QCD effects in $B \to K^{(*)}\ell^+\ell^-$ decays

Nico Gubernari

Based on
arXiv: 2011.09813, 2206.03797, 2305.06301
in collaboration with
Danny van Dyk, Javier Virto, and Méril Reboud

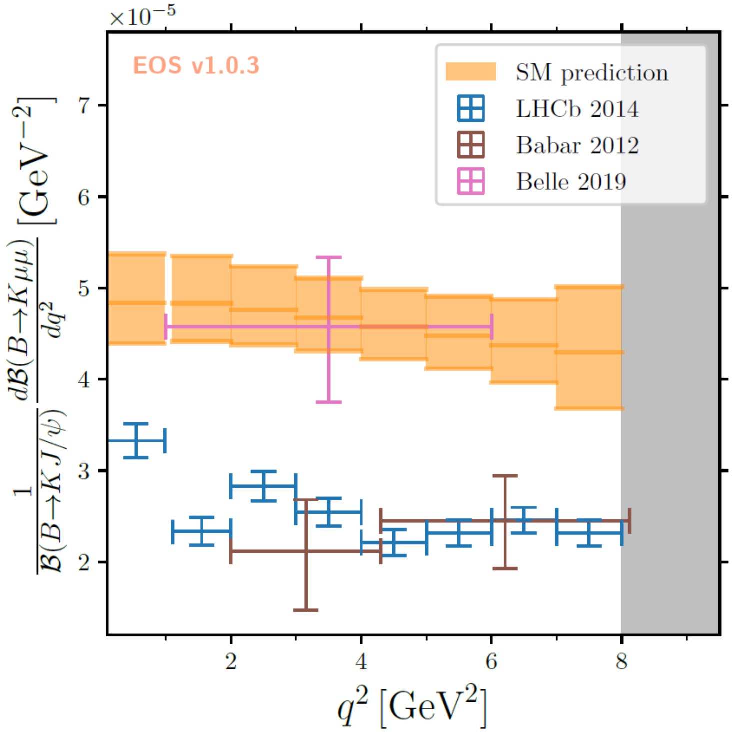
21st Conference on Flavor Physics and CPV
IP2I - Université de Lyon
29-May-2023

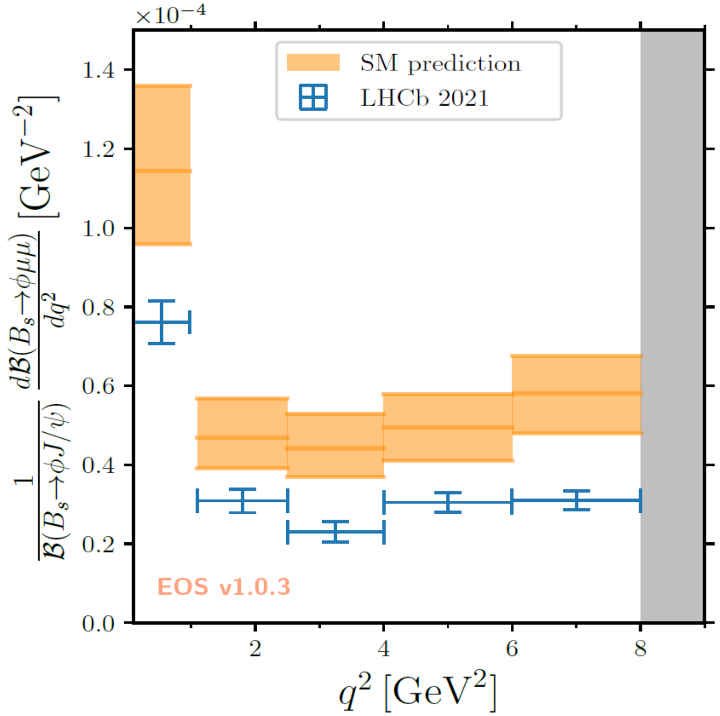
DFG Particle Physics Phenomenology after the Higgs Discovery
Collaborative Research Center TRR 257

Universität Siegen
Center for Particle Physics Siegen

SM predictions for BRs in rare decays

test the SM and constrain new physics by comparing theory predictions and exp. measurements of, e.g., branching ratios $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$





agreement between theory and experiment for LFU ratios R_K and R_{K^*},
but **tension remains for $b \to s\mu^+\mu^-$** observables $\Longrightarrow$ need to understand this tension

focus of this talk: how to obtain these SM predictions and what ingredients are needed

Theoretical framework

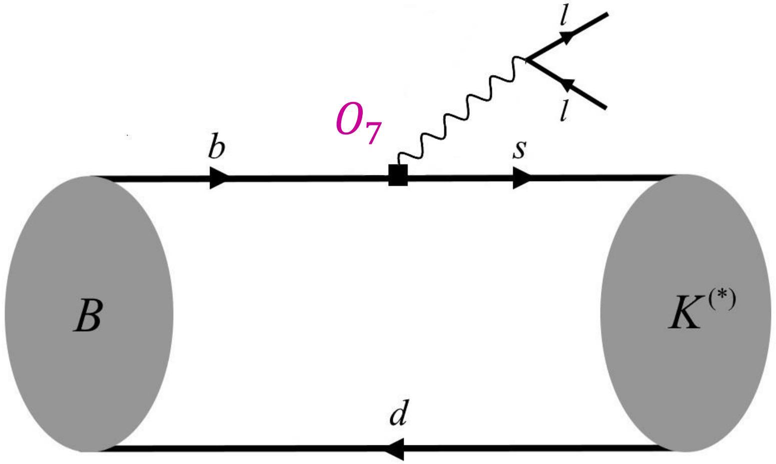
$b \to s\ell^+\ell^-$ effective Hamiltonian

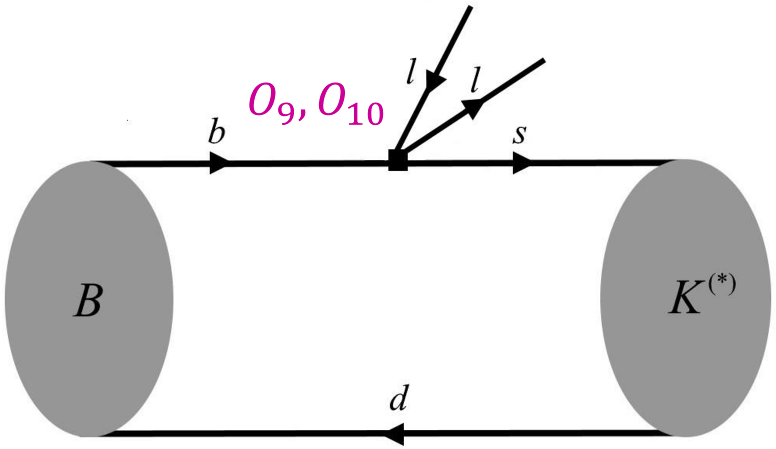
transitions described by the **effective Hamiltonian**

$$\mathcal{H}(b \to s\ell^+\ell^-) = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu) \qquad \mu = m_b$$

main contributions to $B_{(s)} \to \{K^{(*)}, \phi\}\ell^+\ell^-$ in the SM given by local operators O_7, O_9, O_{10}

$$O_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu} \qquad O_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \ell) \qquad O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma^\mu b_L) \sum_\ell (\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

