

Theoretical progress in inclusive penguin decays

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21st Conference on Flavour Physics and CP Violation

21st FPCP 2023

Lyon, May 29 - June 2 2023

FPCP23



Plan of the Talk

- Theoretical framework for exclusive and inclusive modes
- New physics reach of semileptonic penguin decays
- Nonlocal subleading corrections in inclusive modes
- Refactorisation in subleading $\bar{B} \rightarrow X_s \gamma$
- Hadronic mass cut in $\bar{B} \rightarrow X_s \ell \ell$

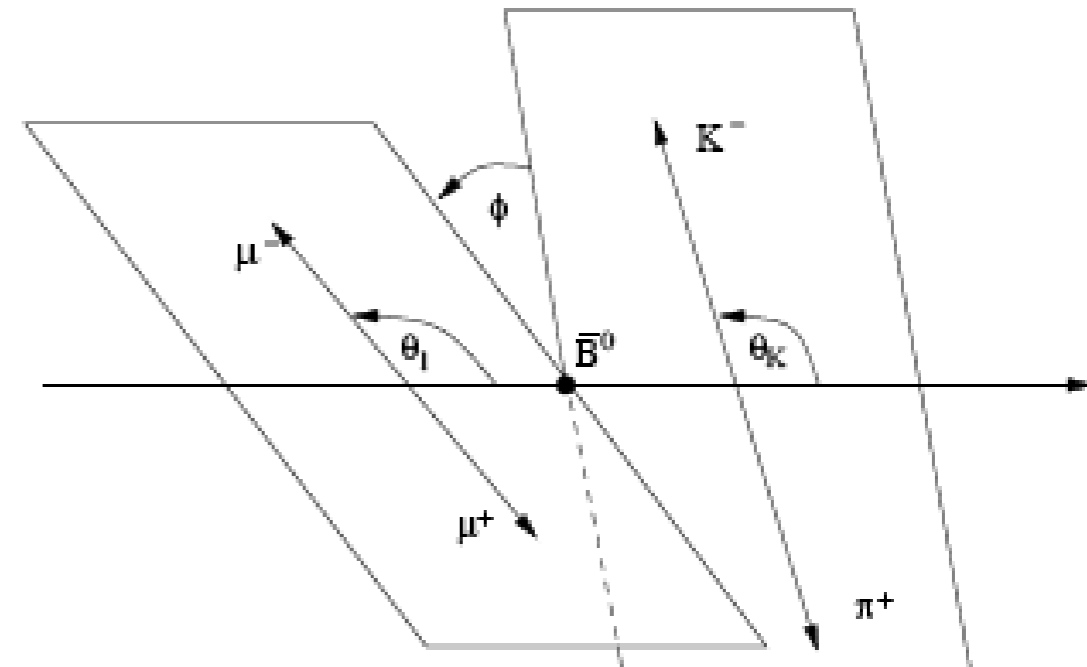
Prologue

$b > s$ anomalies

Differential decay rate of $B \rightarrow K^* \ell \ell$

Assuming the \bar{K}^* to be on the mass shell, the decay $\bar{B}^0 \rightarrow \bar{K}^{*0}(\rightarrow K^- \pi^+) \ell^+ \ell^-$ described by the lepton-pair invariant mass, s , and the three angles θ_l , θ_K , ϕ .

$$\frac{d^4\Gamma}{dq^2 d\cos\theta_l d\cos\theta_K d\phi} = \frac{9}{32\pi} J(q^2, \theta_l, \theta_K, \phi)$$



$$J(q^2, \theta_l, \theta_K, \phi) =$$

$$\begin{aligned} &= J_{1s} \sin^2 \theta_K + J_{1c} \cos^2 \theta_K + (J_{2s} \sin^2 \theta_K + J_{2c} \cos^2 \theta_K) \cos 2\theta_l + J_3 \sin^2 \theta_K \sin^2 \theta_l \cos 2\phi \\ &\quad + J_4 \sin 2\theta_K \sin 2\theta_l \cos \phi + J_5 \sin 2\theta_K \sin \theta_l \cos \phi + (J_{6s} \sin^2 \theta_K + J_{6c} \cos^2 \theta_K) \cos \theta_l \\ &\quad + J_7 \sin 2\theta_K \sin \theta_l \sin \phi + J_8 \sin 2\theta_K \sin 2\theta_l \sin \phi + J_9 \sin^2 \theta_K \sin^2 \theta_l \sin 2\phi \end{aligned}$$

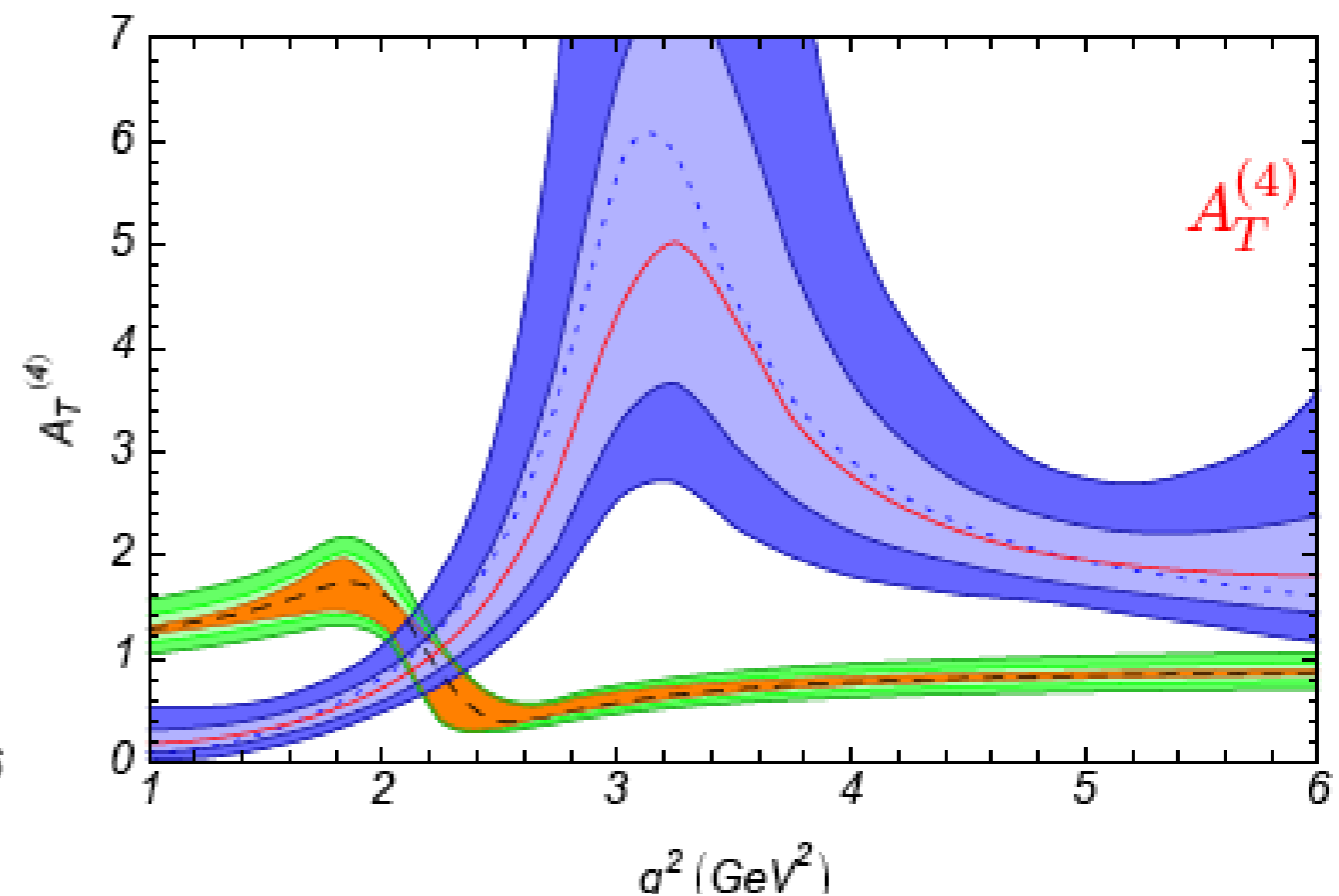
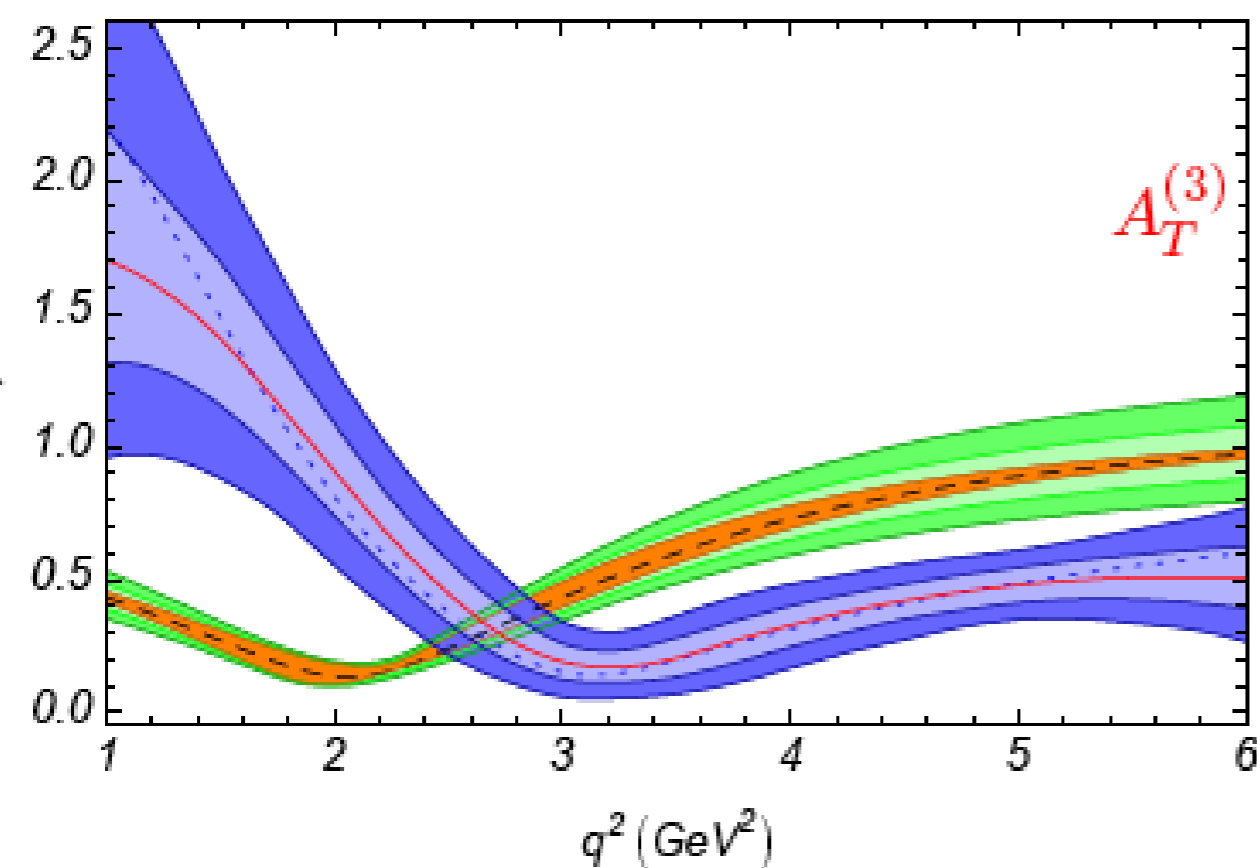
Large number of independent angular observables

Careful design of theoretical clean angular observables

Egede, Hurth, Matias, Ramon, Reece, arXiv:0807.2589, arXiv:1005.0571

- Dependence of soft form factors, ξ_{\perp} and ξ_{\parallel} , to be minimized !
form factors should cancel out exactly at LO, best for all s
- unknown Λ/m_b power corrections

$A_{\perp,\parallel,0} = A_{\perp,\parallel,0}^0 (1 + c_{\perp,\parallel,0})$ vary c_i in a range of $\pm 10\%$ and also of $\pm 5\%$
Guesstimate



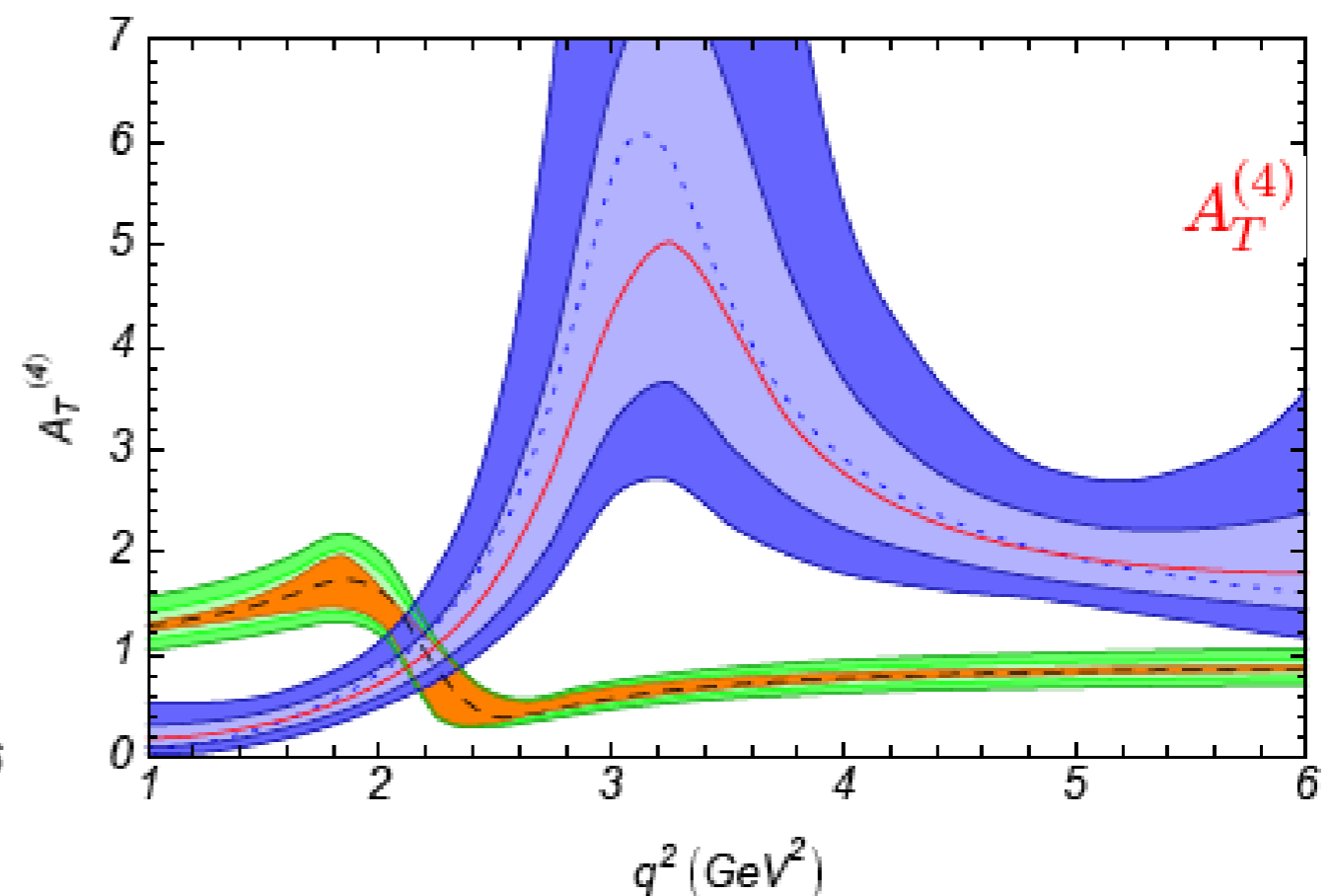
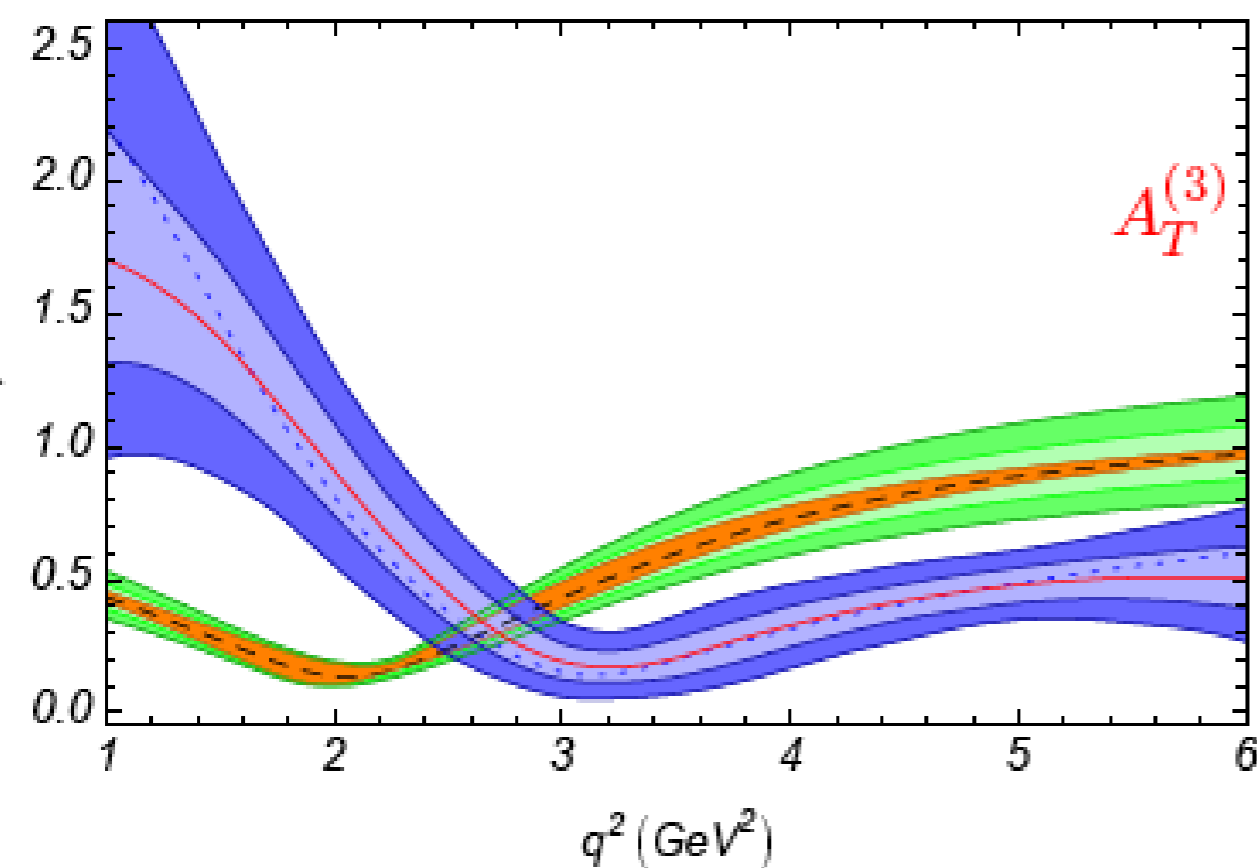
The experimental errors assuming SUSY scenario (b) with large-gluino mass and positive mass insertion, is compared to the theoretical errors assuming the SM.

Careful design of theoretical clean angular observables

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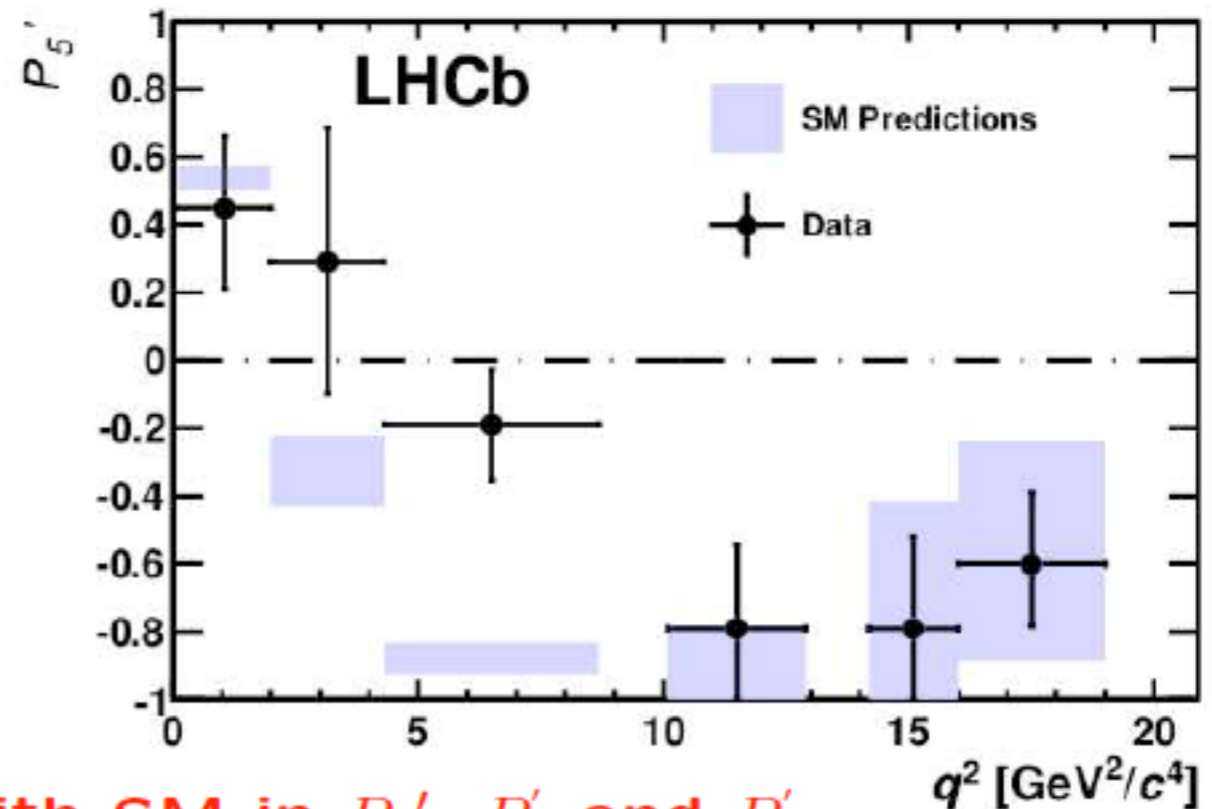
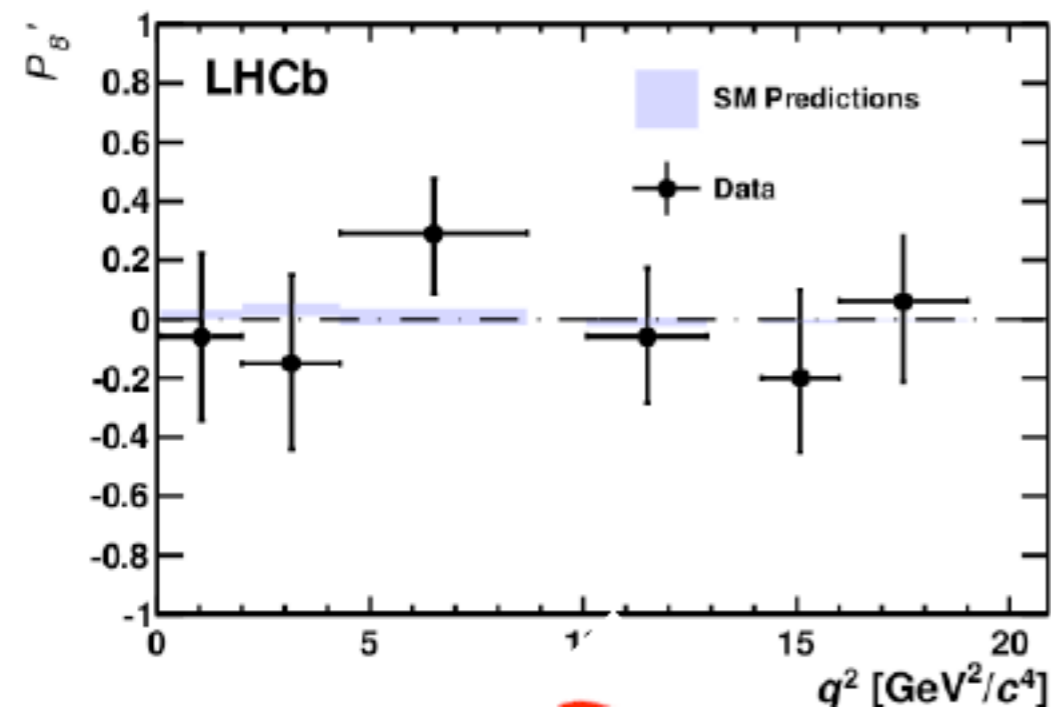
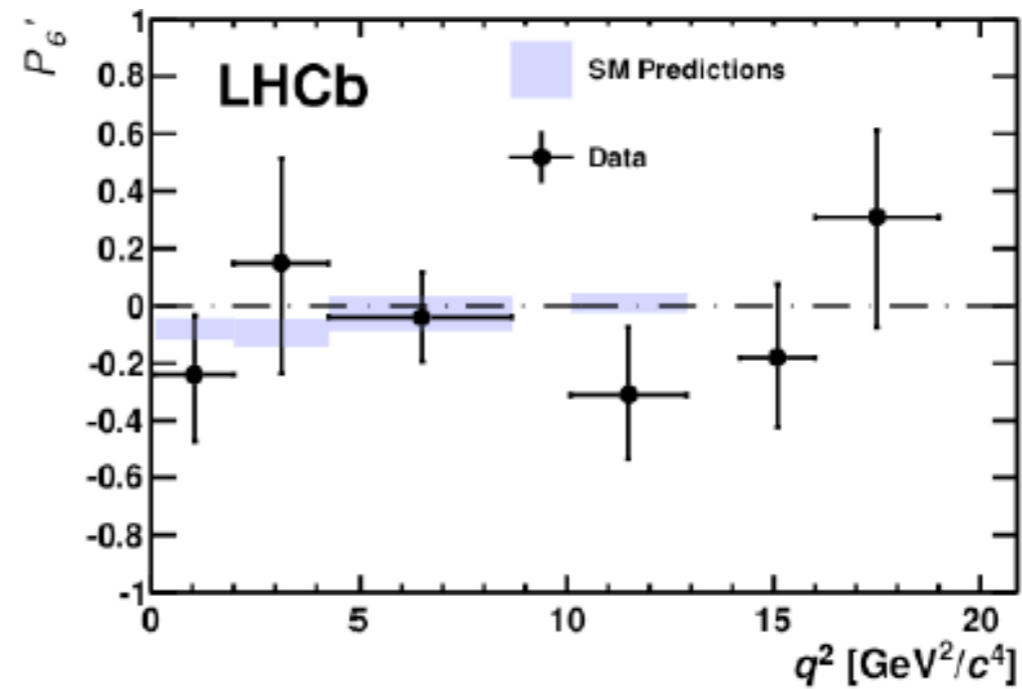
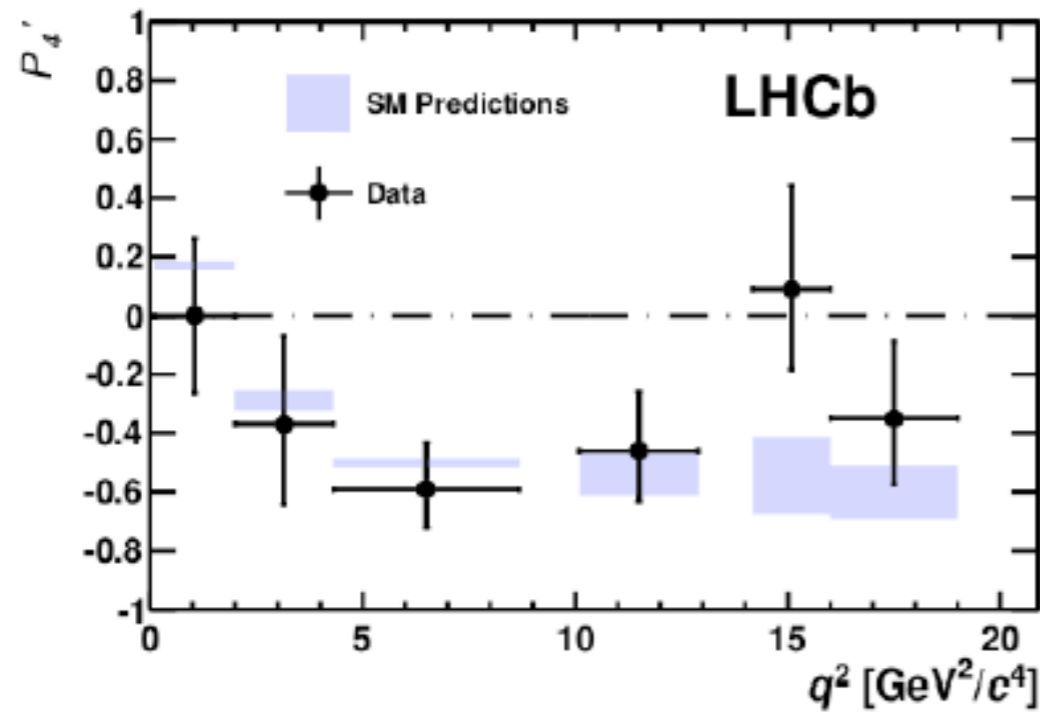
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Guesstimate



