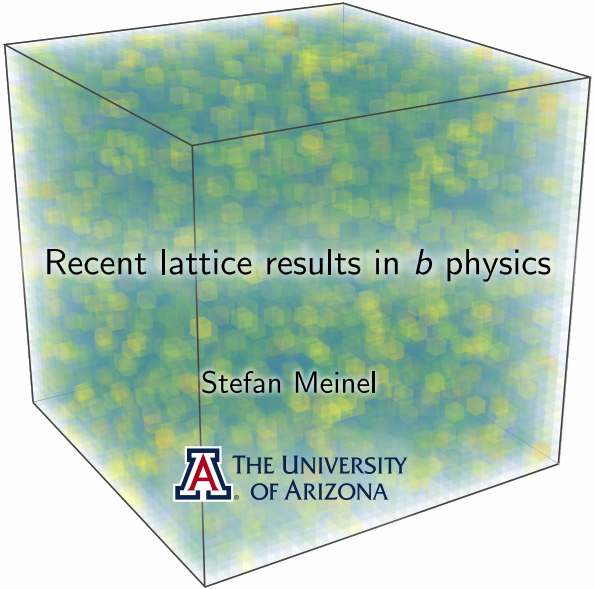

Recent lattice results in b physics

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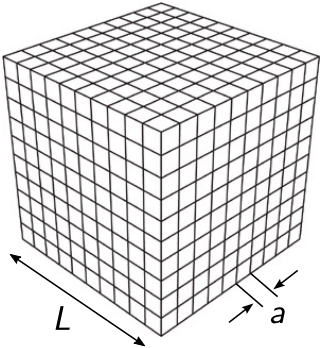
Lattice gauge theory allows us to nonperturbatively compute imaginary-time ($t = -i\tau$, $\tau \in \mathbb{R}$), a.k.a. Euclidean, QCD correlation functions in a finite volume:

$$\langle O_n(\tau_n, \mathbf{x}_n) ... O_1(\tau_1, \mathbf{x}_1) \rangle = \frac{1}{Z} \int \mathcal{D}[\psi, \overline{\psi}, U] \; O_n(\tau_n, \mathbf{x}_n) ... O_1(\tau_1, \mathbf{x}_1) \; e^{-S_E[\psi, \overline{\psi}, U]}$$



We extract the physics of interest by performing fits to the numerical results for these correlation functions.

We know that these correlation functions have spectral representations like

$$
\begin{aligned}
C_2(\tau, \mathbf{p}) &= a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}_{\text{src}})} \langle O_2(\tau_{\text{src}} + \tau, \mathbf{x})\, O_1^\dagger(\tau_{\text{src}}, \mathbf{x}_{\text{src}})\rangle \\
&= \sum_n \frac{1}{2E_n} \langle \Omega | \hat{O}_2(0) | n, \mathbf{p}\rangle \langle n, \mathbf{p} | \hat{O}_1^\dagger(0) | \Omega\rangle e^{-E_n \tau},
\end{aligned}
$$

$$
\begin{aligned}
C_3(\tau, \tau_J, \mathbf{p}', \mathbf{p}) &= a^6 \sum_{\mathbf{y},\mathbf{x}} e^{-i\mathbf{p}'\cdot(\mathbf{y}-\mathbf{x})} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}_{\text{src}})} \langle O_2(\tau_{\text{src}} + \tau, \mathbf{y})\, J(\tau_{\text{src}} + \tau_J, \mathbf{x})\, O_1^\dagger(\tau_{\text{src}}, \mathbf{x}_{\text{src}})\rangle \\
&= \sum_{n',n} \frac{1}{2E_{n'}} \frac{1}{2E_n} \langle \Omega | \hat{O}_2(0) | n', \mathbf{p}'\rangle \langle n', \mathbf{p}' | \hat{J}(0) | n, \mathbf{p}\rangle \langle n, \mathbf{p} | \hat{O}_1^\dagger(0) | \Omega\rangle e^{-E_{n'}(\tau - \tau_J)} e^{-E_n \tau_J}.
\end{aligned}
$$

(These equations are for infinite time extent of the lattice, and for $\tau > \tau_J > 0$.)

Going to large τ, τ_J, $\tau - \tau_J$ suppresses the contributions from excited states. From the above examples, we can extract decay constants and local form factors (still in finite volume).
Computing nonlocal matrix elements (Minkowski time integrals over the positions of multiple currents) requires checking that the integrals over Euclidean time give the same results as those over Minkowski time, which is not the case in general.

Any correlation functions can be computed as averages over previously generated random gauge-link configurations U with probability density

$$\rho[U] = \frac{1}{Z}\left(\prod_F \det D^F[U]\right) e^{-S_E^{\text{gluon}}[U]},$$

where $D^F[U]$ is the lattice Dirac operator for quark flavor F for a given U. Typically, $F = u, d, s$ or $F = u, d, s, c$ are included here.

For example, the two-point function with $O_1 = O_2 = \bar{b}\gamma_5 d$ is given by

$$C_2(\tau, \mathbf{p}) \;=\; \lim_{N_{\text{cfg}}\to\infty} \frac{1}{N_{\text{cfg}}} \sum_{n=1}^{N_{\text{cfg}}} a^3 \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}_{\text{src}})} \text{Tr}\left[\gamma_5 \left(D^d\right)^{-1}_{\tau_{\text{src}}+\tau,\mathbf{x},\tau_{\text{src}},\mathbf{x}_{\text{src}}}[U_n]\, \gamma_5 \left(D^b\right)^{-1}_{\tau_{\text{src}},\mathbf{x}_{\text{src}},\tau_{\text{src}}+\tau,\mathbf{x}}[U_n]\right].$$

Because we have $N_{\text{cfg}} < \infty$ [typically $O(10^2) - O(10^3)$], there is a statistical uncertainty. This uncertainty can also be reduced by averaging over multiple $(\tau_{\text{src}}, \mathbf{x}_{\text{src}})$.

The variance grows exponentially with τ, so one challenge is balancing statistical uncertainties and excited-state contamination.

Lattice gluon actions: discretization errors start at order a^2 or higher for all of them; generally under good control.

Commonly used **lattice fermion actions** suitable for light quarks ($ma \ll 1$):

- *Staggered (e.g. asqtad, HISQ):* has a chiral symmetry that prevents additive mass renormalization and yields automatic order-a improvement, but has the fermion doubling problem (e.g. multiple tastes of pions with different masses, requiring root of fermion determinant) and complicated symmetry properties.
- *Wilson/clover:* no fermion doubling, but no chiral symmetry. Order-a improvement requires tuning.
- *Twisted-mass:* no fermion doubling, automatic order-a improvement, but breaks flavor and parity symmetry.
- *Overlap or domain-wall (DWF):* no fermion doubling, continuum-like symmetries, automatic order-a improvement, most expensive.

In the continuum limit, full QCD with the correct continuum symmetries is obtained with all of these actions.

Common treatments of **heavy quarks on the lattice:**

- *Lattice HQET:* Discretization of continuum HQET with $\mathbf{v} = 0$. Continuum limit is possible when treating $1/m_Q$ corrections as insertions in correlation functions.
- *Lattice NRQCD:* Discretization of continuum NRQCD. Computations must be done with $am_Q > 1$.
- *Fermilab method/RHQ(Columbia/Tsukuba) action/Oktay-Kronfeld action:* Wilson-like action with one or more coefficients tuned to remove heavy-quark discretization errors. Can be used for any am_Q.
- *Use the same action as for the light quarks:* requires very fine lattices to keep $m_Q a < 1$ and typically an extrapolation in $1/m_Q$ to reach the physical bottom mass. Provides some currents that don't require renormalization $\rightarrow$ makes sub-percent precision possible!

The lattice spacing is changed by changing the bare gauge coupling. The quark masses are tuned by matching to hadron masses.

