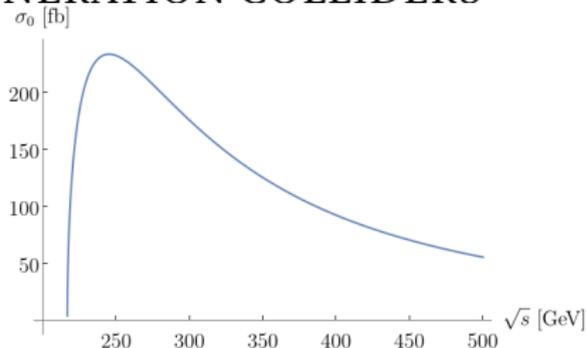




# HIGGSSTRAHLUNG AT NEXT-GENERATION COLLIDERS

**Motivation:** Next-gen colliders could measure  $\sigma(e^+e^- \rightarrow Zh)$  with sub-percent precision

**Question:** Can it serve as a useful probe of neutrino masses?

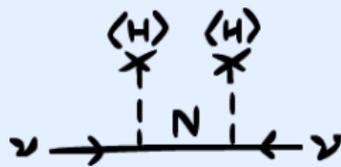


| Collider | $L_{\text{int}} [\text{ab}^{-1}]$ | Z-decay final states                    | $\sqrt{s} [\text{GeV}]$ | Precision   |
|----------|-----------------------------------|---|-------------------------|-------------|
| CEPC     | 20                                | $\ell^+ \ell^-, q\bar{q}, \nu\bar{\nu}$ | 240                     | 0.26%       |
|          | 1                                 | $\ell^+ \ell^-, q\bar{q}, \nu\bar{\nu}$ | 360                     | 1.4%        |
| FCC-ee   | 5                                 | $\ell^+ \ell^-$                         | 240                     | 0.5%        |
|          | 1.5                               | $\ell^+ \ell^-, q\bar{q}, \nu\bar{\nu}$ | 365                     | 0.9%        |
| ILC      | 1.35                              | $\ell^+ \ell^-$                         | 250                     | 1.1%        |
|          | 0.115 (0.5)                       | $\ell^+ \ell^- (q\bar{q})$              | 350                     | 5% (1.63%)  |
|          | 1.6 (0.5)                         | $\ell^+ \ell^- (q\bar{q})$              | 500                     | 2.9% (3.9%) |
| CLIC     | 0.5                               | $\ell^+ \ell^-, q\bar{q}$               | 350                     | 1.65%       |

*H. Cheng et al. 2205.08553; G. Bernardi et al. 2203.06520; J. Yan et al. 1604.07524; M. Thomson 1509.02853; A. Miyamoto 1311.2248; H. Abramowicz et al. 1608.07538*

## FERMIONIC SEESAW MODELS

$$\mathcal{L} = \bar{N}i\not{\partial}N - \left( \tilde{Y}\bar{L}\tilde{H}N + \frac{1}{2}M\bar{N}^c N + \text{h.c.} \right)$$



Mass matrix:

$$\begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix}$$

$$m = \tilde{Y}v/\sqrt{2}$$

- **Type I Seesaw:**  $N$  is an  $SU(2)_L$  singlet

*P. Minkowski (1977) and more...*

- **Type III Seesaw:**  $N$  is an  $SU(2)_L$  triplet

*R. Foot et. al. (1989)*



# APPROACH

- ▶ Consider minimal scenario with two right-handed neutrinos
- ▶ Adopt the following lepton-number conserving texture for the mass matrix:

$$\begin{pmatrix} 0 & m \\ m^T & M \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \theta_e M & 0 \\ 0 & 0 & 0 & \theta_\mu M & 0 \\ 0 & 0 & 0 & \theta_\tau M & 0 \\ \theta_e M & \theta_\mu M & \theta_\tau M & 0 & M \\ 0 & 0 & 0 & M & 0 \end{pmatrix}$$

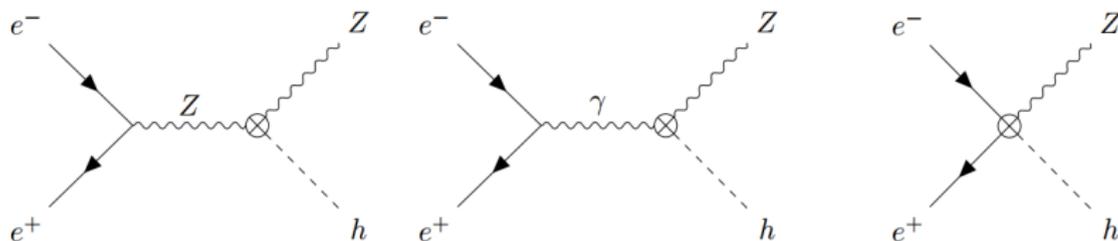
See e.g. *A. Abada, M. B. Gavela et. al.* 0707.4058; *M. B. Gavela et. al.* 0906.1461;  
*O. J. P. Eboli et. al.* 1108.0661; *E. Fernandez-Martinez et. al.* 1508.03051

- ▶ Fix  $M = 1 \text{ TeV}$  and explore the parameter space of the **mixing angles**  $\theta_e, \theta_\mu$  and  $\theta_\tau$ .  
(Results still hold at  $M = 10 \text{ TeV}$ .)
- ▶ Match onto SMEFT at  $\mu = M$  and use `DsixTools` to run to the relevant scales.

