Kaon Decays: Standard Model Tests

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Kaon decays

Important historical role in the shapping of the SM

Charged Currents

• Semi-leptonic

— Cabibbo universality

N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963)

— $V - A$, $\Delta S = \Delta Q$

— form factors

— CVC, PCAC, current algebra, $\pi\pi$ scattering ($K_{\ell 4}$)

L- M. Chounet, J.-M. Gaillard, M. K. Gaillard, Phys. Rep. 4, 199 (1972)

• Non-leptonic

— $K \to \pi\pi$, $K \to \pi\pi\pi$ $\longrightarrow$

Neutral Currents

— FCNC (e.g. $K \to \mu^+\mu^-$) $\longrightarrow$ GIM, charm, CKM

S. L. Glashow, J. Iliopoulos, L. Maiani, Phys. Rev. D 2, 1285 (1970)

M. Kobayashi, T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)

— CP violation (indirect)

J. H. Christenson, J. W. Cronin, V. L. Fitch, R. Turlay, Phys. Rev. Lett. 13, 138 (1964)

OUTLINE

Kaon decays

Still play an important role today as tests of the SM

Charged Currents

- Semi-leptonic

– First-row CKM unitarity

- Non-leptonic

– Direct CPV (ϵ'/ϵ)

Neutral Currents

- Short-distance dominated

– $K \to \pi\nu\bar{\nu}$

- Long-distance dominated

– $K \to \pi\gamma^*$

FIRST-ROW CKM UNITARITY

- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1\ ?$ $\quad |V_{ub}|^2 \sim 1.5 \cdot 10^{-5}$ test at 15ppm
- required by SM (renormalizability)

Kaon decays provide information on V_{ud} and V_{us} from two sources

$$\Gamma_{K_{\ell 3}} \propto |V_{us} f_+(0)|^2 \qquad \frac{\Gamma(K_{\mu 2})}{\Gamma(\pi_{\mu 2})} \propto \left|\frac{V_{us}}{V_{ud}} \frac{F_K}{F_\pi}\right|^2$$

- $f_+(q^2)/f_+(0)$ from data
- $f_+(0)$ from lattice QCD
- Radiative corrections to $K_{\ell 3}$

V. Cirigliano, MK, H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 23, 121 (2002)

V. Cirigliano, M. Giannotti, H. Neufeld, JHEP 11, 006 (2008)

C. Y. Seng, D. Galviz, M. Gorchtein, U. G. Meißner, Phys. Lett. B 820, 136522 (2021); JHEP 11, 172 (2021); JHEP 07, 071 (2022)