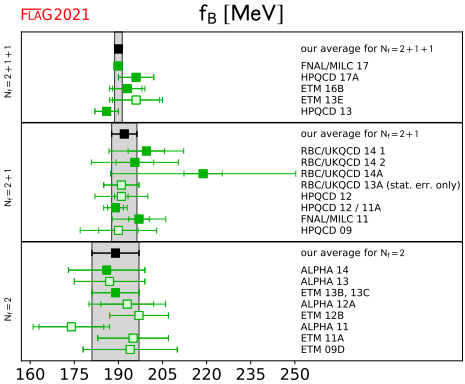
Matching of lattice and continuum weak-decay currents:

- *Fully perturbative:* typically used with NRQCD b quarks – typically leaves several percent systematic uncertainty
- *Mostly nonperturbative:* write $Z_\Gamma = \sqrt{Z_V^{QQ} Z_V^{qq}}\,\rho_\Gamma$ with nonperturbative Z_V^{QQ}, Z_V^{qq} and perturbative $\rho_\Gamma \approx 1$
- *Via intermediate regularization-independent nonperturbative schemes:* RI-SMOM, position-space schemes, ...
- *Use currents that don't require renormalization and Ward identities:* for example, with all-staggered quarks, determine Z_A of local (non-conserved) axial current using

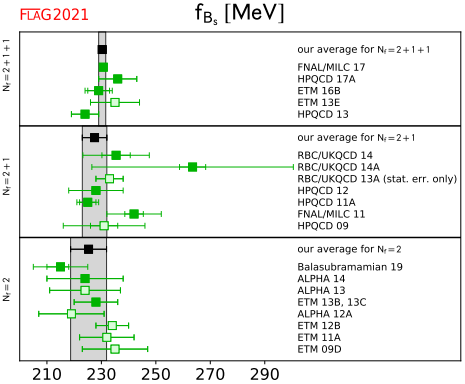
$$\langle\Omega|\underbrace{(m_{Q,0} + m_{q,0})P}_{\text{absolutely normalized}}|H\rangle = M_{\hat{H}} Z_A \langle\Omega|A^0|\hat{H}\rangle$$

$\rightarrow$ makes sub-percent precision possible!

The $B_{(s)}$ decay constants have been determined with sub-percent precision in 2017 (with the "use the same action as for the light quarks" approach); there is more work in progress (including for the $B^*_{(s)}$), but nothing final yet.

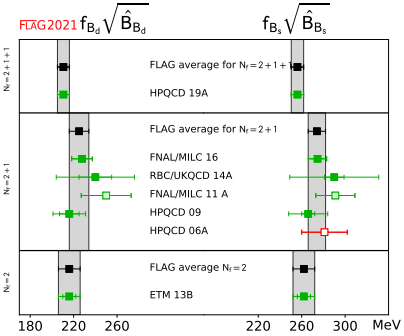


[fB.bib]



[fBs.bib]

The $\Delta B = 2$ four-quark-operator matrix elements have been determined with about 2.5% precision. There is more work in progress (including for $\Delta B = 0$), but nothing final yet.



[fBsqrtBB.bib]

I will focus on form factors for exclusive semileptonic b decays in the following.

Lattice calculations of inclusive decays were discussed at this conference by P. Gambino.

$b \to (u, d, s, c)$ semilept. form factors from lattice QCD: main references

Recent calculations with $N_f \geq 2+1$, journal articles (including preprints) only. ■ = discussed in this talk

Transition	Main references
$B \to \pi$	RBC/UKQCD 1501.05373, FNAL/MILC 1503.07839 (tensor FF: 1507.01618), JLQCD 2203.04938
$B \to K$	HPQCD 1306.2384, 2207.12468, FNAL/MILC 1509.06235
$B \to K^*$	Horgan, Liu, Meinel, Wingate, 1310.3722 (updated in in 1501.00367)
$B \to D$	FNAL/MILC 1503.07237, HPQCD 1505.03925
$B \to D^*$	FNAL/MILC 1403.0635*, 2105.14019, HPQCD 1711.11013*, 2304.03137
$B_s \to K$	HPQCD 1406.2279, RBC/UKQCD 1501.05373 (updated in 2303.11280), FNAL/MILC 1901.02561
$B_s \to K^*$	Horgan, Liu, Meinel, Wingate, 1310.3722 (updated in in 1501.00367)
$B_s \to \phi$	Horgan, Liu, Meinel, Wingate, 1310.3722 (updated in in 1501.00367)
$B_s \to D_s$	HPQCD 1906.00701
$B_s \to D_s^*$	HPQCD 1711.11013*, 1904.02046*, 2105.11433 (updated in 2304.03137)
$B_c \to D$	HPQCD 2108.11242
$B_c \to D_s$	HPQCD 2108.11242
$B_c \to J/\psi$	HPQCD 2007.06957
$\Lambda_b \to p$	Detmold, Lehner, Meinel, 1503.01421
$\Lambda_b \to \Lambda$	Detmold and Meinel, 1602.01399
$\Lambda_b \to \Lambda^*(1520)$	Meinel and Rendon, 2009.09313 (updated in 2107.13140)
$\Lambda_b \to \Lambda_c$	Detmold, Lehner, Meinel, 1503.01421 (tensor FFs in 1702.02243)
$\Lambda_b \to \Lambda_c^*(2595)$	Meinel and Rendon, 2103.08775 (updated in 2107.13140)
$\Lambda_b \to \Lambda_c^*(2625)$	Meinel and Rendon, 2103.08775 (updated in 2107.13140)

*at zero recoil only

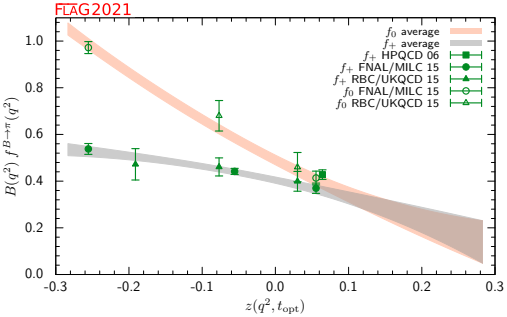
$B \to \pi$: 2023 FLAG web update

http://flag.unibe.ch

New calculation by JLQCD reported at FPCP 2022.
We recently included it in the FLAG average:

HPQCD 06	2+1 Asqtad, NRQCD b (not included in fit)
FNAL/MILC 15	2+1 Asqtad, Fermilab b
RBC/UKQCD 15	2+1 DWF, RHQ b
JLQCD 22	2+1 DWF, DWF b

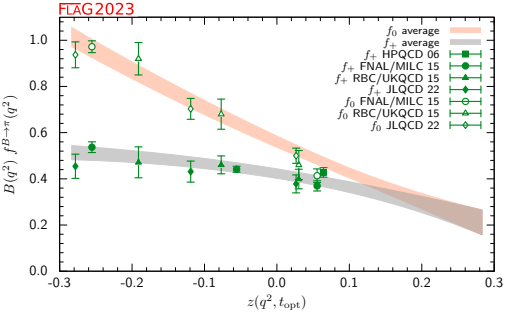
Old



$\chi^2/\text{dof} = 0.82$

[ff_Bpi_latt.bib]

New, including JLQCD 22



$\chi^2/\text{dof} = 3.63$
All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

Uncertainty of $a_0^{0,+}$ increased
Uncertainty of a_1^0, $a_{1,2}^+$ decreased

$B \to \pi$: 2023 FLAG web update

http://flag.unibe.ch

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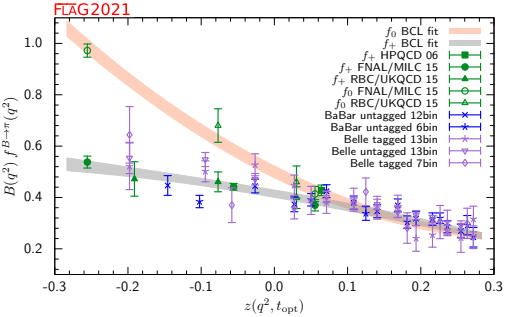
Old

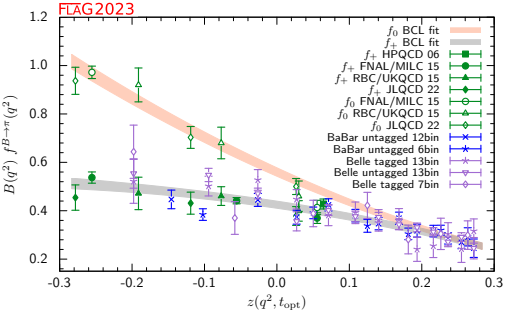


$\chi^2/\text{dof} = 1.41$
All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

$|V_{ub}| = 3.74(17)$

[ff_Bpi_latt+exp.bib]

New, including JLQCD 22



$\chi^2/\text{dof} = 1.88$
All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

$|V_{ub}| = 3.64(16)$

$B_s \to K$: new 2023 calculation by RBC/UKQCD

J. Flynn *et al.*, 2303.11280

The calculation uses $N_f = 2 + 1$ domain-wall fermions, RHQ b, and "mostly nonperturbative" renormalization.

