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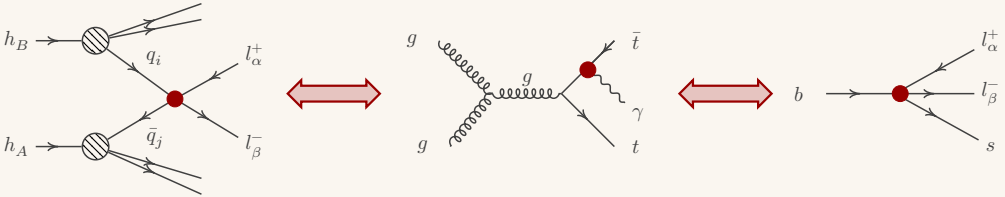
Studienstiftung des deutschen Volkes



More Synergies from Beauty, Top, Z and Drell-Yan in the SMEFT

2304.12837

In collaboration with Cornelius Grunwald, Gudrun Hiller and Kevin Kröninger

Lara Nollen

21st Conference on Flavor Physics and CP Violation 30th May 2023

SMEFT approach to new physics [See Talk by Admir Greljo]

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- No direct measurement of NP at the LHC $\rightarrow$ NP very weakly coupled or **very heavy**
- Scale separation $\rightarrow$ Indirect measurements using **effective field theories**
  $\Longrightarrow$ largely model independent framework
- SMEFT is constructed from all SM fields with the full SM symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_{d=5}^{\infty} \sum_i \frac{C_i^{(d)}}{\Lambda^{d-4}} O_i^{(d)}$$

$O_n^{(j)}$: Local operators, IR-sensitive (SM-fields and symmetries)

$C_n^{(j)}$: Wilson coefficients, UV-sensitive (effective couplings)

Minimal Flavor Violation

- Dimension 6 operators: Warsaw Basis: 59 operators $\rightarrow$ 2499 free parameters

See also e.g. Bruggisser et al. [arXiv:2212.02532] for MFV in SMEFT

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- Flavor patterns reduce degrees of freedom and impose **correlations** among different sectors
- MFV: Impose a $U(3)^5$ symmetry $\mathcal{G}_F = U(3)_{q_L} \times U(3)_{u_R} \times U(3)_{d_R} \times U(3)_{l_L} \times U(3)_{e_R}$
  [JHEP 05 (2021), 257]
- The SM Yukawa matrices are treated as spurions

$$Y_u : \ (3, \bar{3}, 1, 1, 1), \qquad Y_d : \ (3, 1, \bar{3}, 1, 1), \qquad Y_e : \ (1, 1, 1, 3, \bar{3})$$

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$\Longrightarrow$ Expand the quark bilinears

$$\bar{q}_L q_L : a_1 \mathbb{1} + a_2 Y_u Y_u^\dagger + a_3 Y_d Y_d^\dagger + ... \qquad \bar{u}_R u_R : b_1 \mathbb{1} + b_2 Y_u^\dagger Y_u + ...$$

$$\bar{q}_L u_R : (c_1 \mathbb{1} + c_2 Y_u Y_u^\dagger + ...) Y_u \qquad \bar{d}_R d_R : e_1 \mathbb{1} + e_2 Y_d^\dagger Y_d + ...$$

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Minimal Flavor Violation in SMEFT

- Rotating to the mass basis and retaining only y_t yields:

$$C\,\bar{q}_L q_L \supset \left[\bar{u}_L \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_1 & 0 \\ 0 & 0 & a_1 + a_2\, y_t^2 \end{pmatrix} u_L + \bar{d}_L \begin{pmatrix} a_1 + a_2\,|V_{td}|^2\, y_t^2 & a_2\, V_{td}^* V_{ts}\, y_t^2 & a_2\, V_{td}^* V_{tb}\, y_t^2 \\ a_2\, V_{ts}^* V_{td}\, y_t^2 & a_1 + a_2\,|V_{ts}|^2\, y_t^2 & a_2\, V_{ts}^* V_{tb}\, y_t^2 \\ a_2\, V_{tb}^* V_{td}\, y_t^2 & a_2\, V_{tb}^* V_{ts}\, y_t^2 & a_1 + a_2\,|V_{tb}|^2\, y_t^2 \end{pmatrix} d_L\right]$$

$t\bar{t}$ Drell-Yan $b \to s$

- Imposes **correlations** among flavor entries and allows for **down-type FCNCs**

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- Parametrization:

$$\tilde{C}_{q\bar{q}} = \frac{v^2}{\Lambda^2}\, a_1 \qquad \gamma_a = \sum_{n\geq 1} y_t^{2n}\, a_{2n}/a_1$$

- γ_a probes the MFV coefficients $\to$ constrained by combining at least two sectors

$$u_L^i \bar{u}_L^i \sim \tilde{C}_i \qquad d_L^i \bar{d}_L^i \sim \tilde{C}_i(1+\gamma_A|V_{ti}|^2) \qquad \bar{u}_L^i d_L^j \sim \tilde{C}_i\, V_{ij}$$

$$t_L \bar{t}_L \sim \tilde{C}_i(1+\gamma_a) \qquad b_L \bar{s}_L \sim \tilde{C}_i \gamma_a\, V_{ts}^* V_{tb} \qquad \bar{t}_L d_L^j \sim \tilde{C}_i\,(1+\gamma_A) V_{tj}$$