*A. Celis et. al.* 1704.04504

# HIGGSSTRAHLUNG

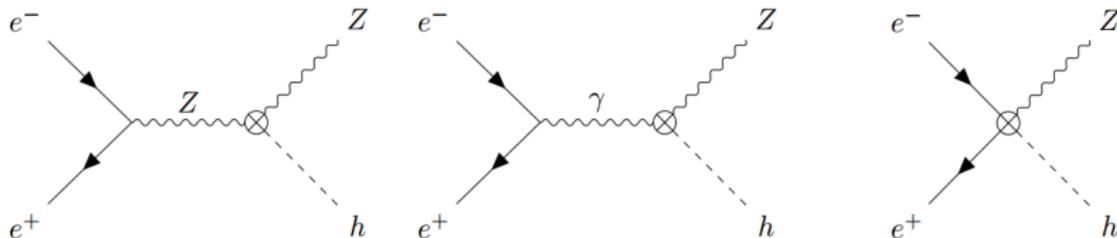
- ▶ Direct contributions from effective operators:



- ▶ Also consider indirect contributions from parameter shifts

## HIGGSSTRAHLUNG

- ▶ Direct contributions from effective operators:



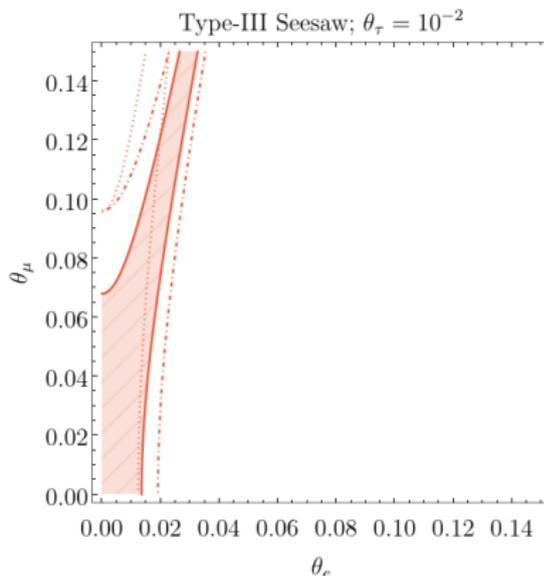
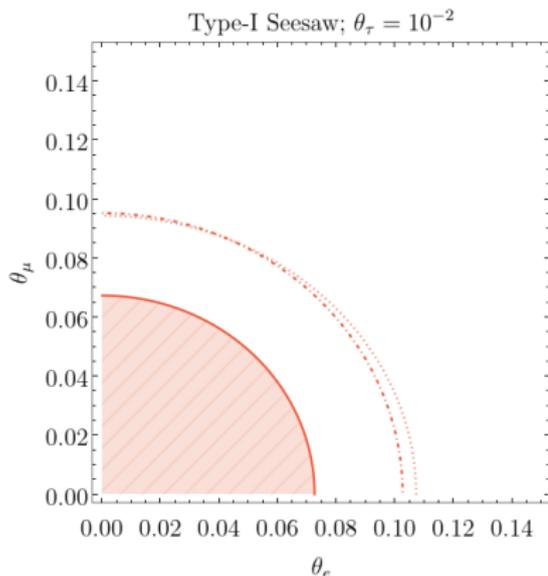
- ▶ Also consider indirect contributions from parameter shifts

$$\frac{\Delta\sigma}{\sigma_0} \approx \begin{cases} 0.90 \hat{C}_{HL,ee}^{(1)} + 0.77 \hat{C}_{HL,ee}^{(3)} - 0.13 \hat{C}_{HL,\mu\mu}^{(3)}, & \sqrt{s} = 240 \text{ GeV} \\ 2.09 \hat{C}_{HL,ee}^{(1)} + 1.96 \hat{C}_{HL,ee}^{(3)} - 0.13 \hat{C}_{HL,\mu\mu}^{(3)}, & \sqrt{s} = 365 \text{ GeV} \end{cases}$$