This was the dream in 2008

SM predictions Descotes-Genon, Hurth, Matias, Virto arXiv:1303.5794

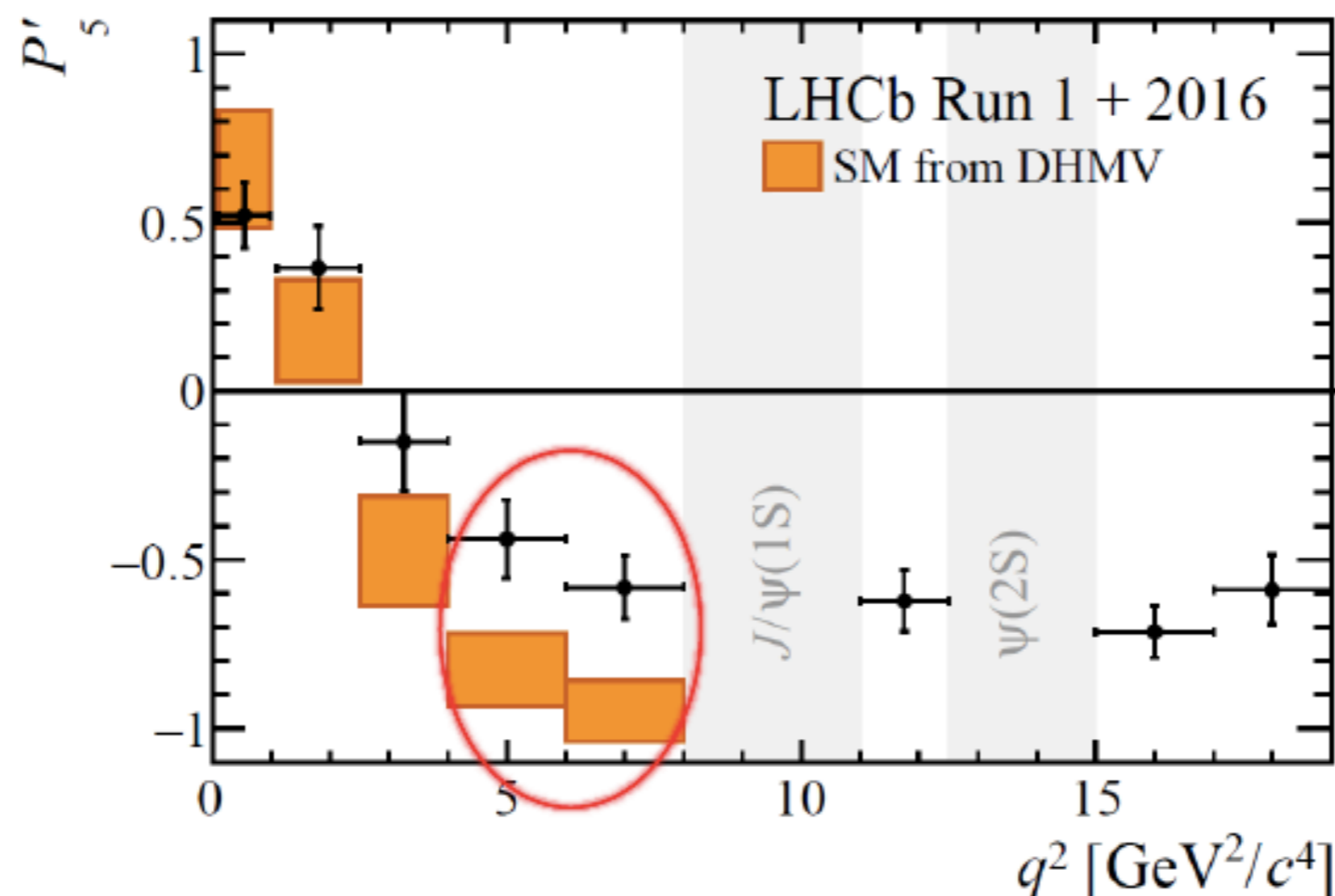


Good agreement with SM in P_4' , P_6' and P_8' ,
but a 3.7σ deviation in the third bin in P_5'

Anomalies in $B \rightarrow K^* \mu^+ \mu^-$ angular observables, in particular P'_5 ; S_5

Long standing anomaly in the $B \rightarrow K^* \mu^+ \mu^-$ angular observable P'_5 / S_5 ($= P'_5 \times \sqrt{F_L(1 - F_L)}$)

- 2013 LHCb (1 fb^{-1})
- 2016 LHCb (3 fb^{-1})
- 2020 LHCb (4.7 fb^{-1})



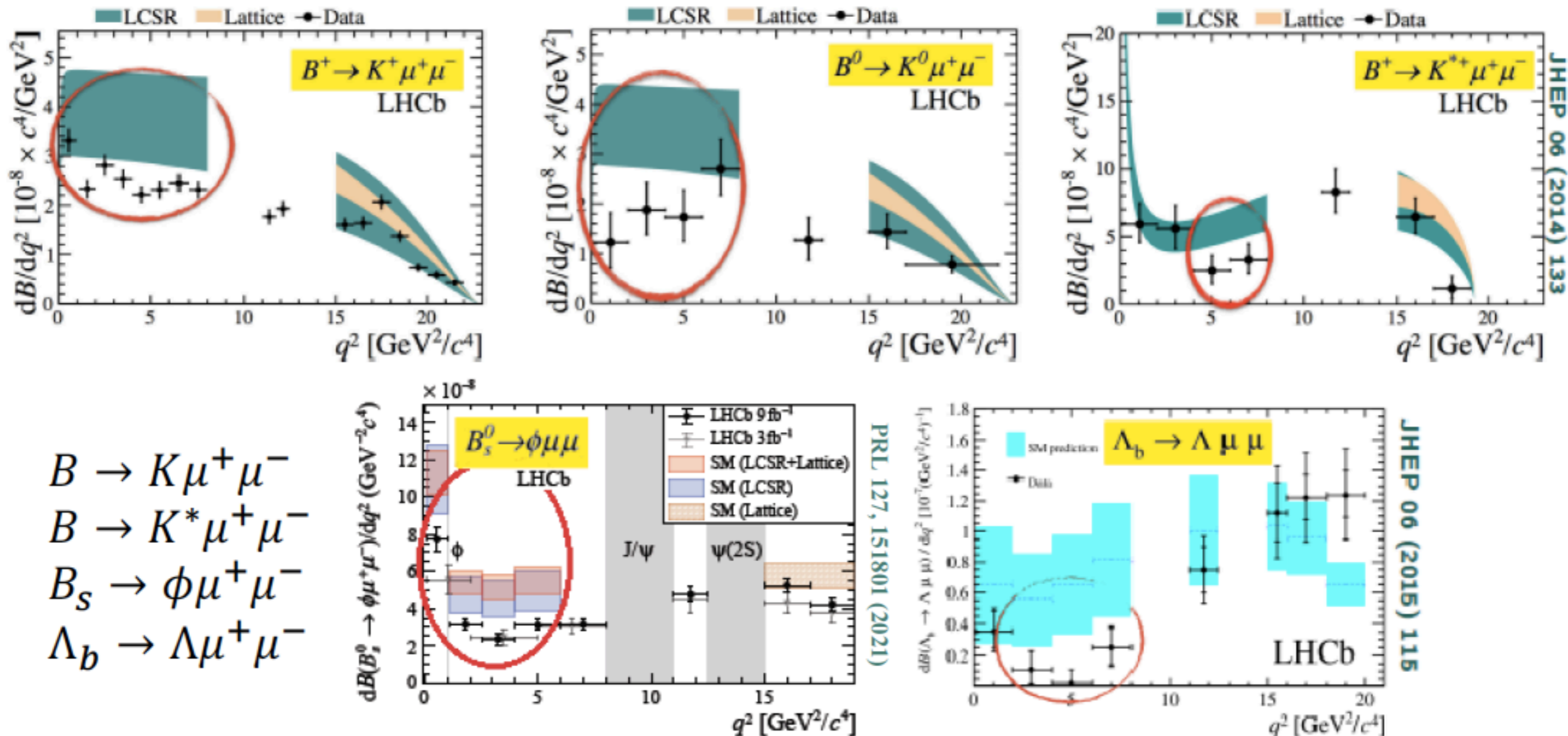
[E. Smith CERN Seminar '20
LHCb 2003.04831]

" $\approx 3\sigma$ " local tension in P'_5 with the respect SM predictions (DHMV)

Also deviations in other angular observables/bins and other decay modes

**New Physics or underestimated hadronic uncertainties
(form factors, power corrections) ?**

More tensions in the $b \rightarrow s$ branching ratios



- deviations with the SM predictions between 1 and 3.5 σ
- general trend: EXP < SM in low q^2

New Physics or underestimated hadronic uncertainties
(form factors, power corrections) ?

Lepton flavour universality in $B \rightarrow K^{(*)} \ell^+ \ell^-$

$$R_{K^{(*)}} = \frac{\mathcal{B}(B \rightarrow K^{(*)} \mu^+ \mu^-)}{\mathcal{B}(B \rightarrow K^{(*)} e^+ e^-)}$$

- Hadronic uncertainties cancel out
 \Rightarrow *theoretically very clean $\mathcal{O}(1\%)$*

Hiller, Kruger hep-ph/0310219

Jun. 2014	May. 2017	Mar. 2019	Mar. 2021	Oct. 2021
LHCb (1 fb ⁻¹) 2.6 σ in [1-6] GeV ² of R_K	LHCb (3 fb ⁻¹) 2.2 σ in [0.045-1.1] GeV ² 2.5 σ in [1.1-6] GeV ² of R_{K^*}	LHCb (5 fb ⁻¹) 2.5 σ in [1.1-6] GeV ² of R_K	LHCb (9 fb ⁻¹) 3.1 σ in [1.1-6] GeV ² of R_K	LHCb (9 fb ⁻¹) < 1.5 σ in [1.1-6] GeV ² of $R_{K^{*+}}, R_{K_S^0}$

- Theoretical prediction very precise
- More than 4 σ significance for New Physics

Would be a spectacular fall of the SM !

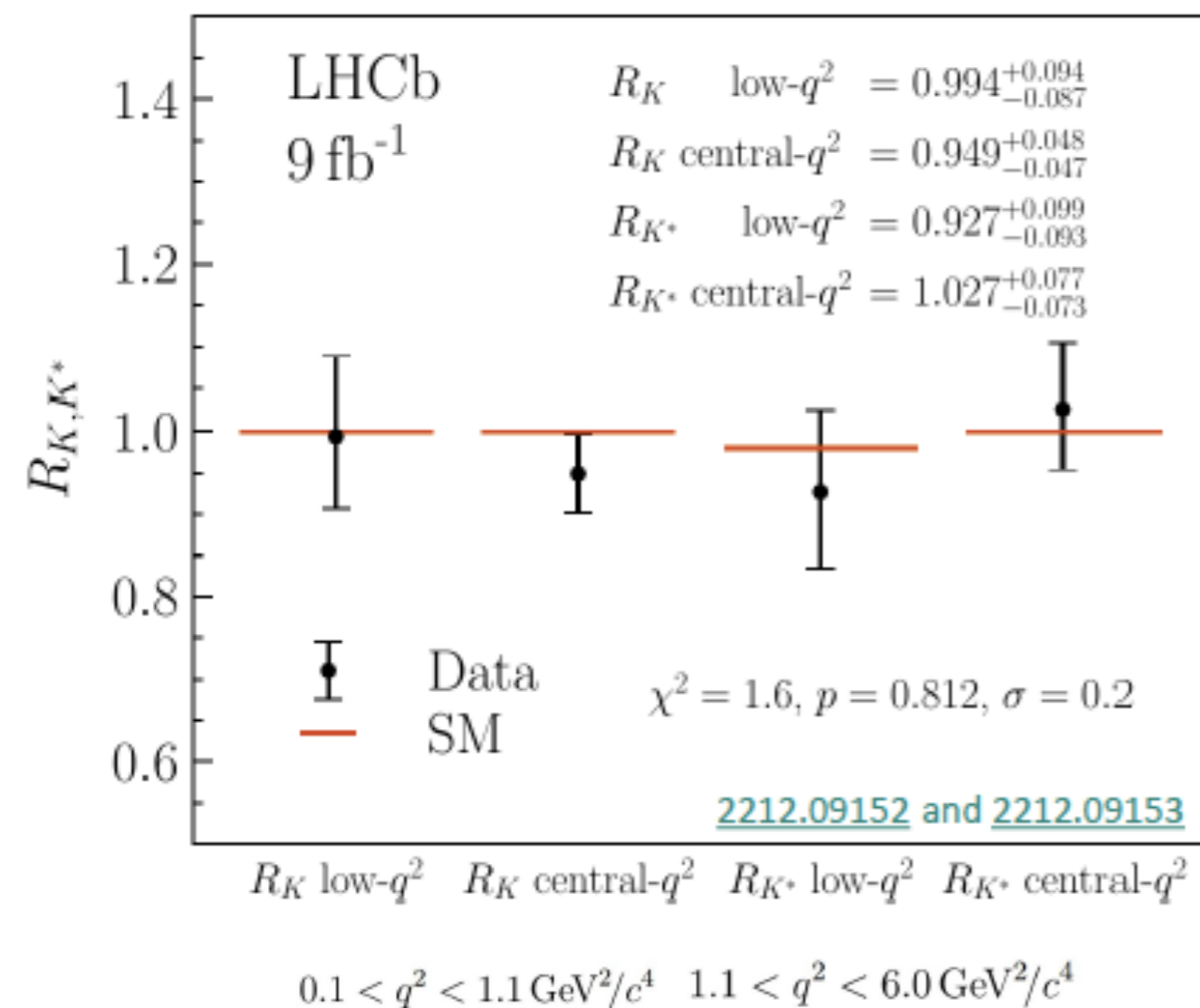
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Hiller, Kruger hep-ph/0310219

December 20th update LHCb



Compatible with SM with a simple χ^2 test on 4 measurement at 0.2σ .

The uncertainties reach 10% to 5% level.

Two-operator fit to clean observables

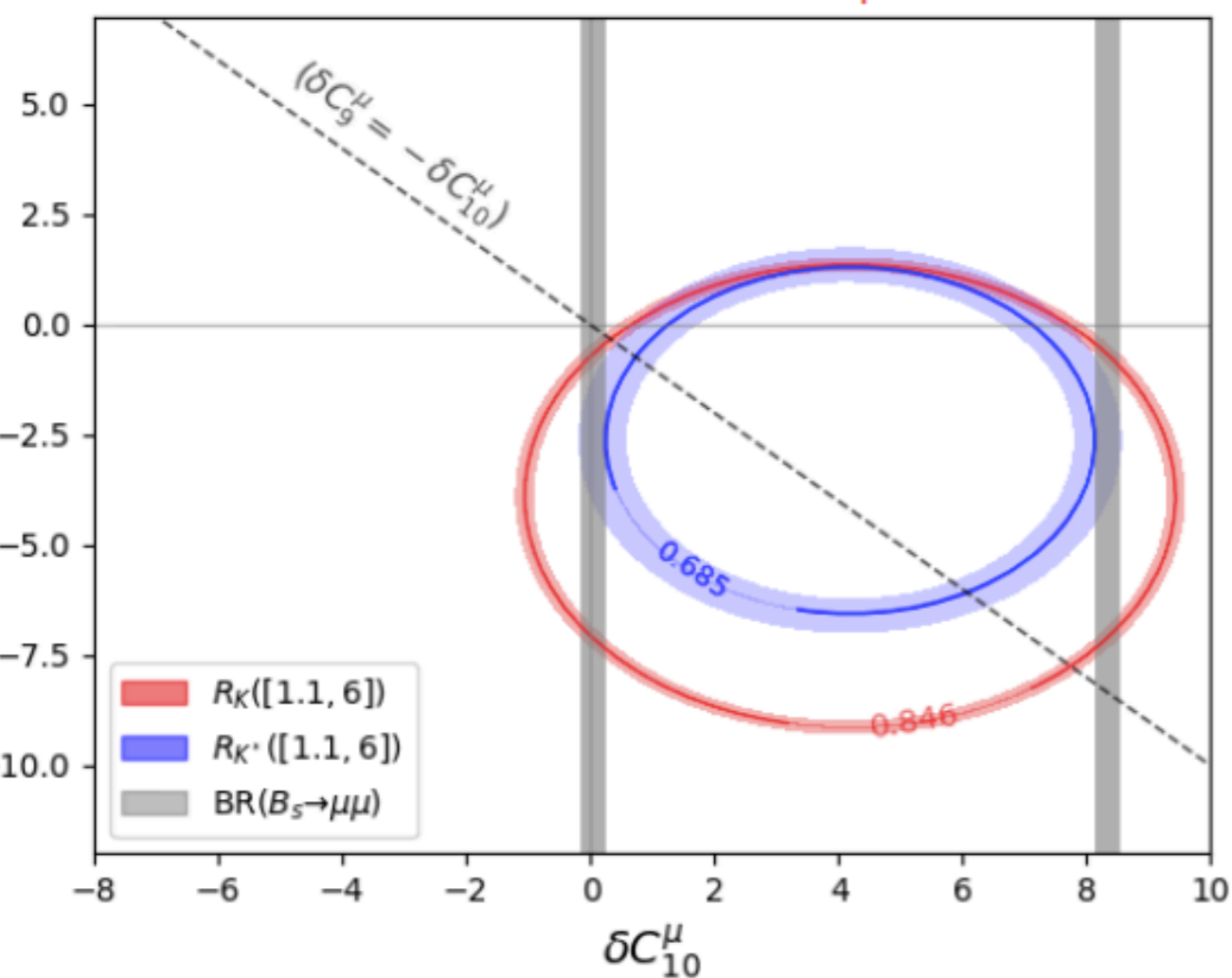
Hurth, Mahmoudi, Martinez-Santos, Neshatpour arXiv:2104.10058

Fit to clean observables $R_K, R_{K^*}, B_s \rightarrow \mu^+ \mu^-$

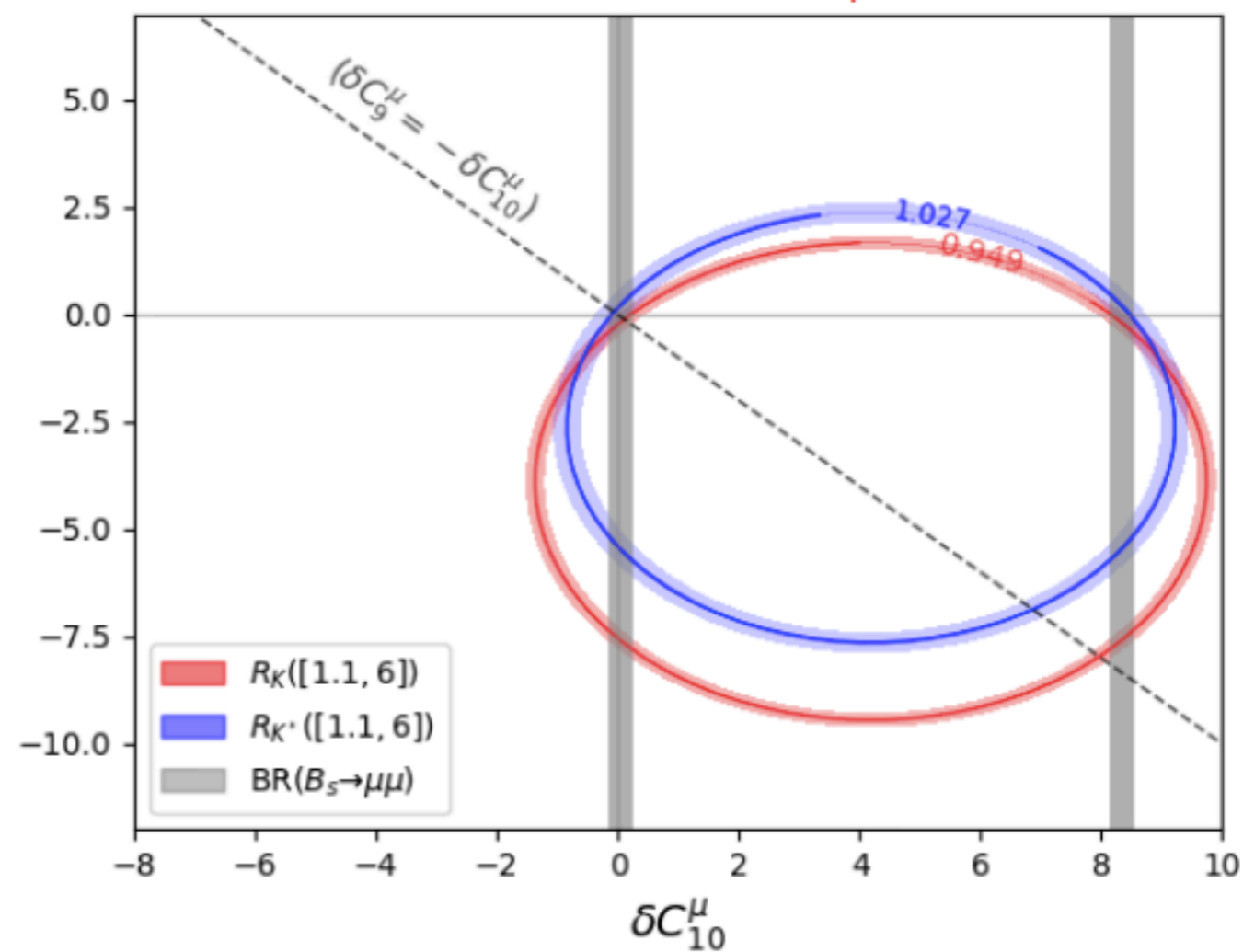
Update for post- R_K era

Coloured regions: 1σ range (th + exp uncertainties added in quadrature) with the experimental central value

