

SEMILEPTONIC B DECAYS

Paolo Gambino

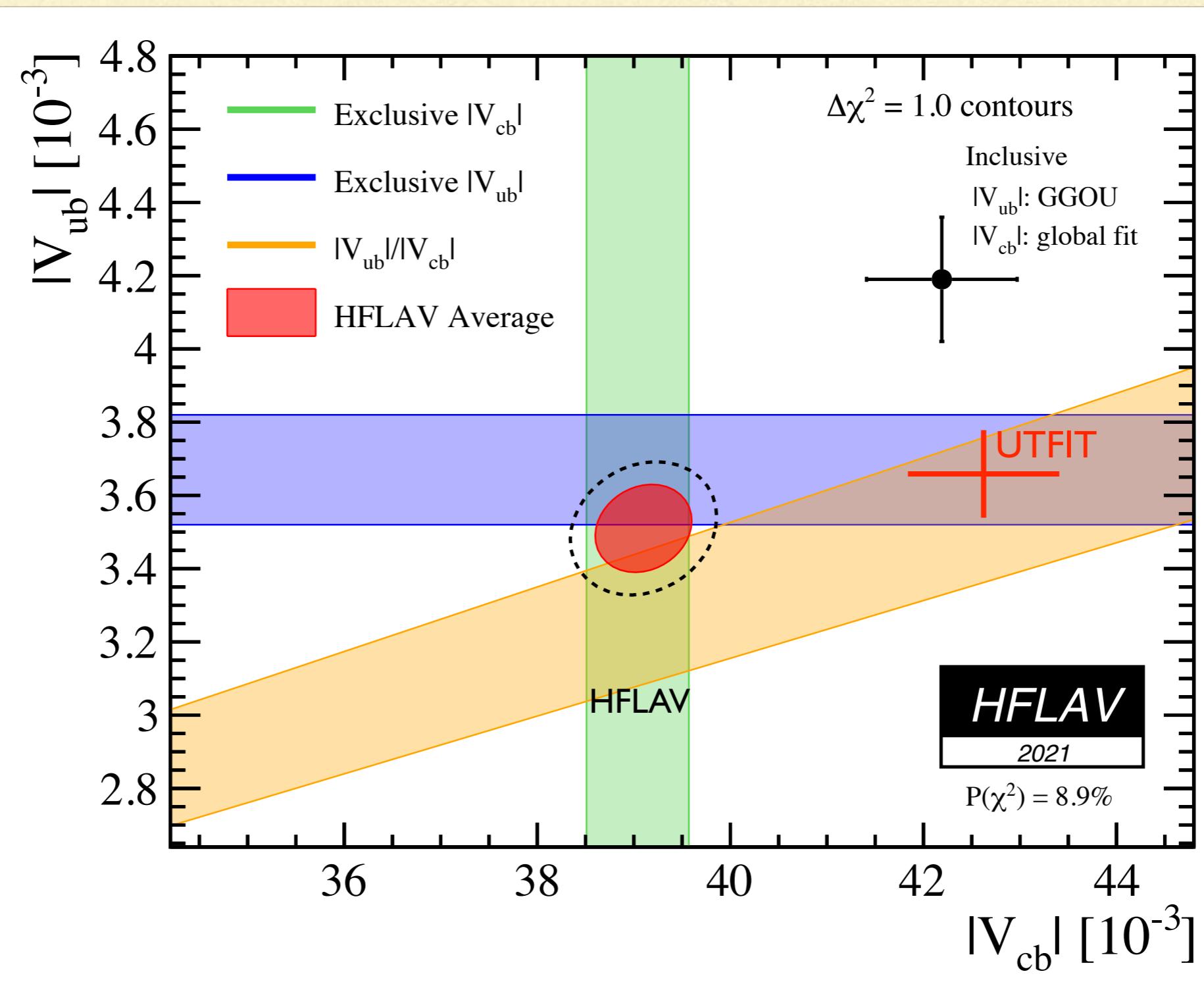
Università di Torino & INFN, Torino
Technische Universitaet Munich

21st FPCP 2023



LYON, 29 May – 2 June 2023

Since many years the inclusive and exclusive determinations of $|V_{cb}|$ and $|V_{ub}|$ diverge



Recently: new calculations of FFs by several lattice collaborations and with light-cone sum rules, new perturbative calculations, all facing the challenges of a precision measurements... and several new measurements as well!

The importance of $|V_{cb}|$

An important CKM unitarity test is the Unitarity Triangle (UT) formed by

$$1 + \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

V_{cb} plays an important role in UT

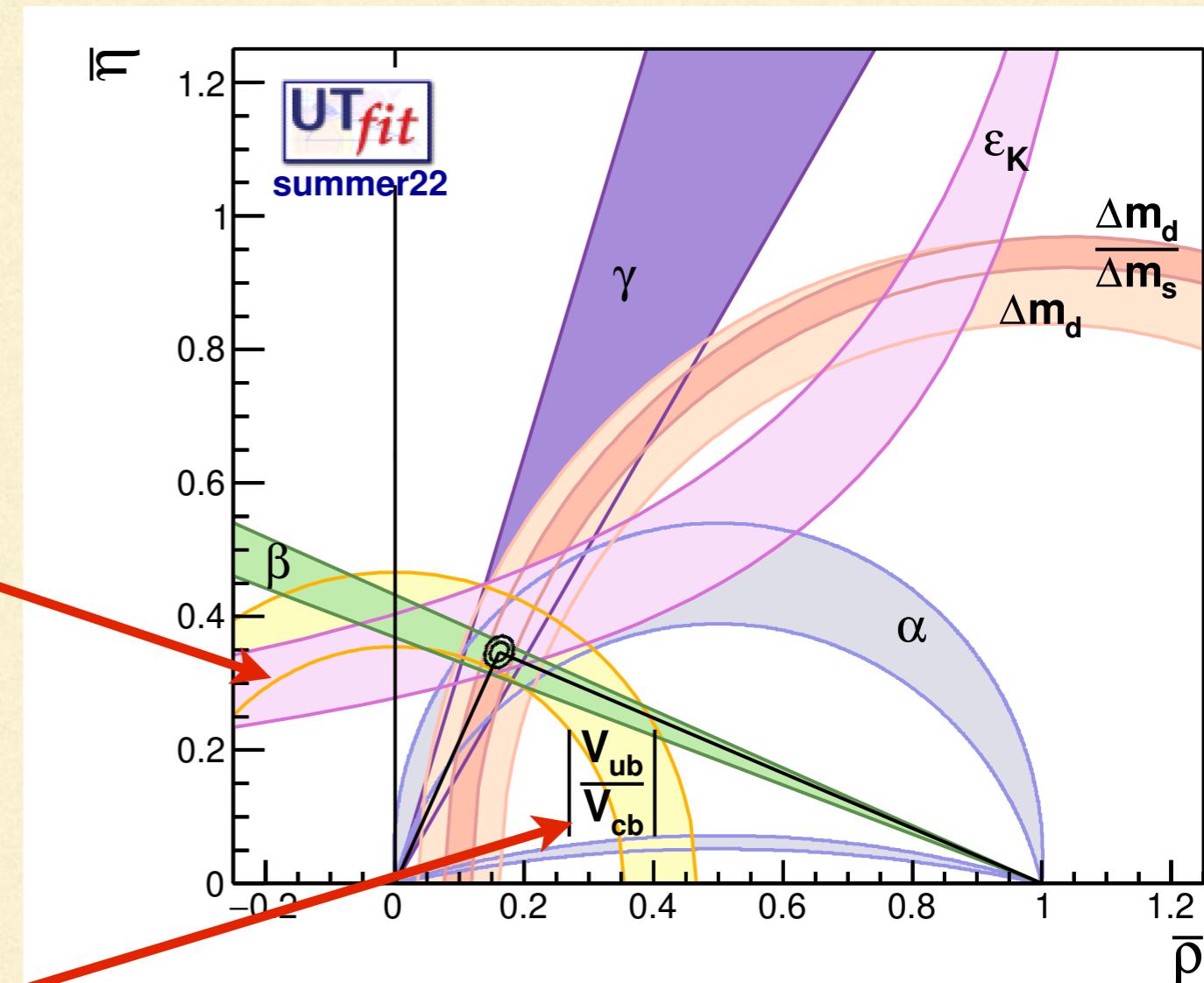
$$\varepsilon_K \approx x|V_{cb}|^4 + \dots$$

and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2 [1 + O(\lambda^2)]$$

where it often dominates the theoretical uncertainty.

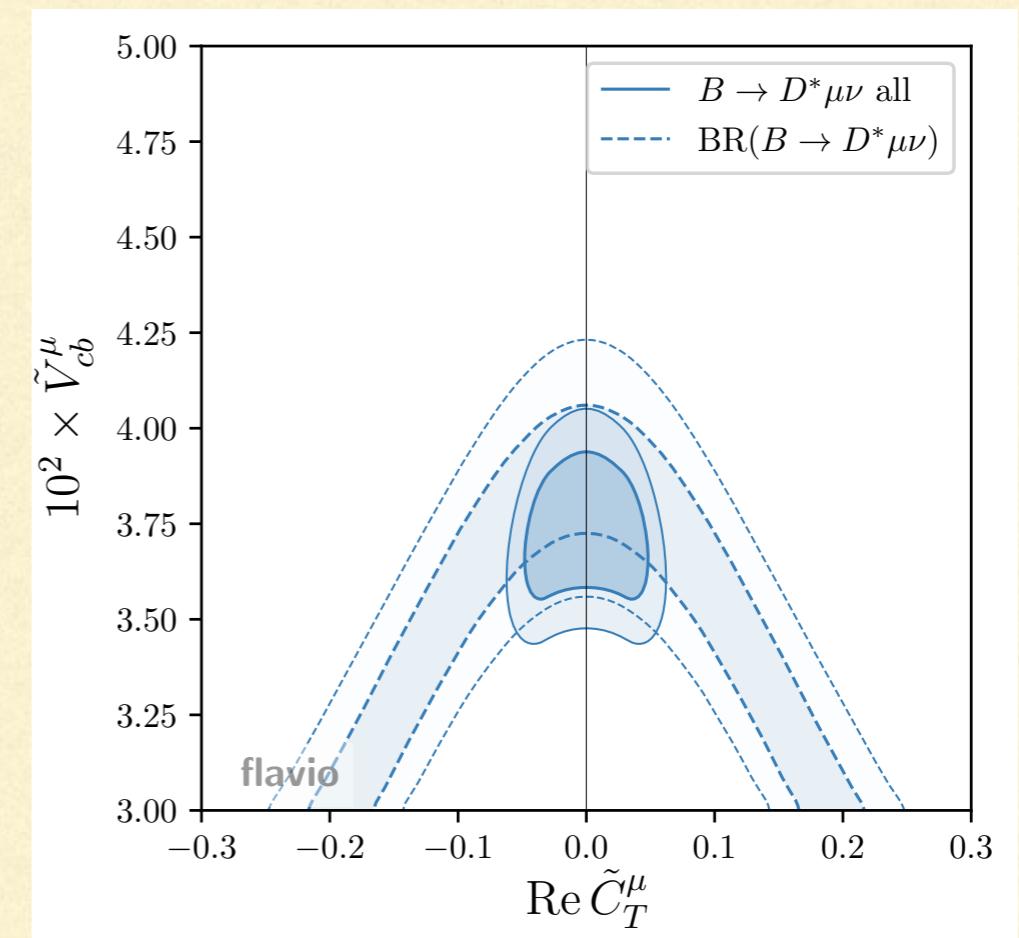
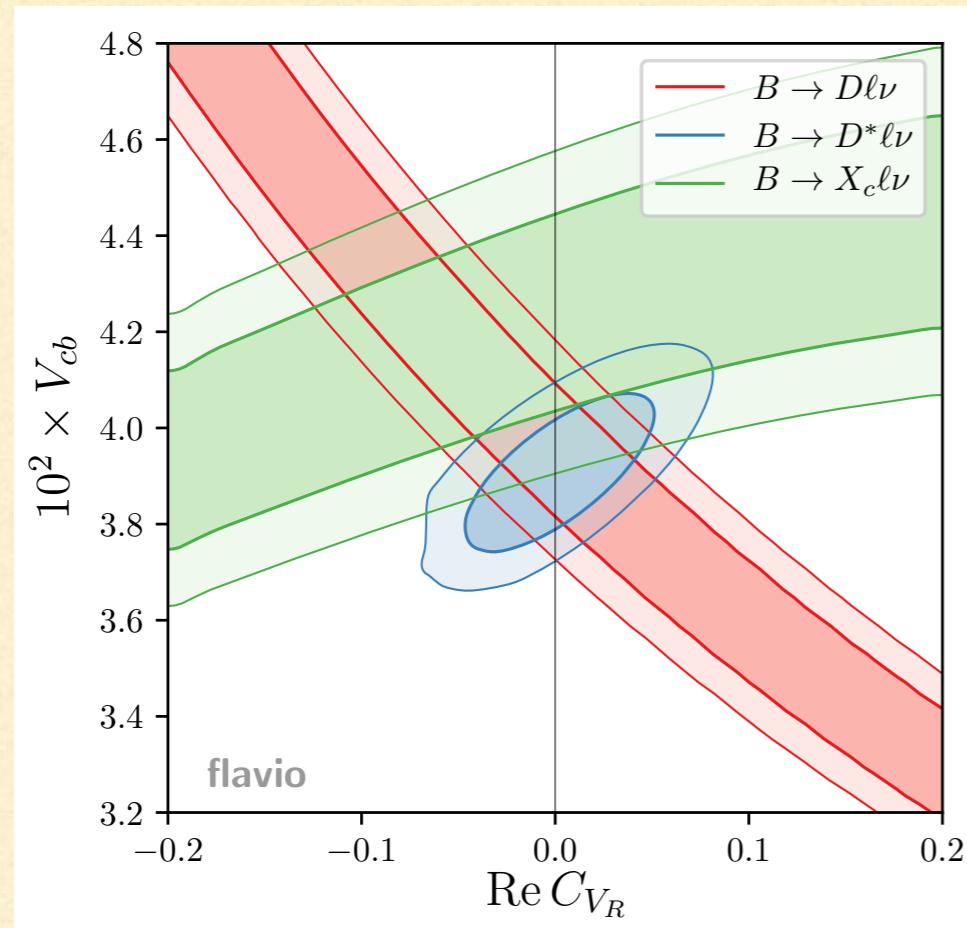
V_{ub}/V_{cb} constrains directly the UT



Our ability to determine precisely V_{cb} is crucial for indirect NP searches

NEW PHYSICS?

Jung & Straub, 1801.01112

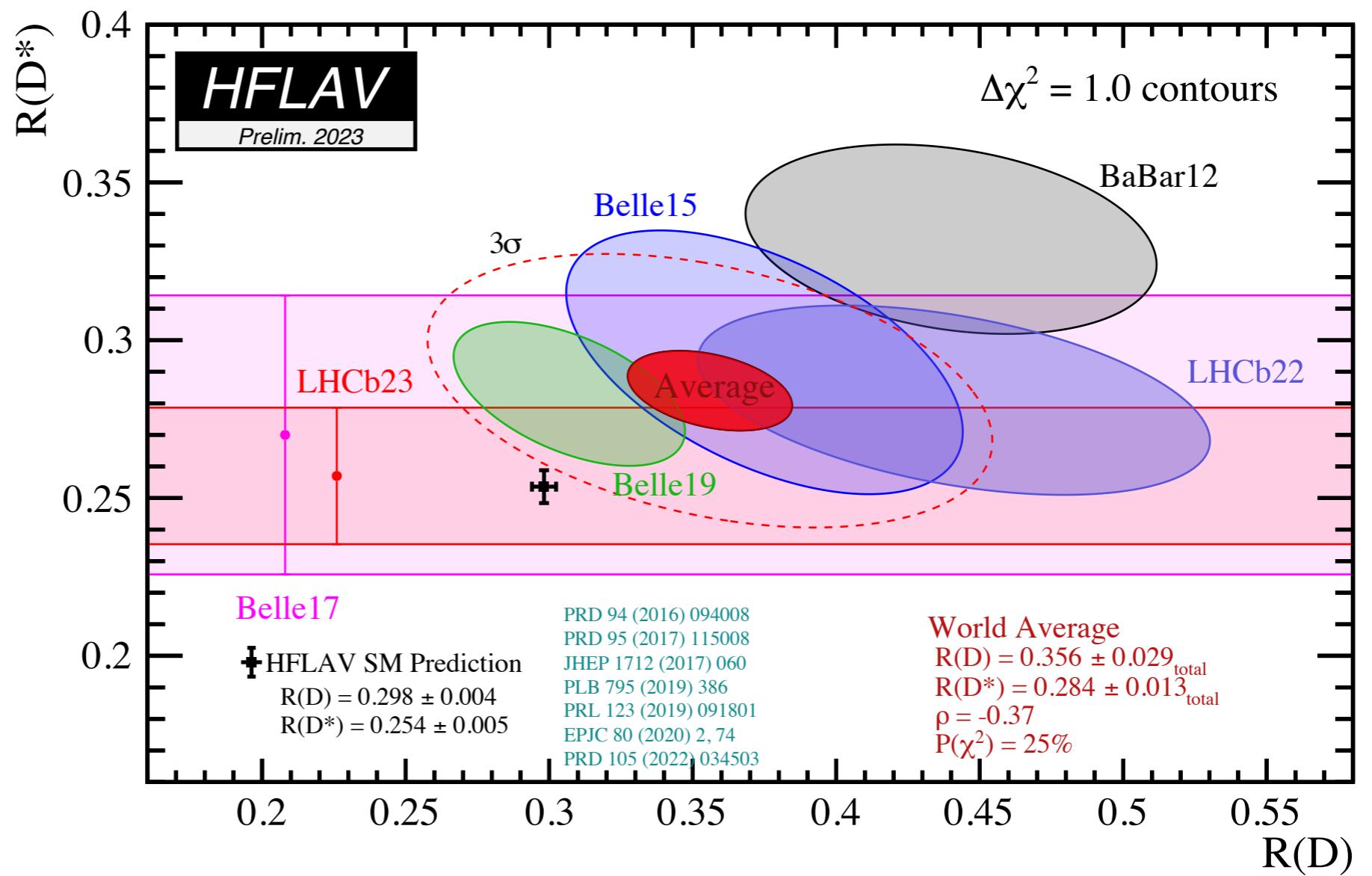


Differential distributions constrain NP strongly, SMEFT interpretation incompatible with LEP data: Crivellin, Pokorski, Jung, Straub...

VIOLATION of LFU with TAUS

SM predictions
based on
same theory
as V_{cb} extraction

$$R(D^{(*)}) = \frac{\mathcal{B}(B \rightarrow D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \rightarrow D^{(*)}\ell\nu_\ell)}$$



INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ/m_b and a_s

$$\begin{aligned} M_i = & M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2} \\ & + \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \dots \end{aligned}$$

Global **shape** parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total **rate** about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks:
they are useful in many applications (rare decays, V_{ub}, \dots)

Reliability of the method depends on our control of higher order effects. Quark-hadron duality violation would manifest itself as inconsistency in the fit.

Kinetic scheme fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice, and recent $O(\alpha_s^3)$ contribution to width.