Here  $\hat{C} = C \times (1 \text{ TeV})^2$

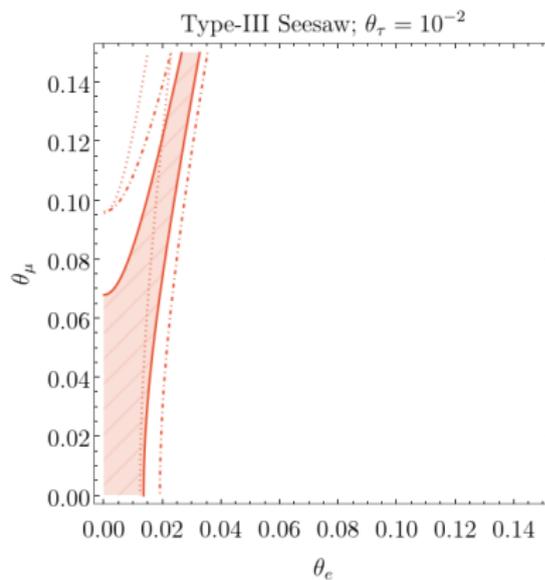
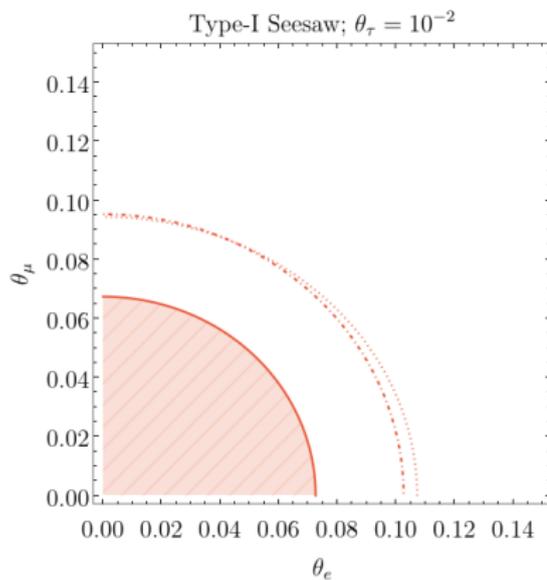
*N. Craig, M. McCullough et. al. 1411.0676*

# HIGGSSTRAHLUNG



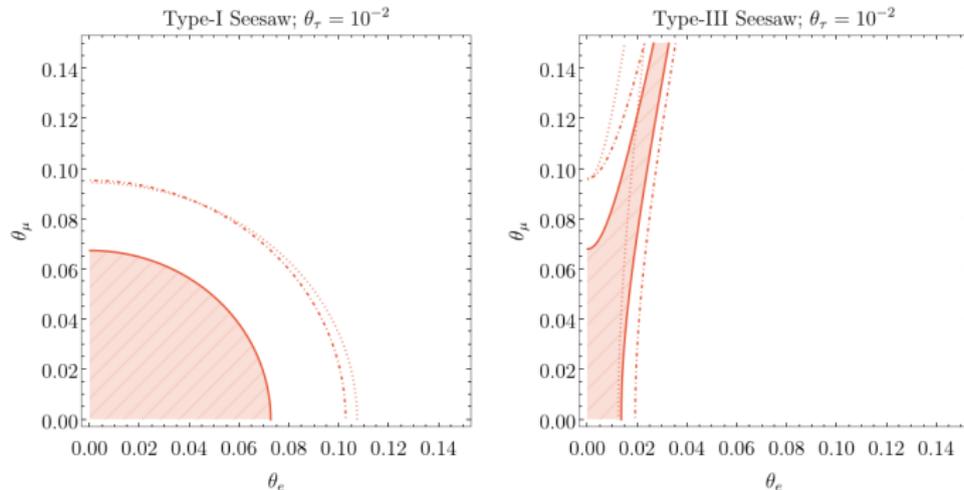
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# HIGGSSTRAHLUNG



- ▶ Red region:  $|\Delta\sigma/\sigma| < 0.5\%$  at  $\sqrt{s} = 240\text{GeV}$
- ▶ Dot-dashed line:  $|\Delta\sigma/\sigma| = 1\%$  at  $\sqrt{s} = 240\text{GeV}$
- ▶ Dotted line:  $|\Delta\sigma/\sigma| = 1\%$  at  $\sqrt{s} = 365\text{GeV}$

# CONSTRAINTS FROM OTHER OBSERVABLES



Consider:

- ▶ Electroweak observables  $m_W$  and  $\sin\theta_w$
- ▶ Lepton flavour universality (LFU) and CKM ratios
- ▶ Lepton flavour violation (LFV) tests

## ELECTROWEAK OBSERVABLES

- W-boson mass and the weak mixing angle,  $s_w = \sin\theta_w$  are shifted by

$$\delta m_W^2 \approx -1.05 \left( \hat{C}_{HL,ee}^{(3)} + \hat{C}_{HL,\mu\mu}^{(3)} \right) \text{GeV}^2$$

