Neutrino masses and leptonic mixing

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- introduction : lepton mixing versus quark mixing
- lepton flavour mixing and CP violation
- 3-flavour interpretation of neutrino oscillation data
- constraints on neutrino masses

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Introduction : lepton mixing versus quark mixing

Lepton mixing is similar in nature to quark mixing : due the fact that the mass eigenstate bases of the leptonic SU(2)L partners (LH charged leptons and neutrinos) do not coincide. The relative (unitary) rotation is called the PMNS (Pontecorvo-Maki-Nakagawa-Sakata) matrix, analogue of the CKM matrix

Also parametrized by 3 mixing angles and 1 CP-violating phase if neutrinos are Dirac fermions (+ 2 additional phases if neutrinos are Majorana fermions)

However, lepton mixing is more elusive than quark mixing. While the CKM angles and phase enter a plethora of electroweak processes involving hadrons, neutrino oscillations are practically the only phenomenon that is sensitive to lepton flavour mixing
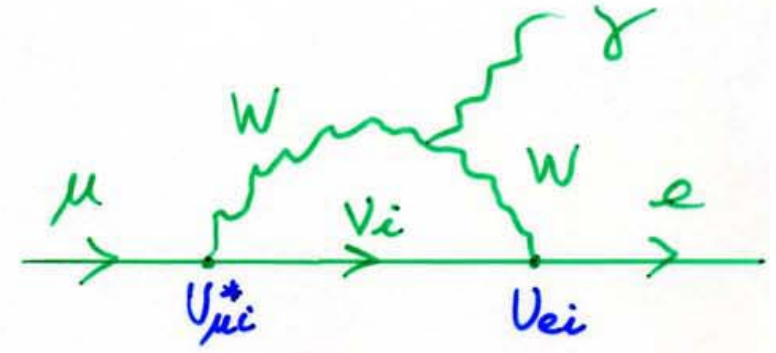
Due to the smallness of neutrino masses: different mass eigenstates cannot be distinguished experimentally and are summed over in processes that produce neutrinos. E.g. cannot tell which mass eigenstates are produced in $\mu \to e\, \nu_i \bar{\nu}_j$

$$\Rightarrow\ \Gamma(\mu \to e\, \nu \bar{\nu}) \equiv \sum_{i,j} \Gamma(\mu \to e\, \nu_i \bar{\nu}_j) \propto \sum_{i,j} |U_{ej} U^*_{\mu i}|^2 = 1 \quad \text{(unitarity)}$$

[$U_{\alpha i}$ = entry of the PMNS matrix]

Processes that change the flavour of charged leptons without involving neutrinos depend non trivially of the PMNS entries, but they are suppressed by the GIM mechanism and unobservable in practice (in the absence of new physics). E.g. for $\mu \to e\, \gamma$:

$$\mathrm{BR}\,(\mu \to e\gamma) \;=\; \frac{3\alpha}{32\pi}\left|\sum_i U^*_{\mu i} U_{ei}\,\frac{m^2_{\nu_i}}{M^2_W}\right|^2$$

Using known oscillations parameters, this gives $\mathrm{BR}\,(\mu \to e\gamma) \lesssim 10^{-54}$: observation of $\mu \to e\,\gamma$ would be an unambiguous signal of new physics !

Instead, the smallness of neutrino masses makes flavour oscillations possible (mass differences much smaller than than energy resolution $\Rightarrow$ can be produced as coherent superposition of mass eigenstates, which gets modified as neutrinos propagate)

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = \sum_i U^*_{\alpha i}|\nu_i\rangle \quad \Rightarrow \quad |\nu(t)\rangle = \sum_i U^*_{\alpha i}\, e^{-iE_i t}|\nu_i\rangle$$

$$\Rightarrow \quad \langle\nu_\beta|\nu(t)\rangle = \sum_j U_{\beta j}\langle\nu_j|\nu(t)\rangle = \sum_i U_{\beta i}U^*_{\alpha i}\, e^{-iE_i t}$$

$\Rightarrow$ oscillation probability $P(\nu_\alpha \to \nu_\beta)$ depends on the PMNS entries $U_{\alpha i}, U_{\beta i}$

Lepton flavour mixing – PMNS matrix

The neutrino to which a given charged lepton (e, μ or τ) couples via the W, called neutrino flavour (or gauge) eigenstate, is not a mass eigenstate. Flavour eigenstates are related to mass eigenstates by the lepton mixing matrix (or PMNS matrix) U

PMNS matrix with entries $U_{\alpha i}$

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\, \nu_i$$

flavour eigenstate ($\alpha = e,\, \mu,\, \tau$)   mass eigenstate with mass m_i ($i = 1,\, 2,\, 3$)

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_L = U_{\mathrm{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L$$

U is parametrized by 3 mixing angles and 1 (resp. 3) physical CP-violating phases if neutrinos are Dirac (Majorana) fermions. Indeed, Majorana fermions cannot be rephased as this would affect the Majorana condition $\psi = -\gamma^0 C \psi^*$

Standard parametrization of the PMNS matrix

Analogous to CKM: written as the product of three rotations with angles θ_{23}, θ_{13} and θ_{12}, the second (complex) rotation depending on the phase δ

$$U \equiv U_{23}U_{13}U_{12}P \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} P$$

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P$$

P is the unit matrix in the Dirac case, and a diagonal matrix of phases containing 2 independent phases ϕ_i in the Majorana case

$$c_{ij} = \cos\theta_{ij}\,, \quad s_{ij} = \sin\theta_{ij}$$

$$\theta_{ij} \in [0, \pi/2]\,, \quad \delta \in [0, 2\pi[\,, \quad \phi_i \in [0, \pi[$$

$$\delta \neq 0, \pi \implies \text{CP violation in oscillations: } P(\nu_\alpha \to \nu_\beta) \neq P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$$

The Majorana phases play a role only in $\Delta L = 2$ processes like neutrinoless double beta decay