3LOOP CALCULATIONS

Fael, Schoenwald, Steinhauser, 2011.11655, 2011.13654, 2205.03410

3loop and 2loop charm mass effects in relation between kinetic and $\overline{\text{MS}}$ b mass

$$m_b^{kin}(1\text{GeV}) = \left[4163 + 259_{\alpha_s} + 78_{\alpha_s^2} + 26_{\alpha_s^3} \right] \text{MeV} = (4526 \pm 15) \text{MeV}$$

Using FLAG $\bar{m}_b(\bar{m}_b) = 4.198(12)\text{GeV}$ one gets $m_b^{kin}(1\text{GeV}) = 4.565(19)\text{GeV}$

3loop correction to **total semileptonic width**

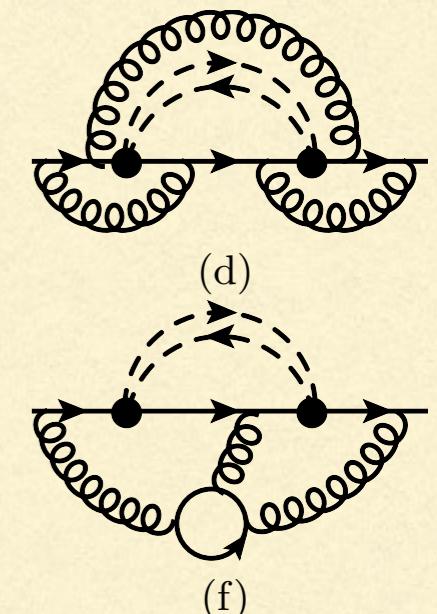
$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[0.9255 - 0.1162_{\alpha_s} - 0.0350_{\alpha_s^2} - 0.0097_{\alpha_s^3} \right]$$

in the kin scheme with $\mu = 1\text{GeV}$ and $\bar{m}_c(3\text{GeV}) = 0.987\text{GeV}$, $\mu_{\alpha_s} = m_b^{kin}$

$$\Gamma_{sl} = \Gamma_0 f(\rho) \left[0.9255 - 0.1140_{\alpha_s} - 0.0011_{\alpha_s^2} + 0.0103_{\alpha_s^3} \right]$$

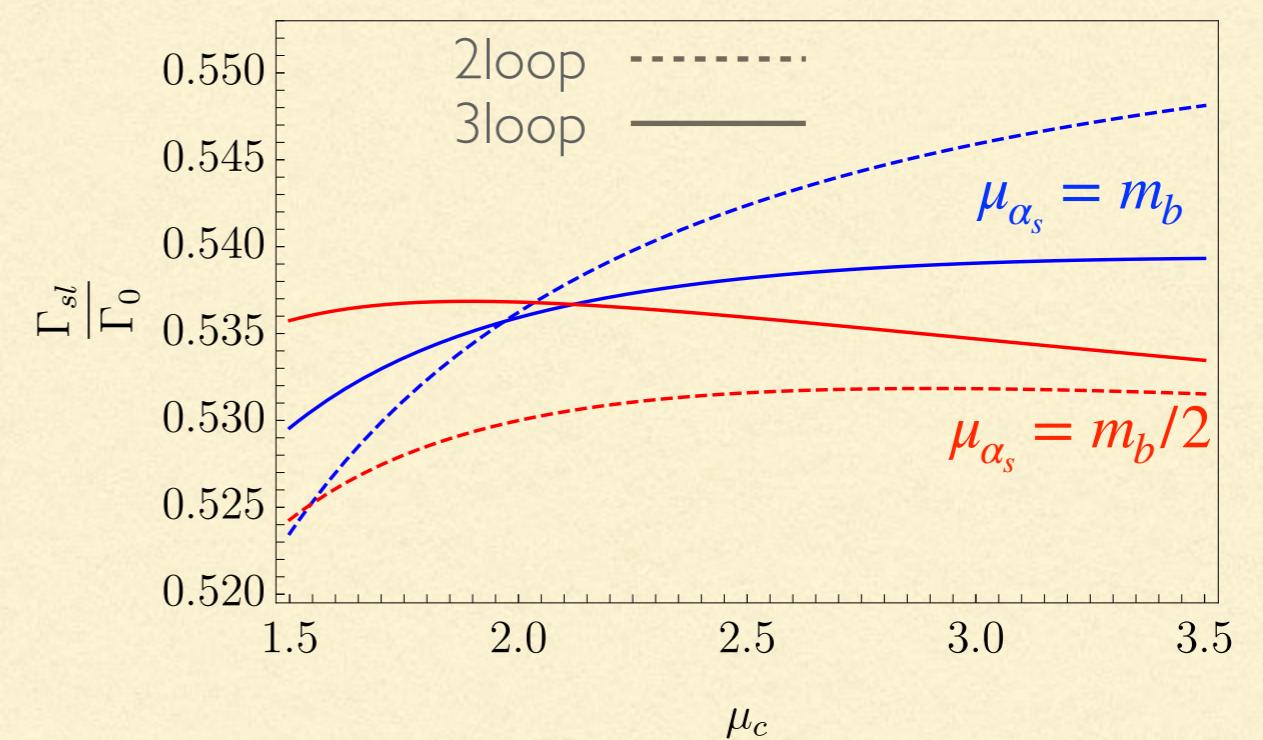
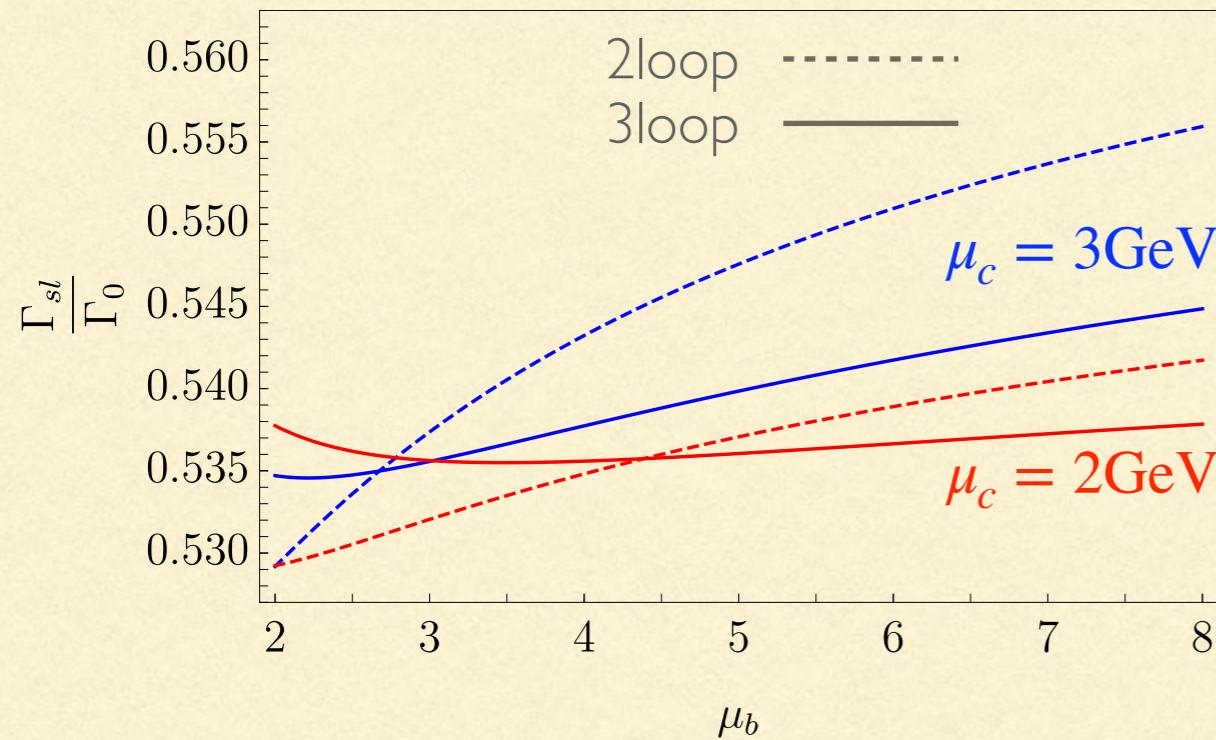
in the kin scheme with $\mu = 1\text{GeV}$ and $\bar{m}_c(2\text{GeV}) = 1.091\text{GeV}$, $\mu_{\alpha_s} = m_b^{kin}/2$

3loop correction tends to lower Γ_{sl} and therefore pushes $|V_{cb}|$ slightly up ($\sim 0.5\%$)



RESIDUAL UNCERTAINTY on Γ_{sl}

Bordone, Capdevila, PG, 2107.00604



Similar reduction in μ_{kin} dependence. Purely perturbative uncertainty $\pm 0.7\%$ (max spread), central values at $\mu_c = 2\text{GeV}$, $\mu_{\alpha_s} = m_b/2$.

$O(\alpha_s/m_b^2, \alpha_s/m_b^3)$ effects in the width are known. Additional uncertainty from higher power corrections, soft charm effects of $O(\alpha_s/m_b^3 m_c)$, duality violation.

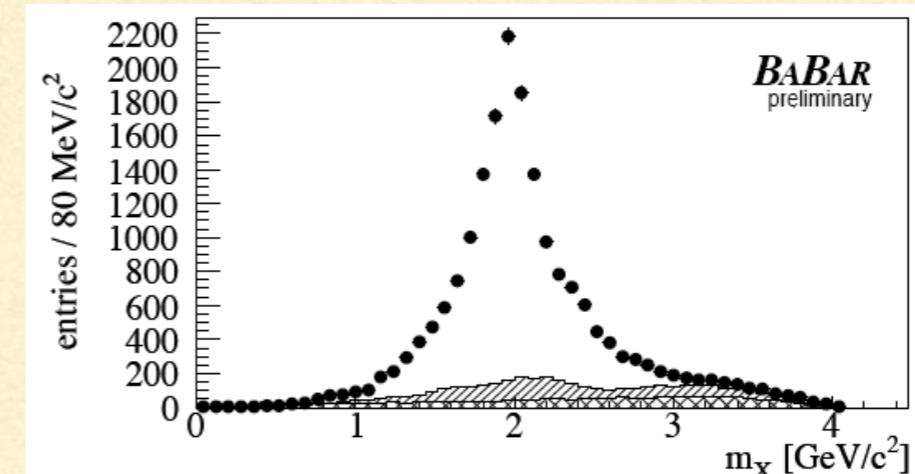
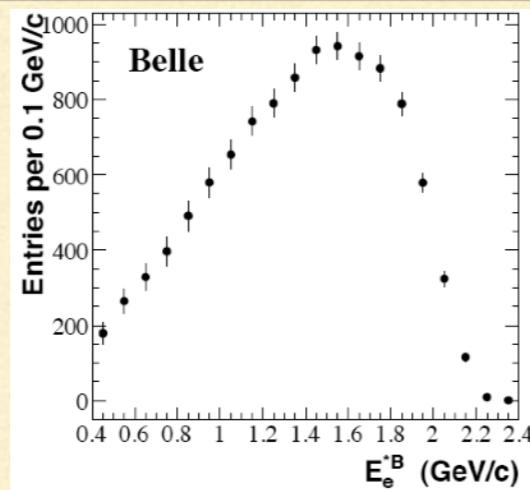
Conservatively: 1.2% overall theory uncertainty in Γ_{sl} (a ~50% reduction)

Interplay with fit to semileptonic moments, known only to $O(\alpha_s^2, \alpha_s \Lambda^2/m_b^2)$

INCLUSIVE SEMILEPTONIC FITS

Bordone, Capdevila, PG, 2107.00604

Electron energy
and invariant
hadronic mass
spectra



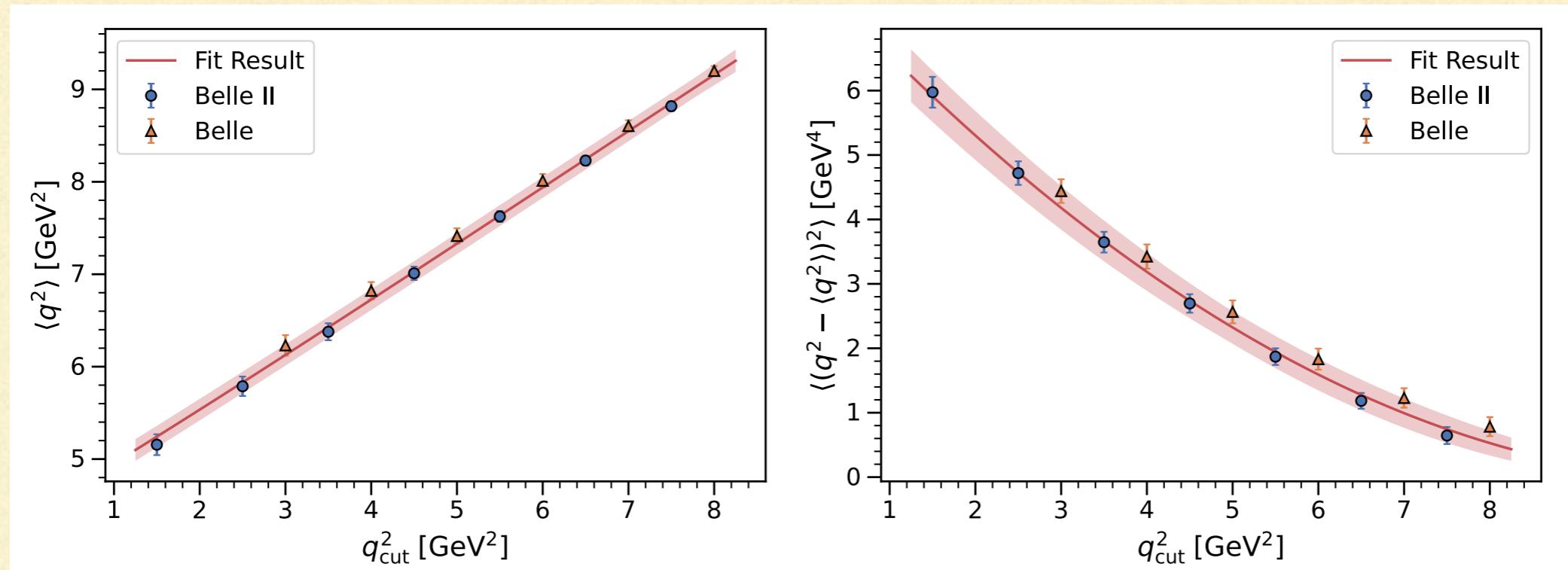
m_b^{kin}	$\bar{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_G^2(m_b)$	ρ_{LS}^3	$\text{BR}_{cl\nu}$	$10^3 V_{cb} $
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

Higher power corrections see a proliferation of parameters but Wilson coefficients are known at LO. We use the Lowest Lying State Saturation Approximation (Mannel,Turczyk,Ural'tsev 1009.4622) as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

$|V_{cb}| = 42.00(53) \times 10^{-3}$

Update of 1606.06174
similar results in IS scheme Bauer et al.

q^2 MOMENTS



2205.10274

New measurements of the q^2 moments by Belle (2109.01685) and Belle II (2205.06372) not yet included in our fit (in progress).

Reparametrisation invariance implies that q^2 moments and total width depend on a smaller set of HQE parameters (Fael, Mannel, Vos), 8 at $O(1/m_b^4)$, but using only the q^2 moments: $|V_{cb}| = 41.99(65) \cdot 10^{-3}$ using the same BR input we employ (Bernlochner et al. 2205.10274)

It would be useful to measure also the FB asymmetry as proposed by Turzcyk

QED CORRECTIONS

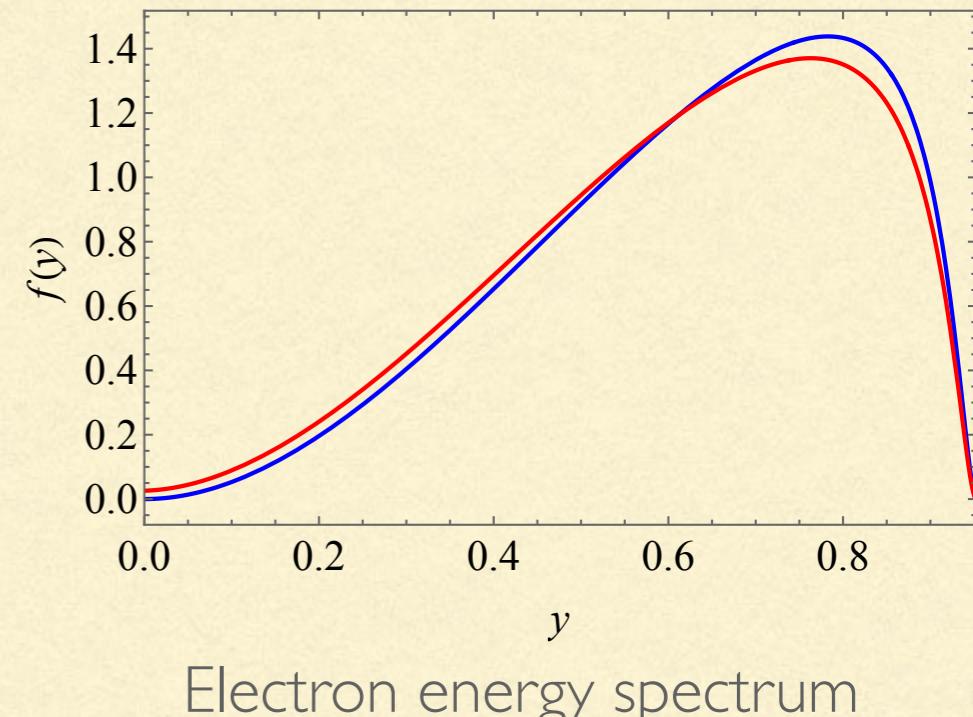
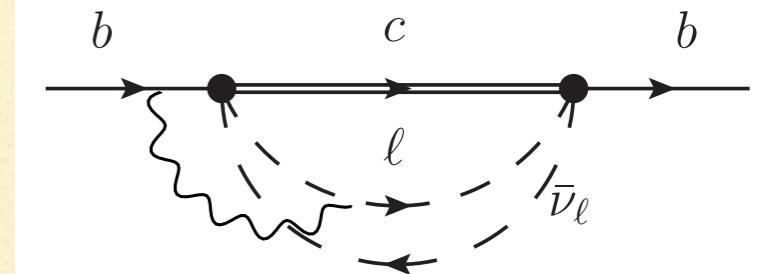
D.Bigi, Bordone, Haisch, PG
in progress

In the presence of photons, **OPE valid only for total width** and moments that do not resolve lepton properties (E_ℓ, q^2). Expect mass singularities and $O(\alpha\Lambda/m_b)$ corrections.

Leading logs $\alpha \ln m_e/m_b$ can be easily computed for simple observables using structure function approach, for ex the lepton energy spectrum

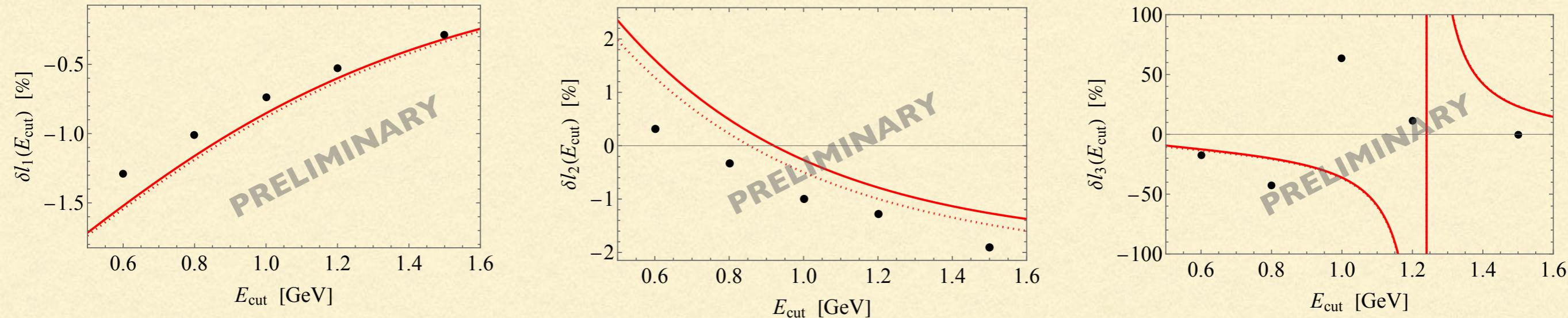
$$\left(\frac{d\Gamma}{dy} \right)^{(1)} = \frac{\alpha}{2\pi} \ln \frac{m_b^2}{m_\ell^2} \int_y^1 \frac{dx}{x} P_{\ell\ell}^{(0)} \left(\frac{y}{x} \right) \left(\frac{d\Gamma}{dx} \right)^{(0)}$$

$$P_{\ell\ell}^{(0)}(z) = \left[\frac{1+z^2}{1-z} \right]_+$$



LEPTONIC MOMENTS

D.Bigi, Bordone, Haisch, PG
in progress

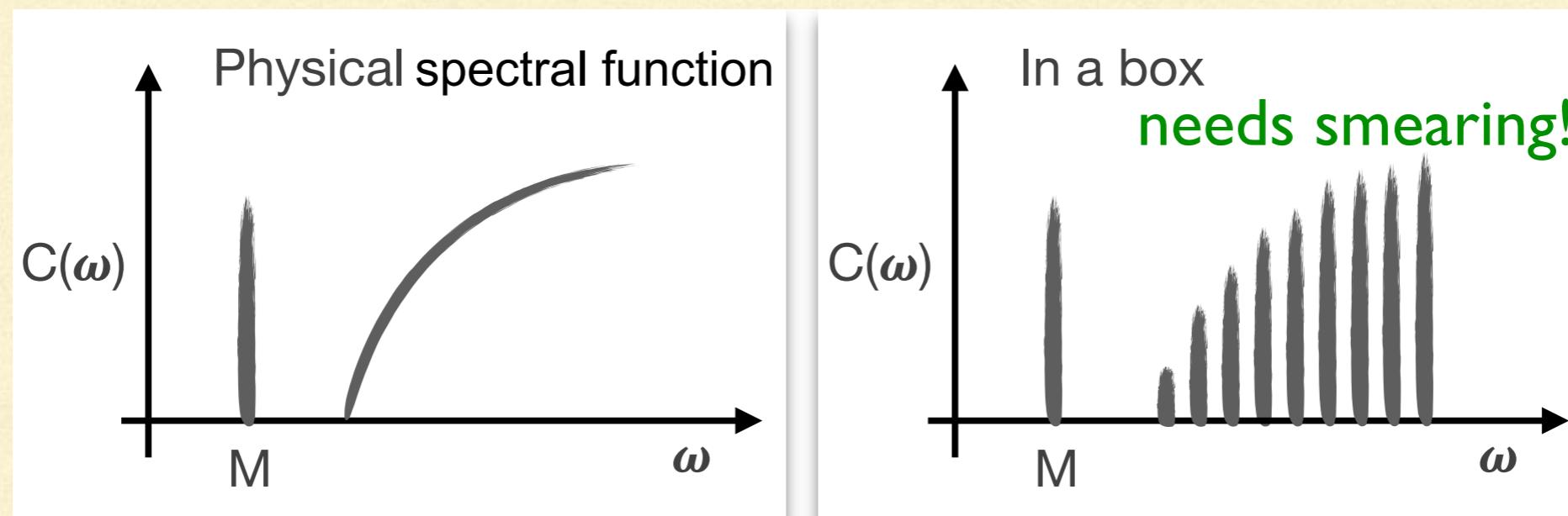


Typically exp measurements are completely inclusive, $B \rightarrow X_c \ell \nu(\gamma)$, but QED radiation is **subtracted** by experiments using **PHOTOS** (soft-collinear photon radiation to MC final states).

BaBar hep-ex/0403030 provides both uncorrected and corrected lepton moments, allowing for comparison with our inclusive LL calculation. Shifts are $0.2\text{-}0.7\sigma_{\text{exp}}$ but NLO logs and power corrections can be also included further reducing the gap. Complete $O(\alpha)$ calculation checks subleading terms and other moments.

