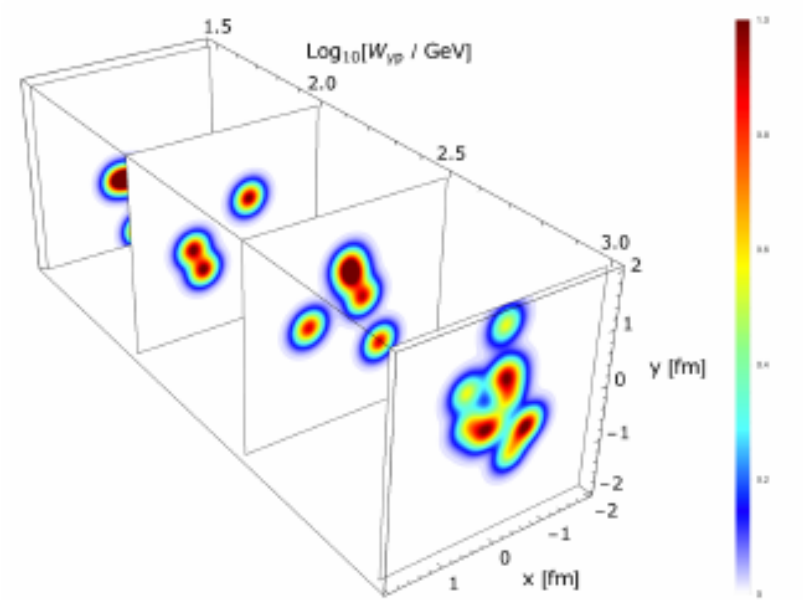
$$r_{\mathrm{rms}} = \sqrt{2(B_{qc} + B_q(x_{I\!P}))}$$

- Arjun Kumar, Tobias Toll : arXiv:2202.06631

Modelling x-dependence: 3. Number of Hotspots

$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q}}$$

$$N_q \to N_q(x_{I\!P}) = p_0\, x_{I\!P}^{p_1}(1+p_2\sqrt{x_{I\!P}})$$



J. Cepila, J. G. Contreras, J. D. Tapia Takaki,
Energy dependence of dissociative J/ψ photoproduction as a signature of gluon saturation at the LHC,
Phys. Lett. B 766 (2017) 186–191.

$$p_0 = 0.011,\ p_1 = -0.56,\ p_2 = 165$$

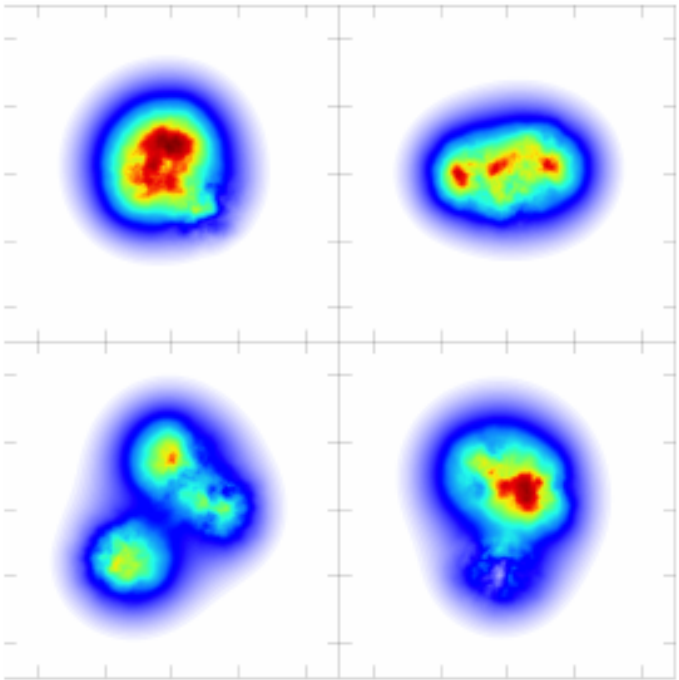
- Arjun Kumar, Tobias Toll : arXiv:2202.06631

Going beyond a round proton

Motivation

- How does the proton geometry fluctuate event-by-event?
- How accurately can we constrain the proton shape fluctuations, and how do these uncertainties propagate from HERA to e.g. flow@LHC
  $\Rightarrow$ Bayesian analysis

$\Rightarrow$

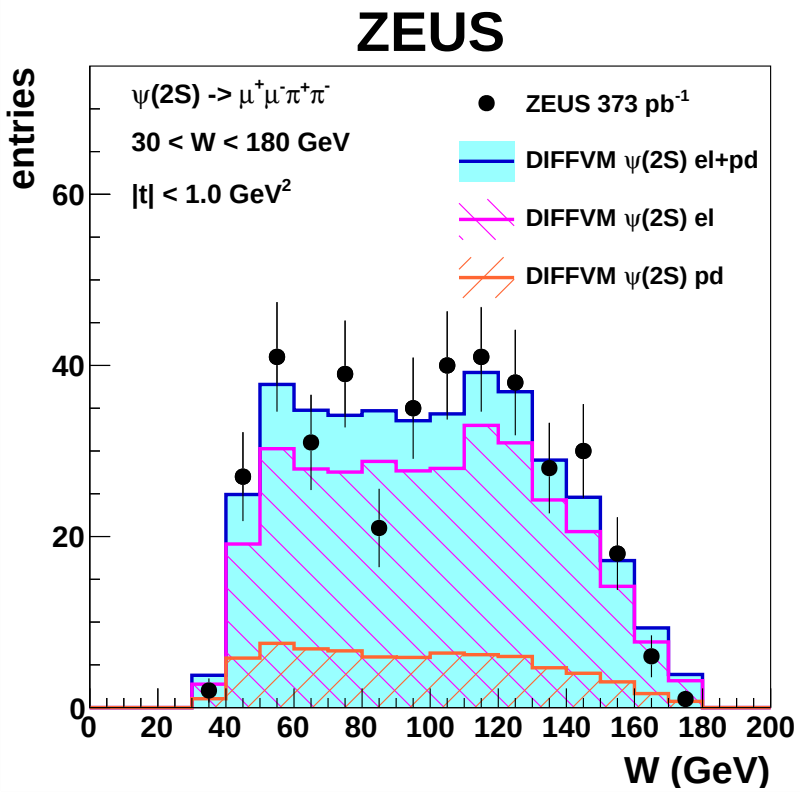


More control plots, ...