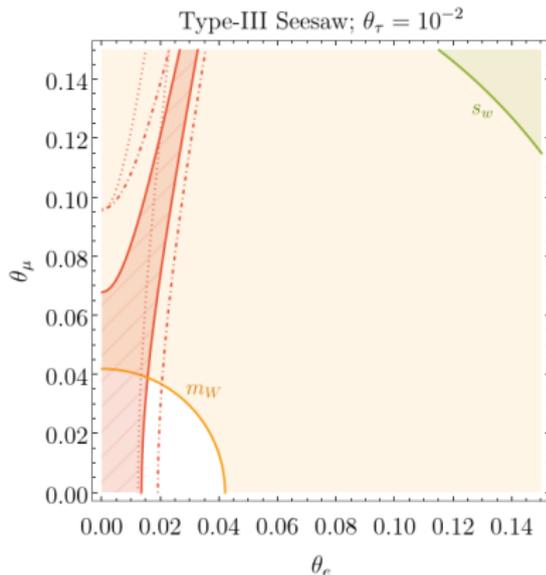
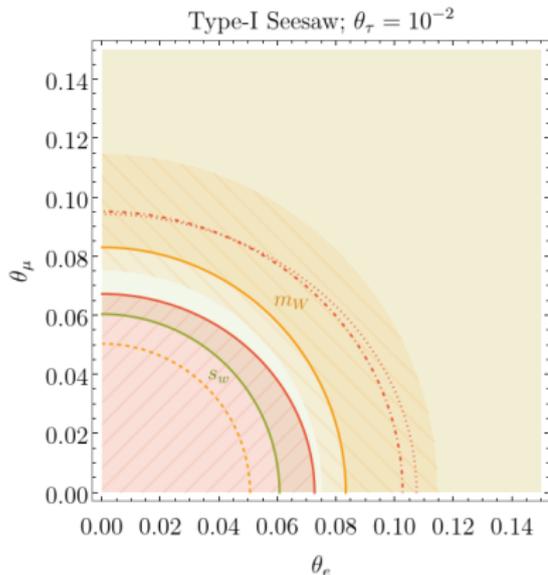
$$\delta s_w^2 \approx 0.020 \left( \hat{C}_{HL,ee}^{(3)} + \hat{C}_{HL,\mu\mu}^{(3)} \right) - 0.005 \sum_i \left( \hat{C}_{HL,ii}^{(1)} + \hat{C}_{HL,ii}^{(3)} \right)$$

- We also know the following:

| Observable                                  | SM prediction           | Measurement           |
|---|-------------------------|-----------------------|
| $\sin^2(\theta_{\text{eff}}^{\text{lept}})$ | $0.231534 \pm 0.000030$ | $0.23153 \pm 0.00026$ |
| $m_W$ [GeV]                                 | $80.356 \pm 0.006$      | $80.377 \pm 0.012$    |

*J. de Blas et. al. 2112.07274; S. Schael et. al. hep-ex/0509008; PDG 2022*

## ELECTROWEAK OBSERVABLES

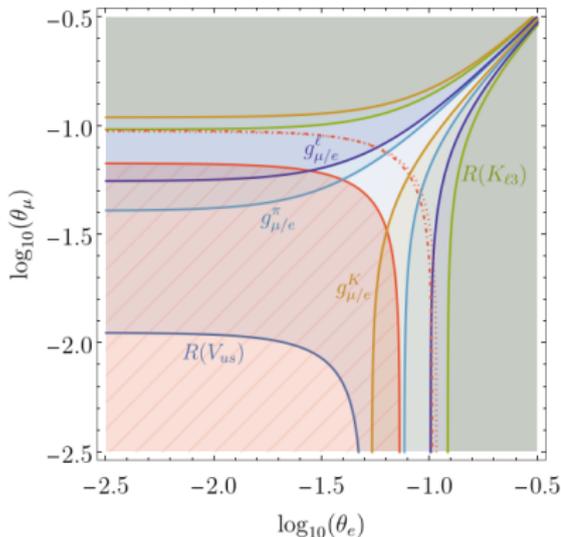
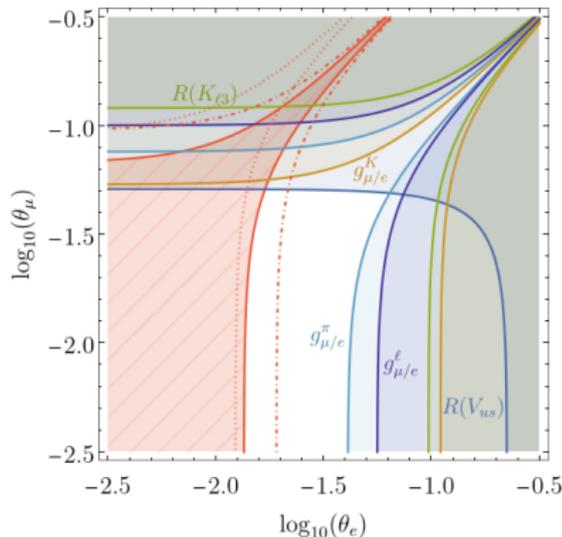


$$\delta m_W^2 \approx -1.05 \left( \hat{C}_{HL,ee}^{(3)} + \hat{C}_{HL,\mu\mu}^{(3)} \right) \text{GeV}$$

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- ▶ Red region:  $|\Delta\sigma/\sigma| < 0.5\%$  at  $\sqrt{s} = 240 \text{ GeV}$
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## LFU AND CKM RATIOS

Type-I Seesaw;  $\theta_\tau = 10^{-2}$ Type-III Seesaw;  $\theta_\tau = 10^{-2}$ 

► Ratios of gauge couplings:  $g_{\mu/e} = \frac{g_\mu}{g_e} \approx 1 + 0.06 \left( \hat{C}_{HL,\mu\mu}^{(3)} - \hat{C}_{HL,ee}^{(3)} \right)$

►  $R(V_{us}) = \frac{V_{us}^{K,\mu 2}}{V_{us}^\beta} \approx 1 - 1.2 \hat{C}_{HL,\mu\mu}^{(3)} - 0.06 \hat{C}_{HL,ee}^{(3)}$  *A. Crivellin and M. Hoferichter 2002.07184*