Neutrino oscillations

Oscillation probability = Σ oscillating terms with different « frequencies » $\Delta m_{ji}^2 \equiv m_j^2 - m_i^2$ and amplitudes (which depend on the PMNS entries)

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4 \sum_{i<j} \text{Re}\left[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right] \sin^2\left(\frac{\Delta m_{ji}^2 L}{4E} \right)$$

$$+ 2 \sum_{i<j} \text{Im}\left[U_{\alpha i} U_{\beta i}^* U_{\alpha j}^* U_{\beta j} \right] \sin\left(\frac{\Delta m_{ji}^2 L}{2E} \right)$$

→ oscillation parameters :

- 2 independent Δm^2 : Δm_{31}^2 (« atmospheric ») and Δm_{21}^2 (« solar »)

- 3 mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and one phase δ [« Dirac » phase of PMNS]

[the « Majorana » phases are relevant only for processes that violate lepton number, such as neutrinoless double beta decay, and have no effect on oscillations]

For antineutrinos, U → U* and the last term changes sign

$\Rightarrow P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) \neq P(\nu_\alpha \to \nu_\beta)$ (if $\delta \neq 0, \pi$) → <u>CP violation</u>

In many experiment setups, oscillations are dominated by a single Δm^2 and can be described to a good approximation as 2-flavour oscillations:
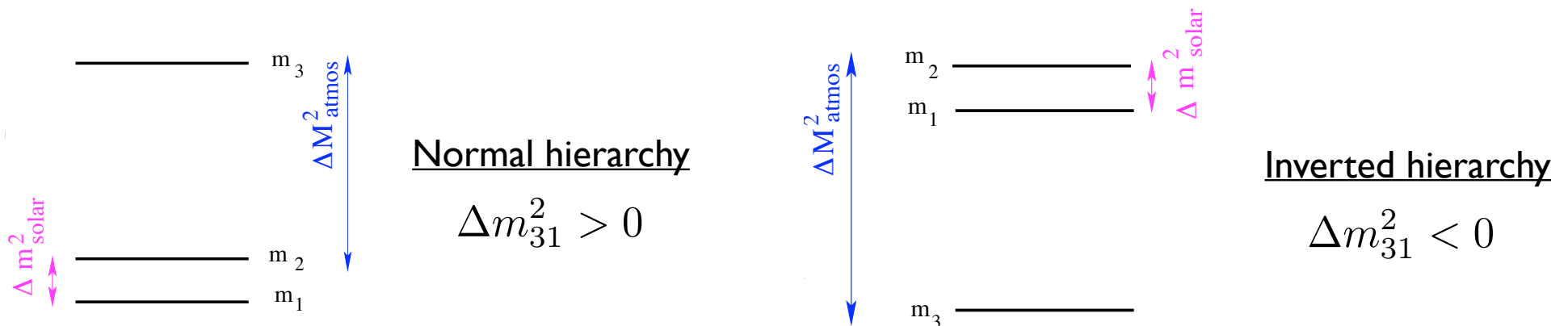
- solar neutrinos (*), LBL reactors $\qquad \Delta m^2_{21}, \ \theta_{12} \qquad \Delta m^2_{21} \simeq 7.4 \times 10^{-5}\,\mathrm{eV}^2$
  ($\nu_e / \bar{\nu}_e$ disappearance)

- atmospheric, LBL accelerators $\qquad \Delta m^2_{31}, \ \theta_{23} \qquad |\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$
  (ν_μ disappearance)

- SBL reactor experiments $\qquad \Delta m^2_{31}, \theta_{13} \qquad \sin^2 \theta_{13} \simeq 0.022$
  ($\bar{\nu}_e$ disappearance)

(*) matter effects dominate for high-energy solar neutrinos

<u>Notes</u>: 1) θ_{13} is the only « small » leptonic angle $\quad \theta_{13} < \theta_{12}, \theta_{23}$

$\sin^2 2\theta_{23} = 1$

2) $\Delta m^2_{21} \ll |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \qquad$ [by convention, $\Delta m^2_{21} > 0$]

sign of Δm^2_{31} still undetermined $\Rightarrow$ two types of spectra allowed

Fig. 15. Mass schemes for 3ν oscillations

• Mixing effects because masses are so different for 3ν oscillations

Two different experimental approaches to determine the mass hierarchy

1) <u>matter effects</u> : neutrino oscillations affected by their scattering on e-, p, n when they propagate through matter (case of $\nu_\mu \to \nu_e$ oscillations at long-baseline experiments), with an opposite effect on antineutrinos

- for $\Delta m_{31}^2 > 0$ (NH), (anti-)neutrino oscillations are enhanced (suppressed)
- for $\Delta m_{31}^2 < 0$ (IH), (anti-)neutrino oscillations are suppressed (enhanced)
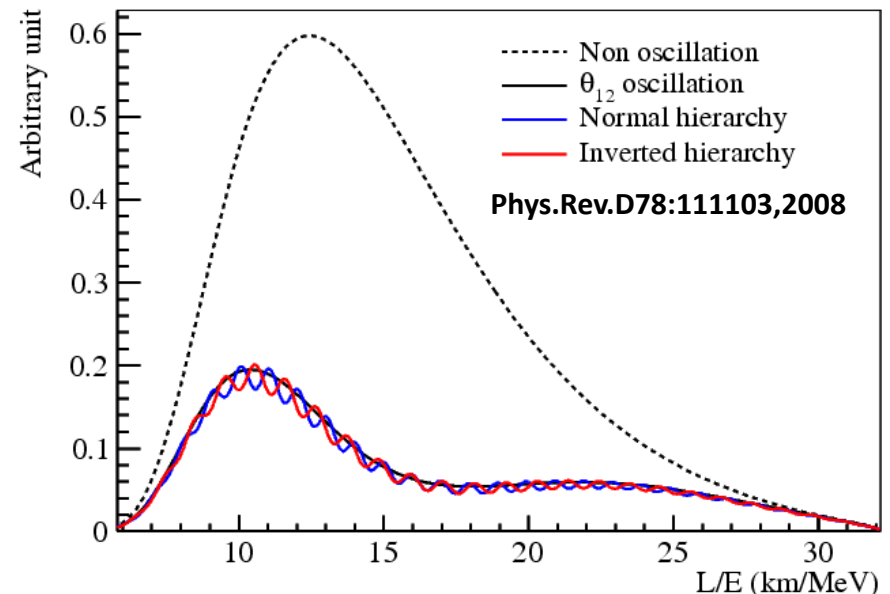
→ DUNE, Hyper-Kamiokande