- IB corrections to $K_{\ell 3}$

J. Gasser, H. Leutwyler, Nucl. Phys. B 250, 517 (1985)

V. Cirigliano, MK, H. Neufeld, H. Rupertsberger, P. Talavera, Eur. Phys. J. C 23, 121 (2002)

- F_K/F_π from lattice QCD

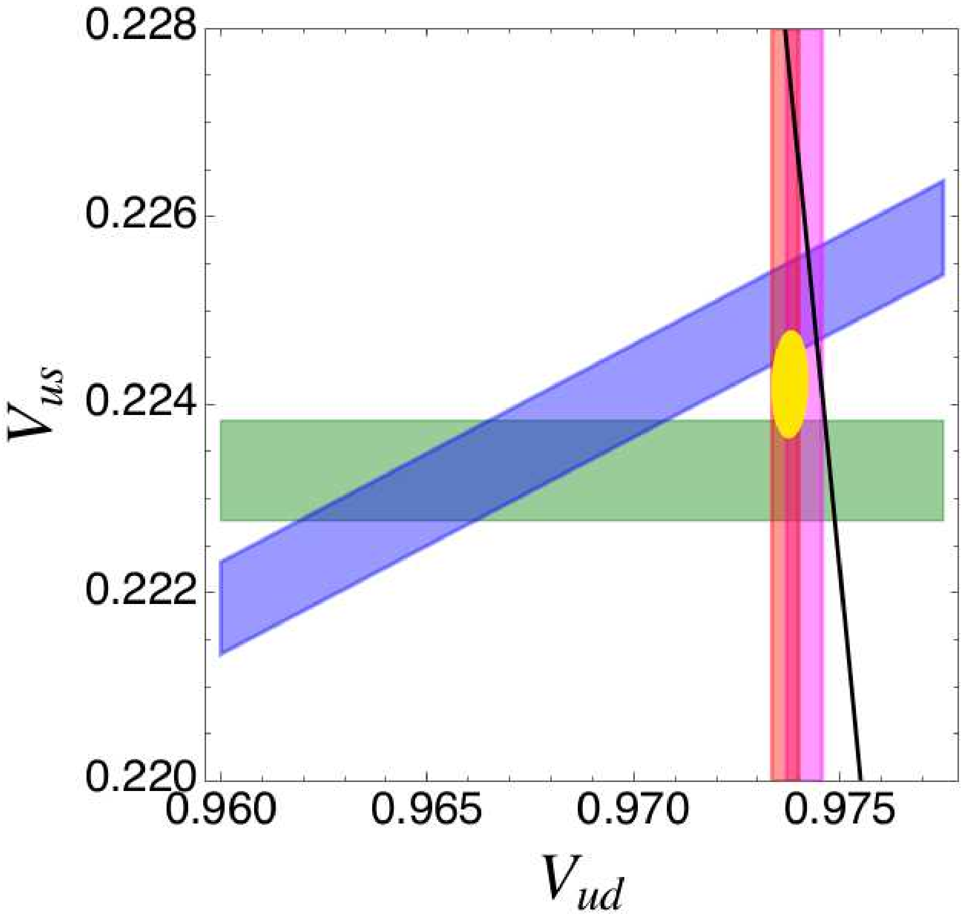
R. J. Dowdall, C. T. H. Davies, G. P. Lepage, C. McNeile, Phys. Rev. D 88, 074504 (2013)

