New LCSR predictions for $b \to s$ hadronic form factors

Yann Monceaux – IP2I – 30/05/2023
In collaboration with Nazila Mahmoudi and Alexandre Carvunis

Motivation: B-anomalies status

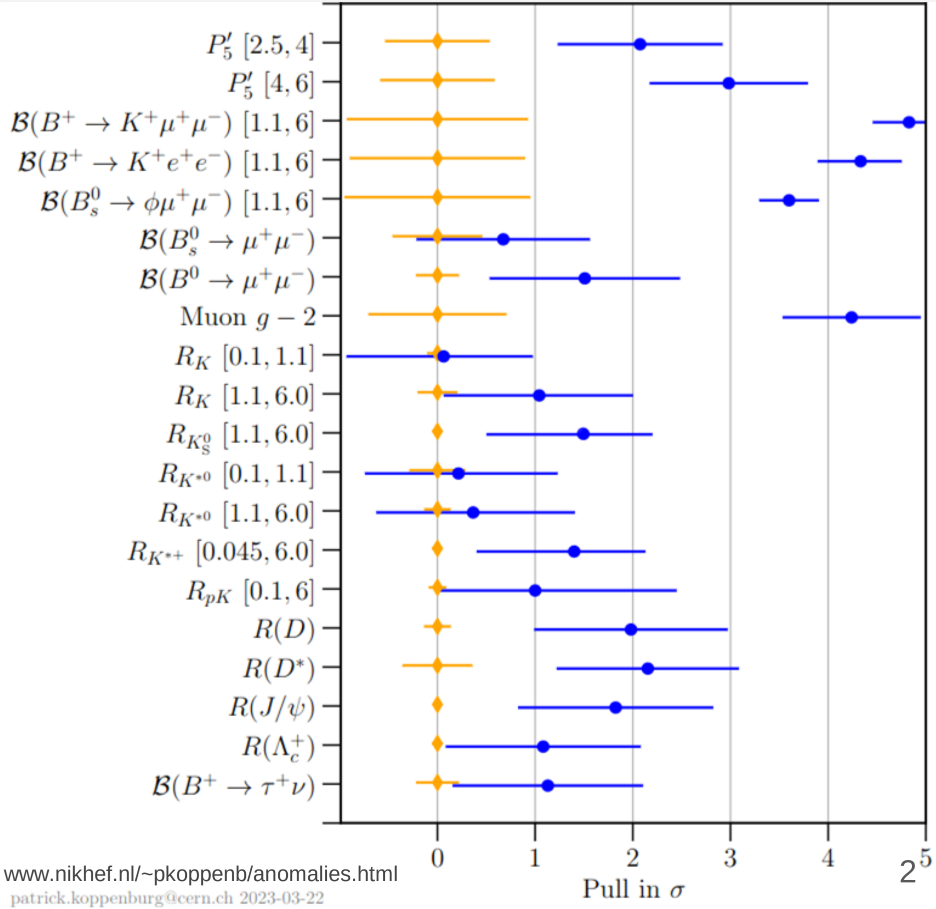
$$b \rightarrow sll$$

Anomalies in 'clean' observables gone :

- ➢ R_K and R_{K^*} (LHCb 2022)
- ➢ $BR(B_s \rightarrow \mu\mu)$ (LHCb 2021)

Deviation in angular observables and Branching fractions at low q² still standing

Theoretically challenging



www.nikhef.nl/~pkoppenb/anomalies.html

Theoretical framework:

$b \longrightarrow sll$ in the weak effective theory

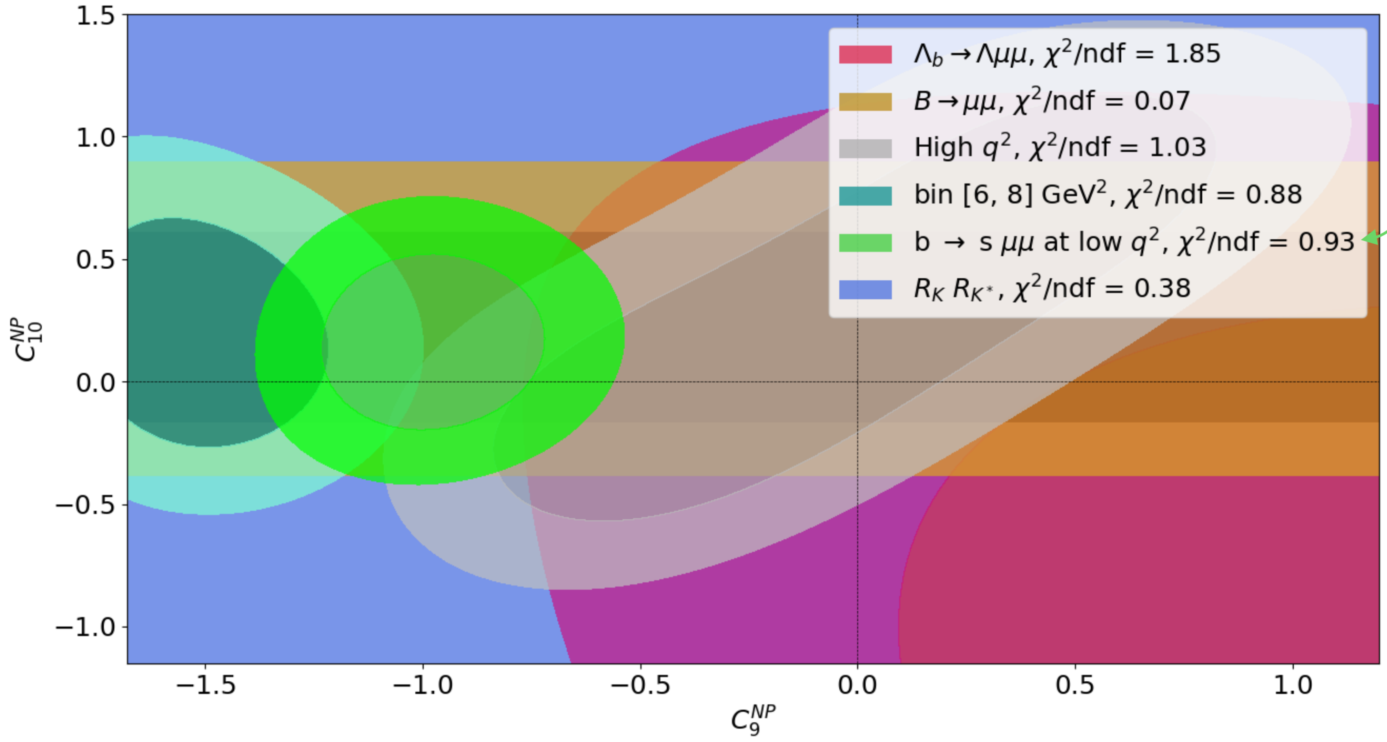
At the scale m_b $\boxed{H_{eff} = H_{eff,sl} + H_{eff,had}}$

- $H_{eff,sl} = -\underbrace{\frac{4G_F\alpha_{em}^2}{\sqrt{2}} V_{tb}V_{ts}^*}_{\mathcal{N}} \sum_{i=7,9,10,S,P} (C_i^l O_i^l + C_i'^l O_i'^l)$ ⟵ $O_7^{(')} = \frac{m_b}{e}(\bar{s}\sigma_{\mu\nu}P_{R(L)}b)F^{\mu\nu}$
$O_9^{(')} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu l)$
$O_{10}^{(')} = (\bar{s}\gamma_\mu P_{R(L)}b)(\bar{l}\gamma^\mu \gamma_5 l)$

- $H_{eff,had} = -\mathcal{N}\frac{1}{\alpha_{em}^2}\left(C_8 O_8 + C_8' + O_8' + \sum_{i=1,\ldots,6} C_i O_i\right) + \mathrm{h.c}$ ⟵ $O_1 = (\bar{s}\gamma_\mu P_L T^a c)(\bar{c}\gamma^\mu P_L T^a b)$
...

C_9-C_{10} Global fit :