2) interference between subleading oscillations governed by Δm_{31}^2 and Δm_{32}^2 in long-baseline reactor experiments ($\bar{\nu}_e \to \bar{\nu}_e$ disappearance)

leads to distorsions of the antineutrino spectrum which depend on the hierarchy

$|\Delta m_{31}^2| > |\Delta m_{32}^2|$ for NH

$|\Delta m_{31}^2| < |\Delta m_{32}^2|$ for IH

→ JUNO

Phys.Rev.D78:111103,2008

CP violation in oscillations

$\Delta P_{\mu e} \equiv P(\nu_\mu \to \nu_e) - P(\bar{\nu}_\mu \to \bar{\nu}_e)$ at leading order in Δm_{21}^2:

$$\Delta P_{\mu e} = -8 J \left(\frac{\Delta m_{21}^2 L}{2E} \right) \sin^2 \left(\frac{\Delta m_{31}^2 L}{4E} \right), \qquad J \equiv \mathrm{Im} \left[U_{e1} U_{\mu 1}^* U_{e2}^* U_{\mu 2} \right]$$

Jarlskog invariant $\quad J = \dfrac{1}{8} \cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} \sin\delta$

$\to$ condition for CP violation : $\quad \delta \neq 0, \pi$

$\to$ for CP violation to be observable, sub-dominant oscillations governed by Δm_{21}^2 must develop $\Rightarrow$ long baseline oscillation experiments (> 100 km), also sensitive to matter effects (which can mimic a CP asymmetry)

CP violation is only possible in appearance channels, such as $\nu_\mu \to \nu_e$

Disappearance experiments, e.g. at reactors, have no sensitivity to δ

First hints of CP violation at T2K

Long baseline accelerator experiment in Japan (295 km)

Observes more events in the neutrino mode $(\nu_\mu \to \nu_e)$ and less events in the antineutrino mode $(\bar{\nu}_\mu \to \bar{\nu}_e)$ than expected if CP conserved

$\Rightarrow$ suggests CP violation (CP conservation excluded at more than 90% C.L.)

[talk at Neutrino 2020]
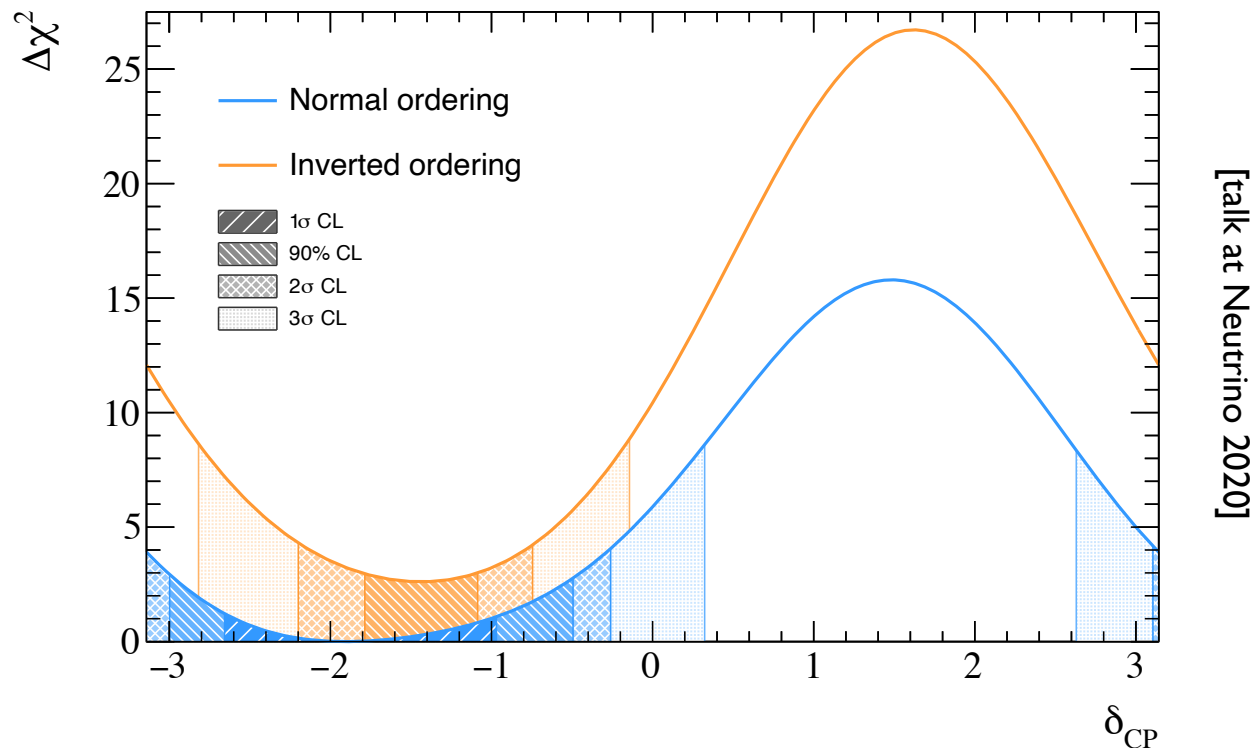
3-flavour interpretation of neutrino oscillation data

(aka « 3-flavour global fit »)

I. Esteban et al., JHEP 09 (2020) 178
NuFIT 5.2 (2022), www.nu-fit.org

(based on data available in November 2022)