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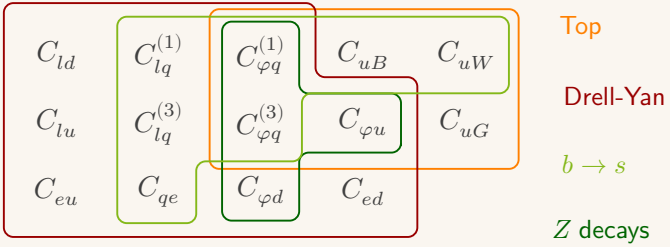
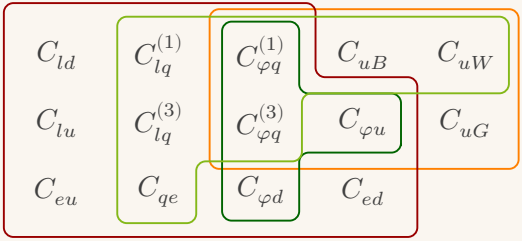
Parameters and Observables



14 Wilson coefficients and $\gamma_{a/b} \to$ 16 degrees of freedom

Parameters and Observables



Top

Drell-Yan

$b \to s$

Z decays

14 Wilson coefficients and $\gamma_{a/b} \to$ 16 degrees of freedom

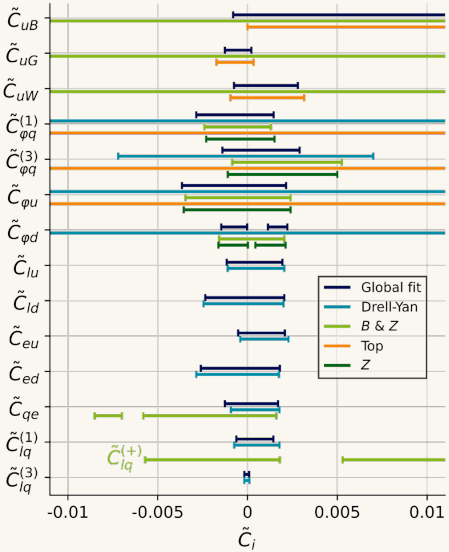
$\sigma_{t\bar{t}}$	$\sigma_{t\bar{t}Z}$	$\sigma_{t\bar{t}\gamma}$	$\sigma_{t\bar{t}W}$
$\sigma_{t\bar{t}H}$	Γ_t	f_0	f_L

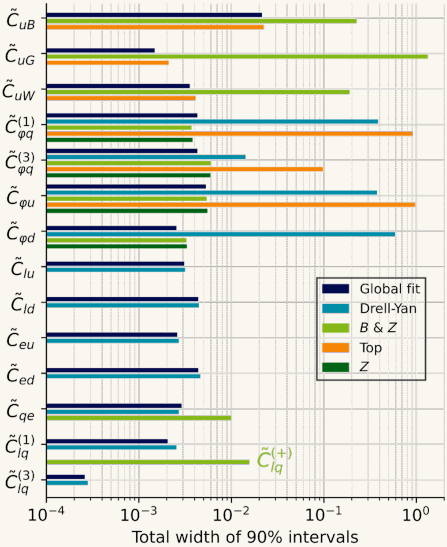
R_b	A_{FB}^b
R_c	A_{FB}^c

e^+e^-	$e\nu$
$\mu^+\mu^-$	$\mu\nu$
$\tau^+\tau^-$	$\tau\nu$

$\mathcal{B}_{\bar{B}\to X_s\gamma}$	$\mathcal{B}_{B^0\to K^*\gamma}$	$\mathcal{B}_{B^+\to K^{*+}\gamma}$	$\mathcal{B}_{\bar{B}\to X_s l^+l^-}$	$\mathcal{B}_{\bar{B}\to X_s l^+l^-}$
$\mathcal{B}_{B_s\to\mu^+\mu^-}$	$\mathsf{F}_{L\ B^0\to K^*\mu^+\mu^-}$	$\mathsf{P}_{i\ B^0\to K^*\mu^+\mu^-}^{(\prime)}$	$\mathcal{B}_{B^0\to K\mu^+\mu^-}$	$\mathcal{B}_{B^+\to K^+\mu^+\mu^-}$
$\mathcal{B}_{B^+\to K^{+*}\mu^+\mu^-}$	$\mathsf{F}_{L\ B_s\to\phi\mu^+\mu^-}$	$\mathsf{S}_{i\ B_s\to\phi\mu^+\mu^-}$	$\mathcal{B}_{\Lambda_b\to\Lambda\mu^+\mu^-}$	$\Delta M_{s\ B_s/\bar{B}_s}$

Synergies from Beauty, Top, Z and Drell-Yan

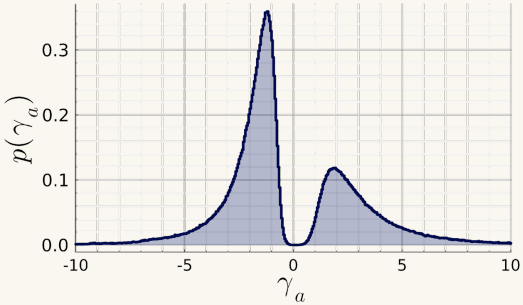


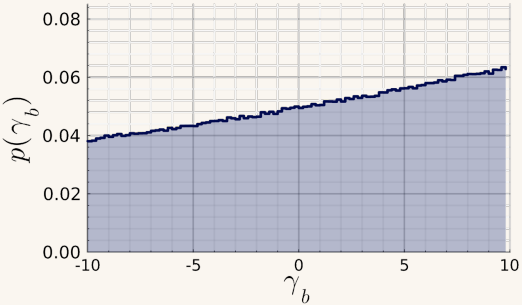


Probing the MFV parameters

$$\gamma_a = \sum_{n\geq 1} y_t^{2n} \frac{a_{2n}}{a_1}$$ left-handed doublets

$$\gamma_b = \sum_{n\geq 1} y_t^{2n} \frac{b_{2n}}{b_1}$$ right-handed up-type quarks