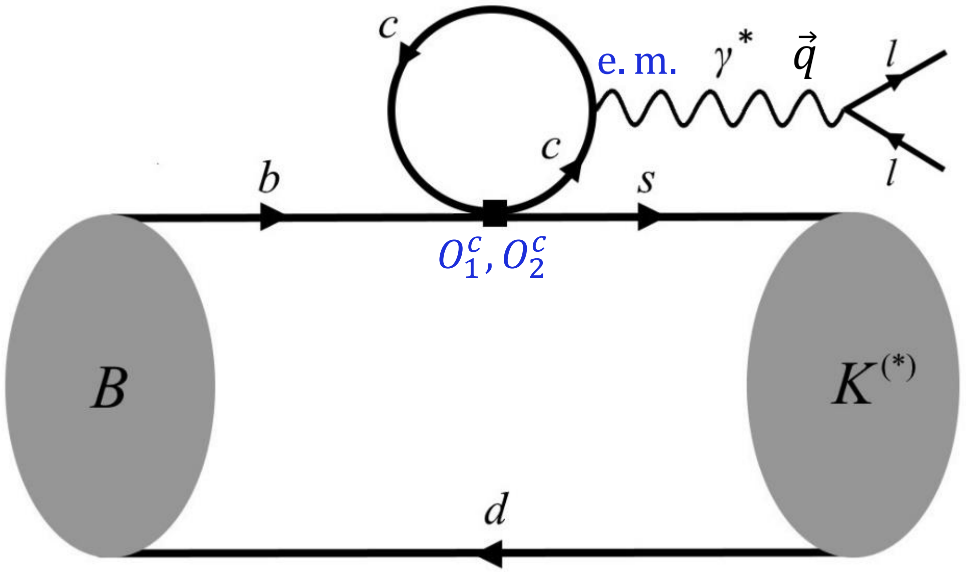




Charm loop in $B \to K^{(*)}\ell^+\ell^-$

additional non-local contributions come from O_1^c and O_2^c combined with the e.m. current (charm-loop contribution)

$$O_1^c = (\bar{s}_L \gamma^\mu c_L)(\bar{c}_L \gamma_\mu b_L) \qquad O_2^c = \left(\bar{s}_L^j \gamma^\mu c_L^i\right)\left(\bar{c}_L^i \gamma_\mu b_L^j\right)$$



Decay amplitude for $B \to K^{(*)}\ell^+\ell^-$ decays

calculate decay amplitudes precisely to probe the SM

$b \to s\mu^+\mu^-$ anomalies: NP or underestimated systematic uncertainties?

(analogous formulas apply to $B_s \to \phi\ell^+\ell^-$ decays)

$$\mathcal{A}\left(B \to K^{(*)}\ell^+\ell^-\right) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\, \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

Decay amplitude for $B \to K^{(*)}\ell^+\ell^-$ decays

calculate decay amplitudes precisely to probe the SM
$b \to s\mu^+\mu^-$ anomalies: NP or underestimated systematic uncertainties?
(analogous formulas apply to $B_s \to \phi\ell^+\ell^-$ decays)

$$\mathcal{A}\left(B \to K^{(*)}\ell^+\ell^-\right) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right)\mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\,\mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

local hadronic matrix elements

$$\mathcal{F}_\mu = \left\langle K^{(*)}(k)\middle| O_{7,9,10}^{\text{had}} \middle| B(k+q)\right\rangle$$

non-local hadronic matrix elements

$$\mathcal{H}_\mu = i\int d^4x\, e^{iq\cdot x}\left\langle K^{(*)}(k)\middle| T\{j_\mu^{\text{em}}(x), (C_1 O_1^c + C_2 O_2^c)(0)\}\middle| B(k+q)\right\rangle$$

Form factors definitions

form factors (FFs) **parametrize hadronic matrix elements**

FFs are functions of the momentum transfer squared q^2

local FFs

$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\, \mathcal{F}_\lambda(q^2)$$

computed with **lattice QCD** and light-cone sum rules with good precision $3\% - 20\%$

Form factors definitions

form factors (FFs) **parametrize hadronic matrix elements**

FFs are functions of the momentum transfer squared q^2

local FFs

$$\mathcal{F}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\,\mathcal{F}_\lambda(q^2)$$

computed with **lattice QCD** and light-cone sum rules with good precision $3\% - 20\%$

non-local FFs

$$\mathcal{H}_\mu(k,q) = \sum_\lambda \mathcal{S}_\mu^\lambda(k,q)\mathcal{H}_\lambda(q^2)$$

calculated using an **Operator Product Expansion (OPE)** or QCD factorization or …
(variety of approaches, most of them model-dependent)

large uncertainties → reduce uncertainties for a better understanding of rare B decays

Local form factors

Methods to compute FFs

non-perturbative techniques are needed to compute FFs

1. Lattice QCD (LQCD)
   numerical evaluation of correlators in a finite and discrete space-time
   more efficient usually at **high q^2**
   reducible systematic uncertainties

2. Light-cone sum rules (LCSRs)
   based on unitarity, analyticity, and quark-hadron duality approximation
   need universal non-perturbative inputs (**light-meson or B-meson** distribution amplitudes)
   only applicable at **low q^2**
   non-reducible systematic uncertainties

complementary approaches to calculate FFs
in the long run LQCD will dominate the theoretical predictions (smaller and reducible syst unc.)

Local form factors predictions

available theory calculations for local FFs $\mathcal{F}_\lambda$

$B \to K$:

- LQCD calculations at **high** q^2
  [HPQCD 2013/2023] [FNAL/MILC 2015]
  and in the **whole** semileptonic region
  [HPQCD 2023]
- LCSR at **low** q^2
  [Khodjamirian/Rusov 2017] [NG/Kokulu/van Dyk 2018]

$B \to K^*$ and $B_s \to \phi$:

- LQCD calculations at **high** q^2
  [Horgan et al. 2015]
- LCSR calculation at **low** q^2
  [Bharucha et al. 2015] [NG/Kokulu/van Dyk 2018]

Local form factors predictions

available theory calculations for local FFs $\mathcal{F}_\lambda$

$B \to K$:

- LQCD calculations at high q^2
  [HPQCD 2013/2023] [FNAL/MILC 2015]
  and in the whole semileptonic region
  [HPQCD 2023]
- LCSR at low q^2
  [Khodjamirian/Rusov 2017] [NG/Kokulu/van Dyk 2018]

$B \to K^*$ and $B_s \to \phi$:

- LQCD calculations at high q^2
  [Horgan et al. 2015]
- LCSR calculation at low q^2
  [Bharucha et al. 2015] [NG/Kokulu/van Dyk 2018]

$B \to K$ **FFs excellent** status (need independent calculation at low q^2)

more LQCD results needed for vector states (for high precision K^* width cannot be neglected)

how to **combine** different calculations for the same channel?
how to obtain result in the **whole** semileptonic region if not available from LQCD?

Map for local FFs

obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by either

- **extrapolating LQCD** calculations to low q^2
- or **combining LQCD** and **LCSRs**

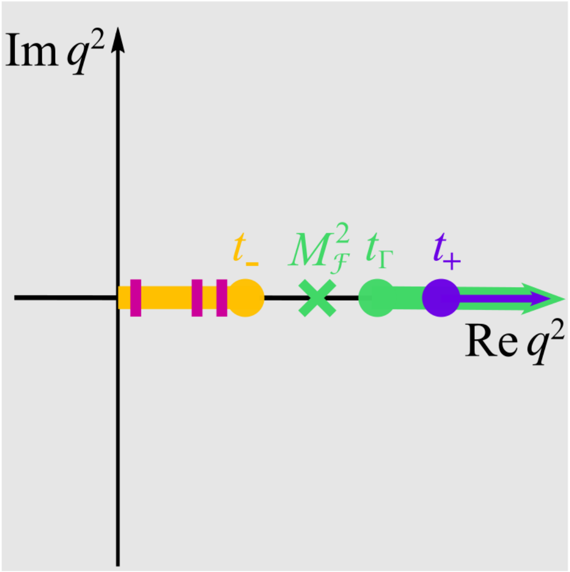
Map for local FFs

obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by either

- **extrapolating LQCD** calculations to low q^2
- or **combining LQCD** and **LCSRs**