## LEPTON FLAVOUR VIOLATION (LFV)

- ▶ Radiative charged-lepton decays:

$$\mu \rightarrow e\gamma, \tau \rightarrow e\gamma$$

$$\text{BR}(\ell_i \rightarrow \ell_j \gamma) = \frac{m_{\ell_i}^3}{4\pi\Gamma_{\ell_i}} \left( |C_{e\gamma,ij}|^2 + |C_{e\gamma,ji}|^2 \right)$$

*Y. Kuno and Y. Okada hep-ph/9909265*

- ▶ Trilepton decays:  $\mu \rightarrow 3e, \tau \rightarrow 3e$

$$\text{BR}(\ell_i \rightarrow 3\ell_j) = f\left(C_{e\gamma}, C_{ee}^{VLL}, C_{ee}^{VLR}, \dots\right)$$

*L. Calibbi et. al. 2107.10273*

- ▶  $\mu$ - $e$  conversion in nuclei

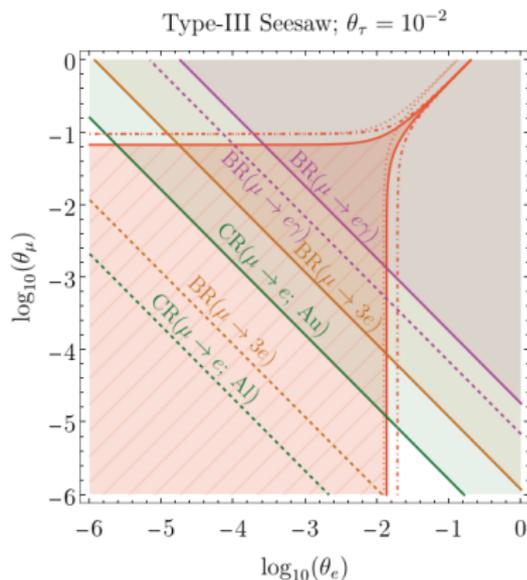
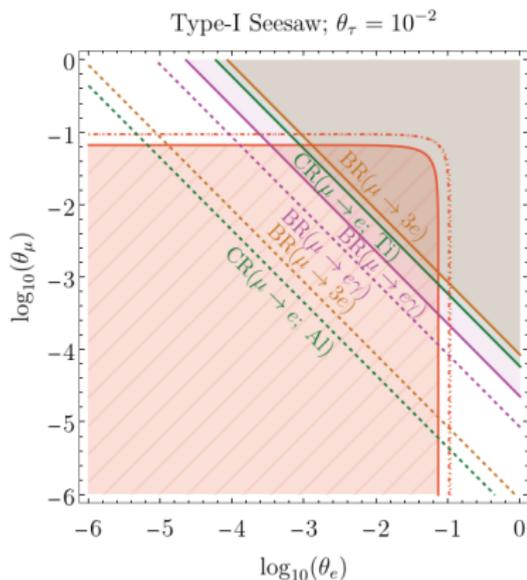
$$\text{CR}(\mu \rightarrow e) = f\left(C_{e\gamma}, C_{ee}^{VLL}, C_{ee}^{VLR}, \dots\right)$$

*R. Kitano et. al. hep-ph/0203110;*

*V. Cirigliano et. al. 0904.0957*

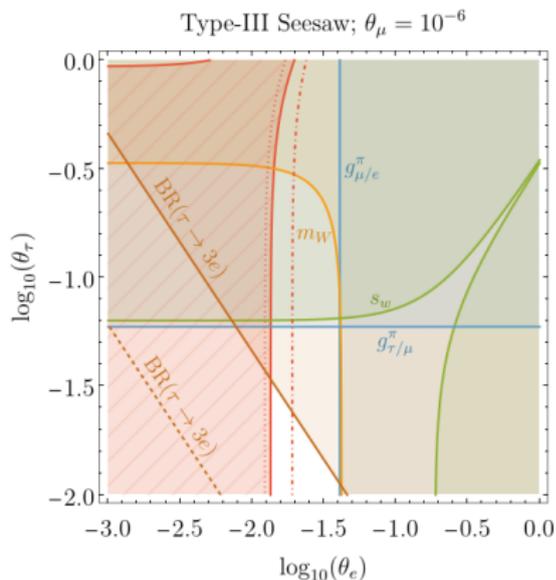
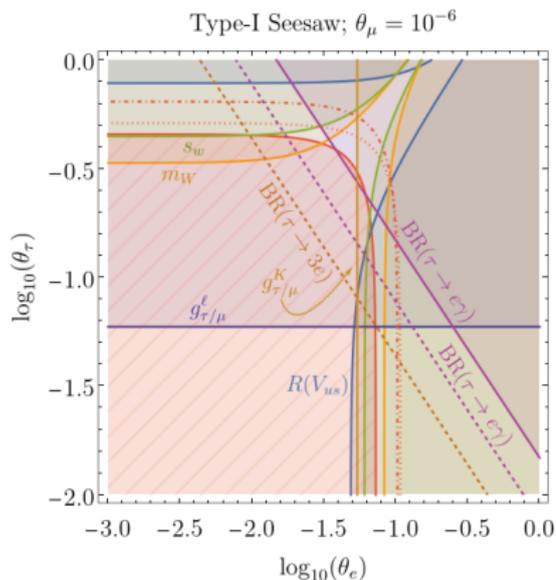
| Observable                       | Current bound         | Future reach            |
|----------------------------------|-----------------------|-------------------------|
| BR( $\mu \rightarrow e\gamma$ )  | $4.2 \times 10^{-13}$ | $6 \times 10^{-14}$     |
| BR( $\tau \rightarrow e\gamma$ ) | $3.3 \times 10^{-8}$  | $9 \times 10^{-9}$      |
| BR( $\mu \rightarrow 3e$ )       | $1 \times 10^{-12}$   | $1 \times 10^{-16}$     |
| BR( $\tau \rightarrow 3e$ )      | $2.7 \times 10^{-8}$  | $4.7 \times 10^{-10}$   |
| CR( $\mu$ - $e$ ; Au)            | $7 \times 10^{-13}$   | -                       |
| CR( $\mu$ - $e$ ; Al)            | -                     | $\mathcal{O}(10^{-17})$ |
| CR( $\mu$ - $e$ ; Ti)            | $6.1 \times 10^{-13}$ | $\mathcal{O}(10^{-18})$ |
| CR( $\mu$ - $e$ ; Pb)            | $4.6 \times 10^{-11}$ | -                       |
| CR( $\mu$ - $e$ ; S)             | $7 \times 10^{-11}$   | -                       |