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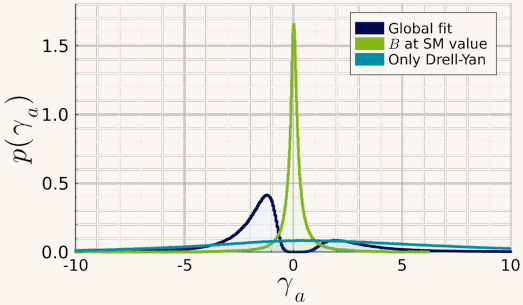
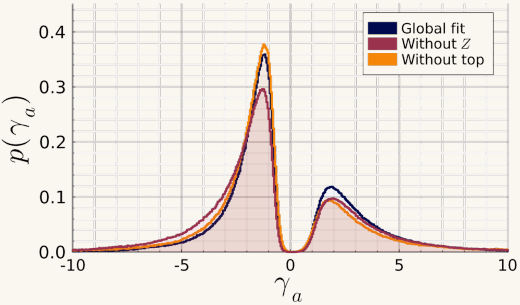
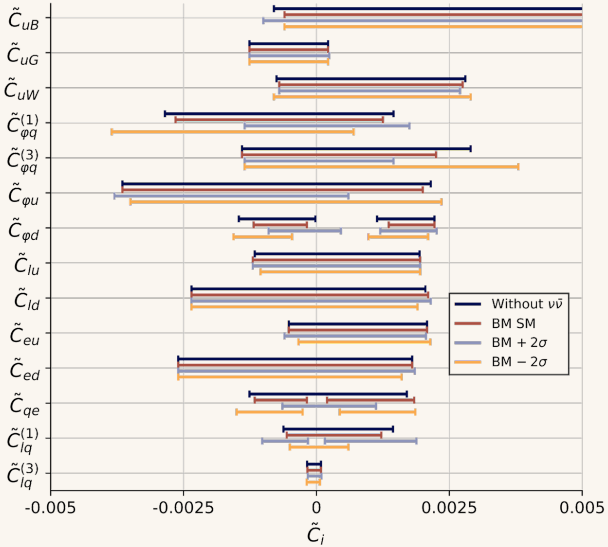
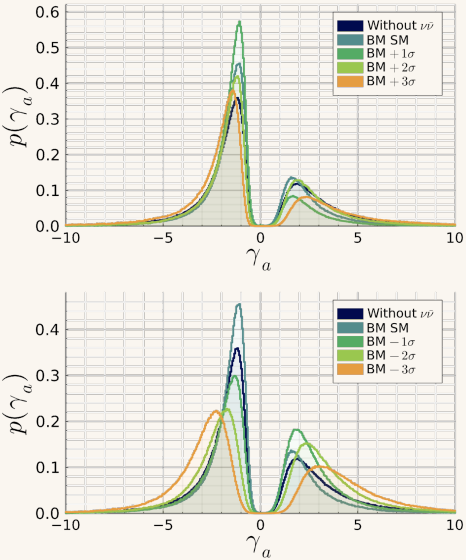


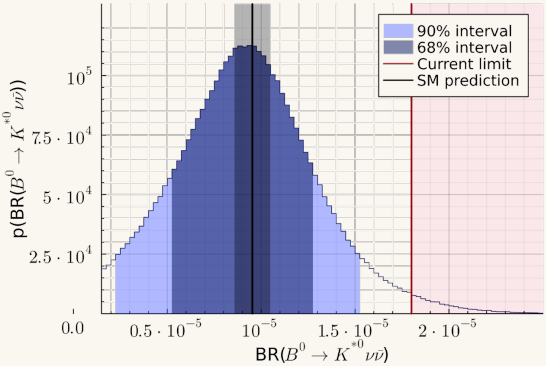


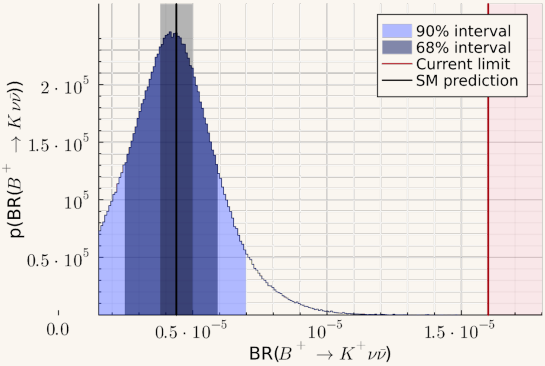
Impact of $B(B^{0/+} \to K^{*0/+}\nu\bar{\nu})$





Predictions of dineutrino branching ratios

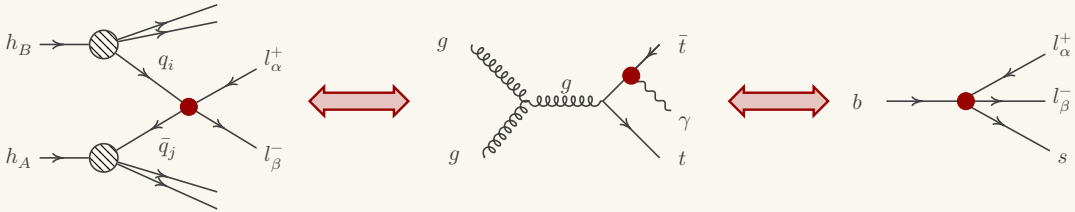




$$\frac{\mathcal{B}(B^+ \to K^+ \nu\bar{\nu})}{\mathcal{B}(B^0 \to K^{*0} \nu\bar{\nu})} = 0.46 \pm 0.08$$