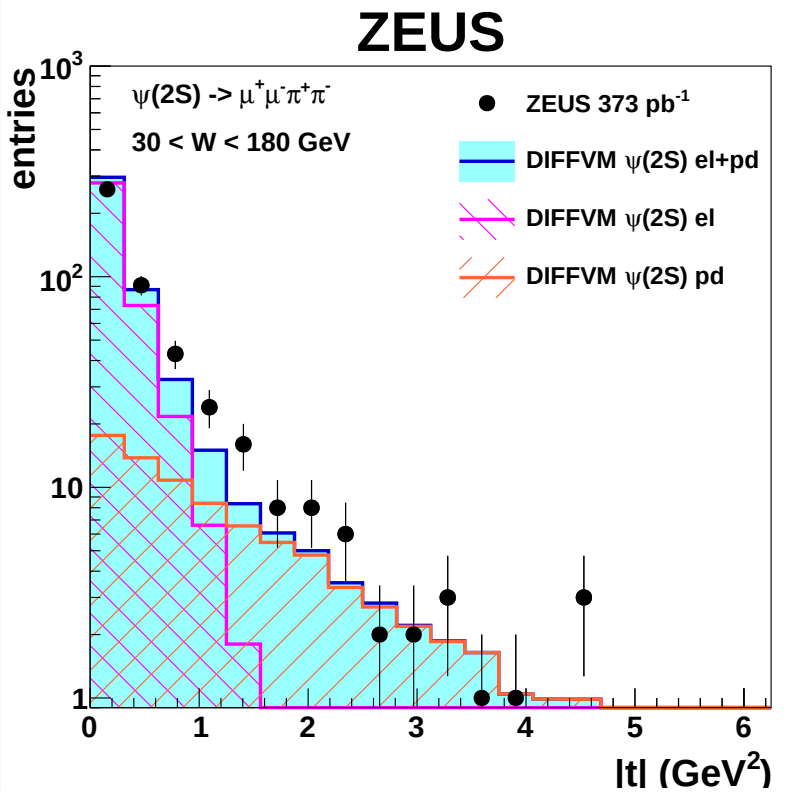
- **More control plots, ...**

4-prongs: W and $|t|$ distributions: $\psi(2S)$ mass window

W: $3.4 < M(\mu^+\mu^-\pi^+\pi^-) < 4.0$ GeV

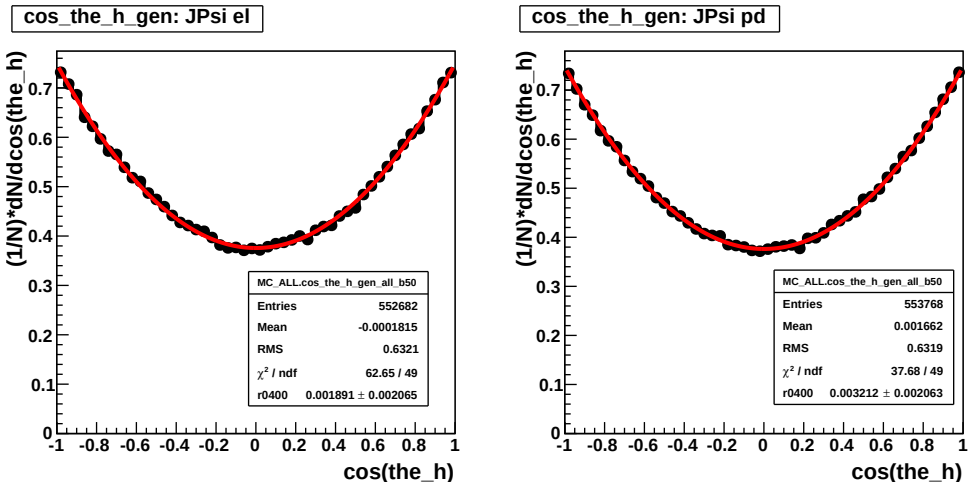


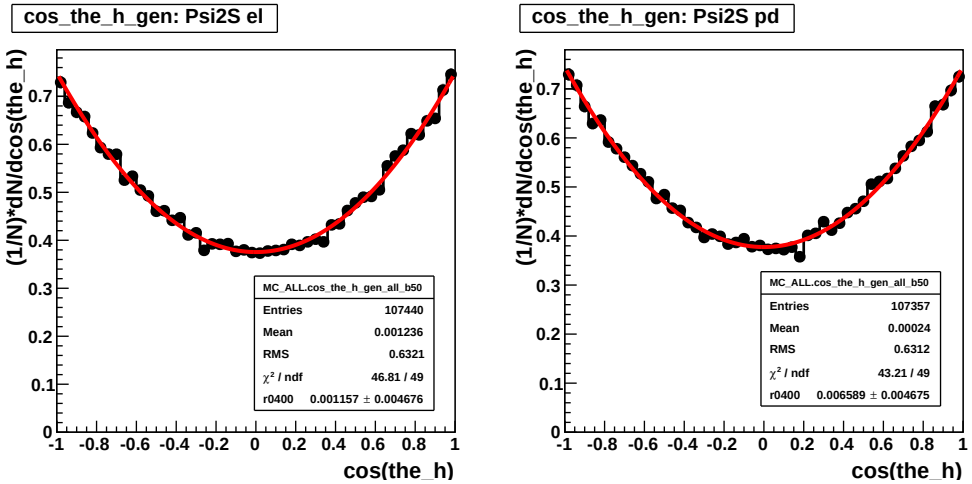
$|t|$: $3.4 < M(\mu^+\mu^-\pi^+\pi^-) < 4.0$ GeV

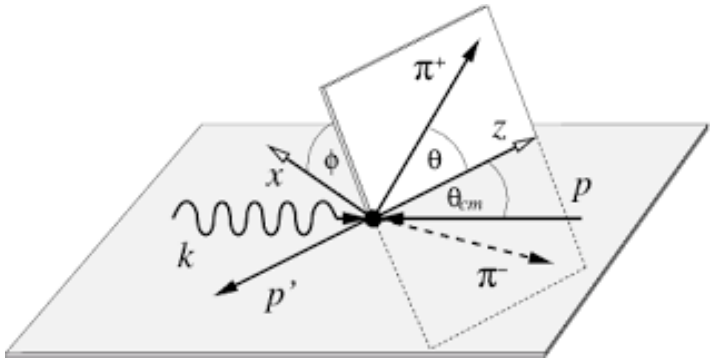


- proton dissociation dominates for $|t| > 1.0$ GeV^2
- proton dissociative fraction: $f_{p.diss} = 0.16 \pm 0.01$ ($|t| < 1.0$ GeV^2) from t-spectra fit

2-prongs: helicity on generator level (before cuts)

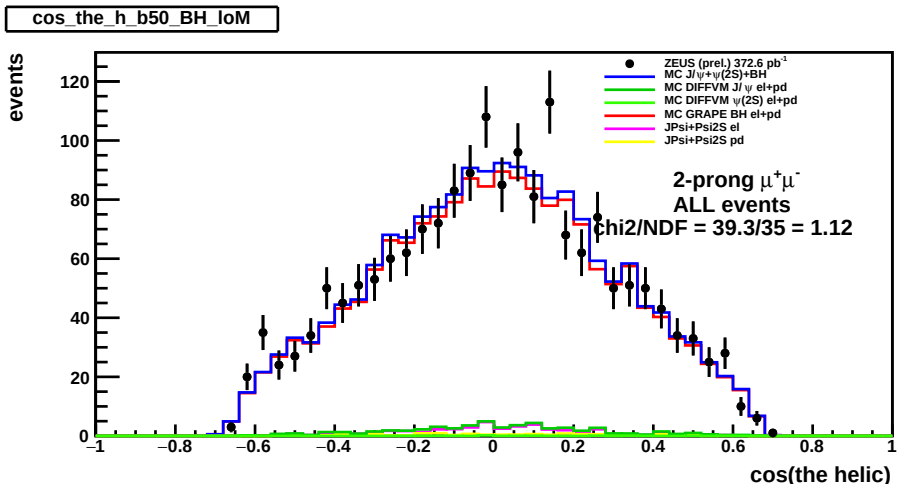


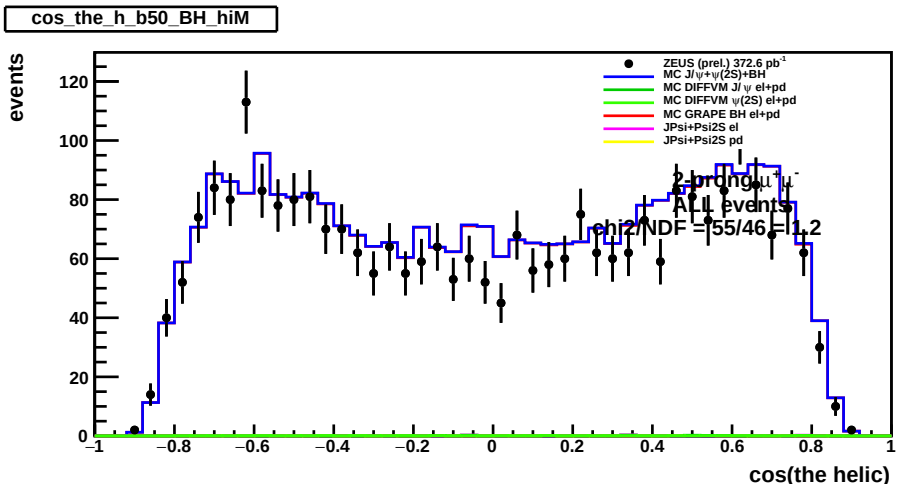


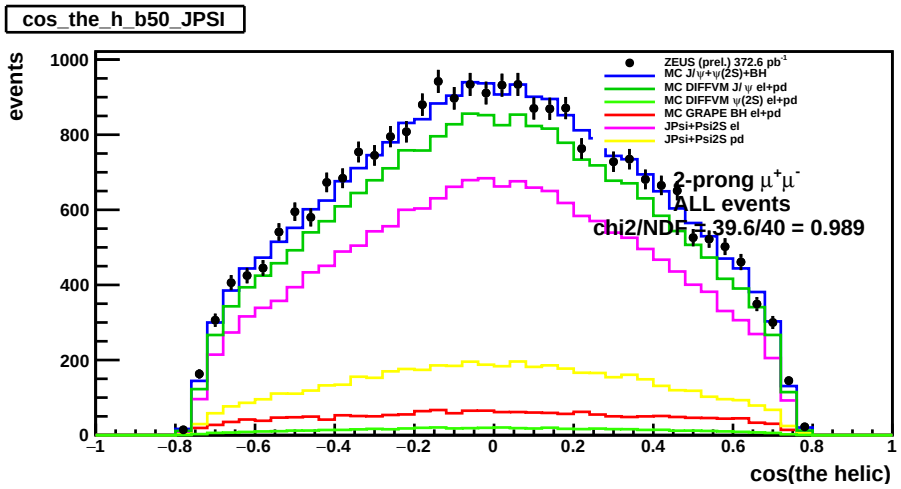


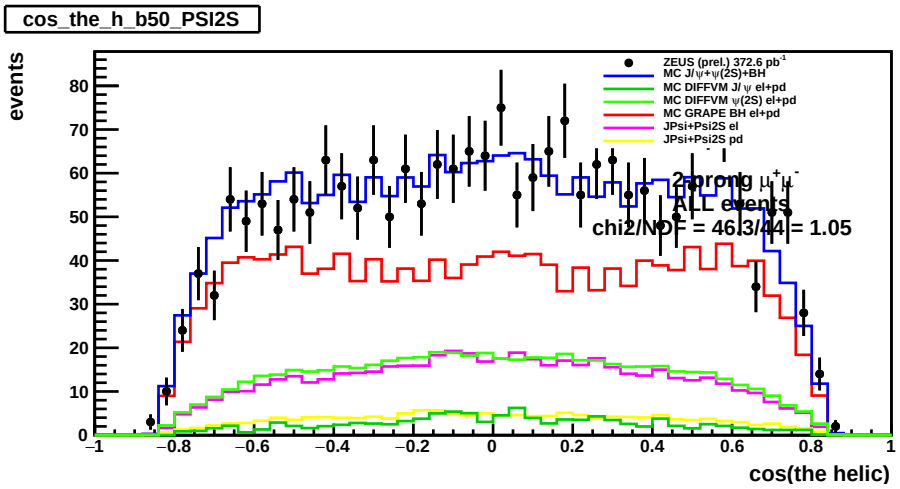
- $\frac{1}{N}\frac{dN}{dcos\theta_h} = \frac{3}{8}(1 + r_{00}^{04} + (1 - 3r_{00}^{04})cos^2\theta_h)$
- for J/ψ and $\psi(2S)$ (el and pd) r_{00}^{04} is 0.0 within errors (as for SCHC)