SuperIso



Angular and Branching fractions

- $B \to K\ \mu\mu$
- $B \to K^*\mu\mu$
- $B_s \to \Phi\mu\mu$

See Bernat Capdevila's talk yesterday

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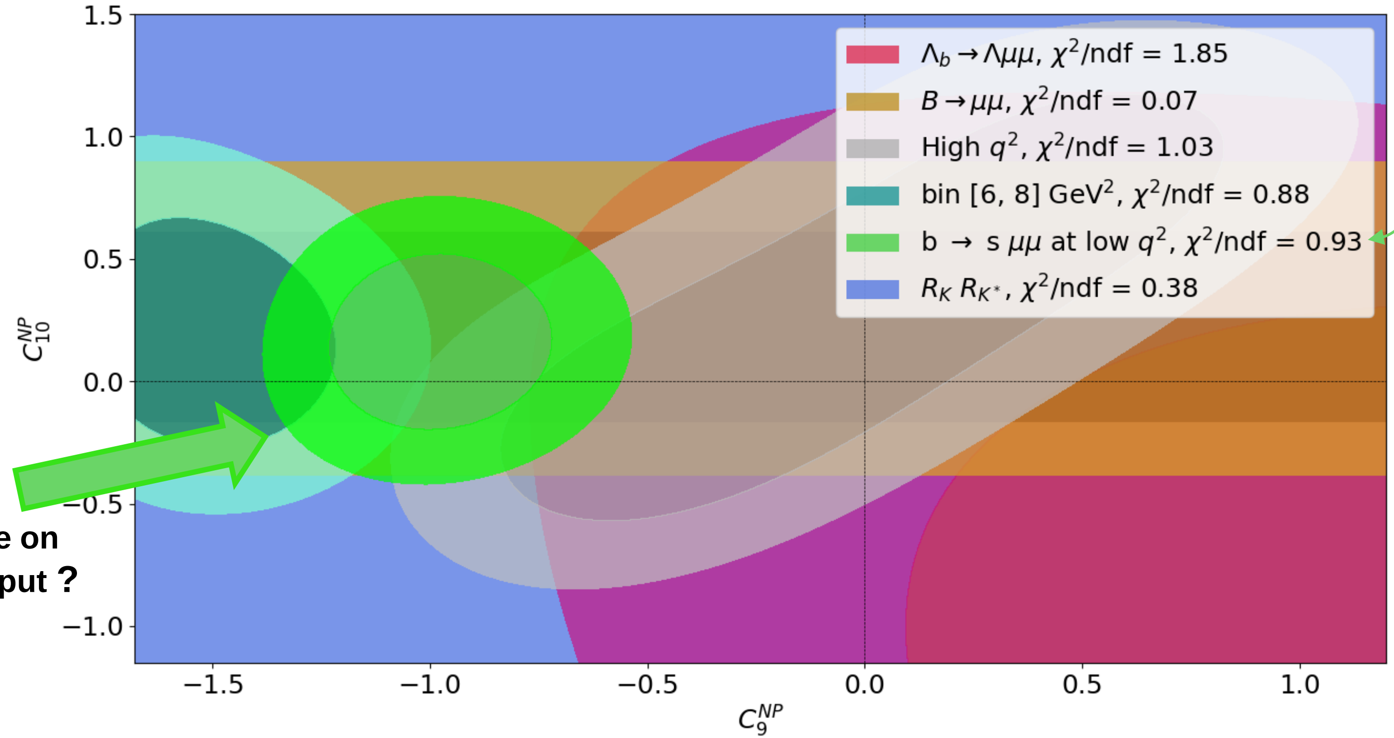


Angular and Branching fractions

- B→K μμ
- B→K*μμ
- B_s → Φμμ

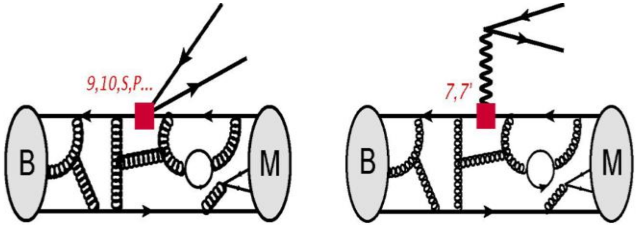
Dependance on Hadronic input ?

See Bernat Capdevila's talk yesterday

Amplitude of B → K(*)ll decays:

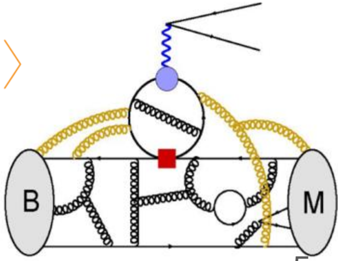
$$\mathcal{A}(B \to K^{(*)} l^+ l^-) = \mathcal{N}\left\{(C_9 L_V^\mu + C_{10} L_A^\mu)\mathcal{F}_\mu(q^2) - \frac{L_V^\mu}{q^2}\left[C_7 \mathcal{F}_\mu{}^T(q^2) + \mathcal{H}_\mu(q^2)\right]\right\}$$

- **Local** $\mathcal{F}_\mu(q^2) = \underbrace{\langle \bar{K}^{(*)}(k)|O_{7,9,10}|\bar{B}(k+q)\rangle}_{\text{Parametrized with local Form Factors}}$



Diagrams by Javier Virto

- **Non-Local** $\mathcal{H}_\mu(q^2) = i\int d^4x e^{iq.x}\langle K^{(*)}(k)|T\{j_\mu^{em}(x), C_i O_i(0)\}|\bar{B}(k+q)\rangle$



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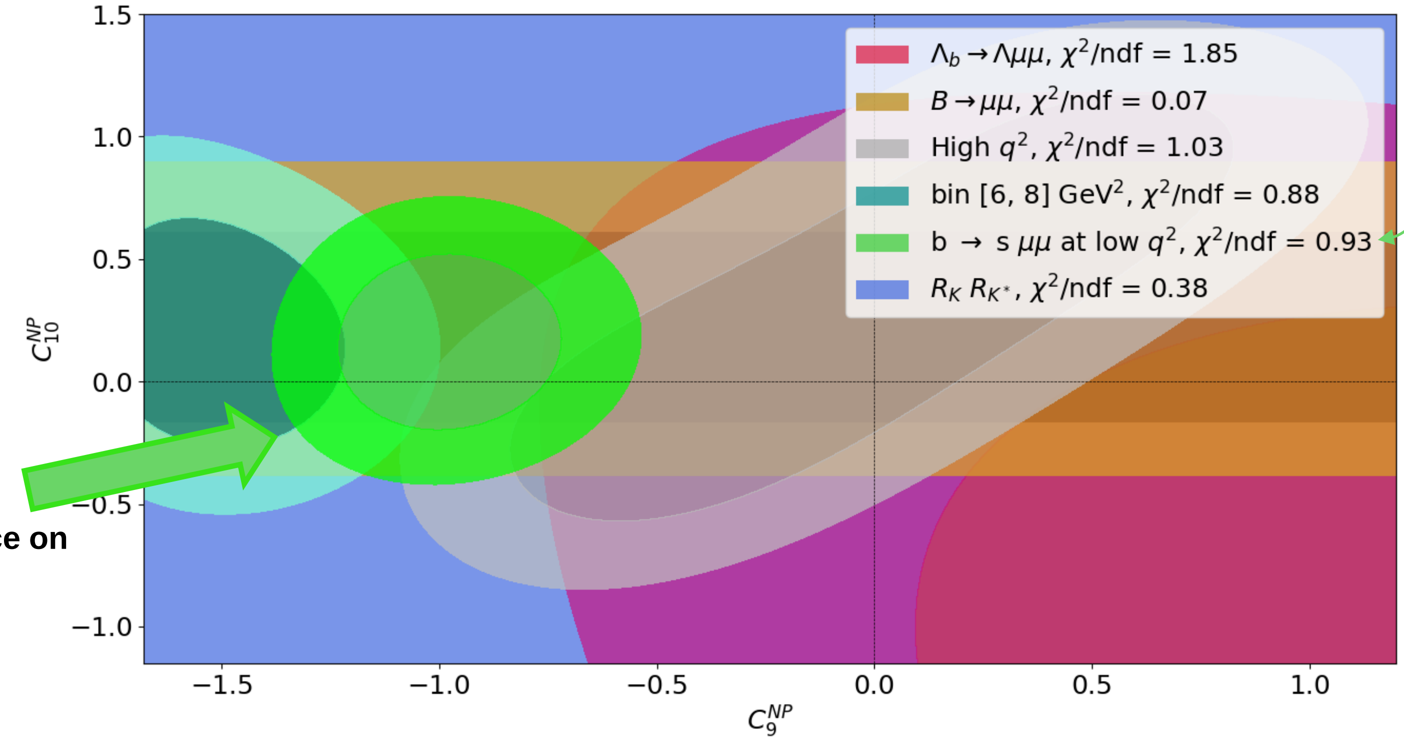