## LEPTON FLAVOUR VIOLATION (LFV)



- ▶ Red region:  $|\Delta\sigma/\sigma| < 0.5\%$  at  $\sqrt{s} = 240\text{ GeV}$
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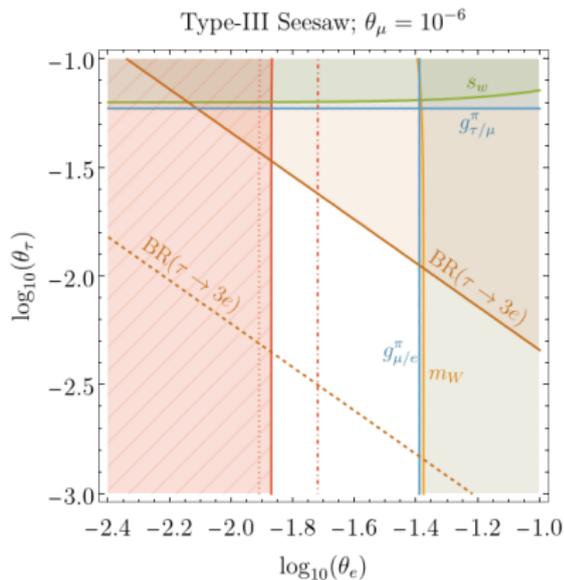
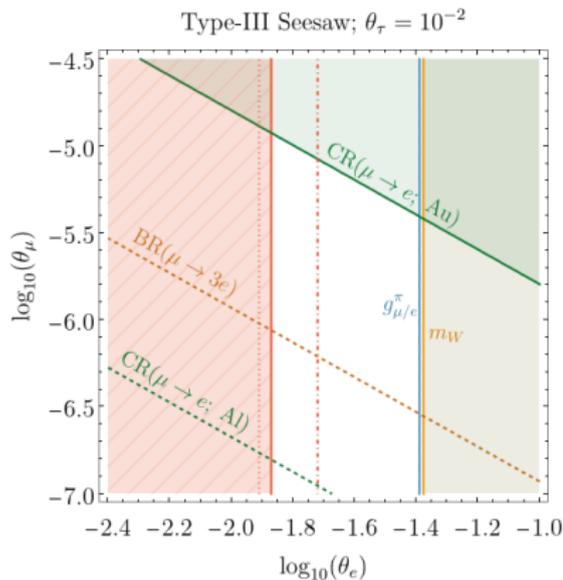


COMBINED PLOTS —  $\theta_e - \theta_\tau$  PLANE

- ▶ Type I Seesaw: observable shift ruled out!
- ▶ Type III Seesaw: observable shift possible!

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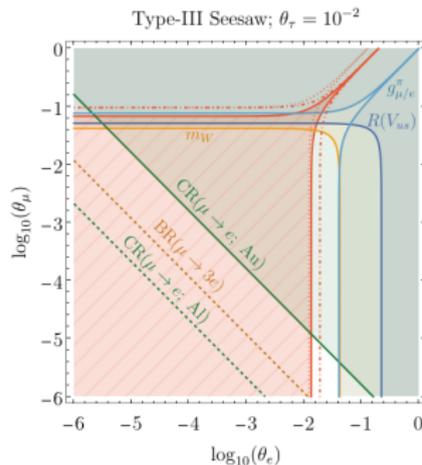
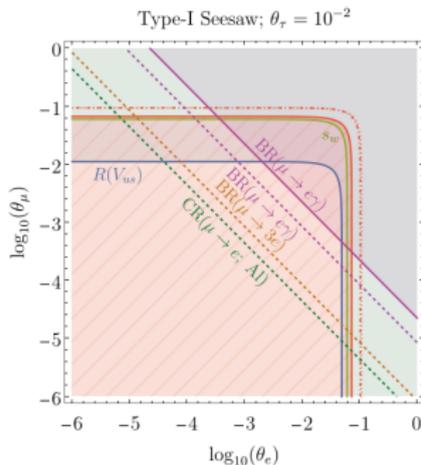
## TYPE-3 ZOOMED IN