All experimental data (leaving aside a few anomalies) is well described in the **3-flavour framework**, and the determination of oscillation parameters is becoming more and more precise

similar conclusions from fits by the Bari group (E. Lisi et al.) and the Valencia group (M. Tortola et al.)
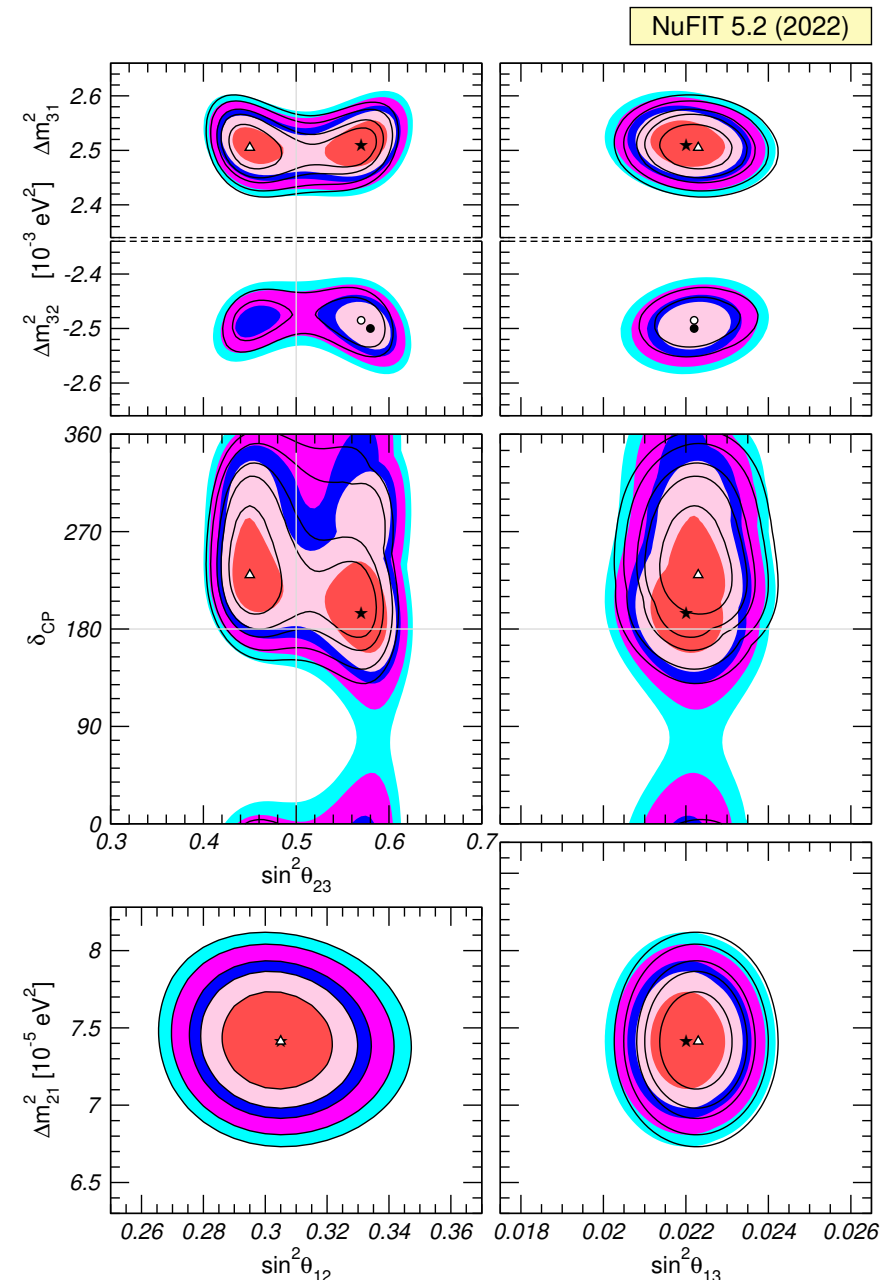
NuFIT 5.2 (2022)

The different contours correspond to 1σ, 90%, 2σ, 99%, 3σ CL (2 dof).

Allowed ranges for the oscillation parameters (November 2022)

	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
$\theta_{12}/°$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
$\theta_{23}/°$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
$\theta_{13}/°$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
$\delta_{CP}/°$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
$\dfrac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
$\dfrac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

(Row label, left margin: with SK atmospheric data)

1σ uncertainty around 5% for θ_{12} and Δm^2_{21} , less than 3% for θ_{13} and $\Delta m^2_{3\ell}$

3σ uncertainty around 15% for θ_{12} and Δm^2_{21} , less than 9% for θ_{13} and $\Delta m^2_{3\ell}$

The best known parameters are θ_{13} and Δm^2_{32} (Δm^2_{31} in the case of normal ordering), with 3σ uncertainties below 9%, and θ_{12} and Δm^2_{21}, with 3σ uncertainties around 15%