Angular and Branching fractions

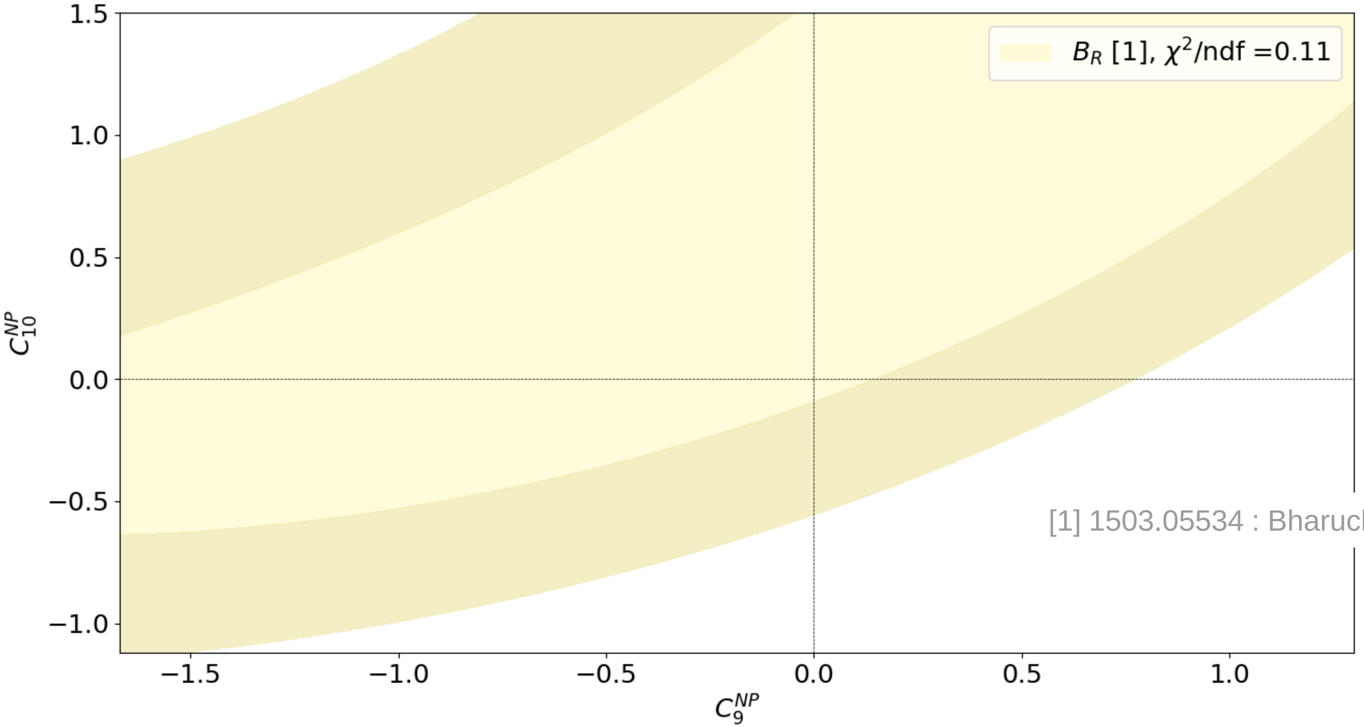
- B→K μμ
- **B→K*μμ**
- B_s → **Φ**μμ

Dependance on Local F.F?

See Bernat Capdevilla's talk yesterday

Fit of B_R of B→K*μμ at low q^2:

Impact of B→K* Local Form Factors



[1] 1503.05534 : Bharucha, Straub and Zwicky

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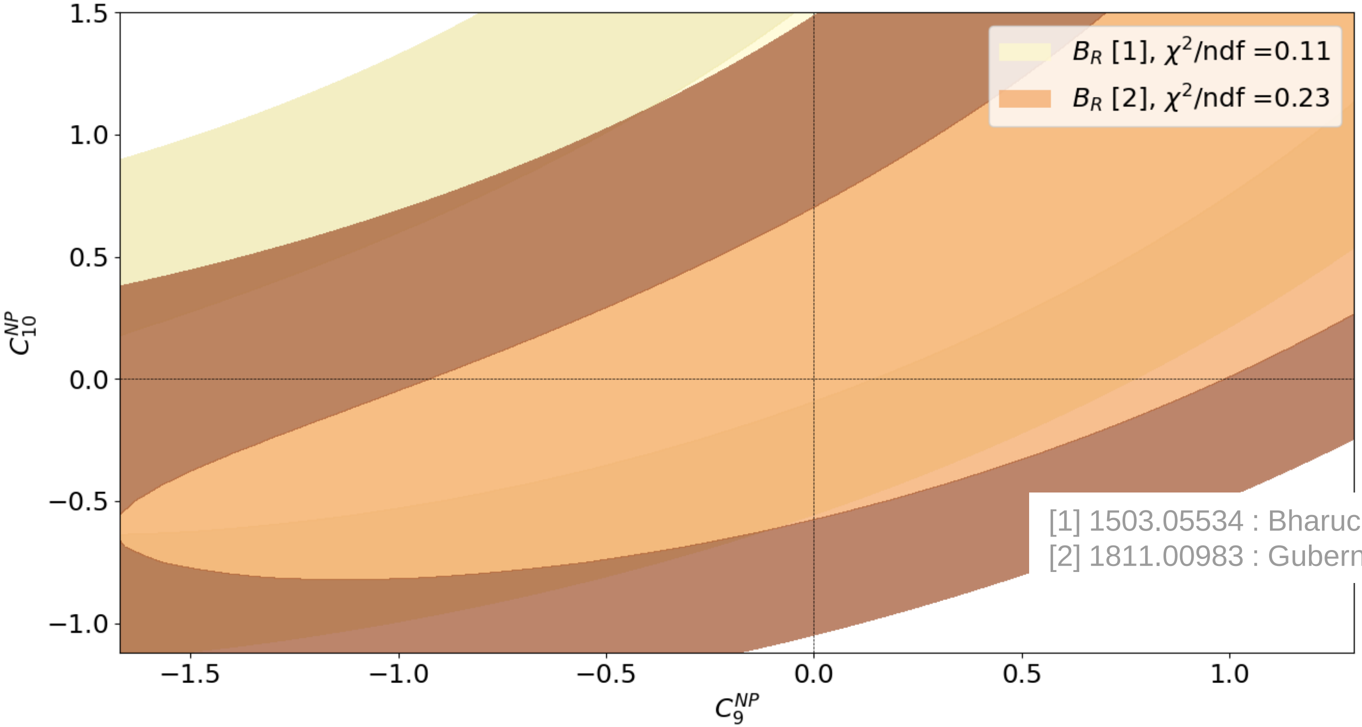

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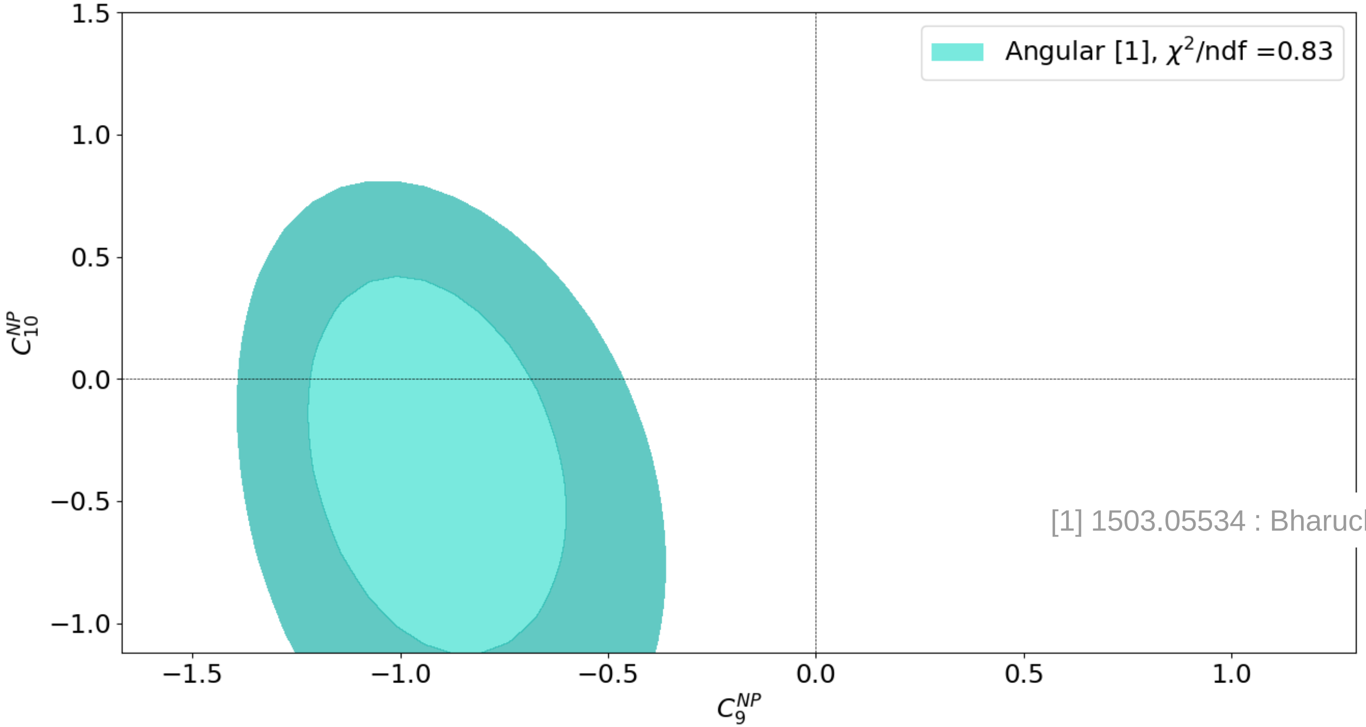