$\mathcal{F}_\lambda$ analytic functions of q^2 except for isolated $s\bar{b}$ poles
and a branch cut for $q^2 > t_\Gamma = \left(M_{B_s} + (2)M_\pi\right)^2$

branch cut differs from the pair production threshold:
$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$ contrary to, e.g., $B \to \pi$



Map for local FFs

obtain local FFs $\mathcal{F}_\lambda$ in the whole semileptonic region by either

- **extrapolating LQCD** calculations to low q^2
- or **combining LQCD** and **LCSRs**

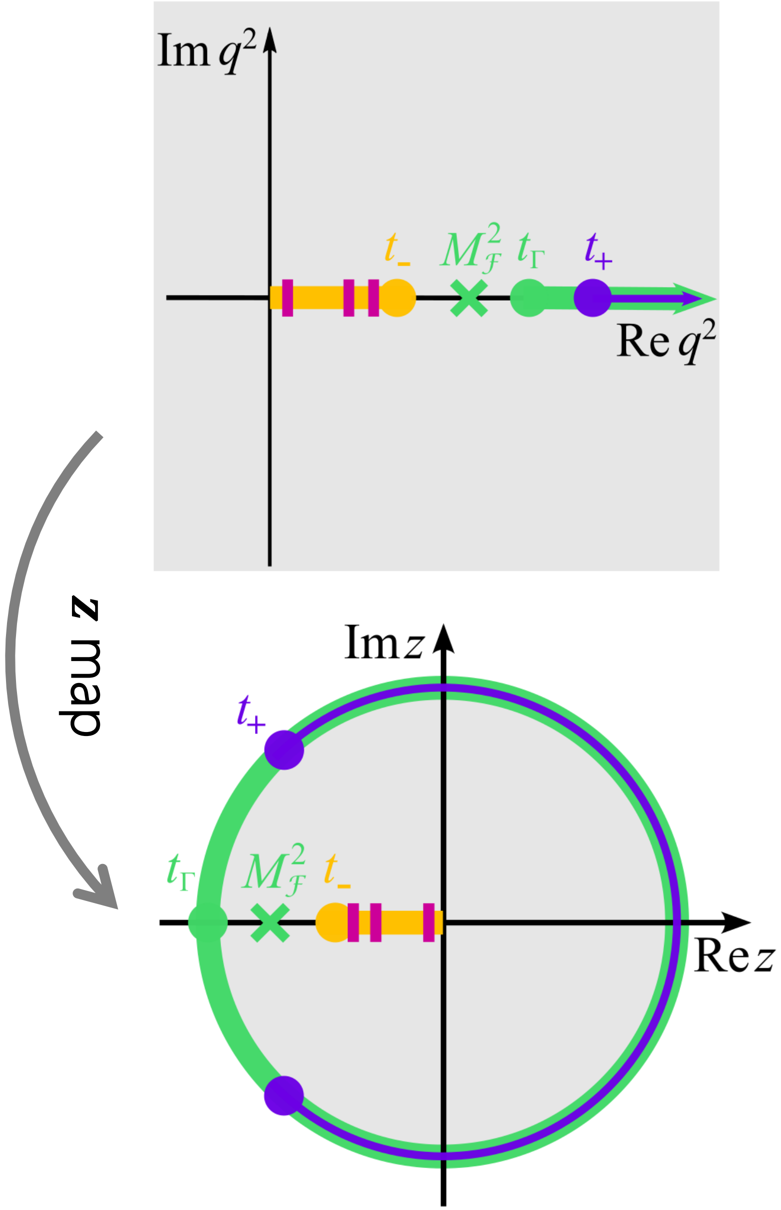
$\mathcal{F}_\lambda$ analytic functions of q^2 except for isolated $s\bar{b}$ poles
and a branch cut for $q^2 > t_\Gamma = \left(M_{B_s} + (2)M_\pi\right)^2$

branch cut differs from the pair production threshold:
$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$ contrary to, e.g., $B \to \pi$

define the map

$$z(q^2) = \frac{\sqrt{t_\Gamma - q^2} - \sqrt{t_\Gamma}}{\sqrt{t_\Gamma - q^2} + \sqrt{t_\Gamma}}$$

previous works on $B \to K^{(*)}$ local FFs always approximated $t_\Gamma = t_+$
non-quantifiable systematic uncertainties



Parametrization for $\mathcal{F}_\lambda$

$\mathcal{F}_\lambda$ analytic in the open unit disk $\Longrightarrow$ **expand $\mathcal{F}_\lambda$ in a Taylor series in z** (up to some known function)

simple (BSZ) z parametrization $\Longrightarrow$ unbounded coefficients [Bharucha/Straub/Zwicky 2015]

$$\mathcal{F}_\lambda = \frac{1}{1 - \frac{q^2}{M_{\mathcal{F}}^2}} \sum_{k=0}^{\infty} a_k\, z^k$$

Parametrization for $\mathcal{F}_\lambda$

$\mathcal{F}_\lambda$ analytic in the open unit disk $\Longrightarrow$ **expand $\mathcal{F}_\lambda$ in a Taylor series in z** (up to some known function)

simple (BSZ) z parametrization $\Longrightarrow$ unbounded coefficients [Bharucha/Straub/Zwicky 2015]

$$\mathcal{F}_\lambda = \frac{1}{1 - \frac{q^2}{M_\mathcal{F}^2}} \sum_{k=0}^{\infty} a_k z^k$$

BGL parametrization $\Longrightarrow$ valid only if $t_\Gamma = t_+$, monomials orthonormal on the unit circle

[Boyd/Grinstein/Lebed 1997]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} b_k z^k \qquad \sum_{k=0}^{\infty} |b_k|^2 < 1$$

Parametrization for $\mathcal{F}_\lambda$

$\mathcal{F}_\lambda$ analytic in the open unit disk $\Longrightarrow$ expand $\mathcal{F}_\lambda$ in a Taylor series in $\boldsymbol{z}$ (up to some known function)

simple (BSZ) z parametrization $\Longrightarrow$ unbounded coefficients [Bharucha/Straub/Zwicky 2015]

$$\mathcal{F}_\lambda = \frac{1}{1 - \frac{q^2}{M_{\mathcal{F}}^2}} \sum_{k=0}^{\infty} a_k \, z^k$$

BGL parametrization $\Longrightarrow$ valid only if $t_\Gamma = t_+$, monomials orthonormal on the unit circle

[Boyd/Grinstein/Lebed 1997]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} b_k \, z^k \qquad \sum_{k=0}^{\infty} |b_k|^2 < 1$$

GvDV parametrization $\Longrightarrow$ valid also for $t_\Gamma \neq t_+$, generalization of BGL, polynomials orthonormal on the arc of the unit circle

[NG/van Dyk/Virto 2020]

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} c_k \, p_k(z) \qquad \sum_{k=0}^{\infty} |c_k|^2 < 1$$