A. Bazavov et al., Phys. Rev. D 98, 074512 (2018)

N. Miller et al, Phys. Rev. D 102, 034507 (2020)

C. Alexandrou et al. [Extended Twisted Mass], Phys. Rev. D 104, 074520 (2021)

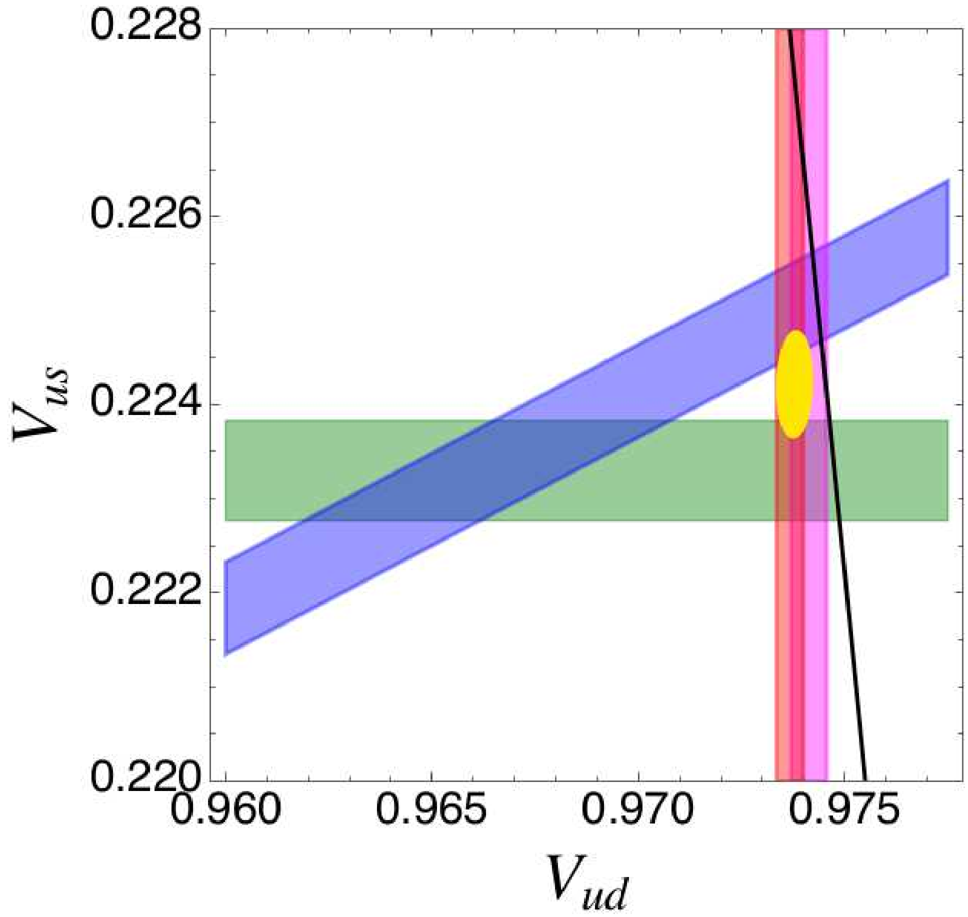
FIRST-ROW CKM UNITARITY



V. Cirigliano, A. Crivellin, M. Hoferichter, M. Moulson, Phys. Lett. B 838, 137748 (2023)

$$
\begin{aligned}
\Delta^{(1)}_{\rm CKM} &= |V^{\beta}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1 = -0.00176(56) \quad [-3.1\sigma] \\
\Delta^{(2)}_{\rm CKM} &= |V^{\beta}_{ud}|^2 + |V^{K_{\ell 3}/K_{\ell 2};\beta}_{us}|^2 - 1 = -0.00098(58) \quad [-1.7\sigma] \\
\Delta^{(3)}_{\rm CKM} &= |V^{K_{\ell 3}/K_{\ell 2};K_{\ell 3}}_{ud}|^2 + |V^{K_{\ell 3}}_{us}|^2 - 1 = -0.0164(63) \quad [-2.6\sigma]
\end{aligned}
$$

FIRST-ROW CKM UNITARITY



- Tension between V_{us}, V_{ud} from kaon decays and V_{ud} from β decay, whether Fermi transitions or n
- Tensions among the data for kaon branching fractions (large scale factors in PDG fit)
- Are the unitarity defects real?
- Possibility to perform new measurements of some Br's

FIRST-ROW CKM UNITARITY

	current	$K_{\mu 3}/K_{\mu 2}$ BR at 5% $+2\sigma$	$K_{\mu 3}/K_{\mu 2}$ BR at 5% -2σ	$K_{\mu 3}/K_{\mu 2}$ BR at 2% $+2\sigma$	$K_{\mu 3}/K_{\mu 2}$ BR at 2% -2σ
$\Delta^{(1)}_{\rm CKM}$	-3.1σ	-2.9σ	-3.3σ	-2.7σ	-3.5σ
$\Delta^{(2)}_{\rm CKM}$	-1.7σ	-1.9σ	-1.5σ	-2.0σ	-1.4σ
$\Delta^{(3)}_{\rm CKM}$	-2.6σ	-1.9σ	-3.2σ	-1.4σ	-3.8σ

V. Cirigliano, A. Crivellin, M. Hoferichter, M. Moulson, Phys. Lett. B 838, 137748 (2023)

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\Delta^{(3)}_{\rm CKM} &= |V_{ud}^{K_{\ell 3}/K_{\ell 2};K_{\ell 3}}|^2 + |V_{us}^{K_{\ell 3}}|^2 - 1 = -0.0164(63) \quad [-2.6\sigma]
\end{aligned}$$

DIRECT CPV: ϵ'/ϵ

Measured at $\sim 15\%$ precision

$$(\epsilon'/\epsilon)^{\mathrm{WA}}_{\mathrm{exp}} = 16.6(2.3) \cdot 10^{-4}$$

NA48 Coll., J. Batley et al., Phys. Lett. B 544, 97 (2002)

KTeV Coll., A. Alavi-Harari et al., Phys. Rev. D 67, 012005 (2003)

KTeV Coll., E. Abouzaid et al., Phys. Rev. D 83, 092001 (2011)

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Theory? $\longrightarrow$ requires non-perturbative evaluation of matrix elements

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Theory? $\longrightarrow$ requires non-perturbative evaluation of matrix elements