2-prongs: helicity: $\cos(\theta_h)$



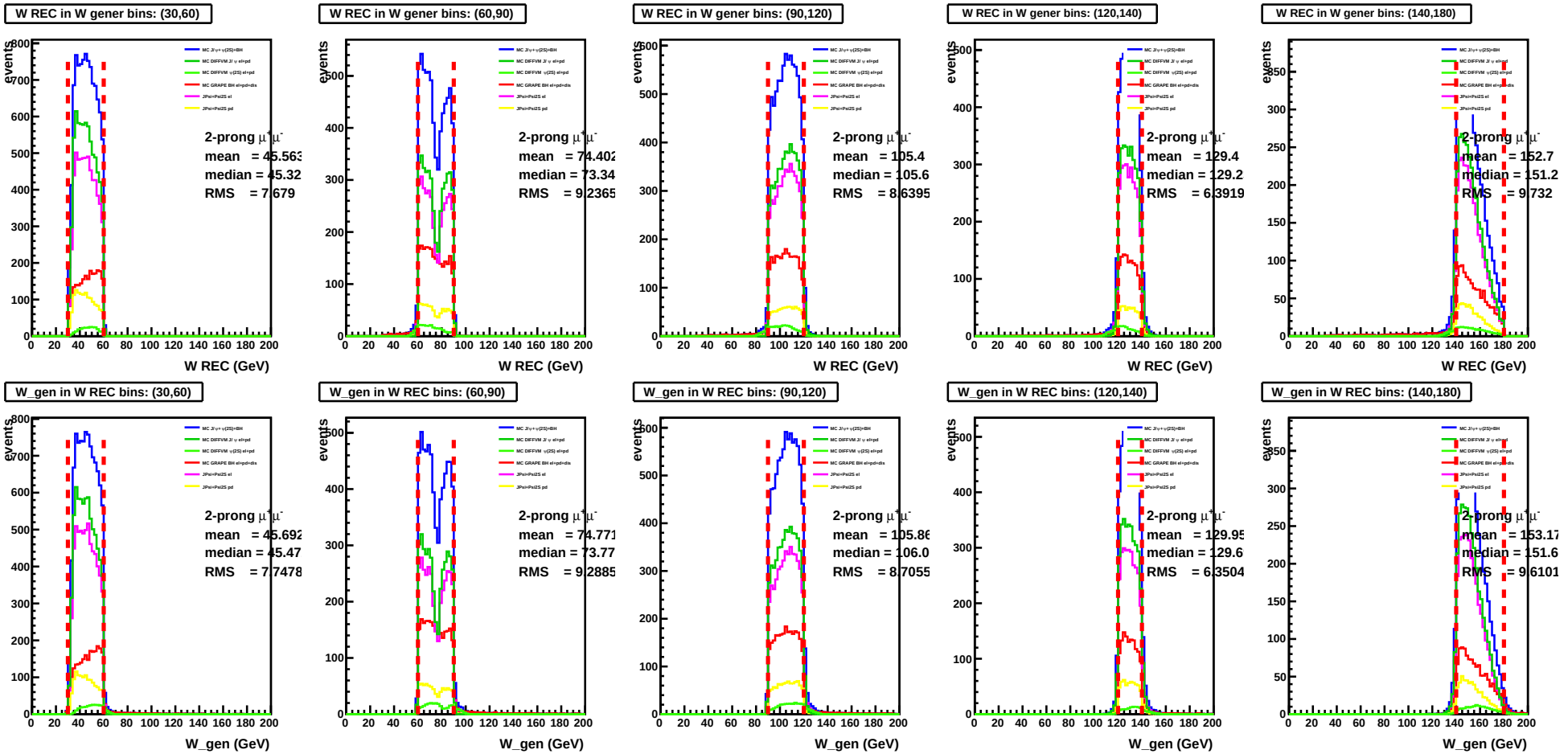






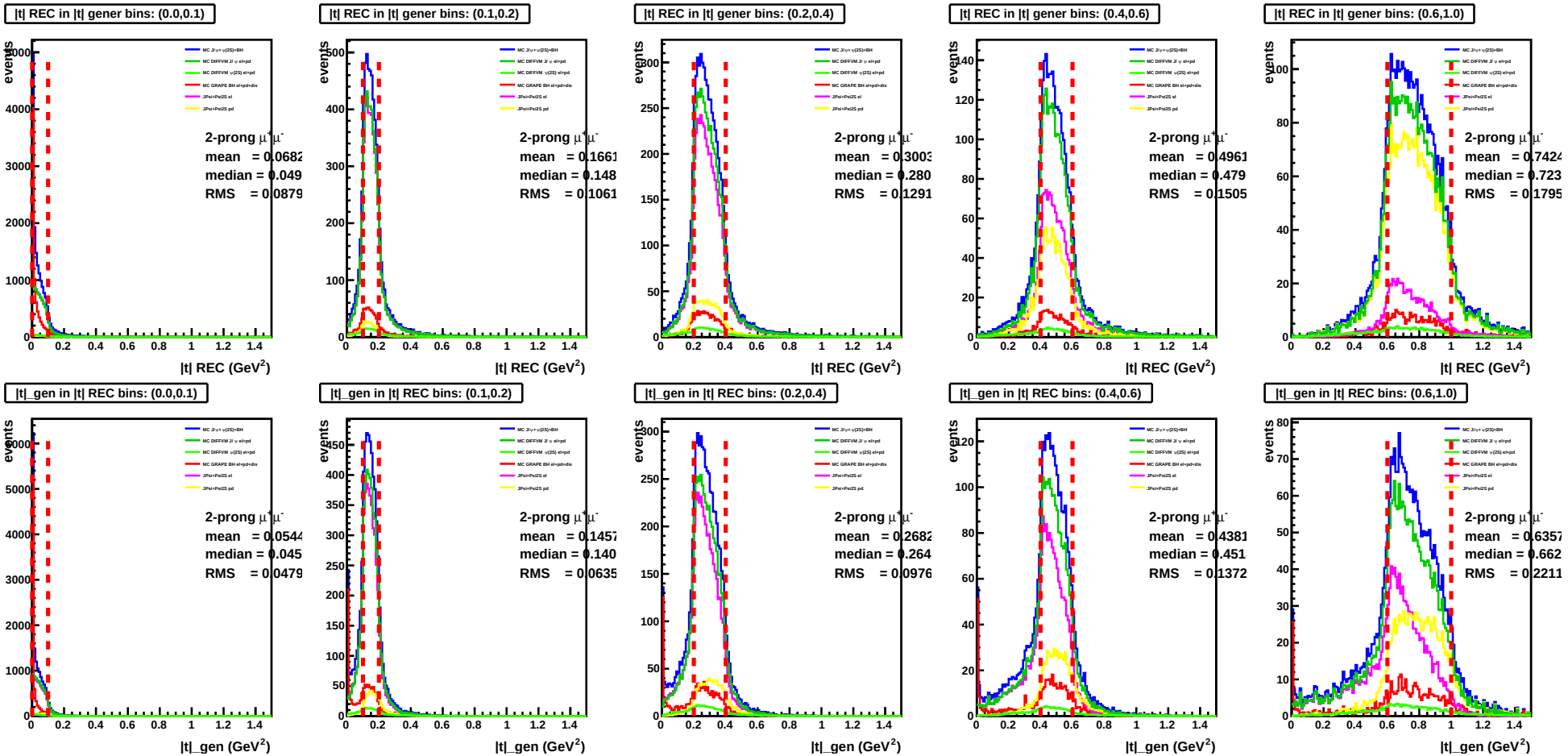
- BH-loM, BH-hiM, J/ψ peak, ψ' peak
- Magenta: elastic contribution, Yellow: p-dissociation, Red: BH
- SCHC: s-channel helicity is conserved for VM !

W binning



- 5 W bins ($W1 \div W5$): $(30 - 60 - 90 - 120 - 140 - 180)$ GeV
- almost equidistant : equal statistic, both dips in $W2$ bin
- W_{REC} in W_{GEN} bins (upper row), W_{GEN} in W_{REC} bins (lower row)
- no migration between bins

$|t|$ binning



- 5 $|t|$ bins ($t1 \div t5$): $(0.0 - 0.1 - 0.2 - 0.4 - 0.6 - 1.0)$ GeV^2
- non-equidistant bins : $\rightarrow$ similar statistic for $\sim$ exp() distribution
- t_{REC} in t_{GEN} bins (upper row), t_{GEN} in t_{REC} bins (lower row)
- some migration to neighbor bins, symmetric, mean value little affected

- DIFFVM **and** GRAPE **Monte Carlo**

DIFFVM – A Monte Carlo Generator for **Diffractive Processes in *ep* Scattering.**

B. List
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Cosenza, Italy
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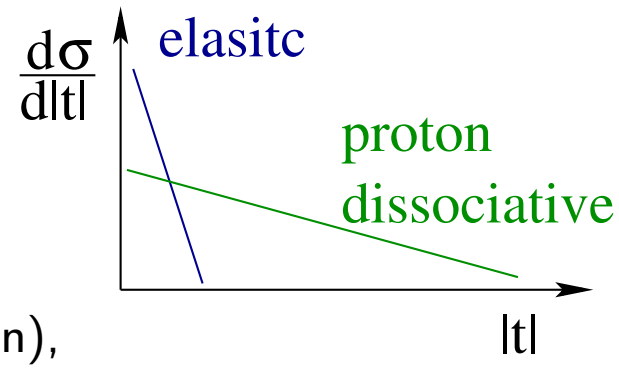
- soft diffractive processes in the Regge framework and Vector Dominance Model
- $\frac{d\sigma}{dQ^2} \propto \frac{1}{(1+Q^2/M_V^2)^{1.5}}$
- $\frac{d\sigma}{d|t|} \propto W_{\gamma p}^{4\epsilon} e^{-b|t|}$ $(4\epsilon = \delta)$ (elastic)
- $\frac{d^2\sigma}{d|t|dM_Y^2} \propto W_{\gamma p}^{4\epsilon} e^{-b'|t|} M_Y^{-\beta}$ (p.diss)
- $\frac{d\sigma}{dM_Y^2} \sim \frac{f(M_Y^2)}{M_Y^{2(1+\epsilon)}}$ for $M_Y^2 < 3.6$ GeV^2 (p resonance region),
  $\frac{d\sigma}{dM_Y^2} \sim \frac{1}{M_Y^{2(1+\epsilon)}}$ for $M_Y^2 \geq 3.6$ GeV^2 (continuum region)
- assuming SCHC: s-channel helicity conservation