INCLUSIVE DECAYS ON THE LATTICE

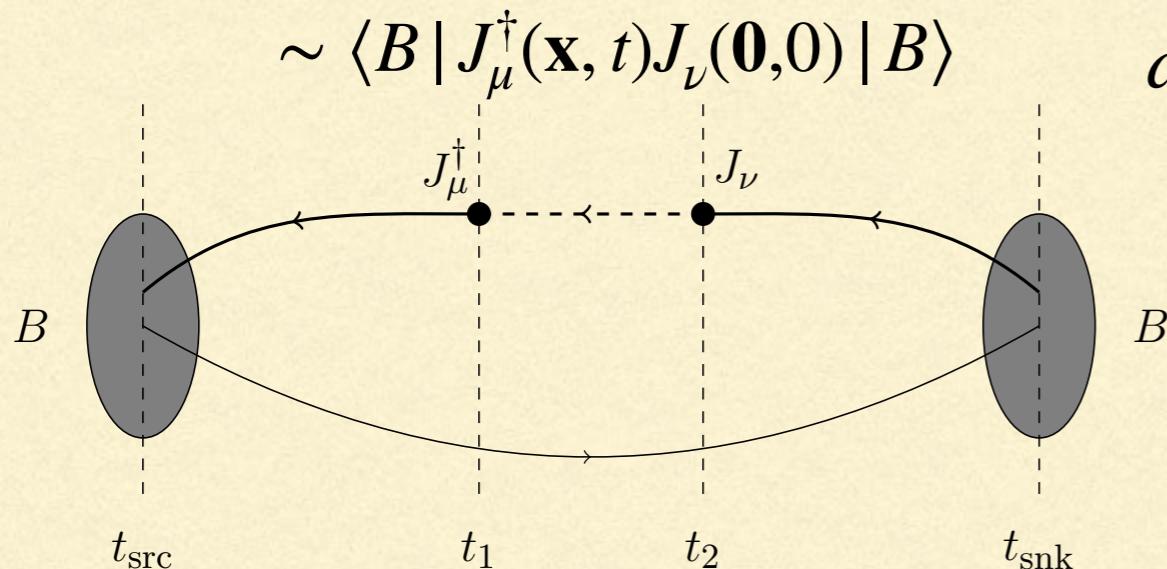
- Inclusive processes *impractical* to treat directly on the lattice. Vacuum current correlators computed in euclidean space-time are related to $e^+e^- \rightarrow$ hadrons or τ decay via analyticity. In our case the correlators have to be computed in the B meson, but analytic continuation more complicated: two cuts, decay occurs only on a portion of the physical cut.
- While the lattice calculation of the spectral density of hadronic correlators is an ***ill-posed problem***, the spectral density is accessible after smearing, as provided by phase-space integration Hansen, Meyer, Robaina, Hansen, Lupo, Tantalo, Bailas, Hashimoto, Ishikawa



A NEW APPROACH

Hashimoto, PG 2005.13730

4-point functions on the lattice are related to the hadronic tensor in euclidean

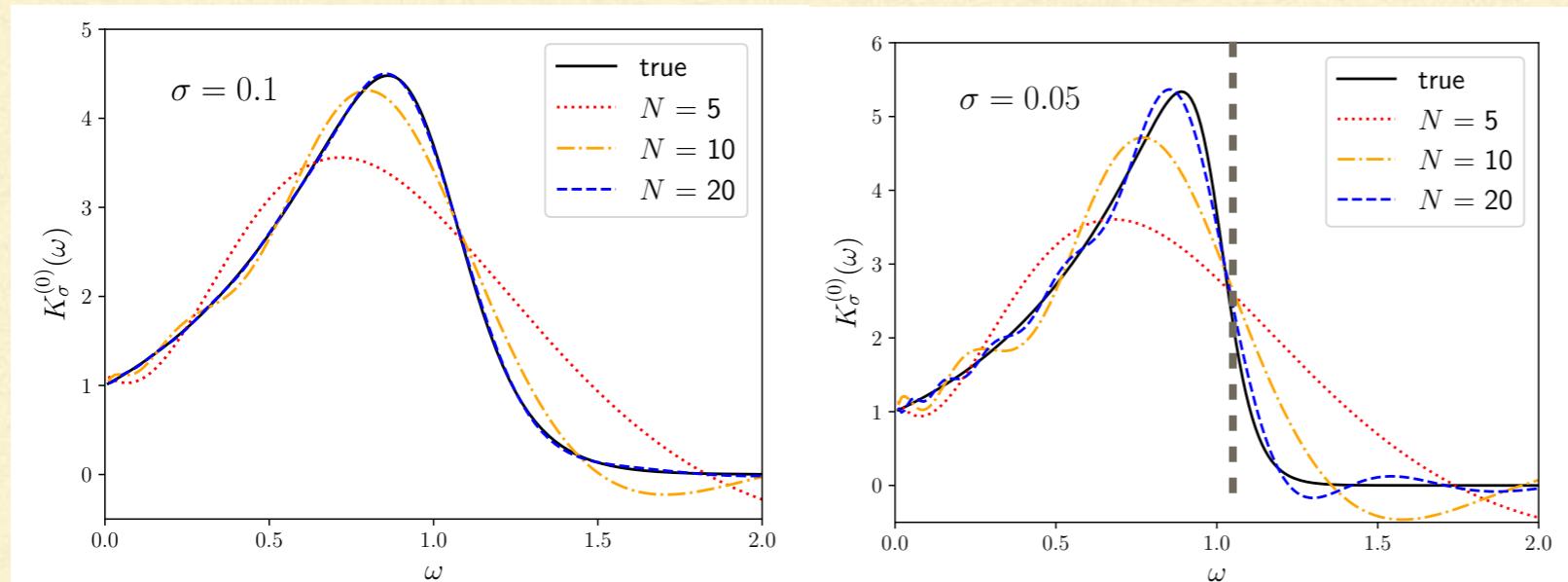


$$d\Gamma \sim L^{\mu\nu} W_{\mu\nu}, \quad W_{\mu\nu} \sim \sum_X \langle B | J_\mu^\dagger | X \rangle \langle X | J_\nu | B \rangle$$

$$\int d^3x \frac{e^{i\mathbf{q}\cdot\mathbf{x}}}{2M_B} \langle B | J_\mu^\dagger(\mathbf{x}, t) J_\nu(\mathbf{0}, 0) | B \rangle \sim \int_0^\infty d\omega W_{\mu\nu} e^{-t\omega}$$

$$\text{smearing kernel } f(\omega) = \sum_n a_n e^{-na\omega}$$

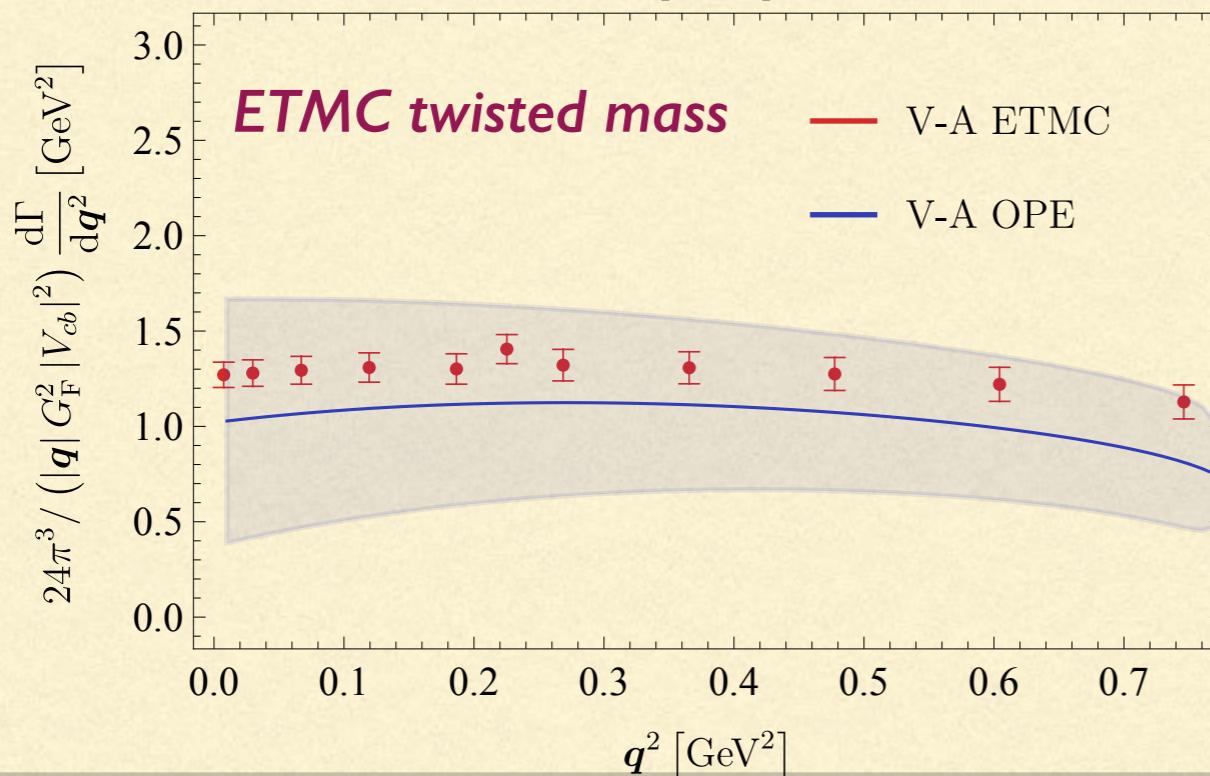
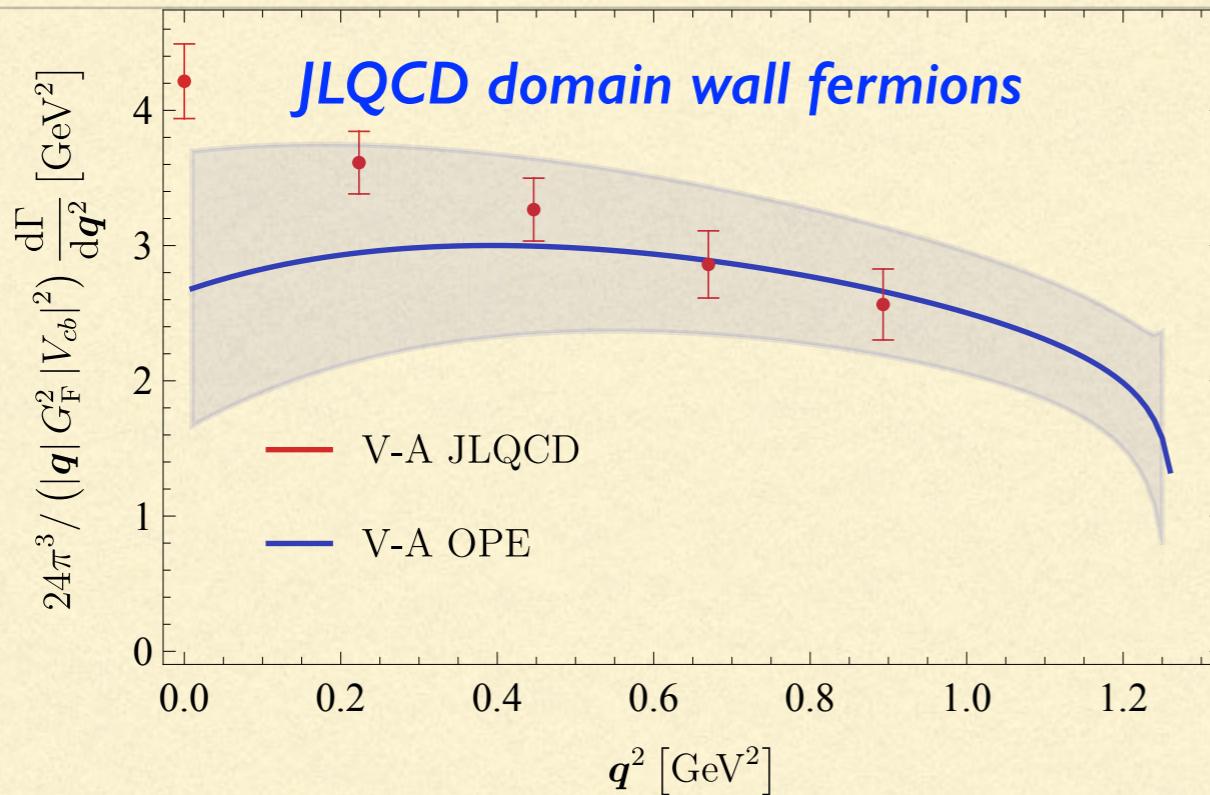
The necessary smearing is provided by phase space integration over the hadronic energy, which is cut by a θ with a sharp hedge: sigmoid $1/(1 + e^{x/\sigma})$ can be used to replace kinematic $\theta(x)$ for $\sigma \rightarrow 0$. Larger number of polynomials needed for small σ



Two methods based on Chebyshev polynomials and Backus-Gilbert. Important:

$$\lim_{\sigma \rightarrow 0} \lim_{V \rightarrow \infty} X_\sigma$$

LATTICE VS OPE



m_b^{kin} (JLQCD)	2.70 ± 0.04
$\overline{m}_c(2 \text{ GeV})$ (JLQCD)	1.10 ± 0.02
m_b^{kin} (ETMC)	2.39 ± 0.08
$\overline{m}_c(2 \text{ GeV})$ (ETMC)	1.19 ± 0.04
μ_π^2	0.57 ± 0.15
ρ_D^3	0.22 ± 0.06
$\mu_G^2(m_b)$	0.37 ± 0.10
ρ_{LS}^3	-0.13 ± 0.10
$\alpha_s^{(4)}(2 \text{ GeV})$	0.301 ± 0.006

OPE inputs from fits to exp data (physical m_b), HQE of meson masses on lattice

1704.06105, J.Phys.Conf.Ser. 1137 (2019) 1, 012005

We include $O(1/m_b^3)$ and $O(\alpha_s)$ terms

Hard scale $\sqrt{m_c^2 + \mathbf{q}^2} \sim 1 - 1.5 \text{ GeV}$

We do not expect OPE to work at high $|\mathbf{q}|$

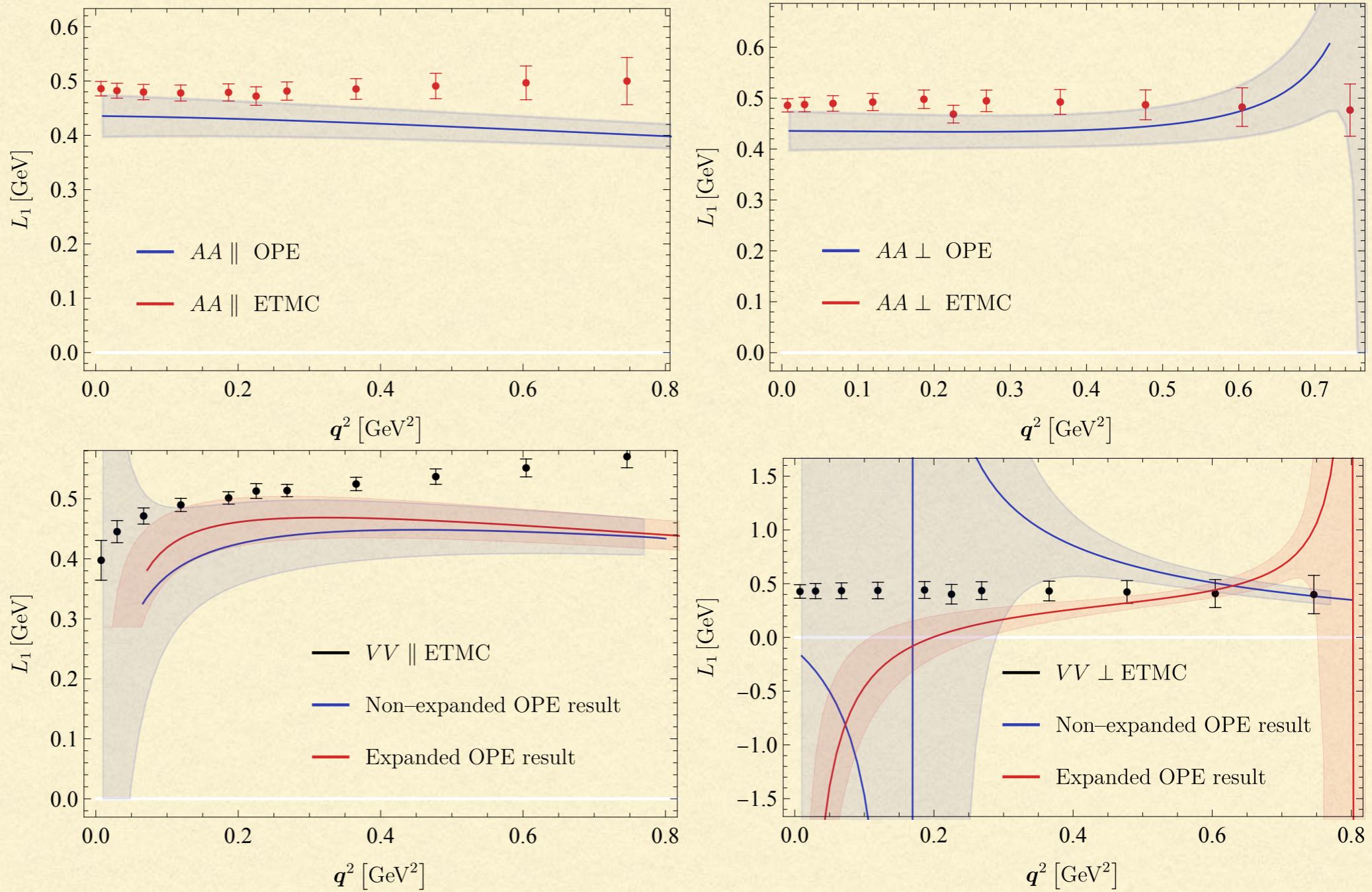
Twisted boundary conditions allow for any value of \vec{q}^2

Smaller statistical uncertainties

MOMENTS

PG, Hashimoto, Maechler, Panero, Sanfilippo, Simula, Smecca, Tantalo, 2203.11762

$$L_1 = \langle E_\ell(\mathbf{q}^2) \rangle$$



smaller errors, cleaner comparison with OPE, individual channels AA, VV, parallel and perpendicular polarization, could help extracting its parameters