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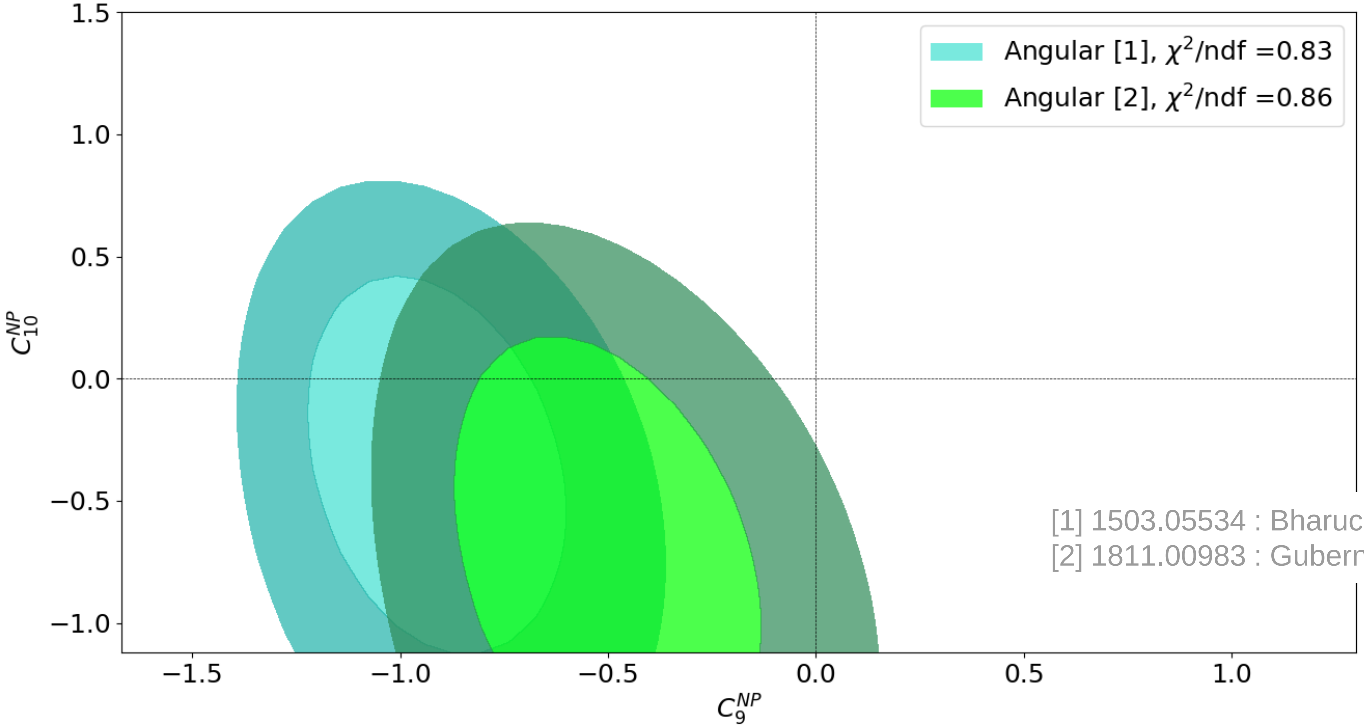

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Local Form Factors computation:

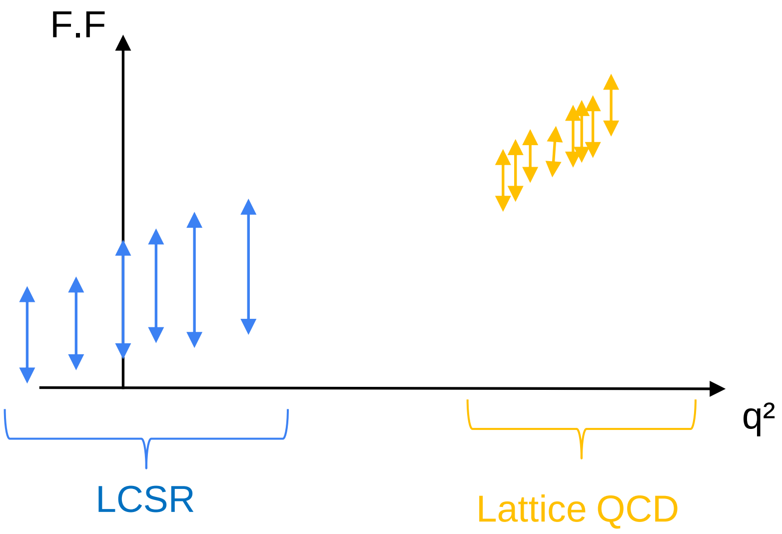
Remaining deviations in FCNC B-decays – both BR and angular observables- are sensitive to local form factors

The Wilson Coefficient fits are impacted !

How are these form factors computed?

Local Form Factors computation:

- At high-q^2 : computed on the lattice
- At low-q^2 : (mostly) Light-Cone Sum Rule (LCSR)



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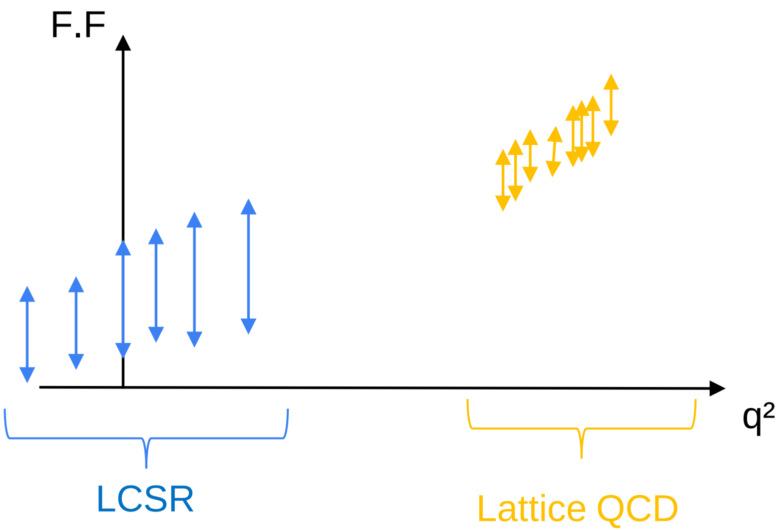
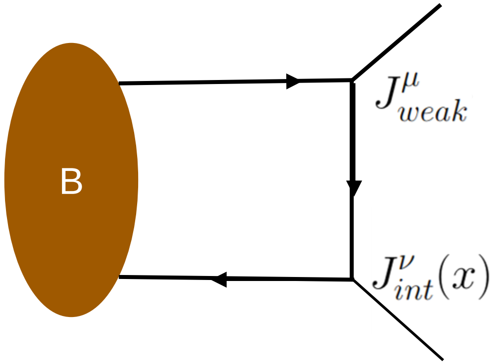
Challenging systematic uncertainties



Procedure for Light-Cone Sum Rules :

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x}\langle 0|\, T J^{\nu}_{int}(x) J^{\mu}_{weak}(0)\,|\bar{B}(q+k)\rangle$$

B to vacuum correlation function



Express it in function of the non-perturbative quantities of interest
(here form factors)

Compute it perturbatively

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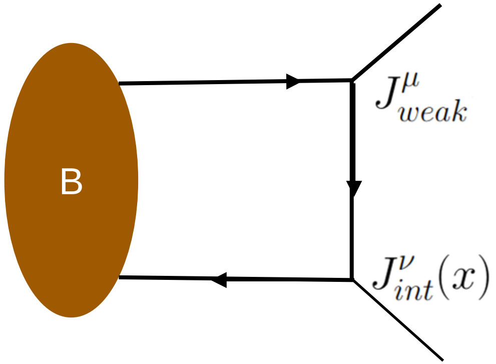

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Match both expression

Procedure for Light-Cone Sum Rules :

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Hadronic unitarity relation
+
Dispersion relation

α F.F

Density of continuum and excited stated

$$\Pi^{\mu\nu}(q,k) = \frac{\langle O|\, J^{\nu}_{int}\,|M(k)\rangle \boxed{\langle M(k)|\, J^{\mu}_{weak}\,|\bar{B}(q+k)\rangle}}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\boxed{\rho^{\mu\nu}(s)}}{s-k^2}$$

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$$K^{(F)}\frac{F(q^2)}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2}$$

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$x^2 \ll 1/\Lambda_{QCD}^2$ → Light-Cone OPE

In growing twist (dimension – spin)

Non perturbative input : B-meson LC distribution amplitudes

$$K^{(F)}\frac{F(q^2)}{m_M^2-k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2} \qquad f_B m_B \int_0^{+\infty} ds \sum_{n=1}^{+\infty} \frac{I_n(s)}{(s-k^2)^n}$$

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s

What we want

What we have

$$K^{(F)}\frac{\boxed{F(q^2)}}{m_M^2-k^2} + \frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds \frac{\rho(s)}{s-k^2} = \boxed{f_B m_B \int_0^{+\infty} ds \sum_{n=1}^{+\infty}\frac{I_n(s)}{(s-k^2)^n}}$$

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In growing twist (dimension – spin)

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What we want **What is this?** **What we have**

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Estimating the density:

At leading twist:

$$\boxed{K^{(F)}\frac{F(q^2)}{m_M^2-k^2}}+\boxed{\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\frac{\rho(s)}{s-k^2}}=\boxed{f_B m_B\int_0^{+\infty}ds\frac{I_1(s)}{(s-k^2)}}$$

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Borel transform

M^2 : Borel parameter

$$F(M^2)\equiv\mathcal{B}_{M^2}F(k^2)=\lim_{-k^2,n\to\infty\text{ and }\frac{-k^2}{n}=M^2}\frac{(-k^2)^{n+1}}{n!}\left(\frac{d}{dk^2}\right)^n F(k^2)$$

$\Big\downarrow$ Supress higher states of unknow contribution

$$\boxed{K^{(F)}F(q^2)e^{-m^2/M^2}}+\boxed{\frac{1}{2\pi}\int_{s_0^h}^{+\infty}ds\rho(s)e^{-s/M^2}}=\boxed{f_B m_B\int_0^{+\infty}dsI_1(s)e^{-s/M^2}}$$

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Semi-Global Quark Hadron duality

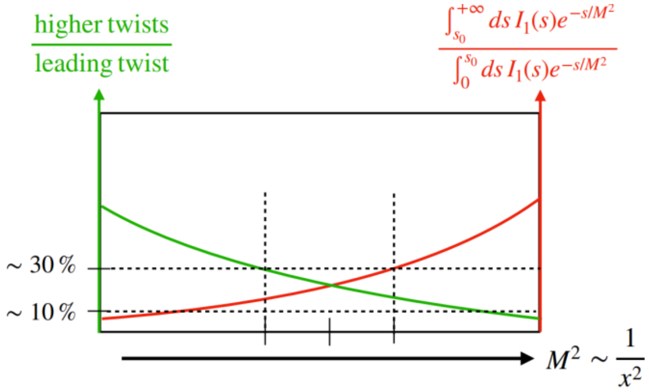
s0 : duality threshold

$$\boxed{\frac{1}{2\pi}\int_{s_0^h}^{+\infty} ds\rho(s)e^{-s/M^2}\approx f_B m_B\int_{s_0}^{+\infty} dsI_1(s)e^{-s/M^2}}$$

