Heavy Quark Physics

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Plan of lectures

- Flavour structure of the SM: CKM matrix and parametrisations, FCNC, CP violation, GIM, examples.
- The Weak Effective Lagrangian, Wilson coefficients, RGE: a simple 1loop example
- Heavy Quark Symmetry and HQET
- Semileptonic decays, form factor parametrisations, unitarity constraints, inclusive decays

References

- Y. Nir, *B physics and CP violation*, Les Houches 2005, hep-ph/0510413
- A. Buras, *Gauge Theory of Weak Decays,* Cambridge Univ. Press, 2020, and hep-ph/9806471
- A. Manohar and M. Wise, *Heavy Quark Physics*, Cambridge Univ. Press, 2000
- A. Manohar, *Les Houches Lectures 2017: introduction to EFT*, 1804.05863
- L. Silvestrini, *Les Houches Lectures 2017*, 1905.00798
- I. Bigi, M. Shifman, N. Uraltsev, *Aspects of Heavy Quark Theory,* hep-ph/9703290
- M. Neubert, *EFT and Heavy Quark Physics*, hep-ph/0512222
- B. Grinstein, *Lectures on Flavor physics and CP violation*, 1701.06916

Flavour in the SM

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}$$

Fermionic fields: Q_L, U_R, D_R, L_L, E_R

$$\mathcal{L}_{\rm kinetic}(Q_L) = i\overline{Q^I_{Li}}\gamma_\mu \left(\partial^\mu + \frac{i}{2}g_s G^\mu_a \lambda_a + \frac{i}{2} g W^\mu_b \tau_b + \frac{i}{6} g' B^\mu\right) Q^I_{Li}$$

Three identical generations: huge flavour symmetry, $U(3)^5 = U(3)_Q$ x $U(3)_U$ x$U(3)_D$ x$U(3)_L$ x$U(3)_{LR}$, broken by Yukawa interactions

$$-\mathcal{L}^{\rm quarks}_{\rm Yukawa} = Y^d_{ij}\overline{Q^I_{Li}}\phi D^I_{Rj} + Y^u_{ij}\overline{Q^I_{Li}}\tilde{\phi} U^I_{Rj} + {\rm h.c.} \qquad \tilde{\phi} = i\tau_2\phi^\dagger$$

$$\mathcal{I}m(\det[Y^d Y^{d\dagger}, Y^u Y^{u\dagger}]) \neq 0. \leftrightarrow \cancel{CP}$$

$Y^{u,d}$ must be complex for CP violation

Indeed, h.c. means $Y_{ij}\overline{\psi_{Li}}\phi\psi_{Rj} + Y^*_{ij}\overline{\psi_{Rj}}\phi^\dagger\psi_{Li}$ *but* $\overline{\psi_{Li}}\phi\psi_{Rj} \underset{CP}{\longleftrightarrow} \overline{\psi_{Rj}}\phi^\dagger\psi_{Li}$

Flavour in the SM (II)

Upon SSB, $\langle\phi\rangle = (0, v/\sqrt{2})^T$ *with* $v \sim 174$ *GeV , the two* $Y^{u,d}$ *become mass matrices: they are arbitrary 3x3 complex matrices, diagonalized by a biunitary transf*

$$-\mathcal{L}_M^q = (M_d)_{ij}\overline{D_{Li}^I}D_{Rj}^I + (M_u)_{ij}\overline{U_{Li}^I}U_{Rj}^I + \text{h.c.}$$

$$V_{qL}M_qV_{qR}^\dagger = M_q^{\text{diag}}$$

$$-\mathcal{L}_{W^\pm}^q = \frac{g}{\sqrt{2}}\overline{u_{Li}}\gamma^\mu(V_{uL}V_{dL}^\dagger)_{ij}d_{Lj}W_\mu^+ + \text{h.c.}.$$

$$\boxed{V = V_{uL}V_{dL}^\dagger, \quad (VV^\dagger = 1)}$$

The CKM matrix

In the quark sector only 3 of the 4 unitary matrices can be chosen arbitrarily, thanks to the flavour symmetry of the kin Lagrangian.
Choosing the basis where Y^u *is diagonal,* $Y^d = V\,\mathrm{diag}(m_d, m_s, m_b)/v$
and the mass eigenstates differ from the "weak" eigenstates d',s',b'.

Weak and mass eigenstates

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} = \hat{V}_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Charged currents

$$\mathcal{L}_{\text{CC}} = \frac{g_2}{2\sqrt{2}}(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu})$$

$$J_\mu^+ = (\bar{u}d')_{V-A} + (\bar{c}s')_{V-A} + (\bar{t}b')_{V-A}$$

Neutral currents (GIM) conserve flavour at lowest order

$$\mathcal{L}_{\text{NC}} = -eJ_\mu^{\text{em}} A^\mu + \frac{g_2}{2\cos\Theta_{\text{W}}} J_\mu^0 Z^\mu$$

$$J_\mu^{em} = \sum_q Q_q \left(\overline{q_L}\gamma_\mu q_L + \overline{q_R}\gamma_\mu q_R \right)$$

does not change flavour; similar for J_μ^Z

How many parameters?

$U(3)_Q$ x $U(3)_U$ x $U(3)_D \rightarrow U(1)_B$ by Yukawa interactions

Y^u and Y^d involve 36 parameters, not all physical. Think of Yukawa couplings as spurions (non-dynamical fields) that break the global symmetry: you can choose freely the elements of three 3x3 unitary matrices except one. 3x9-1=26 components of Y_D,Y_U are unphysical: only 10=6 masses + 3 angles +1 phase remain

Standard Parametrization

$$\hat{V}_{CKM} = \begin{array}{cc} \begin{array}{ccc} d & s & b \end{array} & \\ \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} & \begin{array}{c} u \\ c \\ t \end{array} \end{array}$$

With $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ $\quad 0 < \delta < \pi$

3 angles + 1 phase

The form of V is not unique
a diagonal phase transf changes it $\quad V \longrightarrow P_u V P_d^*$

To excellent accuracy (V_{ub} is $4x10^{-3}$)

$$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta.$$

sine of Cabibbo angle, ~0.2 *~0.004* *~0.04*

Wolfenstein parametrization

V has a hierarchical structure

$$\hat{V} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\varrho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\varrho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

λ~0.22, A, ρ, η are O(1)

To improve the accuracy, <u>define</u> to all orders in λ

$$s_{12} = \lambda\,, \qquad s_{23} = A\lambda^2\,, \qquad s_{13}e^{-i\delta} = A\lambda^3(\varrho - i\eta)$$

$$\hat{V} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda+\mathcal{O}(\lambda^7) & A\lambda^3(\varrho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\varrho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2+\mathcal{O}(\lambda^8) \\ A\lambda^3(1-\overline{\varrho}-i\overline{\eta}) & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\varrho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}$$

$$\overline{\varrho} = \varrho(1-\frac{\lambda^2}{2}), \qquad \overline{\eta} = \eta(1-\frac{\lambda^2}{2})$$

$$V_{ud} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4 + \mathcal{O}(\lambda^6)$$

$$V_{us} = \lambda + \mathcal{O}(\lambda^7)$$

$$V_{ub} = A\lambda^3(\varrho - i\eta)$$

$$V_{cd} = -\lambda + \frac{1}{2}A^2\lambda^5[1 - 2(\varrho + i\eta)] + \mathcal{O}(\lambda^7)$$

$$V_{cs} = 1 - \frac{1}{2}\lambda^2 - \frac{1}{8}\lambda^4(1 + 4A^2) + \mathcal{O}(\lambda^6)$$

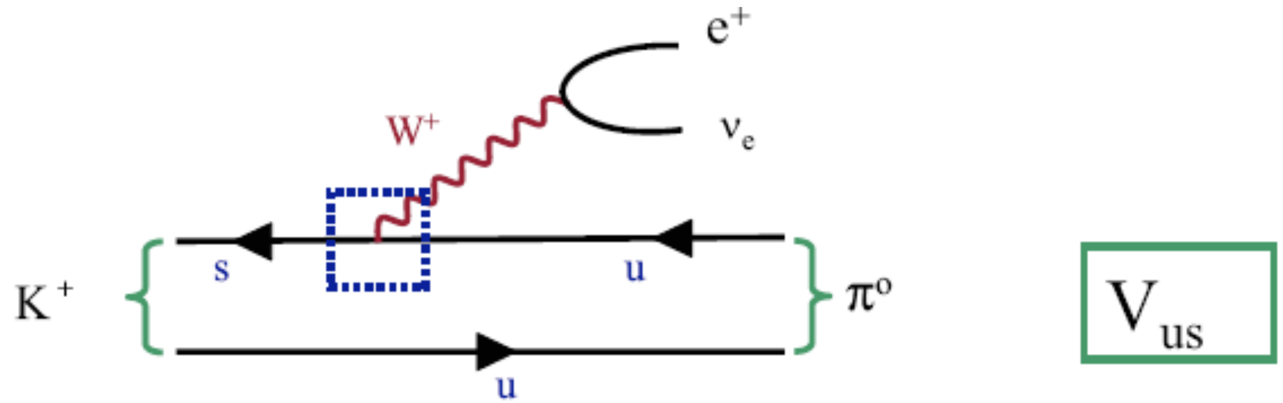
$$V_{cb} = A\lambda^2 + \mathcal{O}(\lambda^8)$$

$$V_{td} = A\lambda^3\left[1 - (\varrho + i\eta)(1 - \frac{1}{2}\lambda^2)\right] + \mathcal{O}(\lambda^7)$$

