Meson light-front wavefunctions-applications to B transition form factors

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Light-Front Wavefunction(LFWF)

$$H^{\mathrm{LF}}_{\mathrm{QCD}}|\Psi(P)\rangle = M^2|\Psi(P)\rangle$$

where $H^{\mathrm{LF}}_{\mathrm{QCD}} = P^+P^- - P_\perp^2$ is the LF QCD Hamiltonian and M is the hadron mass. At equal light-front time ($x^+ = 0$) and in the light-front gauge $A^+ = 0$, the hadron state $|\Psi(P)\rangle$ admits a Fock expansion, i.e.

$$|\Psi(P^+, \mathbf{P}_\perp, S_z)\rangle = \sum_{n,h_i}\int [\mathrm{d}x_i][\mathrm{d}^2\mathbf{k}_{\perp i}]\frac{1}{\sqrt{x_i}}\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)|n : x_iP^+, x_i\mathbf{P}_\perp + \mathbf{k}_{\perp i}, h_i\rangle$$

where $\Psi_n(x_i, \mathbf{k}_{\perp i}, h_i)$ is the LFWF of the Fock state with n constituents and the integration measures are given by

$$[\mathrm{d}x_i] \equiv \prod_i^n \mathrm{d}x_i\delta(1 - \sum_{j=1}^n x_j) \qquad [\mathrm{d}^2\mathbf{k}_{\perp i}] \equiv \prod_{i=1}^n \frac{\mathrm{d}^2\mathbf{k}_{\perp i}}{2(2\pi)^3}16\pi^3\delta^2(\sum_{j=1}^n \mathbf{k}_{\perp i}) \,.$$

$(k_i^+, k_i^-, \mathbf{k}_{\perp i})$ and h_i are the momentum and helicity of the i^{th} constituent and $x_i = k_i^+/P^+$.

The valence meson LFWF

For $n = 2$,

$$\mathbf{k}_{\perp 1} = -\mathbf{k}_{\perp 2} = \mathbf{k}_{\perp}$$

$$x_1 = 1 - x_2 = x$$

The position-space conjugate of $\mathbf{k}_{\perp}$, denoted by $\mathbf{b}_{\perp} = b_{\perp} e^{i\varphi}$, is the transverse separation between the quark and the antiquark.

Introduce a new light-front variable $\boldsymbol{\zeta} = \sqrt{x(1-x)}\mathbf{b}_{\perp} = \zeta e^{i\varphi}$ leads to the meson LFWF in the position-space:

$$\Psi(\zeta, x, \phi) \xlongequal{\text{factorization}} \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}} e^{iL\phi} X(x)$$

$\phi(\zeta)$ and $X(x)$ are referred to as the transverse and longitudinal modes.

Holographic Schrödinger equation

Brodsky, de Téramond (PRL, 09)
Brodsky, de Téramond, Dosch, Erlich (Phys. Rep. 15)

In the semi-classical limit, i.e. zero quark mass and no quantum loop, based on AdS/CFT, one can show that the transverse mode of LFWF of the valence ($n = 2$ for mesons) state can be obtained from a 1-dimensional Schrödinger-like wave equation for the:

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U_\perp(\zeta)\right)\phi(\zeta) = M_\perp^2\phi(\zeta)$$

the potential is uniquely determined from the conformal symmetry breaking mechanism and correspondence with weakly coupled string modes in AdS_5 space, which results in a light-front harmonic oscillator potential in physical spacetime with confinement scale κ:

$$U_\perp(\zeta, J) = \kappa^4\zeta^2 + \kappa^2(J-1)$$

$J = L + S$ is the total meson angular momemtum.

Solving HSE: Eigenvalues and Eigenfunctions

With the confining potential specified, one can solve the holographic Schrödinger equation to obtain the meson mass spectrum,

$$M_\perp^2 = 4\kappa^2 \left(n_\perp + L + \frac{S}{2} \right)$$

and the corresponding normalized eigenfunctions,

$$\phi_{nL}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} \exp\left(\frac{-\kappa^2 \zeta^2}{2} \right) L_n^L(x^2 \zeta^2) \, .$$

- Lightest bound state ($n_\perp = L = S = 0$) is massless ($M_\perp = 0$)
- $M_\perp^2 = 4\kappa^2 L \Rightarrow \kappa = 0.54 \text{ GeV from Regge slope}$

Longitudinal mode

- To completely specify the holographic meson wavefunction, we need the analytic form of the longitudinal mode $X(x)$. This is obtained by matching the expressions for the pion EM or gravitational form factor in physical spacetime and in AdS space resulting in $X(x) = \sqrt{x(1-x)}$
- An alternative *dynamical* method is to obtain $X(x)$ from (1+1) QCD modeled by 't Hooft Equation (PRD 104 (2021) 7, 074013; PLB 836 (2023) 137628)

Light front Wavefunction for pseudoscalar and vector mesons

For pseudoscalar and vector mesons (like π, K, ρ, K^* and ϕ), we set $n_\perp = 0, L = 0$ to obtain

$$\Psi_{0,0}(x,\zeta) = \frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]$$

- The spin structure (helicity wavefunction) separates pseudoscalar from vector mesons (PRD95.074008(2017); PRD98.034010(2018))
- Away from the chiral limit, how to consider quark mass effects?
  - BdT prescription: Consider the shift in kinetic energy due to a small quark mass which consequently leads to modified wavefunction

$$\Psi_\lambda(x,\zeta) = \mathcal{N}_\lambda\sqrt{x(1-x)}\exp\left[-\frac{\kappa^2\zeta^2}{2}\right]\exp\left[-\frac{(1-x)m_q^2 + zm_{\bar{q}}^2}{2\kappa^2 x(1-x)}\right]$$

  - Consider quark mass effects through longitudinal dynamics.