fit this parametrization to LQCD (and LCSR) results and use new improved bounds

Local form factors predictions

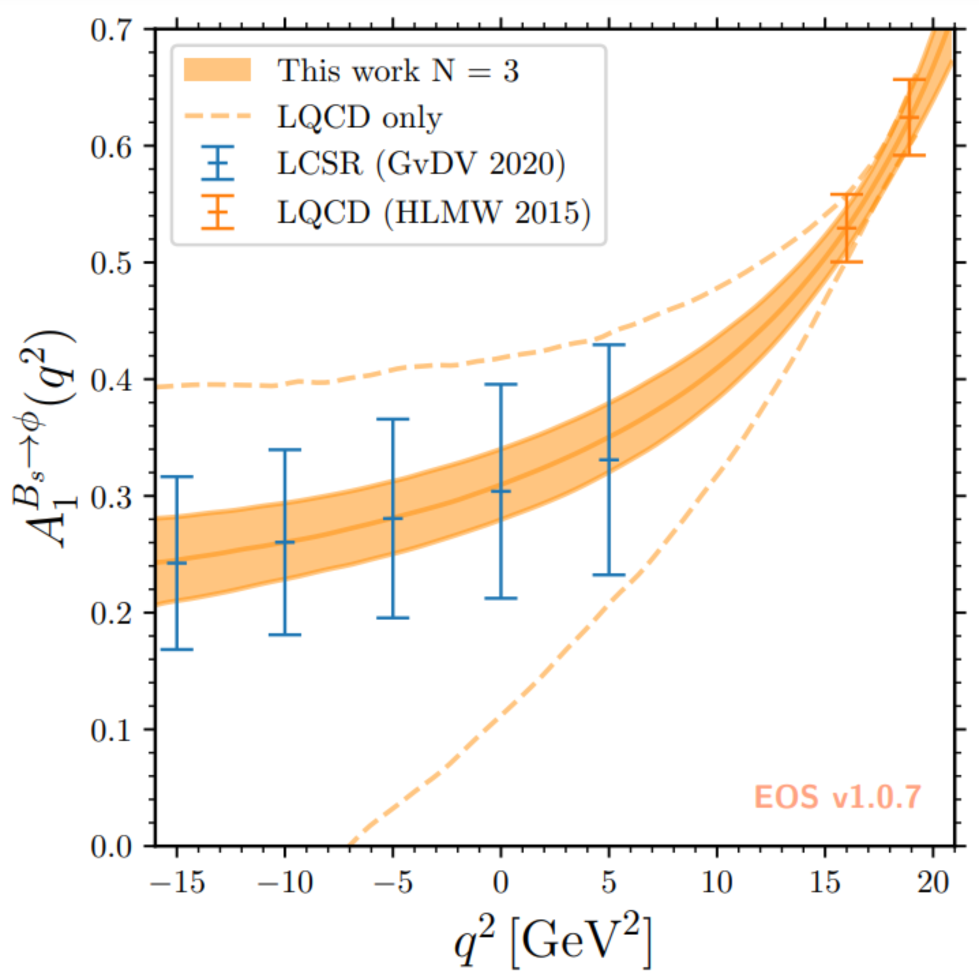
$$\mathcal{A}(B \to K^{(*)}\ell^+\ell^-) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right) \mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\ \mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

fit available inputs to

$$\mathcal{F}_\lambda = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{3} c_k\, p_k(z) \qquad \sum_{k=0}^{3} |c_k|^2 < 1$$

obtain numerical results for the for $B \to K^{(*)}$ and $B_s \to \phi$ in the whole semileptonic region

[NG/Reboud/van Dyk/Virto 2023]



first simultaneous fit of these FFs

systematic uncertainties under control
large p values

results given in machine readable files

Non-local form factors

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

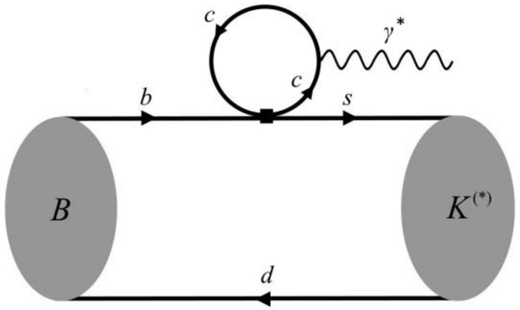
$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

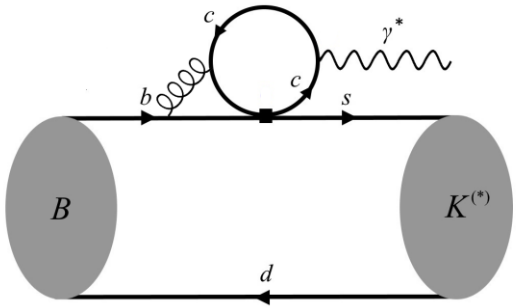
1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

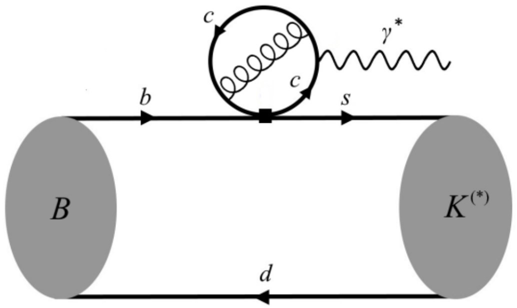
$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

leading power (LO in α_s)



+ hard gluons (α_s) corrections





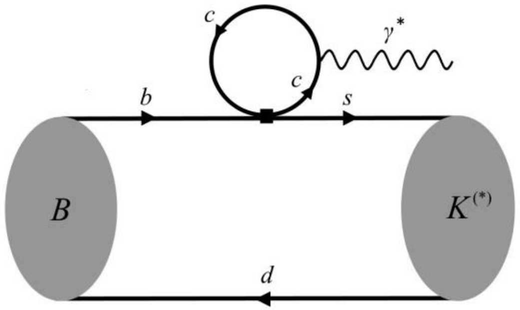
[Bell/Huber 2014] [Asatrian/Greub/Virto 2019]

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

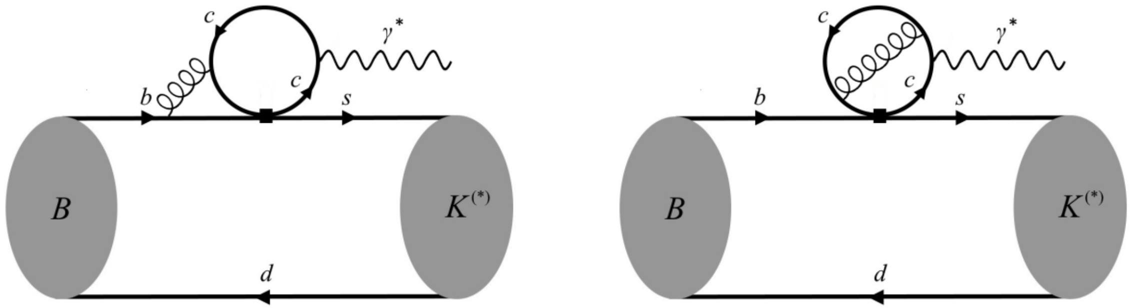
1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = \mathcal{C}_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{\mathcal{C}}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

leading power (LO in α_s)

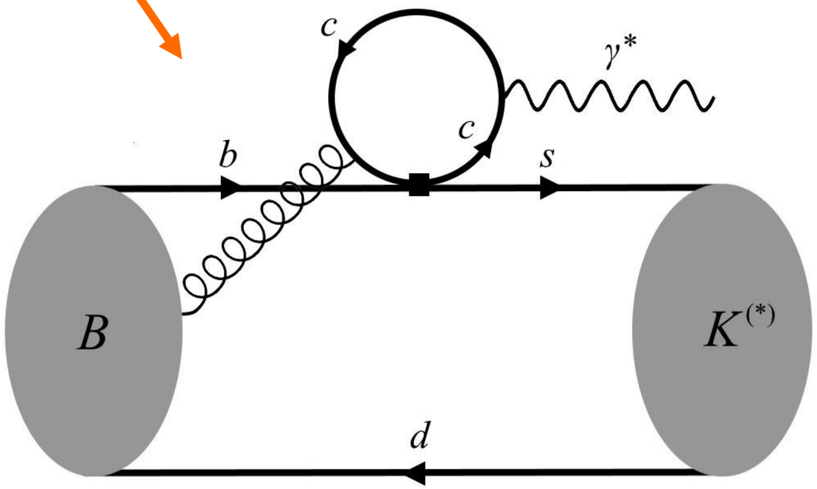


+ hard gluons (α_s) corrections



[Bell/Huber 2014] [Asatrian/Greub/Virto 2019]

soft gluon correction
non-perturbative
$\Longrightarrow$ not α_s suppressed



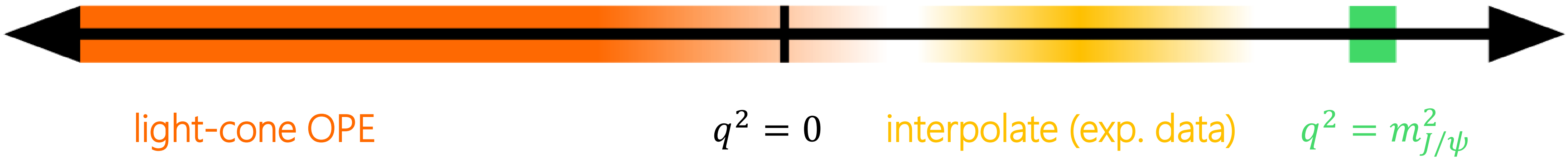
[Khodjamirian et al. 2010]
[NG/van Dyk/Virto 2020]