- ▶ Largest shift at  $\sqrt{s} = 240\text{ GeV}$  is  $\sim 5\%$
- ▶ Largest shift at  $\sqrt{s} = 365\text{ GeV}$  is  $\sim 12\%$

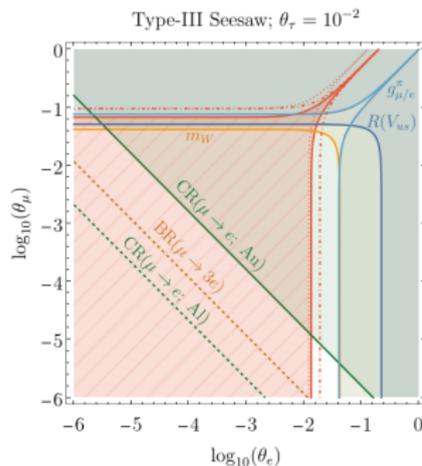
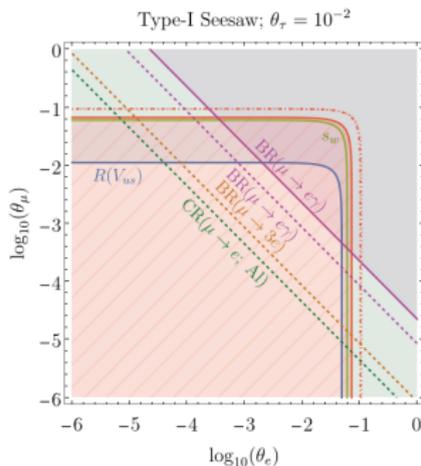
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## SUMMARY &amp; CONCLUSIONS



- ▶  $\sigma(e^+e^- \rightarrow Zh)$  can be measured at future colliders to  $\sim 1\%$  precision
- ▶ If agrees with SM prediction:
  - ▶ Type I OK
  - ▶ Type III constrained
- ▶ If differs from SM prediction:
  - ▶ Type I dead
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Thank you!

# BACKUP: SMEFT OPERATORS

|                             |  |
|-----------------------------|--|
| $\mathcal{O}_{HW}$          | $H^\dagger HW_{\mu\nu}^I W^{I\mu\nu}$  |
| $\mathcal{O}_{HB}$          | $H^\dagger HB_{\mu\nu} B^{\mu\nu}$   |
| $\mathcal{O}_{HWB}$         | $H^\dagger \sigma^I HW_{\mu\nu}^I B^{\mu\nu}$  |
| $\mathcal{O}_{H\Box}$       | $(H^\dagger H)\Box(H^\dagger H)$   |
| $\mathcal{O}_{HD}$          | $(H^\dagger D_\mu H)^* (H^\dagger D^\mu H)$  |
| $\mathcal{O}_{HL,ij}^{(1)}$ | $(H^\dagger i \overleftrightarrow{D}_\mu H)(\overline{L}_i \gamma^\mu L_j)$            |
| $\mathcal{O}_{HL,ij}^{(3)}$ | $(H^\dagger i \overleftrightarrow{D}_\mu^a H)(\overline{L}_i \sigma^a \gamma^\mu L_j)$ |
| $\mathcal{O}_{He,ij}$       | $(H^\dagger i \overleftrightarrow{D}_\mu^a H)(\overline{e}_{Ri} \gamma^\mu e_{Rj})$    |
| $\mathcal{O}_{LL,ijkl}$     | $(\overline{L}_i \gamma^\mu L_j)(\overline{L}_k \gamma_\mu L_l)$                       |

## BACKUP: HIGGSSTRAHLUNG IN THE SM

$$\sigma_0 = \frac{\sqrt{\lambda}}{32\pi s} (g_L^2 + g_R^2) \left( \frac{g_{ZZh}}{s - m_Z^2} \right)^2 \left( 1 + \frac{\lambda}{12sm_Z^2} \right).$$

$$g_{ZZh} = \frac{g_2 m_Z}{\cos\theta_w}$$

$$g_L = g_Z \left( -\frac{1}{2} + \sin^2\theta_w \right)$$

$$g_R = g_Z \sin^2\theta_w$$

$$g_Z = \frac{g_2}{\cos\theta_w}$$

$$\lambda = (s - m_Z^2 - m_h^2)^2 - 4m_Z^2 m_h^2$$

# BACKUP: HIGGSTRAHLUNG SHIFT

$$\frac{\Delta\sigma}{\sigma_0} = 2 \left( \frac{\delta g_{ZZh}}{g_{ZZh}} + \frac{g_L \delta g_L + g_R \delta g_R}{g_L^2 + g_R^2} \right) + \frac{v}{g_{ZZh}} \sum_{i=2}^5 d_i f_i. \quad \text{See Craig et. al. 1411.0676}$$