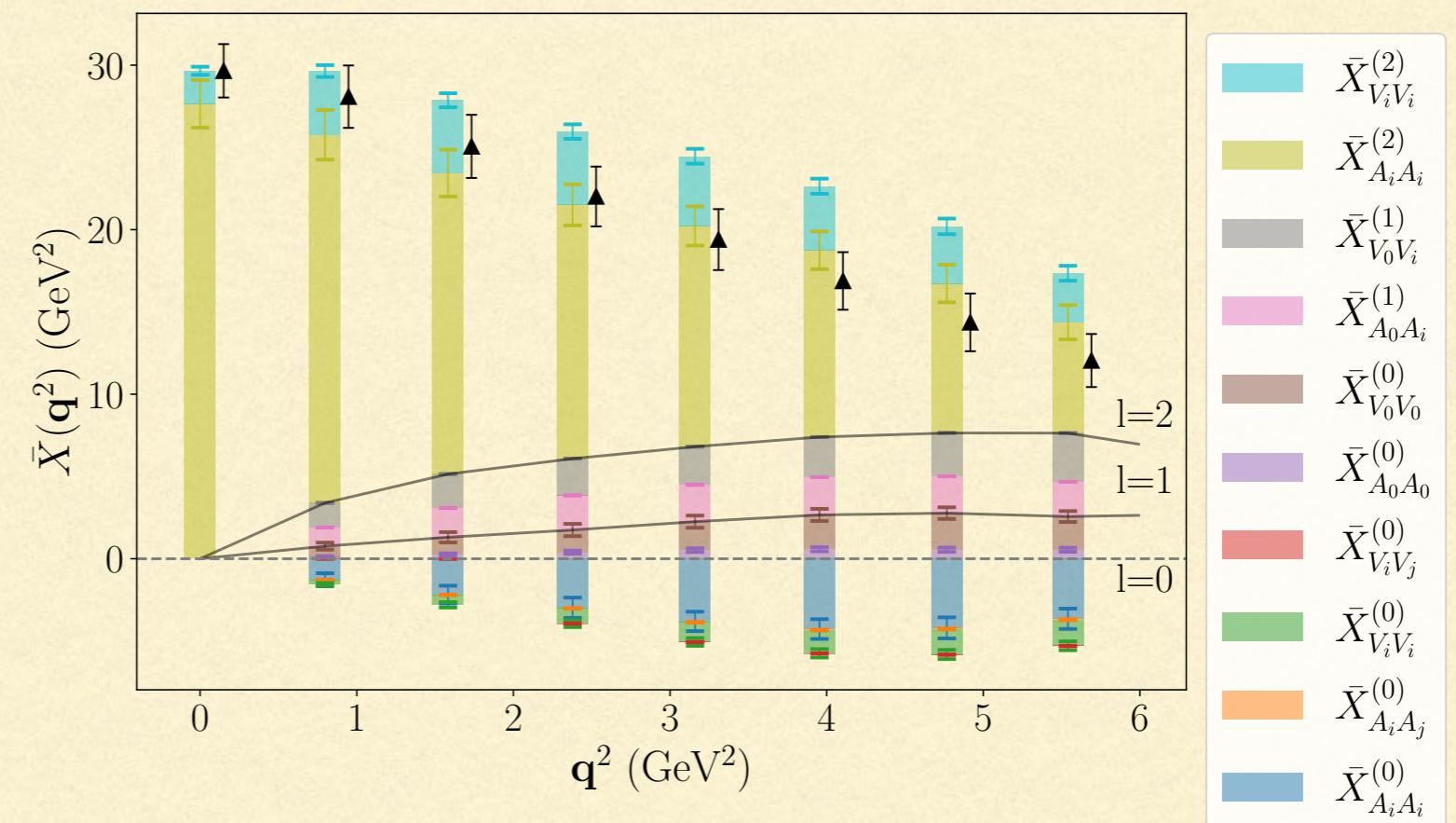
First results at the physical b mass

Relativistic heavy quark
effective action for b

B_s decays,
domain wall fermions,
improved implementation
of Chebychev polynomials
and Backus-Gilbert

qualitative study
~5% statistical uncertainty
on total width

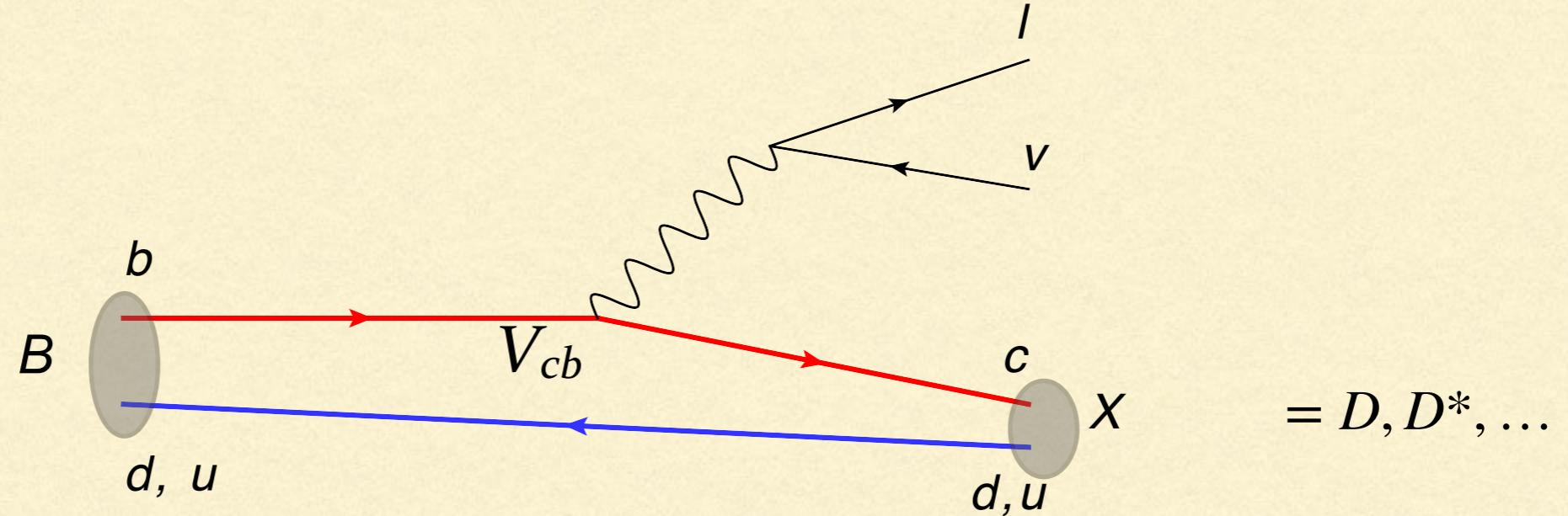
possibly better to compare
with partial width at low \vec{q}^2



Barone, Hashimoto, Juttner, Kaneko, Kellermann, 2305.14092

Ongoing work on **semileptonic D_s decays** by two collaborations

EXCLUSIVE DECAYS



There are 1(2) and 3(4) FFs for D and D^* for light (heavy) leptons, for instance

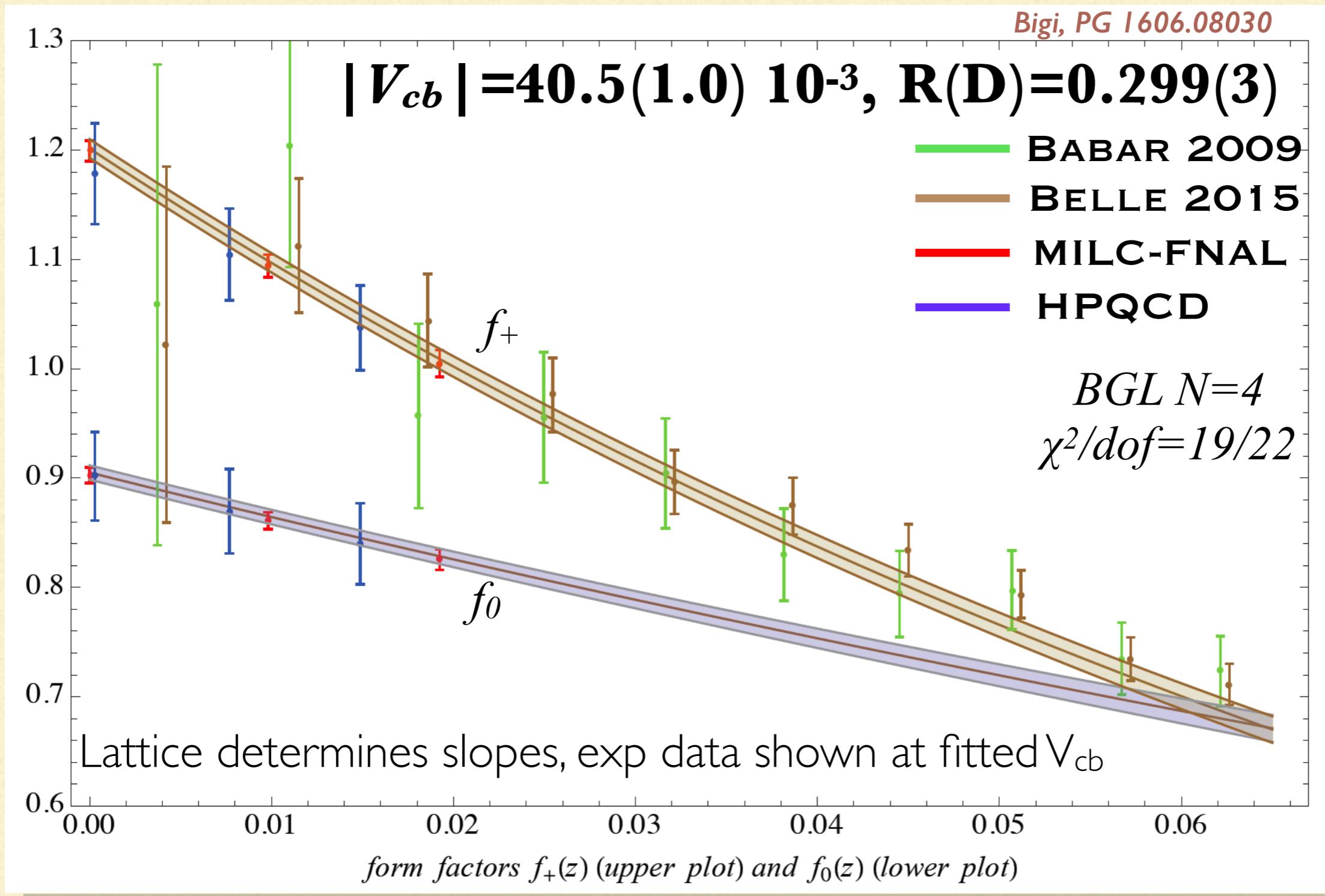
$$\langle D(k) | \bar{c} \gamma^\mu b | \bar{B}(p) \rangle = \left[(p+k)^\mu - \frac{M_B^2 - M_D^2}{q^2} q^\mu \right] f_+^{B \rightarrow D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^\mu f_0^{B \rightarrow D}(q^2)$$

Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp, extrapolation, unitarity constraints, ...

A **model independent parametrization** is very useful

Boyd, Grinstein, Lebed (BGL) and many others

LATTICE + EXP BGL FIT for $B \rightarrow D\ell\nu$



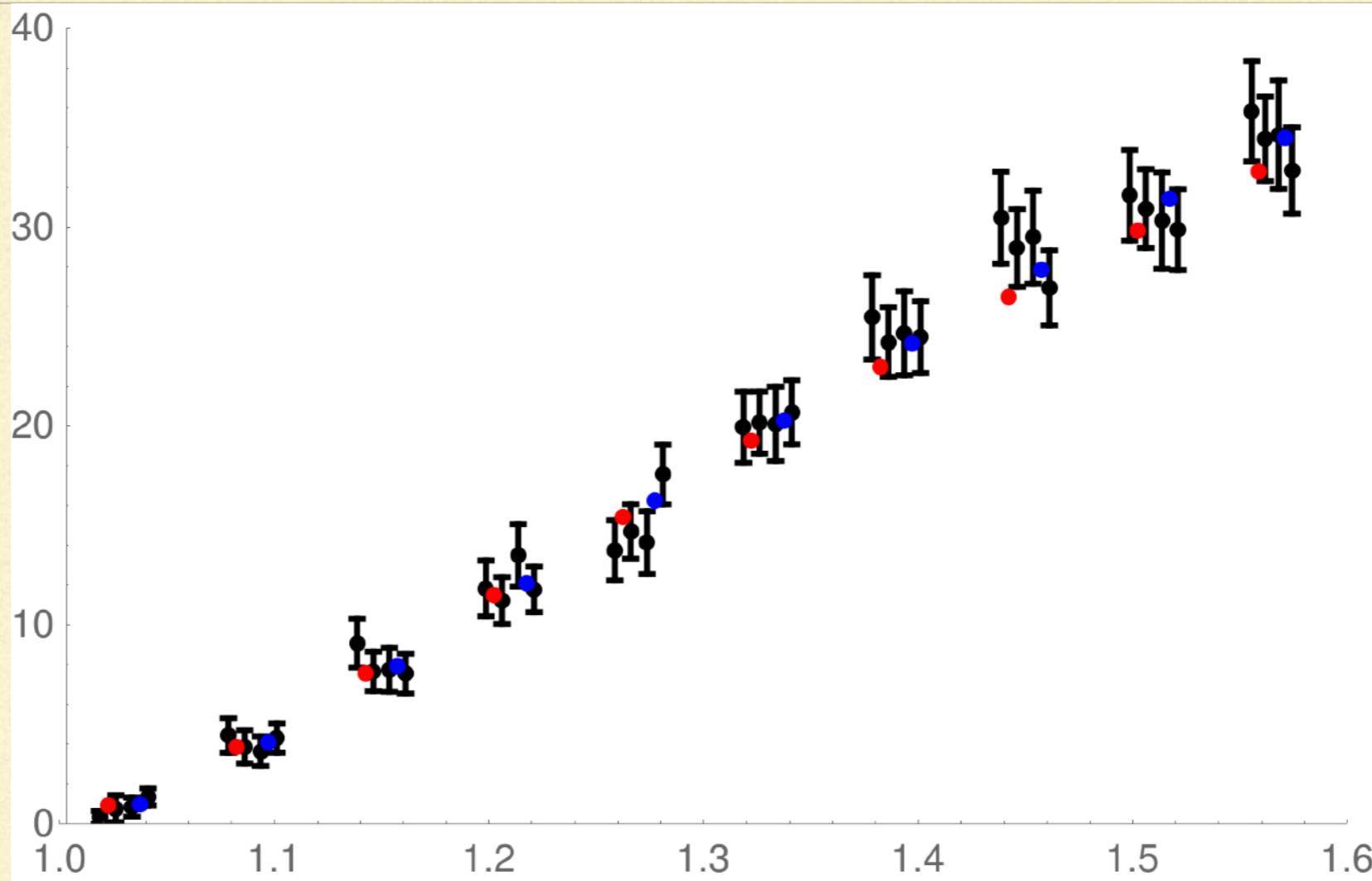
$R(D) = 0.299(3)$

1.3σ from exp

FLAG has
very similar
results

CLN cannot
fit both ff

w DISTRIBUTION for $B \rightarrow D\ell\nu$



Belle 2015 consider 4 channels ($B^{0,+}, e, \mu$) for each bin.
 Average (red points) usually lower than all central values. **D'Agostini bias?**
 Blue points are average of normalised bins.

Standard fit to Belle15+FNAL+HPQCD: $|V_{cb}| = 40.9(1.2) 10^{-3}$

Jung, PG

Fit to normalised bins+width Belle15+FNAL+HPQCD: $|V_{cb}| = 41.9(1.2) 10^{-3}$

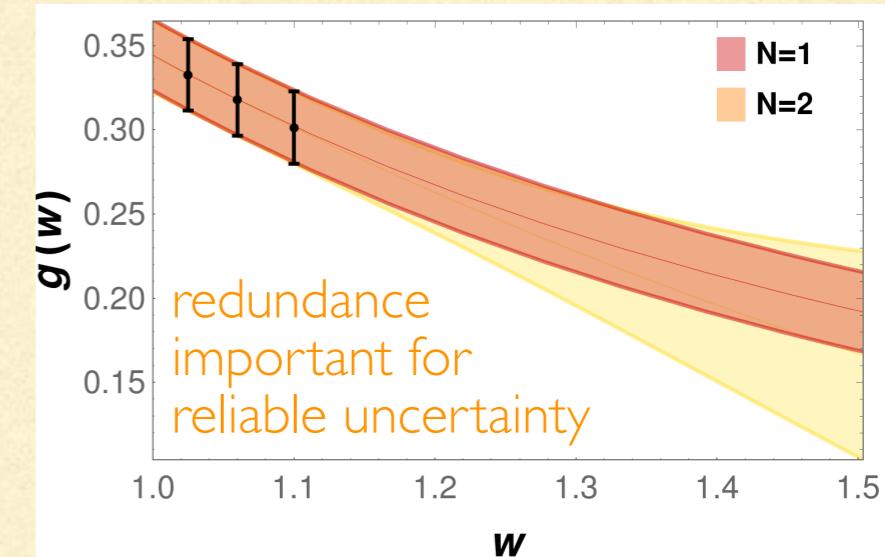
TRUNCATION AND UNCERTAINTY

Fits with BGL parametrisation: **model**

independence vs overfitting. Where do we truncate the series? How can we include unitarity constraints? These questions are related.

Different options with various pro/cons:

1. Frequentist fits with strong χ^2 **penalty** outside unitarity; increase BGL order till χ^2_{min} is stable. Uncertainties from $\Delta\chi^2_{min} = 1$ do not have probabilistic interpretation.
Bigi, PG, 1606.08030, Jung, Schacht, PG 1905.08209
2. Frequentist fit with **Nested Hypothesis Test** to determine optimal truncation order: go to order $N + 1$ if $\Delta\chi^2 = \chi^2_{min,N} - \chi^2_{min,N+1} \geq 1$
Check unitarity a posteriori
Bernlochner et al, 1902.09553
3. **Bayesian inference** using unitarity constraints as prior with BGL Flynn, Jüttner, Tsang 2303.111285 or in the **Dispersive Matrix approach**, Martinelli, Simula, Vittorio et al. 2105.02497



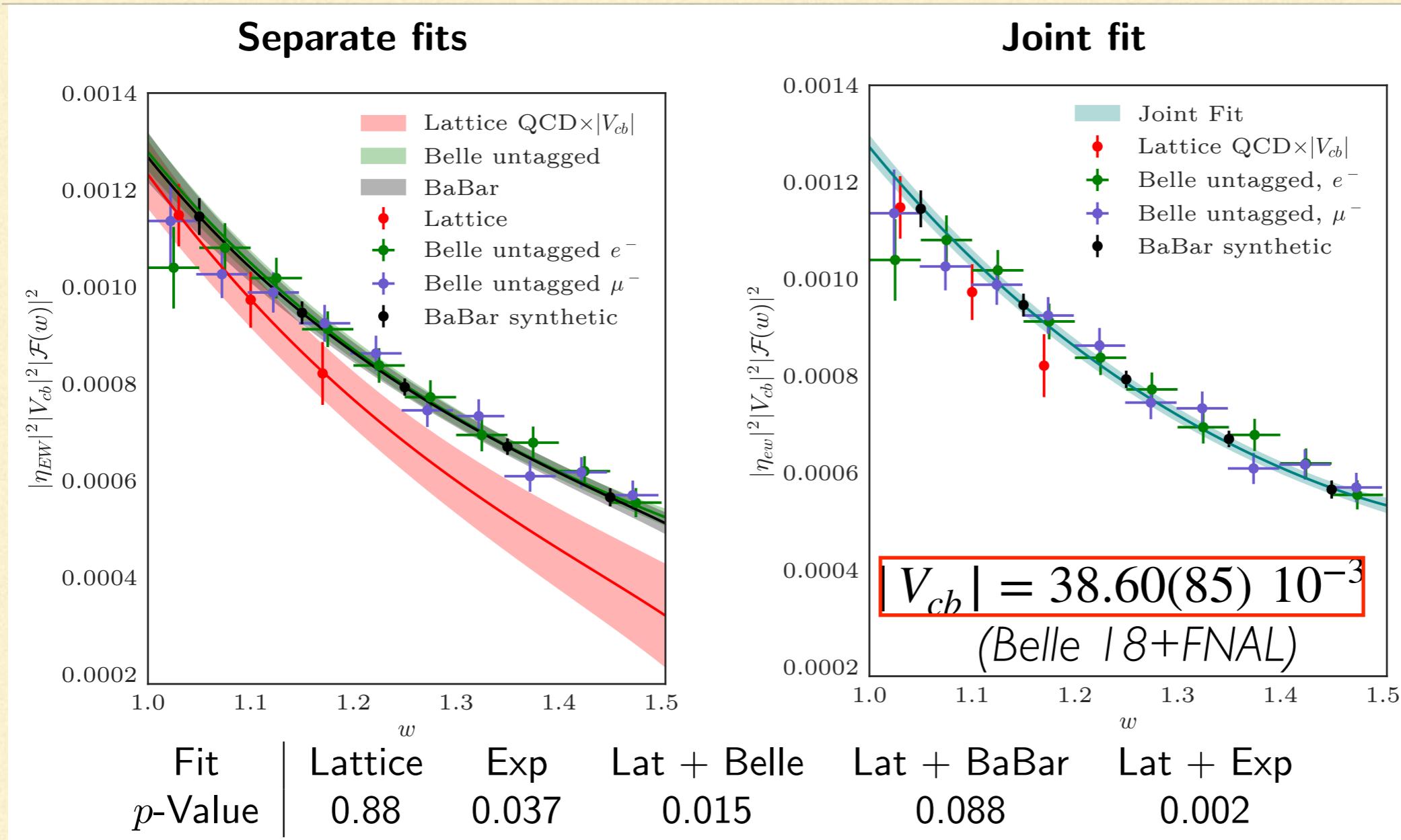
$|V_{cb}|$ from $B \rightarrow D^* l \nu$

More complicated: 4 FFs, angular spectra, D^* unstable. **Present status unclear.**

1. **Parametrisations matter and the related uncertainties require careful consideration.** Belle 2017 dataset analysed with BGL or CLN leads to 6-8% difference in $|V_{cb}|$. Bigi, PG, Schacht, Grinstein, Kobach
Discard old exp results obtained with CLN and provide data in a parametrisation independent way.
2. Despite recent progress, **lattice calculations** are indecisive. Tension between **Fermilab/MILC** 2021 and HPQCD 2023 results at non-zero recoil and **Belle** untagged 2018 data, while **JLQCD** preliminary results give a consistent picture.
3. Most precise data from Belle 2018 untagged (although μ/e 4σ tension in the FB asymmetry)
Bobeth, Bordone, van Dyk, Gubernari, Jung
Belle 2023 (2301.07529) tagged consistent with Belle 2018, *other recent exp analyses (Babar 1903.10002, Belle II presented at Moriond 2023, LHCb 2001.03225) have conflicting results for V_{cb} , see next talk,* but data not yet available for independent fits.