Before December 20th update



After December 20th update



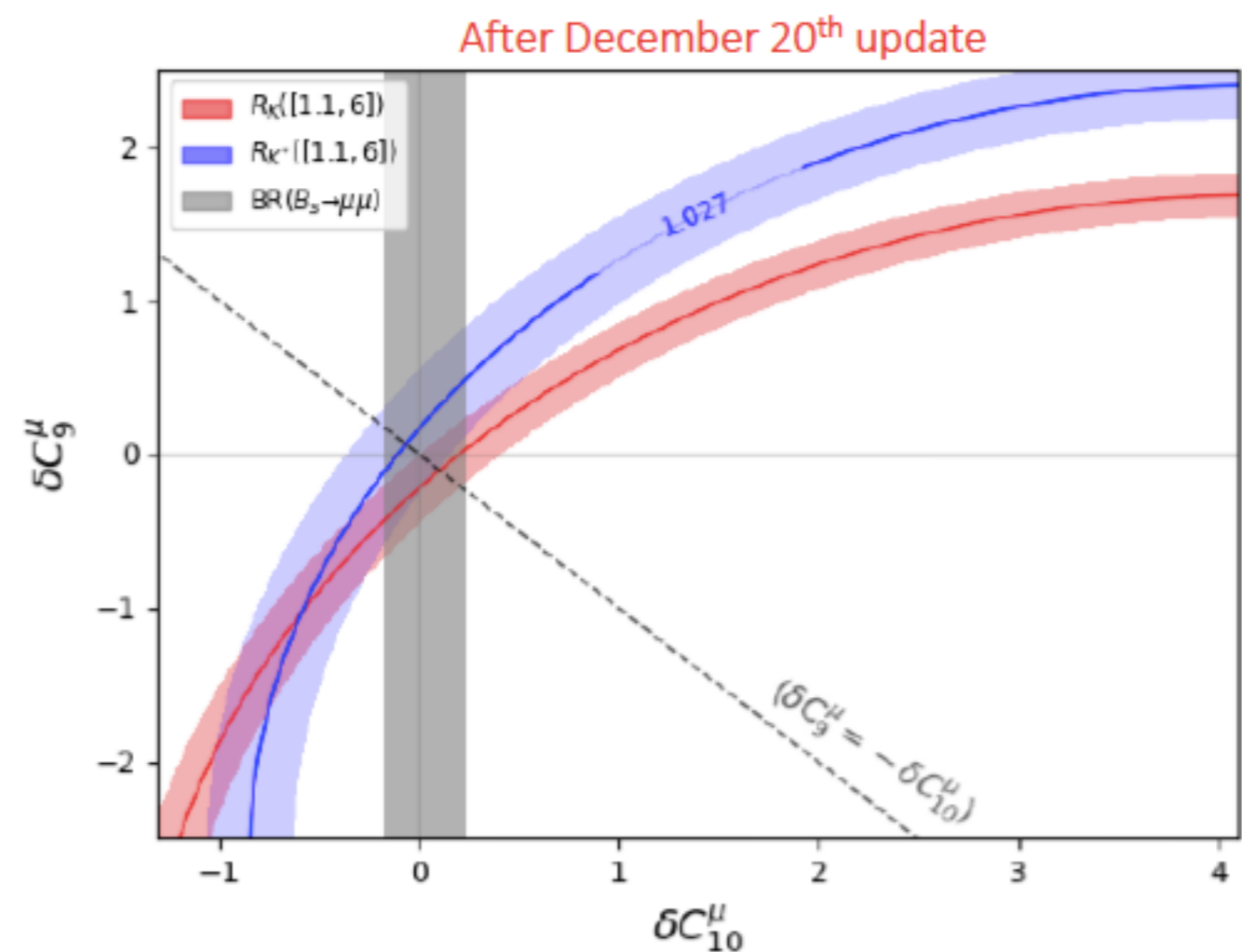
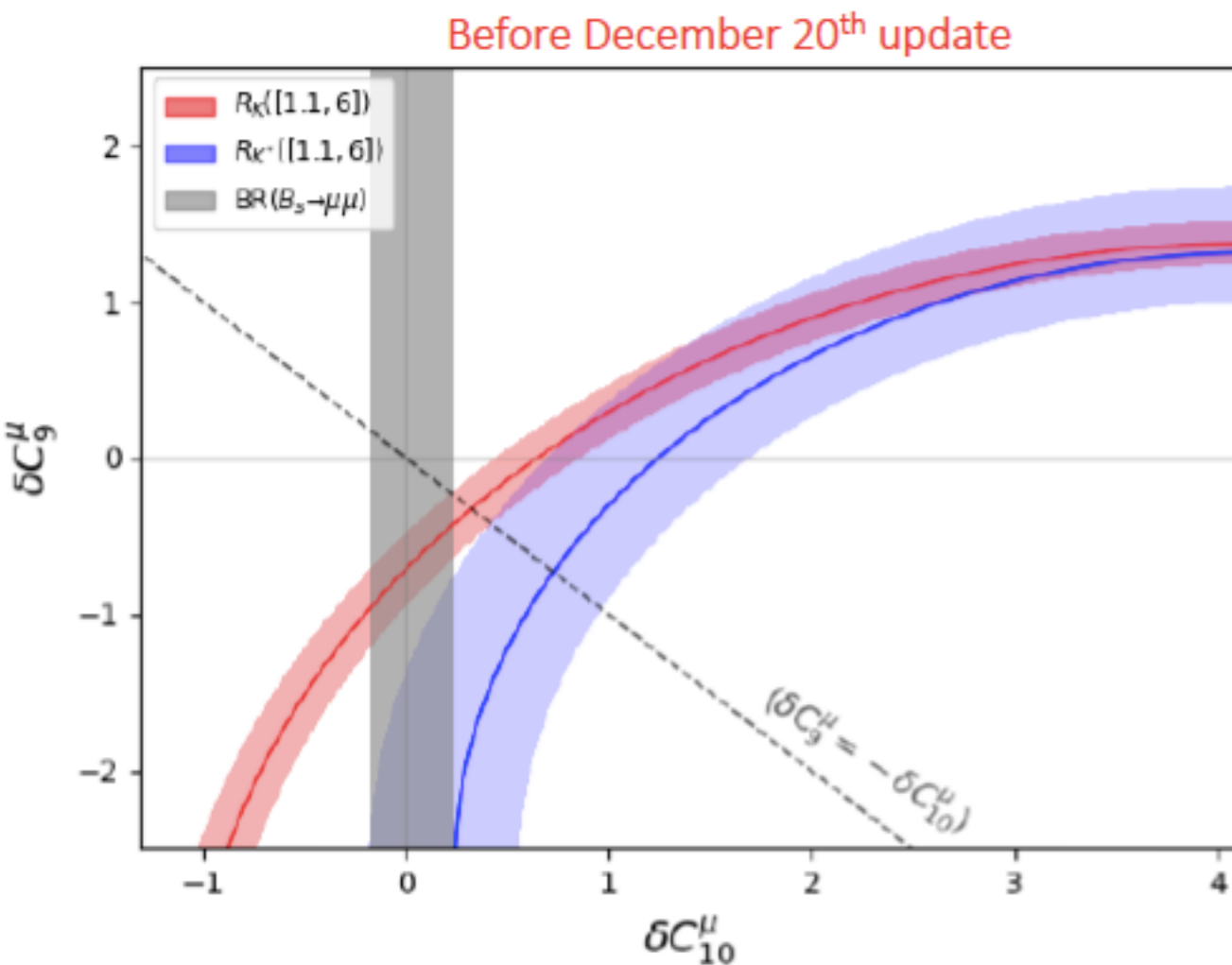
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Fit to clean observables $R_K, R_{K^*}, B_s \rightarrow \mu^+ \mu^-$

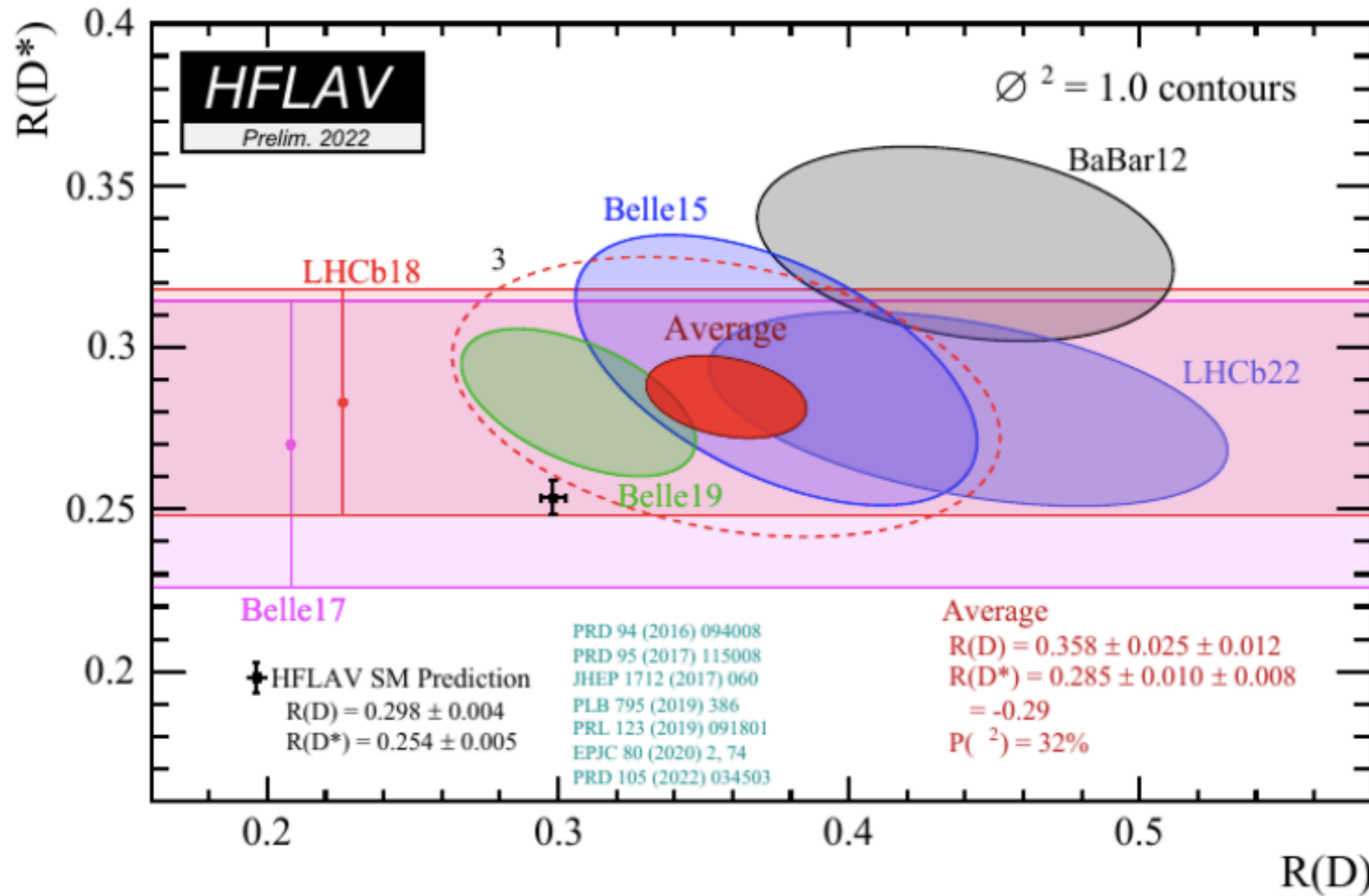
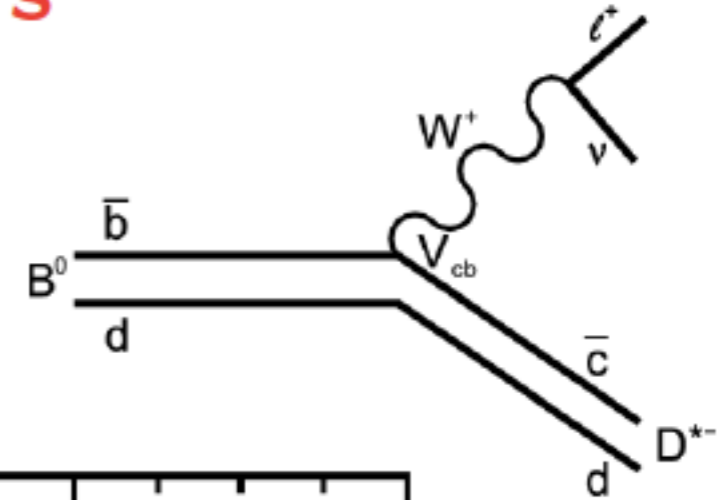
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Anomaly in charged B meson decays

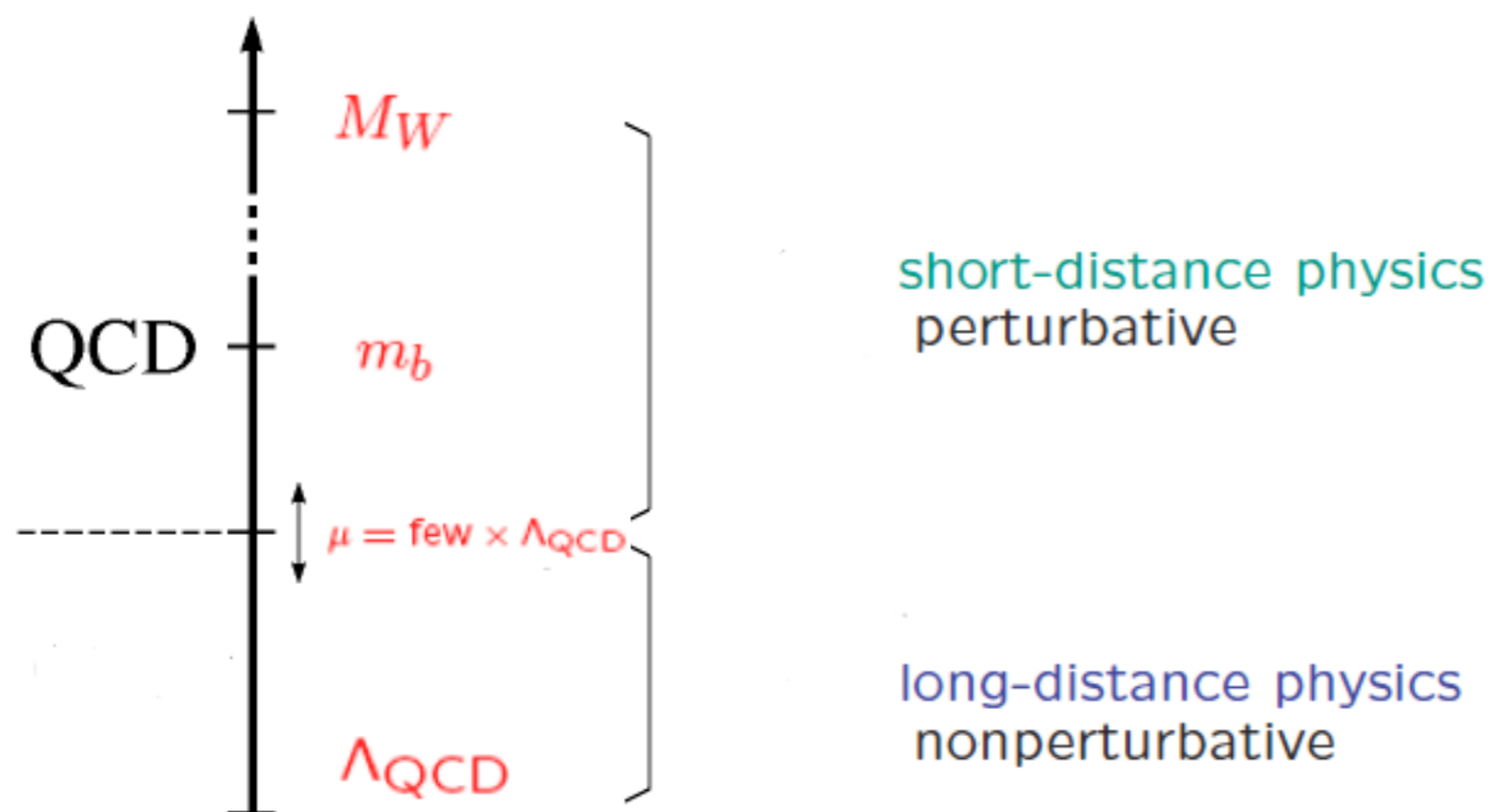
$$R_{D^*} = \frac{\Gamma(\bar{B} \rightarrow D^{(*)} \tau \bar{\nu})}{\Gamma(\bar{B} \rightarrow D^{(*)} \ell \bar{\nu})}; \quad \ell = e, \mu$$



- Average of experimental results in 3.2σ tension with the SM

Theoretical Framework

Theoretical tools for flavour precision observables



Factorization theorems: separating long- and short-distance physics

- Electroweak effective Hamiltonian: $H_{eff} = -\frac{4G_F}{\sqrt{2}} \sum C_i(\mu, M_{heavy}) \mathcal{O}_i(\mu)$
- $\mu^2 \approx M_{New}^2 \gg M_W^2$: 'new physics' effects: $C_i^{SM}(M_W) + C_i^{New}(M_W)$

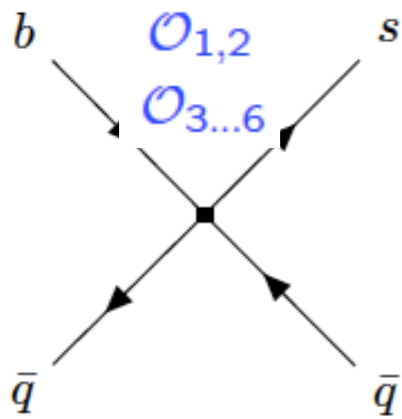
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

HQET, SCET, ...

Effective Weak Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}}V_{tb}V_{ts}^* \left(\sum_{i=1 \dots 10, S, P} \left(C_i(\mu) \mathcal{O}_i(\mu) + C'_i(\mu) \mathcal{O}'_i(\mu) \right) \right)$$

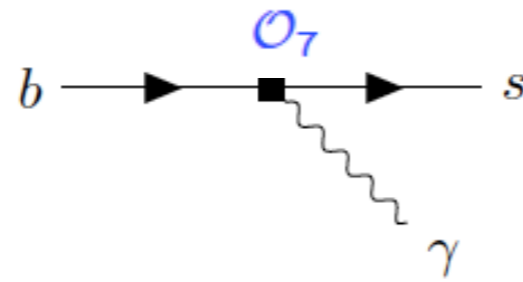
4-quark
operators



$$\mathcal{O}_{1,2} \propto (\bar{s} \Gamma_{\mu} c)(\bar{c} \Gamma^{\mu} b)$$

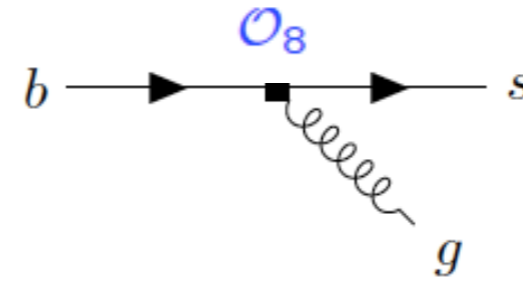
$$\mathcal{O}_{3 \dots 6} \propto (\bar{s} \Gamma_{\mu} b) \sum_q (\bar{q} \Gamma^{\mu} q)$$

electromagnetic
dipole operator



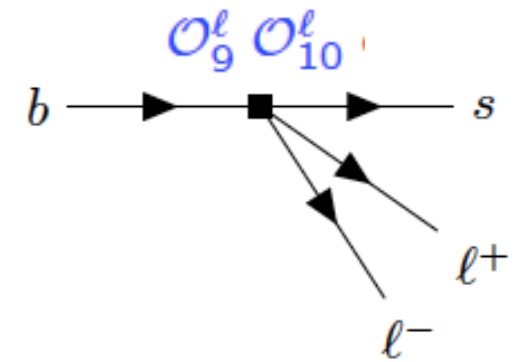
$$\mathcal{O}_7 \propto (\bar{s} \sigma^{\mu\nu} P_R) F_{\mu\nu}^a$$

chromomagnetic
dipole operator



$$\mathcal{O}_8 \propto (\bar{s} \sigma^{\mu\nu} T^a P_R) G_{\mu\nu}^a$$

semileptonic
operators



$$\mathcal{O}_9^l \propto (\bar{s} \gamma^{\mu} b_L)(\bar{l} \gamma_{\mu} l)$$

$$\mathcal{O}_{10}^l \propto (\bar{s} \gamma^{\mu} b_L)(\bar{l} \gamma_{\mu} \gamma_5 l)$$

In the SM: $C_7 = -0.29$ $C_9 = 4.20$ $C_{10} = -4.01$

New physics:

- Corrections to the Wilson coefficients: $C_i \rightarrow C_i^{\text{SM}} + \delta C_i^{\text{NP}}$
- Additional operators: Chirally flipped (\mathcal{O}'_i), (pseudo)scalar (\mathcal{O}_S and \mathcal{O}_P)

Exclusive modes $B \rightarrow K^{(*)} \ell \ell$

Soft-collinear effective theory

$$T_a^{(i)} = C_a^{(i)} \xi_a + \phi_B \otimes T_a^{(i)} \otimes \phi_{a,K^*} + O(\Lambda/m_b)$$

- Separation of **perturbative hard kernels** from **process-independent nonperturbative** functions like form factors
- **Relations between formfactors** in large-energy limit
- **Limitation:** **insufficient information on power-suppressed Λ/m_b terms** (breakdown of factorization: 'endpoint divergences')

The significance of the anomalies depends on the assumptions made for the unknown power corrections!

(This does not affect R_K and R_K^* of course, but does affect combined fits!)

Problem of nonfactorizable power corrections

- Crosscheck with $R_{\mu,e}$ ratios:

OPTION OUT !

NP in the ratios would indirectly confirm the NP interpretation of the anomalies in the angular observables (if there is a coherent picture)

- Ongoing efforts: Estimate of power corrections based on analyticity

van Dyk et al.: arXiv:2011.09813, 2206.03797

- In the long run: Solution with refactorization techniques

New developments in the SCET community

Neubert et al., arXiv:2009.06779

- Crosscheck of the anomalies via inclusive modes

Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

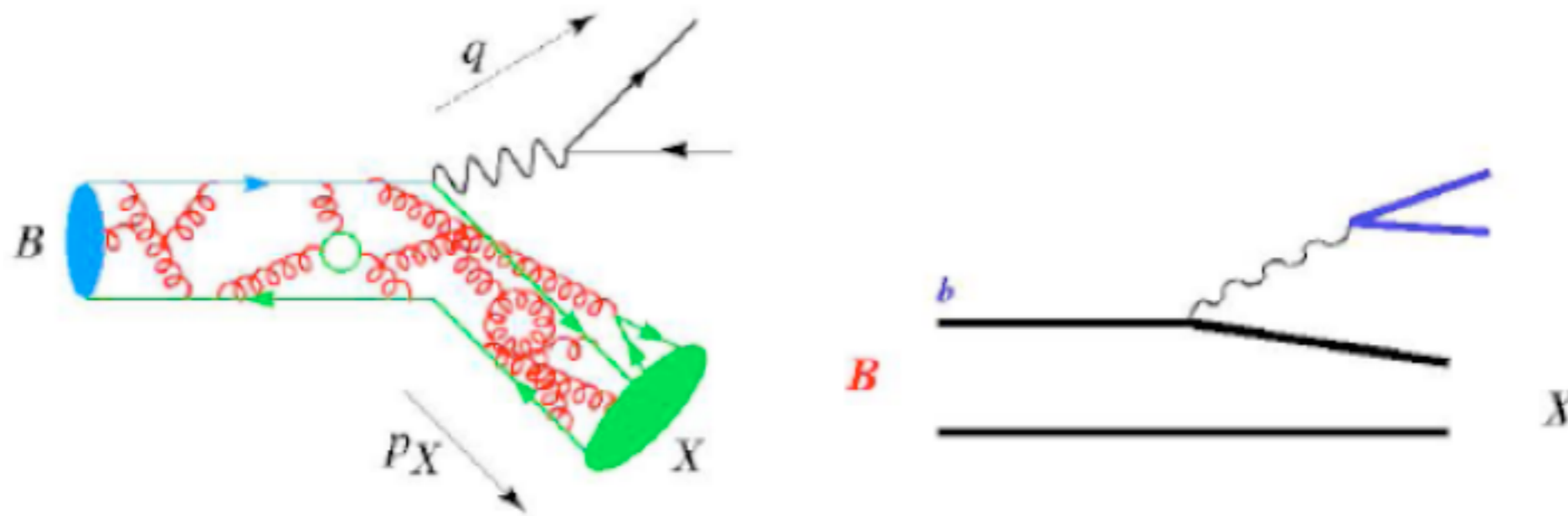
How to compute the hadronic matrix elements $\mathcal{O}_i(\mu = m_b)$?

- Heavy mass expansion for inclusive modes:

$$\Gamma(\bar{B} \rightarrow X_s \gamma) \xrightarrow{m_b \rightarrow \infty} \Gamma(b \rightarrow X_s^{\text{parton}} \gamma), \quad \Delta^{\text{nonpert.}} \sim \Lambda_{QCD}^2/m_b^2$$

No linear term Λ_{QCD}/m_b (perturbative contributions dominant)

Chay, Georgi, Grinstein 1990



Inclusive modes $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$

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Old story:

- If one goes beyond the leading operator (\mathcal{O}_7 , \mathcal{O}_9):
breakdown of local expansion

Dedicated analysis:

naive estimate of non-local matrix elements leads to 5% uncertainty.

$b \rightarrow s \gamma$: Benzke, Lee, Neubert, Paz, arXiv:1003.5012



$b \rightarrow s \ell \ell$: Benzke, Hurth, Turczyk, arXiv:1705.10366

New Physics Reach of Semi-leptonic Penguin Decays

Error of Branching ratio $\bar{B} \rightarrow X_s \ell^+ \ell^-$

BF (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	29 (26,12)	13 (9.7,8.0)	6.6 (3.1,5.8)
[3.5,6.0]	24 (21,12)	11 (7.9,8.0)	6.4 (2.6,5.8)
≥ 14.4	23 (21,9)	10 (8.1,6.0)	4.7 (2.6,3.9)

Error of Normalized Forward-Backward-Asymmetry

AFB_n (%) (stat,syst)	0.7/ab	5/ab	50/ab
[1.0,3.5]	26 (26,2.7)	9.7 (9.7,1.3)	3.1 (3.1,0.5)
[3.5,6.0]	21 (21,2.7)	7.9 (7.9,1.3)	2.6 (2.6,0.5)
≥ 14.4	19 (19,1.7)	7.3 (7.3,0.8)	2.4 (2.4,0.3)

$B \rightarrow (\pi, \rho) \ell^+ \ell^-$, semi-inclusive $\bar{B} \rightarrow X_d \ell^+ \ell^-$ at 50/ab
(uncertainties like $\bar{B} \rightarrow X_s \ell^+ \ell^-$ at 0.7/ab)

Belle-II Extrapolations

Results competitive with LHCb expected with $5ab^{-1}$

Observables	Belle 0.71 ab^{-1}	Belle II 5 ab^{-1}	Belle II 50 ab^{-1}
R_K ($[1.0, 6.0]\text{ GeV}^2$)	28%	11%	3.6%
R_K ($> 14.4\text{ GeV}^2$)	30%	12%	3.6%
R_{K^*} ($[1.0, 6.0]\text{ GeV}^2$)	26%	10%	3.2%
R_{K^*} ($> 14.4\text{ GeV}^2$)	24%	9.2%	2.8%
R_{X_s} ($[1.0, 6.0]\text{ GeV}^2$)	32%	12%	4.0%
R_{X_s} ($> 14.4\text{ GeV}^2$)	28%	11%	3.4%

The Belle II Physics Book, Prog Theor Exp Phys (2019)

Complete angular analysis of inclusive $B \rightarrow X_s \ell \ell$

- Phenomenological analysis to NNLO QCD and NLO QED for all angular observables

Huber,Hurth,Lunghi, arXiv:1503.04849

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} [(1+z^2) H_T(q^2) + 2z H_A(q^2) + 2(1-z^2) H_L(q^2)] \quad (z = \cos \theta_\ell)$$

$$\frac{d\Gamma}{dq^2} = H_T(q^2) + H_L(q^2)$$

$$\frac{dA_{\text{FB}}}{dq^2} = 3/4 H_A(q^2)$$

- Dependence on Wilson coefficients

Lee,Ligeti,Stewart, Tackmann hep-ph/0612156

$$H_T(q^2) \propto 2s(1-s)^2 \left[\left| C_9 + \frac{2}{s} C_7 \right|^2 + |C_{10}|^2 \right]$$

$$H_A(q^2) \propto -4s(1-s)^2 \text{Re} \left[C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \right]$$

$$H_L(q^2) \propto (1-s)^2 \left[\left| C_9 + 2 C_7 \right|^2 + |C_{10}|^2 \right]$$

- Electromagnetic effects due to energetic photons are large and calculated analytically and crosschecked against Monte Carlo generator events

$$\alpha_{\text{em}} \log(m_b^2/m_\ell^2)$$

$$q^2 = (p_{\ell+} + p_{\ell-}) \Rightarrow q^2 = (p_{\ell+} + p_{\ell-} + p_\gamma)$$

Huber,Hurth,Lunghi, arXiv:1503.04849

- In the ratio of the inclusive $b \rightarrow s\ell\bar{\ell}$ decay rate in the high- q^2 region and the semileptonic decay rate large part of the nonperturbative effects cancel out:

Ligeti, Tackmann, arXiv:0707.1694

$$R_{\text{incl}}^{(\ell)}(q_0^2) = \frac{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \rightarrow X_s \bar{\ell} \ell)}{dq^2}}{\int_{q_0^2}^{m_B^2} dq^2 \frac{d\Gamma(B \rightarrow X_u \bar{\ell} \nu)}{dq^2}}$$

Tensions in the inclusive high q^2 decay rate ??