Parameter shifts:

$$\begin{aligned} \frac{\delta g_{ZZh}}{g_{ZZh}} &= v^2 \left( C_{H\Box} + \frac{1}{4} C_{HD} - \frac{1}{\sqrt{2}} \delta G_F \right), \\ \frac{\delta g_L}{g_Z} &= \frac{1}{8(c_w^2 - s_w^2)} v^2 \left( 8s_w c_w C_{HWB} + C_{HD} + 2\sqrt{2} \delta G_F \right) - \frac{1}{2} v^2 \left( C_{HL,11}^{(1)} + C_{HL,11}^{(3)} \right), \quad \text{and} \\ \frac{\delta g_R}{g_Z} &= \frac{s_w^2}{4(c_w^2 - s_w^2)} v^2 \left( 4 \frac{c_w}{s_w} C_{HWB} + C_{HD} + 2\sqrt{2} \delta G_F \right) - \frac{1}{2} v^2 C_{He,11}. \end{aligned}$$

'Direct' contributions:

$$\begin{aligned} d_2 &= 4(s_w^2 C_{HB} + s_w c_w C_{HWB} + c_w^2 C_{HW}) \\ d_3 &= -4s_w c_w C_{HB} - 2(c_w^2 - s_w^2) C_{HWB} + 4s_w c_w C_{HW} \\ d_4 &= -g_Z (C_{HL,11}^{(1)} + C_{HL,11}^{(3)}) \\ d_5 &= -g_Z C_{He,11} \end{aligned}$$

$$\begin{aligned} f_2 &= 12m_Z^2 \frac{s(s+m_Z^2 - m_h^2)}{12sm_Z^2 + \lambda} \\ f_3 &= -12em_Z^2 \frac{g_L + g_R}{g_L^2 + g_R^2} \frac{(s - m_Z^2)(s + m_Z^2 - m_h^2)}{12sm_Z^2 + \lambda} \\ f_4 &= \frac{2g_L}{g_L^2 + g_R^2} (s - m_Z^2) \\ f_5 &= \frac{2g_R}{g_L^2 + g_R^2} (s - m_Z^2) \end{aligned}$$

# BACKUP: WEAK MIXING ANGLE

$$\text{Left-right asymmetry } \mathcal{A}_f = \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} = \frac{2(1 - 4s_w^2)}{1 + (1 - 4s_w^2)^2}$$

$$\delta g_{L,ij}^{\text{direct}} = -\frac{1}{2} g_Z v^2 \left( C_{HL,ij}^{(1)} + C_{HL,ij}^{(3)} \right)$$

$$\delta g_{R,ij}^{\text{direct}} = -\frac{1}{2} g_Z v^2 C_{He,ij},$$

$$\begin{aligned} s_{w,\text{eff}}^2 &= s_{w,\text{SM}}^2 + \delta s_w^2 + \frac{1}{3} \frac{ds_w^2}{d\mathcal{A}_\ell} \left( \frac{\partial \mathcal{A}_\ell}{\partial g_L} \sum_{i=1}^3 \delta g_{L,ii}^{\text{direct}} + \frac{\partial \mathcal{A}_\ell}{\partial g_R} \sum_{i=1}^3 \delta g_{R,ii}^{\text{direct}} \right) \\ &\approx s_{w,\text{SM}}^2 + 0.020 \left( \hat{C}_{HL,11}^{(3)} + \hat{C}_{HL,22}^{(3)} \right) \\ &\quad - 0.005 \sum_{i=1}^3 \left( \hat{C}_{HL,ii}^{(1)} + \hat{C}_{HL,ii}^{(3)} \right) \end{aligned}$$

# BACKUP: SHIFTS IN TERMS OF MIXING ANGLES

|                                   | Type-I   | Type-III  |
|-----------------------------------|--|---|
| $\Delta\sigma/\sigma_0$ (240 GeV) | $0.95 \theta_e ^2 + 1.10 \theta_\mu ^2 + 0.02 \theta_\tau ^2$  | $27.59 \theta_e ^2 - 1.08 \theta_\mu ^2 - 0.01 \theta_\tau ^2$  |
| $\Delta\sigma/\sigma_0$ (365 GeV) | $0.87 \theta_e ^2 + 1.12 \theta_\mu ^2 + 0.04 \theta_\tau ^2$  | $66.15 \theta_e ^2 - 1.09 \theta_\mu ^2 - 0.01 \theta_\tau ^2$  |
| $\Delta\sigma/\sigma_0$ (500 GeV) | $0.80 \theta_e ^2 + 1.14 \theta_\mu ^2 + 0.05 \theta_\tau ^2$  | $126.39 \theta_e ^2 - 1.10 \theta_\mu ^2 - 0.01 \theta_\tau ^2$ |
| Shift                             | Type-I   | Type-III  |
| $\Delta s_W^2$                    | $-0.157( \theta_e ^2 +  \theta_\mu ^2) + 0.003 \theta_\tau ^2$ | $0.017( \theta_e ^2 +  \theta_\mu ^2) - 0.143 \theta_\tau ^2$   |
| $\Delta m_W/\text{GeV}$           | $8.24( \theta_e ^2 +  \theta_\mu ^2) - 0.13 \theta_\tau ^2$    | $-8.51( \theta_e ^2 +  \theta_\mu ^2) - 0.13 \theta_\tau ^2$    |