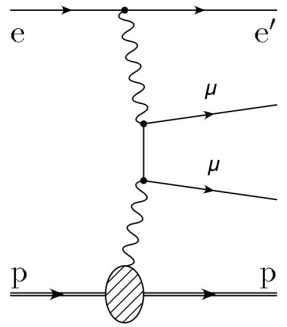


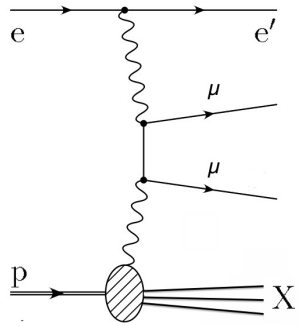
Background Monte Carlo: GRAPE

GRAPE-Dilepton
(Version 1.1)
A generator for dilepton production in *ep* collisions

Tetsuo Abe
Department of Physics, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-8654, Japan

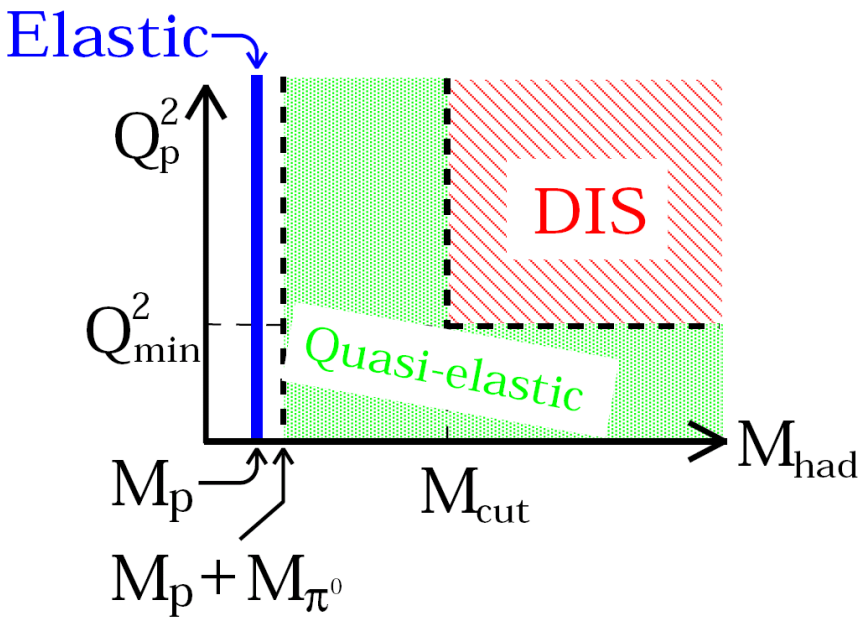
- based on the exact matrix elements in the electroweak theory at tree level via $\gamma\gamma$, γZ^0, $Z^0 Z^0$
  and via photon internal conversion (QED Compton)
- Feynman amplitudes are generated by the automatic calculation system GRACE
- **proton vertex** covers the whole kinematical region
- interface to PYTHIA and SOPHIA
  $\rightarrow$ complete hadronic final state
- **covers elastic, quasi-elastic and DIS processes**





Monte Carlo Simulation: Bethe-Heitler Background ($\mu^+\mu^-$ Continuum)

GRAPE generator - simulate QED lepton pair (Bethe-Heitler)



Important for the shape of the BH $M_{\mu^+\mu^-}$ spectrum (sidebands)

and for the BH t-dependence: low t - elastic BH, higher t - QEL BH

Tuning of signal MC

- **Tuning of DIFFVM Monte Carlo**

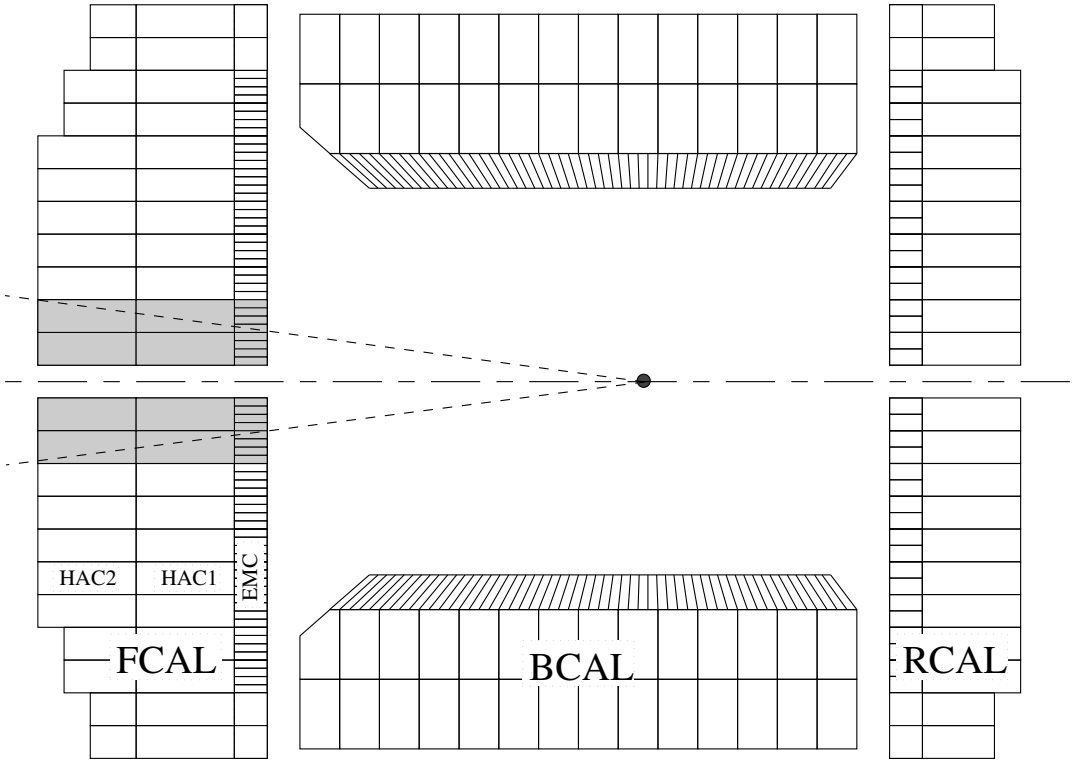
Tuning of DIFFVM MC sample

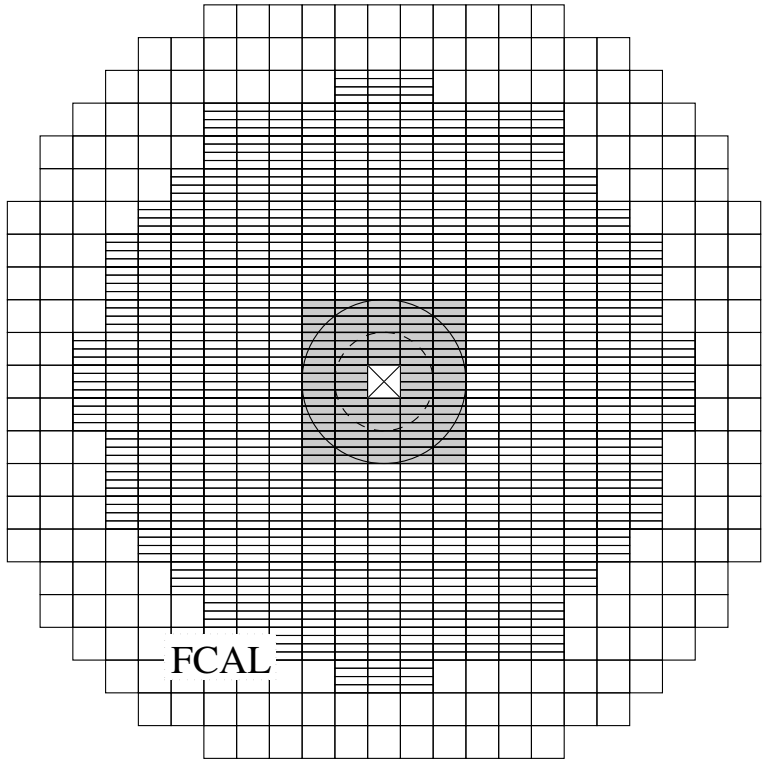
- Reweighting of MC sample at generator level
- $|t|$ dependence: $\sim \exp(-b|t|)$, generated with $b_{el} = 4.0$, $b_{pd} = 1.0$
  reweighted to:
  $b_{el} = 4.6 \pm 0.3$, $b_{pd} = 1.0 \pm 0.1$ (JPSI)
  $b_{el} = 4.3 \pm 0.7$, $b_{pd} = 0.7 \pm 0.2$ (PSI2S)
- shrinkage added by reweighting: $b = b_0 + 4.0\alpha' \log(W/W_0)$;
  $\alpha' = 0.12 \pm 0.04$ GeV^{-2}, $W_0 = 90$ GeV (elastic only)
- W dependence: $\sigma \sim W^{\delta}$,
  generated with $\delta = 0.88$ for both elastic and p.diss
  reweighted to:
  $\delta_{el} = 0.67 \pm 0.10$, $\delta_{pd} = 0.42 \pm 0.15$ (JPSI)
  $\delta_{el} = 1.10 \pm 0.20$, $\delta_{pd} = 0.70 \pm 0.30$ (PSI2S)
- M_V dependence: $\sim \frac{1}{M_V^{\beta}}$, generated with $\beta = 2.5$
  reweighted to $\beta = 2.4 \pm 0.3$ (both JPSI and PSI2S, p.diss only)
- **all parameters are subject to systematics checks**