The main changes compared to the 2015 RBC/UKQCD calculation are

- 1 new ensemble

	L/a	T/a	L_s	a^{-1}/GeV	am_l	$am_s^{\rm sea}$	$am_s^{\rm phys}$	M_π/MeV	# cfgs	# sources
C1	24	64	16	1.7848(50)	0.005	0.040	0.03224(18)	340	1636	1
C2	24	64	16	1.7848(50)	0.010	0.040	0.03224(18)	434	1419	1
M1	32	64	16	2.3833(86)	0.004	0.030	0.02477(18)	301	628	2
M2	32	64	16	2.3833(86)	0.006	0.030	0.02477(18)	363	889	2
M3	32	64	16	2.3833(86)	0.008	0.030	0.02477(18)	411	544	2
F1S	48	96	12	2.785(11)	0.002144	0.02144	0.02167(20)	268	98	24

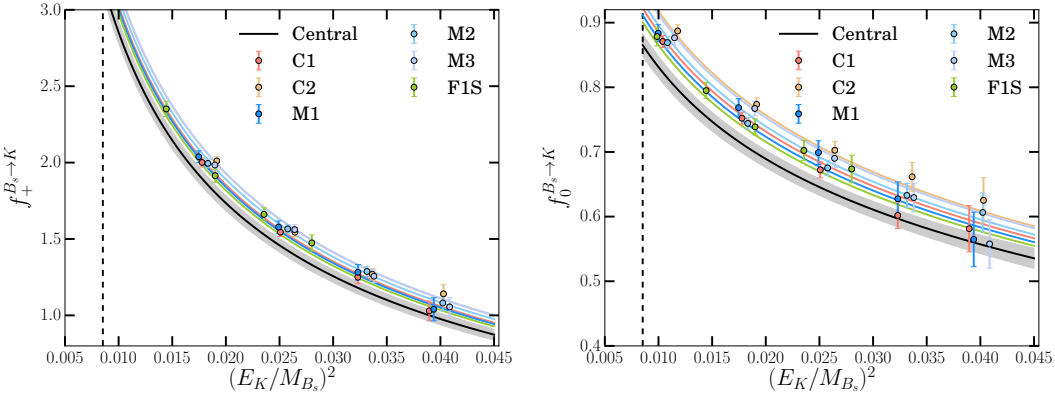
New

- New determination of lattice spacings, new tuning of valence m_s and of b-quark RHQ paramaters
- Chiral-continuum extrapolation performed directly for f_+, f_0, instead of $f_\parallel$, $f_\perp$
- Extrapolation to $q^2 = 0$ using new approach for dispersive bounds

$B_s \to K$: new 2023 calculation by RBC/UKQCD

J. Flynn *et al.*, 2303.11280

$$f_X^{B_s\to K}(M_\pi, E_K, a^2) = \frac{\Lambda}{E_K+\Delta_X}\left[c_{X,0}\left(1+\frac{\delta f(M_\pi^s)-\delta f(M_\pi^p)}{(4\pi f_\pi)^2}\right) + c_{X,1}\frac{\Delta M_\pi^2}{\Lambda^2} + c_{X,2}\frac{E_K}{\Lambda} + c_{X,3}\frac{E_K^2}{\Lambda^2} + c_{X,4}(a\Lambda)^2\right]$$



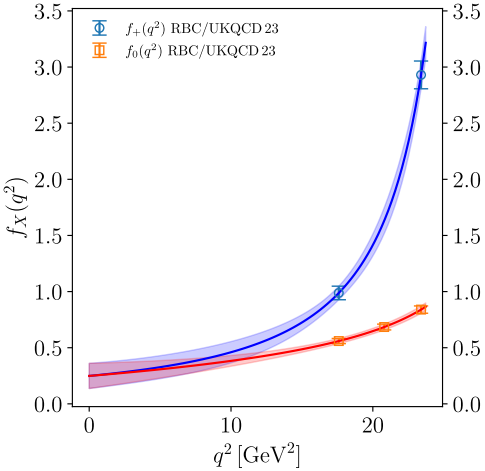
The pole mass differences $\Delta_+ = -42.1$ MeV and $\Delta_0 = 263$ MeV are specific for f_+ and f_0.

RBC/UKQCD 15 and FNAL/MILC 19 used fit models with the same poles for $f_\perp$ and $f_\parallel$.

$B_s \to K$: new 2023 calculation by RBC/UKQCD

J. Flynn *et al.*, 2303.11280

The extrapolation to the full kinematic range uses a generalization of the BGL z-expansion with dispersive bounds [J. Flynn, A. Jüttner, J. Tsang, 2303.11285; T. Blake, S. Meinel, M. Rahimi, D. van Dyk, arXiv:2205.06041; N. Gubernari, D. van Dyk, J. Virto, 2011.09813]



$$\Rightarrow \frac{\Gamma(B_s \to K\mu\nu)}{|V_{ub}|^2} = 6.4(2.4)\ \text{ps}^{-1}.$$

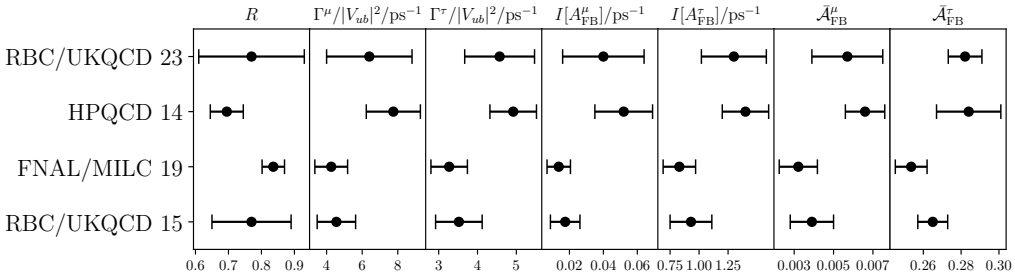
And, using $\mathcal{B}(B_s \to K\mu\nu)/\mathcal{B}(B_s \to D_s\mu\nu)$, $\mathcal{B}(B_s \to D_s\mu\nu)/\mathcal{B}(B \to D\mu\nu)$ from LHCb [2012.05143, 2001.03225] and $\mathcal{B}(B \to D\mu\nu)$ from PDG,

$$|V_{ub}| = 3.78(61) \times 10^{-3}.$$

$B_s \to K$: new 2023 calculation by RBC/UKQCD

J. Flynn *et al.*, 2303.11280

This figure compares predictions for decay rates an angular observables from different lattice calculations.



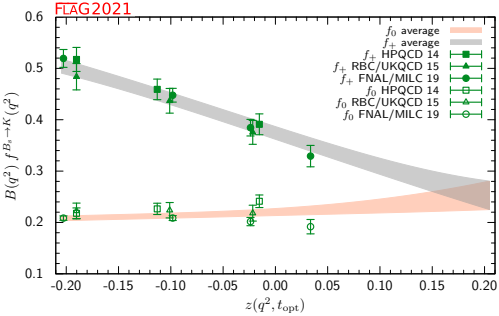
HPQCD 14 and RBC/UKQCD 23 performed the chiral/continuum extrapolation for f_+, f_0.

RBC/UKQCD 15 and FNAL/MILC 15 performed the chiral/continuum extrapolation for $f_\perp$, $f_\parallel$.

$B_s \to K$: my unofficial update of the FLAG average

Replacing RBC/UKQCD 15 by RBC/UKQCD 23

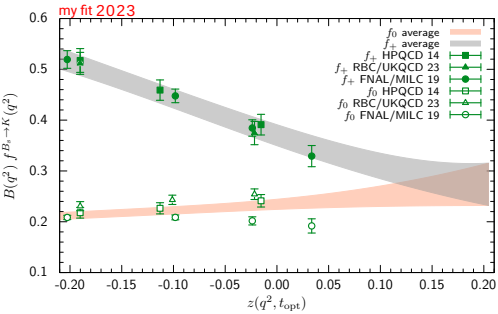
Old



$\chi^2/\text{dof} = 1.54$
All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

$$\frac{\Gamma(B_s \to K\mu\nu)}{|V_{ub}|^2} = 6.28(0.67)\ \text{ps}^{-1}$$

New



$\chi^2/\text{dof} = 3.82$
All uncertainties rescaled by $\sqrt{\chi^2/\text{dof}}$

$$\frac{\Gamma(B_s \to K\mu\nu)}{|V_{ub}|^2} = 6.5(1.1)\ \text{ps}^{-1}$$

$B \to K$: new 2022 HPQCD calculation W. Parrott, C. Bouchard, C. Davies, 2207.12468

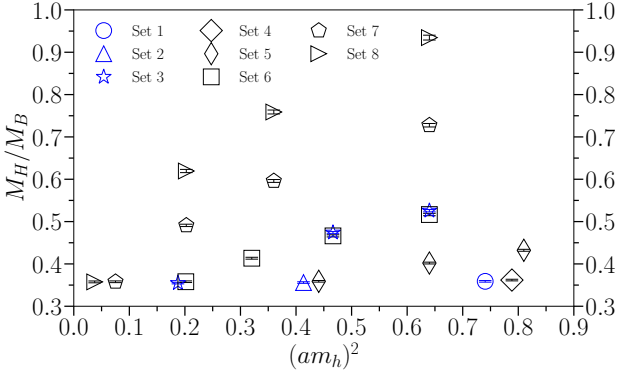
This calculation uses the MILC ensembles of gauge configurations with $N_f = 2+1+1$ highly improved staggered quarks (HISQ), and uses the HISQ action also for the b quark. The renormalization factors Z_V and Z_A were obtained using Ward identities, and Z_T using RI-SMOM.