By contrast, θ_{23} (first [$\theta_{23} < \pi/4$] or second [$\theta_{23} > \pi/4$] octant ?), the mass ordering (or mass hierarchy), and the CP-violating phase δ depend on subleading 3-flavour effects and are poorly known
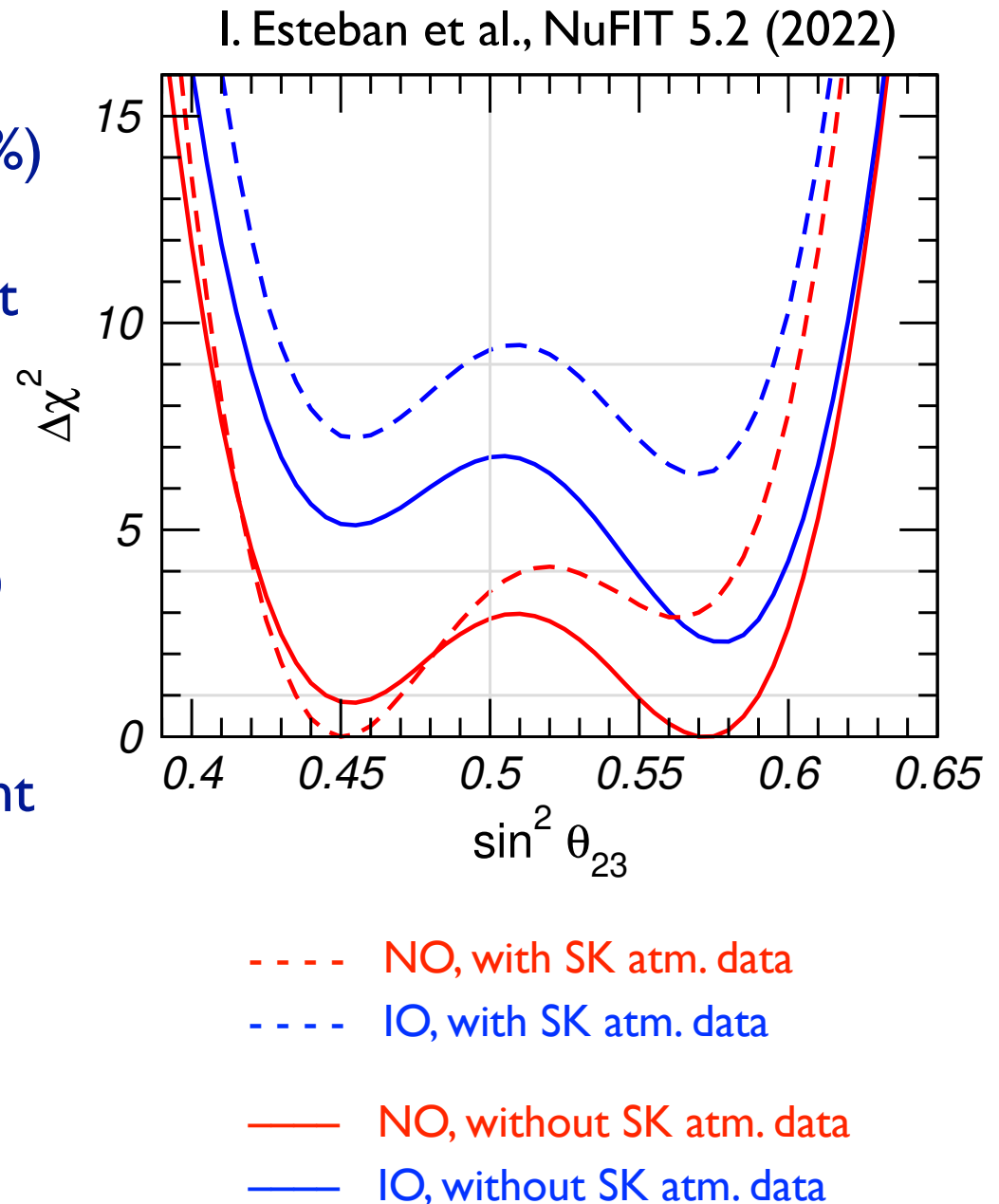
$\Rightarrow$ not yet statistically significant

θ₂₃ mixing angle

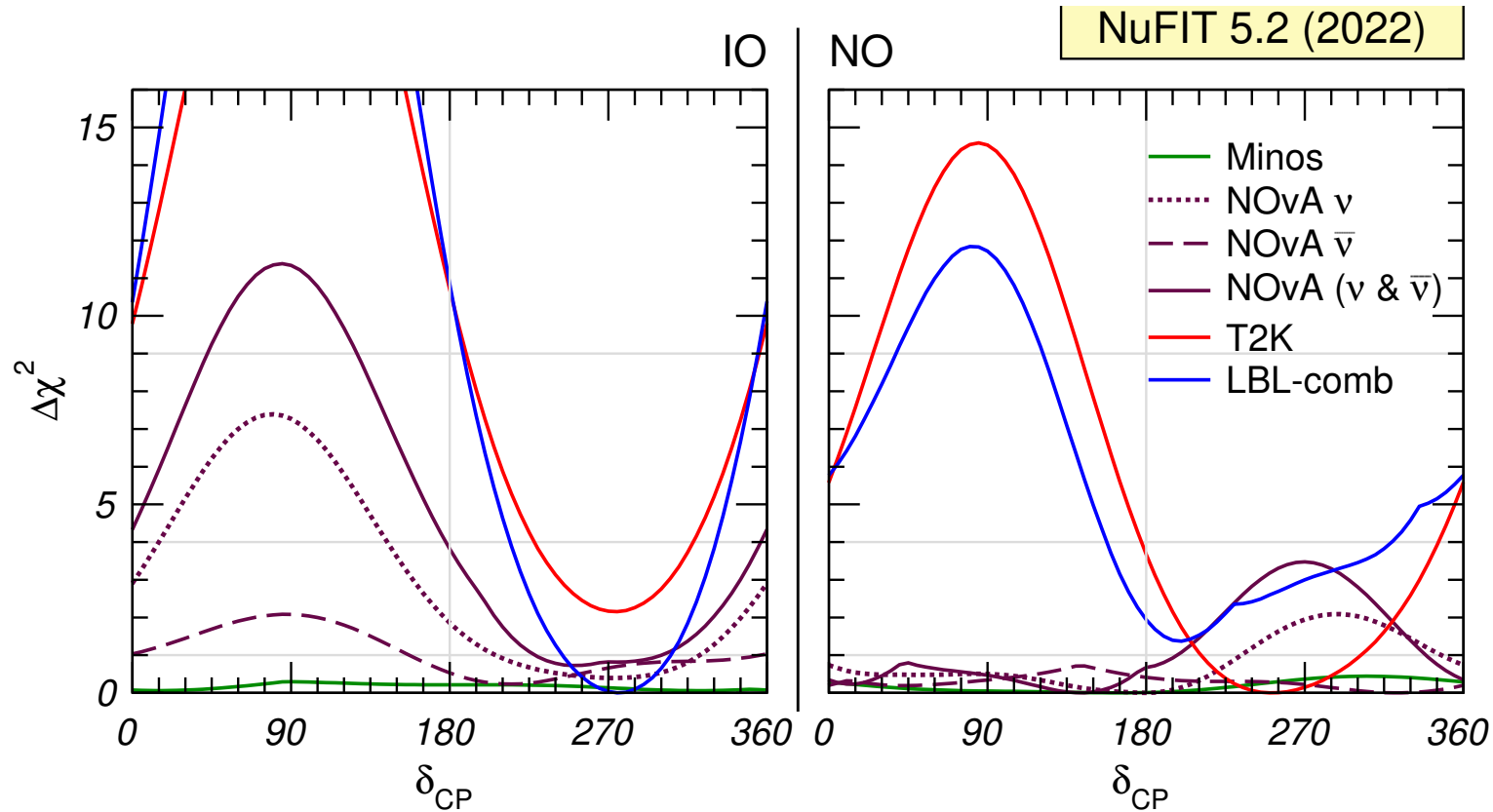Known with much less precision than θ_{13} and θ_{12} (3σ uncertainty: 24%)