Setting the parameters:

$$F(q^2) = \frac{f_B m_B}{K^{(F)}} \int_0^{s_0} ds I_1(s) e^{-(s - m^2)/M^2}$$

- Borel parameter M² : compromise between supression of higher twists, and continuum and excited states contribution



Range of the Borel parameter
E.g. for $B \to K$: $M^2 \in [0.5, 1.5]\,\mathrm{GeV}^2$

- Duality threshold s0 : Independence of F(q²) w.r.t M² :

Daughter Sum Rule : $\frac{d}{dM^2} F(q^2) = 0$

(Preliminary) results:

s_0 from SVZ sum rules Khodjamirian-Mannel hep-ph/0308297

Form Factor $q^2 = 0$	Our Result	Gubernari et al. 2018	Other results
$f_+^{B\to\pi}$	0.249 ± 0.064 PRELIMINARY	0.21 ± 0.07	0.258 ± 0.031 [1] 0.25 ± 0.05 [2] 0.301 ± 0.023 [3] 0.280 ± 0.037 [4]
$f_T^{B\to\pi}$	0.259 ± 0.065 PRELIMINARY	0.19 ± 0.06	0.253 ± 0.028 [1] 0.21 ± 0.04 [2] 0.273 ± 0.021 [3] 0.26 ± 0.06 [4]
$f_+^{B\to K}$	0.376 ± 0.068 PRELIMINARY	0.27 ± 0.08	0.331 ± 0.041 [1] 0.31 ± 0.04 [2] 0.395 ± 0.033 [3] 0.364 ± 0.05 [4]
$f_T^{B\to K}$	0.367 ± 0.053 PRELIMINARY	0.25 ± 0.07	0.358 ± 0.037 [1] 0.27 ± 0.04 [2] 0.381 ± 0.027 [3] 0.363 ± 0.08 [4]

- Following a very similar procedure to Gubernari et al 2018
- Results in agreement with previous calculations

[1] Ball and Zwicky 2005, light meson DA's

[2] Khodjamirian, Mannel, Offen 2007, B meson DA's

[3] Khodjamirian, Rusov, LCSR + CKM

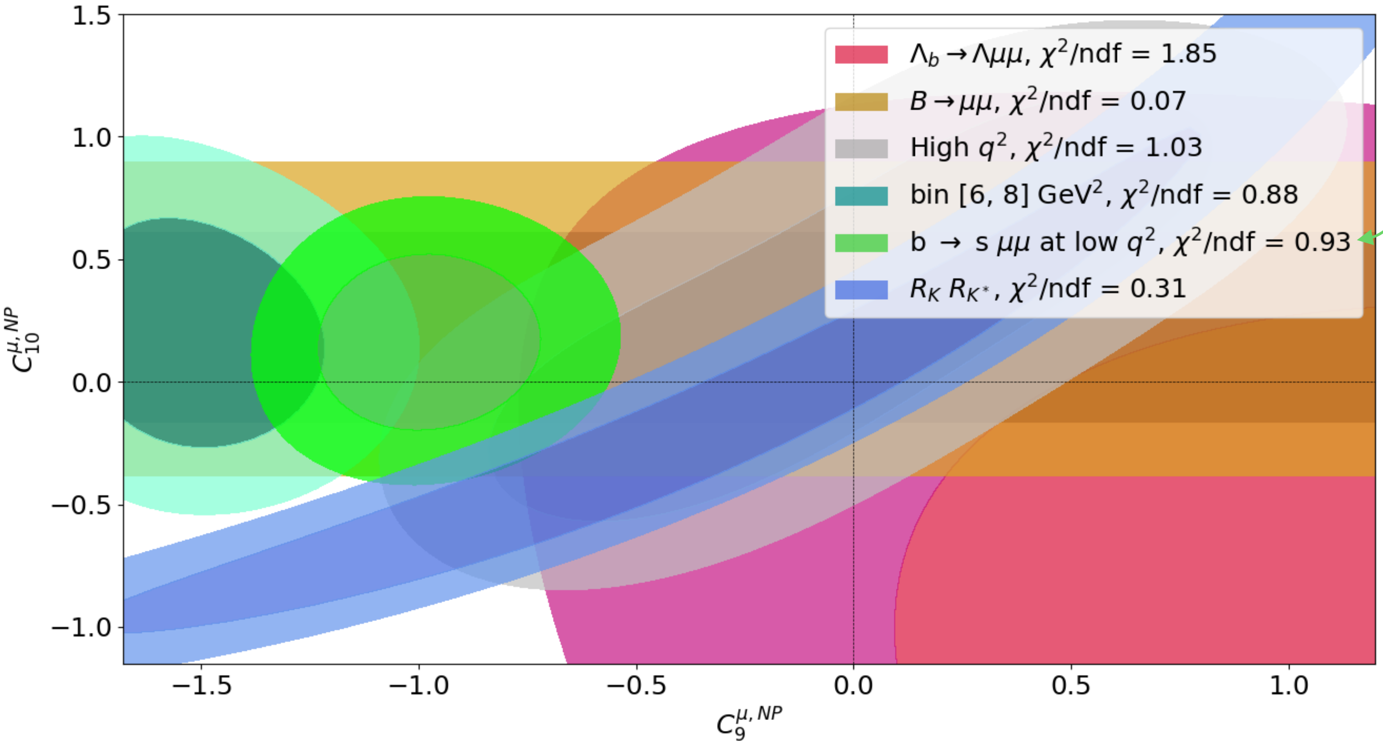
[4] Lu, Shen, Wang, Wei, LCSR + QCD SR up to twist 6

Conclusion :

- Remaining deviations in FCNC B-decays – both BR and angular observables- are sensitive to form factors
- LCSR can be used to predict form factors at low q^2 but suffer from large uncertainties
- Coming results : $B \rightarrow K^{(*)}$, $D^{(*)}$ local form factors

Backup:

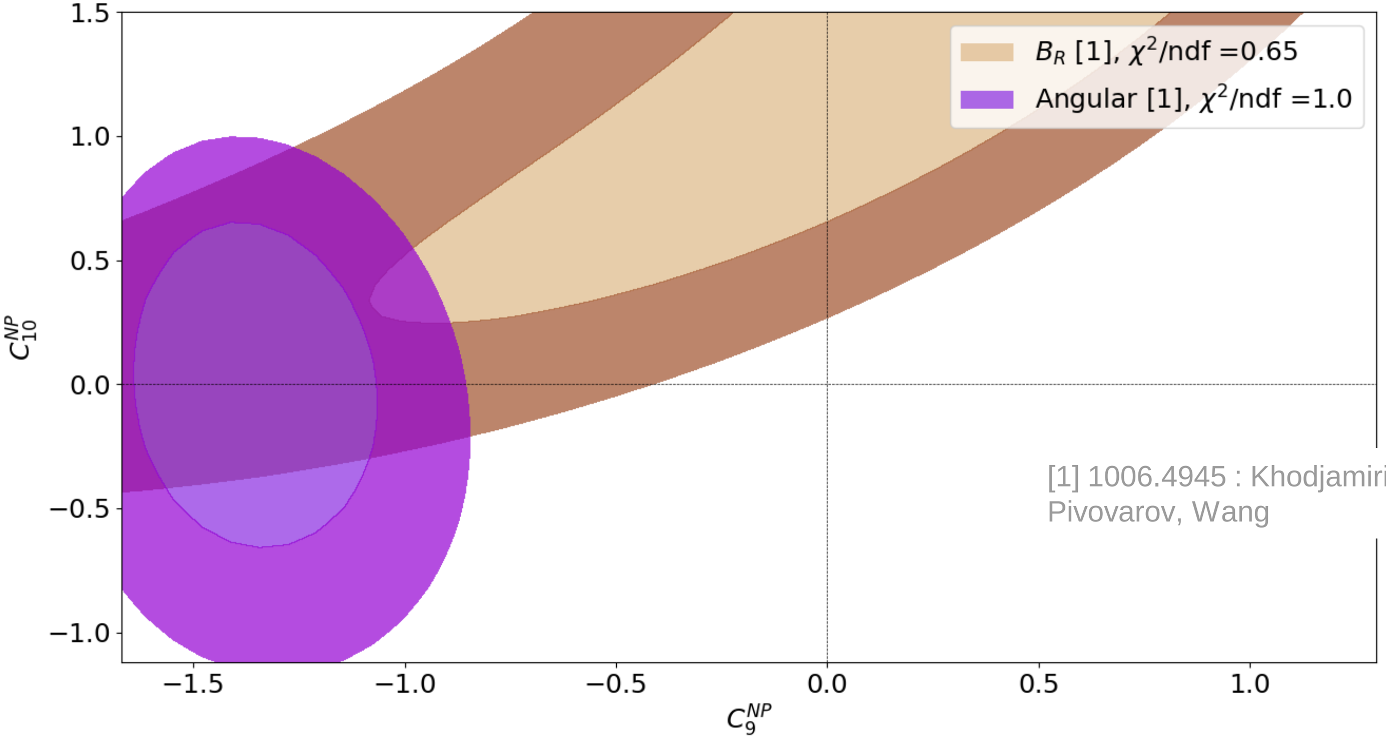
C_9^μ-C_{10}^μ fit :