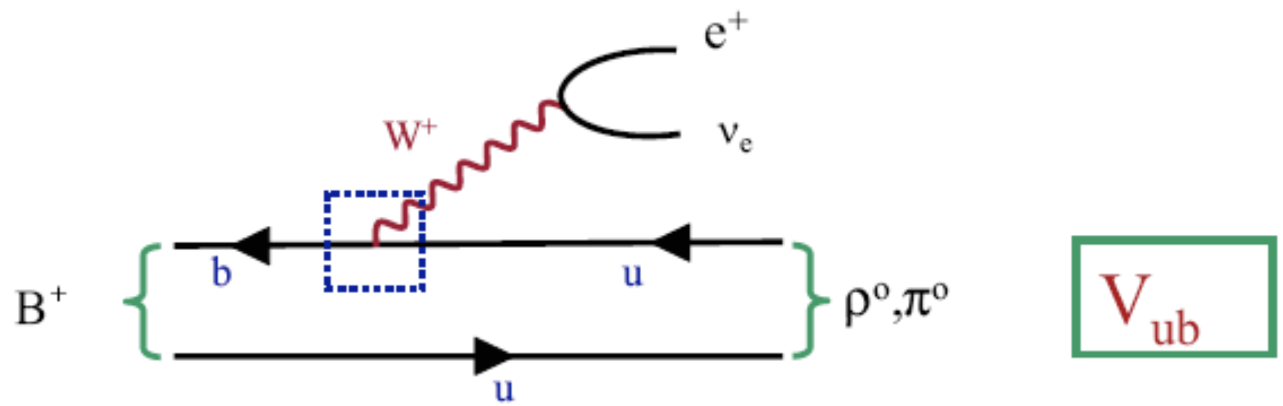
$$V_{ts} = -A\lambda^2 + \frac{1}{2}A(1 - 2\varrho)\lambda^4 - i\eta A\lambda^4 + \mathcal{O}(\lambda^6)$$

$$V_{tb} = 1 - \frac{1}{2}A^2\lambda^4 + \mathcal{O}(\lambda^6)$$

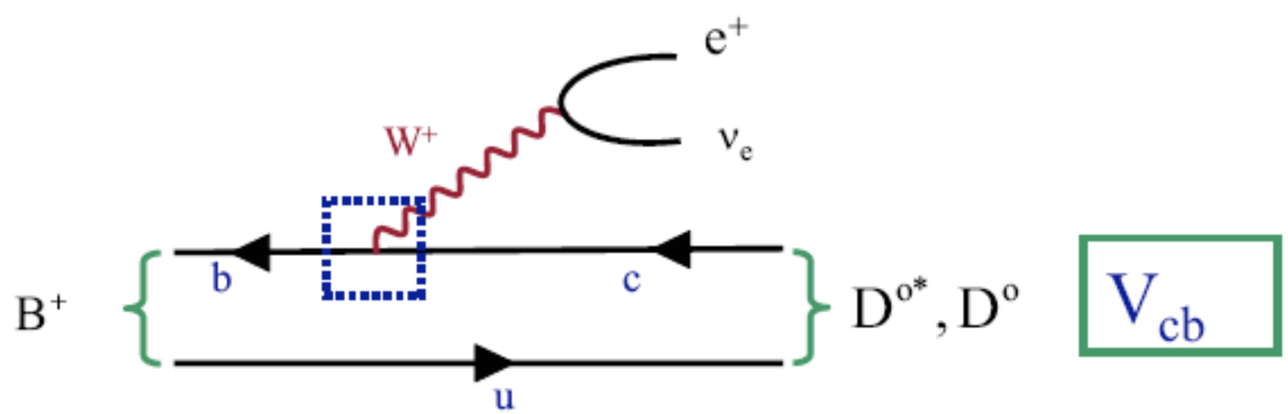
Tree-level decays



and similarly neutron or nuclear β decay for V_{ud}



and similarly D decay for V_{cd} and V_{cs}



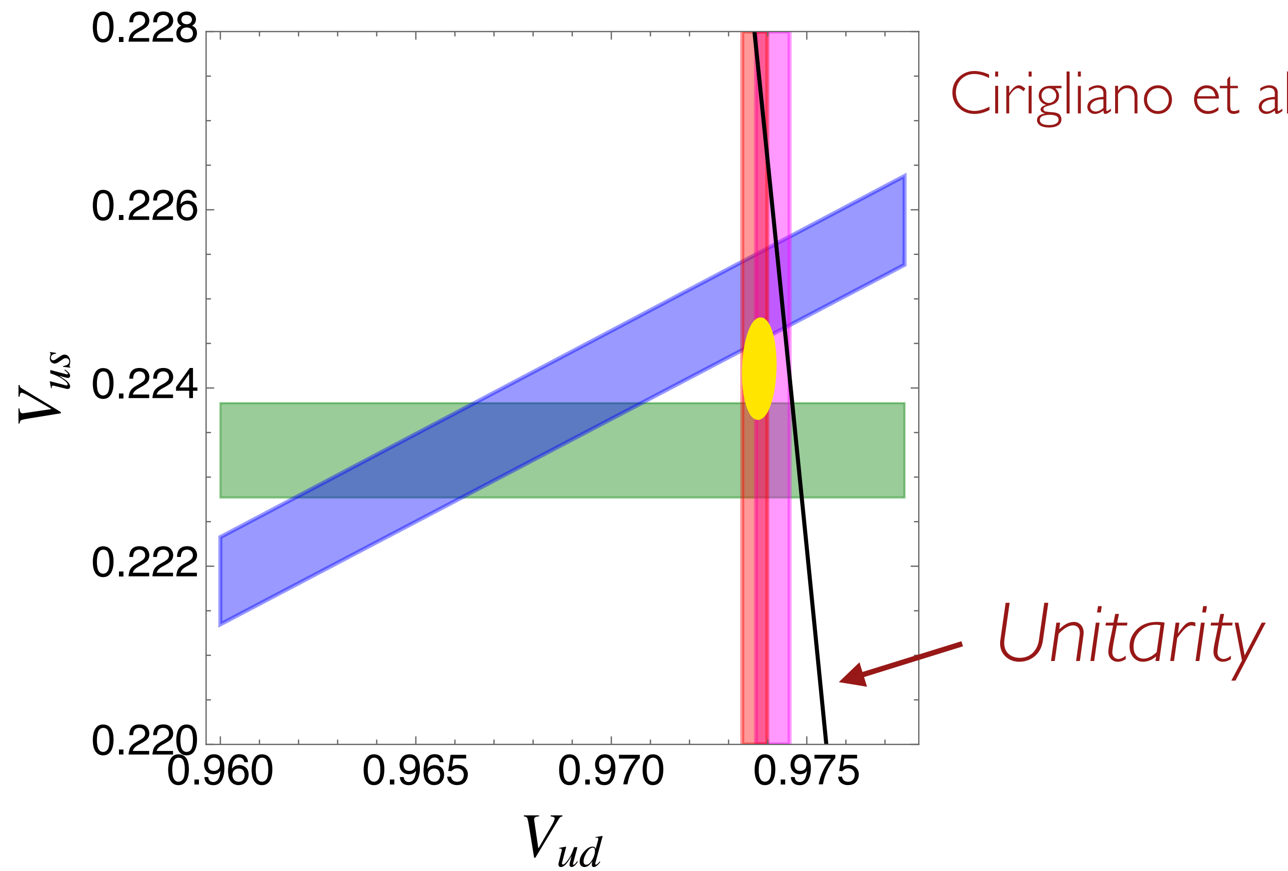
diagrams from A. Buras lectures

Unitarity relations, e.g. $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

experiment + lattice QCD etc

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$$
$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1 + 0.001(11)$$

A Cabibbo anomaly?