No strong preference for one octant rather than the other (preferred octant depends on the mass ordering and on whether SK atmospheric data is included or not)

For NO, the slight preference (around 90% C.L.) for the first octant is due to atmospheric data

I. Esteban et al., NuFIT 5.2 (2022)

$\Delta\chi^2$ vs $\sin^2\theta_{23}$

- - - - NO, with SK atm. data

- - - - IO, with SK atm. data

——— NO, without SK atm. data

——— IO, without SK atm. data

CP violation (LBL experiments)

Tension between T2K and NOvA : T2K's best fit (with NO and δ close to $255°$) is disfavoured by NOvA
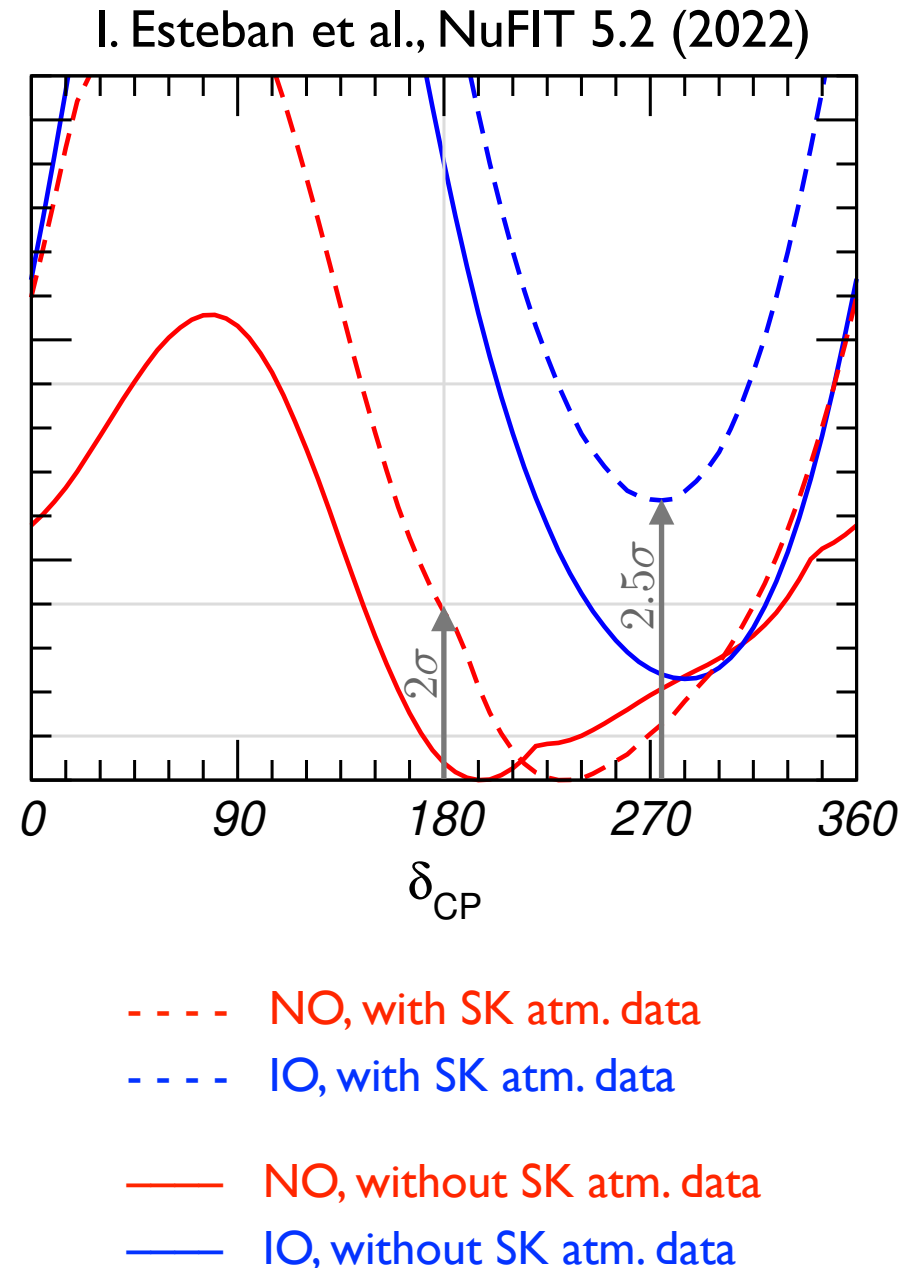
T2K and NOvA in better agreement for IO, with δ close to $3\pi/2$

However, when combining LBL with all other data, NO is preferred

Mass ordering and CP violation

Even without SK atmospheric data, the global fit prefers NO

With SK atmospheric data, NO is favoured at the 2.5σ level, and CP conservation is excluded at the 2σ level, with a best fit value $\delta \approx 230°$

I. Esteban et al., NuFIT 5.2 (2022)

- - - - NO, with SK atm. data
- - - - IO, with SK atm. data

——— NO, without SK atm. data
——— IO, without SK atm. data

Constraints on neutrino masses

Oscillation experiments measure only mass squared differences
→ information on the neutrino mass scale from beta decay or cosmology