FERMILAB/MILC CALCULATION

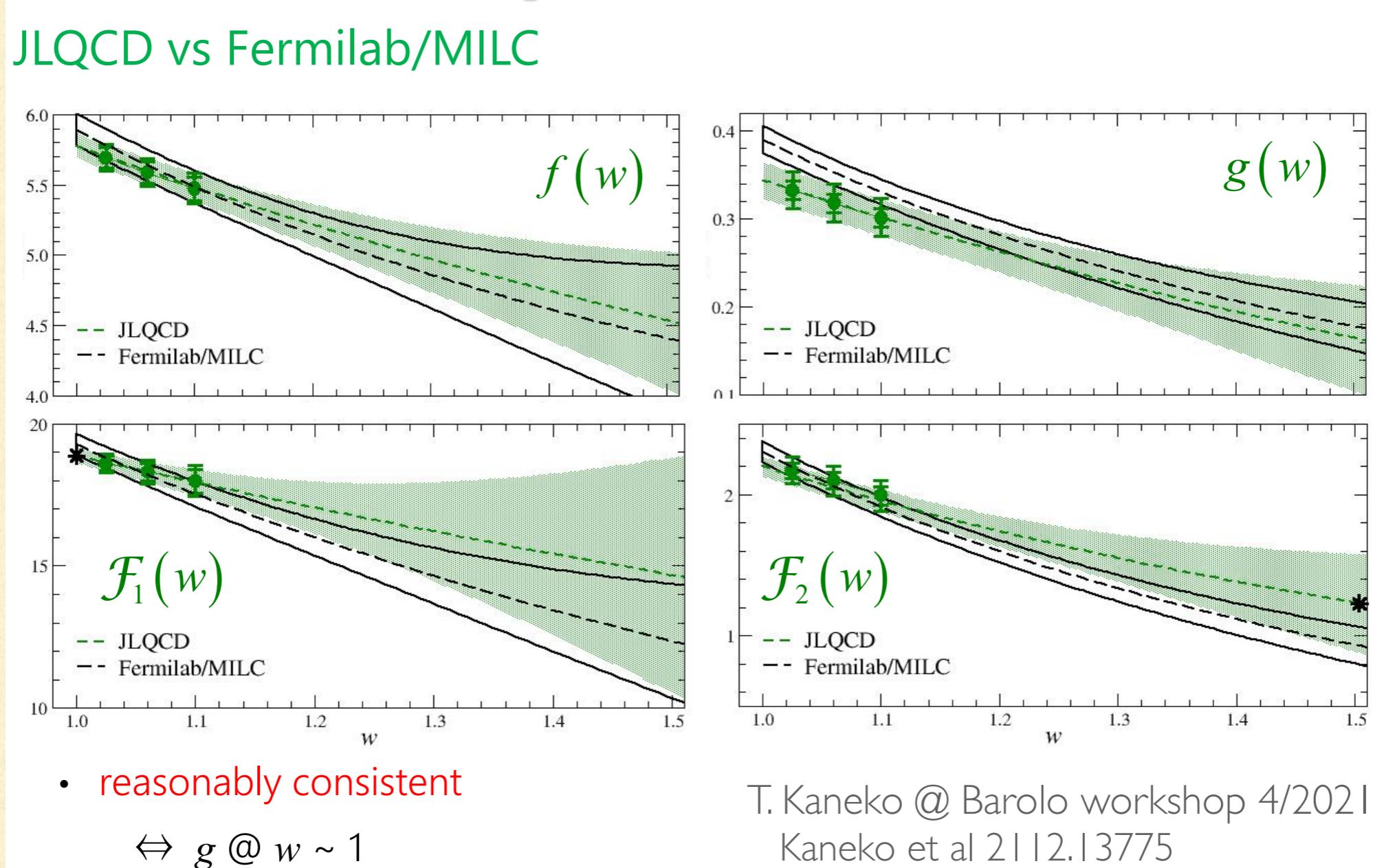
2105.14019



First lattice calculation beyond zero recoil for this mode

Our analysis of Belle I8+ FNAL data (Jung, PG):
 $|V_{cb}| = 39.4(9) 10^{-3} (\chi^2_{min} = 50)$ using only total rate $|V_{cb}| = 42.2^{+2.8}_{-1.7} 10^{-3}$

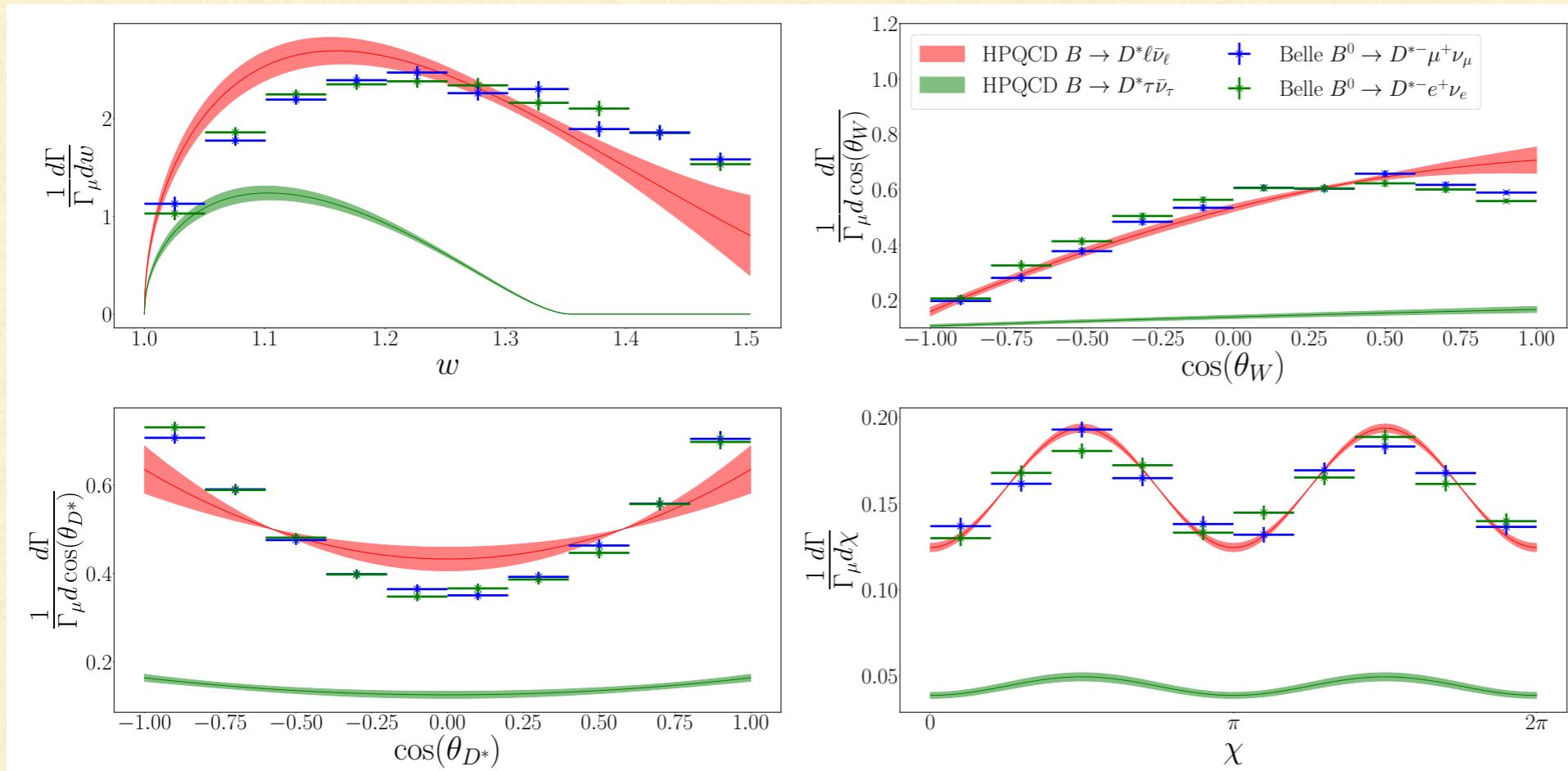
JLQCD PRELIMINARY RESULTS



Our analysis of Belle I8+ JLQCD data (Jung, PG):
 $|V_{cb}| = 40.7(9) \cdot 10^{-3}$ ($\chi^2_{min} = 33$) using only total rate $|V_{cb}| = 40.8^{+1.8}_{-2.3} \cdot 10^{-3}$

NEW HPQCD FFS CALCULATION

2304.03137



Extrapolation in m_h , data cover the whole w region

Our analysis of Belle 18+ HPQCD data (Jung, PG):
 $|V_{cb}| = 40.4(8) \cdot 10^{-3}$ using only total rate $|V_{cb}| = 44.4 \pm 1.6 \cdot 10^{-3}$

HPQCD and FNAL are not well compatible: adding 16 FNAL points increases χ^2 by 35

Tension with Belle 2018
data similar to FNAL

Belle 18+HPQCD

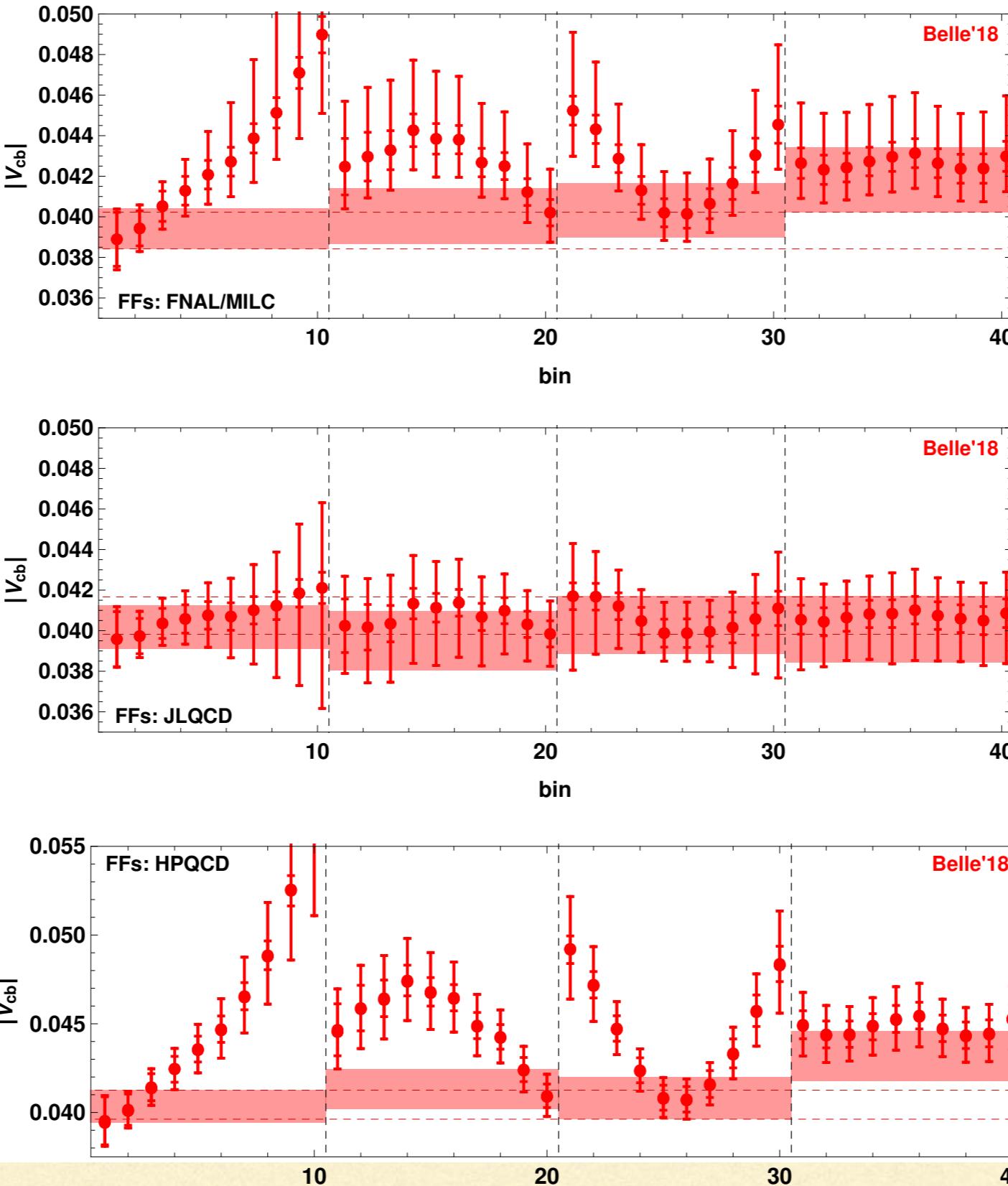
BGL exp	χ^2	$ V_{cb} $
0001	78	41.0(8)
0101	68	41.2(8)
0111	57	40.8(8)
1111	57	40.8(8)
1121	54	40.6(8)
1222	52	40.6(8)
2222	50	40.4(8)
2232	50	40.4(8)
3333	50	40.4(8)

FNAL/MILC

JLQCD

HPQCD

Binned V_{cb} from Belle'18 data: FNAL/MILC vs JLQCD



Binned analysis
proposed
by Martinelli,
Simula,Vittorio
in DM approach
2105.08674
2109.15248

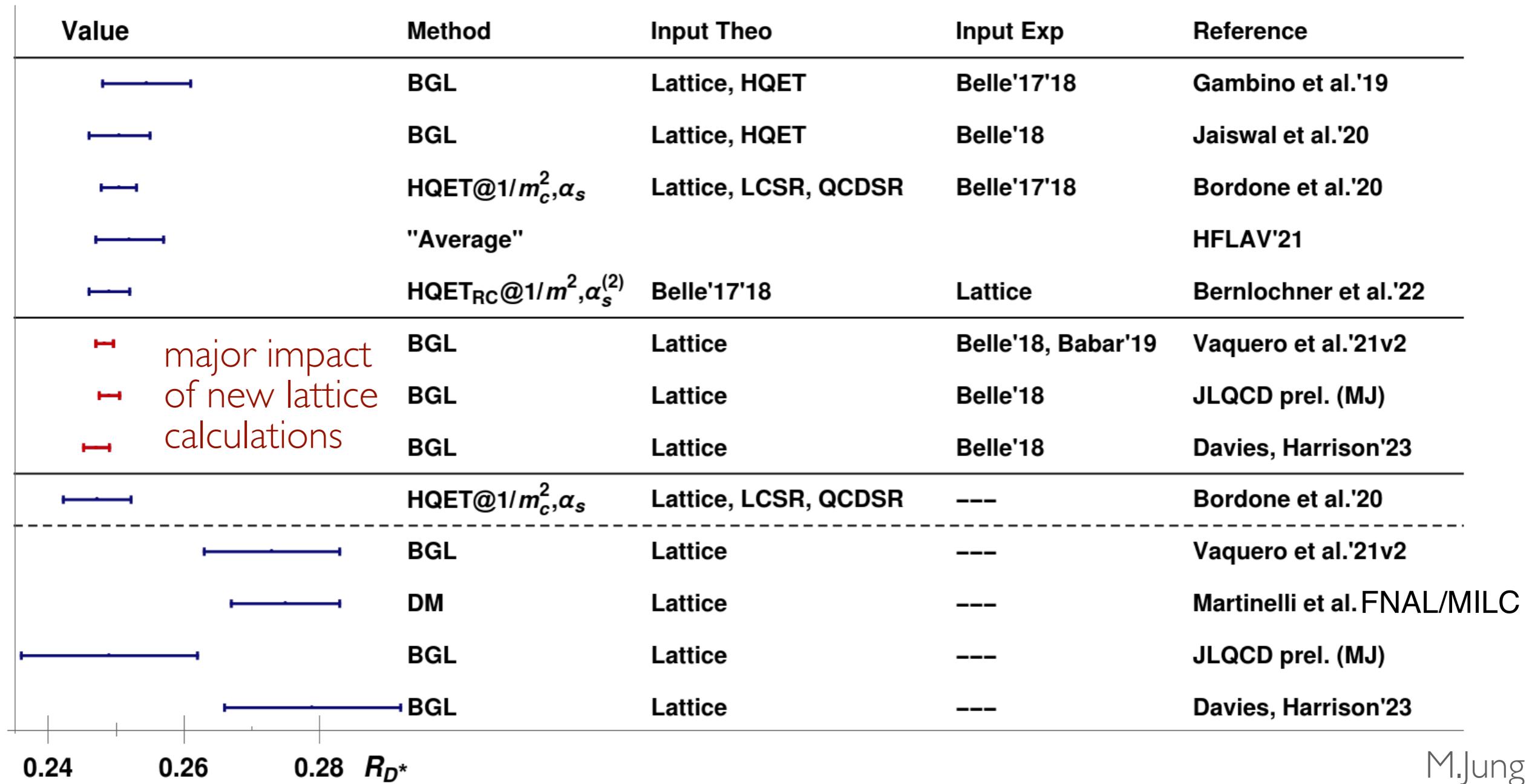
Extracting V_{cb}
from each bin,
FFs only
determined
by lattice QCD

M. Jung

Global BGL fit to Belle'18+FNAL+JLQCD+HPQCD data:

$|V_{cb}| = 40.3(7) \cdot 10^{-3} (\chi^2_{min} = 91.2)$ using only total rate $|V_{cb}| = 42.4(1.0) \cdot 10^{-3}$

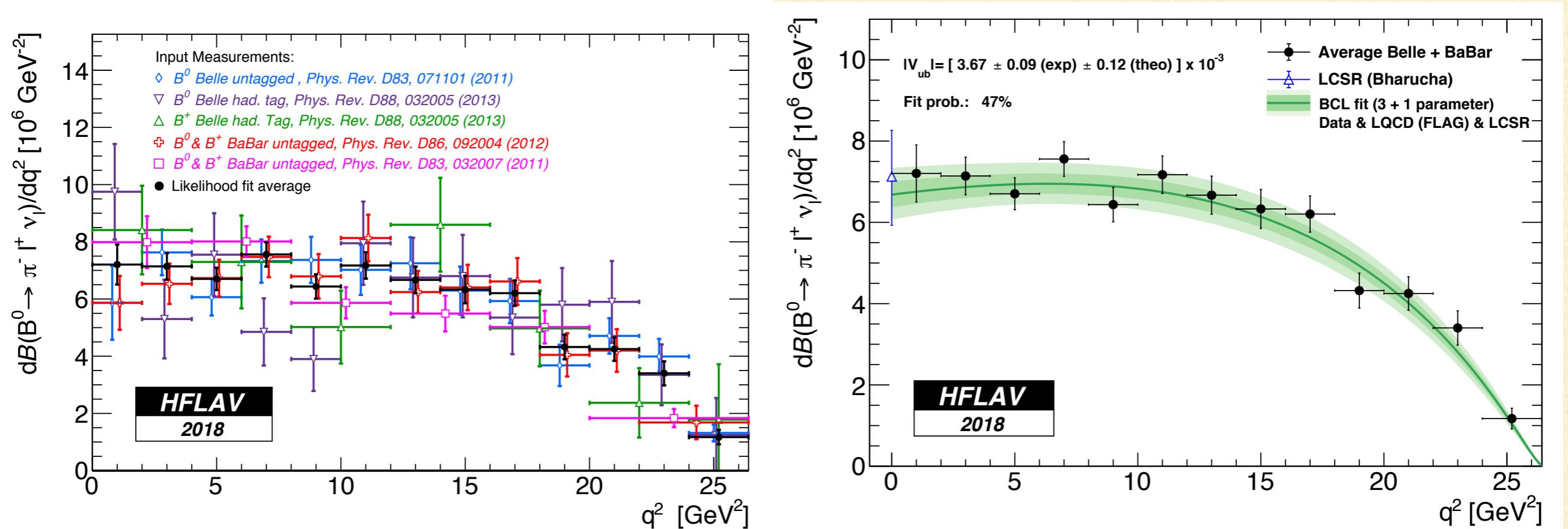
Overview over predictions for $R(D^*)$



Predictions based only on Fermilab & HPQCD lead to larger $R(D^*)$, in better agreement with exp, mostly because of the suppression at high w of the denominator.

No reason not to use experimental data for a SM test, especially in presence of tensions in lattice data.