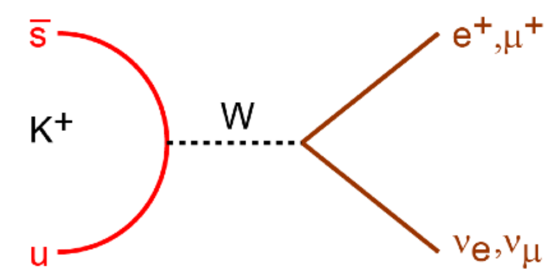
Cirigliano et al., 2208.11707

Most precise det. of V_{ud} *comes from superallowed Fermi* ($0^+ \to 0^+$) *transitions:*

$$|V_{ud}| = 0.97367(32)$$

Semileptonic K decays ($K_{\ell 3}$) + *LQCD give* $|V_{us}| = 0.2231(4)(4)$

Muonic K ($K_{\mu 2}$) *over* π *decays + LQCD give* $|V_{us}/V_{ud}| = 0.2311(5)$



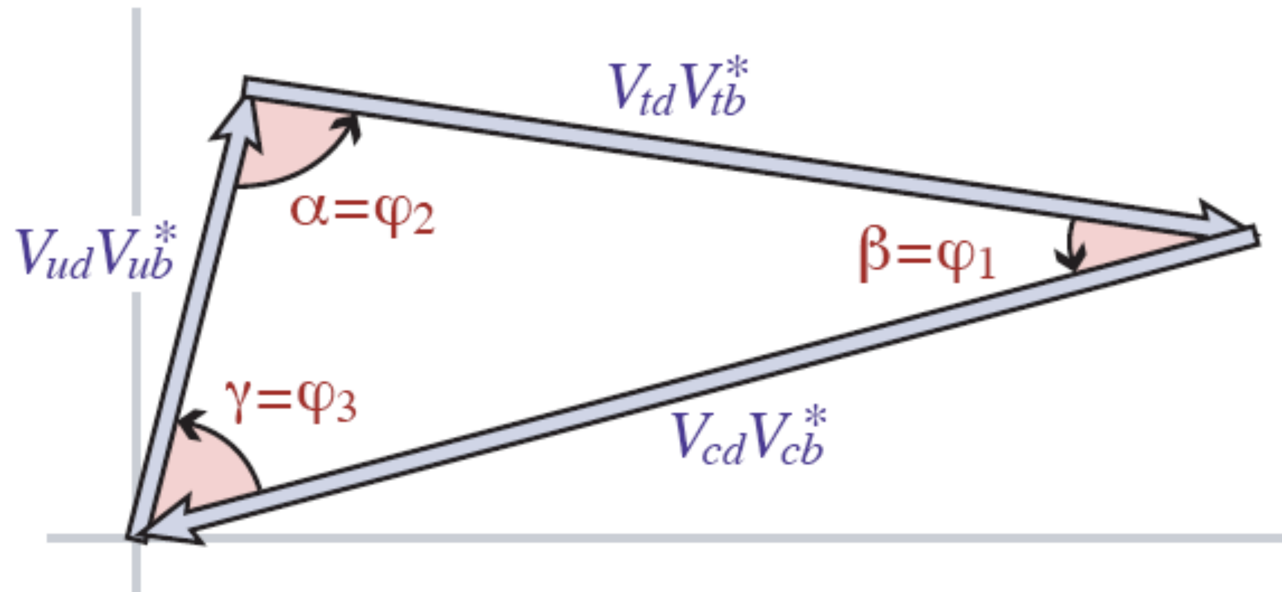
Deviations from unitarity could be due to NP, underestimated uncertainties (nuclear structure, QED, …) or experimental problems.

Unitarity Triangles

$$V_{ji}V^*_{jk} = \delta_{ik}$$

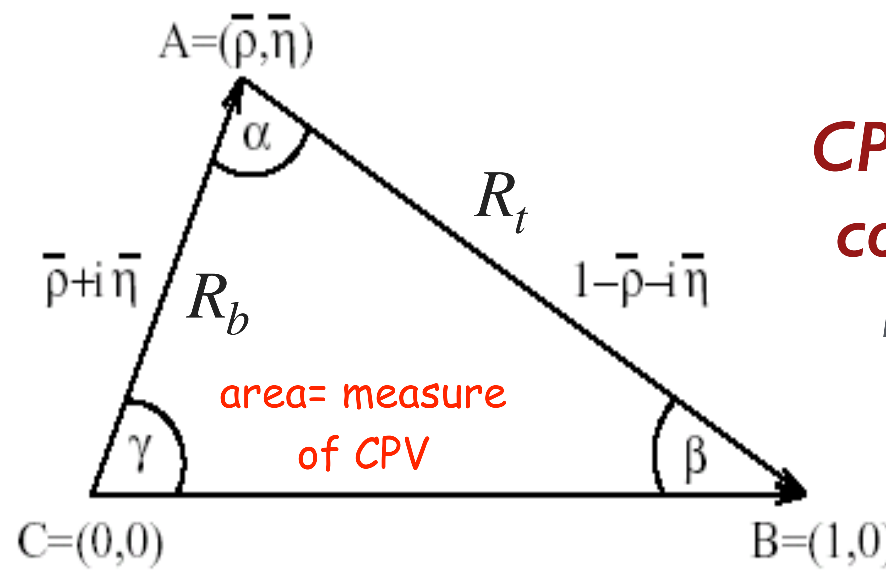
$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0$$

In this case all sides are O(λ³)



$$1 + \frac{V_{ud}V^*_{ub}}{V_{cd}V^*_{cb}} + \frac{V_{td}V^*_{tb}}{V_{cd}V^*_{cb}} = 0$$

$$R_b = \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| \sim 0.4$$



CP conserving processes constrain CP violation!

mixing and CP linked

In the phase convention of the Standard Parametrisation, the angles β and γ correspond to the phases of V_{td} and V_{ub} : $V_{ub} = |V_{ub}|e^{-i\gamma}, V_{td} = |V_{td}|e^{-i\beta}$