Conclusion

- Flavor pattern allow to connect different sectors and exploit correlations
- Synergies arise in the global fit due to the flavor-link established by MFV
- The second order in the MFV expanion is tested by combining beauty and collider measurements
- Dineutrino branching ratios can provide crucial information in the search for BSM physics
- We predicted the dineutrino ratios to be centred around the SM prediction, and within the reach of Belle II



Supplementary Slides

Dineutrino benchmarks, limits and SM predictions

- Experimental upper limits [Phys. Rev. D 96, 091101 (2017)]

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{exp}} < 1.8 \cdot 10^{-5} \qquad B(B^+ \to K^+\nu\bar{\nu})_{\text{exp}} < 1.6 \cdot 10^{-5}$$

- SM prediction [arXiv:1810.08132]

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{SM}} = (9.53 \pm 0.95) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{SM}} = (4.39 \pm 0.60) \cdot 10^{-6}$$

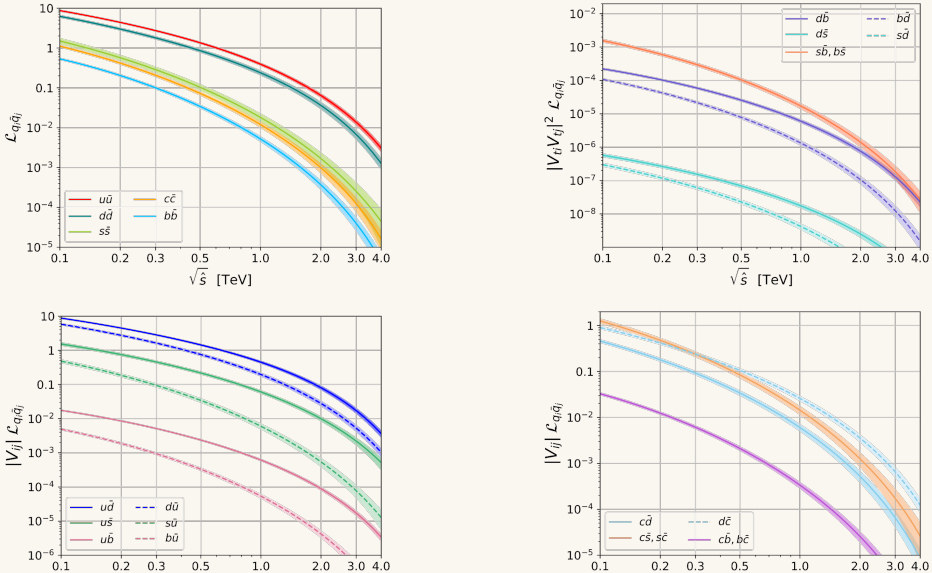
- Benchmark measurements

$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{BM SM}} = (9.5 \pm 2.5) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{BM SM}} = (4.4 \pm 1.3) \cdot 10^{-6}$$

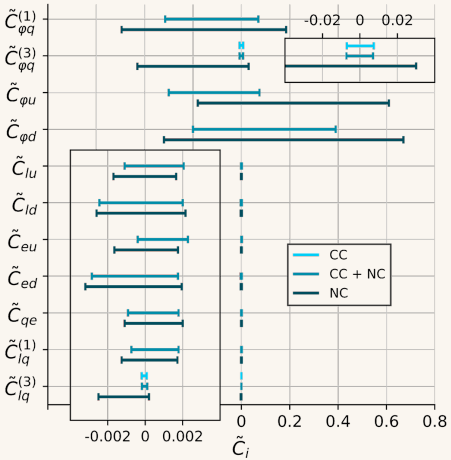
$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{BM}+2\sigma} = (14.5 \pm 2.5) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{BM}+2\sigma} = (7.0 \pm 1.3) \cdot 10^{-6}$$

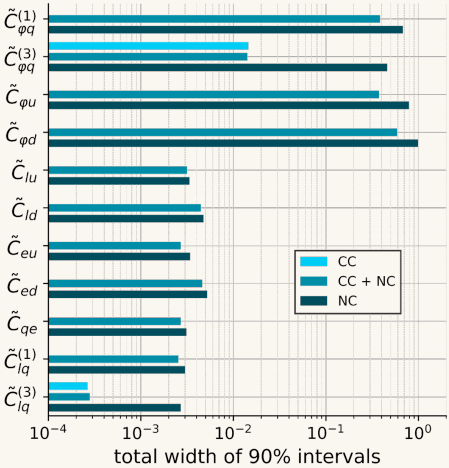
$$B(B^0 \to K^{*0}\nu\bar{\nu})_{\text{BM}-2\sigma} = (4.6 \pm 2.5) \cdot 10^{-6} \quad B(B^+ \to K^+\nu\bar{\nu})_{\text{BM}-2\sigma} = (1.8 \pm 1.3) \cdot 10^{-6}$$