Obtaining theoretical predictions for $\mathcal{H}_\lambda$

1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

2. extract $\mathcal{H}_\lambda$ at $q^2 = m_{J/\psi}^2$ from $B \to K^{(*)}J/\psi$ and $B_s \to \phi\, J/\psi$ measurements (decay amplitudes independent of the local FFs)



Obtaining theoretical predictions for $\mathcal{H}_\lambda$

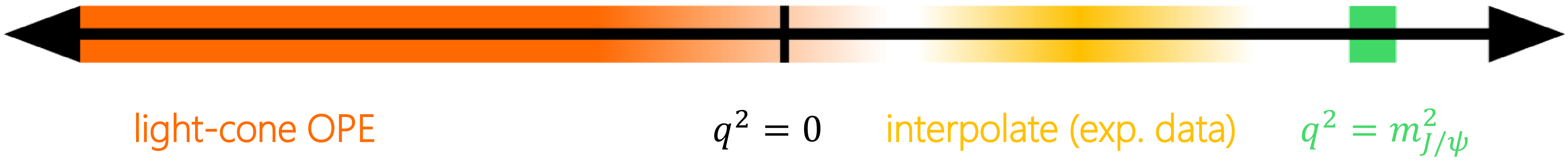
1. compute the non-local FFs $\mathcal{H}_\lambda$ using a light-cone OPE at negative q^2

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2)\mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2)\mathcal{V}_\lambda(q^2) + \cdots$$

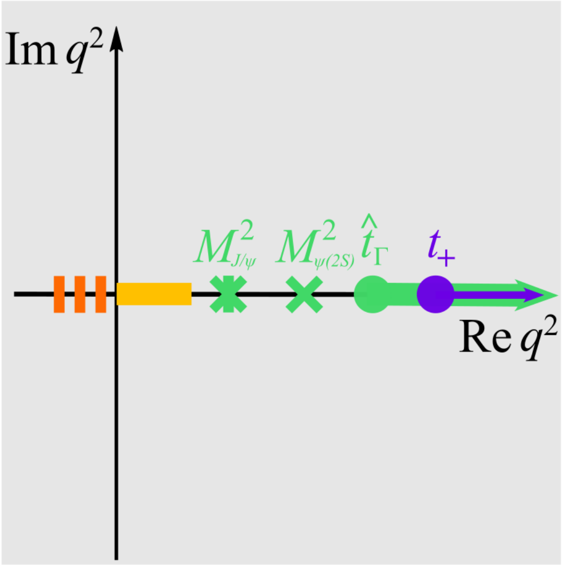
2. extract $\mathcal{H}_\lambda$ at $q^2 = m_{J/\psi}^2$ from $B \to K^{(*)}J/\psi$ and $B_s \to \phi\, J/\psi$ measurements (decay amplitudes independent of the local FFs)

3. **new approach: interpolate** these two results to obtain theoretical predictions in the low q^2 $(0 < q^2 < 8\ \mathrm{GeV}^2)$ region $\Longrightarrow$ compare with experimental data

need a parametrization to interpolate $\mathcal{H}_\lambda$: which is the optimal parametrization?



Map for non-local FFs



similar situation with respect to $\mathcal{F}_\lambda$

$\mathcal{H}_\lambda$ analytic functions of q^2 except for isolated $c\bar{c}$ poles (J/ψ and $\psi(2S)$) and a branch cut for $q^2 > \hat{t}_\Gamma = 4M_D^2$

branch cut differs from the pair production threshold:

$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$

Map for non-local FFs

similar situation with respect to $\mathcal{F}_\lambda$

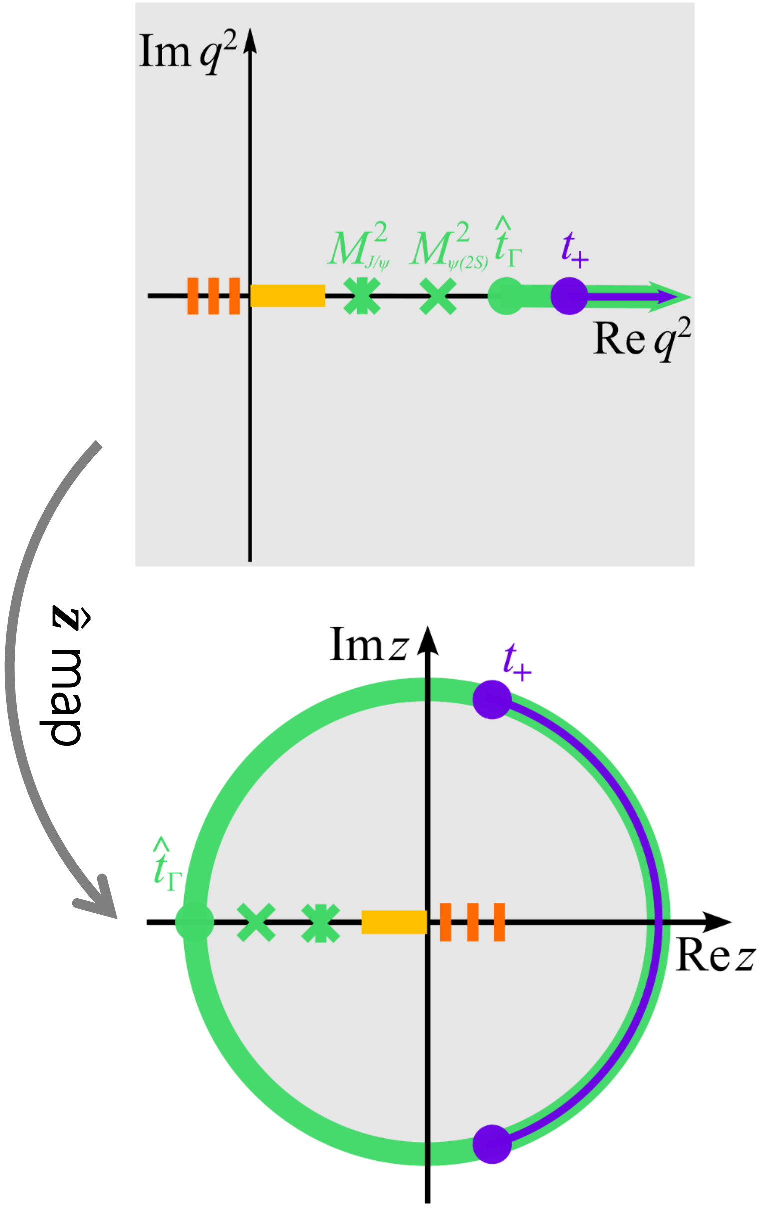
$\mathcal{H}_\lambda$ analytic functions of q^2 except for isolated $c\bar{c}$ poles (J/ψ and $\psi(2S)$) and a branch cut for $q^2 > \hat{t}_\Gamma = 4M_D^2$

branch cut differs from the pair production threshold:
$t_\Gamma \neq t_+ = \left(M_B + M_{K^{(*)}}\right)^2$

define the map

$$\hat{z}(q^2) = \frac{\sqrt{\hat{t}_\Gamma - q^2} - \sqrt{\hat{t}_\Gamma}}{\sqrt{\hat{t}_\Gamma - q^2} + \sqrt{\hat{t}_\Gamma}}$$

only difference between $\mathcal{F}_\lambda$ and $\mathcal{H}_\lambda$ is the threshold $\hat{t}_\Gamma$ and the poles due to more complicate structure of the operator