NB $\gamma = \delta$ of the standard parametrisation

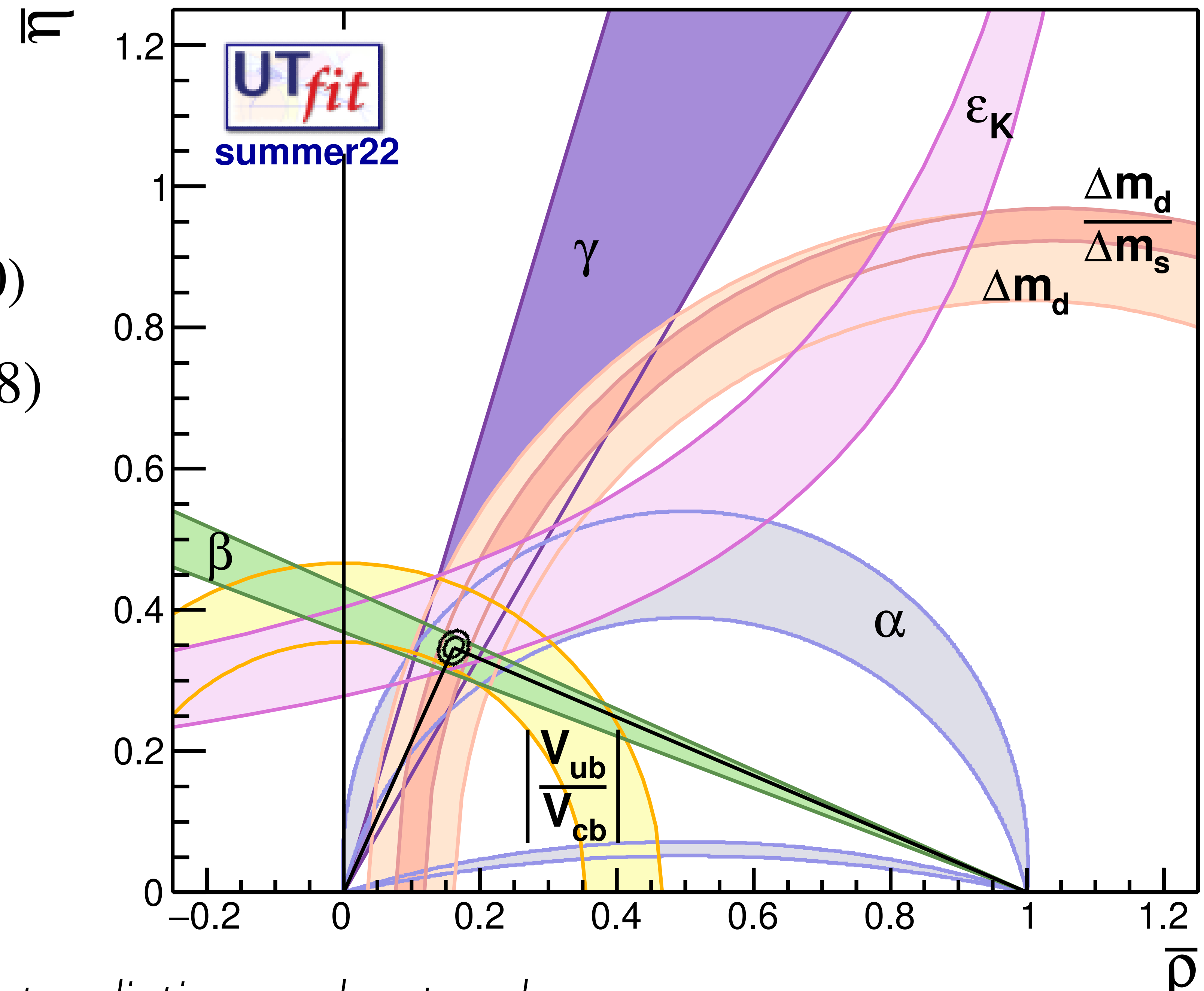
UT fit 2022

2212.03894

$\bar\rho = 0.161(10) \qquad \bar\eta = 0.347(10)$

$A = 0.828(11) \qquad \lambda = 0.2252(8)$

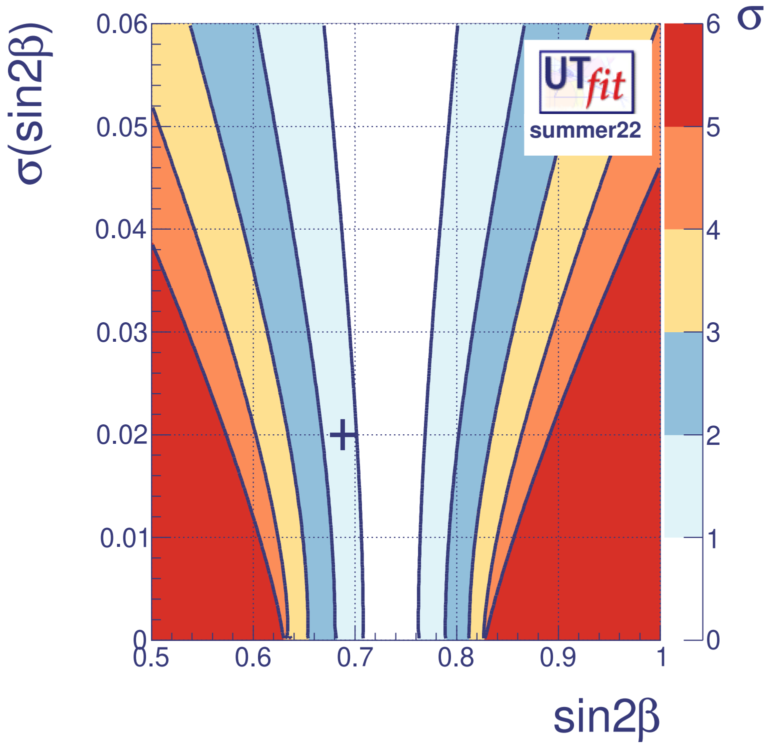
Similar results from
CKMFitter collaboration
http://ckmfitter.in2p3.fr

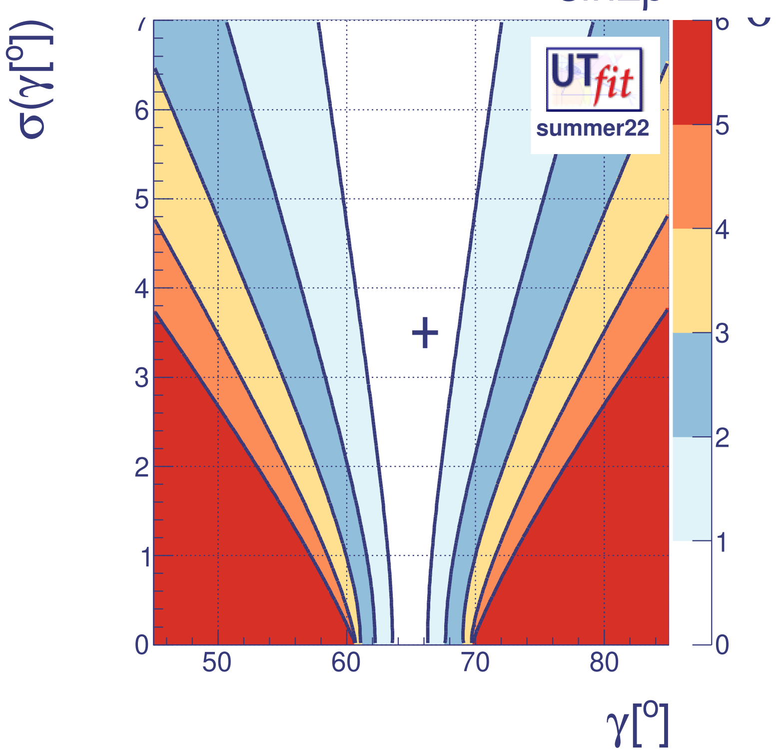


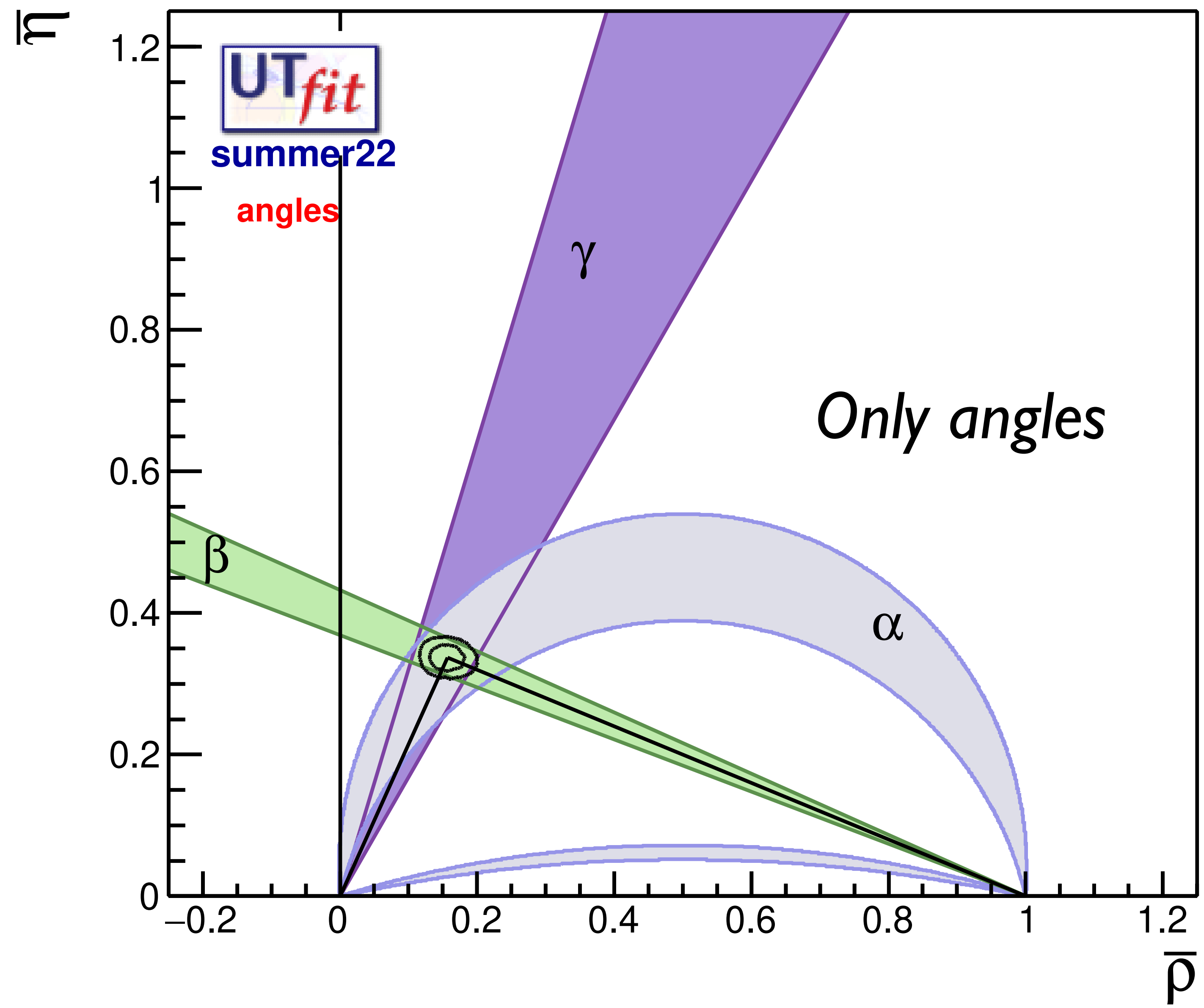
Example of comparison between prediction and exp value

$\sin 2\beta_{fit} = 0.736(28) \qquad \sin 2\beta_{exp} = 0.688(20)$

from CP asymmetry in
B→J/Ψ Ks (mostly)