Parton-Parton Luminosities

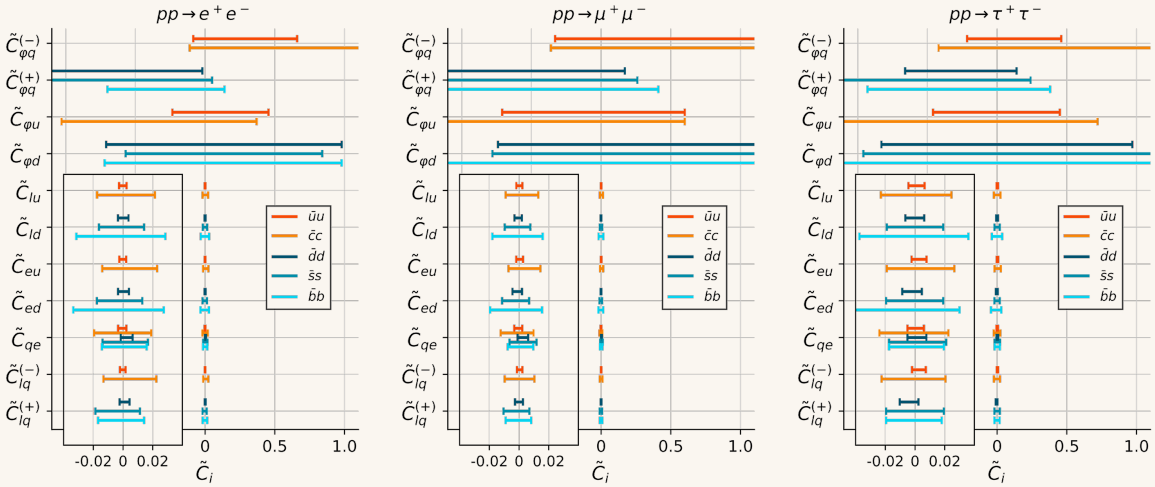


MFV Drell-Yan Fits

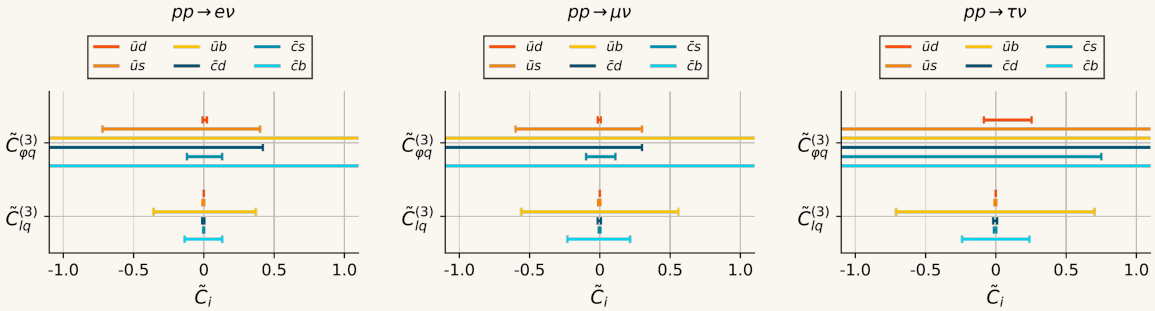




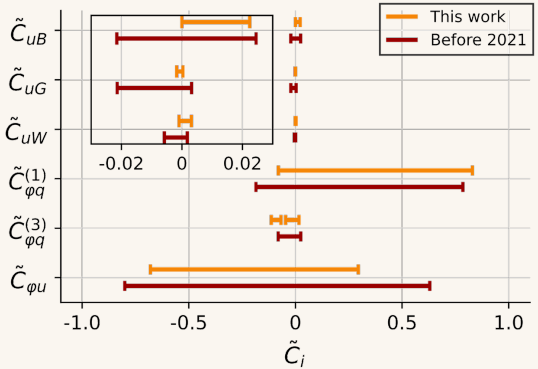
Flavor-Specific NC Drell-Yan Fits

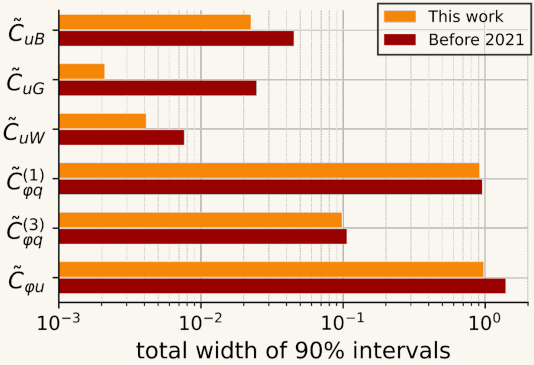


Flavor-Specific CC Drell-Yan Fits



Top Fits


This work
Before 2021
$\tilde{C}_{uB}$
$\tilde{C}_{uG}$
$\tilde{C}_{uW}$
$\tilde{C}_{\varphi q}^{(1)}$
$\tilde{C}_{\varphi q}^{(3)}$
$\tilde{C}_{\varphi u}$
-0.02
0
0.02
-1.0
-0.5
0
0.5
1.0
$\tilde{C}_i$


This work
Before 2021
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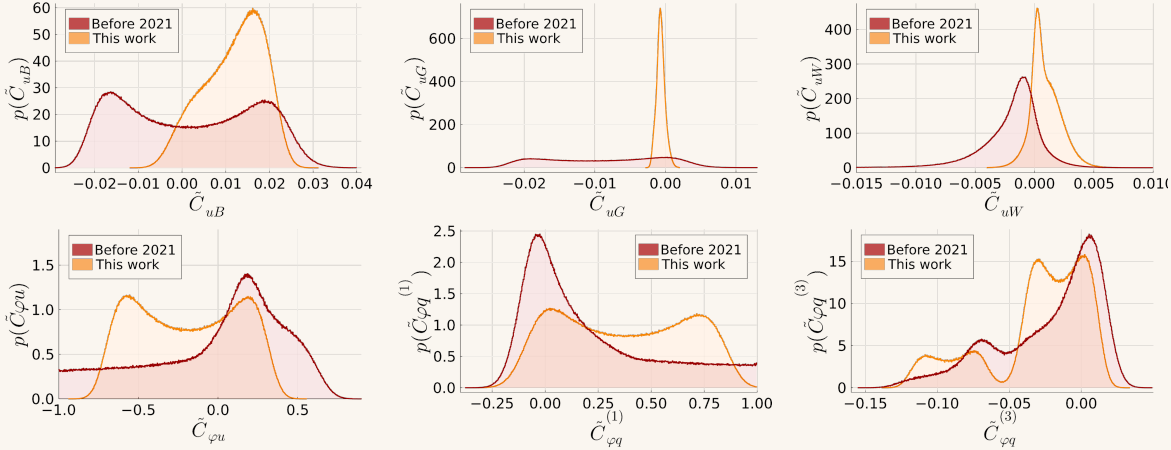
$\tilde{C}_{\varphi q}^{(1)}$
$\tilde{C}_{\varphi q}^{(3)}$
$\tilde{C}_{\varphi u}$
10^{-3}
10^{-2}
10^{-1}
10^{0}
total width of 90% intervals