Parametrizations for $\mathcal{H}_\lambda$

simple q^2 parametrization [Jäger/Camalich 2012, Ciuchini et al. 2015]

$$\mathcal{H}_\lambda(q^2) = \mathcal{H}_\lambda^{\text{QCDF}}(q^2) + \mathcal{H}_\lambda^{\text{rest}}(0) + \frac{q^2}{M_B^2}\mathcal{H}_\lambda^{\text{rest},\prime}(0) + \frac{(q^2)^2}{M_B^4}\mathcal{H}_\lambda^{\text{rest},\prime\prime}(0) + \cdots$$

simple z parametrization [Bobeth/Chrzaszcz/van Dyk/Virto 2017]

$$\mathcal{H}_\lambda(z) \propto \sum_{k=0}^{\infty} \alpha_k z^k$$

GvDV parametrization $\Longrightarrow$ new (bounded) parametrization, $\hat{z}$ polynomials [NG/van Dyk/Virto 2020]

$$\mathcal{H}_\lambda(\hat{z}) = \frac{1}{\mathcal{P}(z)\phi(z)} \sum_{k=0}^{\infty} \beta_k p_k(\hat{z}) \qquad \sum_{k=0}^{\infty} |\beta_k|^2 < 1$$

fit this parametrization to OPE result and $B \to K^{(*)} J/\psi$ data

Non-local form factors predictions

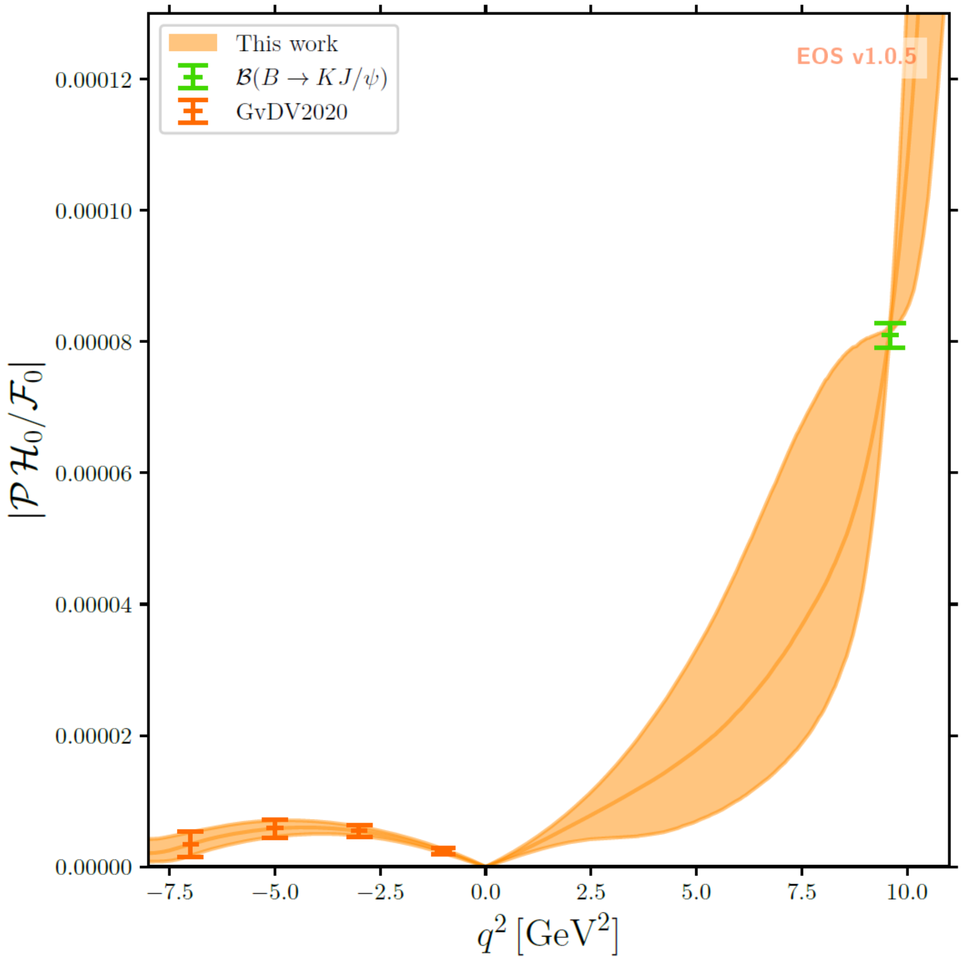
$$\mathcal{A}(B \to K^{(*)}\ell\ell) = \mathcal{N}\left[\left(C_9 L_V^\mu + C_{10} L_A^\mu\right)\mathcal{F}_\mu - \frac{L_V^\mu}{q^2}\left(C_7\,\mathcal{F}_{T,\mu} + \mathcal{H}_\mu\right)\right]$$

obtain numerical results for the non-local FFs $\mathcal{H}_\lambda$

$$\mathcal{H}_\lambda \cong \sum_{n=0}^{5} \beta_n p_n(\hat{z}) \qquad \sum_{k=0}^{5} |\beta_k|^2 < 1$$

fit the $\hat{z}$ parametrization

- light-cone OPE calculation at negative q^2
- $B \to K^{(*)} J/\psi$ and $B_s \to \phi\, J/\psi$ measurements at $q^2 = m_{J/\psi}^2$
- unitarity bound



new approach to obtain non-local FFs
one fit per decay channel (all p values $> 11\%$)