Cosmology

Upper bound on sum of neutrino masses from CMB and large structure data
[eV-scale SM neutrinos would be hot dark matter and affect structure formation, leading
to fewer small structures than observed ⇒ must be a subdominant DM component]

$$\sum m_\nu < 0.12 \text{ eV} \quad \begin{array}{l}(95\,\%,\, Planck \text{ TT,TE,EE+lowE} \\ \text{+lensing+BAO}).\end{array} \qquad \text{[Planck 2018]}$$

[adding Lyman-a, Palanque-Delabrouille et al. obtain < 0.09 eV, 95% CL (JCAP04 (2020) 038)]

Kinematic measurements (beta decay)

The non-vanishing neutrino mass leads to a distortion of the Ee spectrum
close to the endpoint

Best bound (KATRIN) : $\qquad m_\nu < 0.8 \text{ eV} \quad (95\% \text{ C.L.})$
[Nature Phys. 18 (2022) 160]

Present bound (KATRIN) : $\qquad m_\nu < 0.8 \text{ eV} \quad (95\% \text{ C.L.})$

KATRIN will reach a final sensitivity of about 0.3 eV (95% CL)
(5σ discovery potential 0.35 eV)

All three neutrino mass eigenstates are produced in beta decay. However the
experimental energy resolution does not allow to resolve them, and what is
measured is the effective mass

$$m_\beta^2 \equiv \sum_i m_i^2 |U_{ei}|^2$$
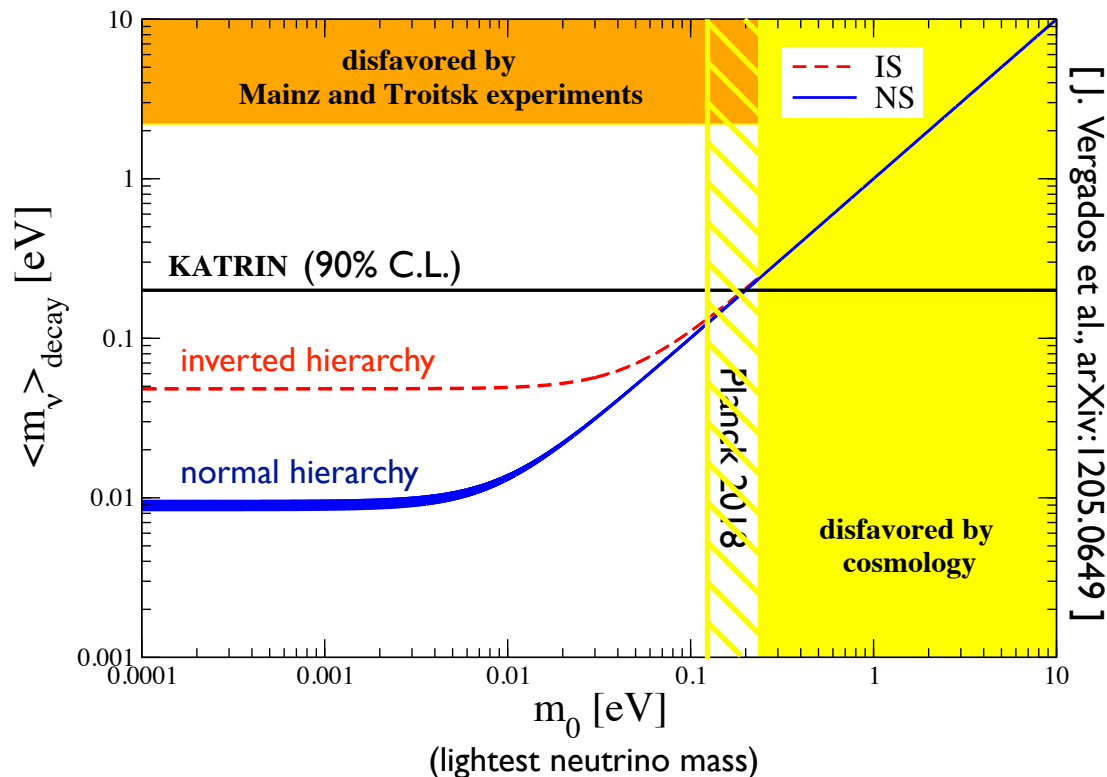
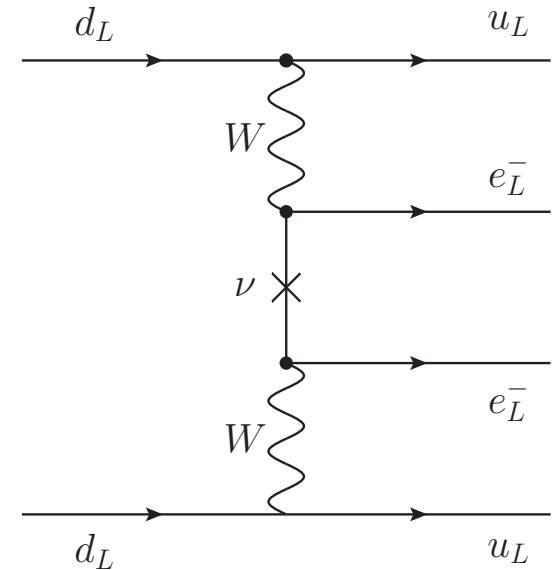
KATRIN will test only
the degenerate case