Top Fit Posterior Probability Distributions



SMEFT Operators in Warsaw basis

$$O_{uG} = \left(\bar{q}_L \sigma^{\mu\nu} T^A u_R\right) \tilde{\varphi} G^A_{\mu\nu}\,,$$

$$O_{uB} = \left(\bar{q}_L \sigma^{\mu\nu} u_R\right) \tilde{\varphi} B_{\mu\nu}\,,$$

$$O^{(1)}_{lq} = \left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{q}_L \gamma^\mu q_L\right)\,,$$

$$O_{eu} = \left(\bar{e}_R \gamma_\mu e_R\right)\left(\bar{u}_R \gamma^\mu u_R\right)\,,$$

$$O_{lu} = \left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{u}_R \gamma^\mu u_R\right)\,,$$

$$O^{(1)}_{\varphi q} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)\left(\bar{q}_L \gamma^\mu q_L\right)\,,$$

$$O_{\varphi u} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)\left(\bar{u}_R \gamma^\mu u_R\right)\,,$$

$$O_{uW} = \left(\bar{q}_L \sigma^{\mu\nu} u_R\right) \tau^I \tilde{\varphi} W^I_{\mu\nu}\,,$$

$$O_{qe} = \left(\bar{q}_L \gamma_\mu q_L\right)\left(\bar{e}_R \gamma^\mu e_R\right)\,,$$

$$O^{(3)}_{lq} = \left(\bar{l}_L \gamma_\mu \tau^I l_L\right)\left(\bar{q}_L \gamma^\mu \tau^I q_L\right)\,,$$

$$O_{ed} = \left(\bar{e}_R \gamma_\mu e_R\right)\left(\bar{d}_R \gamma^\mu d_R\right)\,,$$

$$O_{ld} = \left(\bar{l}_L \gamma_\mu l_L\right)\left(\bar{d}_R \gamma^\mu d_R\right)\,,$$

$$O^{(3)}_{\varphi q} = \left(\varphi^\dagger i \overleftrightarrow{D}^I_\mu \varphi\right)\left(\bar{q}_L \tau^I \gamma^\mu q_L\right)\,,$$

$$O_{\varphi d} = \left(\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi\right)\left(\bar{d}_R \gamma^\mu d_R\right)\,,$$

Weak Effective Theory

Effective Lagrangian for $b \to sll$

$$\mathcal{L}_{\text{WET}}^{bs} = \frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$Q_8 = \frac{g_s}{16\pi^2} m_b (\bar{s}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

$$Q_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \ell)$$

$$Q_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

$$Q_L = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\nu} \gamma^\mu (1 - \gamma_5) \nu)$$

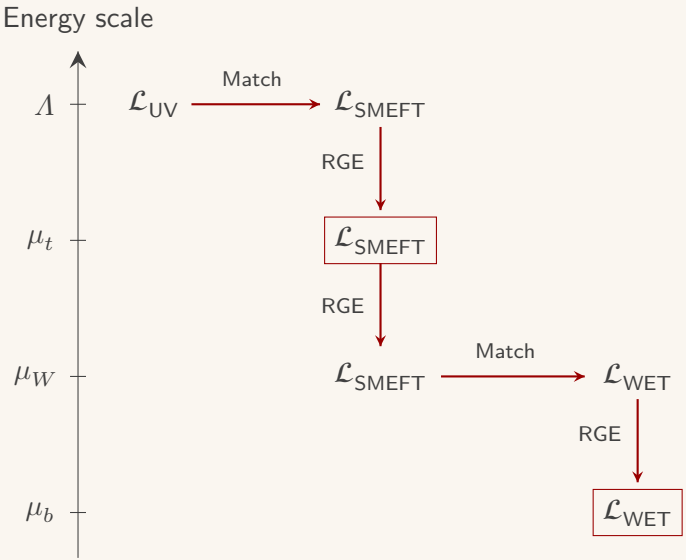
Weak Effective Theory for Meson mixing

Effective Lagrangian for $B_s\bar{B}_s$

$$\mathcal{L}_{\mathsf{WET}}^{\mathsf{mix}} = \frac{G_F^2 m_W^2}{16\pi^2} \left|V_{tb}V_{ts}^*\right|^2 \sum_i Q_i^{\mathsf{mix}}(\mu) C_i^{\mathsf{mix}}(\mu)\,,$$

$$Q_{V,LL}^{\mathsf{mix}} = \left(\bar{s}_L \gamma_\mu b_L\right)\left(\bar{s}_L \gamma^\mu b_L\right)$$