Isidori, Polonsky, Tinari, arXiv:2305.03076

$$R_{\text{incl}}^{SM}(\text{15}) = \frac{\int_{\text{15}}^{m_B^2} dq^2 \frac{d\Gamma(B \rightarrow X_s \bar{\ell} \ell)}{dq^2}}{\int_{\text{15}}^{m_B^2} dq^2 \frac{d\Gamma(B \rightarrow X_u \bar{\ell} \nu)}{dq^2}} \quad \times \quad \mathcal{B}(B \rightarrow X_u \bar{\ell} \nu)_{\text{15}}^{\text{exp}} = (1.50 \pm 0.24) \times 10^{-4}$$

Belle, arXiv:2107.13855

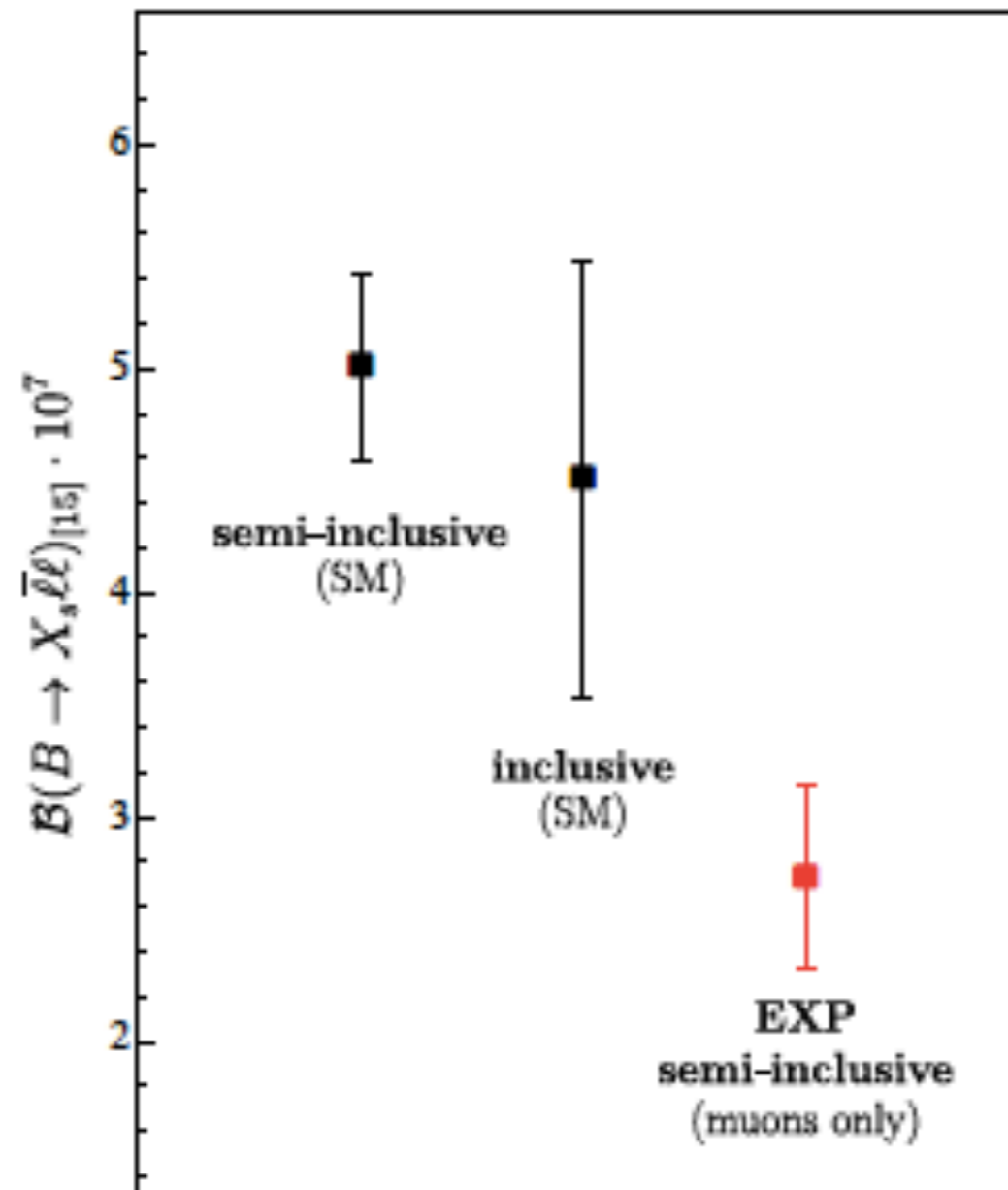
$$= \text{"}\mathcal{B}(B \rightarrow X_s \bar{\ell} \ell)_{\text{15}}^{SM}\text{"} \stackrel{!}{=} \sum_i \mathcal{B}(B \rightarrow X_s^i \bar{\mu} \mu)_{\text{15}}^{\text{exp}} = (2.74 \pm 0.41) \times 10^{-7}$$

Isidori, Polonsky, Tinari, arXiv:2305.03076

- Experimental semi-inclusive rate is estimated by the sum of the $B \rightarrow K$ and $B \rightarrow K^*$ modes and a correction factor for the two-body final states $B \rightarrow K\pi$.

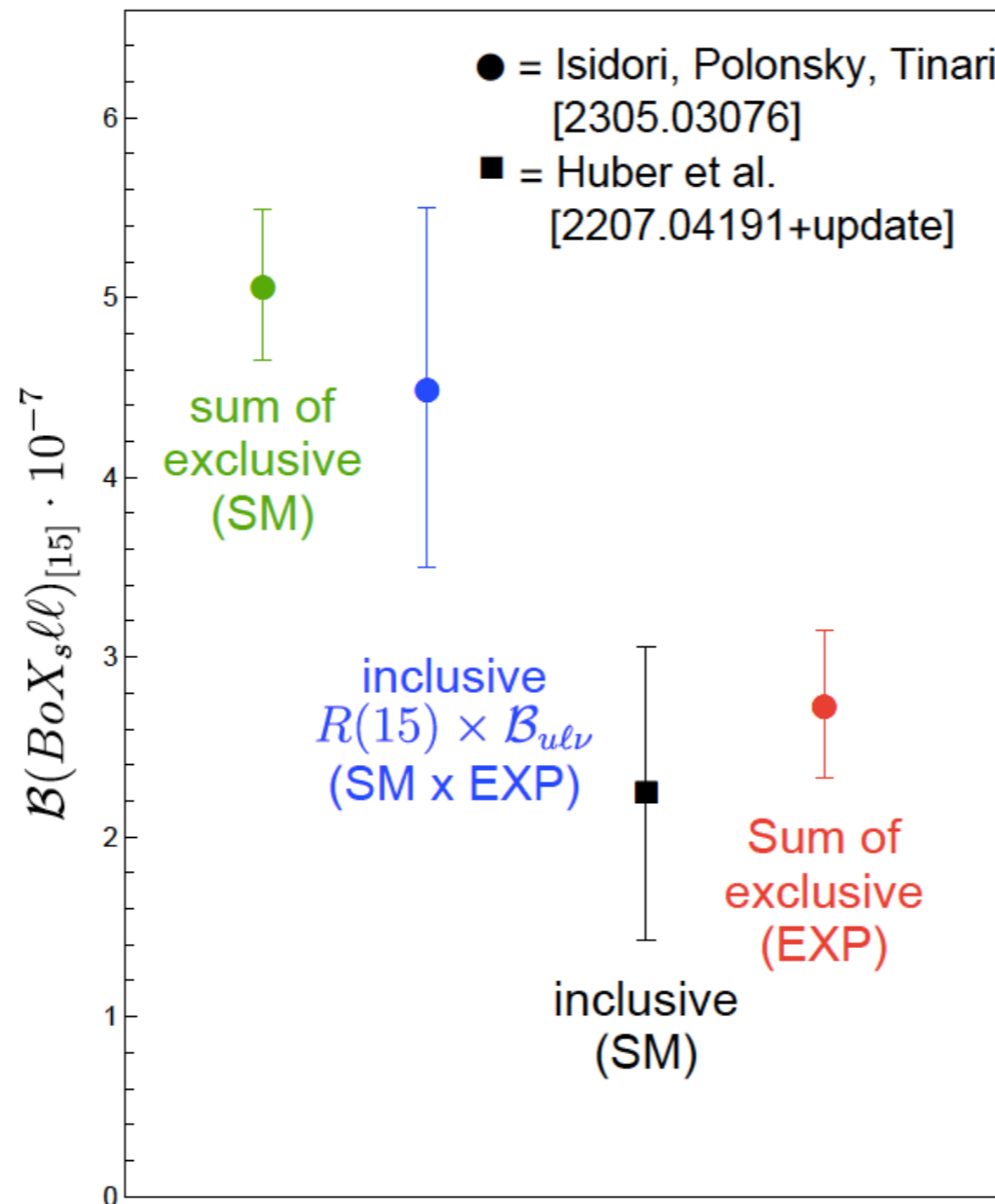
- Isidori et al. claim a tension up to 2σ – confirming analogous results in the exclusive modes.

Isidori, Polonsky, Tinari, arXiv:2305.03076



- We do not find any tension if we compare our direct result for the branching $\mathcal{B}(B \rightarrow X_s \ell \ell)_{[15]}^{\text{SM}}$ with the estimated experimental semi-inclusive rate at all.

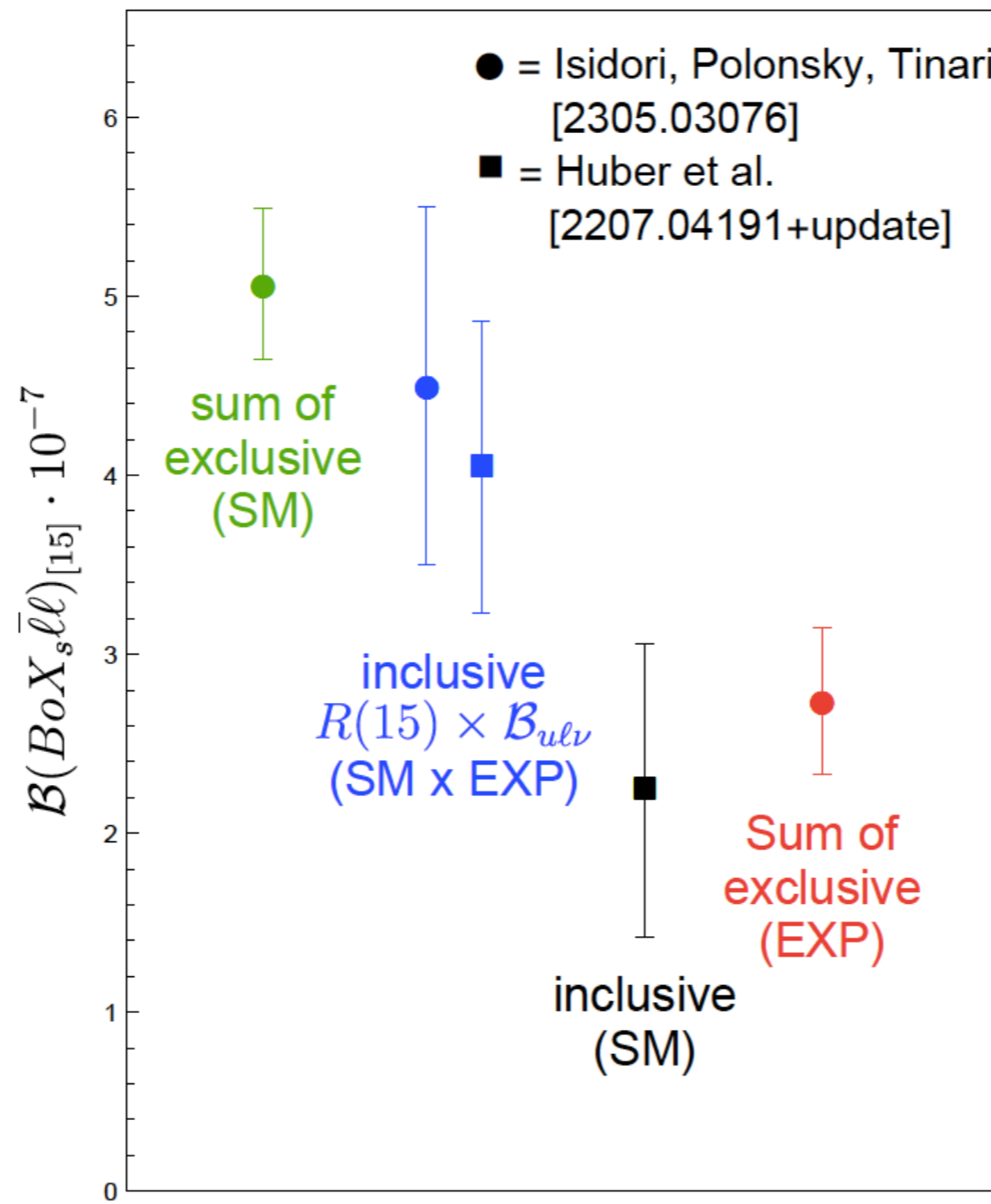
Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191
Update



- We do not find any tension if we compare our direct result for the branching $\mathcal{B}(B \rightarrow X_s \ell \bar{\ell})_{[15]}^{\text{SM}}$ with the estimated experimental semi-inclusive rate at all.
- We find a slight tension when we compare our results of the ratio $R_{\text{incl}}^{\text{SM}}(15)$ and our direct result for $\mathcal{B}(B \rightarrow X_s \ell \bar{\ell})_{[15]}^{\text{SM}}$.

Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191

Update



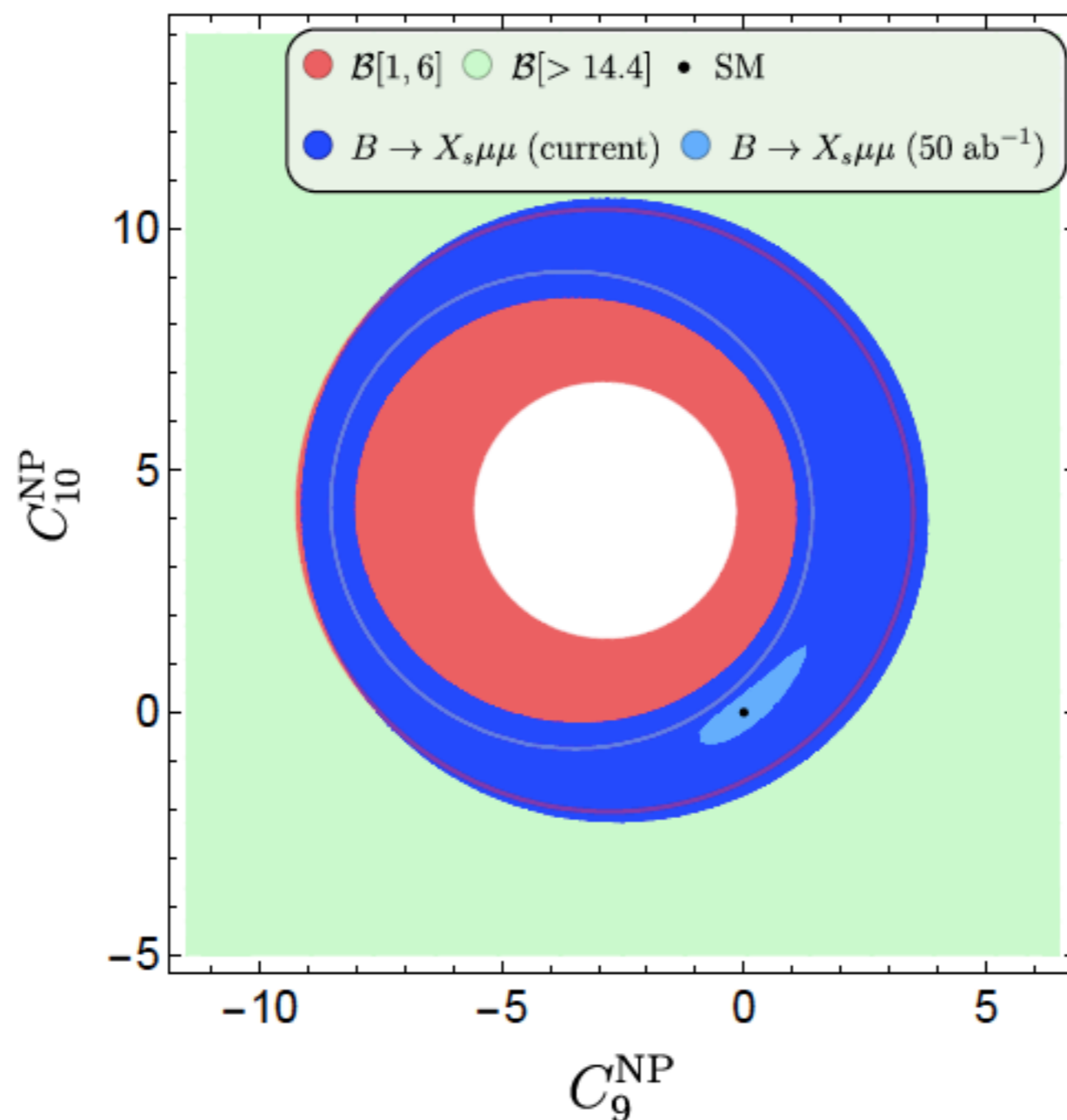
New physics sensitivity

Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191

Constraints on Wilson coefficients C_9^{NP} and C_{10}^{NP}

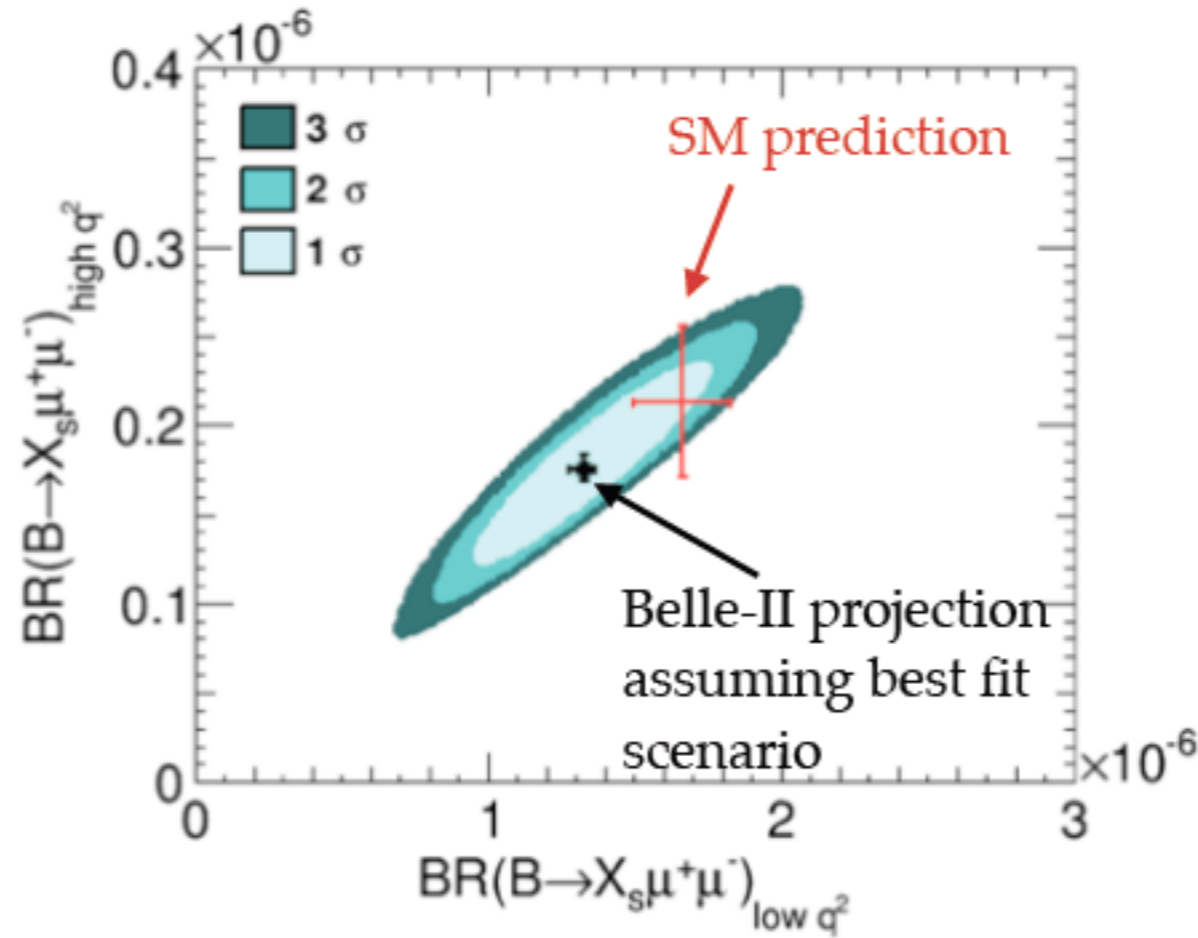
that we obtain at 95% C.L. from present experimental data
(red low q^2 , green high q^2)

that we will obtain at 95% C.L. from 50ab^{-1} data at Belle-II
(light blue)



Crosscheck of LHCb anomalies with inclusive modes

Hurth, Mahmoudi, Neshatpour, arXiv:1410.4545



If NP then the effect of C_9 and C'_9 are large enough to be checked at Belle-II with theoretically clean modes.

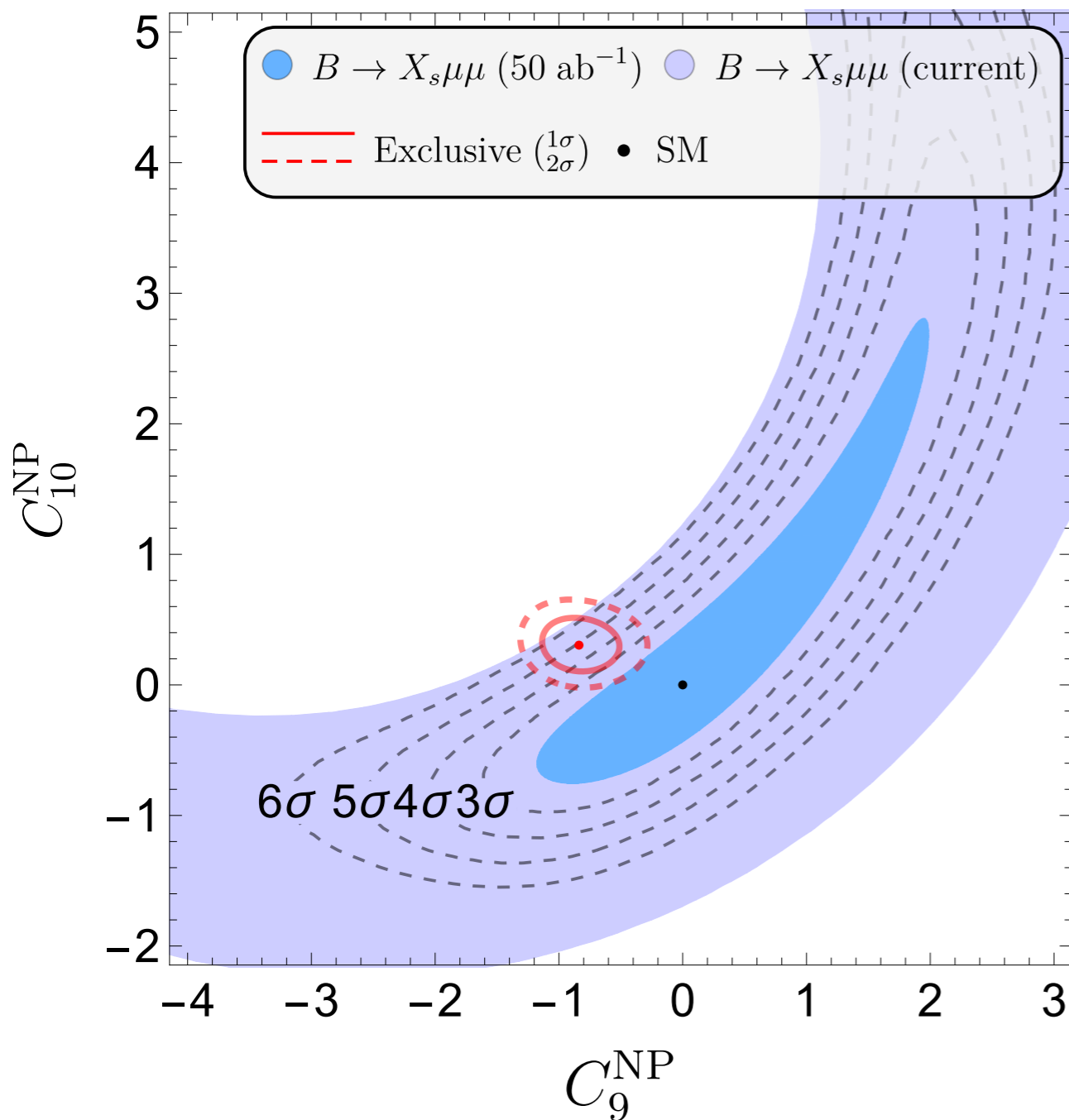
Hurth, Mahmoudi, arXiv:1312.5267 Experimental extrapolation by Kevin Flood