Future experiments like Project 8 aim at the 40 meV level

Neutrinoless double beta decay

$$(A, Z) \rightarrow (A, Z+2) + e^- + e^-$$

violates lepton number by 2 units

$\Rightarrow$ possible only for Majorana neutrinos

$$Q_{\beta\beta} \equiv M_i - M_f - 2m_e = T_{e_1} + T_{e_2}$$

Half-life: $\left[T_{1/2}^{0\nu}\right]^{-1} = \Gamma_{0\nu} = G_{0\nu}(Q_{\beta\beta}, Z) \, |M_{0\nu}|^2 \, |m_{\beta\beta}|^2$
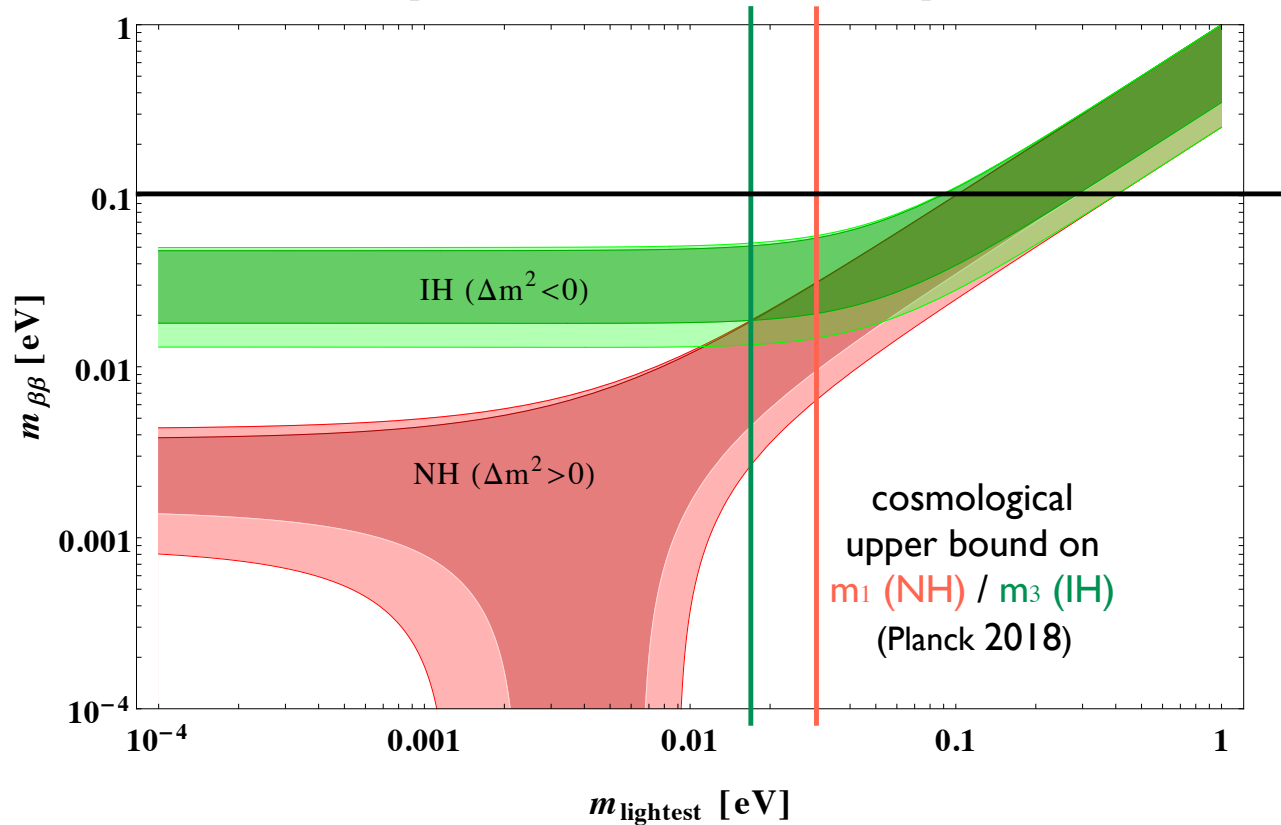
integrated phase-space factor

nuclear matrix element (NME)
(large theoretical uncertainty)

Sensitive to the effective mass parameter:

$$m_{\beta\beta} \equiv \sum_i m_i U_{ei}^2 = m_1 c_{13}^2 c_{12}^2 e^{2i\alpha_1} + m_2 c_{13}^2 s_{12}^2 e^{2i\alpha_2} + m_3 s_{13}^2$$

possible cancellations in the sum (Majorana phases α_1, α_2 in U)

[Dell'Oro et al., arXiv:1404.2616]

IH ($\Delta m^2 < 0$)

NH ($\Delta m^2 > 0$)

cosmological
upper bound on
m₁ (NH) / m₃ (IH)
(Planck 2018)

currently here, around 100 meV
(experimental upper bounds
depend on NME calculations
⇒ 2 - 4 uncertainty factor)

IH ($\Delta m^2 < 0$)

Current best limit (90% C.L.)
KamLAND-Zen

NH ($\Delta m^2 > 0$)

$T_{1/2}^{0\nu} > 2$
$T_{1/2}^{0\nu} > 2.3 \times 10^{26}$ yr
$m_{\beta\beta} <$
$m_{\beta\beta} < (36-156)$ meV
(uncertainty from NMEs)

dark shaded areas = best fit values of oscillation parameters (only α_1, α_2 vary)

light shaded areas = 3σ regions due to uncertainties on oscillation parameters
(+ dependence on α_1, α_2)

$m_{\beta\beta}$

Conclusions

Within the current experimental precision, the « PMNS paradigm » is very successful : all existing oscillation data (apart from a few unconvincing anomalies) is well described by 3-flavour oscillations governed by a unitary lepton mixing matrix

The increased precision of upcoming oscillations experiments (which should establish CP violation in the lepton sector and determine the neutrino mass ordering) will make it possible to challenge this paradigm and to look for possible hints of new physics

Several kinds of new physics might invalidate the PMNS paradigm :

- light sterile neutrinos [see J. Gehrlein's talk], even though the parameter space that could explain the LSND and MiniBooNE anomalies has been essentially excluded by other experiments

- non-unitarity of the PMNS matrix (due to the existence of sterile neutrinos that are too heavy to be produced in low-energy experiments). Can be tested by determining more precisely the oscillation parameters (also constrained by a variety of electroweak processes)

- non-standard neutrino interactions with quarks and leptons (NSIs) may affect either neutrino production and detection, or neutrino propagation in matter (also constrained by other processes like coherent elastic neutrino-nucleus scattering)