Running and Matching



- Match a potential UV theory onto the SMEFT at the matching scale Λ
- Apply the SMEFT RGE to evolve the Wilson coefficients to lower energy scales
- Compute top and collider observables at the scale μ_t
- Match the SMEFT onto the WET at the scale μ_W
- Use the WET RGE to compute the Wilson coefficients at the scale μ_b of flavor-observables

EFT cross section and Simulation Chain

$$\mathcal{M} = \mathcal{M}^{\text{SM}} + \frac{1}{\Lambda^2}\sum_i C_i \mathcal{M}_i^{\text{BSM}} \xrightarrow{\sigma \propto |\mathcal{M}|^2} \sigma = \sigma^{\text{SM}} + \frac{1}{\Lambda^2}\sum_i C_i \sigma_i^{\text{int}} + \frac{1}{\Lambda^4}\sum_{i\leq j} C_i C_j \sigma_{ij}^{\text{BSM}}$$

model definition	SMEFTsim [arXiv:2012.11343]
parton level Monte Carlo	MadGraph5 [arXiv:1405.0301]
hadronisation and parton shower	Pythia8 [arXiv:1410.3012]
detector simulation	Delphes3 [arXiv:1307.6346]

Top quark measurements included in the fit

Process	Observable	SMEFT operators	Experiment
$t\bar{t}$	$\frac{\mathrm{d}\sigma}{\mathrm{dm}(t\bar{t})}$	$\tilde{C}_{uG}$	CMS
$t\bar{t}Z$	$\frac{\mathrm{d}\sigma}{\mathrm{dp_T}(Z)}$	$\tilde{C}_{uG}$ $\tilde{C}_{uZ}$ $\tilde{C}_{\varphi u}$ $\tilde{C}^{-}_{\varphi q}$	ATLAS
$t\bar{t}\gamma$	$\frac{\mathrm{d}\sigma}{\mathrm{dp_T}(\gamma)}$	$\tilde{C}_{uG}$ $\tilde{C}_{u\gamma}$	ATLAS
$t\bar{t}W$	$\sigma_{t\bar{t}W}$	$\tilde{C}_{uG}$	ATLAS
$t\bar{t}H$	$\sigma_{t\bar{t}H} \times B_{\gamma\gamma}$	$\tilde{C}_{uG}$	ATLAS
$t \to Wb$	$f_0,\ f_L$	$\tilde{C}_{uW}$	ATLAS
$t \to Wb$	Γ_t	$\tilde{C}_{uW}$ $\tilde{C}^{3}_{\varphi q}$	ATLAS

Drell-Yan measurements included in the fit

Process	Observable	Experiment	$\sqrt{s}$	Int. luminosity
$pp \to e^+e^-$	Events, 68 bins	CMS	13 TeV	137 fb^{-1}
$pp \to \mu^+\mu^-$	Events, 36 bins	CMS	13 TeV	140 fb^{-1}
$pp \to \tau^+\tau^-$	Events, 17 bins	ATLAS	13 TeV	139 fb^{-1}
$pp \to e\nu$	Events, 40 bins	ATLAS	13 TeV	139 fb^{-1}
$pp \to \mu\nu$	Events, 35 bins	ATLAS	13 TeV	139 fb^{-1}
$pp \to \tau\nu$	Events, 10 bins	ATLAS	13 TeV	139 fb^{-1}

Flavor measurements included in the fit

Process	Observable	Experiment	q^2 bin [GeV^2]
$B \to X_s \gamma$	$B_{E_\gamma > 1.6 \text{ GeV}}$	HFLAV	/
$B^0 \to K^* \gamma$	B	HFLAV	/
$B^+ \to K^{*+} \gamma$	B	HFLAV	/
$\bar{B} \to X_s \ell^+ \ell^-$	B	BaBar	$[1, 6]$
	A_{FB}	Belle	$[1, 6]$
$B_s \to \mu^+ \mu^-$	B	CMS	/
$B^0 \to K^* \mu^+ \mu^-$	F_L, P_1, P_2, P_3, P_4', P_5', P_6', P_8'	LHCb	$[1.1, 6]$
$B^0 \to K \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$	LHCb	$[1, 6]$
$B^+ \to K^+ \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$ LHCb	$[1, 6]$	
$B^+ \to K^{+*} \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$	LHCb	$[1, 6]$
$B_s \to \phi \mu^+ \mu^-$	F_L, S_3, S_4, S_7	LHCb	$[1, 6]$
$\Lambda_b \to \Lambda \mu^+ \mu^-$	$\mathrm{d}B/\mathrm{d}q^2$	LHCb	$[15, 20]$
$B_s - \bar{B}_s$ mixing	ΔM_s	HFLAV	/

Tree-level Matching

$$\begin{aligned}\Delta C_9^{\text{tree}} &= \frac{\pi}{\alpha}\gamma_a\left[\tilde{C}_{lq}^{+} + \tilde{C}_{qe} + \left(-1 + 4\sin^2\theta_w\right)\tilde{C}_{\varphi q}^{+}\right] \\ &= \gamma_a \cdot \left(430.511\ \left(\tilde{C}_{qe} + \tilde{C}_{lq}^{+}\right) - 45.858\tilde{C}_{\varphi q}^{+}\right)\ ,\end{aligned}$$

$$\begin{aligned}\Delta C_{10}^{\text{tree}} &= \frac{\pi}{\alpha}\gamma_a\left[-\tilde{C}_{lq}^{+} + \tilde{C}_{qe} + \tilde{C}_{\varphi q}^{+}\right] \\ &= \gamma_a \cdot 430.511\left(\tilde{C}_{\varphi q}^{+} + \tilde{C}_{qe} - \tilde{C}_{lq}^{+}\right)\ ,\end{aligned}$$

$$\begin{aligned}\Delta C_{L}^{\text{tree}} &= \frac{\pi}{\alpha}\gamma_a\left[\tilde{C}_{lq}^{-} + \tilde{C}_{\varphi q}^{+}\right] \\ &= \gamma_a \cdot 430.511\ \left(\tilde{C}_{\varphi q}^{+} + \tilde{C}_{lq}^{-}\right)\end{aligned}$$