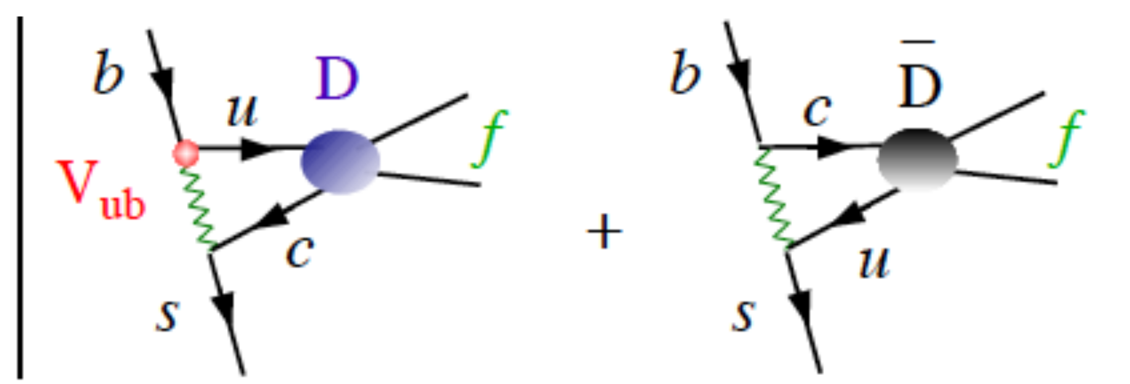




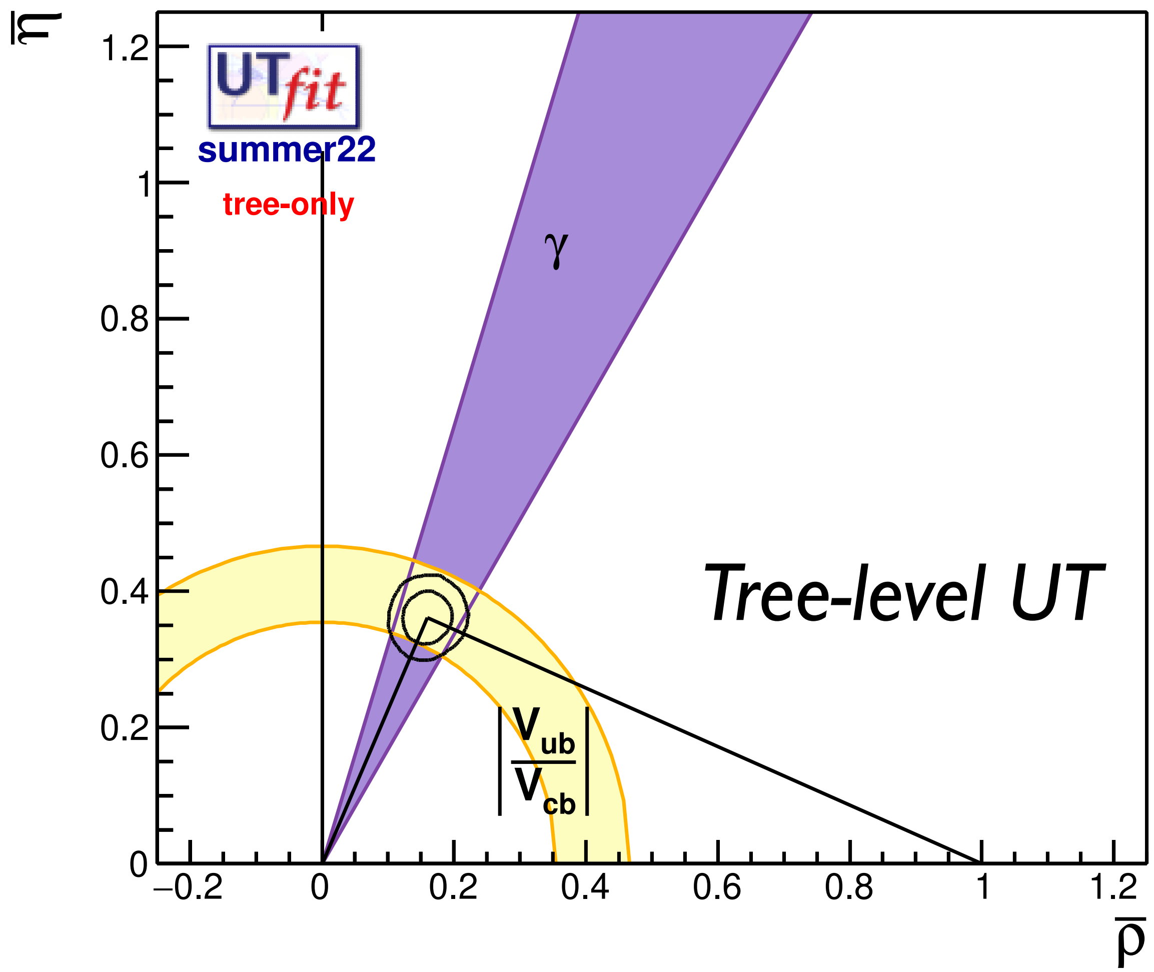


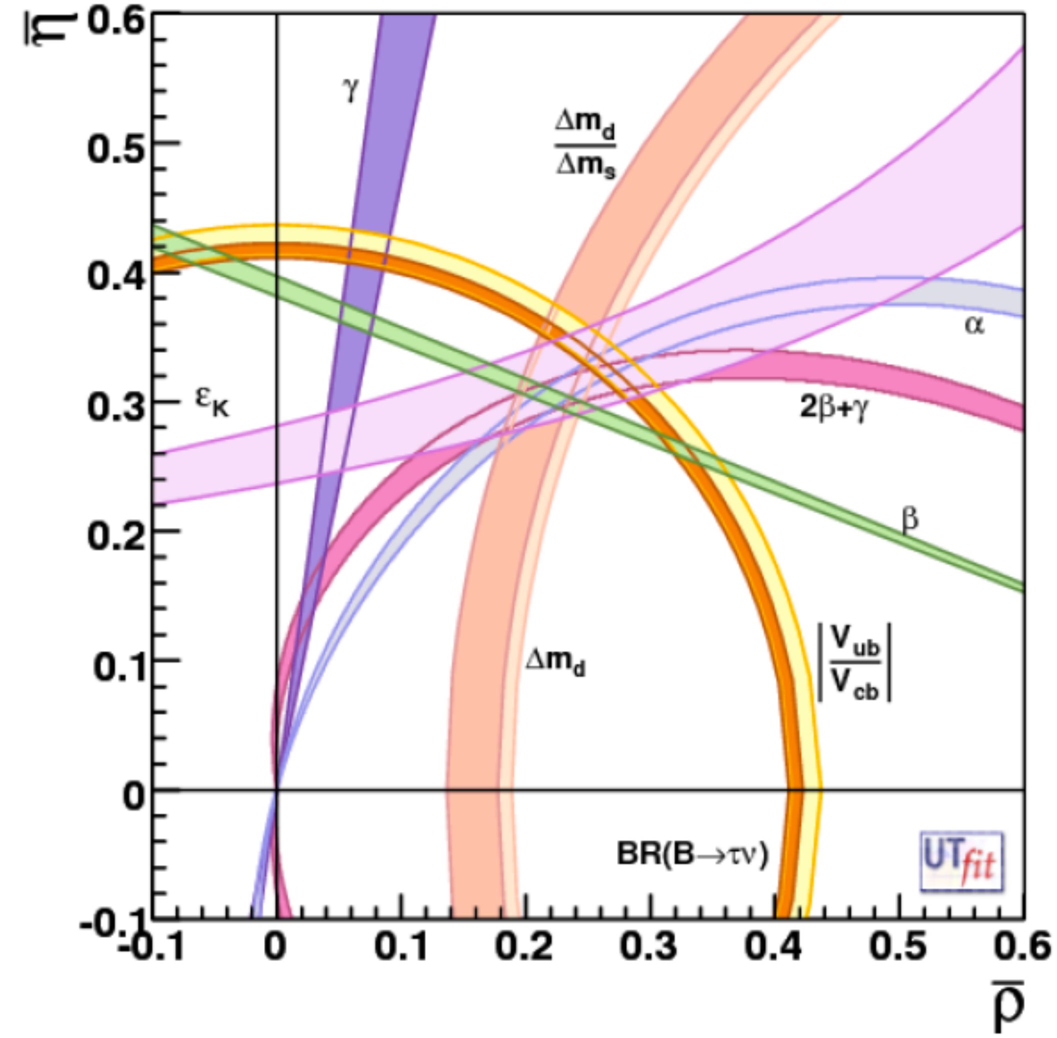
The phase $\gamma = \arg(V_{ub})$ can be obtained, for instance, by non-leptonic processes of the type

$$B \to D\,(\bar{D}) + K \to f + K \;:$$



All hadronic parameters are extracted from data on BR and CP asymmetries





<u>Tree level UT</u>: γ and $|V_{ub}|/|V_{cb}|$ can determine the UT precisely in a way independent of loop induced New Physics

Belle-II+LHCb dream scenario

CP violation in the SM

- It is non-trivially linked to flavour violation, generally NP has flavour diagonal CPV
- CP is not violated in the SM lepton sector, although with ν masses CPV becomes inevitable
- CKM phase is the *only source of CPV in the quark sector*. In particular, the phase is the *same in K and B decays*. NP can only lead to small modifications.
- CPV appears in charged currents and FCNC only. Again this is a strong constraint on NP (eg. no electric dipole moment has been measured yet)
- the CKM phase is *not* sufficient to explain baryogenesis and the asymmetry between matter and antimatter in the Universe. *There must be sources of CPV beyond the CKM phase.*