Set	β	w_0/a	a (fm)	$N_x^3 \times N_t$	$n_{\rm cfg} \times n_{\rm src}$	$am_l^{\rm sea/val}$	$am_s^{\rm sea}$	$am_c^{\rm sea}$	$am_s^{\rm val}$	$am_c^{\rm val}$	$Z_T(m_b)$
1	5.8	1.1367(5)	0.15	$32^3 \times 48$	998×16	0.00235	0.0647	0.831	0.0678	0.8605	-
2	6.0	1.4149(6)	0.12	$48^3 \times 64$	985×16	0.00184	0.0507	0.628	0.0527	0.643	-
3	6.3	1.9518(7)	0.088	$64^3 \times 96$	620×8	0.00120	0.0363	0.432	0.036	0.433	1.0029(43)
4	5.8	1.1119(10)	0.15	$16^3 \times 48$	1020×16	0.013	0.065	0.838	0.0705	0.888	0.9493(42)
5	6.0	1.3826(11)	0.12	$24^3 \times 64$	1053×16	0.0102	0.0509	0.635	0.0545	0.664	0.9740(43)
6	6.3	1.9006(20)	0.09	$32^3 \times 96$	499×16	0.0074	0.037	0.440	0.0376	0.449	1.0029(43)
7	6.72	2.896(6)	0.059	$48^3 \times 144$	413×8	0.0048	0.024	0.286	0.0234	0.274	1.0342(43)
8	7.0	3.892(12)	0.044	$64^3 \times 192$	375×4	0.00316	0.0158	0.188	0.0165	0.194	1.0476(42)

Sets 1-3 have $m_l \approx m_l^{\rm phys}$
Sets 4-8 have $m_l = 0.2 m_s$

$B \to K$: new 2022 HPQCD calculation W. Parrott, C. Bouchard, C. Davies, 2207.12468

The calculation uses multiple heavy-quark masses m_h satisfying $am_h < 1$ on each ensemble. At $a \approx 0.044$ fm (Set 7), the heavy-light pseudoscalar meson mass reaches $\approx 0.94\ M_B$.



$B \to K$: new 2022 HPQCD calculation W. Parrott, C. Bouchard, C. Davies, 2207.12468

The dependence on kinematics, heavy-quark mass, light and strange masses, and lattice spacing is fitted with a modified z expansion.

$$f_0(q^2) = \frac{\mathcal{L}}{1 - \frac{q^2}{M^2_{H^*_{s0}}}} \sum_{n=0}^{N-1} a_n^0 z^n$$

$$f_+(q^2) = \frac{\mathcal{L}}{1 - \frac{q^2}{M^2_{H^*_s}}} \sum_{n=0}^{N-1} a_n^+ \left(z^n - \frac{n}{N}(-1)^{n-N} z^N \right)$$

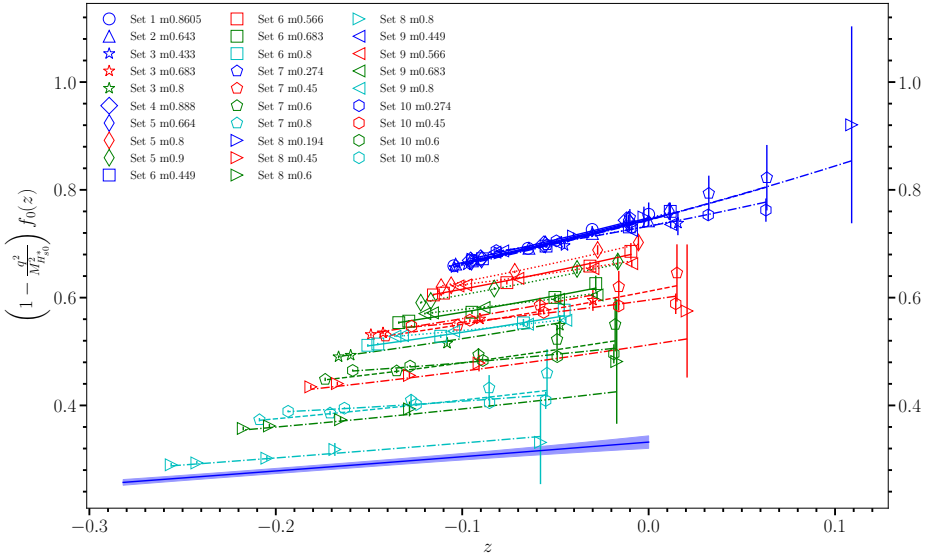
$$f_T(q^2) = \frac{\mathcal{L}}{1 - \frac{q^2}{M^2_{H^*_s}}} \sum_{n=0}^{N-1} a_n^T \left(z^n - \frac{n}{N}(-1)^{n-N} z^N \right)$$

This work sets $t_0 = 0$, so $q^2 = 0$ corresponds to $z = 0$.

$$\mathcal{L} = 1 - \frac{9g^2}{8} x_\pi \left(\log x_\pi + \delta_{FV} \right) - \left(\frac{1}{2} + \frac{3g^2}{4} \right) x_K \log x_K - \left(\frac{1}{6} + \frac{g^2}{8} \right) x_\eta \log x_\eta$$

$$a_n^{0,+,T} = \left(\frac{M_D}{M_H} \right)^{\zeta_n} \left(1 + \rho_n^{0,+,T} \log \left(\frac{M_H}{M_D} \right) \right) \times (1 + \mathcal{N}_n^{0,+,T}) \times \sum_{i,j,k,l=0}^{N_{ijkl}-1} d_{ijkln}^{0,+,T} \left(\frac{\Lambda_{\rm QCD}}{M_H} \right)^i \left(\frac{a m_h^{\rm val}}{\pi} \right)^{2j} \times \left(\frac{a \Lambda_{\rm QCD}}{\pi} \right)^{2k} (x_\pi - x_\pi^{\rm phys})^l$$

$B \to K$: new 2022 HPQCD calculation W. Parrott, C. Bouchard, C. Davies, 2207.12468



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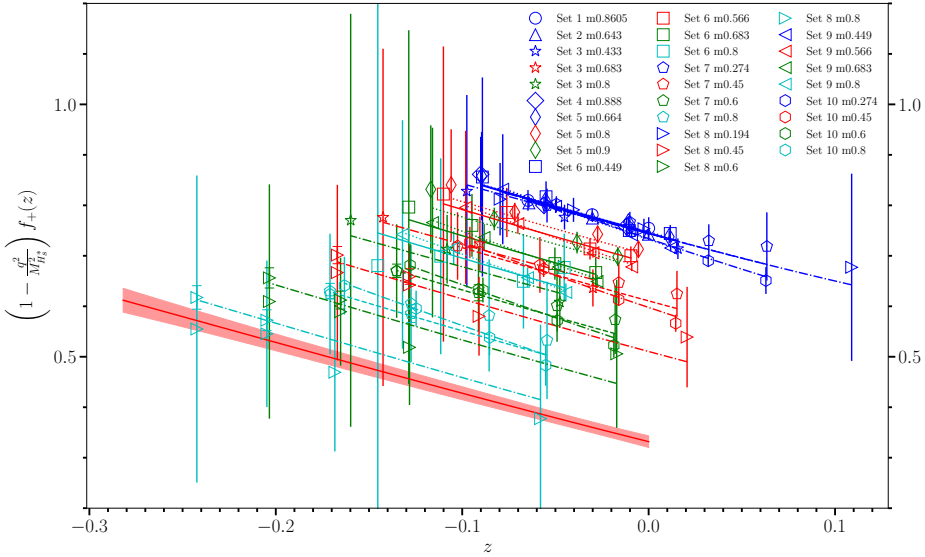