Lattice QCD

$$(\epsilon'/\epsilon)_{\mathrm{LQCD}} = 21.7(8.4) \cdot 10^{-4}$$

RBC-UKQCD Coll., R. Abbott et al., Phys. Rev. D 102, 054509 (2020)

– Uncertainty dominated by systematics, no IB included so far

$$(8.4) \longrightarrow (2.6)_{\mathrm{stat}}(6.2)_{\mathrm{syst}}(5)_{\mathrm{IB}}$$

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Theory? $\longrightarrow$ requires non-perturbative evaluation of matrix elements

Large N_c limit $\longrightarrow$ factorization of matrix elements

$$\langle\pi\pi|Q_i(\nu)|K\rangle = B_i(\nu)\langle\pi\pi|Q_i(\nu)|K\rangle^\infty \qquad B_i(\nu) = 1 + \frac{b_i(\nu)}{N_c}$$

$$\epsilon' = \frac{1}{\sqrt{2}} e^{i(\chi_2-\chi_0-\frac{\pi}{2})}\frac{{\rm Re}A_2}{{\rm Re}A_0}\cdot\frac{{\rm Im}A_0}{{\rm Re}A_0}\underbrace{\left[1-\frac{{\rm Re}A_0}{{\rm Re}A_2}\cdot\frac{{\rm Im}A_2}{{\rm Im}A_0}\right]}_{\equiv x} \qquad x \sim 1 - \frac{1}{2}\frac{B_8^{3/2}}{B_6^{1/2}}$$

$$x = \frac{1}{2} - \frac{1}{3}(b_8 - b_6) + {\rm IB}$$

Sensitive to values of b_6, b_8 and IB, large cancellations possible

H. Gisbert, [arXiv:2012.09483 [hep-ph]]

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Theory? $\longrightarrow$ requires non-perturbative evaluation of matrix elements

Need to go beyond large N_c limit

- Chiral perturbation theory
  - Importance of $\pi\pi$ final-state interactions
  - Many low-energy constants not known $\longrightarrow$ use factorization (i.e. large-N_c limit)
  - IB corrections included

- Dual QCD model
  - Replace quark-gluon evolution by meson (0^- and 1^-) evolution below 1 GeV
  - $1/N_c$ corrections due to loops, loop integrals with a cut-off scale
  - matching to scale dependence in the C_i's
  - IB corrections included

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KTeV Coll., E. Abouzaid et al., Phys. Rev. D 83, 092001 (2011)

	$B_6^{1/2}$	$B_8^{3/2}$	$\hat{\Omega}_{\text{eff}}$	ϵ'/ϵ
RBC-UKQCD	$1.49(25)$	$0.85(5)$	$--$	$21.7(8.4) \cdot 10^{-4}$
ChPT	$1.35(20)$	$0.55(20)$	$17(9) \cdot 10^{-2}$	$14(5) \cdot 10^{-4}$
Dual QCD	≤ 0.6	$0.80(10)$	$29(7) \cdot 10^{-2}$	$5(2) \cdot 10^{-4}$

ChPT:

E. Pallante, A. Pich, Phys. Rev. Lett. 84, 2568 (2000); Nucl.Phys.B 592, 294 (2001)
E. Pallante, A. Pich, I. Scimemi, Nucl. Phys. B **617**, 441 (2001)
V. Cirigliano, H. Gisbert, A. Pich, A. Rodríguez-Sánchez, JHEP 02, 032 (2020)

Dual QCD:

W. A. Bardeen, A. J. Buras, J. M. Gérard, Phys. Lett. B 192, 138 (1987); B 211, 343 (1988); Eur. Phys. J. C 74, 2871 (2014)

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KTeV Coll., E. Abouzaid et al., Phys. Rev. D 83, 092001 (2011)

Progress requires either evaluation of low-energy constants beyond the large-N_c limit and/or

reduction of systematic uncertainties in LQCD (e.g. including IB)

$\longrightarrow$ strategies to reduce error below 30% (10%) in 5 (10) years

T. Blum et al., RBC-UKQCD Coll., arXiv:2203.10998 [hep-lat].

FCNC: $K \to \pi\nu\bar{\nu}$, $K \to \pi\ell^+\ell^-$

Rare kaon decays proceed through FCNC

$\longrightarrow$ they are suppressed in the SM

$\longrightarrow$ they provide an interesting window into new physics

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$\longrightarrow$ requires reliable prediction of SM contribution