Strong CP Problem

- *Nonperturbative* QCD effects induce

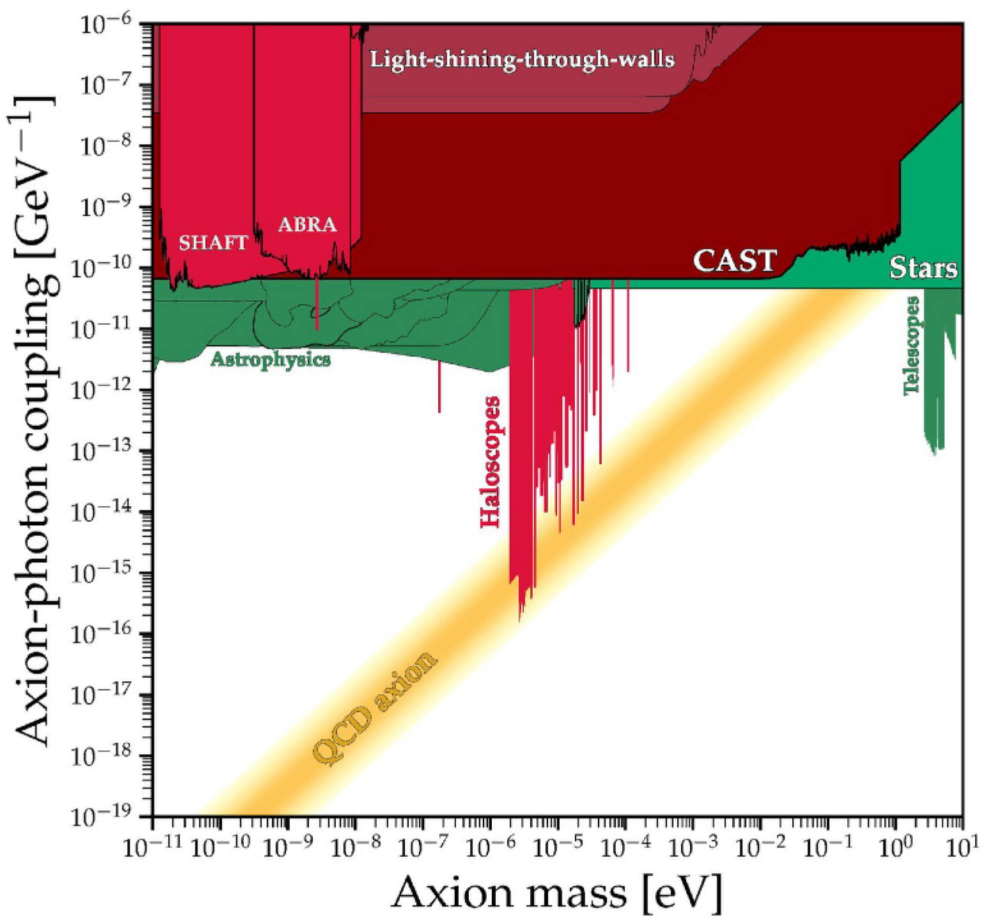
$$\mathcal{L}_\theta = \frac{\theta_{\rm QCD}}{32\pi^2}\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu a}F^{\rho\sigma a}$$

- This term violates CP, is flavour diagonal, and induces an electric dipole moment in the neutron, $d_N < 10^{-25}$ e cm, from which

$$\theta_{\rm QCD} \lesssim 10^{-10}$$

- Such a small value is *unnatural* and is not protected by any symmetry. One would expect $\vartheta_{\rm QCD}=O(1)$, unless one of the quark mass is zero, but this cannot be.

- Possible solutions involve a new spontaneously broken global symmetry, whose Goldstone boson is the ***axion*** (Peccei-Quinn mechanism) or spontaneously broken CP.

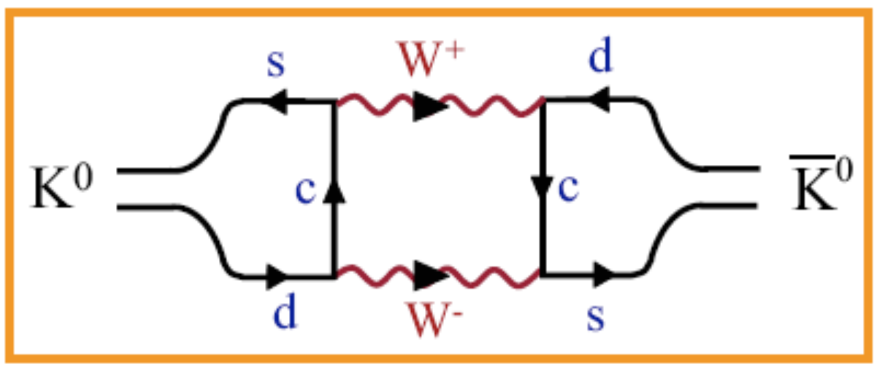


FCNC

- The CKM mechanism forbids FCNC at tree level (GIM, *Glashow Iliopoulos Maiani 1970*)
- Beyond tree level, GIM guarantees additional suppression of $\Delta S=1,2$ transitions, that are $O(G_F^2\, m^2)$ rather than $O(G_F\, \alpha)$.

Gaillard-Lee (March 1974)

$\boxed{\lambda \cong 0.22}$



$$\left\{ \begin{array}{l} \Delta M_K = \left[1.4\ \mathrm{GeV}^5\right] G_F^2 \lambda^2 \left[\dfrac{m_c^2}{M_W^2}\right] \\ G_F \cong 1.2 \cdot 10^{-5}\, \mathrm{GeV}^{-2} \end{array} \right\}$$

$$\boxed{\Delta M_K = 10^{-11}\,\mathrm{GeV}\left[\frac{m_c^2}{M_W^2}\right] = 3.5 \cdot 10^{-15}\,\mathrm{GeV}}$$

$$\boxed{m_c = \sqrt{3.5 \cdot 10^{-2}}\, M_W = 1.5\ \mathrm{GeV}}$$

(Prediction !!)

$$\left\{ \begin{array}{c} \text{November Revolution} \\ 1974 \\ \text{Discovery of } \bar{c}c \text{ State} \\ \text{(SLAC, Brookhaven)} \end{array} \right\}$$

$$\boxed{M_{\bar{c}c} \cong 3.1\ \mathrm{GeV}}$$

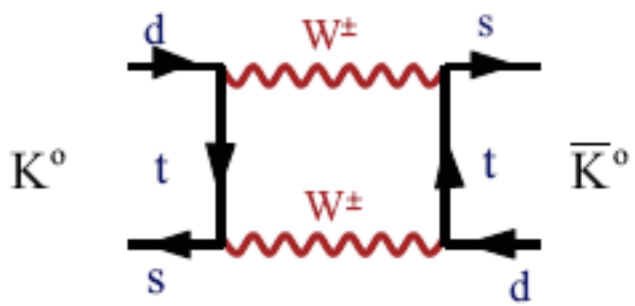
$$\boxed{m_c \cong 1.5\ \mathrm{GeV}} \quad !!$$

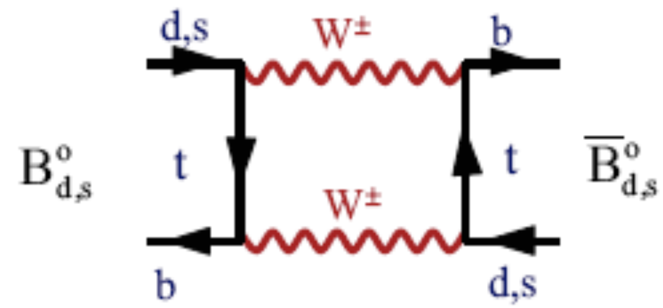
Prediction confirmed !

A. Buras

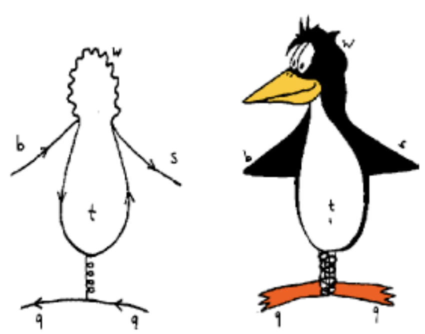
Systematics of FCNC: boxes & penguins

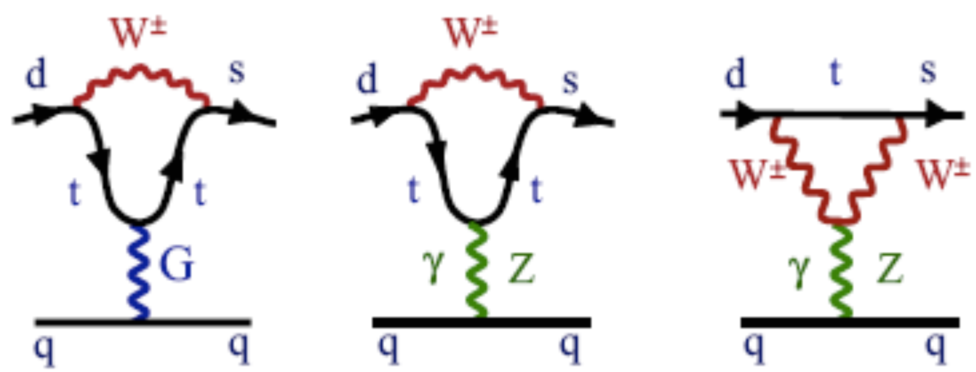
$\not{CP}$ ε_K-Parameter
ΔM (K_L-K_S)