The 't Hooft Equation

G. 't Hooft, A Two-Dimensional Model for Mesons, Nucl. Phys. B 75 (1974) 461470

$$\left(\frac{m_q^2}{x} + \frac{m_{\bar{q}}^2}{1-x}\right)\chi(x) + U_L(x)\chi(x) = M_L^2\chi(x)$$

$$U_L(x)\chi(x) = \frac{g^2}{\pi}\mathcal{P}\int \mathrm{d}y \frac{\chi(x)-\chi(y)}{(x-y)^2}$$

The longitudinal mode $\longrightarrow X(x) = \sqrt{x(1-x)}\chi(x)$, $M^2 = M_\perp^2 + M_L^2$
Hadron spectroscopy using Holographic QCD plus longitudinal dynamics:
PLB 836 (2023) 137628; PRD 104 (2021) 7, 074013

Light cone DAs in terms of LFWF

PRD87.054013(2013)

$$\phi_\rho^{\parallel}(z,\mu) = \frac{N_c}{\pi f_\rho M_\rho} \int \mathrm{d}r\ \mu J_1(\mu r)[M_\rho^2 z(1-z) + m_f^2 - \nabla_r^2] \frac{\Psi_L(r,z)}{z(1-z)} ,$$

$$\phi_\rho^{\perp}(z,\mu) = \frac{N_c m_f}{\pi f_\rho^{\perp}} \int \mathrm{d}r\ \mu J_1(\mu r) \frac{\Psi_T(r,z)}{z(1-z)} ,$$

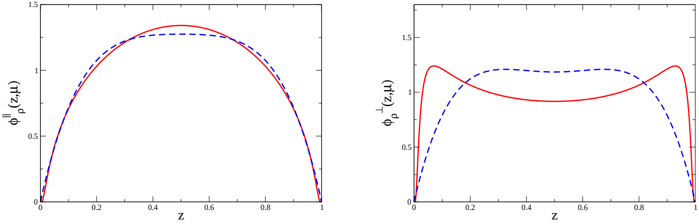
$$g_\rho^{\perp(v)}(z,\mu) = \frac{N_c}{2\pi f_\rho M_\rho} \int \mathrm{d}r\ \mu J_1(\mu r) \left[m_f^2 - (z^2 + (1-z)^2)\nabla_r^2\right] \frac{\Psi_T(r,z)}{z^2(1-z)^2}$$

$$\frac{\mathrm{d}g_\rho^{\perp(a)}}{\mathrm{d}z}(z,\mu) = \frac{\sqrt{2}N_c}{\pi f_\rho M_\rho} \int \mathrm{d}r\ \mu J_1(\mu r)(1-2z)[m_f^2 - \nabla_r^2] \frac{\Psi_T(r,z)}{z^2(1-z)^2} .$$

Distribution amplitudes are normalized:

$$\int_0^1 \mathrm{d}u\ \phi_\rho^{\perp,\parallel}(u,\mu) = \int_0^1 \mathrm{d}u\ g_\rho^{\perp(a,v)}(u,\mu) = 1$$

AdS/QCD DAs for ρ:comparison to Sum Rules



(c) Twist-2 DA for the longitudinally polarized ρ meson

(d) Twist-2 DA for the transversely polarized ρ meson

Figure: Twist-2 DAs for the ρ meson. Solid Red: AdS/QCD DA; Dashed Blue: Sum Rules DA.

DAs for K^*:comparison to Sum Rules

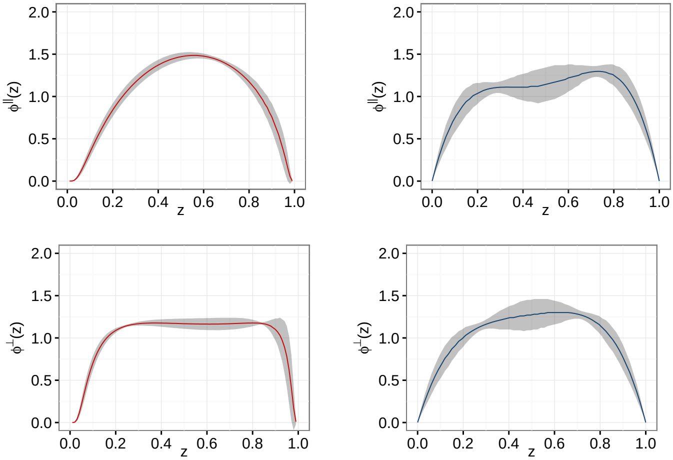


Figure: Twist-2 DAs predicted by AdS/QCD (graphs on the left) and SR (graphs on the right). The uncertainty band is due to the variation of the quark masses for AdS/QCD and the error bar on Gegenbauer coefficients for SR.