Assuming Belle II measures SM values

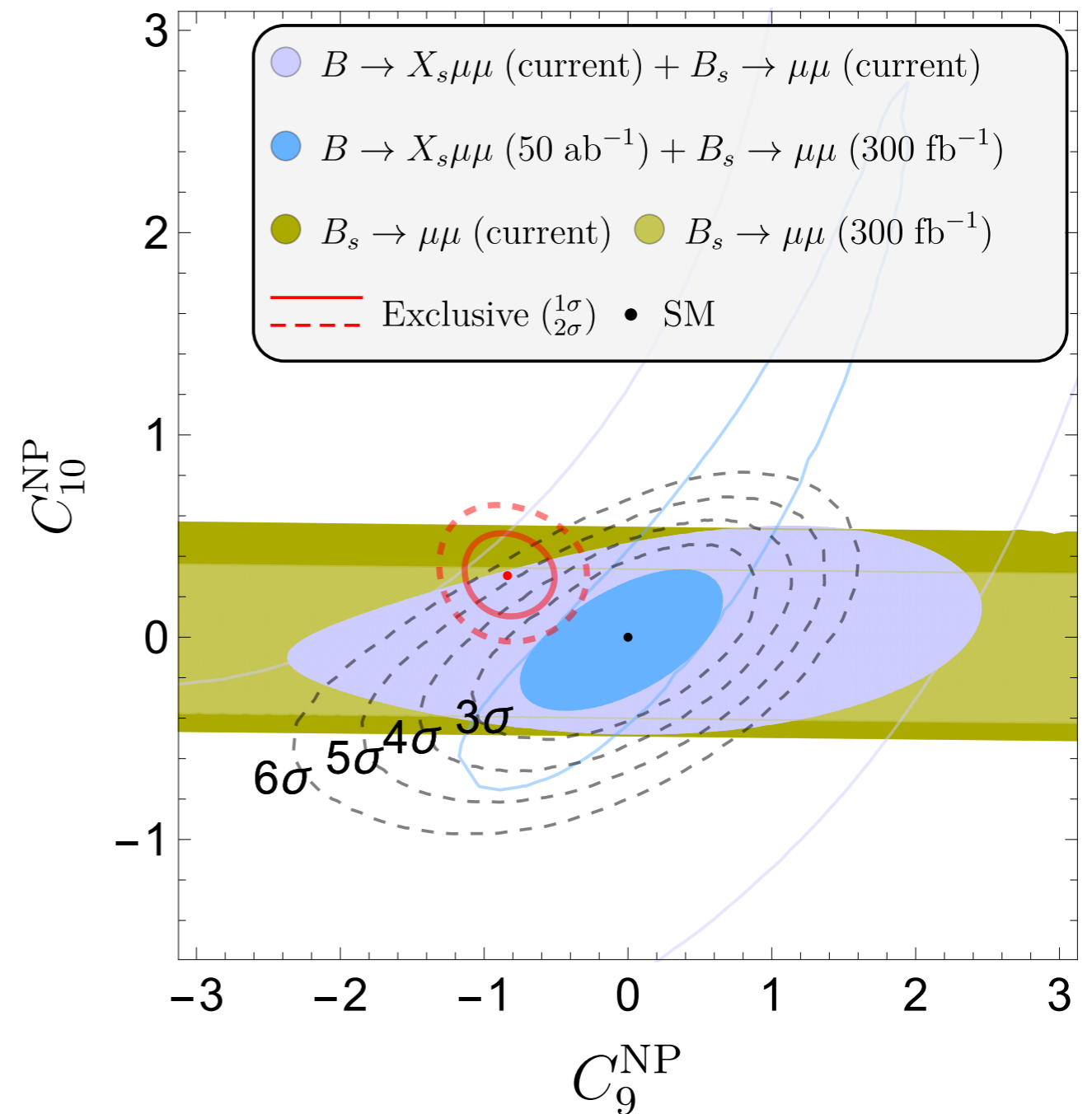
Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191

Update for post- R_K era

Exclusive vs Inclusive



Exclusive vs Inclusive

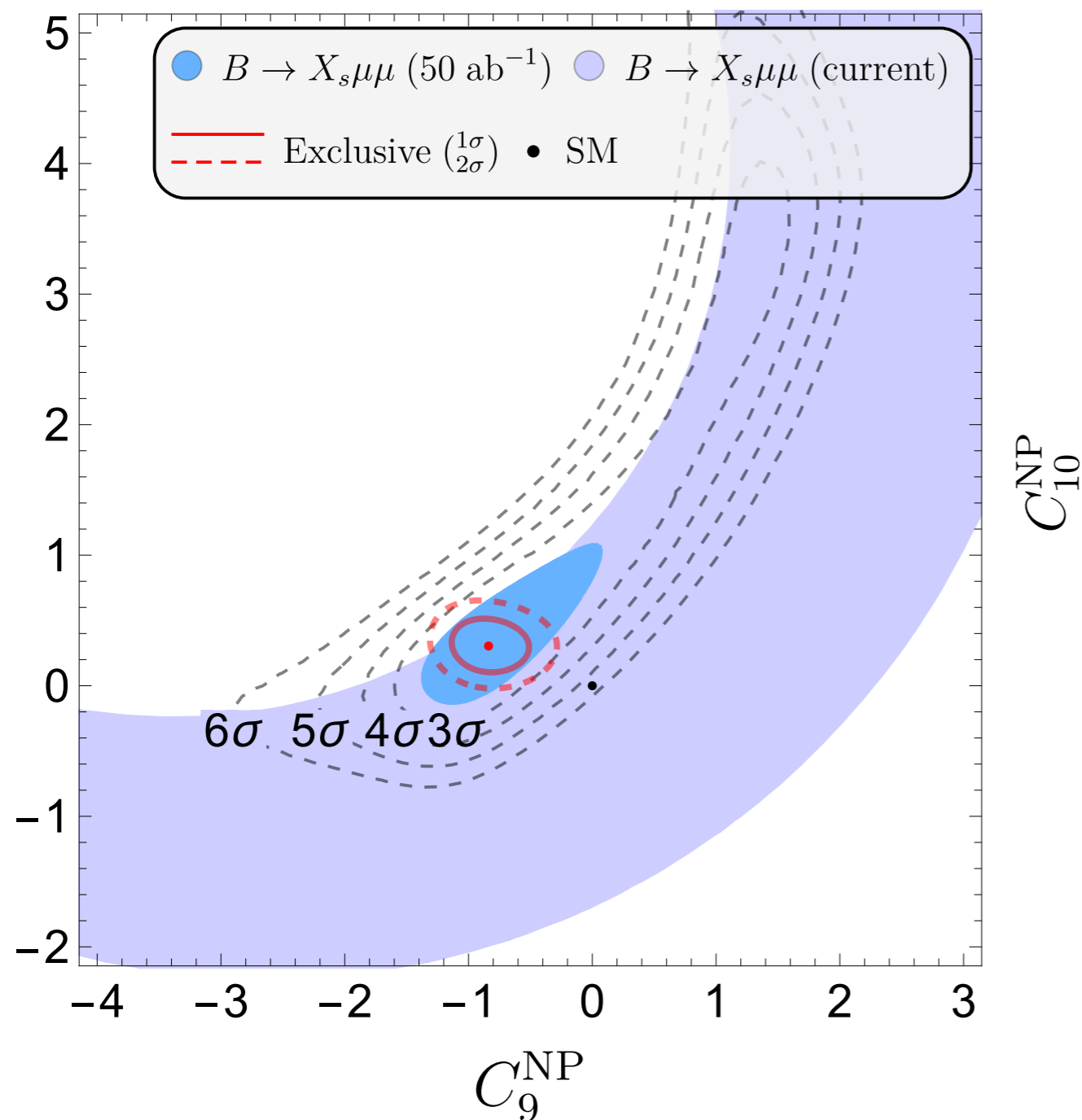


Assuming Belle II measures best fit point of exclusive fit

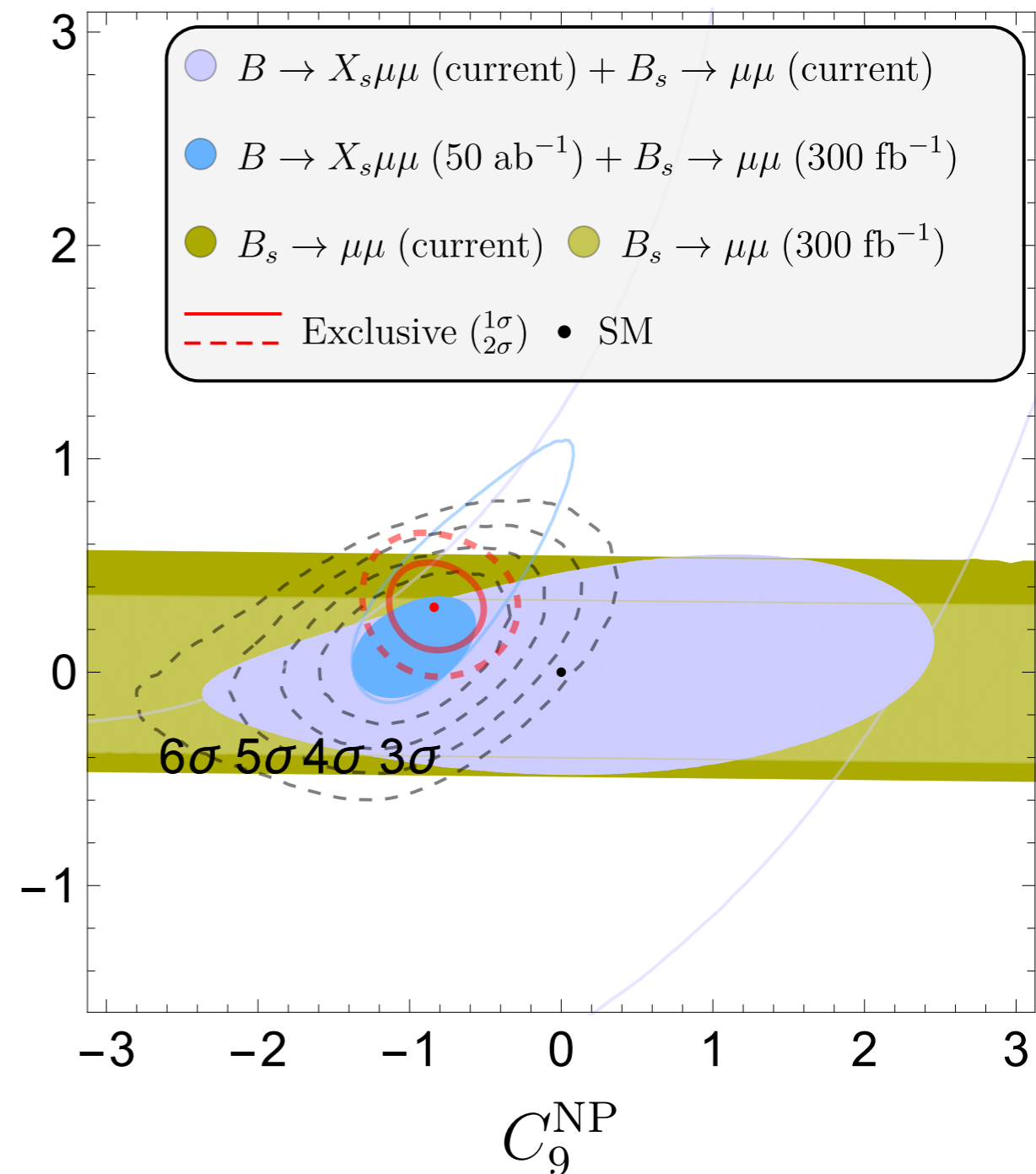
Huber, Hurth, Jenkins, Lunghi, Qin Qin, Vos, arXiv:2007.04191

Update for post- R_K era

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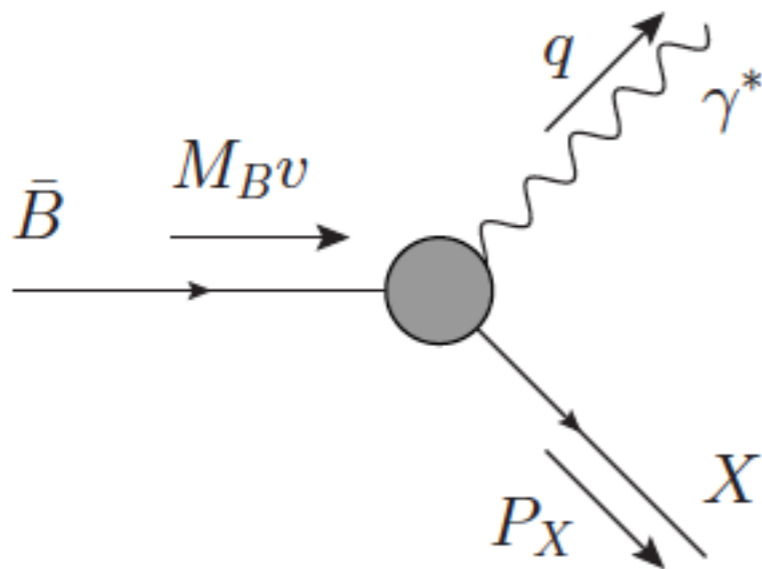


Nonlocal subleading contributions,
refactorisation and hadronic mass cut

Subleading power factorization in $B \rightarrow X_s \ell^+ \ell^-$

Benzke, Hurth, Turczyk, arXiv:1705.10366; Benzke, Hurth, arXiv:2006.00624

- Cuts in the dilepton mass spectrum necessary due to $c\bar{c}$ resonances
- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \rightarrow c(\rightarrow s e^+ \nu) e^- \bar{\nu}$)
- Kinematics: X_s is jetlike and $m_X^2 \leq m_b \Lambda_{QCD}$ (shapefunction region)
- Multiscale problem \Rightarrow SCET with scaling Λ_{QCD}/m_b



$$M_B^2 \sim m_b^2 \gg m_X^2 \sim \Lambda_{QCD} m_b \gg \Lambda_{QCD}^2$$

Little calculation

- B meson rest frame $q = p_B - p_X$ $2 m_B E_X = m_B^2 + M_X^2 - q^2$
 X_s system is jet-like with $E_X \sim m_B$ and $m_X^2 \ll E_X^2$

- $p_X^- p_X^+ = m_X^2$ two light-cone components

$$\bar{n} p_X = p_X^- = E_X + |\vec{p}_X| \sim \mathcal{O}(m_B)$$

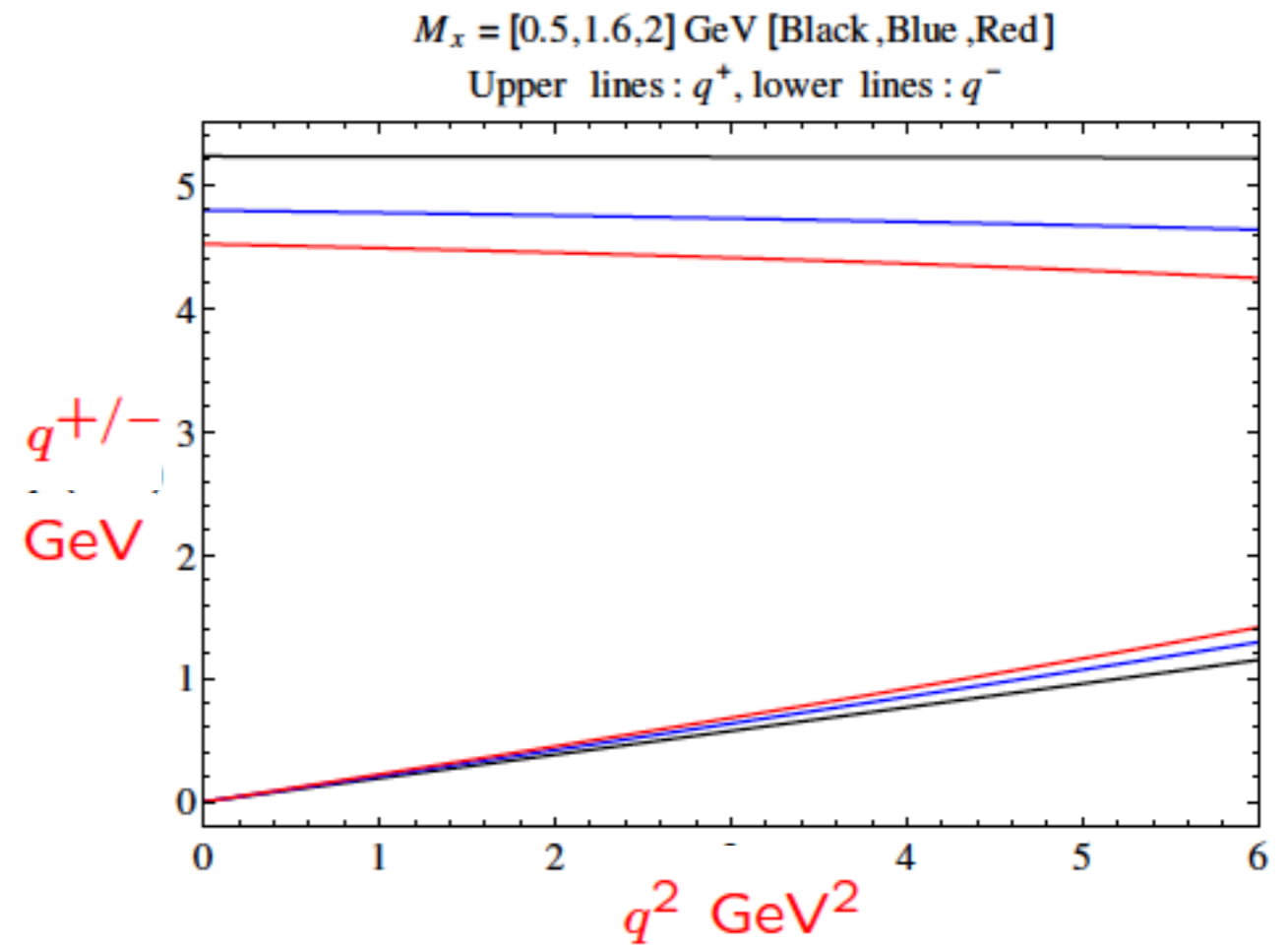
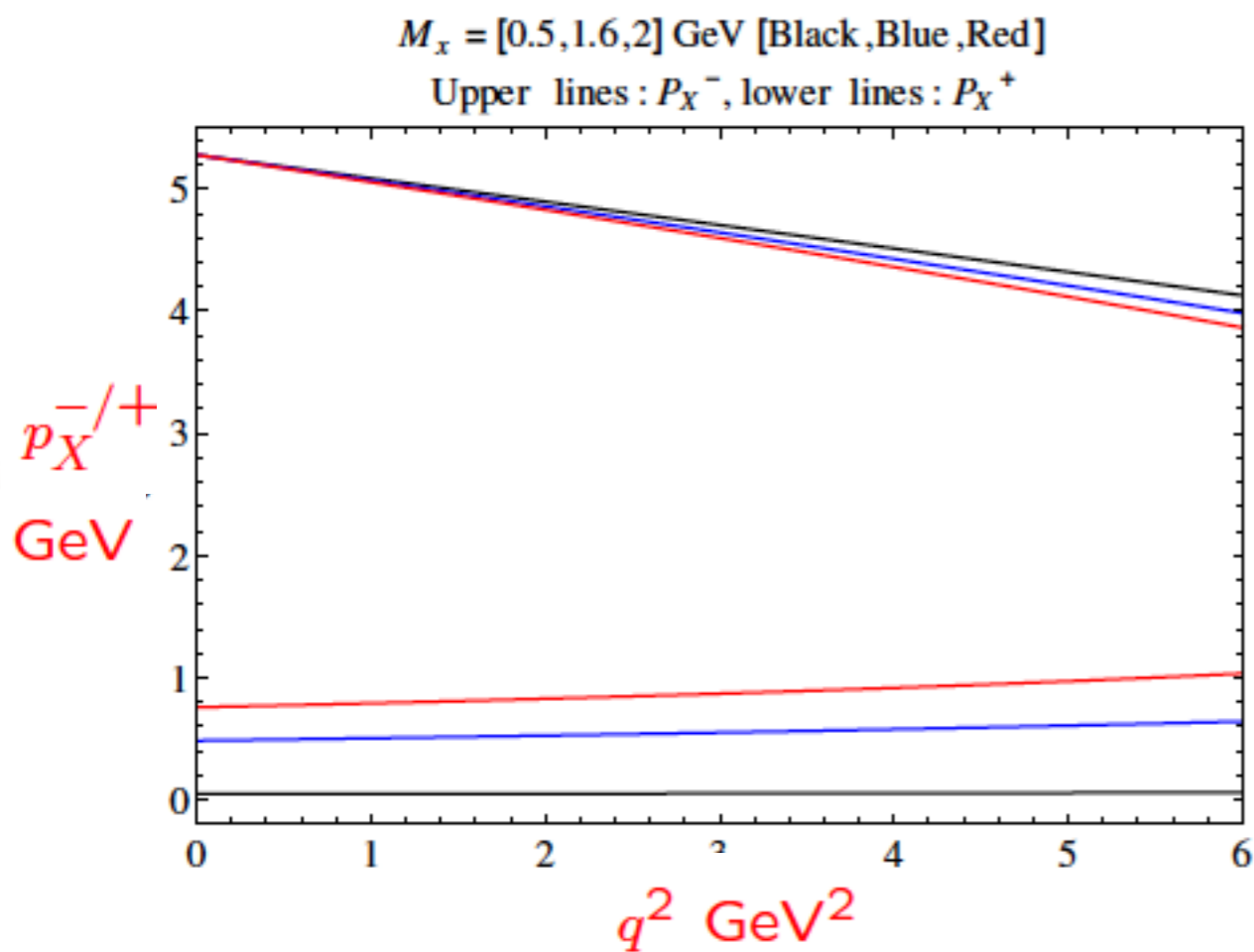
$$n p_X = p_X^+ = E_X - |\vec{p}_X| \sim \mathcal{O}(\Lambda_{\text{QCD}})$$

- $q^+ = n q = m_B - p_X^+$ $q^- = \bar{n} q = m_B - p_X^-$

$$m_X^2 = P_X^2 = (M_B - n \cdot q)(M_B - \bar{n} \cdot q)$$

$$\lambda = \Lambda_{\text{QCD}}/m_b \quad m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$$

Scaling of p_X and q



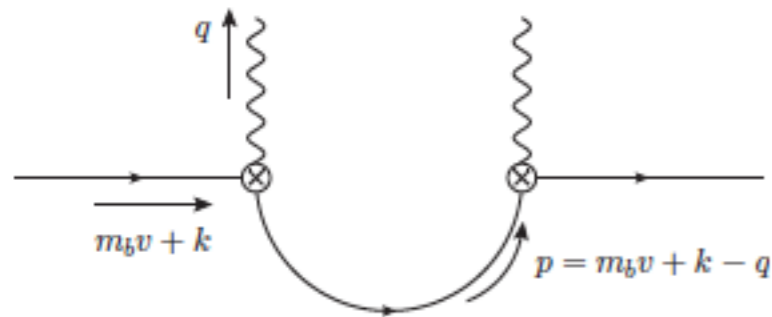
For $q^2 < 6 \text{ GeV}^2$ the scaling of np_X and $\bar{n}p_X$ implies $\bar{n}q$ is of order λ , means q anti-hard-collinear (just kinematics).

Stewart and Lee assume $\bar{n}q$ to be order 1, means q is hard.
This problematic assumption implies a different matching of SCET/QCD.

[Lee, Stewart hep-ph/0511334](#)

Shapefunction region

Local OPE breaks down for $m_X^2 \sim \lambda \Rightarrow m_b - n \cdot q \sim \lambda$



$$\frac{1}{(m_b v + k - q)^2} = \frac{1}{m_b - n \cdot q} \left(1 - \frac{n \cdot k}{m_b - n \cdot q} + \dots \right) \frac{1}{m_b - \bar{n} \cdot q}$$

Resummation of leading contributions into a shape function.

(scaling of $\bar{n}q$ does not matter here; zero in case of $B \rightarrow X_s \gamma$)

Factorization theorem $d\Gamma \sim H \cdot J \otimes S$

The hard function H and the jet function J are perturbative quantities.

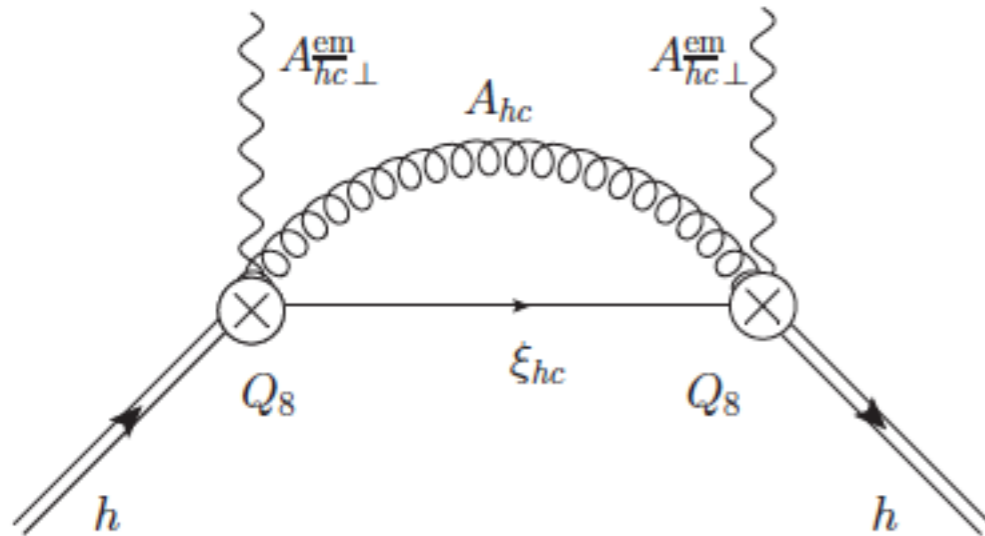
The shape function S is a non-perturbative non-local HQET matrix element.