- SD dominated rare decays $\Longrightarrow K \to \pi\nu\bar{\nu}$, $K \to \pi\pi\nu\bar{\nu}$

- LD dominated rare decays $\Longrightarrow K \to \pi\ell^+\ell^-$, $K \to \pi\pi\ell^+\ell^-$, $K \to \pi\gamma\ell^+\ell^-$, $K \to \gamma^*\gamma^*$,...]

FCNC: $K \to \pi\nu\bar{\nu}$, $K \to \pi\ell^+\ell^-$

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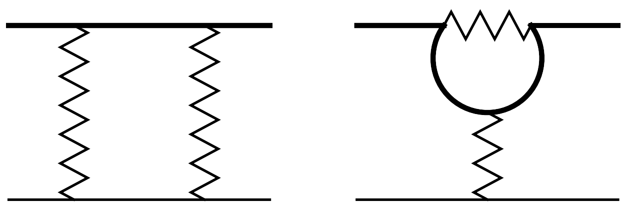
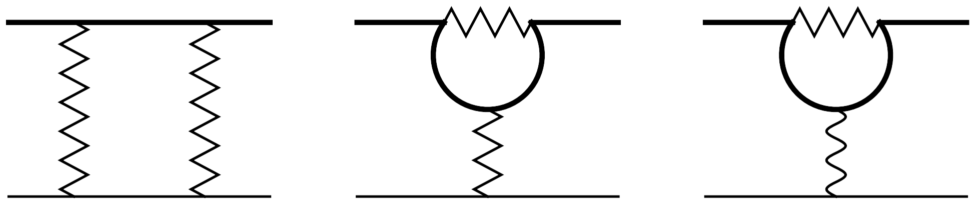


- LD dominated rare decays $\Longrightarrow K \to \pi\ell^+\ell^-$, $K \to \pi\pi\ell^+\ell^-$, $K \to \pi\gamma\ell^+\ell^-$, $K \to \gamma^*\gamma^*$,...]



For reviews, see V. Cirigliano et al, Rev. Mod. Phys. 84, 399 (2012)

L. Littenberg, G. Valencia in PDG

$K \to \pi\nu\bar{\nu}$

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{NA62}} = (10.6^{+4.0}_{-3.5} \pm 0.9) \cdot 10^{-11} \qquad \mathcal{B}(K^0 \to \pi^0\nu\bar{\nu})_{\mathrm{KOTO}} \leq 3.0 \cdot 10^{-9}$$

NA62 Coll., E. Cortina Gil et al., JHEP 06, 093 (2021)

KOTO Coll., J. Ahn et al., Phys. Rev. Lett. 122, 021802 (2019)

$K \to \pi\nu\bar{\nu}$

Theory predictions

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = 8.4(1.0)\cdot 10^{-11} \quad \mathcal{B}(K^0 \to \pi^0\nu\bar{\nu})_{\rm SM} = 3.4(6)10^{-11}$$

A. J. Buras et al., JHEP 11, 033 (2015)

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = 7.7(6)\cdot 10^{-11} \quad \mathcal{B}(K^0 \to \pi^0\nu\bar{\nu})_{\rm SM} = 2.6(3)10^{-11}$$

J. Brod, M. Gorbahn, E. Stamou, arXiv:2105.02868

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M. Bordone, B. Capdevilla, P. Gambino, Phys. Lett. B 822, 136679 (2021)

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J. Brod, M. Gorbahn, E. Stamou, arXiv:2105.02868

M. Bordone, N. Gubernari, D. van Dyck, M. Jung, Eur. Phys. J. 80, 347 (2020)

Sizeable dependence on input parameters V_{cb}, V_{ub}, γ

viz.