SM predictions and confrontation with data

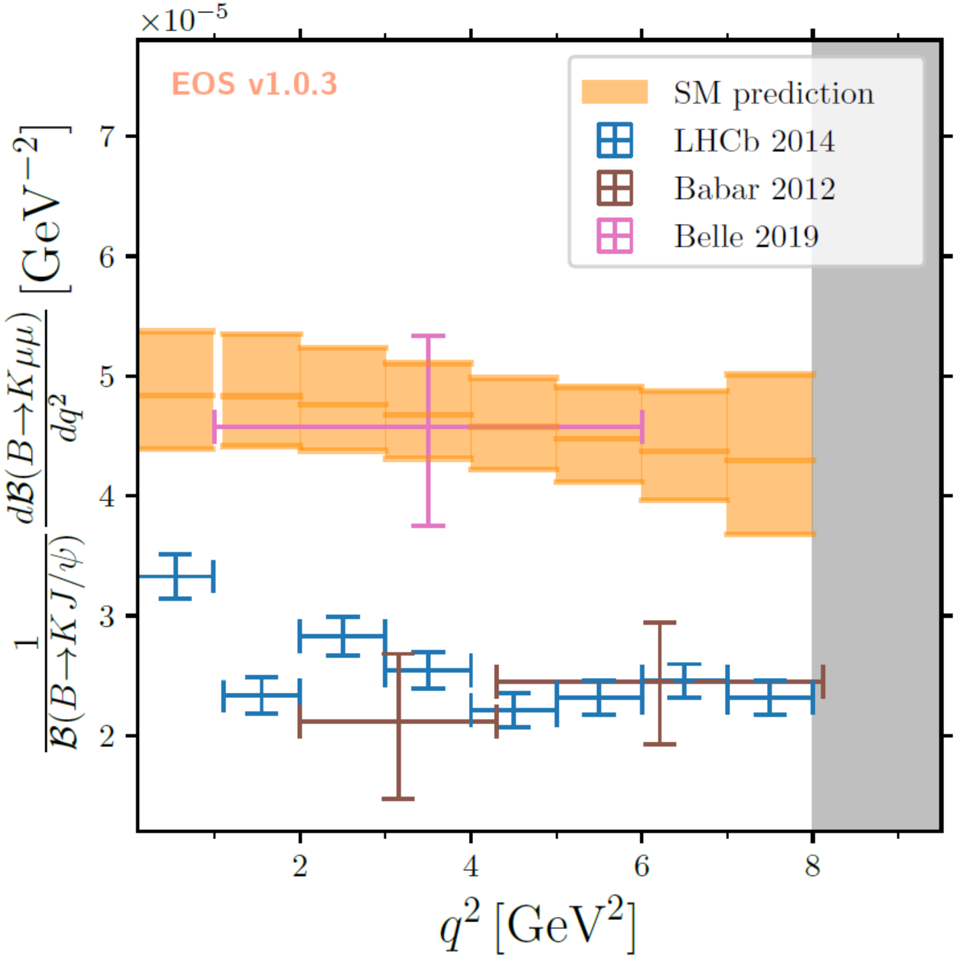
Standard Model predictions

using our local $\mathcal{F}_\lambda$ and non-local $\mathcal{H}_\lambda$ FFs values
we predict observables (BRs and angular observables)
for $B \to K^{(*)}\mu^+\mu^-$, and $B_s \to \phi\mu^+\mu^-$ in the SM

- we do not use QCD factorization (QCDF) like all previous SM predictions (non-quantifiable systematic uncertainty)

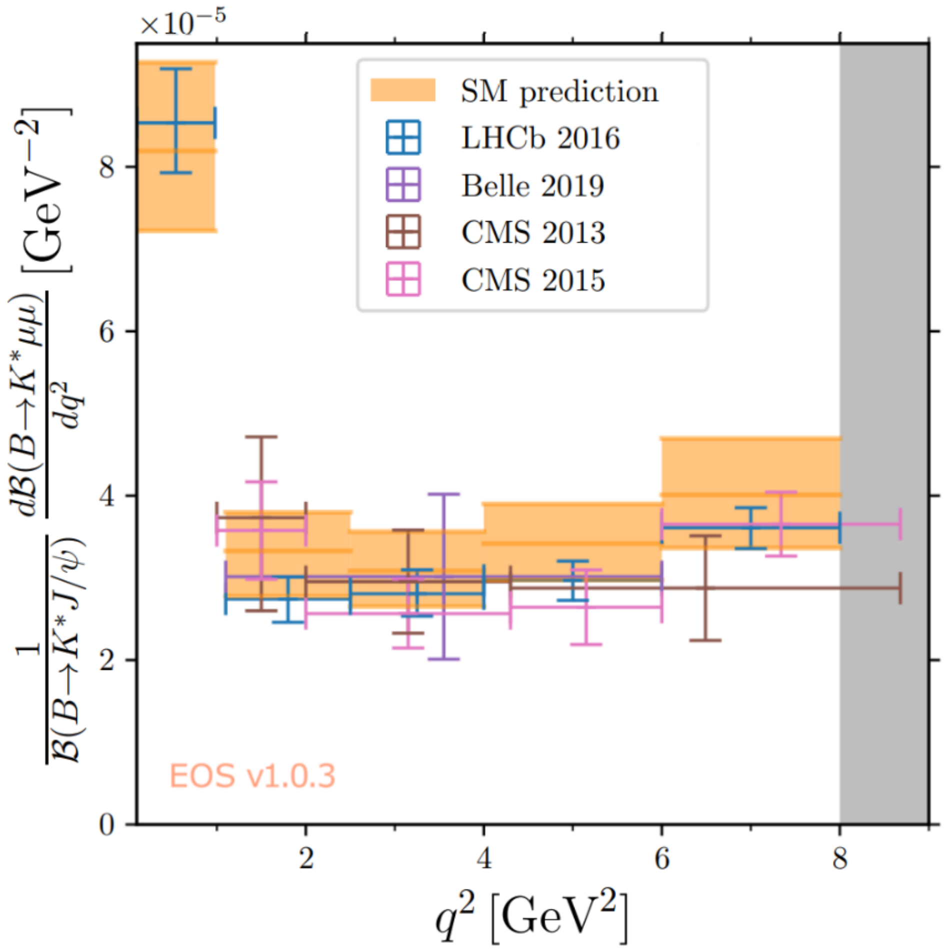
- theory uncertainties mostly due to local FFs

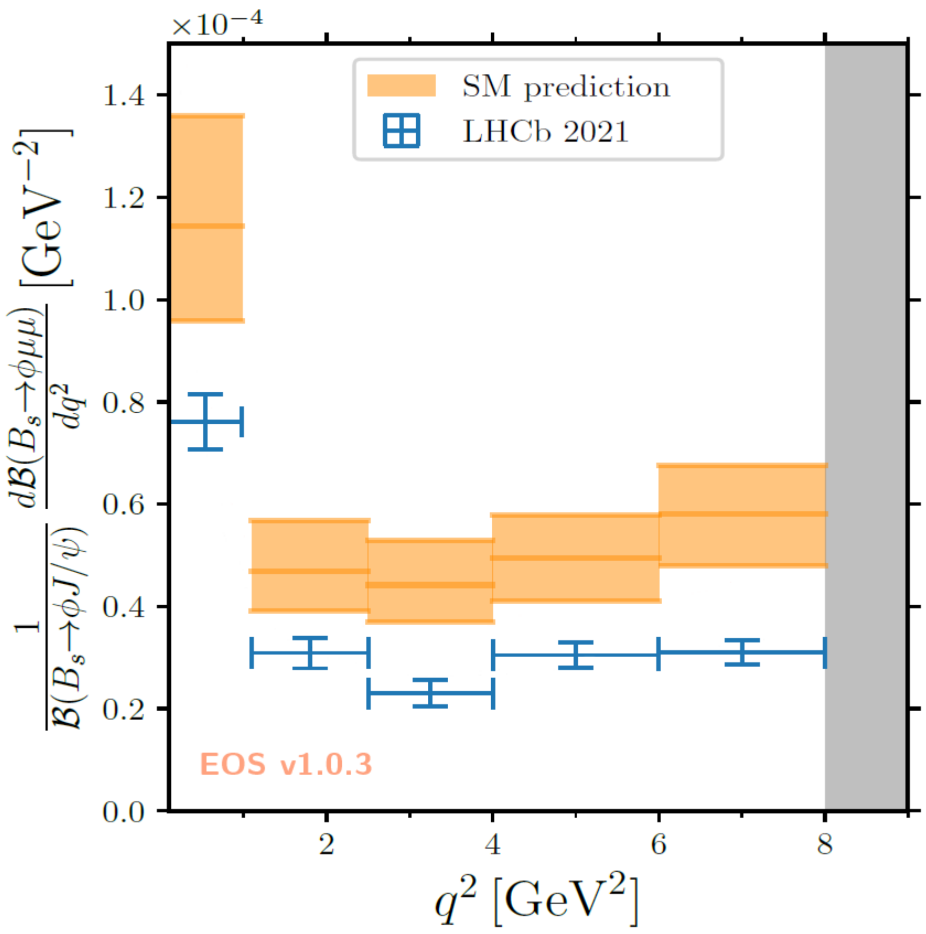
- coherent tension between SM predictions and experimental data



[TO BE UPDATED]

Comparison with measurements for $B \to K^*\mu^+\mu^-$





larger theory uncertainties due to less precise inputs for local FFs

$\Longrightarrow$ smaller tension but coherent shift w.r.t. data

Summary and conclusion

Summary and conclusion

1. reassess BGL parametrization for local FFs $\mathcal{F}_\lambda$ to consider below threshold branch cut and improved unitarity bounds

   combine LQCD (and LCSRs) inputs to get new results for $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ local FFs $\mathcal{F}_\lambda$

Summary and conclusion

1. reassess BGL parametrization for local FFs $\mathcal{F}_\lambda$ to consider below threshold branch cut and improved unitarity bounds

   combine LQCD (and LCSRs) inputs to get new results for $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ local FFs $\mathcal{F}_\lambda$

2. new theoretical predictions using our OPE calculation for the non-local FFs $\mathcal{H}_\lambda$ at $q^2 < 0$, experimental data for $B \to K^{(*)}J/\psi$, and a unitarity bound

   new approach — $\mathcal{H}_\lambda$ uncertainties can be systematically reduced with unitarity bound
   (more local form factors $\mathcal{F}_\lambda$ calculations, saturating the unitarity bound...)