(universality of the shape function, uncertainties due to subleading shape functions)

Calculation at subleading power

Example of **direct** photon contribution which factorizes

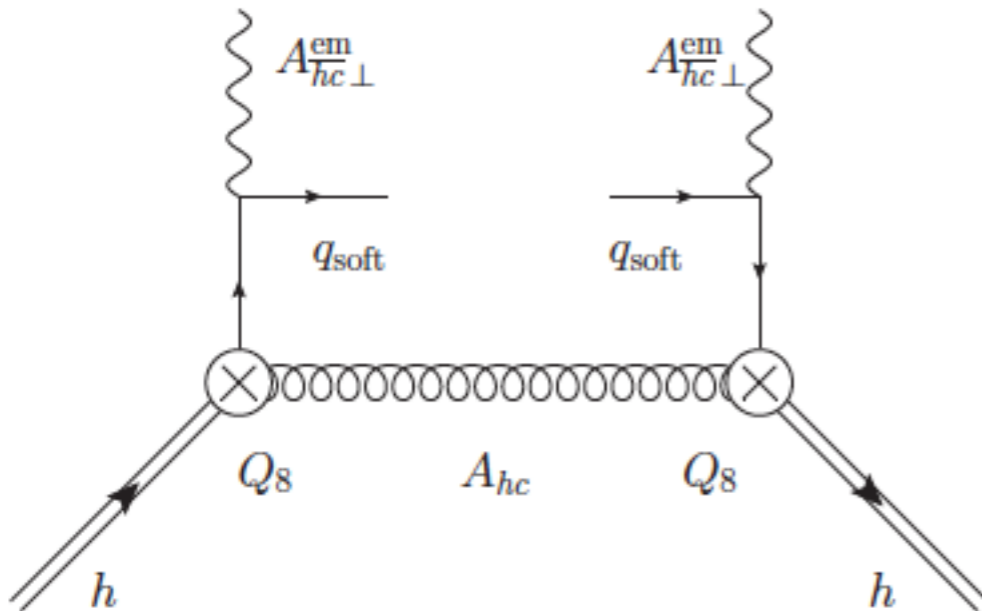
$$d\Gamma \sim H \cdot j \otimes S$$



$$\rightarrow \frac{\alpha_s}{m_b} \text{ in low } m_\chi^2 \text{ region}$$

Example of **resolved** photon contribution (double-resolved) which factorizes

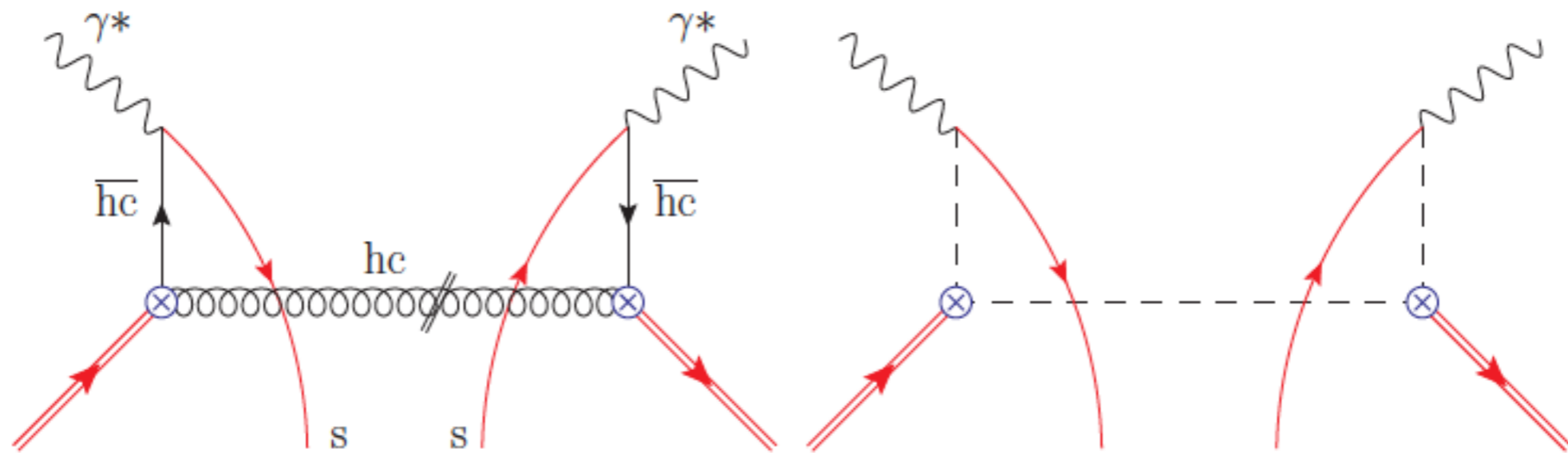
$$d\Gamma \sim H \cdot J \otimes s \otimes \bar{J} \otimes \bar{J}$$



$$\rightarrow \frac{\Lambda}{m_b}$$

In the resolved contributions the photon couples to light partons instead of connecting directly to the effective weak-interaction vertex.

Interference of Q_8 and Q_8

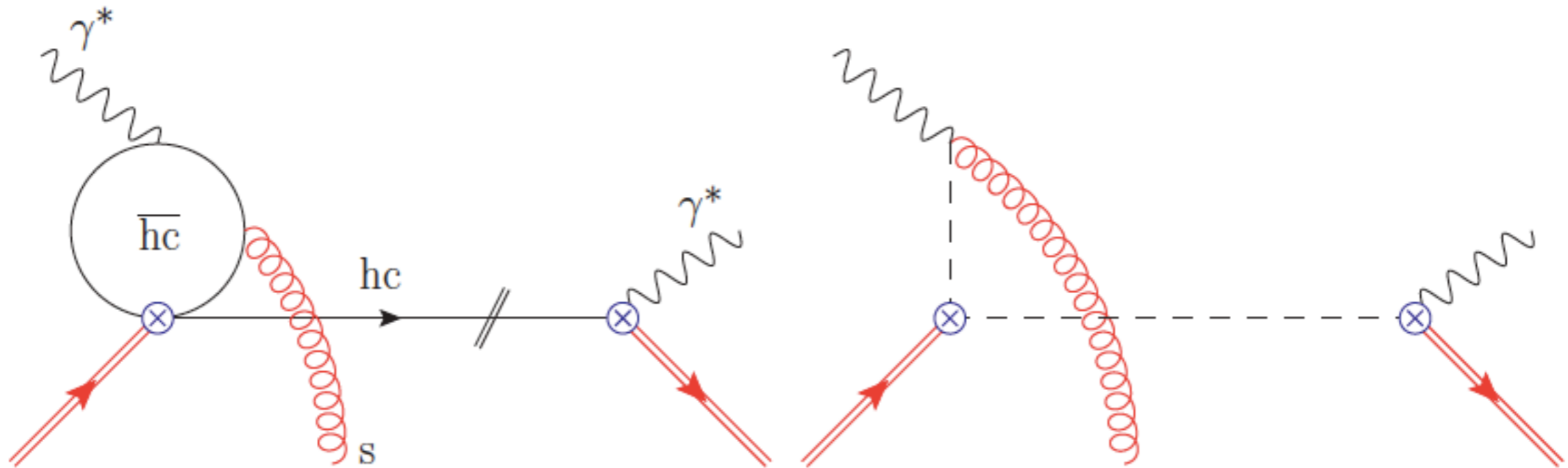


$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\epsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\epsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \dots s(\mathbf{tn} + \mathbf{u}\bar{n}) \bar{s}(\mathbf{r}\bar{n}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

- No resolved contribution if the photon is assumed to be hard !

Interference of Q_1 and Q_7



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{1}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + i\epsilon}$$

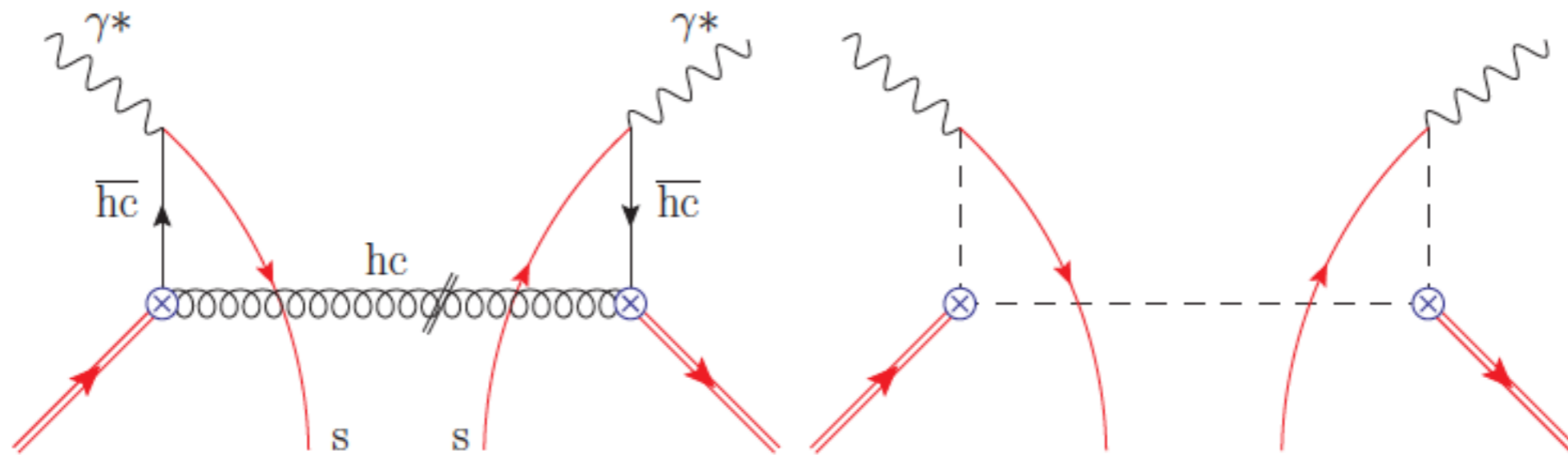
$$\frac{1}{\omega_1} \left[\bar{n} \cdot q \left(F \left(\frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - 1 \right) - (\bar{n} \cdot q + \omega_1) \left(F \left(\frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) - 1 \right) \right.$$

$$\left. + \bar{n} \cdot q \left(G \left(\frac{m_c^2}{n \cdot q \bar{n} \cdot q} \right) - G \left(\frac{m_c^2}{n \cdot q (\bar{n} \cdot q + \omega_1)} \right) \right) \right] g_{17}(\omega, \omega_1)$$

$$g_{17}(\omega, \omega_1) = \int \frac{dr}{2\pi} e^{-i\omega_1 r} \int \frac{dt}{2\pi} e^{-i\omega t} \frac{1}{M_B} \langle \bar{B} | \bar{h}(\text{tn}) \dots G_s^{\alpha\beta}(r\bar{n}) \dots h(0) | \bar{B} \rangle$$

- Shape function is nonlocal in both light cone directions
- It survives $M_X \rightarrow 1$ limit (irreducible uncertainty)

Interference of Q_8 and Q_8



$$\frac{d\Gamma^{\text{res}}}{dn \cdot q d\bar{n} \cdot q} \sim \frac{e_s^2 \alpha_s}{m_b} \int d\omega \delta(\omega + p_+) \int \frac{d\omega_1}{\omega_1 + \bar{n} \cdot q + i\epsilon} \int \frac{d\omega_2}{\omega_2 + \bar{n} \cdot q - i\epsilon} g_{88}(\omega, \omega_1, \omega_2)$$

$$g_{88}(\omega, \omega_1, \omega_2) = \frac{1}{M_B} \langle \bar{B} | \bar{h}(\mathbf{tn}) \dots s(\mathbf{tn} + \mathbf{u}\bar{n}) \bar{s}(\mathbf{r}\bar{n}) \dots h(\mathbf{0}) | \bar{B} \rangle_{\text{F.T.}}$$

- Subtlety in the Q_8 - Q_8 contribution: convolution integral is UV divergent
 - This implies that there is no complete proof of the factorization formula yet.
 - Nevertheless one shows that scale dependence of direct and resolved contribution cancel. [Benzke, Lee, Neubert, Paz, arXiv:1003.5012](#)
 - Refactorization methods allow to resolve the problem and reestablish factorization formula.

Numerical evaluation of the resolved contributions

Strategy:

- Use explicit definition of shape function as HQET matrix element to derive properties
 - PT invariance implies that soft functions are real
 - Moments of shape functions are related to HQET parameters
 - Soft functions have no significant structure outside the hadronic range
 - Values of soft functions are within the hadronic range
- Perform convolution integrals with model functions

Numerical evaluation $\mathcal{O}_1^c - \mathcal{O}_{7\gamma}$

$$\mathcal{F}_{17}^q = \frac{1}{m_b} \frac{C_1(\mu) C_{7\gamma}(\mu)}{C_{\text{OPE}}} e_c \operatorname{Re} \left[\frac{-(\lambda_t^q)^* \lambda_c^q}{|\lambda_t^q|^2} \right] \int_{-\infty}^{+\infty} d\omega_1 J_{17}(q_{\min}^2, q_{\max}^2, \omega_1) h_{17}(\omega_1, \mu)$$

Systematic analysis with the Hermite polynomials:

$$h_{17}(\omega_1, \mu) = \sum_n a_{2n} H_{2n} \left(\frac{\omega_1}{\sqrt{2}\sigma} \right) e^{-\frac{\omega_1^2}{2\sigma^2}}$$

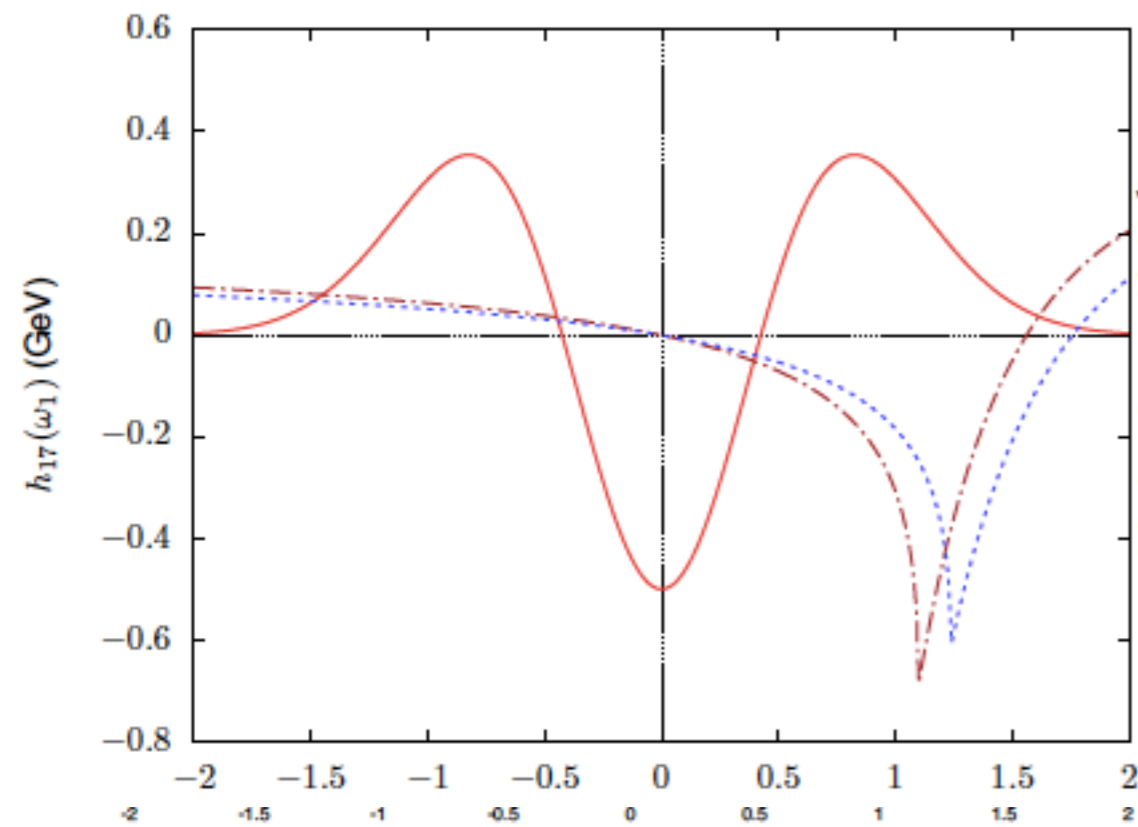
Further constraints from higher moments of soft function:

$$\begin{aligned} \int_{-\infty}^{\infty} d\omega_1 \omega_1^0 h_{17}(\omega_1, \mu) &= 0.237 \pm 0.040 \text{ GeV}^2 \\ \text{New input: } \int_{-\infty}^{\infty} d\omega_1 \omega_1^2 h_{17}(\omega_1, \mu) &= 0.15 \pm 0.12 \text{ GeV}^4 \\ &\text{Paz et al. arXiv:1908.02812} \end{aligned}$$

Updated result for $\bar{B} \rightarrow X_s \gamma$

Benzke,Hurth,arXiv:2006.00624

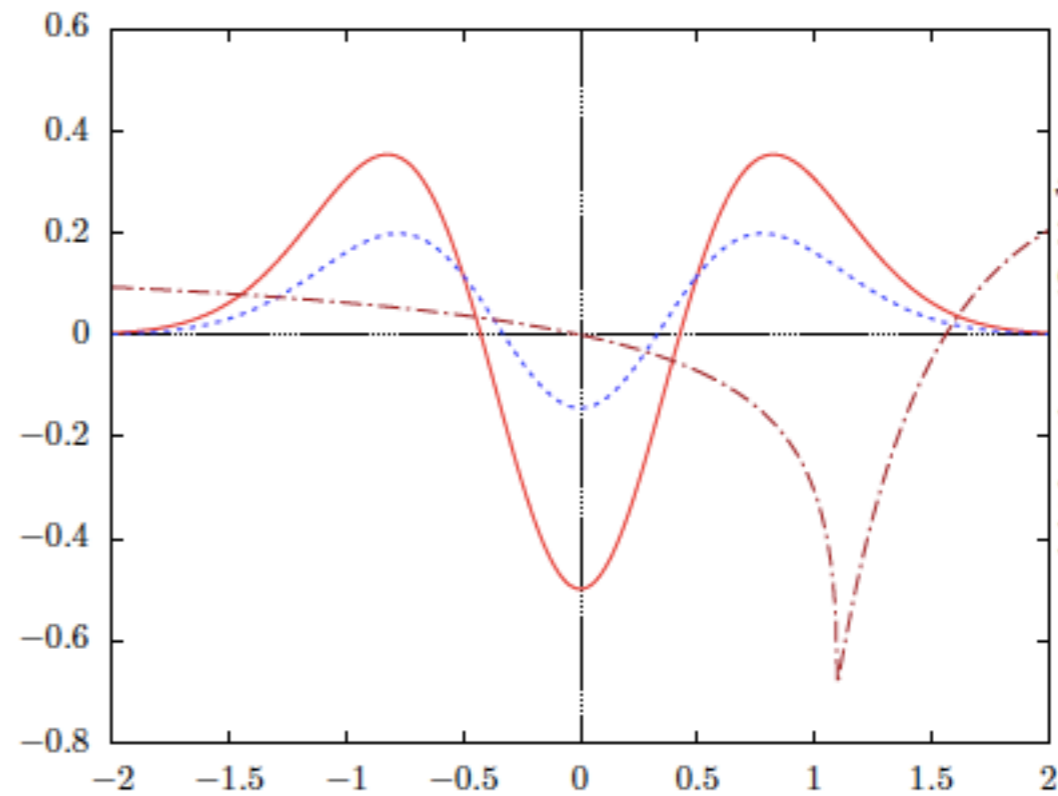
Charm dependence of jet function: Constraint on shape function:



Benzke,Hurth,arXiv:2006.00624

$$\mathcal{F}_{b \rightarrow s \gamma}^{17} \in [-0.4\%, 4.7\%]$$

$$\mathcal{F}_{b \rightarrow s \gamma}^{\text{total}} \in [-3.7\%, 6.5\%]$$



Neubert et al., arXiv: 1003.5012

$$\mathcal{F}_{b \rightarrow s \gamma}^{17} \in [-1.9\%, 4.7\%]$$

$$\mathcal{F}_{b \rightarrow s \gamma}^{\text{total}} \in [-5.2\%, 6.5\%]$$

(In addition: large scale dependence)

Still: Largest uncertainty in the prediction of the decay rate of $\bar{B} \rightarrow X_s \gamma$

Remarks

- There is a significant scale dependence of around 40% if one chooses the hard-collinear instead of the hard scale at LO.
- A NLO analysis will significantly reduce large scale dependence and also the dependence on the charm mass.
- Comparison with the numerical analysis in [Paz et al. arXiv:1908.02812](#)

$$\mathcal{F}_{b \rightarrow s \gamma}^{17} \in [-0.4\%, 1.9\%] \quad \text{versus} \quad \mathcal{F}_{b \rightarrow s \gamma}^{17} \in [-0.4\%, 4.7\%]$$

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Reason for significantly smaller error is twofold:

Comparison with the numerical analysis in Paz et al. arXiv:1908.02812

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Reason for significantly smaller error is twofold:

- For charm dependence only the parametric uncertainty was used

$$1.17 \text{ GeV} \leq m_c \leq 1.23 \text{ GeV}$$

We use scale variation of the hard-collinear scale

$$\mu_{\text{hc}} \sim \sqrt{m_b \Lambda_{\text{QCD}}} \quad \text{from} \quad 1.3 \text{ GeV} \text{ to } 1.7 \text{ GeV} \quad \text{and get}$$

$$1.14 \text{ GeV} \leq m_c \leq 1.26 \text{ GeV}$$

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- Numerically large $1/m_b^2$ term due to kinematic factors was dropped compared to the original analysis in 2010 Neubert et al., arXiv: 1003.5012

This kinematic $1/m_b^2$ term has a $1/m_b$ shape function, all other $1/m_b^2$ contributions have a shape function of order $1/m_b^2$. So no cancellation expected. Benzke, Hurth, arXiv:2303.06447

The large kinematic $1/m_b^2$ term can be used as conservative estimate of all $1/m_b^2$ contributions to resolved $\mathcal{O}_{7\gamma} - \mathcal{O}_1$.

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Underestimation of the uncertainty due to the resolved contribution.

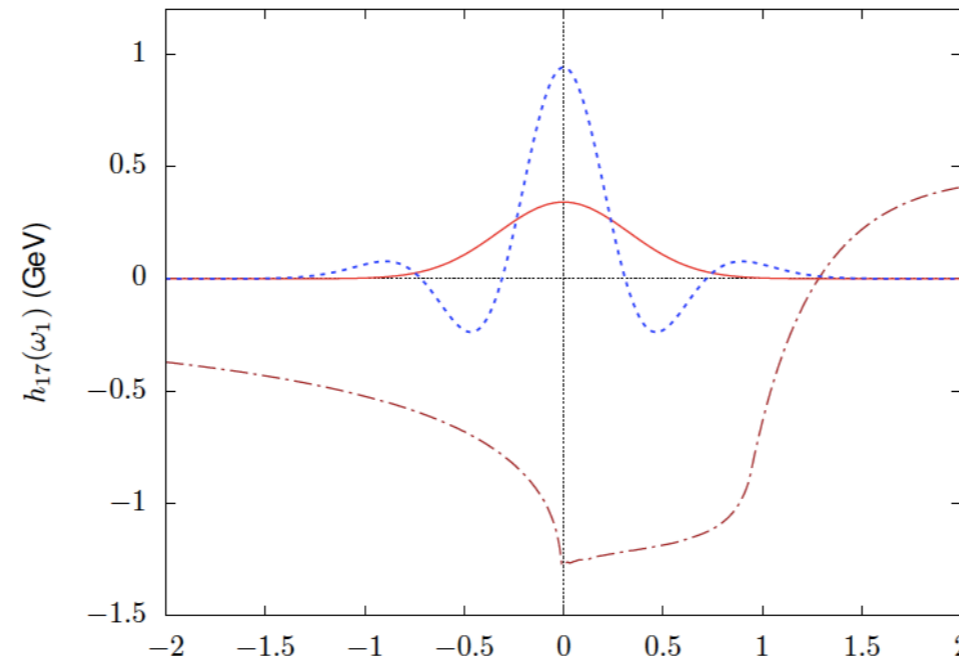
But used in recent $b \rightarrow s\gamma$ analysis. Misiak, Rehman, Steinhauser, arXiv:2002.01548v2

Updated result for $\bar{B} \rightarrow X_s \ell \ell$

Benzke, Hurth, arXiv:2006.00624

Rather symmetric jet function \rightarrow

Various shape functions lead to very similar values of the convolution



arXiv:2006.00624

$$\mathcal{F}_{b \rightarrow s \ell \ell}^{17} \in [+0.2\%, +2.6\%]$$

arXiv:1705.10366

$$\mathcal{F}_{b \rightarrow s \ell \ell}^{17}|_{1/m_b} \in [-0.5\%, +3.4\%]$$

We find large scale dependence of the results in both penguins
 $\Rightarrow \alpha_s$ corrections desirable

Numerical relevant contributions to $O(1/m_b^2)$

$$\mathcal{F}_{19}: O(1/m_b^2) \text{ but } |C_{9/10}| \sim 13|C_{7\gamma}|$$

Refactorisation in subleading $\bar{B} \rightarrow X_s \gamma$

Hurth, Szafron, arXiv:2301.01739

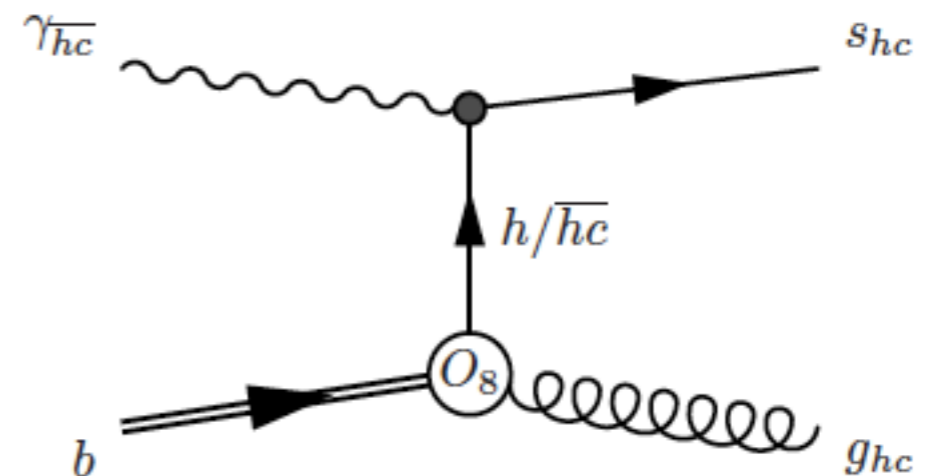
- Naive factorisation theorem with anti-hardcollinear Jet functions \bar{J}

$$d\Gamma(\bar{B} \rightarrow X_s \gamma) = \sum_{n=0}^{\infty} \frac{1}{m_b^n} \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \\ + \sum_{n=1}^{\infty} \frac{1}{m_b^n} \left[\sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} + \sum_i H_i^{(n)} \otimes J_i^{(n)} \otimes S_i^{(n)} \otimes \bar{J}_i^{(n)} \otimes \bar{J}_i^{(n)} \right]$$