Angular and
Branching fractions

Angular and B_R at low q^2:

Impact of B→K* Non-Local Contributions



[1] 1006.4945 : Khodjamirian, Mannel, Pivovarov, Wang

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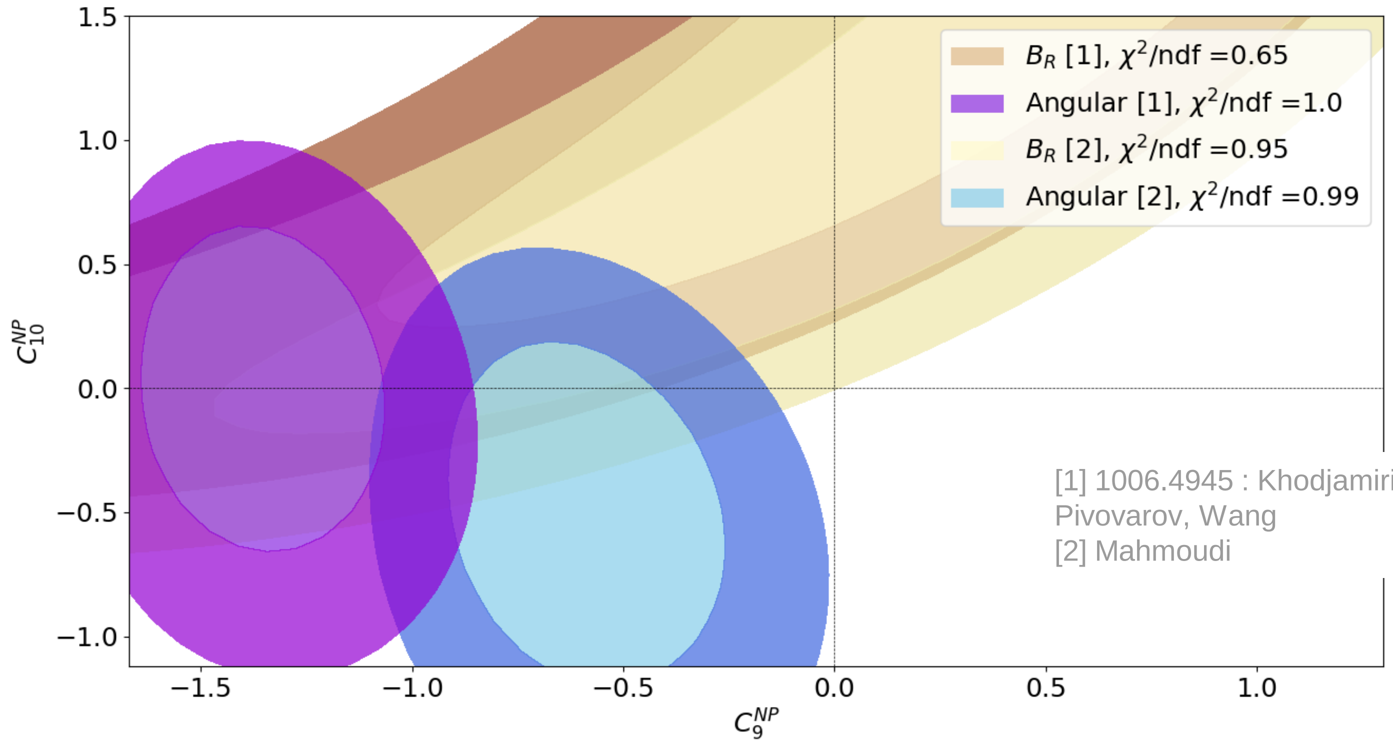

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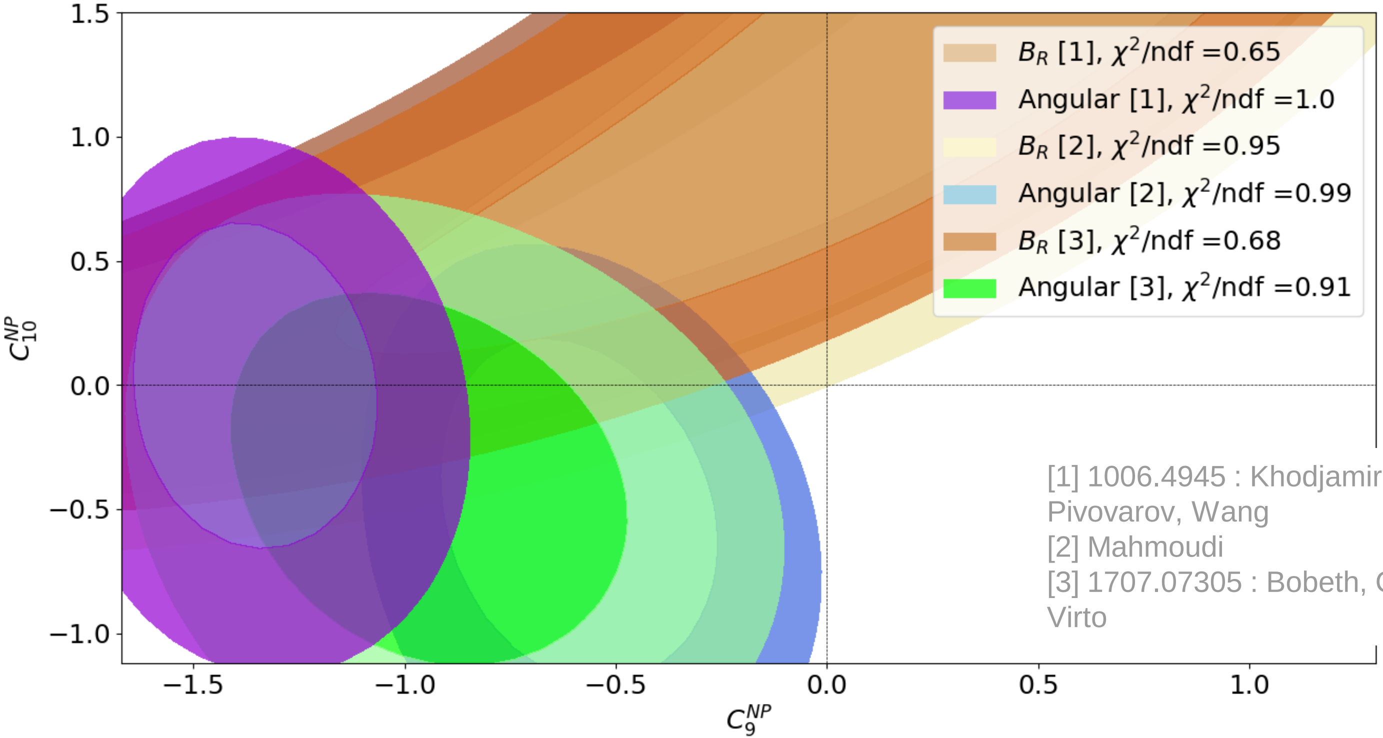
[2] Mahmoudi

Angular and B_R at low q^2:

Impact of B→K* Non-Local Contributions



[1] 1006.4945 : Khodjamirian, Mannel, Pivovarov, Wang
[2] Mahmoudi
[3] 1707.07305 : Bobeth, Chrzaszcz, van Dyk, Virto

Amplitude of $\mathbf{B} \to \mathbf{M}\ell\ell$ decays

Local contributions - definition of the form factors

- 3 independent f.f. for B to pseudoscalar meson:

$$\left\langle P(k) \middle| \bar{q}_1 \gamma^\mu b \middle| B(p) \right\rangle = \left[(p+k)^\mu - \frac{m_B^2 - m_P^2}{q^2} q^\mu \right] f_+^{B\to P} + \frac{m_B^2 - m_P^2}{q^2} q^\mu f_0^{B\to P}$$

$$\left\langle P(k) \middle| \bar{q}_1 \sigma^{\mu\nu} q_\nu b \middle| B(p) \right\rangle = \frac{i f_T^{B\to P}}{m_B + m_P} \left[q^2 (p+k)^\mu - \left(m_B^2 - m_P^2 \right) q^\mu \right]$$

- 7 independent f.f. for B to vector meson:

$$\left\langle V(k,\eta) \middle| \bar{q}_1 \gamma^\mu b \middle| B(p) \right\rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma \frac{2 V^{B\to V}}{m_B + m_V}$$

$$\left\langle V(k,\eta) \middle| \bar{q}_1 \gamma^\mu \gamma_5 b \middle| B(p) \right\rangle = i\eta_\nu^* [g^{\mu\nu} \left(m_B + m_V \right) A_1^{B\to V} - \frac{(p+k)^\mu q^\nu}{m_B + m_V} A_2^{B\to V} - q^\mu q^\nu \frac{2 m_V}{q^2} \left(A_3 - A_0 \right)]$$

$$\left\langle V(k,\eta) \middle| \bar{q}_1 i \sigma^{\mu\nu} q_\nu b \middle| B(p) \right\rangle = \epsilon^{\mu\nu\rho\sigma} \eta_\nu^* p_\rho k_\sigma 2 T_1^{B\to V}$$

$$A_3^{B\to V} \equiv \frac{m_B + m_V}{2 m_V} A_1^{B\to V} - \frac{m_B - m_V}{2 m_V} A_2^{B\to V}.$$

$$\left\langle V(k,\eta) \middle| \bar{q}_1 i \sigma^{\mu\nu} q_\nu \gamma_5 b \middle| B(p) \right\rangle = i\eta_\nu^* [\left(g^{\mu\nu} \left(m_B^2 - m_V^2 \right) - (p+k)^\mu q^\nu \right) T_2^{B\to V} + q^\nu \left(q^\mu - \frac{q^2}{m_B^2 - m_V^2} (p+k)^\mu \right) T_3^{B\to V}]$$