EXCLUSIVE V_{ub} $B \rightarrow \pi \ell \nu$



HFLAV $|V_{ub}| = (3.70 \pm 0.10 (\text{exp}) \pm 0.12 (\text{theo})) \times 10^{-3}$ (data + LQCD),
 $|V_{ub}| = (3.67 \pm 0.09 (\text{exp}) \pm 0.12 (\text{theo})) \times 10^{-3}$ (data + LQCD + LCSR),

- **New LCSR results** (1811.00983) have been included for the first time in global fits to lattice and experimental data on $B \rightarrow \pi \ell \nu$ in 2103.01809 and 2102.07233, leading to $|V_{ub}| = 3.77(15)10^{-3}$ and $|V_{ub}| = 3.88(13)10^{-3}$. The latter removes outliers and is within 1σ from most recent inclusive results.
- HFLAV adopts a 2stage procedure, first making averages at different q^2 (low p) and fitting to extract V_{ub}

FLAG5 Web update

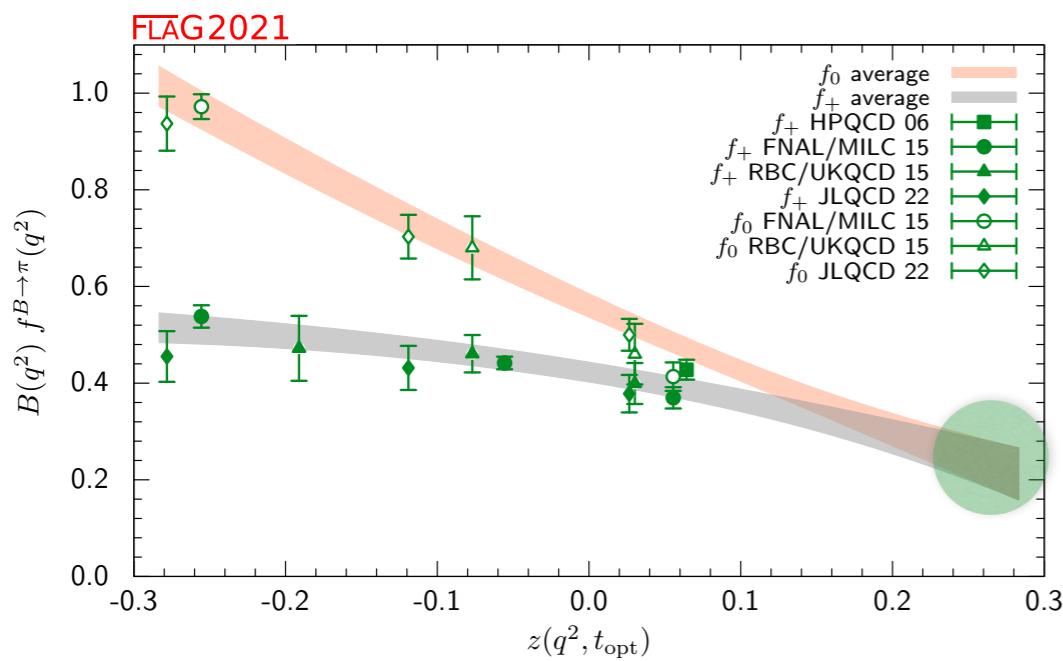
http://flag.unibe.ch/2021/Media?action=AttachFile&do=get&target=FLAG_2023_webupdate.pdf

FNAL/MILC (15) + RBC/UKQCD (15) + JLQCD (22)

$B \rightarrow \pi$ ($N_f = 2 + 1$)

	Central Values	Correlation Matrix				
a_0^+	0.423 (21)	1	-0.00506	-0.0740	0.402	0.0923
a_1^+	-0.508 (93)	-0.00506	1	0.497	-0.0557	0.659
a_2^+	-0.74 (34)	-0.0740	0.497	1	-0.152	0.677
a_0^0	0.561 (24)	0.402	-0.0557	-0.152	1	-0.548
a_1^0	-1.42 (11)	0.0923	0.659	0.677	-0.548	1

$\chi^2/\text{dof} = 43.6/12$: error rescaled by $\sqrt{\chi^2/\text{dof}} = 1.9$



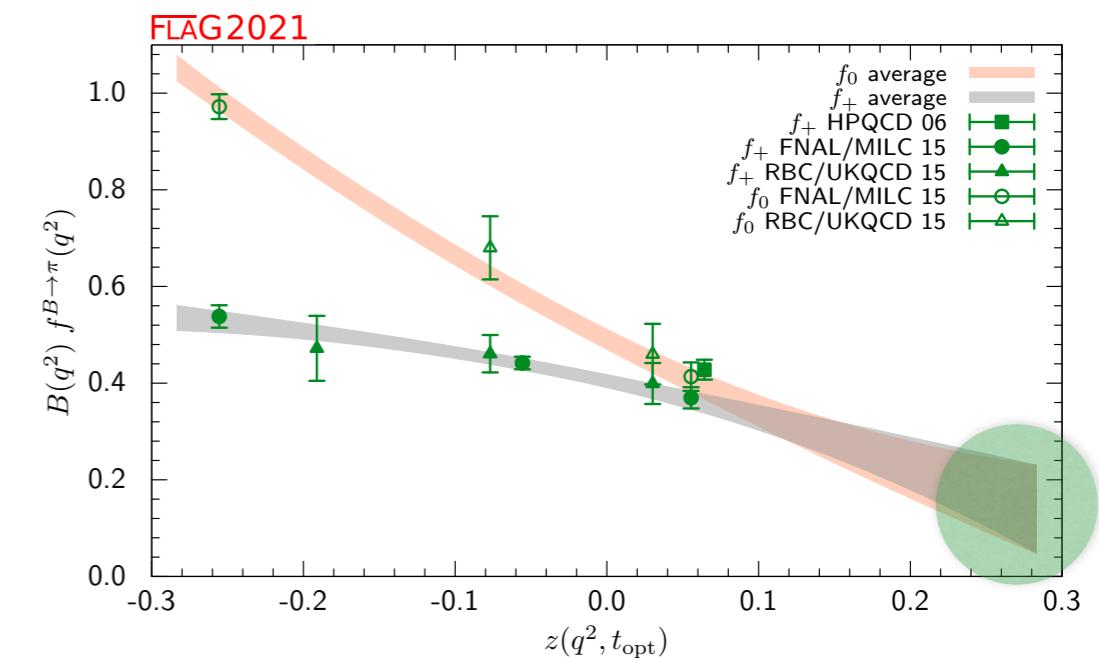
FLAG5 (2111.09849)

FNAL/MILC (15) + RBC/UKQCD (15)

$B \rightarrow \pi$ ($N_f = 2 + 1$)

	Central Values	Correlation Matrix				
a_0^+	0.404 (13)	1	0.404	0.118	0.327	0.344
a_1^+	-0.68 (13)	0.404	1	0.741	0.310	0.900
a_2^+	-0.86 (61)	0.118	0.741	1	0.363	0.886
a_0^0	0.490 (21)	0.327	0.310	0.363	1	0.233
a_1^0	-1.61 (16)	0.344	0.900	0.886	0.233	1

$\chi^2/\text{dof} = 0.82$: no error rescaling



- Error on slope parameters **decreased** ($a_{1,2}^+$ and a_1^0)

- Error on normalization **increased** ($a_0^{+,0}$)

$B \rightarrow \pi$ form factors

Small impact on $|V_{ub}|$ after including experimental data (information at small q^2 / large z)

FLAG5 Web update

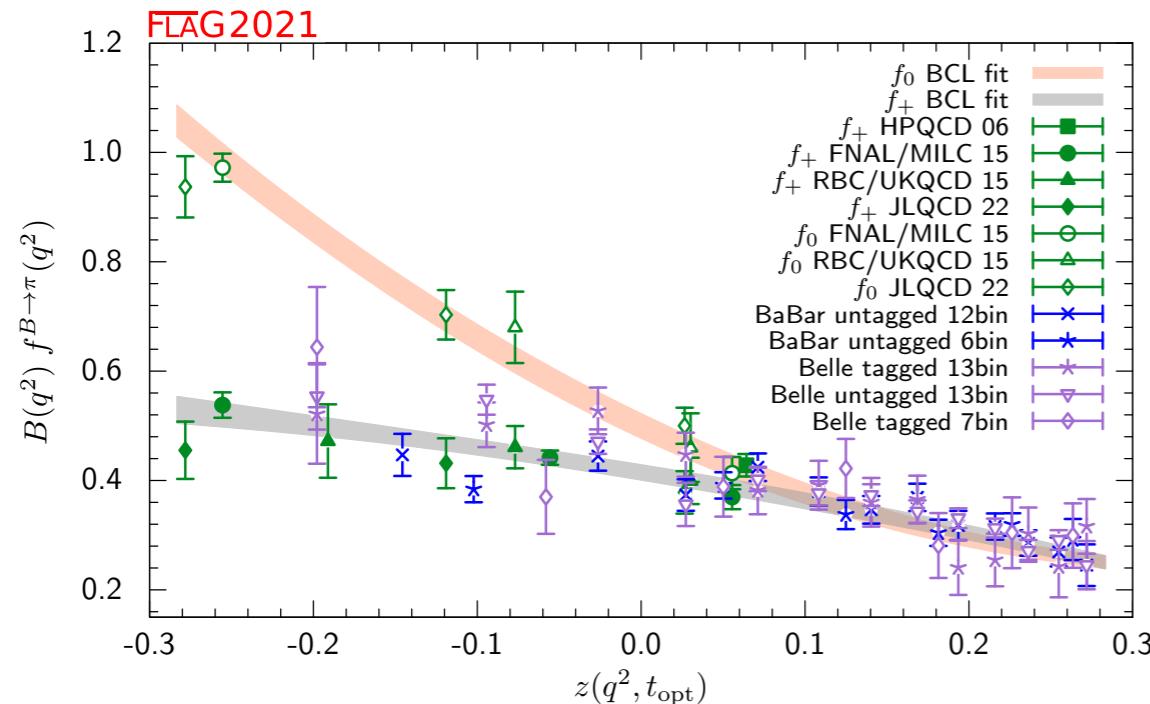
http://flag.unibe.ch/2021/Media?action=AttachFile&do=get&target=FLAG_2023_webupdate.pdf

$B \rightarrow \pi \ell \nu$ ($N_f = 2 + 1$)	
	Central values
$ V_{ub} \times 10^3$	3.64 (16)
a_0^+	0.425 (15)
a_1^+	-0.443 (39)
a_2^+	-0.51 (13)
a_0^0	0.560 (17)
a_1^0	-1.346 (53)

Correlation Matrix

	1	-0.812	-0.107	0.127	-0.325	-0.151
$ V_{ub} \times 10^3$	3.64 (16)	-0.812	1	-0.189	-0.308	0.409
a_0^+	0.425 (15)	-0.107	-0.189	1	-0.499	-0.0345
a_1^+	-0.443 (39)	0.127	-0.308	-0.499	1	0.150
a_2^+	-0.51 (13)	0.127	-0.308	-0.499	-0.189	0.128
a_0^0	0.560 (17)	-0.325	0.409	-0.0345	-0.189	1
a_1^0	-1.346 (53)	-0.151	0.00937	0.150	0.128	-0.772

$$\chi^2/\text{dof} = 116.6/62: \text{error rescaled by } \sqrt{\chi^2/\text{dof}} = 1.37$$

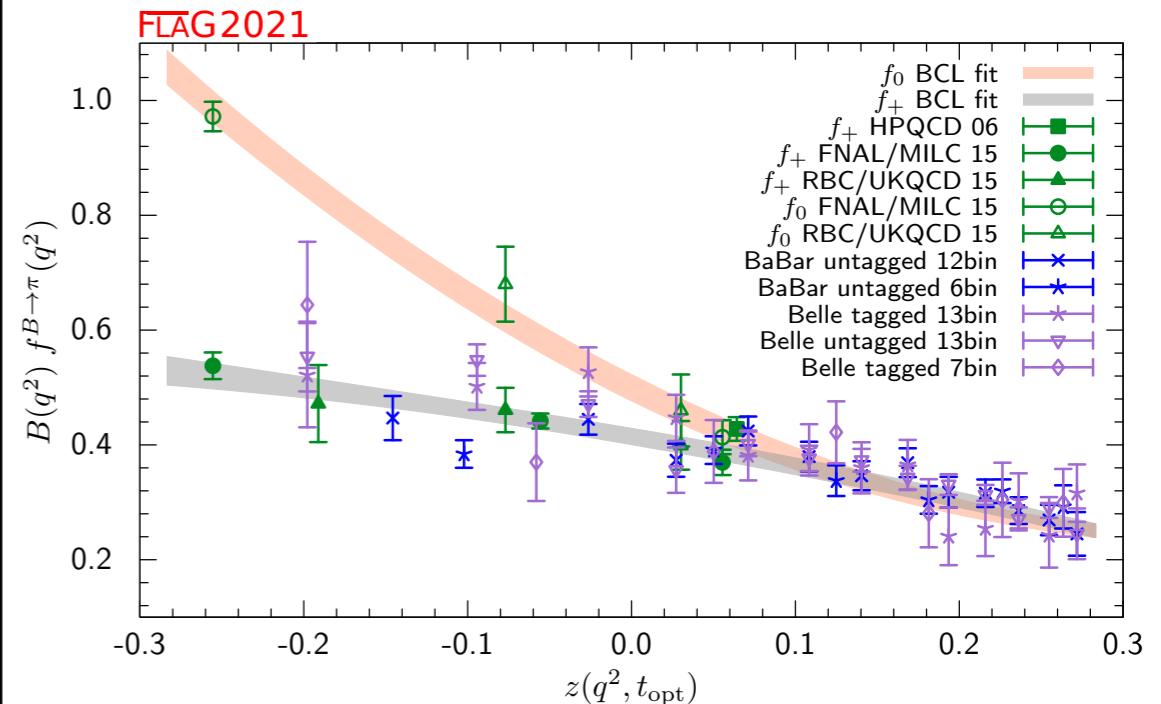


FLAG5 (2111.09849)

$B \rightarrow \pi \ell \nu$ ($N_f = 2 + 1$)

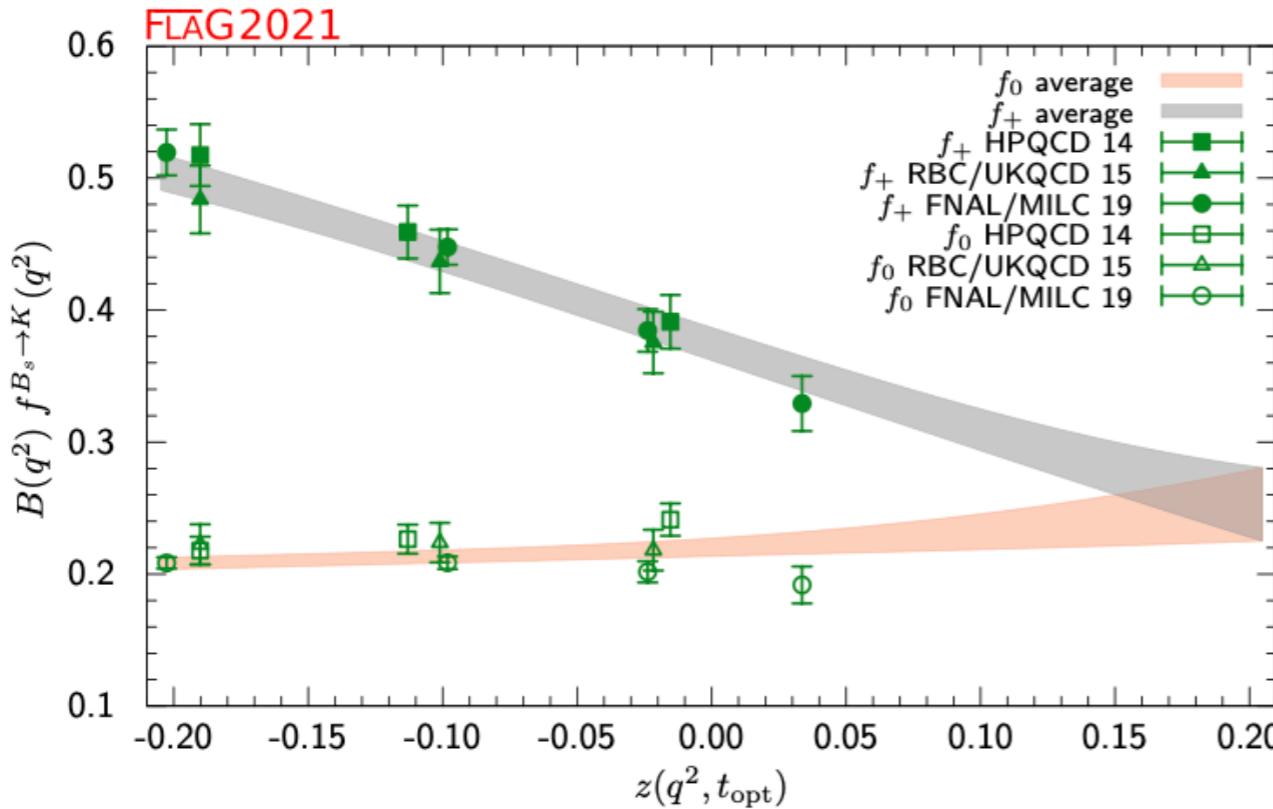
	Central Values	Correlation Matrix					
$ V_{ub} \times 10^3$	3.74 (17)	1	-0.851	-0.349	0.375	-0.211	-0.246
a_0^+	0.415 (14)	-0.851	1	0.155	-0.454	0.260	0.144
a_1^+	-0.488 (53)	-0.349	0.155	1	-0.802	-0.0962	0.220
a_2^+	-0.31 (18)	0.375	-0.454	-0.802	1	0.0131	-0.100
a_0^0	0.500 (23)	-0.211	0.260	-0.0962	0.0131	1	-0.453
a_1^0	-1.424 (54)	-0.246	0.144	0.220	-0.100	-0.453	1

$$\chi^2/\text{dof} = 78.7/56 = 1.41: \text{error rescale by } \sqrt{\chi^2/\text{dof}} = 1.19$$



$B_s \rightarrow K$ form factors and $|V_{ub}/V_{cb}|$

- FLAG5 combined form factors:



[LHCb 2012.05143]

$$\frac{1}{|V_{ub}|^2} \int_{q_{\min}^2 = m_\mu^2}^{7 \text{ GeV}^2} \frac{d\Gamma(B_s \rightarrow K^- \mu^+ \nu_\mu)}{dq^2} = (2.26 \pm 0.38) \text{ ps}^{-1}$$

$$\frac{1}{|V_{ub}|^2} \int_{7 \text{ GeV}^2}^{q_{\max}^2 = (m_{B_s} - m_K)^2} \frac{d\Gamma(B_s \rightarrow K^- \mu^+ \nu_\mu)}{dq^2} = (4.02 \pm 0.31) \text{ ps}^{-1}$$

↓

$$\frac{|V_{ub}|}{|V_{cb}|} (\text{low}) = 0.0819 \pm 0.0072_{\text{lat.}} \pm 0.0029_{\text{exp}}$$

$$\frac{|V_{ub}|}{|V_{cb}|} (\text{high}) = 0.0860 \pm 0.0037_{\text{lat.}} \pm 0.0038_{\text{exp}}$$

$$\frac{|V_{ub}|}{|V_{cb}|} (\text{low}) = 0.061(4) \quad \text{using LCSR}$$

Khodjamirian, Rusov

Note: RBC/UKQCD provides synthetic data points, HPQCD and FNAL/MILC only z-fit results

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.079(4)(4) \quad (\Lambda_b \rightarrow p \mu \nu) \quad \text{LHCb + Meinl et al}$$

My average of inclusive $\frac{|V_{ub}|}{|V_{cb}|} = 0.094(6)$ **and exclusive** $\frac{|V_{ub}|}{|V_{cb}|} = 0.094(4)$

INCLUSIVE $|V_{ub}|$

Important Belle measurement 2102.00020

In my opinion, the cleanest
measurement is the most inclusive one
with $M_X < 1.7\text{GeV}$, $E_\ell > 1\text{GeV}$:

$$|V_{ub}| = (3.97 \pm 0.08 \pm 0.16 \pm 0.16) 10^{-3}$$

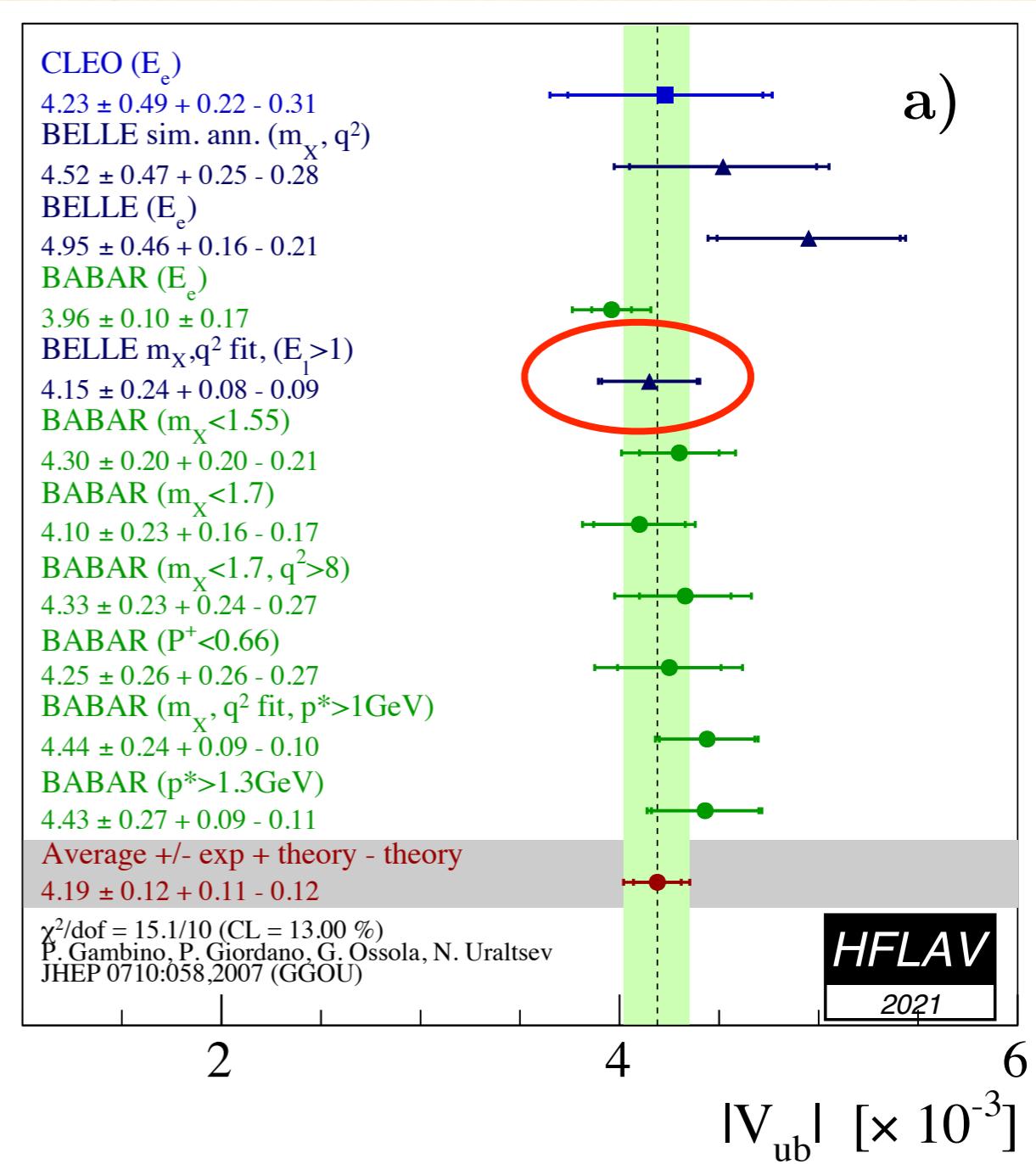
Framework	$ V_{ub} [10^{-3}]$
BLNP	$4.28 \pm 0.13^{+0.20}_{-0.21}$
DGE	$3.93 \pm 0.10^{+0.09}_{-0.10}$
GGOU	$4.19 \pm 0.12^{+0.11}_{-0.12}$
ADFR	$3.92 \pm 0.1^{+0.18}_{-0.12}$
BLL (m_X/q^2 only)	$4.62 \pm 0.20 \pm 0.29$

Not all approaches at the same level
Some discrepancy hidden in the average

Recent calculation of the $O(\alpha_s/m_b^2)$ effects
in $B \rightarrow X_u \ell \nu$, Capdevila, Nandi, PG