p.diss fractions

- **extracting fractions of proton dissociation**

Proton dissociation taggers

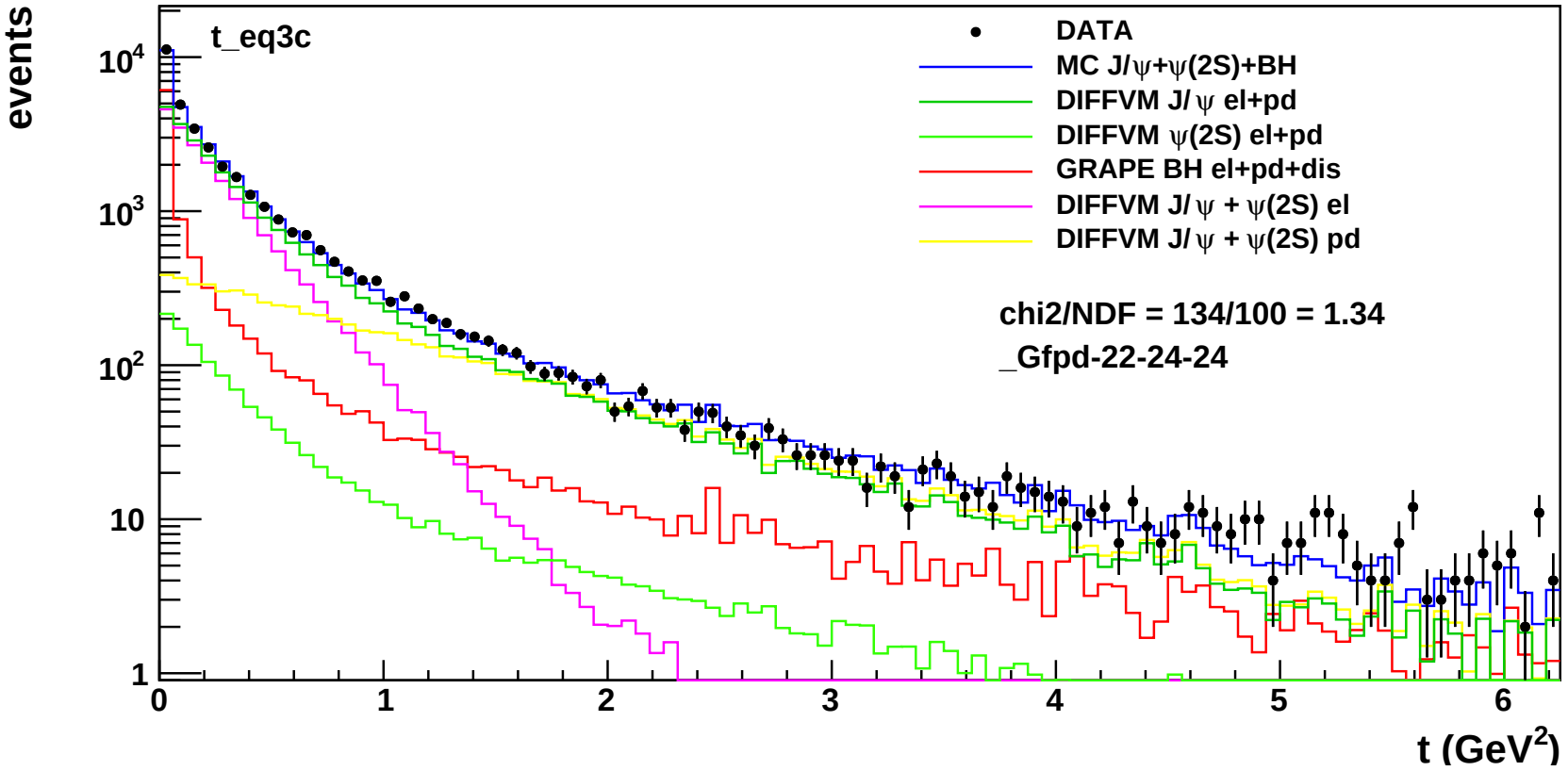




- Energy in forward cone to suppress p.diss events: $\theta_{max} = 0.12\,rad$
- using EFO : "Energy Flow Objects" (trackers + CAL info):

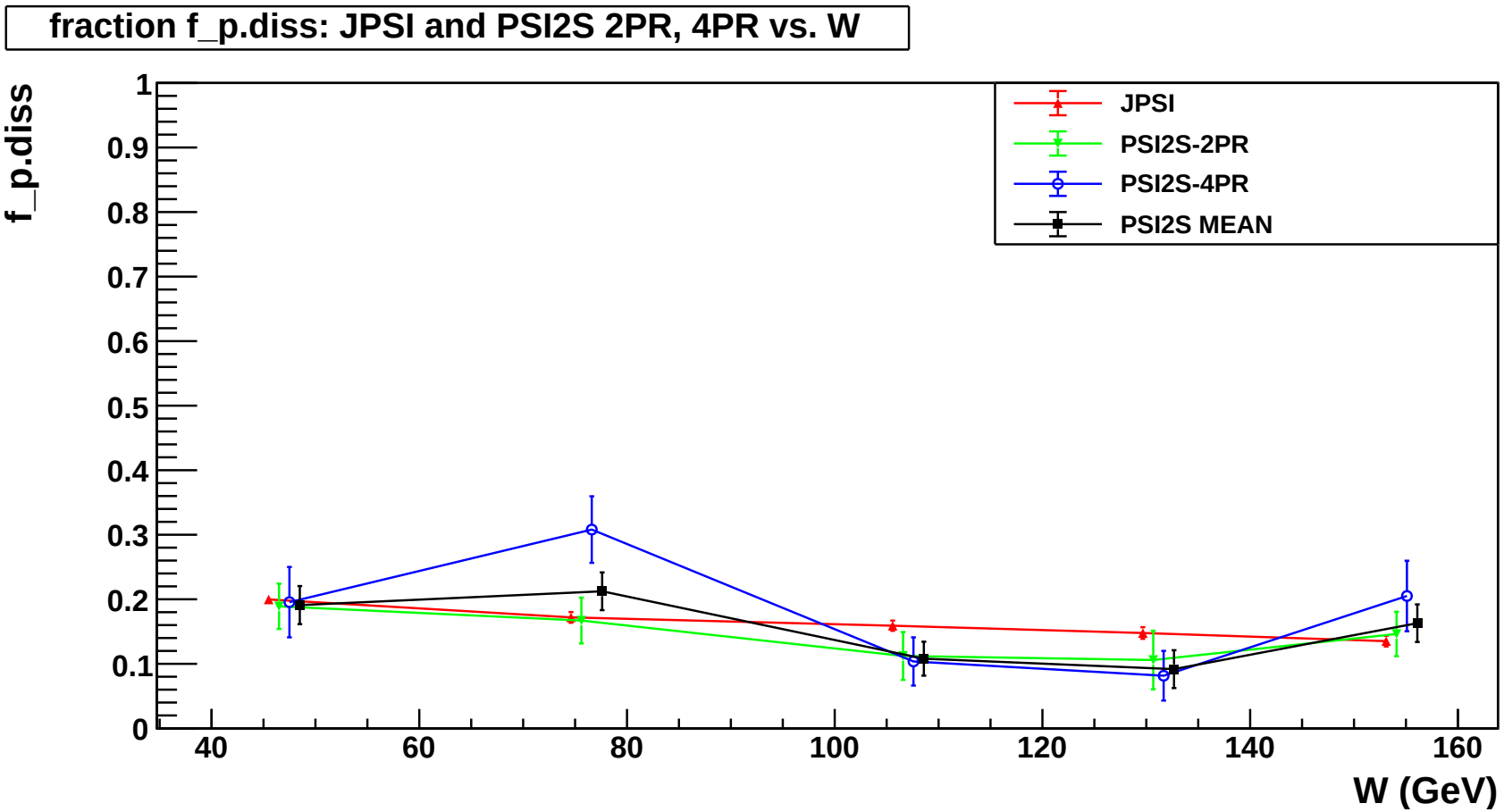
$$\left(\sum_{EFOs} E(\theta_{EFO} < \theta_{max}) \right) < 1\,\mathrm{GeV}$$

2-prongs: $|t|$ distribution: all 2-prong events



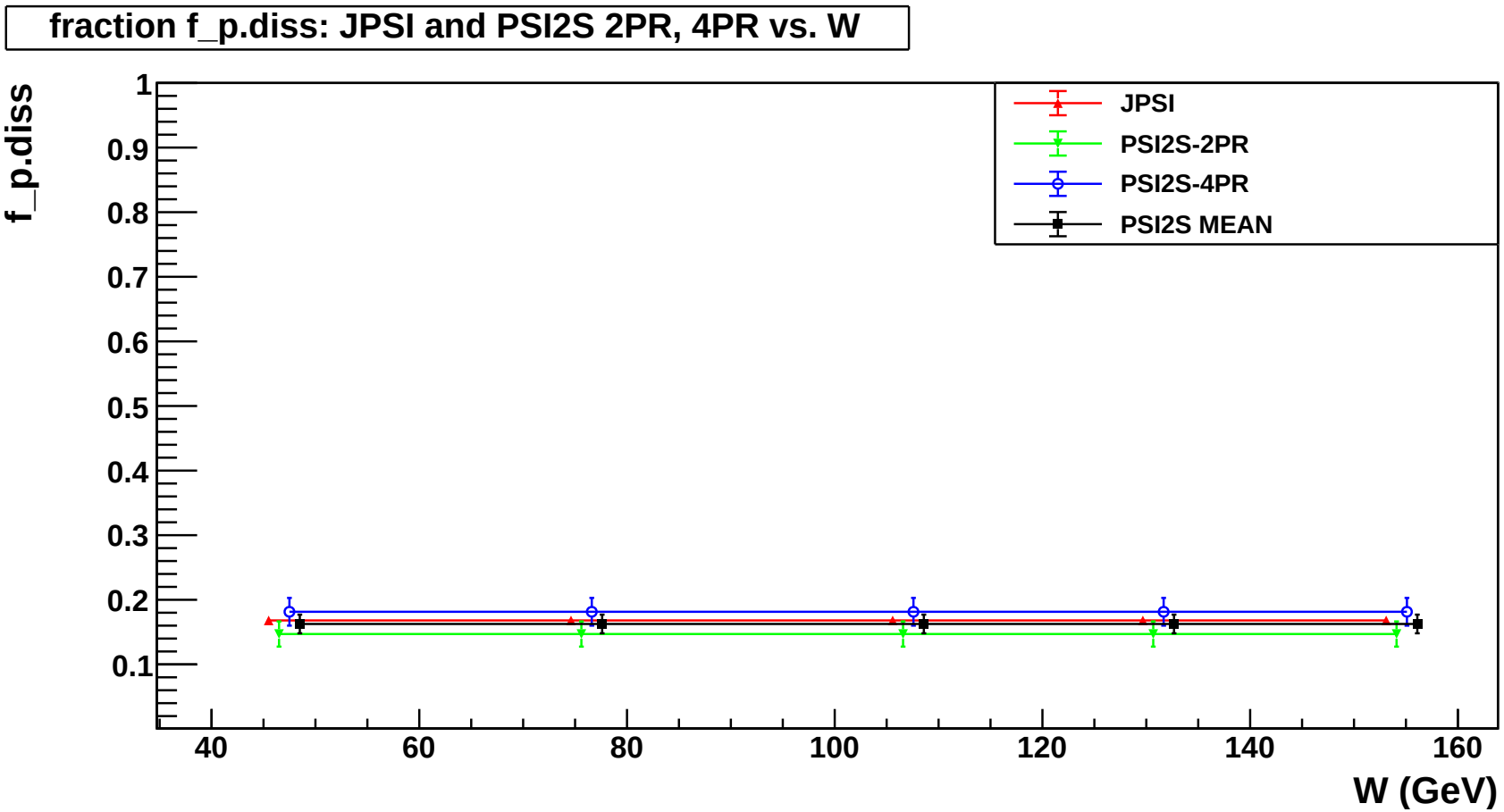
- **spectra like this are** used to evaluate the p.diss fractions (use longer "lever arm" then integrate it up to $|t| = 1.0$ GeV^2)
- using root package `TFractionalFitter` (TFF)
- fitted $f_{p.diss} = 0.17$ and $= 0.16$ (JPSI and PSI2S, BH subtracted)
- **p.diss take over elastic around $\sim$ 1 GeV^2** (yellow and magenta histos)