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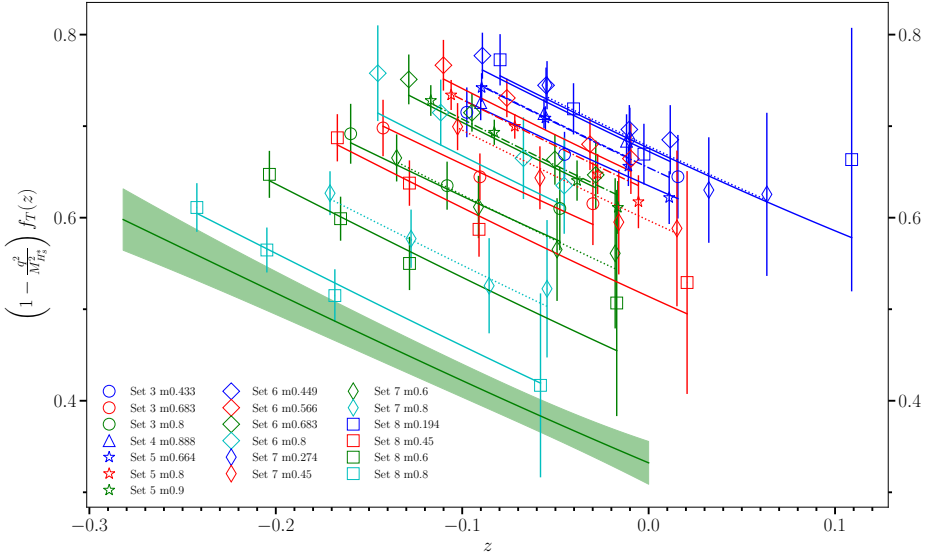

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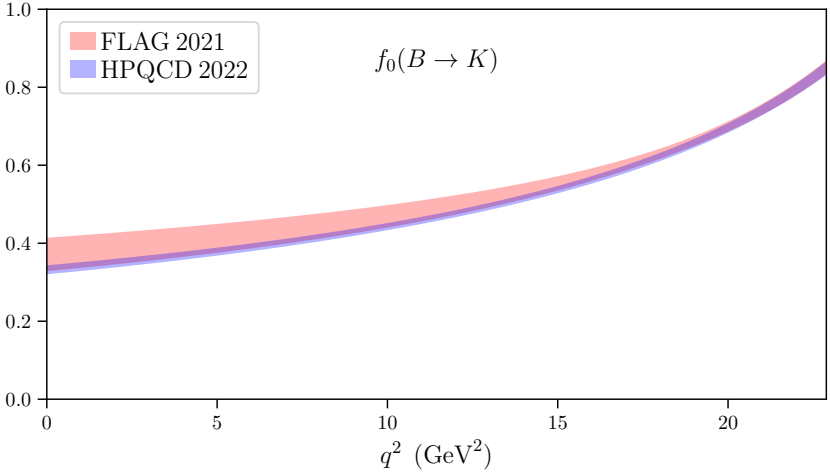
Shown here is my comparison to the $N_f = 2 + 1$ FLAG average (which is based on HPQCD 13 with NRQCD+asqtad and FNAL/MILC 16 with Fermilab meth.+asqtad).



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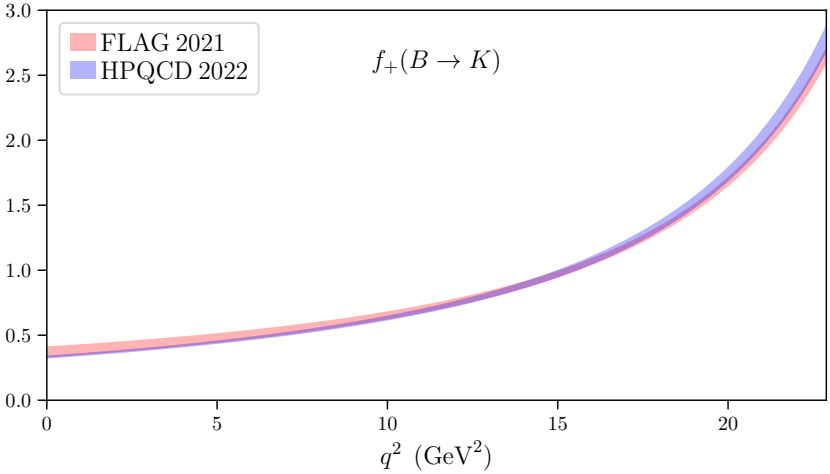

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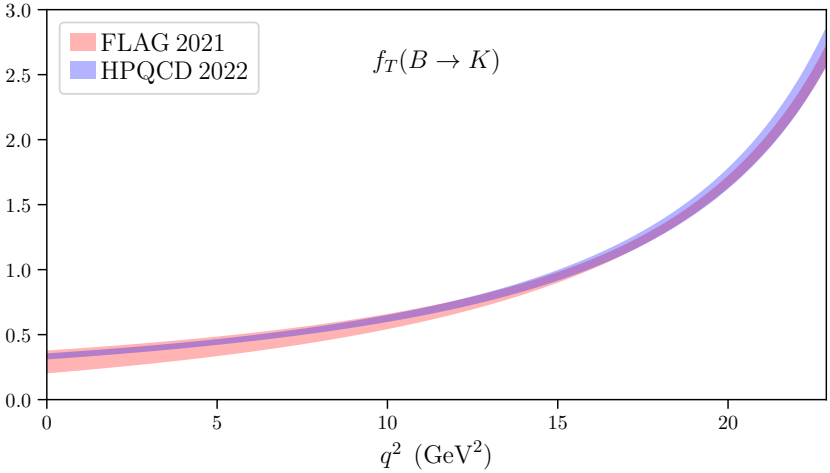
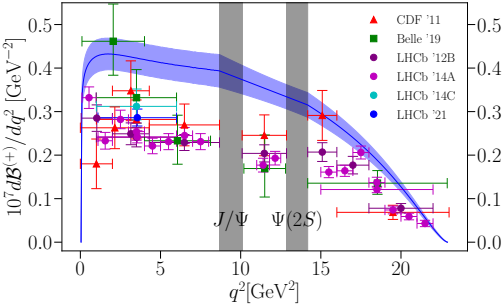
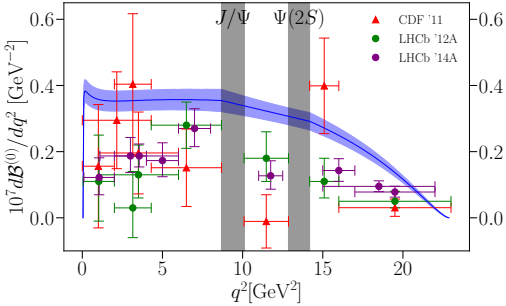


$B \to K$: new 2022 HPQCD calculation W. Parrott, C. Bouchard, C. Davies, 2207.13371

The blue curves below are HPQCD's Standard-Model predictions for the differential branching fractions of $B^+ \to K^+\ell^+\ell^-$ (left) and $B^0 \to K^0\ell^+\ell^-$ (right), averaged over $\ell = e, \mu$.

The experimental results Belle '19, LHCb '14C and LHCb '21 are for $\ell = e$, the others are for $\ell = \mu$. The LHCb electron result has since moved farther downward [arXiv:2212.09153].





$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137

This calculation also uses the MILC ensembles of gauge configurations with $N_f = 2 + 1 + 1$ highly improved staggered quarks (HISQ), and uses the HISQ action also for the b quark, for a range of lower-than-physical masses. Again, the renormalization factors Z_V and Z_A were obtained using Ward identities, and Z_T using RI-SMOM.

Set	a (fm)	$N_x \times N_t$	am_{l0}	am_{s0}	am_{c0}	M_π (MeV)	$n_{\rm cfg} \times n_{\rm t}$
1	0.0902	32×96	0.0074	0.037	0.440	316	1000×16
2	0.0592	48×144	0.0048	0.024	0.286	329	500×4
3	0.0441	64×192	0.00316	0.0158	0.188	315	375×4
4	0.0879	64×96	0.0012	0.0363	0.432	129	600×8
5	0.0568	96×192	0.0008	0.0219	0.2585	135	100×4

Set	$am_h^{\rm val}$	$am_c^{\rm val}$	$am_s^{\rm val}$
1	$0.65, 0.725, 0.8$	0.449	0.0376
2	$0.427, 0.525, 0.65, 0.8$	0.274	0.0234
3	$0.5, 0.65, 0.8$	0.194	0.0165
4	$0.65, 0.725, 0.8$	0.433	0.036
5	$0.427, 0.525, 0.65, 0.8$	0.2585	0.0165

$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137

The dependence on kinematics, heavy-quark mass, light and strange masses, and lattice spacing is fitted with a modified expansion in $(w-1)$.