$B \to \rho$ transition form factors

Form factors are defined as:

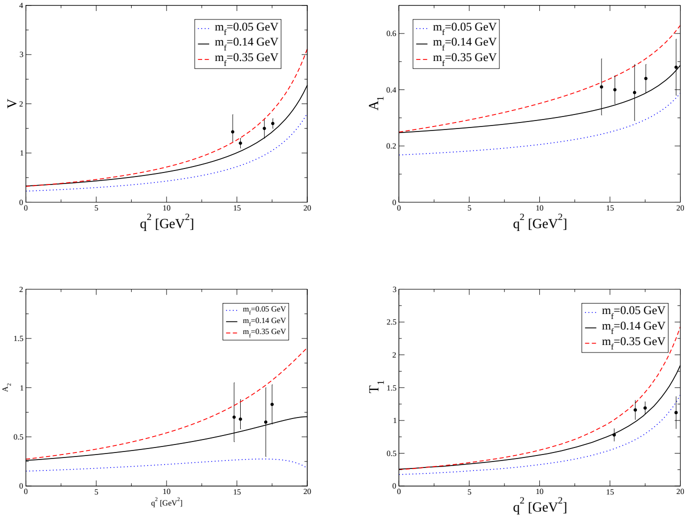
$$\begin{aligned}
\langle \rho(k,\varepsilon)|\bar{q}\gamma^\mu(1-\gamma^5)b|B(p)\rangle &= \frac{2iV(q^2)}{m_B+m_\rho}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma - 2m_\rho A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu \\
&- (m_B+m_\rho)A_1(q^2)\left(\varepsilon^{\mu *} - \frac{\varepsilon^*\cdot q q^\mu}{q^2}\right) \\
&+ A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_\rho}\left[(p+k)^\mu - \frac{m_B^2-m_\rho^2}{q^2}q^\mu\right]
\end{aligned}$$

$$\begin{aligned}
q_\nu\langle \rho(k,\varepsilon)|\bar{d}\sigma^{\mu\nu}(1-\gamma^5)b|B(p)\rangle &= 2T_1(q^2)\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu p_\rho k_\sigma \\
&- iT_2(q^2)[(\varepsilon^*\cdot q)(p+k)_\mu - \varepsilon^*_\mu(m_B^2-m_\rho^2)] \\
&- iT_3(q^2)(\varepsilon^*\cdot q)\left[\frac{q^2}{m_B^2-m_\rho^2}(p+k)_\mu - q_\mu\right]
\end{aligned}$$

AdS/QCD prediction for $B \to \rho$ transition form factors

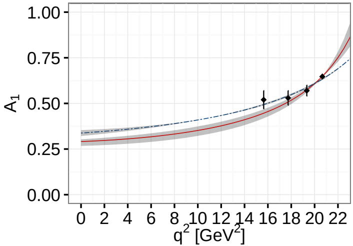
PRD88.074031(2013)

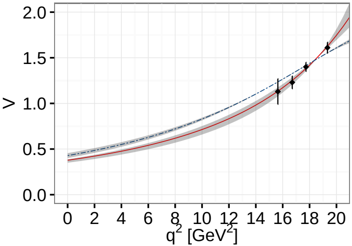
Using light-cone sum rules with holographic DAs

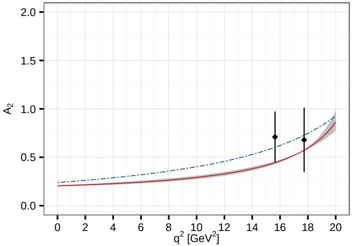


AdS/QCD prediction for $B \to K^*$ transition form factors

PRD.98.053002(2018)

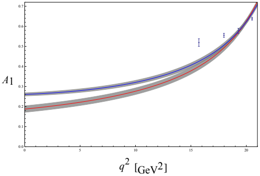


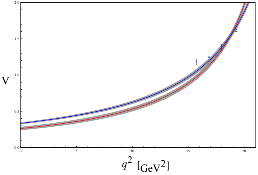


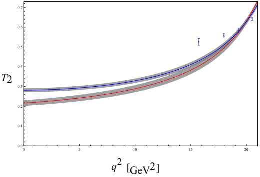


AdS/QCD prediction for $B_s \to \Phi$ transition form factors

PRD.100.113005(2019)







Summary and outlook

- DAs obtained from AdS/QCD LFWF can be used in LCSR for $B \to P, V$ form factors.
- These LFWFs should satisfy constraints from spectroscopy, decay constant and other observables.
- We are currently looking into a proper LFWF for heavy-light meson that is compatible with the above constraints.
- The goal is to directly calculate B transition form factors valid for the full kinematical range.