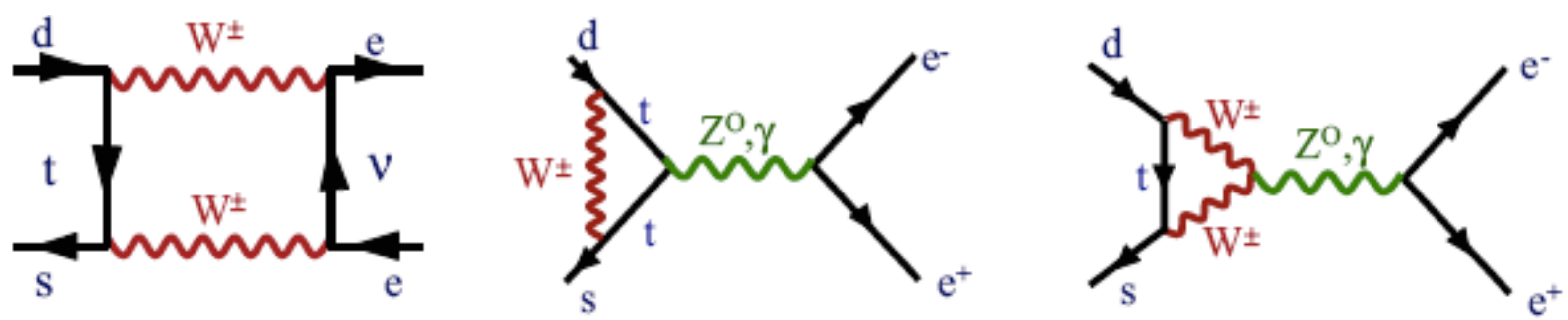


$B^o_d - \overline{B}^o_d$ Mixing



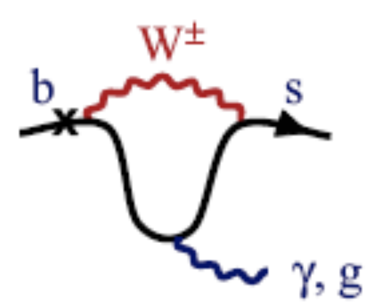


eg CP violation in K decay (ε')



$K_L \rightarrow \mu\overline{\mu}$,

rare K and B decays
New Physics may contribute
at the same level as SM
and may not undergo the same
CKM suppression…



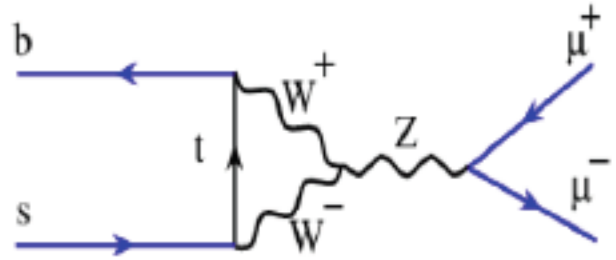
$B \rightarrow X_s\gamma \qquad B \rightarrow K^*\gamma$

A. Buras, lectures

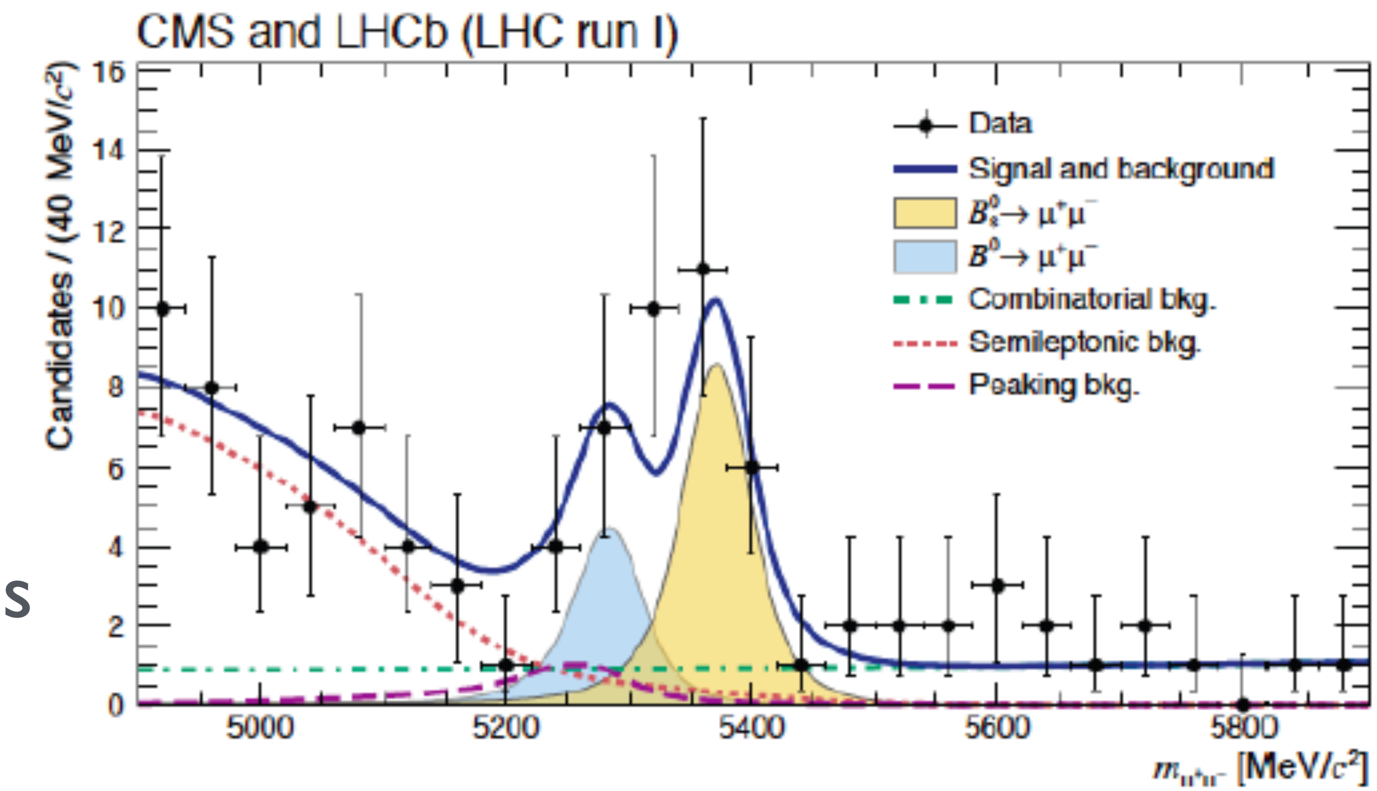
LETTER First observation of $B_s \rightarrow \mu^+\mu^-$

OPEN
doi:10.1038/nature14474

Standard Model



+ other similar diagrams



$$\mathcal{B}(B_s^0 \rightarrow \mu^+\mu^-) = 2.8^{+0.7}_{-0.6} \cdot 10^{-9}$$

SM: 3.66±0.23 x10^{-9}

6.2σ significance → first observation

- compatible with SM at 1.2σ

$$\mathcal{B}(B^0 \rightarrow \mu^+\mu^-) = 3.9^{+1.6}_{-1.4} \cdot 10^{-10}$$

SM: 1.06±0.09 x10^{-10}

3.0σ significance → first evidence

- compatible with SM at 2.2σ

SM: Bobeth et al: PRL 112 101801 (2014)

Decoupling

- ***Decoupling theorem***: the effects of heavy particles in a renormalisable theory are power-suppressed (up to a redefinition of the couplings) if the theory remains renormalizable and no coupling is proportional to the heavy masses. Appelquist,Carazzone, 1974

- Examples: QCD with/without top, QED at low energy. These are QFT valid at energies below a scale Λ, they are ***effective field theories***