$f_{p.diss}$ fractions in W bins (TFF estimator)



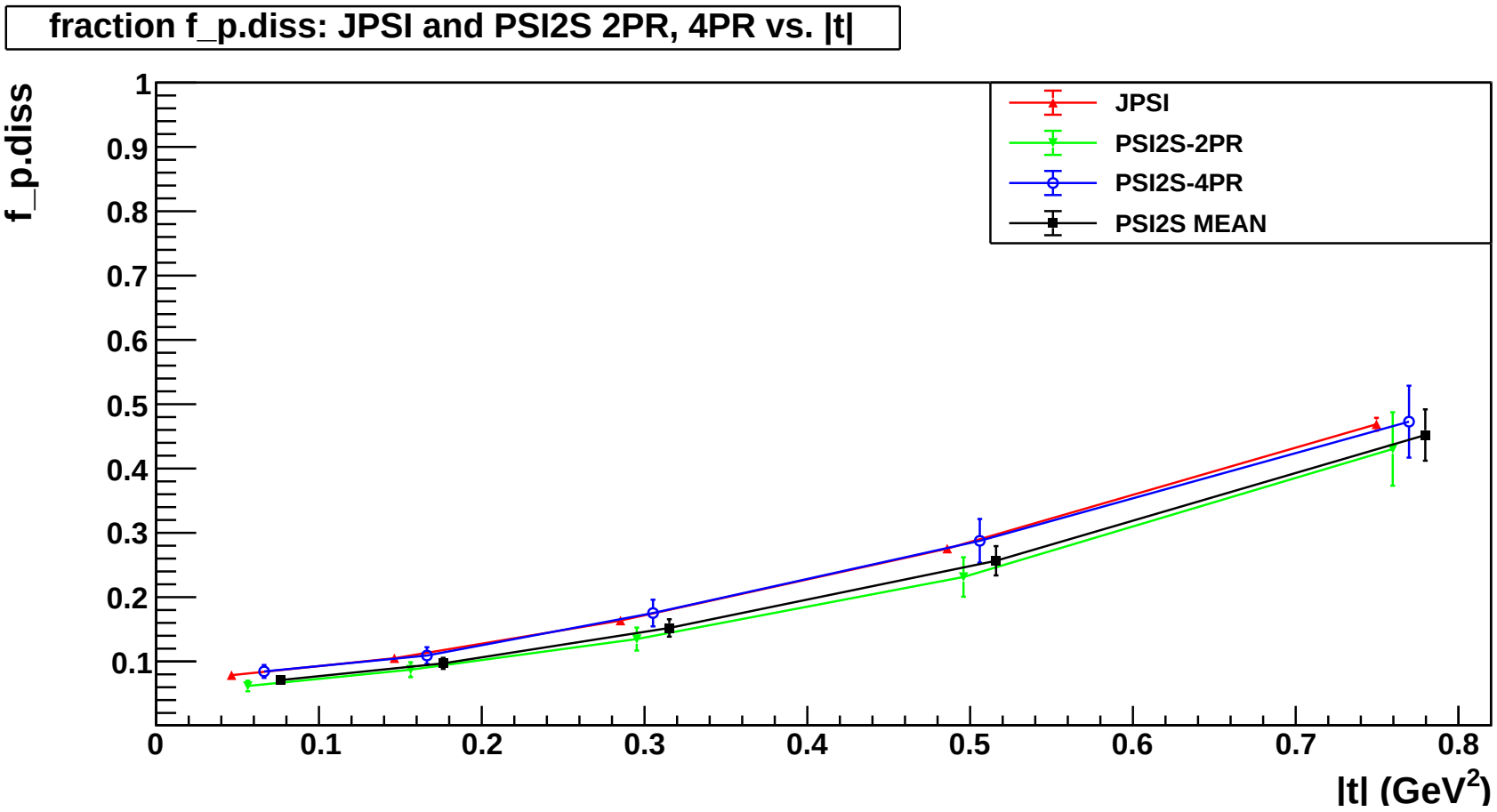
- average value $\sim$ 17% JPSI and $\sim$ 16% PSI2S (mean)
- compatible results for 2- and 4-prong channels, no W dependence
- **black**: weighted mean for PSI2S 2- and 4-prong
- bigger fluctuations for PSI2S 2- and 4-prongs

$f_{p.diss}$ fractions in W bins (TFF estimator)



- average value $\sim$ 17% JPSI and $\sim$ 16% PSI2S (mean)
- **black**: weighted mean for PSI2S 2- and 4-prong (used in analysis)
- for R analysis: the same mean value is used for all W bins
- no significant impact on final ratio R ($f_{p.diss}$ fractions cancels out)

$f_{p.diss}$ fractions in $|t|$ bins (TFF estimator)



- compatible results for 2- and 4-prong channels
- **black**: weighted mean for PSI2S 2- and 4-prong (used in analysis)
- negligible effect on final R analysis ($f_{p.diss}$ fractions cancels out)
- bigger impact on systematics for large $|t|$ due to the b-slope variation !

Modeling of nucleon resonances ($N*$, ...)

- **Modeling of nucleon resonance states**
- (low M_Y proton dissociation)

Modeling of nucleon resonance state: DIFFVM

- $\frac{d\sigma}{dM_Y^2} \sim \frac{1}{M_Y^{2(1+\epsilon)}}$ for $M_Y^2 \geq 3.6$ GeV^2 (continuum region)
- $\frac{d\sigma}{dM_Y^2} \sim \frac{f(M_Y^2)}{M_Y^{2(1+\epsilon)}}$ for $M_Y^2 < 3.6$ GeV^2 (p resonance region)
- $f(M_Y^2)$ from the **fit the the p.diss cross section on deuterium**: $pD \to YD$ (Phys. Rep. **101** (3) (1983), 169)
- for $M_Y < 1.9$ GeV several resonances are included (Pomeron carries quantum numbers of the vacuum (I=0, G = P = C= +) only N^{*+} states with $J^{\mathcal{P}} = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^-, ...)$
- $N^{*+} = N(1440), N(1520), N(1680), N(1700)$
- N^{*+} decays into: $N\pi, \Delta\pi, N\rho, N\pi\pi$ included (BR from PGD 1992)
- N^{*+} decays isotropically in their rest frame

- dissociation in the continuum state carried by JETSET (splitting proton into $q - qq$ system, q couples to $\mathbb{P}$, leaving qq spectator)