LCSR: The correlation function

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0\,|\,TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)\,|\,\bar{B}(P_B = q+k) >$$

Unitarity relation

$$2\text{Im}(\Pi^{\mu\nu}) = \sum_{\text{X}}\int \text{d}\tau_{\text{X}} < 0\,|\,\text{J}^{\nu}_{\text{int}}\,|\,\text{X} > < \text{X}\,|\,\text{J}^{\mu}_{\text{weak}}\,|\,\overline{\text{B}} > (2\pi)^4\delta^{(4)}(\text{k}-\text{P}_\text{X})$$

Dispersion relation

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{1}{\pi}\int_{t_{min}}^{+\infty} ds\frac{\text{Im}\,\Pi^{\mu\nu}(\text{q}^2,\text{s})}{s-k^2}$$

$\propto$ decay constant of the light meson

What we want to compute

$$\Pi^{\mu\nu}(q^2,k^2) = \frac{< 0\,|\,j_\nu\,|\,M(k) > < M(k)\,|\,j_\mu\,|\,B >}{m_M^2 - k^2} + \frac{1}{2\pi}\int_{s_0^h}^{\infty} ds\frac{\rho^{\mu\nu}}{s-k^2}$$

Continuum, a priori unknown

$$\left\langle 0 \left| \bar{q}_2\gamma^\nu\gamma_5 q_1 \right| P(k) \right\rangle = ik^\nu f_P$$

$$\left\langle 0 \left| \bar{q}_2\gamma^\nu q_1 \right| V(k,\eta) \right\rangle = i\eta^\nu m_V f_V$$

Light-Cone Sum Rules

B-meson distribution amplitude

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0|TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)|\bar{B}(P_B = q+k) >$$

Heavy Quark Effective Theory

$$\Pi^{\mu\nu}(q,k) = i\int d^4x e^{ik.x} < 0|TJ^{\nu}_{int}(x)J^{\mu}_{\text{weak}}(0)|\bar{B}_v(h_v = \tilde{q}+k) > + \mathcal{O}(1/m_b)$$

$$\Pi^{\mu\nu} = \int d^4x \int \frac{d^4p'}{(2\pi)^4} e^{i(k-p').x} \left[\Gamma^{\nu}_2 \frac{\not{p}' + m_1}{m_1^2 - p'^2}\Gamma^{\mu}_1\right]_{\alpha\beta} < 0|\bar{q}_2{}^{\alpha}(x)h_v^{\beta}(0)|\bar{B}(v) >$$

Condition for Perturbativity and Light-Cone dominance:

$$\tilde{q} \leq m_b^2 + m_b k^2/\Lambda_{\text{had}}$$
$$k^2 \ll \Lambda^2_{\text{had}}$$

Perturbative piece (Fully calculable)

Can be expressed as a function of B-meson distribution amplitudes

$$< 0|\bar{q}_2{}^{\alpha}(x)h_v^{\beta}(0)|\bar{B}(v) > = -\frac{if_B m_B}{4}\int_0^{+\infty} dw e^{-iwv.x}\Phi_{2p}(w)^{\beta\alpha}$$
$$= \sum_t -\frac{if_B m_B}{4}\int_0^{+\infty} dw e^{-iwv.x}\Phi^t_{2p}(w)^{\beta\alpha}$$

Near the light-cone ($x^2 \ll 1/\Lambda^2_{\text{QCD}}$) the DA's are expanded in a series of operators with increasing (twist = dimension - spin)
At $x^2 = 0$, the only non-zero contribution is twist 2

List of Form Factors used

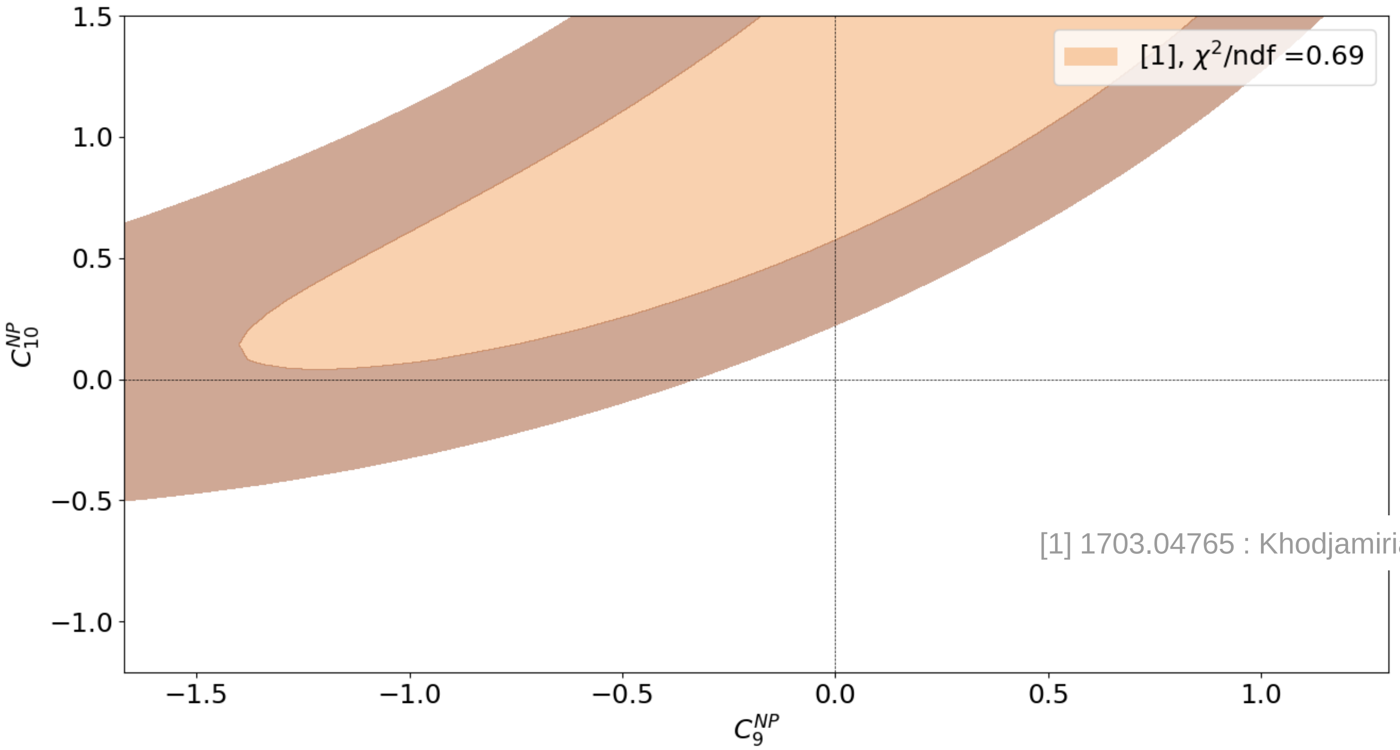
B → K : 2018 Gubernari et al

B→K* : 2015 BSZ

Bs→**Φ** : 2015 BSZ

B_R at low q^2:

Impact of B→K Local Form Factors



[1] 1703.04765 : Khodjamirian and Rusov

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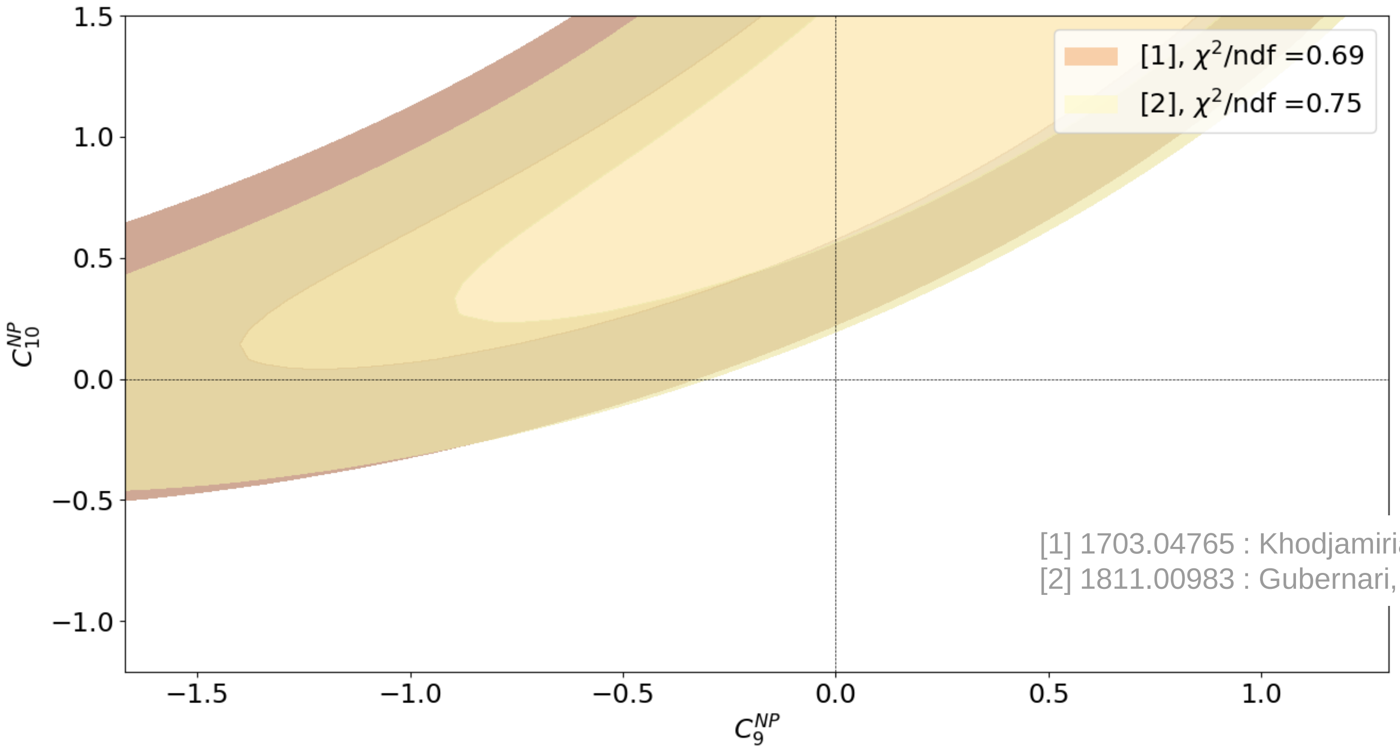

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[2] 1811.00983 : Gubernari, Kokulu and van Dyk

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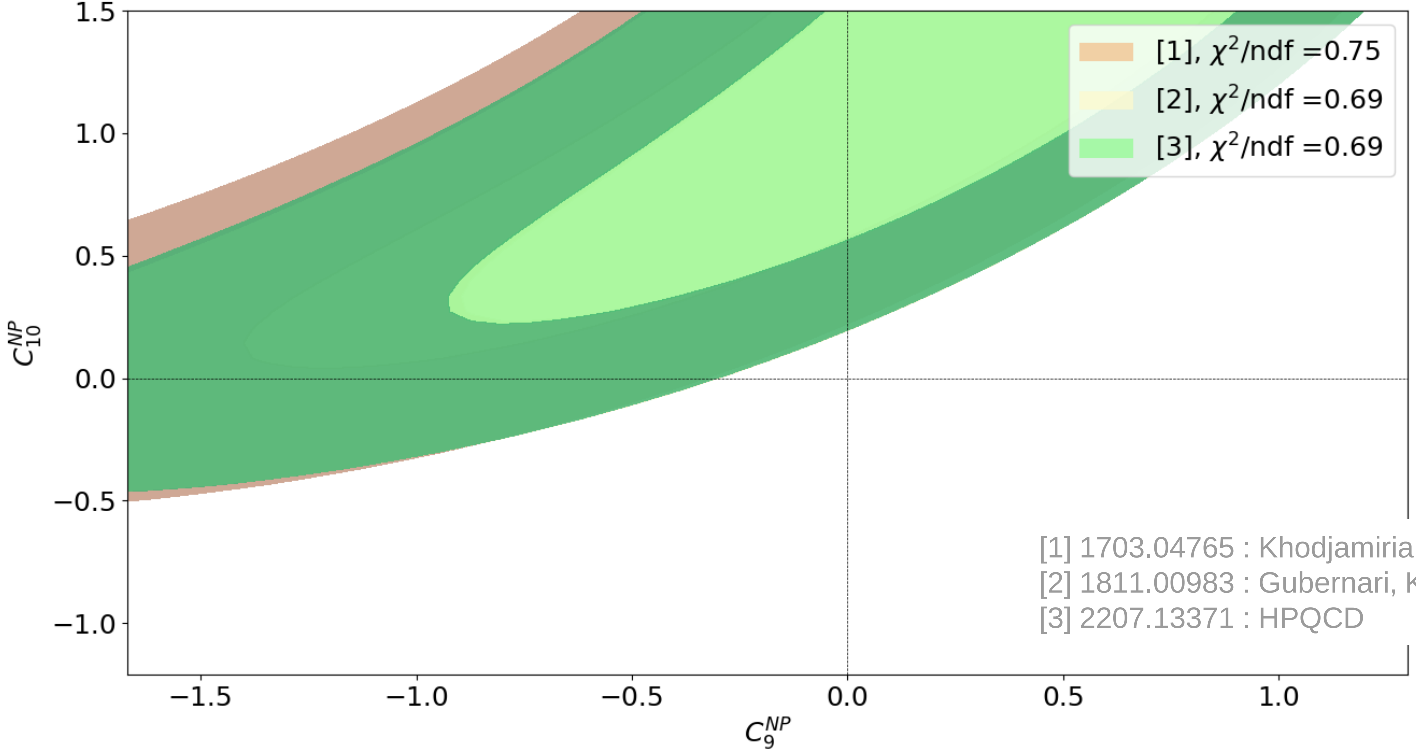
Impact of B→K Local Form Factors



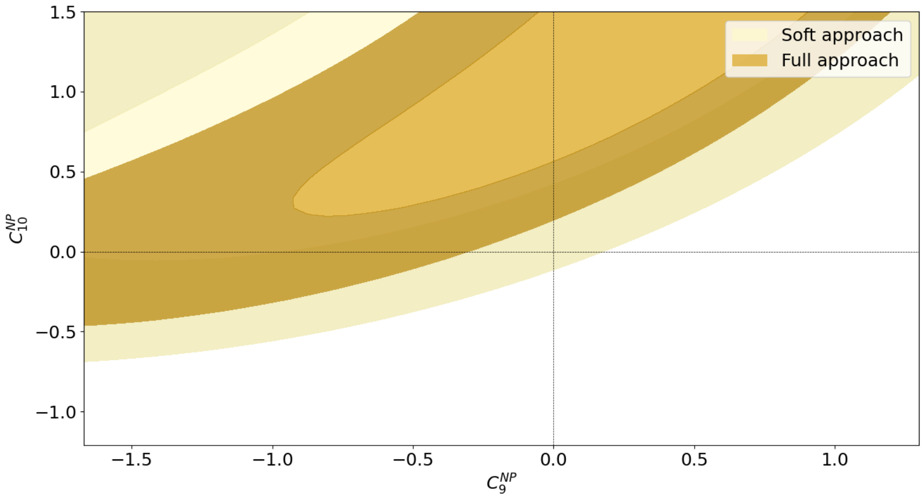
[1] 1703.04765 : Khodjamirian and Rusov
[2] 1811.00983 : Gubernari, Kokulu and van Dyk
[3] 2207.13371 : HPQCD

List of b→sll operators

$$
\begin{aligned}
O_1 &= (\bar{s}\gamma_\mu T^a P_L c)(\bar{c}\gamma^\mu T^a P_L b)\ , \\
O_2 &= (\bar{s}\gamma_\mu P_L c)(\bar{c}\gamma^\mu P_L b)\ , \\
O_3 &= (\bar{s}\gamma_\mu P_L b)\sum_q (\bar{q}\gamma^\mu q)\ , \\
O_4 &= (\bar{s}\gamma_\mu T^a P_L b)\sum_q (\bar{q}\gamma^\mu T^a q)\ , \\
O_5 &= (\bar{s}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} P_L b)\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} q)\ , \\
O_6 &= (\bar{s}\gamma_{\mu_1}\gamma_{\mu_2}\gamma_{\mu_3} T^a P_L b)\sum_q (\bar{q}\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3} T^a q)\ , \\
O_7 &= \frac{e}{16\pi^2}\Big[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)b\Big] F_{\mu\nu}\ , \\
O_8 &= \frac{g}{16\pi^2}\Big[\bar{s}\sigma^{\mu\nu}(m_s P_L + m_b P_R)T^a b\Big] G^a_{\mu\nu}\ ,
\end{aligned}
$$

Full and Soft approach:

- Full : using the whole set of form factors
- Soft : using the symmetry relations between form factors to eliminate form factors ratios



$$\frac{f_0}{f_+} = \frac{2E_K}{M_B}\left(1 + \frac{\alpha_s C_F}{4\pi}[2-2L] + \frac{\alpha_s C_F}{4\pi}\frac{M_B(M_B-2E_K)}{(2E_K)^2}\frac{\Delta F_P}{\xi_P}\right), \tag{739}$$

$$\frac{f_T}{f_+} = \frac{M_K+M_B}{M_B}\left(1 + \frac{\alpha_s C_F}{4\pi}\left[\ln\frac{m_b^2}{\mu^2} + 2L\right] - \frac{\alpha_s C_F}{4\pi}\frac{M_B}{2E_K}\frac{\Delta F_P}{\xi_P}\right) \tag{740}$$

where

$$\Delta F_P = \frac{8\pi^2 f_B f_P}{N_C M_B}\int \frac{d\omega}{\omega}\Phi_{B,+}(\omega)\int_0^1 du \frac{\Phi_K(u)}{\bar{u}}. \tag{741}$$

and

$$L \equiv -\frac{m_b^2 - q^2}{q^2}\ln\left(1 - \frac{q^2}{m_b^2}\right) \tag{742}$$