$$Br(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = 8.39(30)\cdot 10^{-11}\left[\frac{|V_{cb}|}{40.7\cdot 10^{-3}}\right]^{2.8}\left[\frac{\gamma}{73.2^{\rm o}}\right]^{0.74}$$

$$Br(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = 3.36(5)\cdot 10^{-11}\left[\frac{|V_{ub}|}{3.88\cdot 10^{-3}}\right]^{2}\left[\frac{|V_{cb}|}{40.7\cdot 10^{-3}}\right]^{2}\left[\frac{\sin\gamma}{\sin 73.2^{\rm o}}\right]^{2}$$

J. Brod et al., Phys Rev D 83, 034030 (2011)

A. J. Buras et al, JHEP 1511, 33 (2011)

$K \to \pi\nu\bar{\nu}$

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A. J. Buras et al., JHEP 11, 033 (2015)

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J. Brod, M. Gorbahn, E. Stamou, arXiv:2105.02868

M. Bordone, N. Gubernari, D. van Dyck, M. Jung, Eur. Phys. J. 80, 347 (2020)

Reduce dependence on input parameters V_{cb}, V_{ub}, γ upon expressing observables in terms of observables that have also strong dependence on e.g. V_{cb}, but otherwise small theoretical uncertainties, e.g. ϵ_K, $\mathcal{B}(K_S \to \mu^+\mu^-)$

$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = 8.60(42) \cdot 10^{-11}$ $\mathcal{B}(K^0 \to \pi^0\nu\bar{\nu})_{\rm SM} = 2.94(15)10^{-11}$ $60^{\rm o} \leq \gamma \leq 75^{\rm o}$

A. J. Buras, E. Venturini, Acta Phys. Polon. B53 6, A1

$K \to \pi\gamma^*$: Experimental situation

exp.	mode	number of events	a_+	b_+
BNL-E865	$K^+ \to \pi^+ e^+ e^-$	$10\,300$	$-0.587(10)$	$-0.655(44)$
NA48/2	$K^\pm \to \pi^\pm e^+ e^-$	$7\,253$	$-0.578(16)$	$-0.779(66)$
NA48/2	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120	$-0.575(39)$	$-0.813(145)$
NA62	$K^+ \to \pi^+ \mu^+ \mu^-$	27679	$-0.575(13)$	$-0.722(43)$

E. Cortina Gil et al. [NA62 Collaboration], JHEP 11, 011 (2022)

exp.	mode	number of events
NA48/1	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	$K_S \to \pi^0 \mu^+ \mu^-$	6

$$a_S = -1.29(3.15) \quad b_S = +17.8(10.6)$$

or

$$a_S = +1.28(3.16) \quad b_S = -17.6(10.6)$$

G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data
- Sign of a_S important for contribution to $K_L \to \pi^0\gamma^* \to \pi^0\ell^+\ell^-$ from indirect CPV

$$\mathcal{B}(K_L \to \pi^0\mu^+\mu^-)_{\text{CPV}} = 10^{-12} \cdot \left[3.7|a_S|^2 + 1.6 a_S \left(\frac{\text{Im}\lambda_t}{10^{-4}}\right) + 1.0 \left(\frac{\text{Im}\lambda_t}{10^{-4}}\right)^2\right]$$

G. Buchalla, G. D'Ambrosio, G. Isodori, Nucl. Phys B 672, 387 (2003)

G. Isidori, C. Smith, R. Unterdorfer, Eur. Phys. J C 36, 57 (2004)

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- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data
- $\left(\frac{a_+}{b_+}\frac{M_K^2}{M_V^2}\right)^{\text{exp}} \sim 0.4$ deviates strongly from the VMD prediction $\left(\frac{a_+}{b_+}\frac{M_K^2}{M_V^2}\right)^{\text{VMD}} \sim 1$

$K \to \pi\gamma^*$: Theory description

$$\mathcal{W}_{+;S}(s) = G_F M_K^2 \left(a_{+;S} + b_{+;S} \frac{s}{M_K^2} \right) + \mathcal{W}_{\pi\pi}(s; \alpha_{+;S}, \beta_{+;S})$$