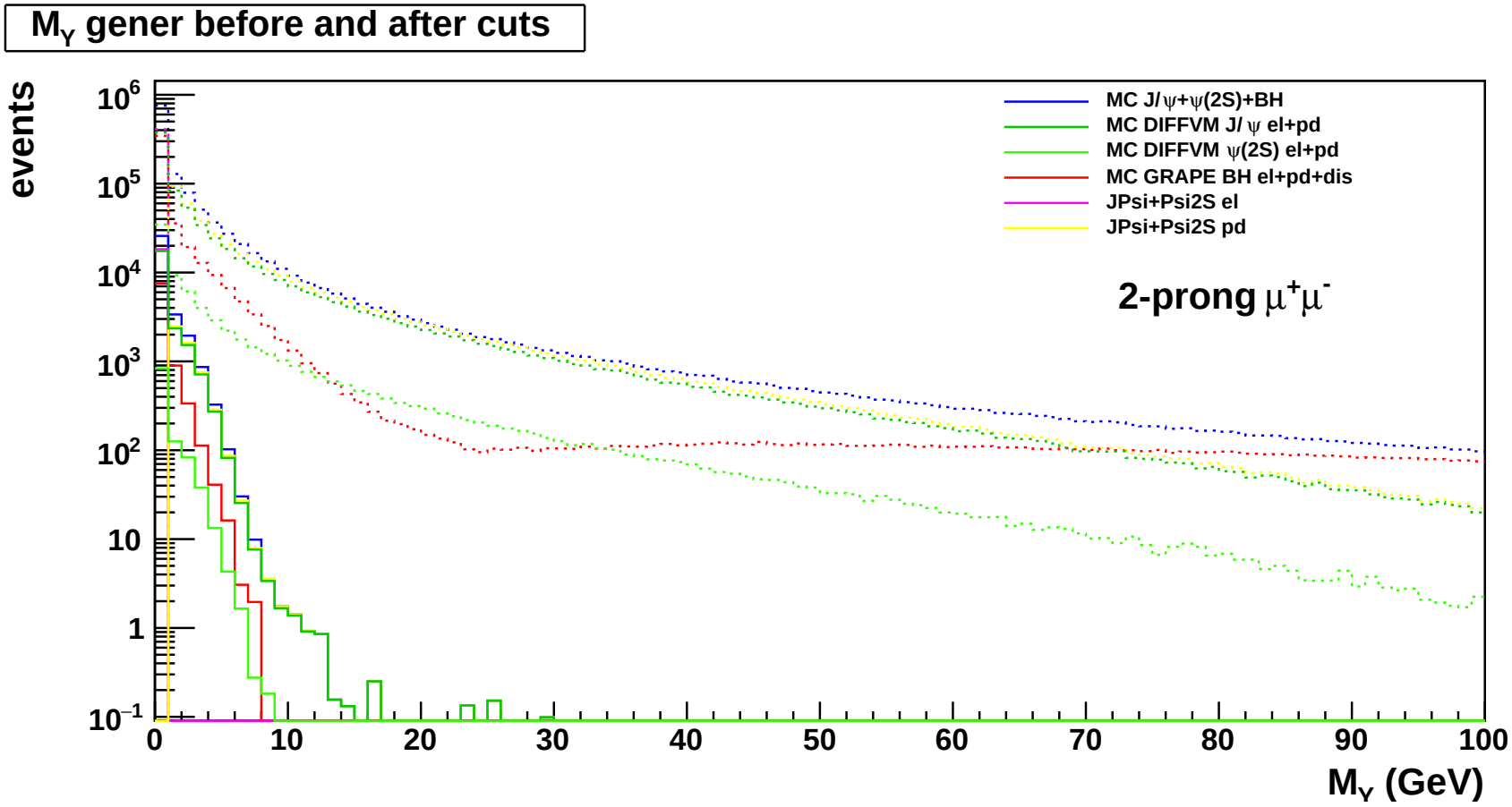
Modeling of nucleon resonance state: GRAPE

- $d\sigma \sim L_{\mu\nu} W^{\mu\nu}$
- hadron tensor:

$$W^{\mu\nu} = W_1 \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) + W_2 \frac{1}{M_P^2} \left(p_P^\mu - \frac{p_P \cdot q}{q^2} q^\mu \right) \left(p_P^\nu - \frac{p_P \cdot q}{q^2} q^\nu \right)$$

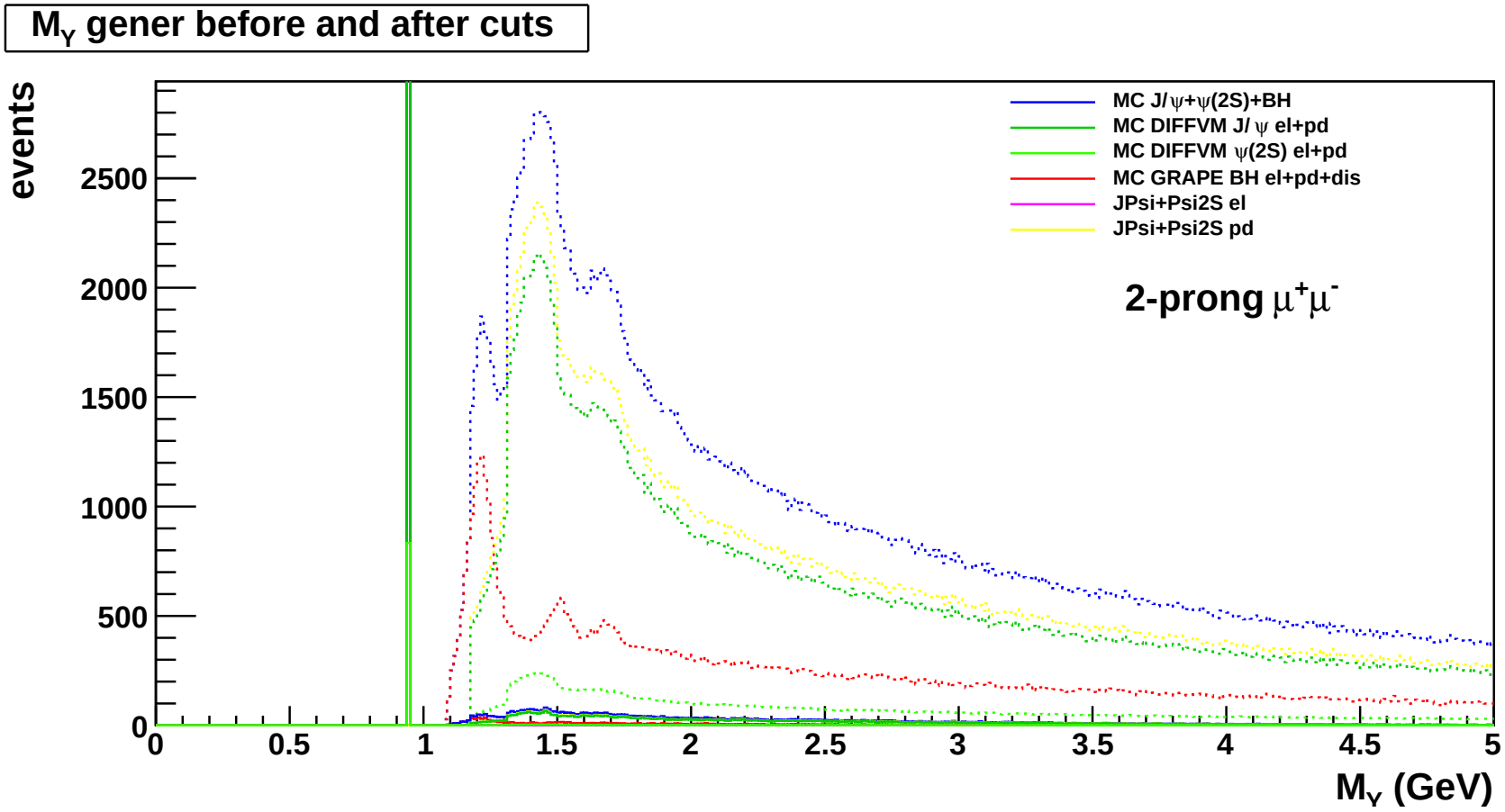
- $W_{1,2}(Q_P^2, M_{had})$ are proton electromagnetic structure functions
- for $M_{had} < 2$ GeV $W_{1,2}$ parameterized by Brasse at al. (Nucl. Phys. **B 110** (1976) 413.) (resonance region)
- for $M_{had} > 2$ GeV $W_{1,2}$ parameterized by ALLM97 (hep-ph/9712415) (continuum)
- both parameterizations from **fits to experimental total $\gamma^* p$ cross sections**
- exclusive hadronic final state generated by SOPHIA
- (plus DIS di-leptons diagrams, in the framework of QPM, using PDF's)

MC generator level: M_Y before and after selection cuts



- M_Y at generator level (not measured quantity ! $\rightarrow$ lost in beam-pipe)
- before and after selection cuts
- GRAPE (BH) does include DIS scattering $\rightarrow$ rise of xsec. for large M_Y
- DIFFVM in DIS mode generates only **electroproduction** (with proton dissociation, "rapidity gap events") $\rightarrow$ this is OK

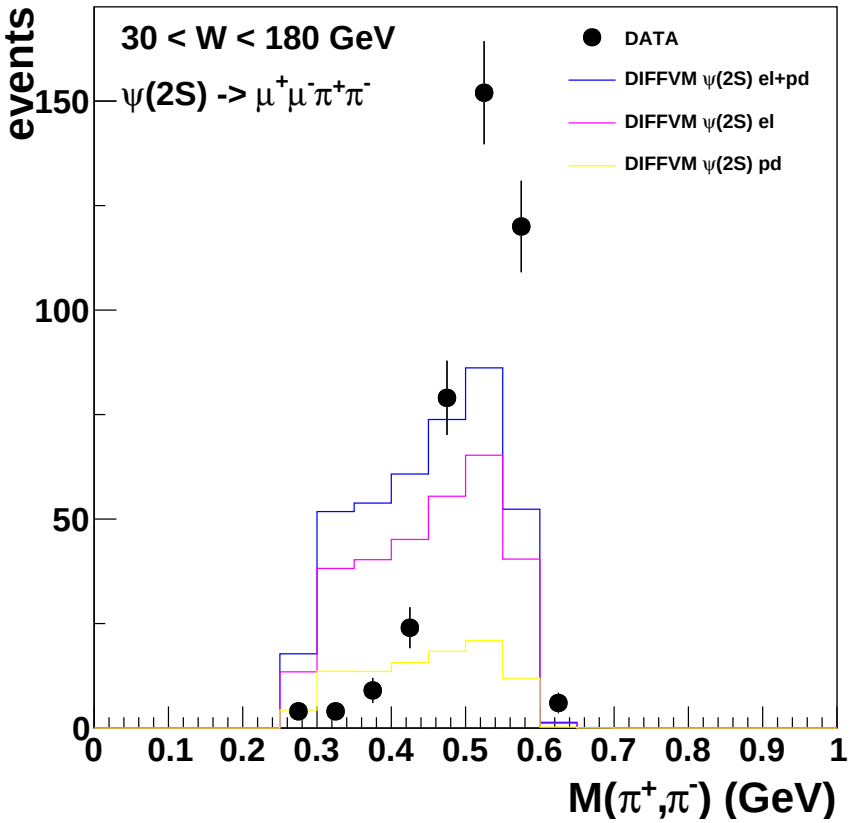
MC generator level: zoom at low $M_Y < 5$ GeV (lin scale)