- Contribution of the gluon dipole operator does not factorise

$$O_{8g} = -\frac{g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} b$$

$$q^\mu = E_\gamma \bar{n}^\mu \quad \text{and} \quad p_B^\mu = M_B v^\mu$$



Refactorisation in subleading $\bar{B} \rightarrow X_s \gamma$

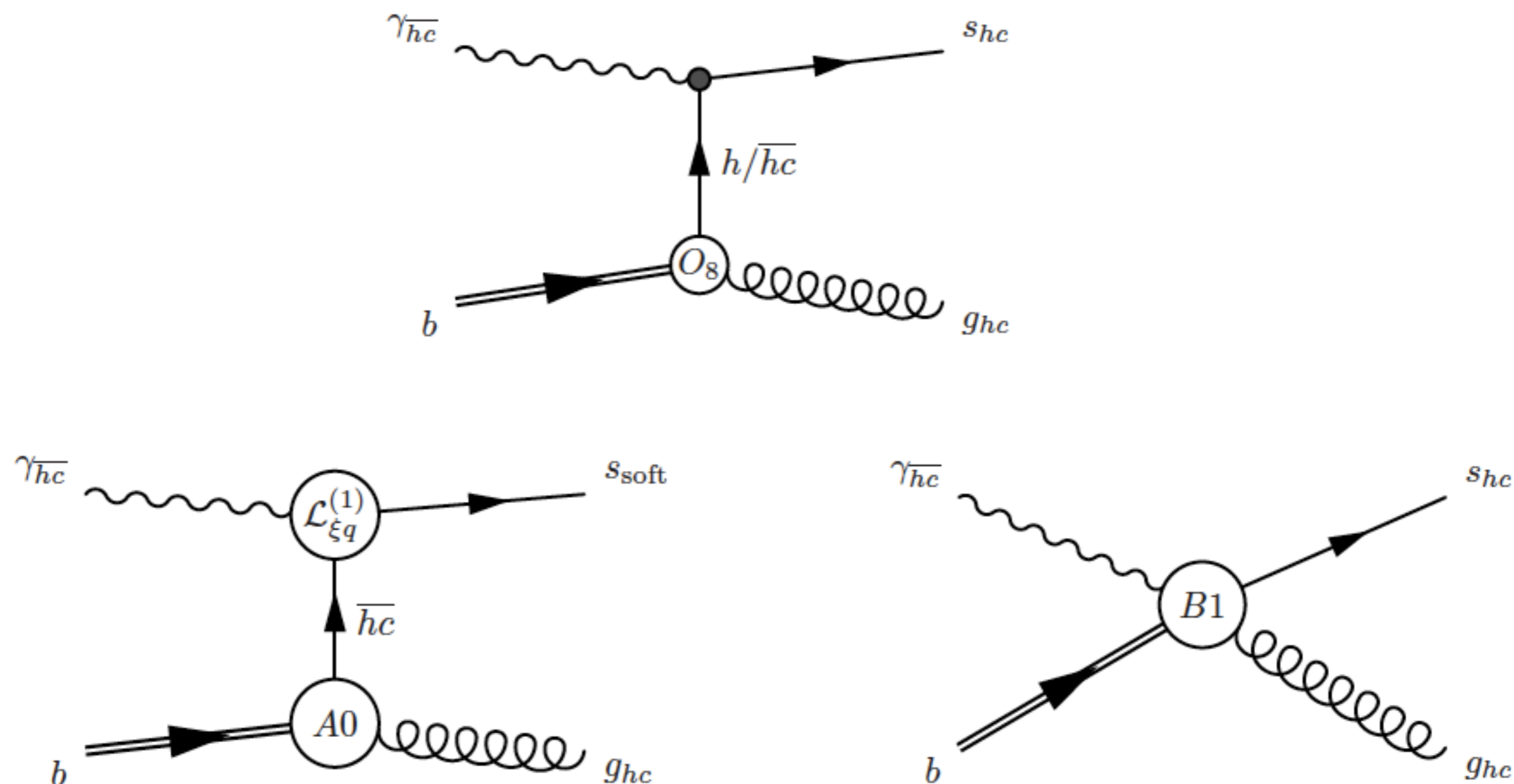
Hurth, Szafron, arXiv:2301.01739

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- Contribution of the gluon dipole operator does not factorise
- One can identify divergences in resolved *and* direct contribution in SCET-I as endpoint-divergences
- One can use refactorisation techniques developed in collider examples
Neubert et al., arXiv:2009.06779
- First QCD application with nonperturbative objects in flavour physics

Degeneracy in EFT leads to endpoint divergences



$$\mathcal{O}_{8g}^{A0}(0) = \bar{\chi}_{\bar{h}c}(0) \frac{\not{n}}{2} \gamma_{\mu\perp} \mathcal{A}^{\mu}_{hc\perp}(0) (1 + \gamma_5) h(0)$$

$$\mathcal{O}_{8g}^{B1}(u) = \int \frac{dt}{2\pi} e^{-i u m_b t} \bar{\chi}_{hc}(t\bar{n}) \gamma_{\nu\perp} Q_s \mathcal{B}^{\nu}_{\bar{h}c\perp}(0) \gamma_{\mu\perp} \mathcal{A}^{\mu}_{hc\perp}(0) (1 + \gamma_5) h(0)$$

Factorisation of direct contribution

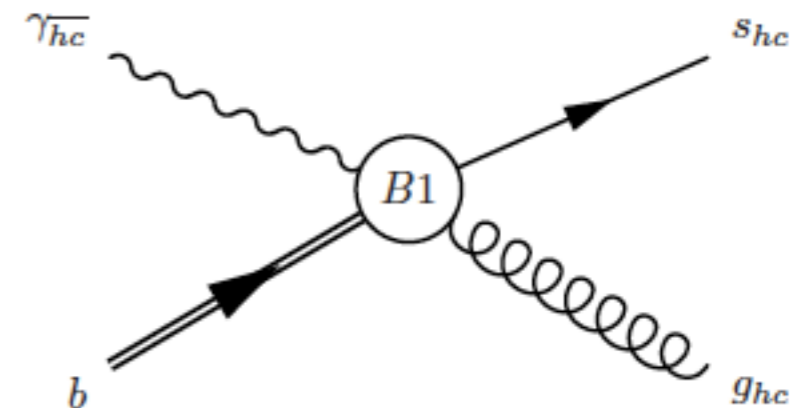
$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_B \int_0^1 du C^{B1}(m_b, u) \int_0^1 du' C^{B1*}(m_b, u') \int_{-p_+}^{\bar{\Lambda}} d\omega J(M_B(p_+ + \omega), u, u') \mathcal{S}(\omega)$$

$$C_{LO}^{B1}(m_b, u) = (-1) \frac{\bar{u}}{u} \frac{m_b^2}{4\pi^2} \frac{G_F}{\sqrt{2}} \lambda_t C_{8g} = (-1) \frac{\bar{u}}{u} C_{LO}^{A0}(m_b)$$

$$J(p^2, u, u') = \frac{(-1)}{2N_c} \frac{1}{2\pi} \int \frac{dt dt'}{(2\pi)^2} d^4x e^{-im_b(ut - u't') + ipx} (d-2)^2$$

$$\text{Disc} \left[\langle 0 | \text{tr} \left[\frac{\not{n}}{4} (1 - \gamma_5) \mathcal{A}_{hc\perp}^\mu(x) \chi_{hc}(t'\bar{n} + x) \bar{\chi}_{hc}(t\bar{n}) \mathcal{A}_{\mu}^{hc\perp}(0) (1 + \gamma_5) \right] | 0 \rangle \right]$$

$$\mathcal{S}(\omega) = \frac{1}{2m_B} \int \frac{dt}{2\pi} e^{-i\omega t} \langle B | h(tn) S_n(tn) S_n^\dagger(0) h(0) | B \rangle$$



Endpoint divergence in direct contribution at leading order

Hard matching coefficients

$$C_{LO}^{B1}(m_b, u) = (-1) \frac{\bar{u}}{u} \frac{m_b^2}{4\pi^2} \frac{G_F}{\sqrt{2}} \lambda_t C_{8g} = (-1) \frac{\bar{u}}{u} C_{LO}^{A0}(m_b)$$

convoluted with jet function

$$J(p^2, u, u') = C_F \frac{\alpha_s}{4\pi m_b} \theta(p^2) A(\epsilon) \delta(u - u') u^{1-\epsilon} (1-u)^{-\epsilon} \left(\frac{p^2}{\mu^2} \right)^{-\epsilon}$$

lead to endpoint divergence in the $u \rightarrow 0$ limit

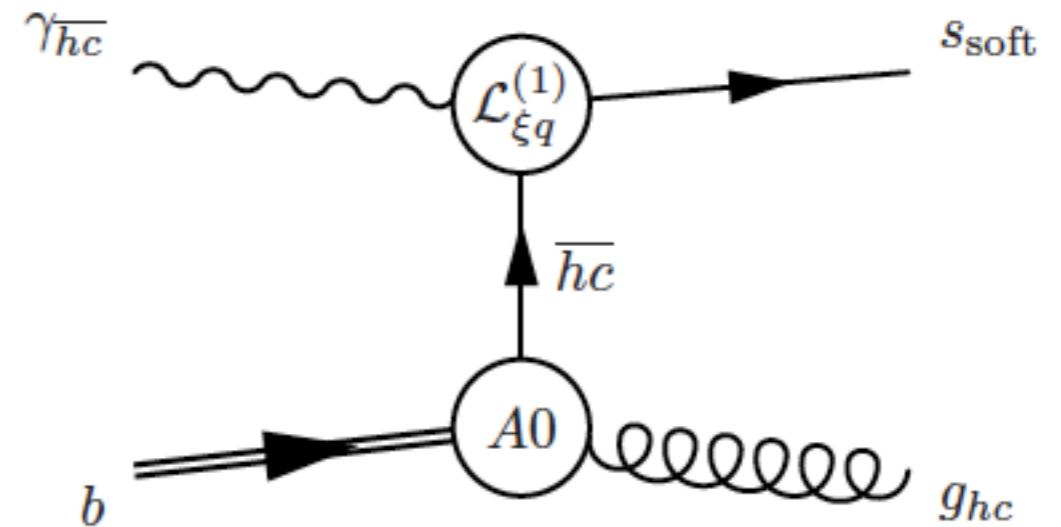
$$\int_0^1 du \frac{1}{u} \int_u^1 du' \frac{1}{u'} u^{1-\epsilon} \delta(u - u') \sim \int_0^1 du \frac{1}{u^{1+\epsilon}}$$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left| C^{A0}(m_b) \right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \int d\omega_1 \int d\omega_2 \bar{J}(\omega_1) \bar{J}^*(\omega_2) \mathcal{S}(\omega, \omega_1, \omega_2)$$

Anti-hardcollinear jet function $\bar{J}(\omega)$ is defined on the amplitude level.

$$\mathcal{O}_{T\xi q} = i \int d^d x T [\mathcal{L}_{\xi q}(x), \mathcal{O}_{8g}^{A0}(0)]$$



$$\mathcal{O}_{T\xi q} = \int d\omega \int \frac{dt}{2\pi} e^{-it\omega} [\bar{q}_s]_\alpha(tn) [\bar{J}(\omega)]_{\alpha\beta}^{a\nu\mu} Q_s \mathcal{B}_{\bar{h}c\perp}^\nu(0) \mathcal{A}_{hc\perp}^{\mu a}(0) [h(0)]_\beta$$

Decomposition to all orders:

$$[\bar{J}(\omega)]_{\alpha\beta}^{a\nu\mu} = \bar{J}(\omega) t^a \left[\gamma_\perp^\nu \gamma_\perp^\mu \frac{\not{n} \not{\bar{n}}}{4} \right]_{\alpha\beta}$$

Factorisation of resolved contribution

$$\frac{d\Gamma}{dE_\gamma} = \mathcal{N}_A \left| C^{A0}(m_b) \right|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \int d\omega_1 \int d\omega_2 \bar{J}(\omega_1) \bar{J}^*(\omega_2) \mathcal{S}(\omega, \omega_1, \omega_2)$$

Operatorial definition of the soft function in position space $\mathcal{S}(u, t, t')$

$$\mathcal{S}(u, t, t') = (d-2)^2 g_s^2 \langle B | \bar{h}(un) (1 - \gamma_5) [S_n(un) t^a S_n^\dagger(un)] S_{\bar{n}}(un) S_{\bar{n}}^\dagger(t'\bar{n} + un) \\ \frac{\not{n} \not{\bar{n}}}{4} q_s(t'\bar{n} + un) \bar{q}_s(t\bar{n}) \frac{\not{\bar{n}} \not{n}}{4} S_{\bar{n}}(t\bar{n}) S_{\bar{n}}^\dagger(0) [S_n(0) t^a S_n^\dagger(0)] (1 + \gamma_5) h(0) | B \rangle / (2m_B)$$

$$\mathcal{S}(\omega, \omega_1, \omega_2) = \int \frac{du}{2\pi} e^{-iu\omega} \int \frac{dt}{2\pi} e^{-it\omega_1} \int \frac{dt'}{2\pi} e^{it'\omega_2} \mathcal{S}(u, t, t')$$

Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N}_A |C_{LO}^{A0}(m_b)|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega \delta(m_b(p_+ + \omega)) \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \frac{1}{(\omega_1 - i\epsilon)} \frac{1}{(\omega_2 + i\epsilon)} \mathcal{S}(\omega, \omega_1, \omega_2)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
- As a consequence the structure of the soft function corresponds to the leading power shape function $\mathcal{S}(\omega)$

$\omega_{1,2} \rightarrow \infty$ corresponds to $t, t' \rightarrow 0$ and $q_s(un) \rightarrow S_n(un)q_{hc}(un)$, $\bar{q}_s(0) \rightarrow q_{hc}S_n^\dagger(0)$

$$\mathcal{S}(u, t, t') = (d-2)^2 g_s^2 \langle B | \bar{h}(un) (1 - \gamma_5) [S_n(un) t^a S_n^\dagger(un)] S_{\bar{n}}(un) S_{\bar{n}}^\dagger(t'\bar{n} + un) \\ \frac{\not{t}'\not{\bar{n}}}{4} q_s(t'\bar{n} + un) \bar{q}_s(t\bar{n}) \frac{\not{t}\not{\bar{n}}}{4} S_{\bar{n}}(t\bar{n}) S_{\bar{n}}^\dagger(0) [S_n(0) t^a S_n^\dagger(0)] (1 + \gamma_5) h(0) | B \rangle / (2m_B)$$

$$\mathcal{S}(u) = \langle B | \bar{h}(un) S_n(un) S_n^\dagger(0) h(0) | B \rangle / (2m_B)$$

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More general:

Asymptotic ($\omega_1 \sim \omega_2 \leq \omega$) soft function $\tilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$ is a convolution of a perturbative kernel K and the leading power soft function.

$$\tilde{\mathcal{S}}(\omega, \omega_1, \omega_2) = \int d\omega' K(\omega, \omega', \omega_1, \omega_2) \mathcal{S}(\omega')$$

Endpoint divergence in resolved contribution at leading order

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N}_A |C_{LO}^{A0}(m_b)|^2 \int_{-p_+}^{\bar{\Lambda}} d\omega \delta(m_b(p_+ + \omega)) \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \frac{1}{(\omega_1 - i\epsilon)} \frac{1}{(\omega_2 + i\epsilon)} \mathcal{S}(\omega, \omega_1, \omega_2)$$

- Endpoint divergence occurs only for asymptotic $\omega_1 \sim \omega_2 \gg \omega$
- For $\omega_1 \sim \omega_2 \gg \omega$ light quarks become "hard-collinear" and can be decoupled from the soft gluons
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$$\tilde{\mathcal{S}}(\omega, \omega_1, \omega_2) = \int d\omega' K(\omega, \omega', \omega_1, \omega_2) \mathcal{S}(\omega')$$

Leading order in α_s :

$$\tilde{\mathcal{S}}(\omega, \omega_1, \omega_2) = C_F A(\epsilon) \frac{\alpha_s}{(4\pi)} \omega_1^{1-\epsilon} \delta(\omega_1 - \omega_2) \int_{\omega}^{\bar{\Lambda}} d\omega' \mathcal{S}(\omega') \left(\frac{(\omega' - \omega)}{\mu^2} \right)^{-\epsilon}$$

Refactorisation at leading order

$$\frac{d\Gamma}{dE_\gamma}|_{B}^{u,u'\rightarrow 0} = -\mathcal{N} \left| C_{LO}^{A0}(m_b) \right|^2 \frac{\alpha_s C_F}{(4\pi) m_b} \frac{1}{\epsilon} A(\epsilon) \int_{-p_+}^{\bar{\Lambda}} d\omega \mathcal{S}_{LO}(\omega) \left(\frac{m_b(\omega + p_+)}{\mu^2} \right)^{-\epsilon}$$

$$\frac{d\Gamma}{dE_\gamma}|_A^{\text{asy}} = : \mathcal{N} \left| C_{LO}^{A0}(m_b) \right|^2 \frac{\alpha_s C_F}{(4\pi) m_b} \frac{1}{\epsilon} A(\epsilon) \int_{-p_+}^{\bar{\Lambda}} d\omega \mathcal{S}_{LO}(\omega') \left(\frac{m_b(\omega + p_+)}{\mu^2} \right)^{-\epsilon}$$

One verifies that

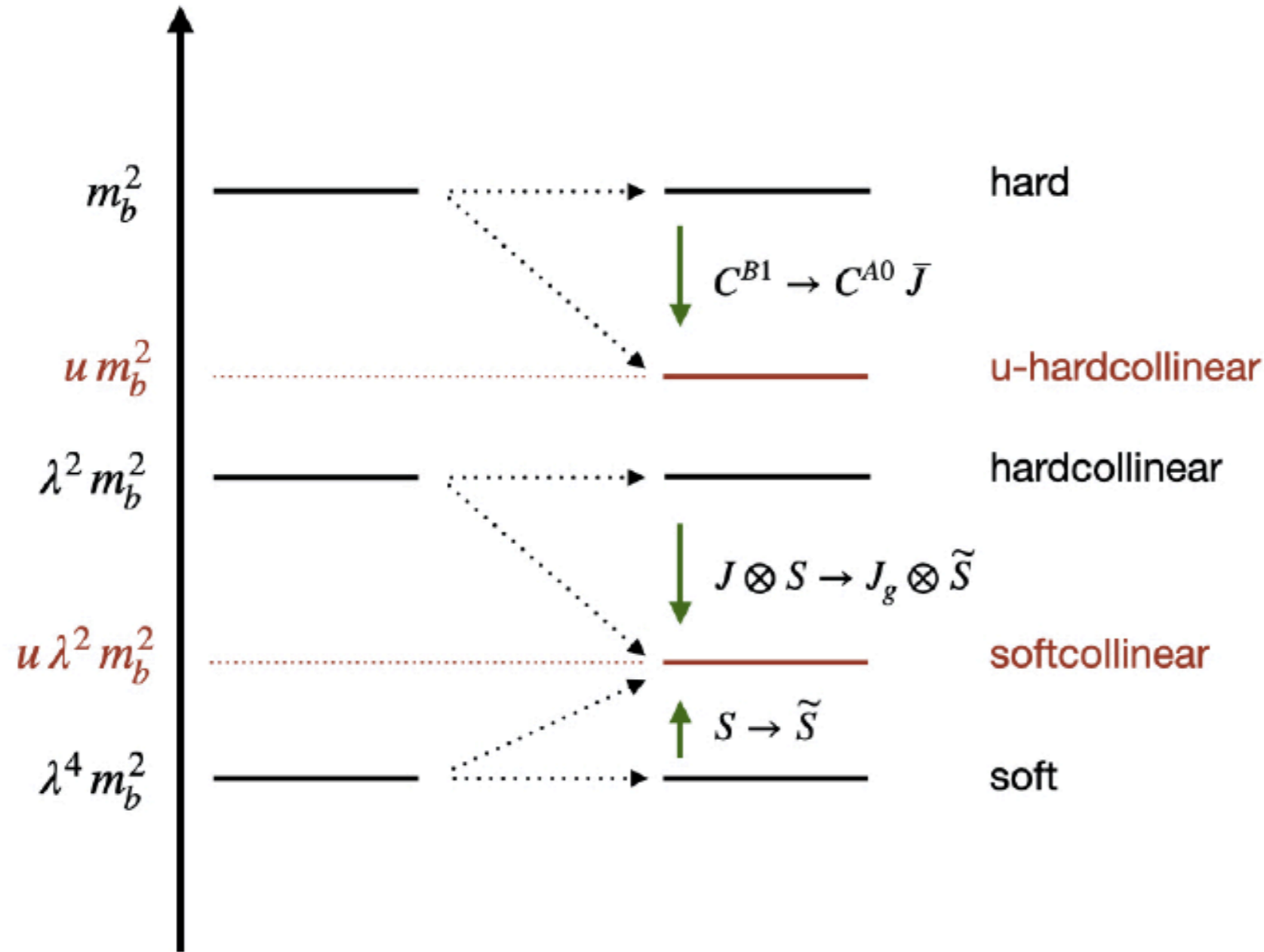
$$\frac{d\Gamma}{dE_\gamma}|_A^{\text{asy}} = (-1) \frac{d\Gamma}{dE_\gamma}|_B^{u,u'\rightarrow 0}$$

Refactorisation conditions can be formulated on the operator level

Express the fact that in the limits $u \sim u' \ll 1$ and $\omega_1 \sim \omega_2 \gg \omega$ the two terms of the subleading $\mathcal{O}_8 - \mathcal{O}_8$ contribution have the same structure.

- $\llbracket C^{B1}(m_b, u) \rrbracket = (-1) C^{A0}(m_b) m_b \bar{J}(um_b)$
($\llbracket g(u) \rrbracket$ only denotes the leading term of a function $g(u)$ in the limit $u \rightarrow 0$)
- $\tilde{\mathcal{S}}(\omega, \omega_1, \omega_2)$ corresponds to $\mathcal{S}(\omega, \omega_1, \omega_2)$ in the limit $\omega_1 \sim \omega_2 \gg \omega$
(In this limit: $q_s \rightarrow q_{sc}$ and higher power corrections in $\omega/\omega_{1,2}$ are neglected)
- $\int_{-p_+}^{\bar{\Lambda}} d\omega \llbracket J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \rrbracket = \int_{-p_+}^{\bar{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \tilde{\mathcal{S}}(\omega, m_b u, m_b u')$
(In this limit $\chi_{hc} \rightarrow q_{sc}$, brackets indicate again that the $u \rightarrow 0$ and $u' \rightarrow 0$ limits)

The refactorisation relations are operatorial relations that guarantee the cancellation of endpoint divergences between the two terms to all orders in α_s .



Near the endpoint, u is no longer $u \sim O(1)$, i.e. $u \ll 1$, we introduce additional, unphysical scales which make it possible to factorise further objects appearing in the bare factorisation theorem.

Refactorised (endpoint finite) factorisation theorem

We subtract the two asymptotic terms

$$0 = 2\mathcal{N} |C^{A0}(m_b)|^2 \int_{-p_+}^{\Lambda} d\omega J_g(m_b(p_+ + \omega)) \int_{m_b}^{\infty} d\omega_1 \bar{J}(\omega_1) \int_0^{\omega_1} d\omega_2 \bar{J}^*(\omega_2) \tilde{\mathcal{S}}(\omega, \omega_1, \omega_2) \\ + 2\mathcal{N} \int_0^1 du \llbracket C^{B1}(m_b, u) \rrbracket \int_u^1 du' \llbracket C^{B1*}(m_b, u') \rrbracket \int_{-p_+}^{\bar{\Lambda}} d\omega \llbracket J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \rrbracket$$

with

$$\llbracket J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \rrbracket = J_g(m_b(p_+ + \omega)) \tilde{\mathcal{S}}(\omega, m_b u, m_b u')$$

$$\llbracket C^{B1}(m_b, u') \rrbracket = (-1) C^{A0}(m_b) m_b \bar{J}(u m_b)$$

from the all-order factorisation theorems we derived

$$\frac{d\Gamma}{dE_\gamma} = 2\mathcal{N} |C^{A0}(m_b)|^2 \int_{-\infty}^{\infty} d\omega_1 \bar{J}(\omega_1) \int_{-\infty}^{\omega_1} d\omega_2 \bar{J}^*(\omega_2) \int_{-p_+}^{\bar{\Lambda}} d\omega J_g(m_b(p_+ + \omega)) \mathcal{S}(\omega, \omega_1, \omega_2) \\ + 2\mathcal{N} \int_0^1 du C^{B1}(m_b, u) \int_u^1 du' C^{B1*}(m_b, u') \int_{-p_+}^{\bar{\Lambda}} d\omega J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega)$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$\begin{aligned} \frac{d\Gamma}{dE_\gamma}|_{A+B} = & 2\mathcal{N} \int_{-p_+}^{\bar{\Lambda}} d\omega \left\{ J_g(m_b(p_+ + \omega)) |C^{A0}(m_b)|^2 \right. \\ & \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \bar{J}(\omega_1) \bar{J}^*(\omega_2) \left[\mathcal{S}(\omega, \omega_1, \omega_2) - \theta(\omega_1 - m_b) \theta(\omega_2) \tilde{\mathcal{S}}(\omega, \omega_1, \omega_2) \right] \\ & + \int_0^1 du \int_u^1 du' \left[C_{LO}^{B1}(m_b, u) C^{B1*}(m_b, u') J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \right. \\ & \left. \left. - \llbracket C^{B1}(m_b, u) \rrbracket \llbracket C^{B1*}(m_b, u') \rrbracket \llbracket J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \rrbracket \right] \right\}, \end{aligned}$$

Refactorised (endpoint finite) factorisation theorem

and end up with the factorisation theorem without endpoint divergences:

$$\begin{aligned} \frac{d\Gamma}{dE_\gamma}|_{A+B} = & 2\mathcal{N} \int_{-p_+}^{\bar{\Lambda}} d\omega \left\{ J_g(m_b(p_+ + \omega)) |C^{A0}(m_b)|^2 \right. \\ & \times \int_{-\infty}^{\infty} d\omega_1 \int_{-\infty}^{\omega_1} d\omega_2 \bar{J}(\omega_1) \bar{J}^*(\omega_2) \left[\mathcal{S}(\omega, \omega_1, \omega_2) - \theta(\omega_1 - m_b) \theta(\omega_2) \tilde{\mathcal{S}}(\omega, \omega_1, \omega_2) \right] \\ & + \int_0^1 du \int_u^1 du' \left[C_{LO}^{B1}(m_b, u) C^{B1*}(m_b, u') J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \right. \\ & \left. \left. - \llbracket C^{B1}(m_b, u) \rrbracket \llbracket C^{B1*}(m_b, u') \rrbracket \llbracket J(m_b(p_+ + \omega), u, u') \mathcal{S}(\omega) \rrbracket \right] \right\}, \end{aligned}$$

Finally we show that refactorisation and renormalisation commute.