G. D'Ambrosio, G. Ecker, G. Isidori, J. Portoles, JHEP 08, 004 (1998)

$\alpha_{+;S}$, $\beta_{+;S} \longrightarrow$ linear and quadratic slopes of $K \to \pi\pi^+\pi^-$ amplitudes

$$\alpha_+ = -20.40(18) \cdot 10^{-8} \qquad \beta_+ = -2.05(6) \cdot 10^{-8}$$
$$\alpha_S = -6.42(23) \cdot 10^{-8} \qquad \beta_S = -0.19(43) \cdot 10^{-8}$$

G. D'Ambrosio, MK, S. Neshatpour, Phys. Lett. B 835, 137594 (2022)

- $\mathcal{W}_{+;S}(s)$ contains a non-local contribution that is UV singular $\longrightarrow$ LD-SD matching

SD: $C_{7V}(\nu) Q_{7V} = (\bar{s}d)_{V-A}(\bar{\ell}\ell)_V$

G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

G. D'Ambrosio, D. Greynat, MK, Phys. Lett. B 797, 134891 (2019)

- No real prediction for $a_{+;S}$, $b_{+;S}$
  - Unknown low-energy constants

    G. Ecker, A. Pich, E. de Rafael, Nucl. Phys. B 291, 692 (1987); Nucl. Phys. B 303, 665 (1988)

  - VMD leads to $\left(\frac{a_+}{b_+} \frac{M_K^2}{M_V^2} \right)^{\text{VMD}} \sim 1$
- Challenge for lattice QCD

G. Isidori, G. Martinelli, P. Turchetti, Phys. Lett. B 633, 75 (2006)

RBC-UKQCD coll., Phys. Rev. D 92, 094512 (2015)

$$\frac{\mathcal{W}_+(s = 0.013(2) M_K^2)}{G_F M_K^2} = -0.87(4.44)$$

P. A. Boyle et al. [RBC-UKQCD coll.], Phys. Rev. D 107, L011503 (2023)

$K \to \pi\gamma^*$: Theory description from a large N_c perspective

- Large N_c should provide a first realistic quantitative estimate of $a_{+;S}$ and $b_{+;S}$

  $\longrightarrow$ no accidental cancellations expected

- Large-N_c inspired phenomenological models have been tried in the past...

H.-Y. Cheng, Phys. Rev. D 42, 72 (1990)

S. Friot, D. Greynat, E. de Rafael, Phys. Lett B 595, 301 (2004)

E. Coluccio Leskow, G. D'Ambrosio, D. Greynat, A. Nath, Phys. Rev. D 93, 094031 (2016)

- ... but never a full analysis, including SD-LD matching
- A complete large-N_c analysis is under way

G. D'Ambrosio, MK, S. Neshatpour [arXiv:2206.?????]

- Preliminary results give

$$\left(\frac{a_+^\infty}{b_+^\infty}\frac{M_K^2}{M_V^2}\right) \sim +0.34 \qquad \left(\frac{a_S^\infty}{b_S^\infty}\frac{M_K^2}{M_V^2}\right) \sim +0.68 \qquad a_+^\infty < 0 \qquad a_S^\infty > 0$$

- Next application: $K \to \gamma^*\gamma^*$

G. D'Ambrosio, MK, S. Neshatpour, in preparation

CONCLUSION

- Kaon physics important source of precision tests of the SM
- Will remain the case in the future

E. Goudzovski, E. Passemar, et al., Snowmass 2021 [arXiv:2209.07156 [hep-ex]]

- Contributions from lattice QCD are essential, will further increase in importance, e.g. for ϵ'/ϵ and especially for rare kaon decays

T. Blum, P. Boyle, M. Bruno, N. Christ, et al., Snowmass 2021 [arXiv:2203.10998 [hep-lat]]

- There are tensions in the data, revealed by constrained fit (PDG) on partial widths (scale factors)

e.g. between $\Gamma(K_L \to \pi^0\pi^0\pi^0)$ and $\Gamma(K_L \to \pi^+\pi^-\pi^0)$ or between $\Gamma(K^+ \to \pi^0\pi^0\pi^+)$ and $\Gamma(K^+ \to \pi^+\pi^+\pi^-)$

G. D'Ambrosio, MK, S. Neshatpour, Phys. Lett. B 835, 137594 (2022)

It is important to reduce these tensions

- V_{cb} tension in excl. vs. incl.: on the way toward an experimental solution?