Summary and conclusion

1. reassess BGL parametrization for local FFs $\mathcal{F}_\lambda$ to consider below threshold branch cut and improved unitarity bounds

   combine LQCD (and LCSRs) inputs to get new results for $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ local FFs $\mathcal{F}_\lambda$

2. new theoretical predictions using our OPE calculation for the non-local FFs $\mathcal{H}_\lambda$ at $q^2 < 0$, experimental data for $B \to K^{(*)}J/\psi$, and a unitarity bound

   new approach — $\mathcal{H}_\lambda$ uncertainties can be systematically reduced with unitarity bound
   (more local form factors $\mathcal{F}_\lambda$ calculations, saturating the unitarity bound…)

3. new and precise SM predictions for observables in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays

   coherent deviations between SM and data in $B \to K^{(*)}\ell^+\ell^-$ and $B_s \to \phi\ell^+\ell^-$ decays

Thank you!

Backup slides

Global fit to $b \to s\mu^+\mu^-$ (results)

we **obtain good fits**, agreement between the three fits

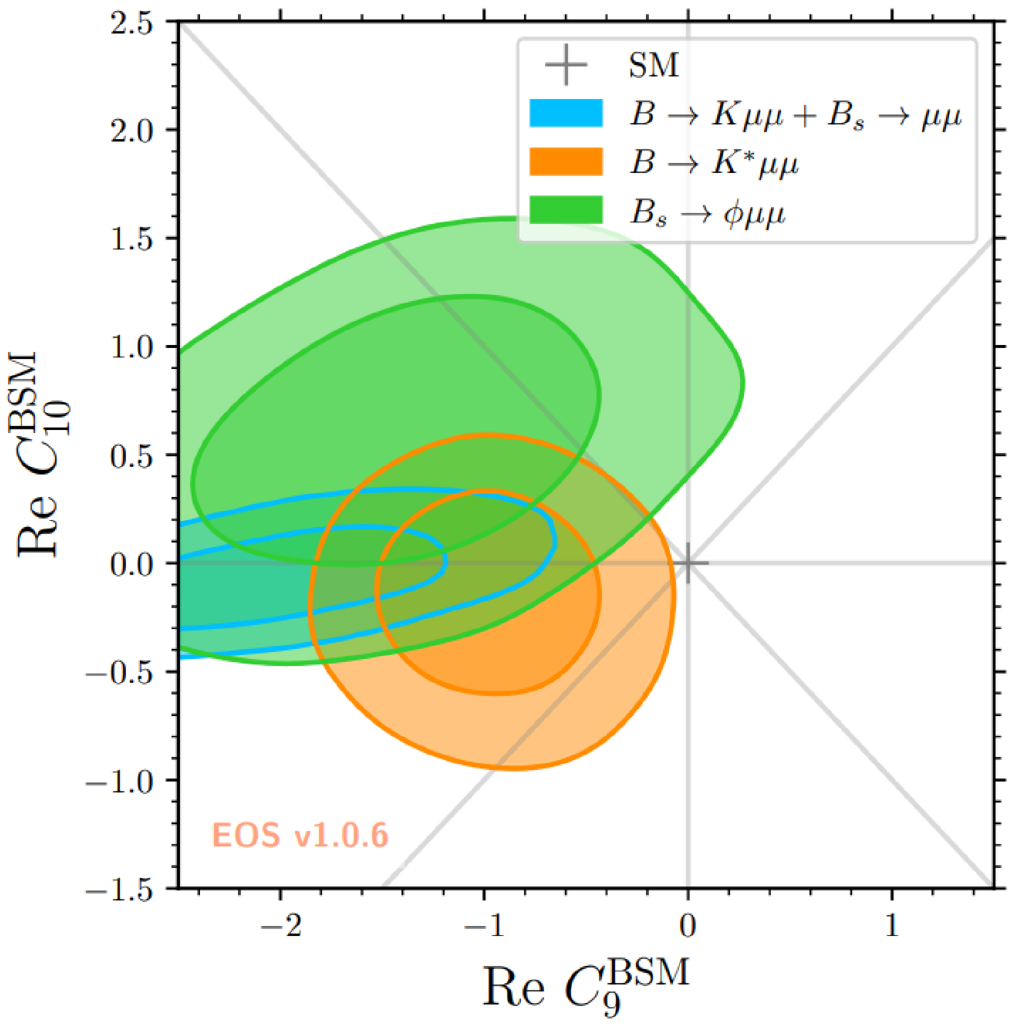
substantial tension w.r.t. SM (in agreement with the literature)

pulls (p value of the SM hypothesis):

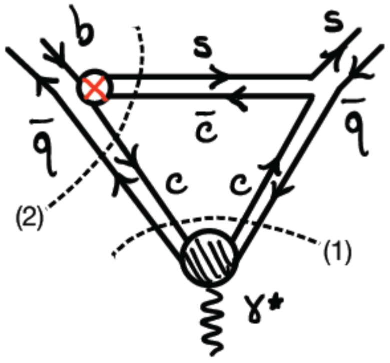
- 5.7σ for $B \to K\mu^+\mu^- + B_s \to \mu^+\mu^-$
- 2.7σ for $B \to K^*\mu^+\mu^-$
- 2.6σ for $B_s \to \phi\mu^+\mu^-$

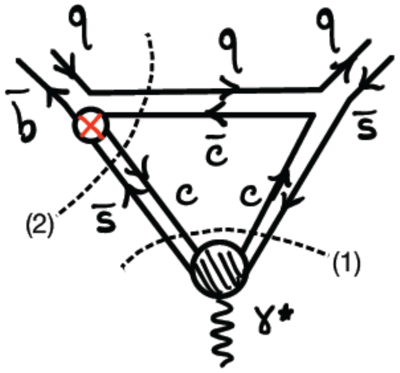
local FFs $\mathcal{F}_\lambda$ main uncertainties

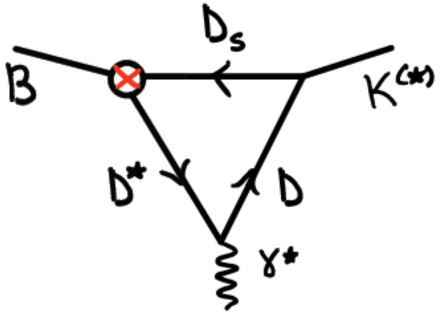
present theory predictions for
non-local FFs $\mathcal{H}_\lambda$ cannot explain this tension



Missing something?







Ciuchini et al. 2022 (also way before) claim that $B \to \bar{D} D_s \to K^{(*)} \ell^+ \ell^-$ rescattering might have a sizable contribution $O(20\%)$

is a **mesonic** estimate the best way to go? (many states contributing, interferences even harder to compute)

partonic calculation doesn't yield large contribution (LP OPE and NLO α_s) [Asatrian/Greub/Virto 2019]

$$\mathcal{H}_\lambda(q^2) = C_\lambda(q^2) \mathcal{F}_\lambda(q^2) + \tilde{C}_\lambda(q^2) \mathcal{V}_\lambda(q^2) + \cdots$$

C_λ is complex valued for any q^2 value due to branch cut in $p^2 = M_B^2$ as expected

large duality violations? large NLP OPE or α_s^2 corrections? spectator scattering?