NEW! $|V_{ub}^{\text{excl.}}| / |V_{ub}^{\text{incl.}}| = 0.97 \pm 0.12$

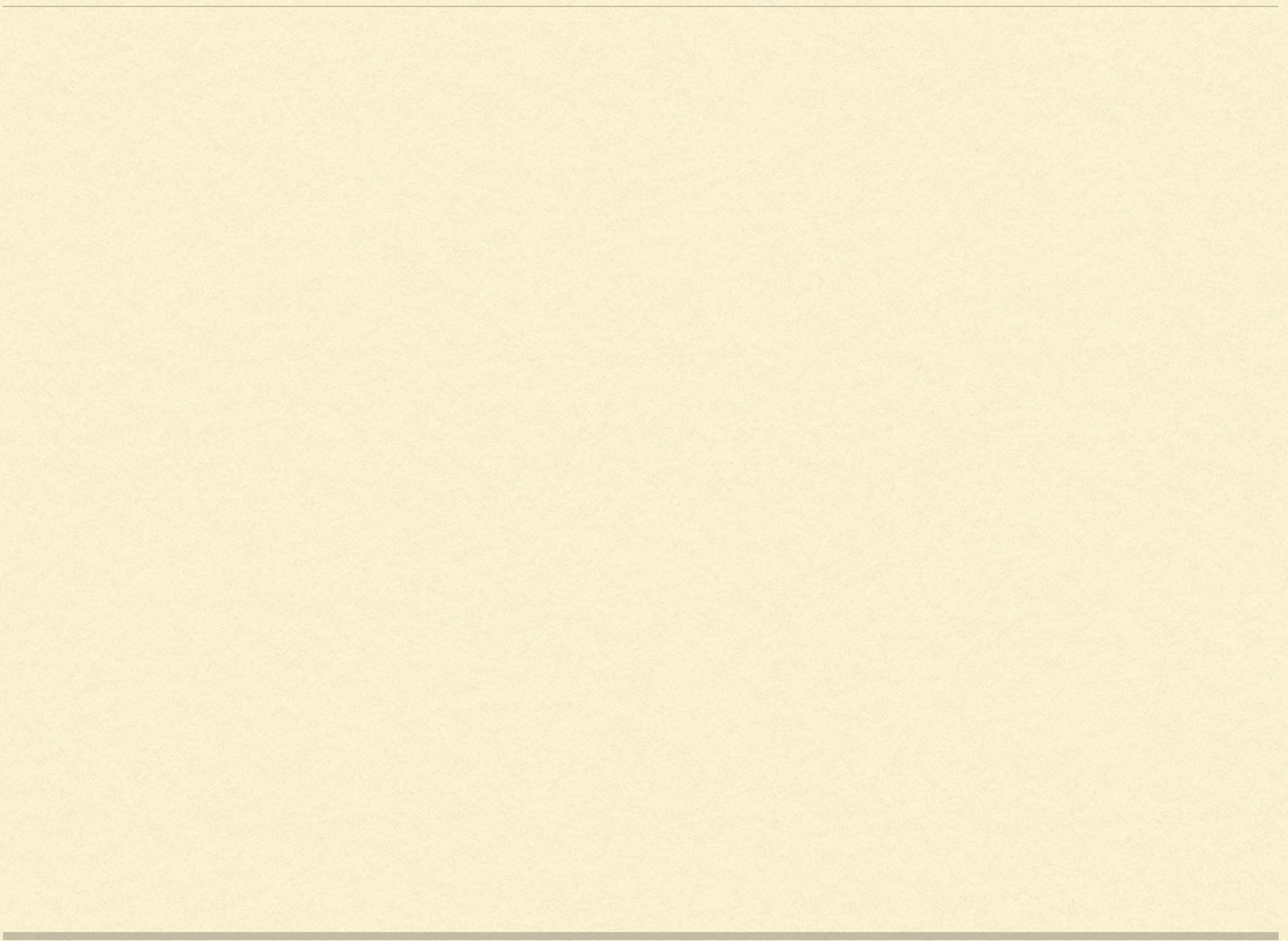
2303.17309



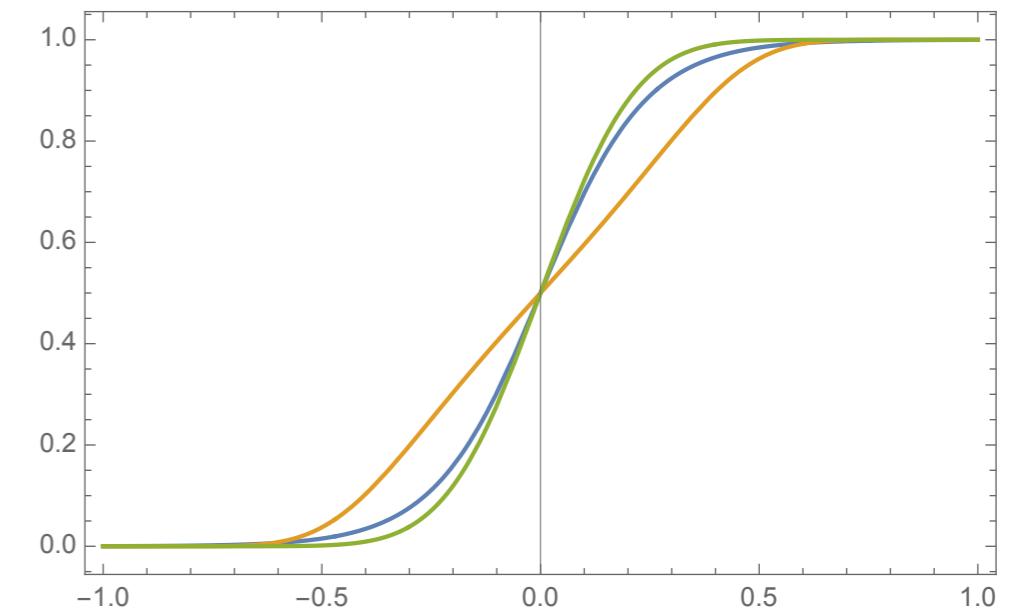
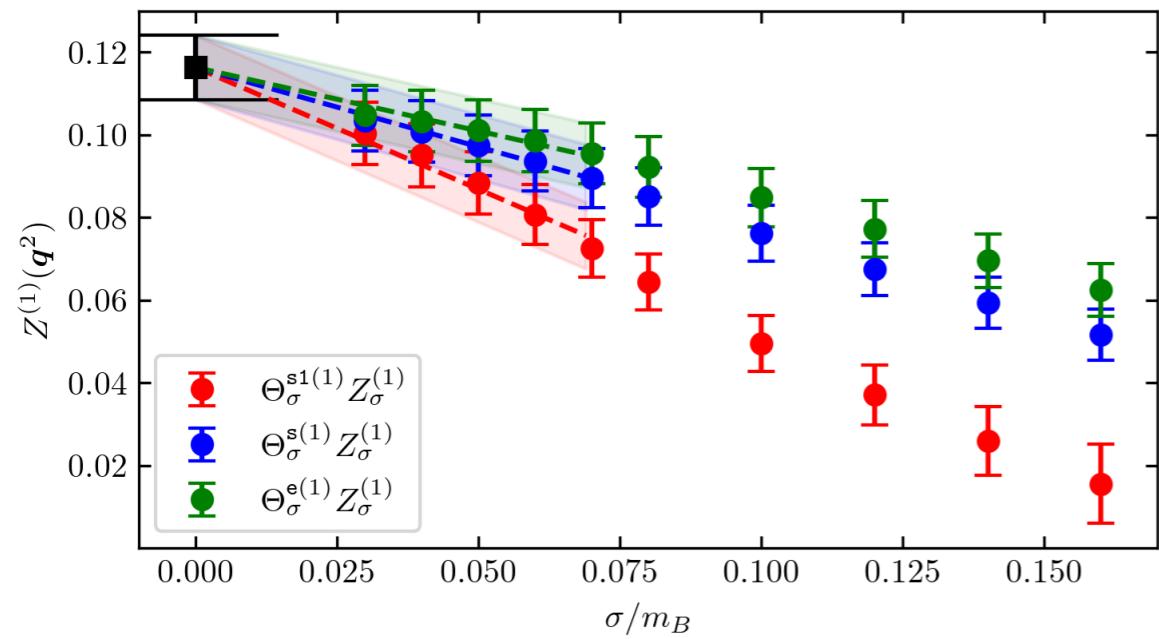
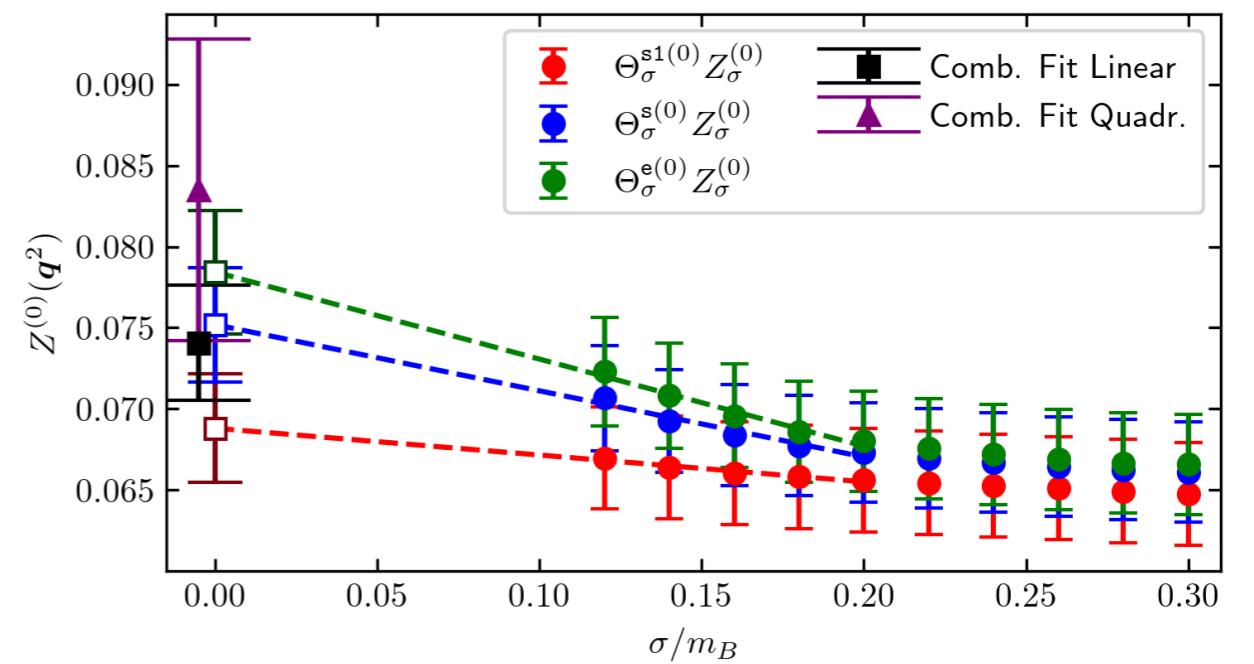
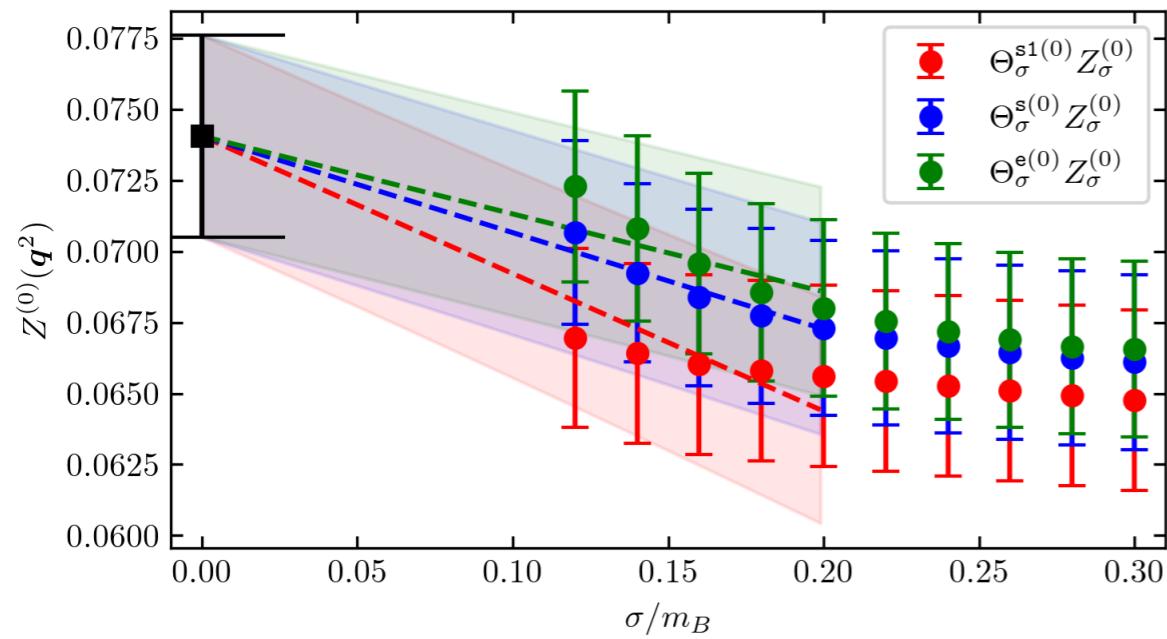
Look forward to validating approaches on Belle II data (SIMBA, NNVUB)!

SUMMARY

- **Despite new theoretical and exp results the V_{cb} puzzle persists. However, the intense th and exp activity gives hope.**
- **Inclusive $b \rightarrow c$:** new 3loop calculations show pert effects under control, 1.2% accuracy on $|V_{cb}|$, theory uncertainty no longer dominant.
- New method to study **inclusive semileptonic meson decays on the lattice**. Exploratory calculations for $m_b \sim 2.5\text{GeV}$ in good agreement with OPE. Promising way to complement/validate the OPE, but still a long way to go
- **Exclusive $b \rightarrow c$:** in the past uncertainties have been underestimated; several lattice groups are computing necessary FFs at non-zero recoil and new exp analyses have appeared but the **situation is still unclear**. FNAL & HPQCD in tension with exp spectra, JLQCD gives a more consistent picture with reduced tension with inclusive.
- Tensions in the LQCD calculations of $B \rightarrow \pi$ and $B_s \rightarrow K$ FFs.
- **New exp results suggest inclusive and exclusive V_{ub} are getting closer...**



in detail

ETMC at $|q| = 0.5\text{GeV}$

Using different approx to the kernel
improves the $\sigma \rightarrow 0$ extrapolation
Interplay with continuum and infinite volume limits

D'AGOSTINI BIAS

Standard χ^2 fits sometimes lead to paradoxical results

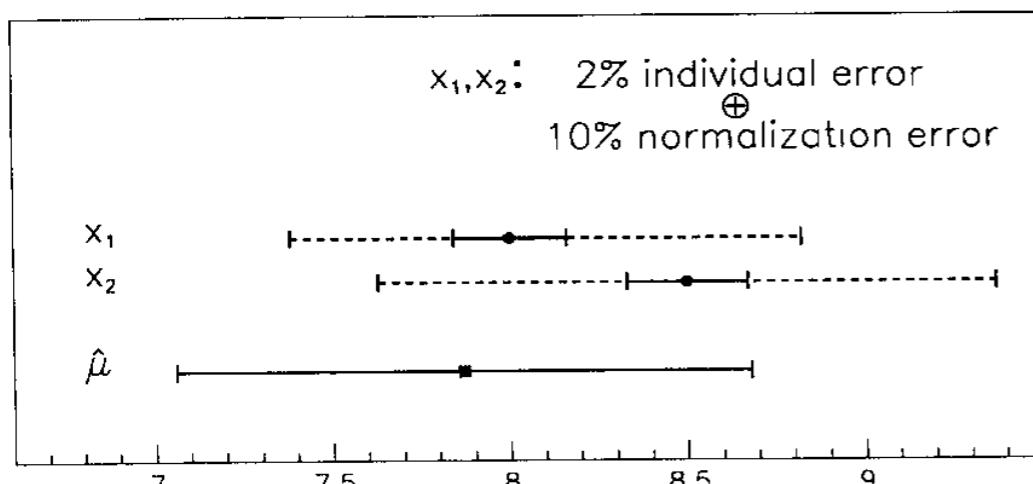


Fig. 1. Best estimate of the true value from two correlated data points, using in the χ^2 the empirical covariance matrix of the measurements. The error bars show individual and total errors.

$$\hat{k} = \frac{x_1 \sigma_2^2 + x_2 \sigma_1^2}{\sigma_1^2 + \sigma_2^2 + (x_1 - x_2)^2 \sigma_f^2},$$

Many exp systematics are highly correlated. Bias is stronger with more bins

On the use of the covariance matrix to fit correlated data

G. D'Agostini

Dipartimento di Fisica, Università "La Sapienza" and INFN, Roma, Italy

(Received 10 December 1993; revised form received 18 February 1994)

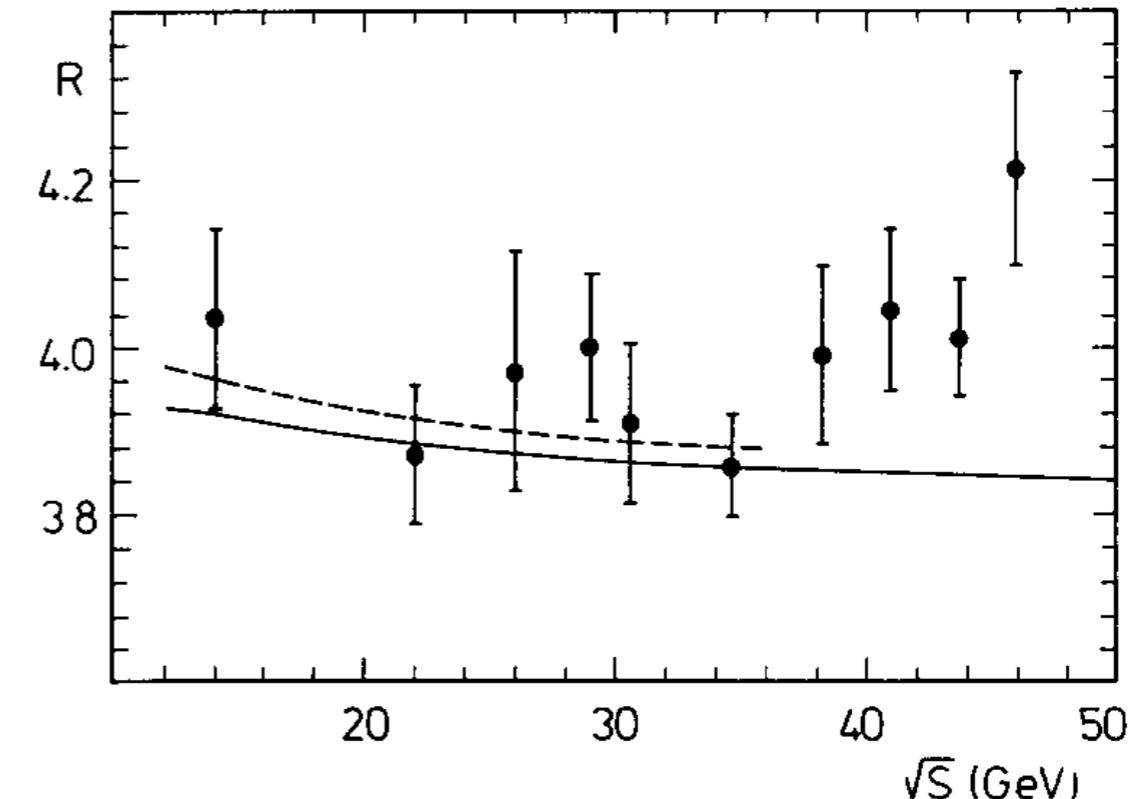
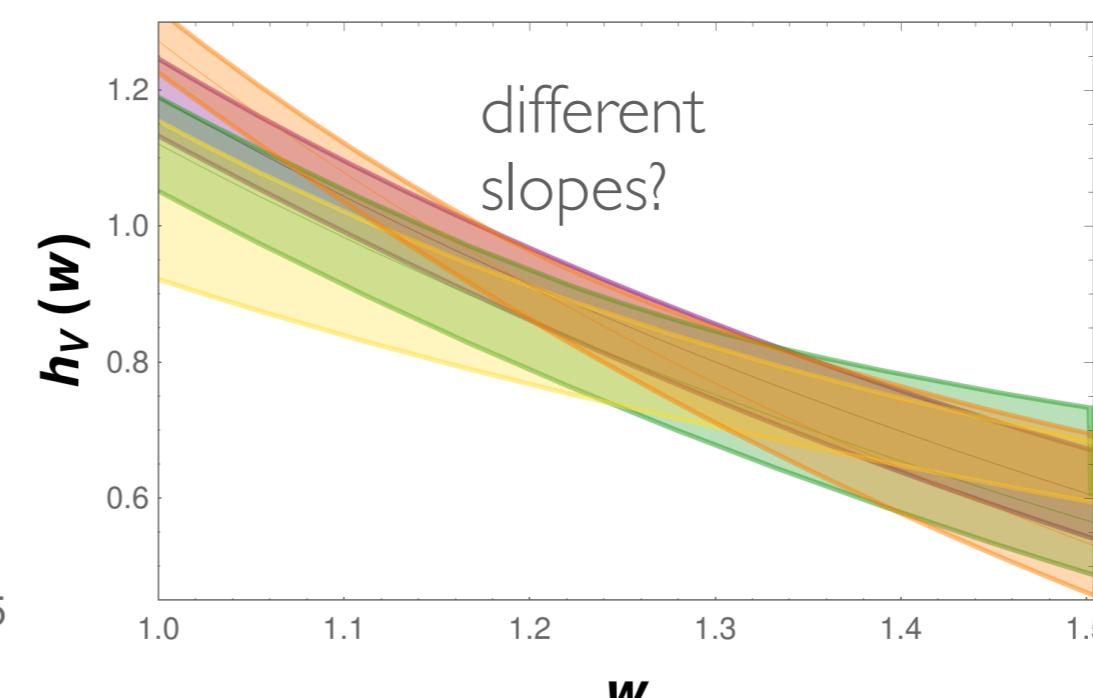
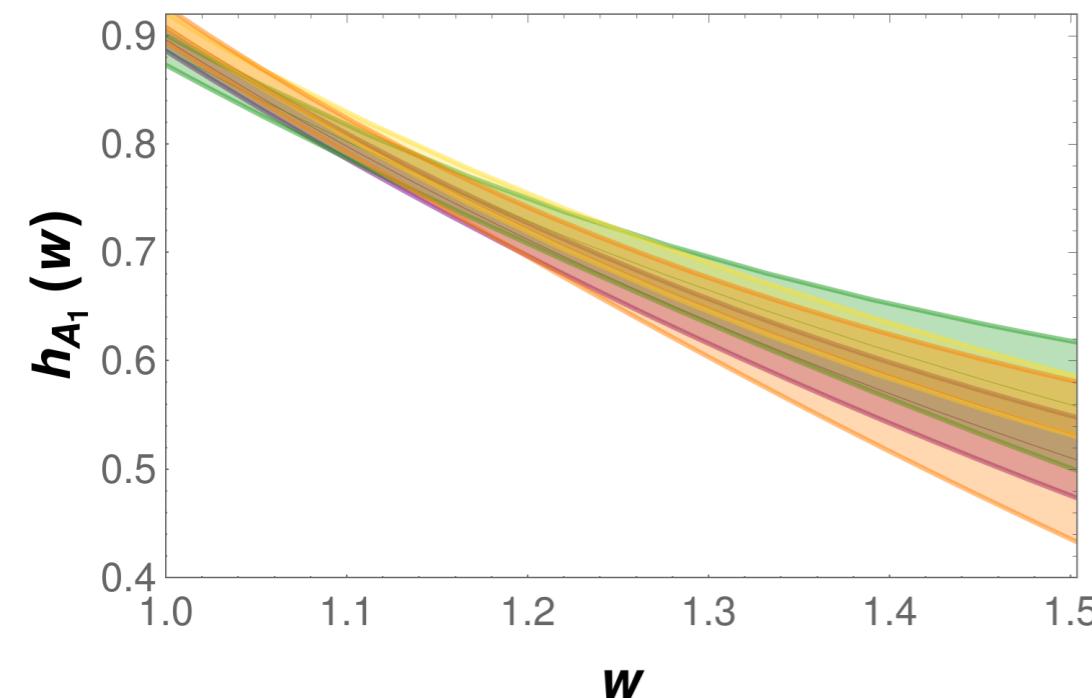