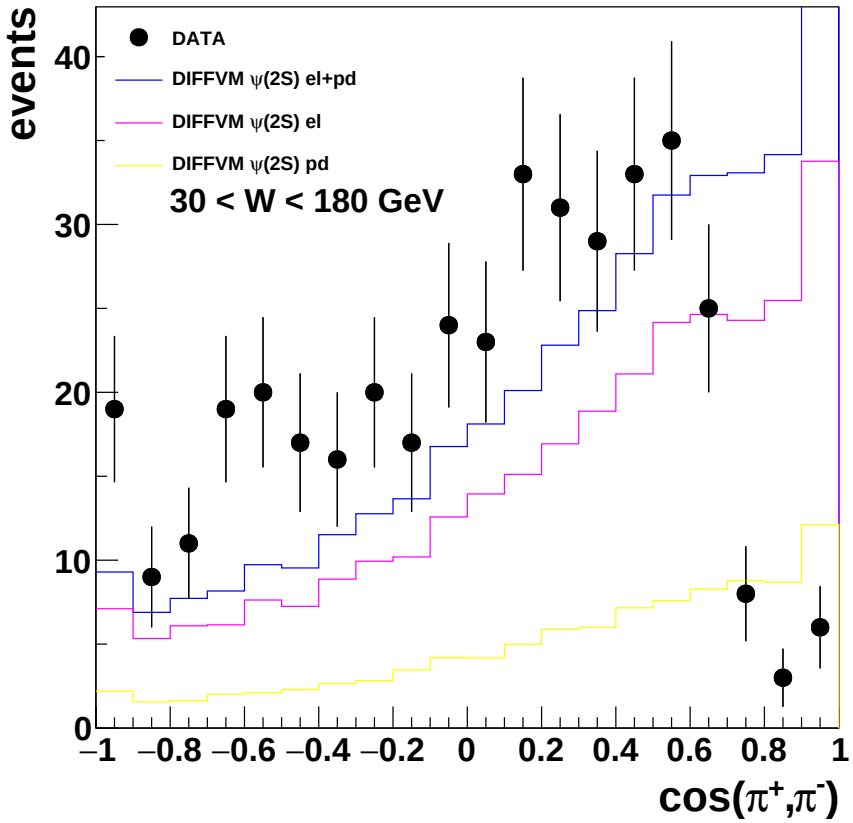


- **different structure of nucleon resonances between GRAPE and DIFFVM (!?)**
- which is right ?
- **how much it is important for MC based p.diss BG subtraction ?**

- **Pions phase space reweighting**

4-PRONGS: $M(\pi^-, \pi^+)$, $\cos(\pi^-, \pi^+)$





- $\psi' \to J/\psi + \pi^+\pi^-$
- $M(\pi^-, \pi^+)$, $\cos(\pi^-, \pi^+)$
- DIFFVM MC before pions phase space reweighting

Pions phase space reweighting (DIFFVM 4-prongs)

- $weight = (M(\pi^-, \pi^+)^2 - 4.0 * M_\pi^2)^2$
- ref: `Phys_Lett_B61_1976_183.pdf`
- final $\pi^+\pi^-$ interaction is not in pure S-state
- $\rightarrow$ for the impact of this correction see next 2 pages

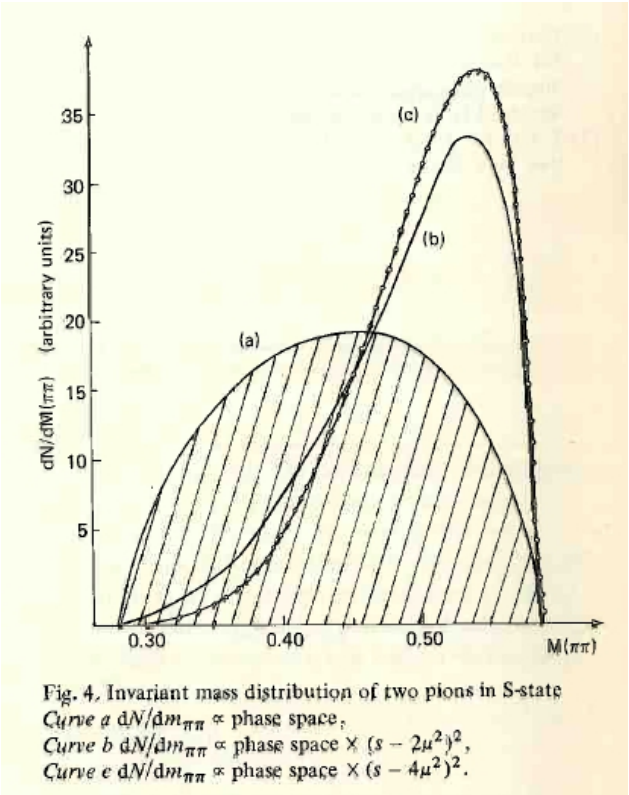
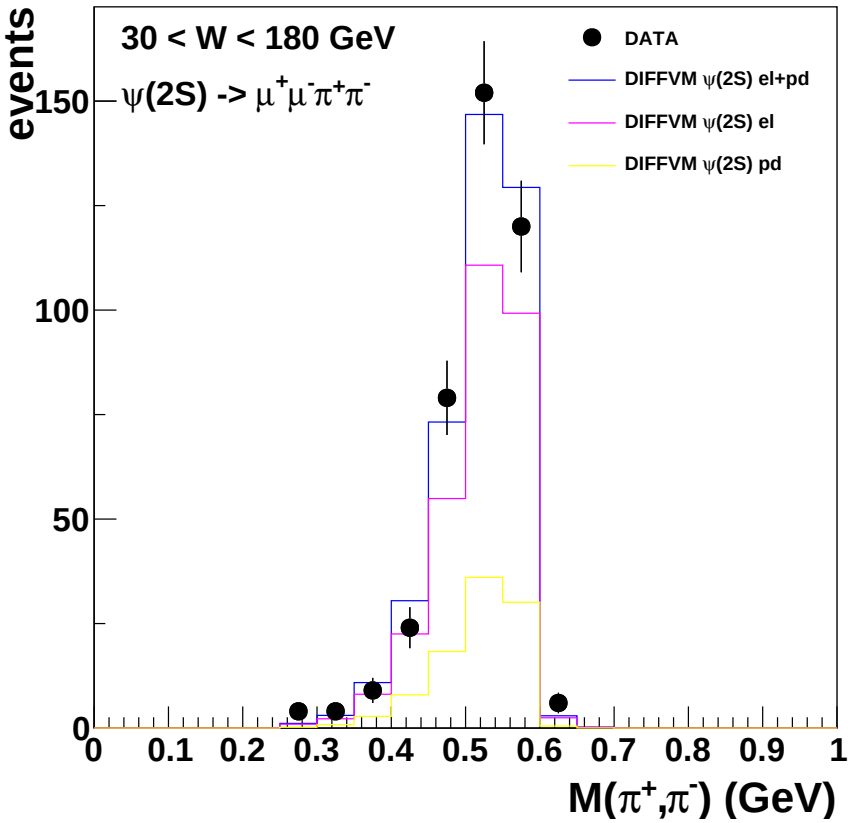
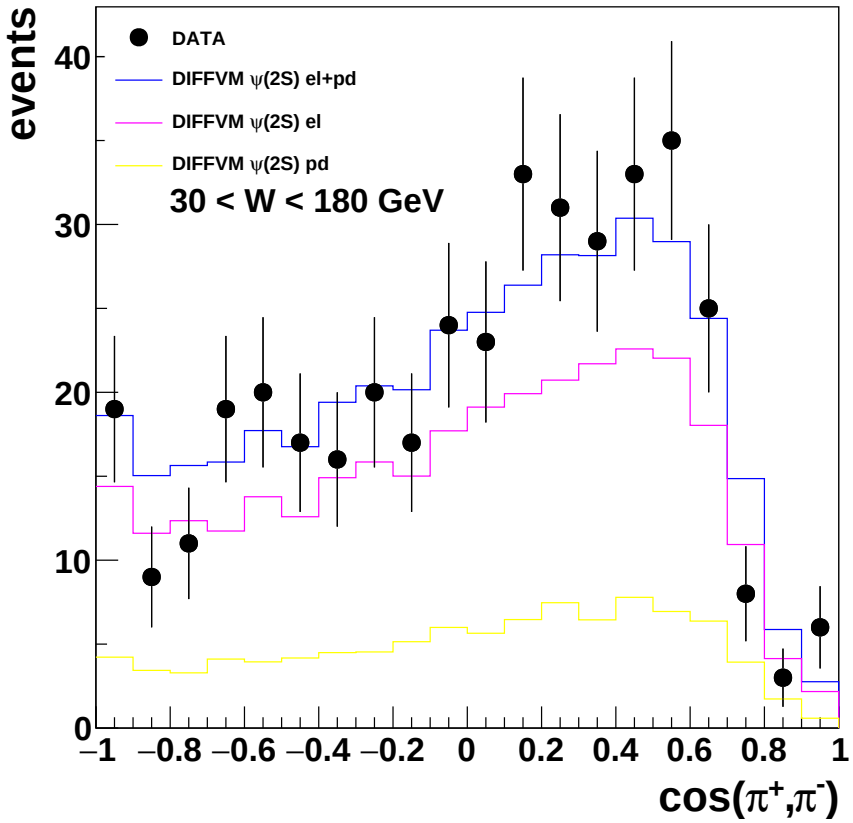


Fig. 4. Invariant mass distribution of two pions in S-state
Curve a $dN/dm_{\pi\pi} \propto$ phase space,
Curve b $dN/dm_{\pi\pi} \propto$ phase space $\times (s - 2\mu^2)^2$,
Curve c $dN/dm_{\pi\pi} \propto$ phase space $\times (s - 4\mu^2)^2$.

4-PRONGS: $M(\pi^-, \pi^+)$, $\cos(\pi^-, \pi^+)$





- $\psi' \to J/\psi + \pi^+\pi^-$
- $M(\pi^-, \pi^+)$, $\cos(\pi^-, \pi^+)$
- DIFFVM MC after pions phase space reweighting