One-Loop Matching

$$C_7 = -2.351\,\tilde{C}_{uB} + 0.093\,\tilde{C}_{uW} + \gamma_a \cdot \left(-0.095\,\tilde{C}^{+}_{\varphi q} + 1.278\,\tilde{C}^{(3)}_{\varphi q}\right) + (1+\gamma_a)\cdot\left(-0.388\,\tilde{C}^{(3)}_{\varphi q}\right)$$

$$C_8 = -0.664\,\tilde{C}_{uG} + 0.271\,\tilde{C}_{uW} + \gamma_a \cdot \left(0.284\,\tilde{C}^{+}_{\varphi q} + 0.667\,\tilde{C}^{(3)}_{\varphi q}\right) + (1+\gamma_a)\cdot\left(-0.194\,\tilde{C}^{(3)}_{\varphi q}\right)$$

$$C_9 = 2.506\,\tilde{C}_{uB} + 2.137\,\tilde{C}_{uW} + (1+\gamma_b)\left(0.213\,\tilde{C}_{\varphi u} + 2.003\left(-\tilde{C}_{lu} - \tilde{C}_{eu}\right)\right)$$
$$+ (1+\gamma_a)\cdot\left(-0.213\,\tilde{C}^{(1)}_{\varphi q} + 4.374\,\tilde{C}^{(3)}_{\varphi q} + 2.003\left(\tilde{C}_{qe} + \tilde{C}^{(1)}_{lq}\right) - 3.163\,\tilde{C}^{(3)}_{lq}\right)$$

$$C_{10} = -7.515\,\tilde{C}_{uW} + (1+\gamma_b)\cdot\left(2.003\left(-\tilde{C}_{\varphi u} - \tilde{C}_{eu} + \tilde{C}_{lu}\right)\right)$$
$$+ (1+\gamma_a)\cdot\left(2.003\left(\tilde{C}^{(1)}_{\varphi q} + \tilde{C}_{qe} - \tilde{C}^{(1)}_{lq}\right) - 17.884\,\tilde{C}^{(3)}_{\varphi q} + 3.163\,\tilde{C}^{(3)}_{lq}\right)$$

$$C_L = 12.889\,\tilde{C}_{uW} + (1+\gamma_a)\cdot\left(2.003\left(\tilde{C}^{(1)}_{\varphi q} + \tilde{C}^{(1)}_{lq}\right) - 22.830\tilde{C}^{(3)}_{\varphi q} - 16.275\tilde{C}^{(3)}_{lq}\right)$$
$$+ (1+\gamma_b)\cdot 2.003\left(-\tilde{C}_{\varphi u} - \tilde{C}_{lu}\right)$$

$$C^{\text{mix}}_{V,LL} = -22.023\,\tilde{C}_{uW} + \gamma_a \cdot \left(14.317\,\tilde{C}^{(1)}_{\varphi q} + 11.395\,\tilde{C}^{(3)}_{\varphi q}\right).$$

MFV Spurion Expansion

$$\bar{q}_L q_L :\sim a_1 \mathbb{1} + a_2 Y_u Y_u^\dagger + a_3 Y_d Y_d^\dagger + \ldots \qquad \bar{u}_R u_R :\sim b_1 \mathbb{1} + b_2 Y_u^\dagger Y_u + \ldots$$

$$\bar{q}_L u_R :\sim (c_1 \mathbb{1} + c_2 Y_u Y_u^\dagger + \ldots) Y_u \qquad \bar{q}_L d_R :\sim (d_1 \mathbb{1} + d_2 Y_u Y_u^\dagger + \ldots) Y_d$$

$$\bar{d}_R d_R :\sim e_1 \mathbb{1} + e_2 Y_d^\dagger Y_d + \ldots$$

After rotation to mass basis:

$$\begin{aligned}
\bar{q}_L q_L : \bar{d}_{Li} d_{Lj} &\to a_1 \delta_{ij} + a_2 y_t^2 V_{ti}^* V_{tj} \\
\bar{u}_{Li} u_{Lj} &\to a_1 \delta_{ij} + a_2 y_t^2 \delta_{3i} \delta_{3j} \\
\bar{u}_{Li} d_{Lj} &\to a_1 V_{ij} + a_2 y_t^2 \delta_{3i} V_{tj} \\
\bar{d}_{Li} u_{Lj} &\to a_1 V_{ji}^* + a_2 y_t^2 V_{ti}^* \delta_{3j}
\end{aligned}$$

$$\begin{aligned}
\bar{q}_L u_L : \bar{u}_{Li} u_{Rj} &\to (c_1 + c_2 y_t^2) y_t \delta_{3i} \delta_{3j} \,, \\
\bar{d}_{Li} u_{Rj} &\to (c_1 + c_2 y_t^2) y_t V_{ti}^* \delta_{3j} \,, \\
\bar{q}_L d_R : \quad &\simeq 0 \\
\bar{u}_R u_R : \bar{u}_{Ri} u_{Rj} &\to b_1 \delta_{ij} + b_2 y_t^2 \delta_{3i} \delta_{3j} \\
\bar{d}_R d_R : \bar{d}_{Ri} d_{Rj} &\to e_1 \delta_{ij}
\end{aligned}$$