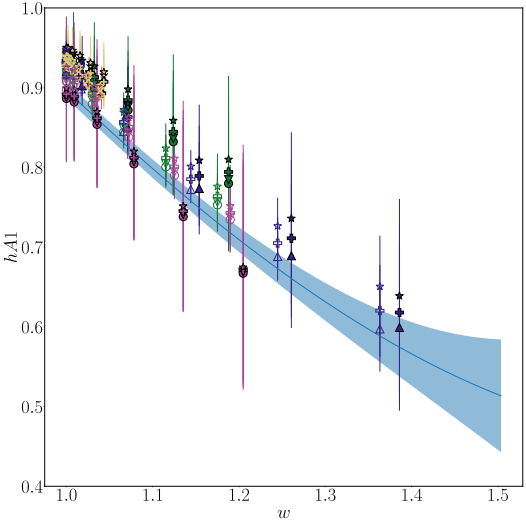
$$F^{Y^{(s)}}(w) = \sum_{n=0}^{3} a_n^Y (w-1)^n \mathcal{N}_n^Y + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2}\left(\log s_{SU(3)}^{Y^{(s)}} - \log s_{SU(3)\mathrm{phys}}^{Y}\right) + \tilde{a}^Y\left(\left(\frac{M_\pi^{\mathrm{phys}}}{\lambda_\chi}\right)^2 - \left(\frac{M_{\pi(K)}}{\lambda_\chi}\right)^2\right)$$

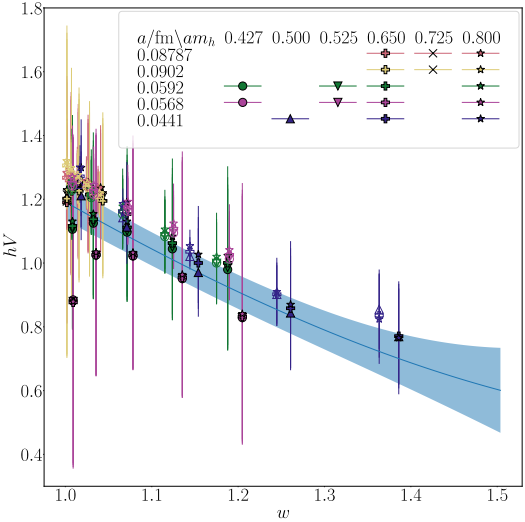
$$a_n^Y = \sum_{j,k,l=0}^{3} b_n^{Y,jkl}\Delta_h^{(j)}\left(\frac{am_c^{\mathrm{val}}}{\pi}\right)^{2k}\left(\frac{am_h^{\mathrm{val}}}{\pi}\right)^{2l}$$

$$\Delta_h^{(j\neq 0)} = \left(\frac{\Lambda}{M_{H_s}}\right)^j - \left(\frac{\Lambda}{M_{B_s}^{\mathrm{phys}}}\right)^j$$

$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137





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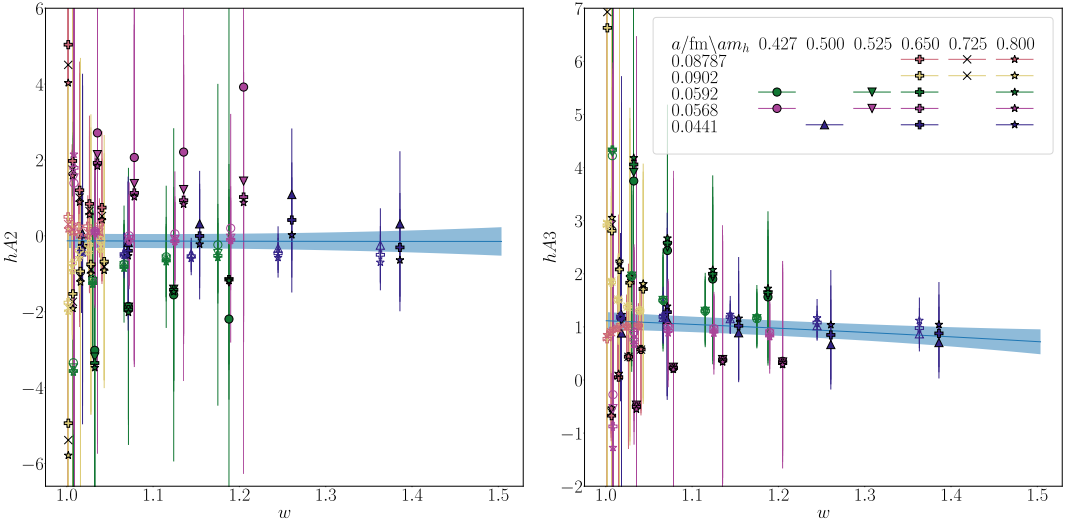
J. Harrison, C. Davies, 2304.03137



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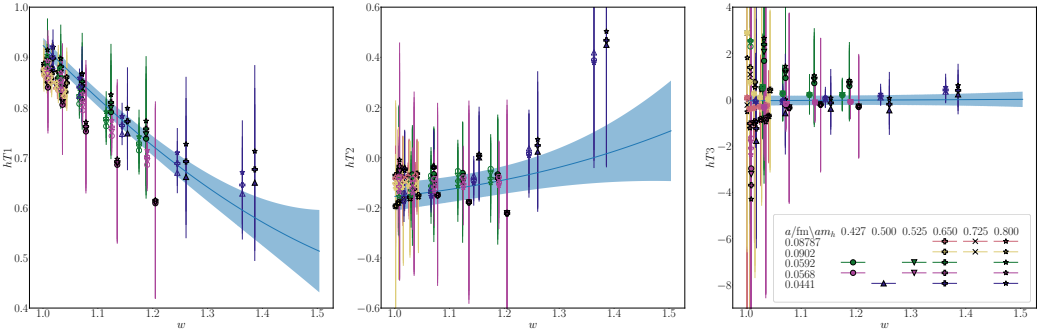
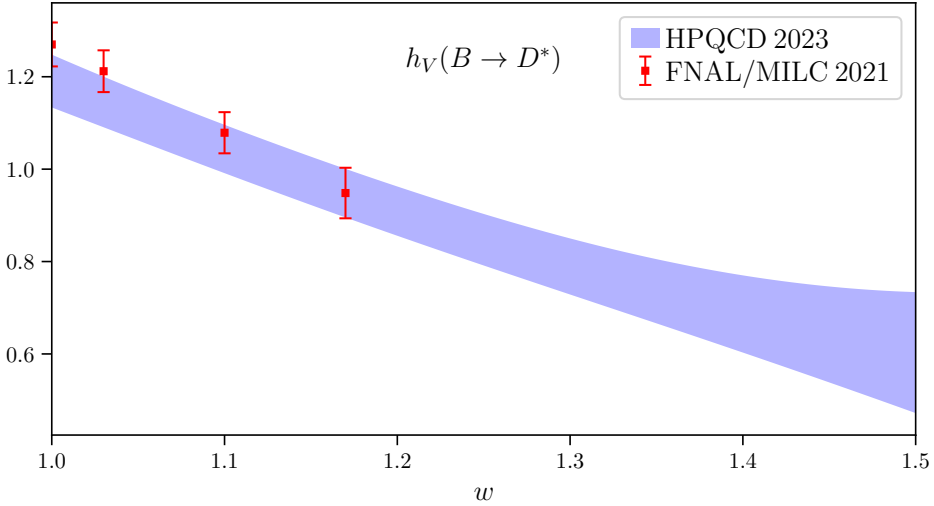


$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137

Shown here is a comparison with the 2021 FNAL/MILC results (red points).