Fig. 2. R measurements from PETRA and PEP experiments with the best fits of QED + QCD to all the data (full line) and only below 36 GeV (dashed line). All data points are correlated (see text).

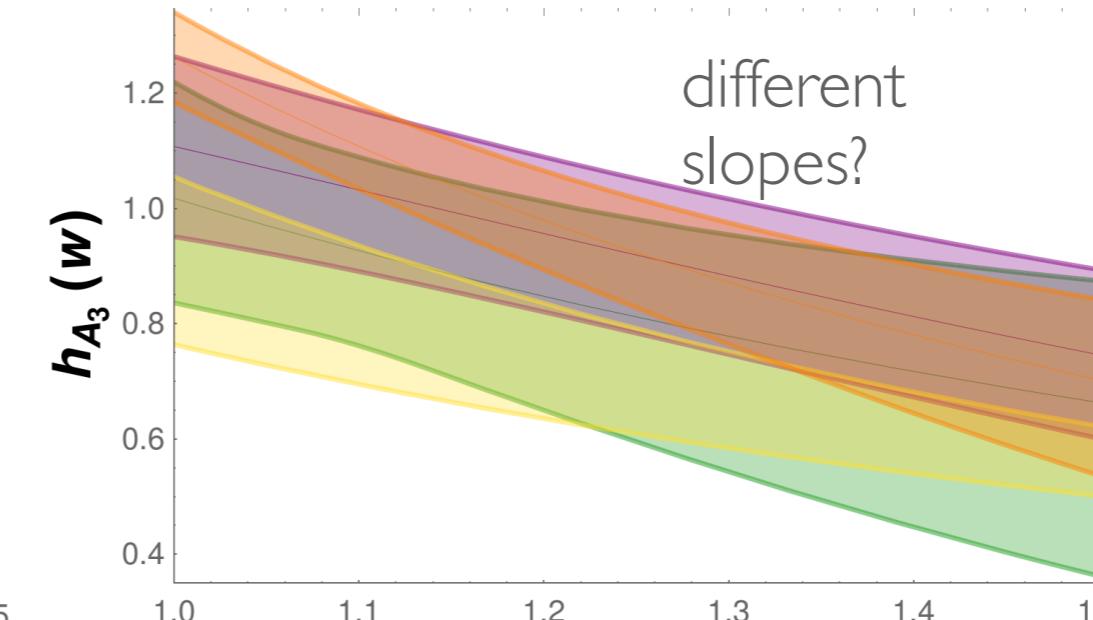
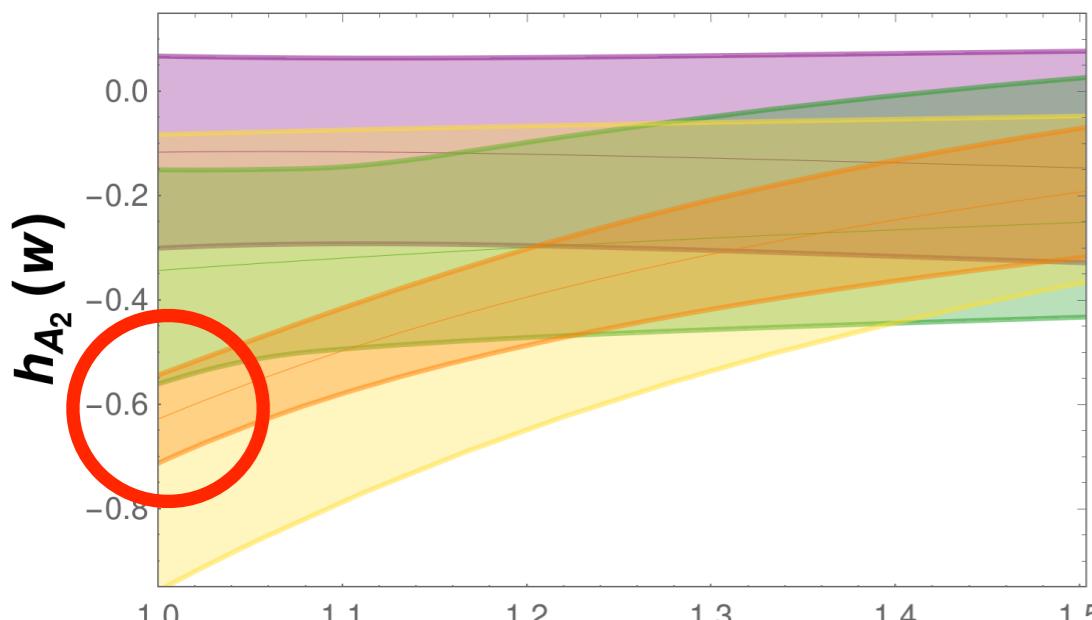
LATTICE FORM FACTORS AT NONZERO RECOIL

2105.14019, 2112.13775, 2304.03137



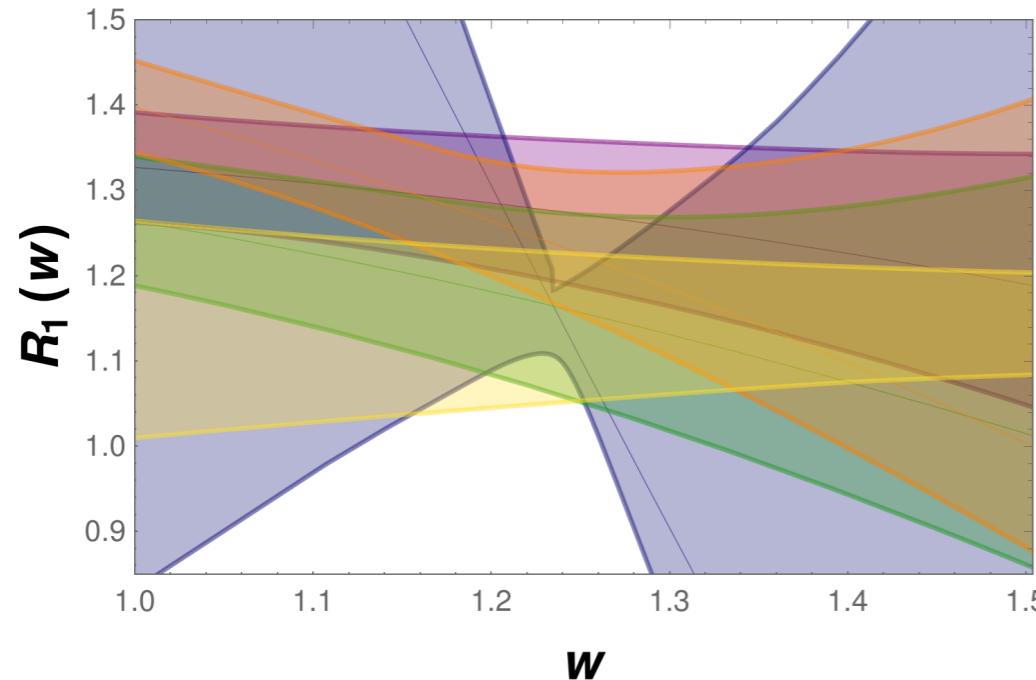
M.Jung

FERMILAB/MILC
JLQCD
HPQCD
HQE
(LCSR+SR+lat<2019)

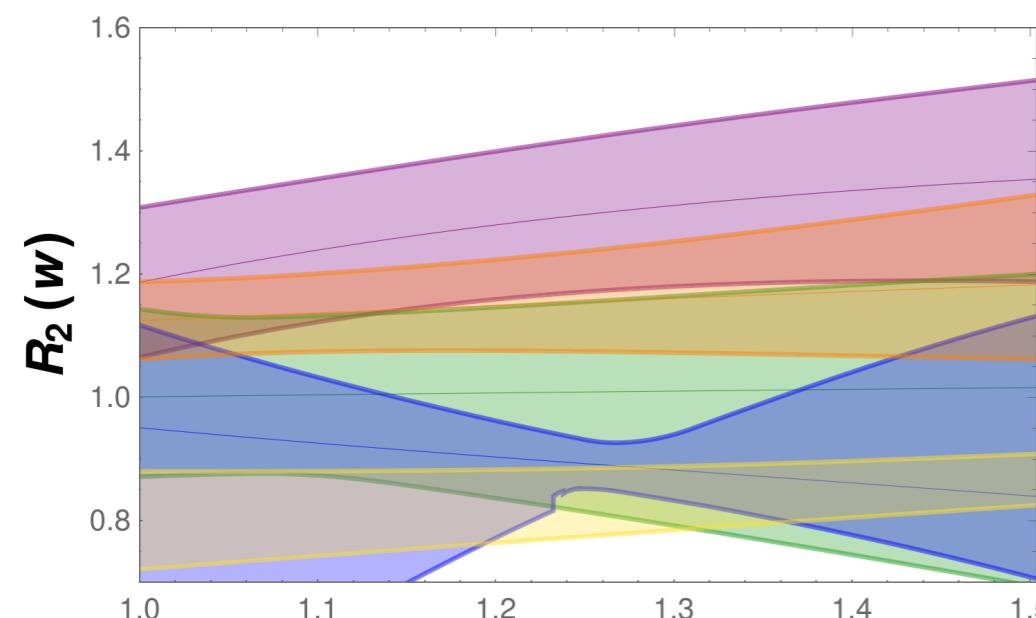


BGL fits with weak unitarity. General good agreement, but a few exceptions

RATIOS OF FORM FACTORS



FERMILAB/MILC
JLQCD
HPQCD
HQE (LCSR+SR+lat<2019)
EXP (Belle 2018)



Form factor ratios more sensitive to differences. Stark disagreement between FERMILAB & HPQCD and HQE & EXP in R_2

$B^0 \rightarrow D^{*-} \ell^+ \nu$ untagged (189/fb)

preliminary [to be submitted to Phys. Rev. D] 

BGL fit result

BGL truncation order determined by
Nested Hypothesis Test [Phys. Rev. D100, 013005]

	Values	Correlations				χ^2/ndf
$\tilde{a}_0 \times 10^3$	0.89 ± 0.05	1.00	0.26	-0.27	0.07	
$\tilde{b}_0 \times 10^3$	0.54 ± 0.01	0.26	1.00	-0.41	-0.46	
$\tilde{b}_1 \times 10^3$	-0.44 ± 0.34	-0.27	-0.41	1.00	0.56	40/31
$\tilde{c}_1 \times 10^3$	-0.05 ± 0.03	0.07	-0.46	0.56	1.00	

Preliminary

Relative uncertainty (%) Preliminary

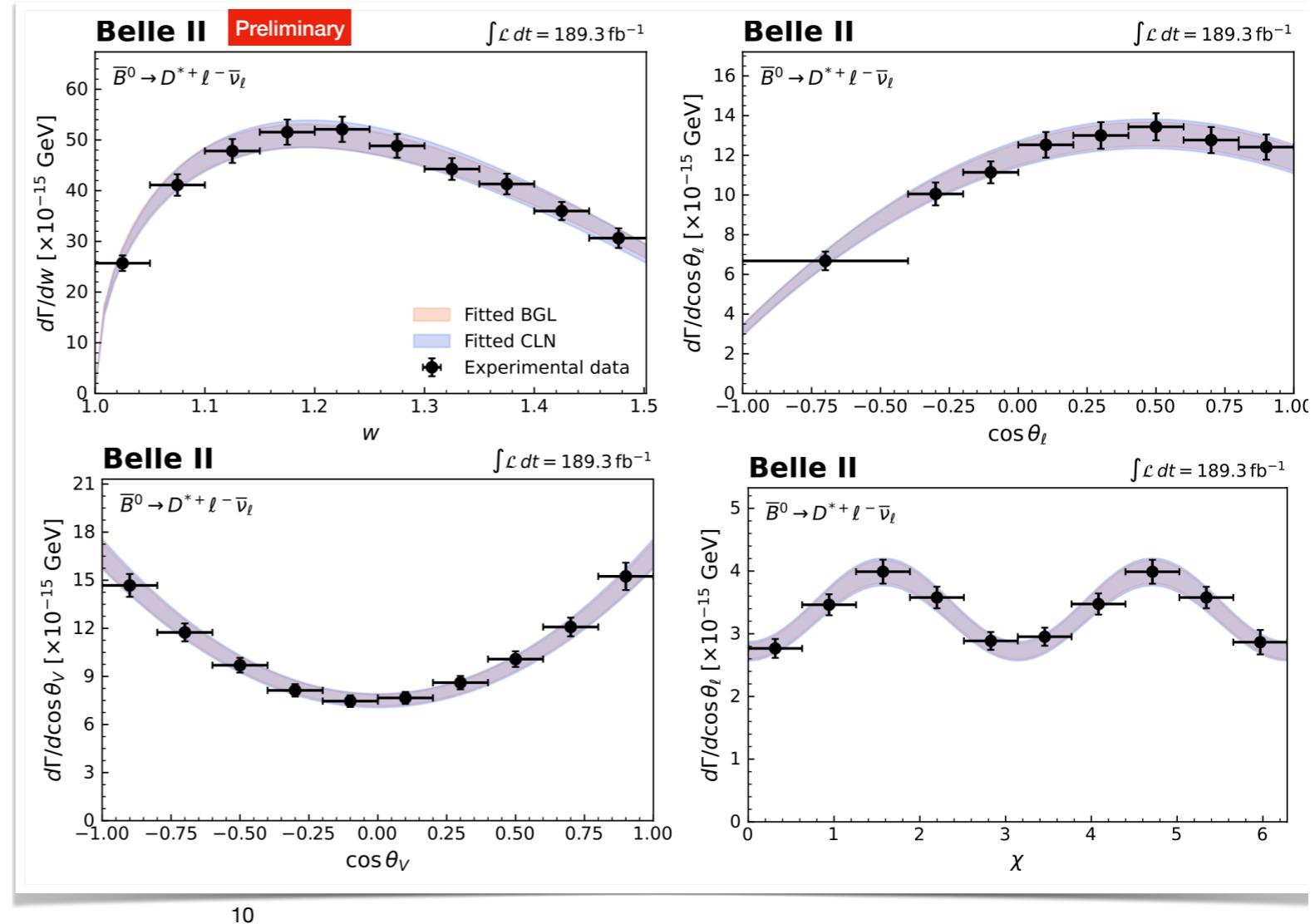
	\tilde{a}_0	\tilde{b}_0	\tilde{b}_1	\tilde{c}_1
Statistical	3.3	0.7	44.8	35.4
Finite MC samples	3.0	0.7	39.4	33.0
Signal modelling	3.0	0.4	40.0	30.8
Background subtraction	1.2	0.4	24.8	18.1
Lepton ID efficiency	1.5	0.3	3.1	2.5
Slow pion efficiency	1.5	1.5	18.4	22.0
Tracking of K, π, ℓ	0.5	0.5	0.6	0.5
$N_{B\bar{B}}$	0.8	0.8	1.1	0.8
f_{+-}/f_{00}	1.3	1.3	1.7	1.3
$\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+)$	0.4	0.4	0.5	0.4
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	0.4	0.4	0.5	0.4
B^0 lifetime	0.1	0.1	0.2	0.1
Total	6.1	2.5	78.3	64.1

C. Schwanda, Moriond '23

LQCD used only for normalisation at zero recoil ($w = 1$)

Preliminary $|V_{cb}| \eta_{\text{EW}} \mathcal{F}(1) = \frac{1}{\sqrt{m_B m_{D^*}}} \left(\frac{|\tilde{b}_0|}{P_f(0) \phi_f(0)} \right) \quad \mathcal{F}(1) = 0.906 \pm 0.013$

$|V_{cb}|_{\text{BGL}} = (40.9 \pm 0.3_{\text{stat}} \pm 1.0_{\text{syst}} \pm 0.6_{\text{theo}}) \times 10^{-3}$

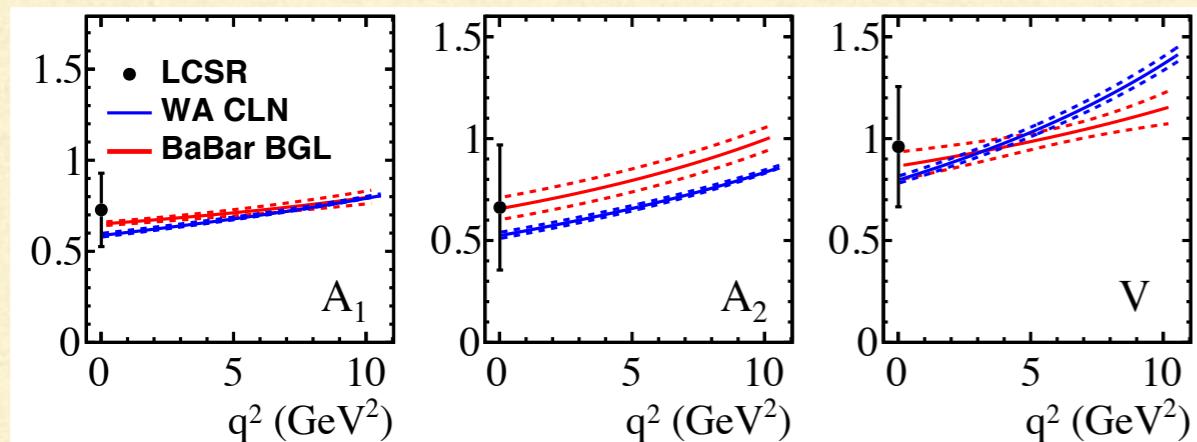


$|V_{cb}|_{\text{CLN}} = (40.4 \pm 0.3_{\text{stat}} \pm 1.0_{\text{syst}} \pm 0.6_{\text{theo}}) \times 10^{-3}$

RESULTS BY BABAR AND LHCb

1903.10002, 2001.03225

Reanalysis of tagged B^0 and B^+ data, unbinned 4 dimensional fit with simplified BGL and CLN
About 6000 events
No data provided yet



No clear BGL(111)/CLN difference but disagreement with HFLAV CLN ffs

$V_{cb}=0.0384(9)$



Measurement of $|V_{cb}|$ with $B_s^0 \rightarrow D_s^{(*)-} \mu^+ \nu_\mu$ decays

$$\mathcal{R} \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^- \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^- \mu^+ \nu_\mu)},$$
$$\mathcal{R}^* \equiv \frac{\mathcal{B}(B_s^0 \rightarrow D_s^{*-} \mu^+ \nu_\mu)}{\mathcal{B}(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu)}$$

$V_{cb}=0.0414(16)$ CLN
 $V_{cb}=0.0423(17)$ BGL(222)

Fit to exp data and lattice FFs based on HFLAV BRs, employs BGL(222)