Hadronic cut dependence in $\bar{B} \rightarrow X_s \ell \ell$

- Additional cut in the hadronic mass spectrum (X_s) needed for background suppression (i.e. $b \rightarrow c(\rightarrow s e^+ \nu) e^- \bar{\nu}$)
- Previous SCET calculation with some simplifications and certain problems with SCET scaling (q assumed to be hard)
Uncertainty due to subleading shape functions estimated to 5 – 10%

Lee, Ligeti, Stewart, Tackmann hep-ph/0512191

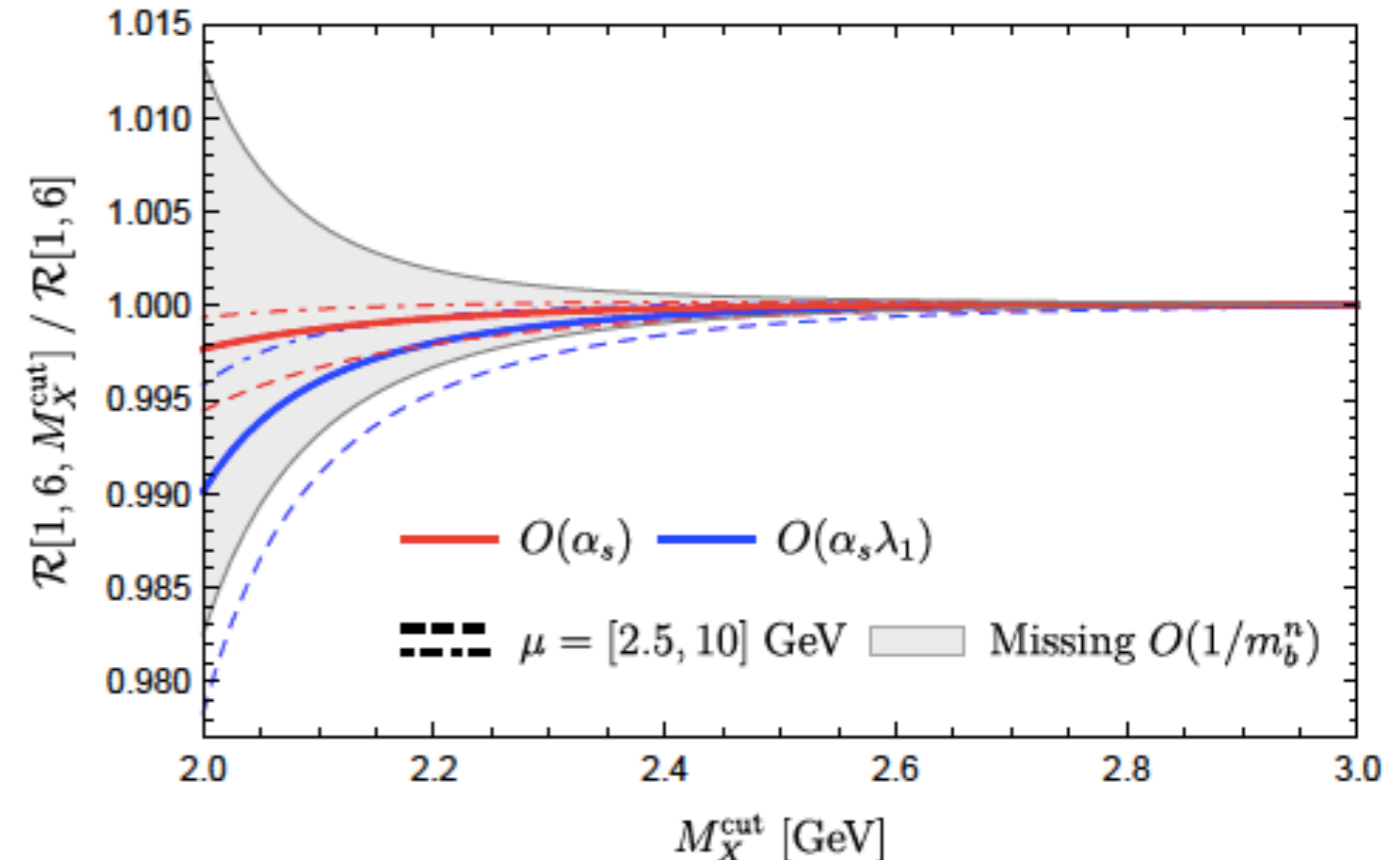
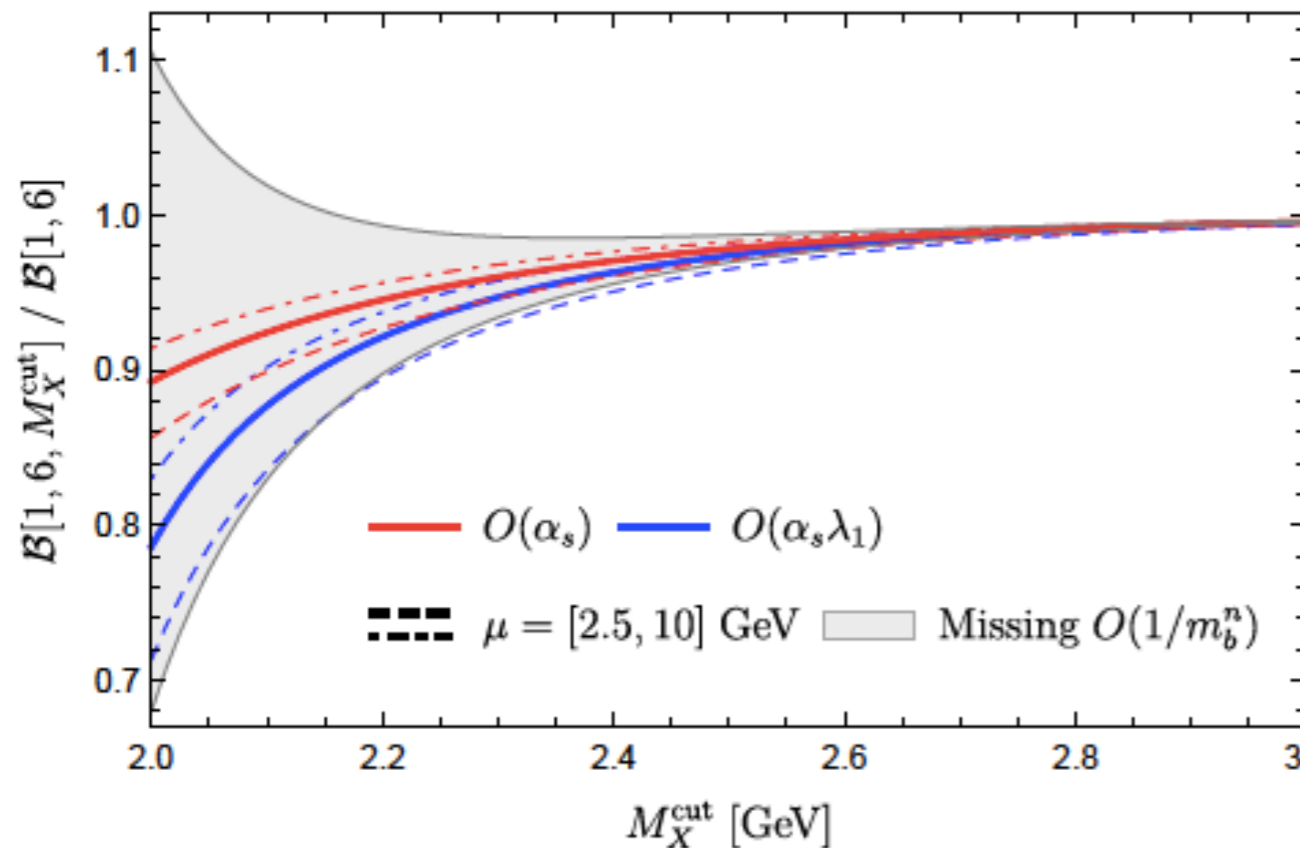
Lee, Tackmann arXiv:0812.0001

- **New Strategy to minimise uncertainty** Huber, Hurth, Jenkins, Lunghi
 - Calculation of cut dependence using OPE for mild hadronic cuts
 - Analyse breakdown of OPE via λ_1 power corrections
 - Try to interpolate between SCET and OPE calculation
 - Use cut-independent ratios in OPE and SCET to analyse interpolation

Hadronic cut dependence in $\bar{B} \rightarrow X_s \ell \ell$

Huber, Hurth, Jenkins, Lunghi, to appear

- We computed the fully differential distribution of $\bar{B} \rightarrow X_s \ell^+ \ell^-$ at $O(\alpha_s)$ in the OPE
- Also the three $\bar{B} \rightarrow X_s \ell^+ \ell^-$ angular observables, together with the $\bar{B} \rightarrow X_u \ell^- \nu$ branching fraction, all with the same hadronic mass cut
- We find effective Independence of the hadronic mass cut



Summary

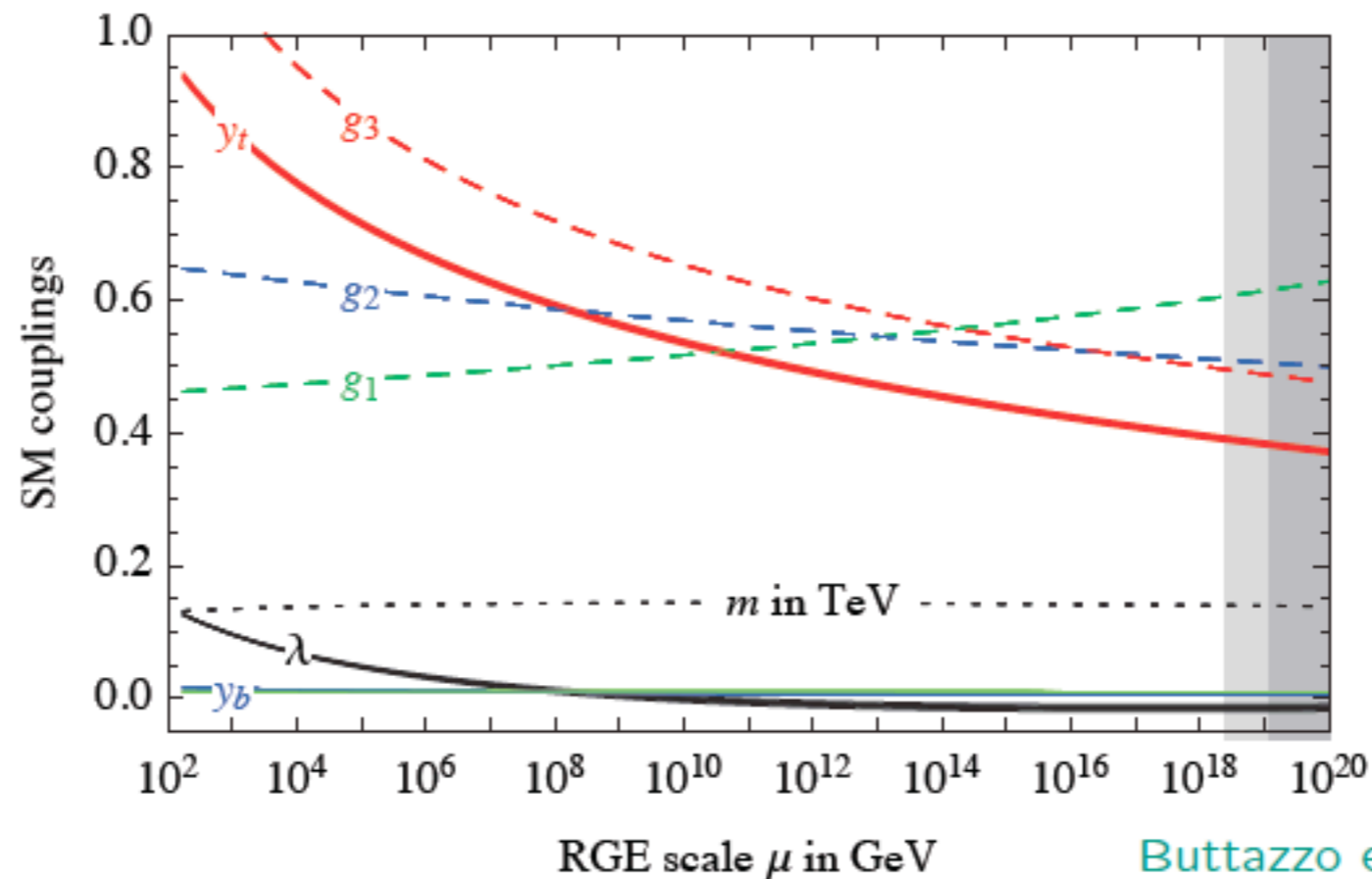
- In the post- R_K era we still have significant tensions in exclusive $b \rightarrow s$ angular observables and branching ratios.
- Inclusive semi-leptonic decays require Belle-II for full exploitation, but are theoretically very clean and allow for crosschecks of the present tensions.
- Refactorisation techniques allow to solve the problem of endpoint divergences, in particular in subleading $\bar{B} \rightarrow X_s \gamma$.
- Nonlocal power corrections presently belong to the largest uncertainties in the inclusive modes $\bar{B} \rightarrow X_s \gamma$ and $\bar{B} \rightarrow X_s \ell \ell$ (higher moments of shape functions and α_s corrections needed).
- Uncertainties due to the hadronic mass cut in $\bar{B} \rightarrow X_s \ell \ell$ may be reduced in the near future.

Epilogue

Self-consistency of the SM

Do we need new physics beyond the SM ?

- It is possible to extend the validity of the SM up to the M_P as weakly coupled theory.



High-energy extrapolation shows that the Yukawa couplings, weak gauge couplings and the Higgs self coupling remain perturbative in the entire energy domain between the electroweak and Planck scale (no Landau poles !).

- Renormalizability implies no constraints on the free parameters of the SM Lagrangian.

Experimental evidence beyond SM

- **Dark matter** (visible matter accounts for only 4% of the Universe)
- **Neutrino masses** (Dirac or Majorana masses ?)
- **Baryon asymmetry of the Universe** (new sources of CP violation needed)

Experimental evidence beyond SM

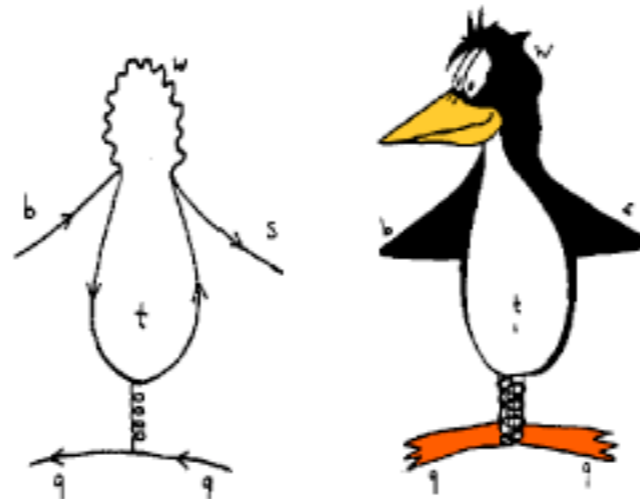
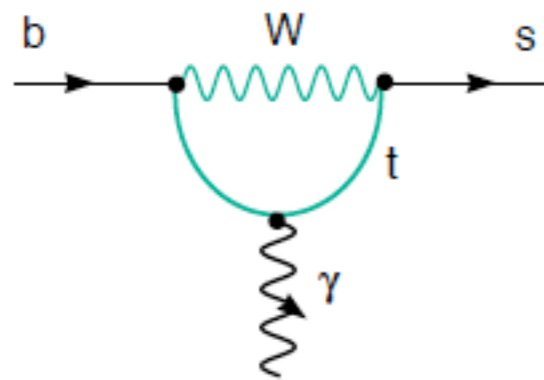
- **Dark matter** (visible matter accounts for only 4% of the Universe)
- **Neutrino masses** (Dirac or Majorana masses ?)
- **Baryon asymmetry of the Universe** (new sources of CP violation needed)

Caveat:

Answers perhaps wait at energy scales which we do not reach with present experiments.

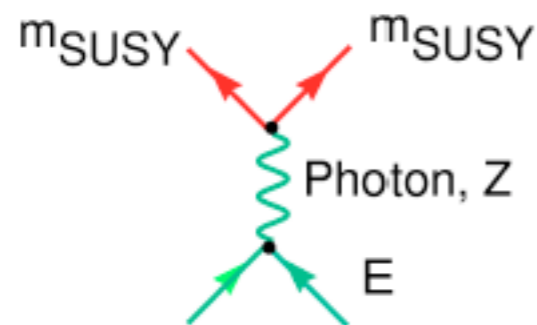
Indirect exploration of higher scales via flavour

- Flavour changing neutral current processes like $b \rightarrow s \gamma$ or $b \rightarrow s \ell^+ \ell^-$ directly probe the SM at the one-loop level.

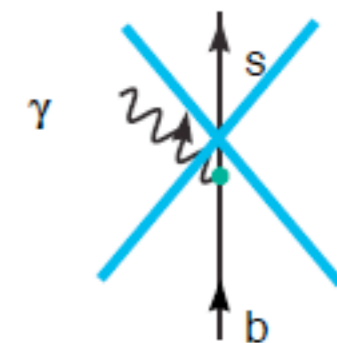
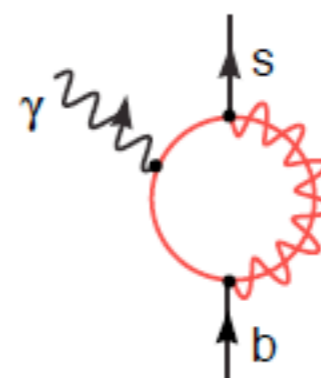
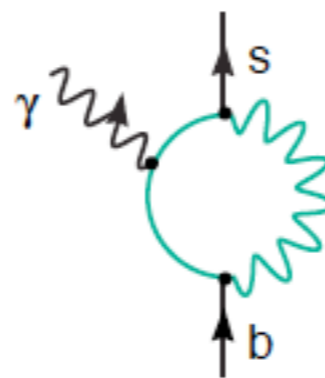


- Indirect search strategy for new degrees of freedom beyond the SM

Direct:



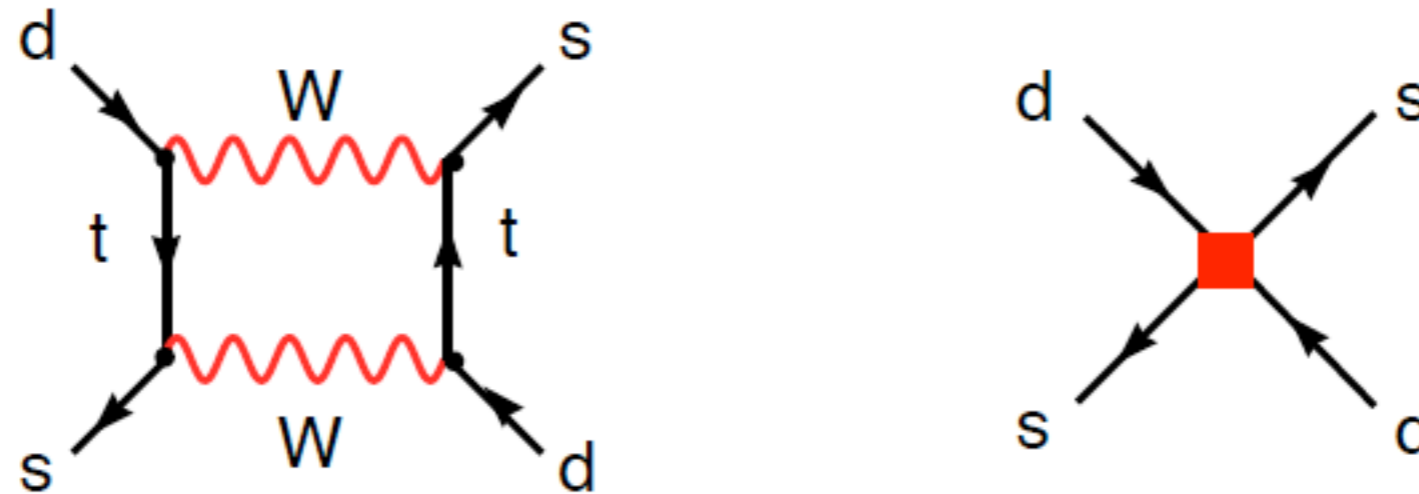
Indirect:



Flavour problem of New Physics or how FCNCs hide?

$$\mathcal{L} = \mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \sum_i \frac{c_i^{New}}{\Lambda} \mathcal{O}_i^{(5)} + \dots$$

- SM as effective theory valid up to cut-off-scale Λ
- $K^0 - \bar{K}^0$ -mixing $\mathcal{O}^6 = (\bar{s}d)^2$: $c^{SM}/M_W^2 \times (\bar{s}d)^2 + c^{New}/\Lambda^2 \times (\bar{s}d)^2 \Rightarrow \Lambda > 10^4 \text{ TeV}$



- Natural stabilisation of Higgs boson mass $\Rightarrow \Lambda \sim 1\text{TeV}$

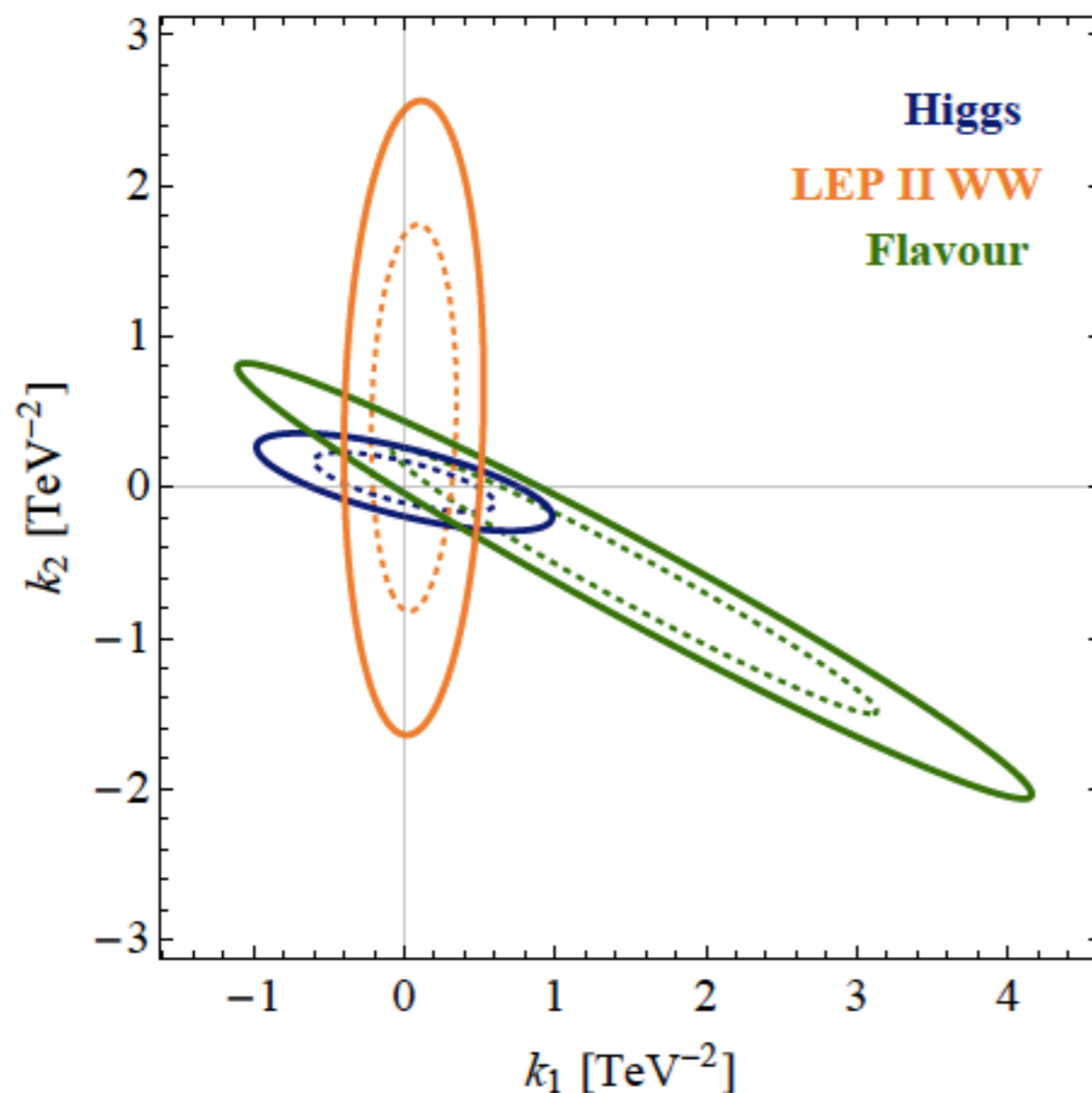
Ambiguity of new physics scale from flavour data

$$(C_{SM}^i/M_W + C_{NP}^i/\Lambda_{NP}) \times \mathcal{O}_i$$

Flavour matters

Aoude, Hurth, Renner, Shepherd arXiv:1903.00500 and arXiv2003.5432

Role of flavour data in global SMEFT fits using the leading term in spurionic Yukawa expansion at the new physics scale as initial conditions (there are no FCNC at the tree level at the NP scale) "*leading MFV*"



Example: 2 flat directions when fitting Z -pole data

Flavour data is competitive with existing constraints.

Michelangelo Mangano

- The days of "guaranteed" discoveries or no-lose theorems in particle physics are over, at least for the time being
- but the big questions of our field remain open (hierarchy problem, flavour, neutrinos, dark matter, baryogenesis,...)
- This simply implies that, more than for the past 30 years, future HEP's progress is to be driven by experimental exploration, possibly renouncing/reviewing deeply rooted theoretical bias.

Experimental flavour opportunities

- **LHCb:** allows for wide range of analyses, highlights: B_s mixing phase, angle γ , $B \rightarrow K^* \mu \mu$, $B_s \rightarrow \mu \mu$, $B_s \rightarrow \phi \phi$ then upgrades to 50 and 300 fb^{-1}
- Dedicated kaon experiments J-PARC E14 and CERN P-326/NA62: rare kaon decays $K_L^0 \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
- Super-B factory Belle-II at KEK (50 ab^{-1})
Belle-II is a Super Flavour factory: besides precise B measurements CP violation in charm, lepton flavour violating modes $\tau \rightarrow \mu \gamma$, ...

Spares

Inclusive versus exclusive $b \rightarrow s$ penguin modes

- Exclusive decays
 - Leptonic: $B_s \rightarrow \mu^+ \mu^-$
 - ✓ Accurate theory prediction (decay constant known with good precision)
 - Semileptonic: $B \rightarrow K^* \mu^+ \mu^-$, $B \rightarrow K \mu^+ \mu^-$, $B_s \rightarrow \phi \mu^+ \mu^-$
 - ✓ Hadronic uncertainties difficult to assess (form factor and non-local effects)
 - ✓ Many observables experimentally available
- Inclusive decays $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \ell^+ \ell^-$:
 - Precise theoretical calculations based on heavy mass expansion
 - Perturbative contributions dominant
 - Theoretical calculations available for non-perturbative power corrections
 - Experimentally more challenging → Belle II can offer further data

Lepton flavour universality in $B \rightarrow K^{(*)} \ell^+ \ell^-$

LHCb The results presented here differ from previous LHCb measurements of R_K [32] and R_{K^*} [29]. For R_K central- q^2 , the difference is partly due to the use of tighter electron identification criteria and partly due to the modeling of the residual misidentified hadronic backgrounds; statistical fluctuations make a smaller contribution to the difference since the same data are used as in Ref. [32].

December 20th update **LHCb**