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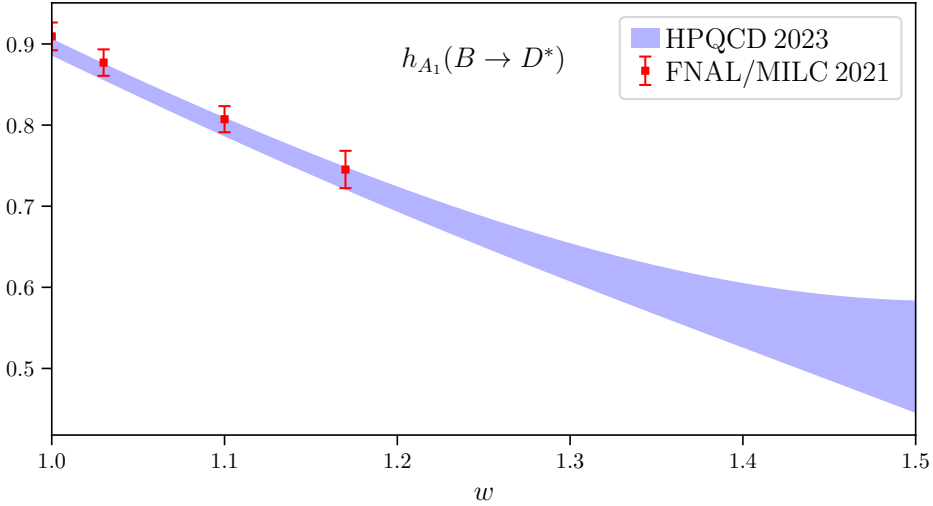

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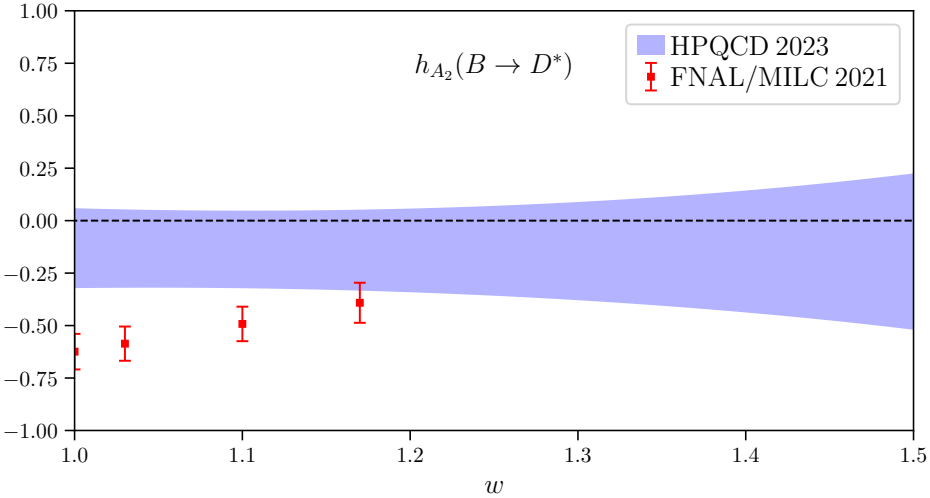

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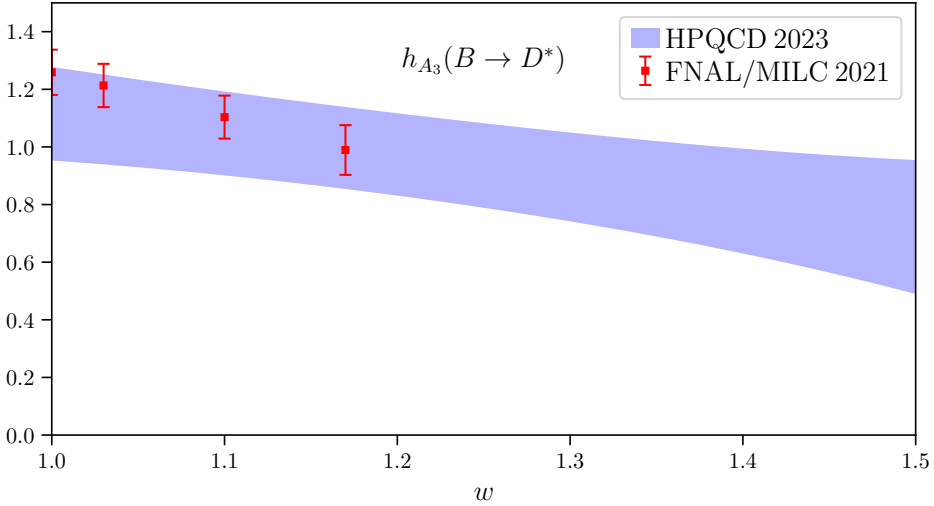
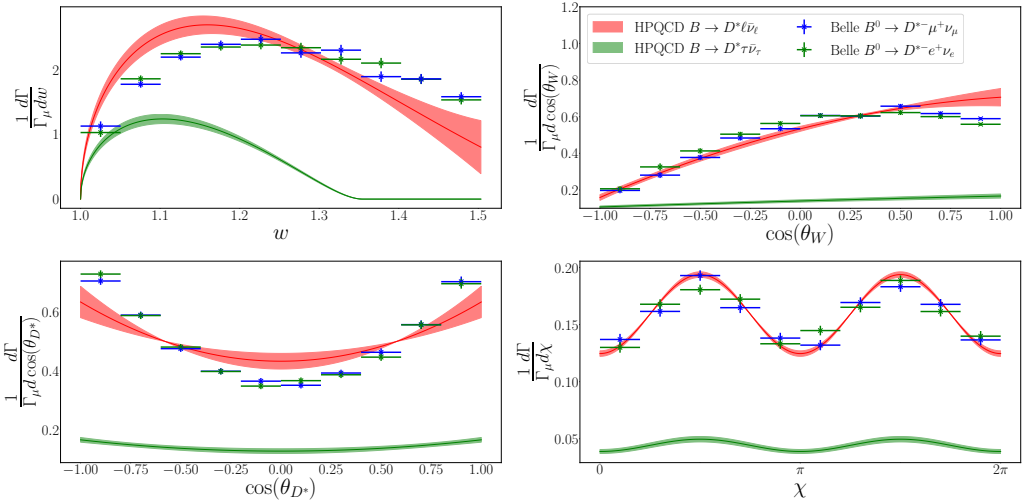


$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137

HPQCD's SM predictions for the $B \to D^* \ell \bar{\nu}$ differential decay rates and angular observables (red curves) are in tension with the 2018 Belle results [1809.03290]



$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137

From a combined fit with the Belle differential distributions [1809.03290], the authors obtain

$$V_{cb} = 39.31(54)_{\text{exp}}(51)_{\text{latt}} \times 10^{-3}.$$

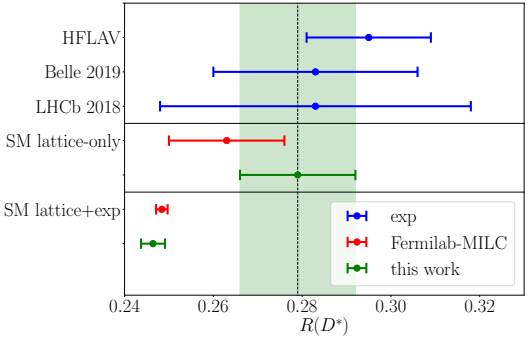
Using instead the total rate gives the higher value

$$V_{cb} = 44.2(1.7)_{\text{latt}}(0.5)_{\text{exp}} \times 10^{-3}.$$

$B \to D^*$: new 2023 HPQCD calculation

J. Harrison, C. Davies, 2304.03137

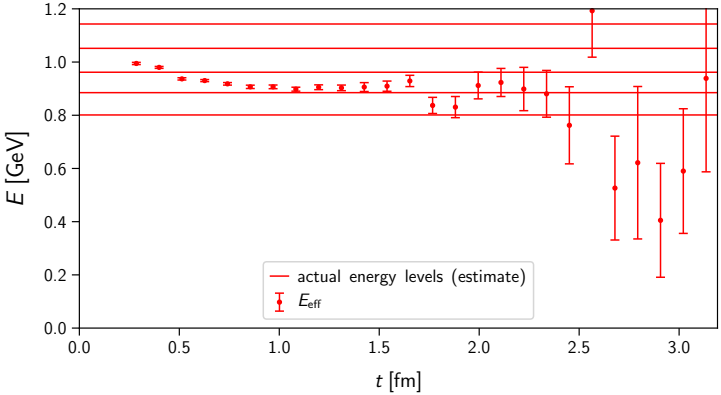
This figure shows measurements (blue) and SM predictions (red=FNAL/MILC, green=HPQCD) of $R(D^*) = \frac{\Gamma(B \to D^* \tau \bar{\nu})}{\Gamma(B \to D^* \mu \bar{\nu})}$.



Outlook: B semileptonic decays with two-hadron/resonance final states

Past lattice calculations of the $B \to K^*$ and $B \to \rho$ form factors were performed with heavy quark masses for which the K^*/ρ is stable, or neglected the unstable nature of the K^*/ρ.

What happens at low quark masses? Here is the effective-energy plot of a zero-momentum two-point correlation function of a simple quark-antiquark field $O_{K^*} = \bar{s}\gamma^j d$ computed on a 5.5 fm lattice with approximately physical quark masses (RBC/UKQCD 48I ensemble):



*now denoting Euclidean time by t

$E_{\text{eff}}(t) = a^{-1} \log[C(t)/C(t+a)]$

Outlook: B semileptonic decays with two-hadron/resonance final states

The correct way to study an unstable K^*/ρ in a finite volume is known, building upon seminal work by Lüscher and Lellouch. See, e.g., the review article R. A. Briceño, J. J. Dudek, R. D. Young, `arXiv:1706.06223`.

With collaborators Luka Leskovec, Marcus Petschlies, *et al.*, we are working on lattice calculations of the $B/D \to \pi\pi\ell\bar{\nu}$ ($I = 1$ P-wave only) and $B/D \to K\pi\ell\bar{\nu}/K\pi\ell\bar{\ell}$ ($I = \frac{1}{2}$ P-wave and S-wave) form factors.

I will show some preliminary results for the $B \to \pi\pi(I = 1, L = 1)\ell\bar{\nu}$ vector form factor.

Outlook: B semileptonic decays with two-hadron/resonance final states

- Gauge configurations generated by the JLab/W&M/LBNL/LANL/MIT lattice groups with clover-improved Wilson light and strange quarks
- RHQ b and c quarks
- Currently mostly nonperturbative renormalization with tree-level ρ_Γ, but considering other options

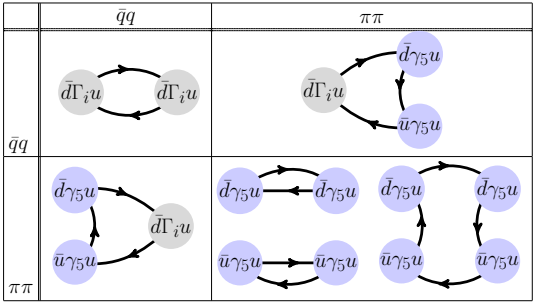
Ensemble	$N_s^3 \times N_t$	a (fm)	m_π (MeV)	Computation of correlation functions
C13	$32^3 \times 96$	≈ 0.114	≈ 320	Complete
D6	$48^3 \times 96$	≈ 0.088	≈ 180	Complete
D5	$32^3 \times 64$	≈ 0.088	≈ 280	Planned
E5	$48^3 \times 128$	≈ 0.073	≈ 270	In progress

The results shown in the following are from C13.

Outlook: $B \to \pi\pi(I=1, L=1)\ell\bar{\nu}$ form factors

L. Leskovec, S. Meinel, M. Petschlies *et al.*, 2212.08833 and *in preparation*

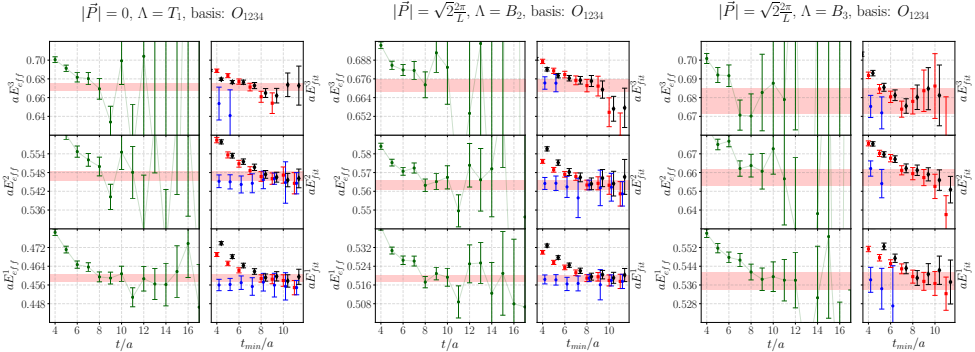
Step 1: compute two-point correlation matrix



Outlook: $B \to \pi\pi(I=1, L=1)\ell\bar{\nu}$ form factors

L. Leskovec, S. Meinel, M. Petschlies *et al.*, 2212.08833 and *in preparation*

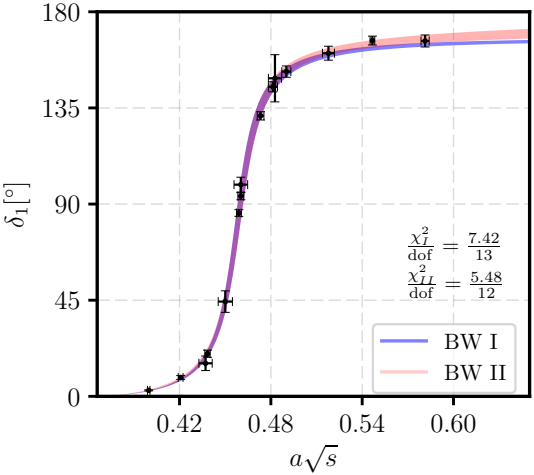
Step 2: extract finite-volume energy levels from time dependence



Outlook: $B \to \pi\pi(I=1, L=1)\ell\bar{\nu}$ form factors

L. Leskovec, S. Meinel, M. Petschlies *et al.*, 2212.08833 and *in preparation*

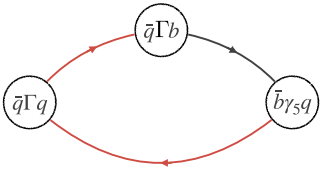
Step 3: determine $\pi\pi$ scattering amplitude from finite-volume effects in energy levels using the Lüscher quantization condition

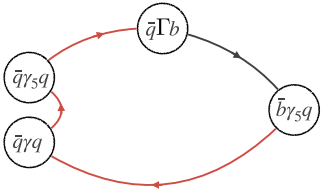


Outlook: $B \to \pi\pi(I = 1, L = 1)\ell\bar{\nu}$ form factors

L. Leskovec, S. Meinel, M. Petschlies *et al.*, 2212.08833 and *in preparation*

Step 4: compute three-point correlation matrix

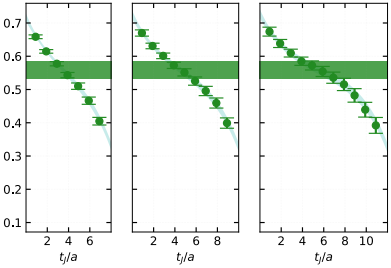




Outlook: $B \to \pi\pi(I = 1, L = 1)\ell\bar{\nu}$ form factors

L. Leskovec, S. Meinel, M. Petschlies *et al.*, 2212.08833 and *in preparation*

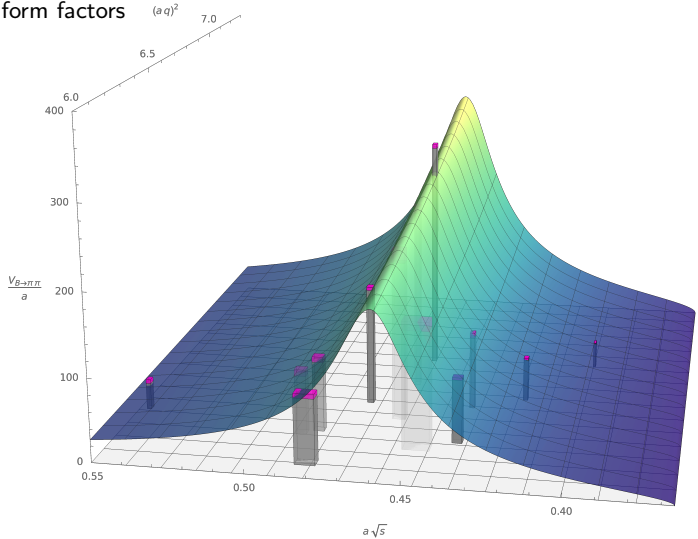
Step 5: fit projected three-point functions to extract finite-volume $B \to \pi\pi$ matrix elements



Outlook: $B \to \pi\pi(I=1, L=1)\ell\bar{\nu}$ form factors

L. Leskovec, S. Meinel, M. Petschlies *et al.*, 2212.08833 and *in preparation*

Step 6: use Lellouch-Lüscher formalism to obtain infinite-volume $B \to \pi\pi$ form factors, and, from pole residues, $B \to \rho$ form factors



Summary

- Recently, new lattice calculations of the $B \to \pi$, $B \to K$, $B_s \to K$, and $B \to D^*$ form factors have been published, with significant impact on flavor physics. There are some tensions between the results of different groups that need to be understood. Possible origins include the assumptions in the chiral/continuum/kinematic extrapolations and underestimated excited-state contamination. More calculations by several groups are underway.

- Lattice calculations of $B/D \to \pi\pi\ell\bar{\nu}$ and $B/D \to K\pi\ell\bar{\nu}/K\pi\ell\bar{\ell}$ local form factors are possible (for not too large two-hadron invariant mass) and are underway.

- There are many other exciting recent lattice results that I could not cover, for example in nucleon physics, kaon physics, charm physics, QED effects, hadronic contributions to muon $g-2$, inclusive decays.