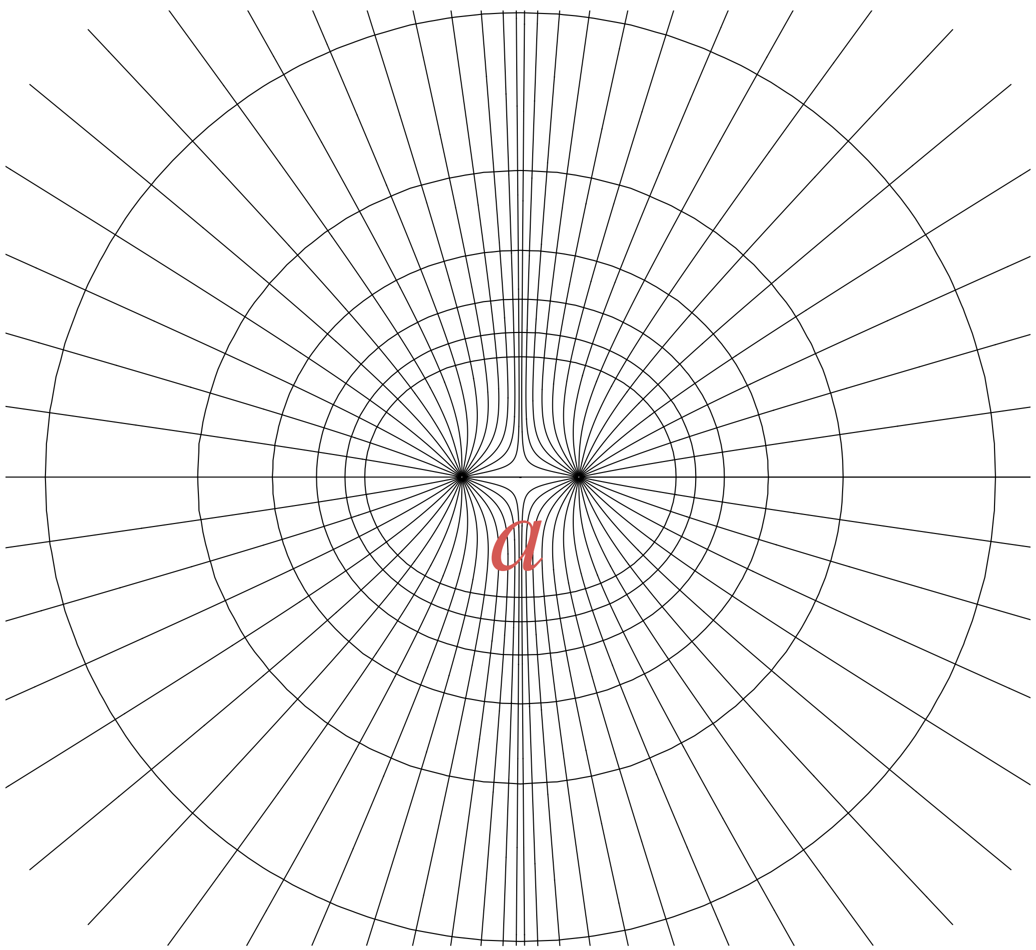
- The SM is a ***renormalizable effective theory***: it is screened from new physics effects.

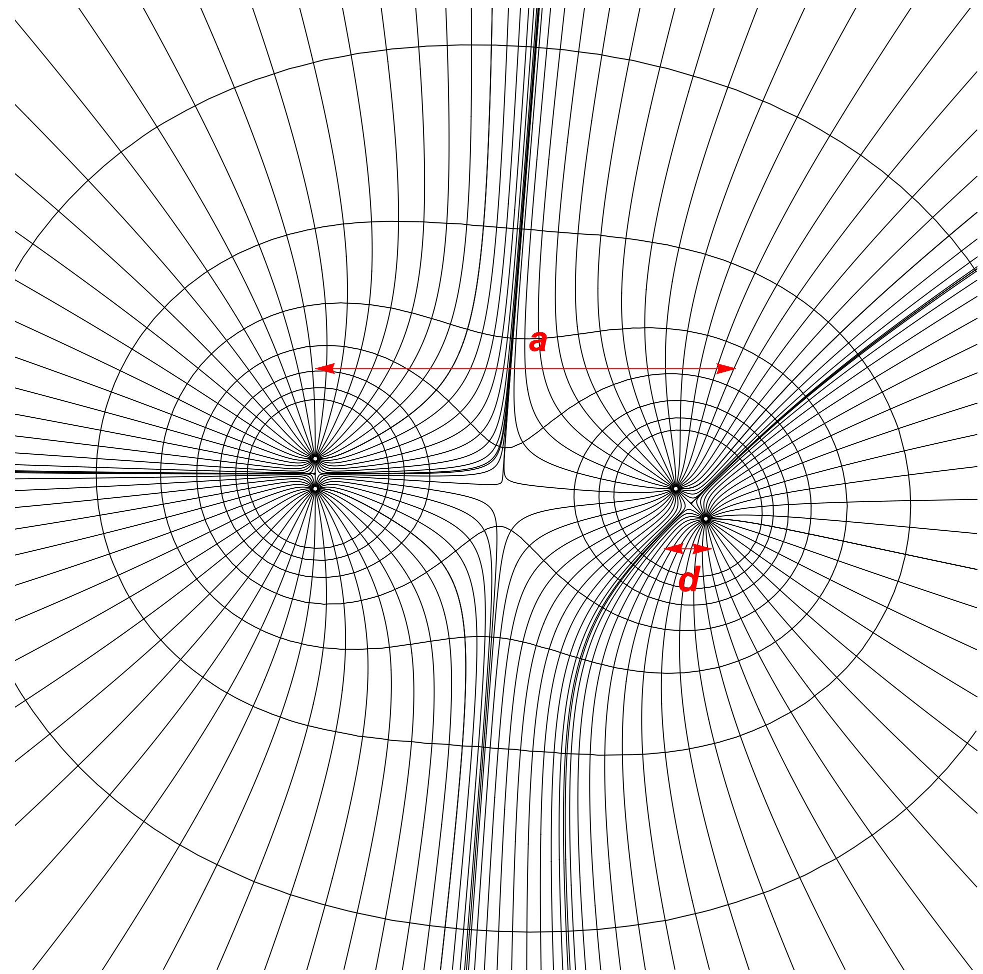
$$\mathcal{L}_{\rm eff} = \mathcal{L}^{\rm SM}_{\rm gauge} + \mathcal{L}^{\rm SM}_{\rm Higgs} + \mathcal{L}^{\rm SM}_{\rm Yukawa} + \Delta\mathcal{L}_{d>4} \ ,$$

$$\Delta\mathcal{L}_{d>4} = \sum_{d>4}\sum_{n=1}^{N_d} \frac{c_n^{(d)}}{\Lambda^{d-4}} \mathcal{O}_n^{(d)}(\text{SM fields})$$ SMEFT

Wilson coefficients

Separating scales: the idea behind effective theories





Manohar, 1804.05863

Multipole expansion in EM

$$V(\mathbf{r}) = \frac{1}{r}\sum_{l,m} c_{lm}\left(\frac{a}{r}\right)^l Y_{lm}(\Omega)$$

Works for $a \ll r$. We can measure $c_{lm}a^l$ and get info on a and charge distribution, or (when a is known) simplify the potential

Multiscale problem:

$d \ll a \ll r$

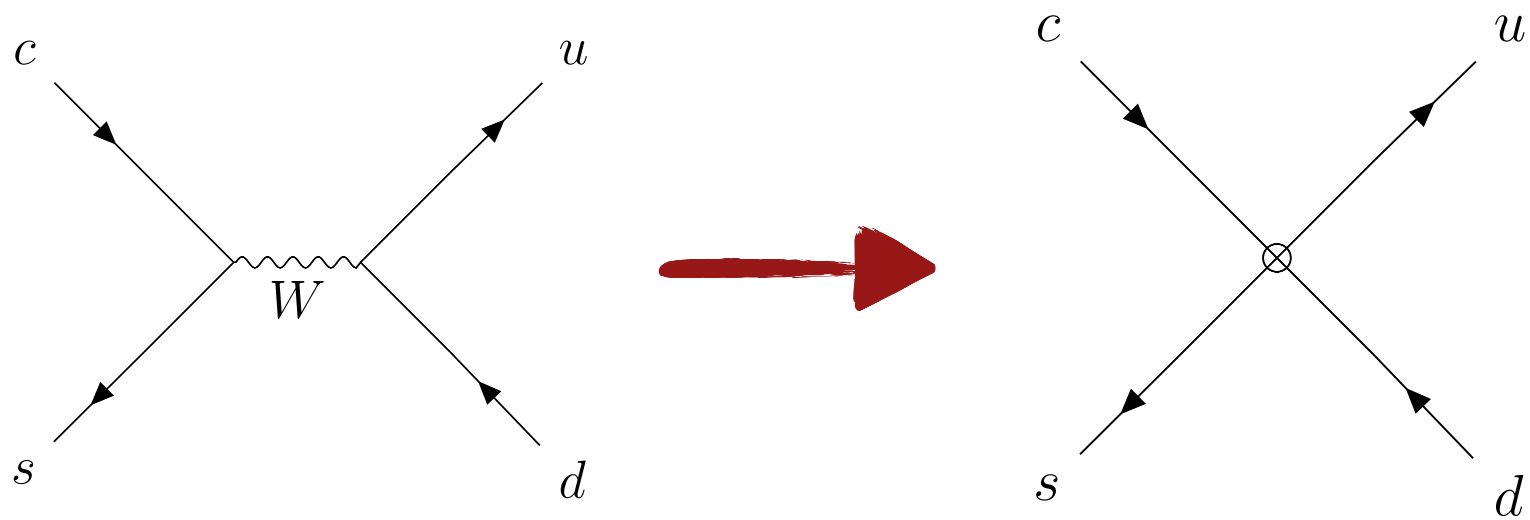
After Fourier transf

$\Lambda_{NP} \gg M_W \gg m_b$

Info on d requires more precise measurements

Fermi Lagrangian

Describes very well weak decays of muons, kaons, charm…



Taylor expanding the W propagator

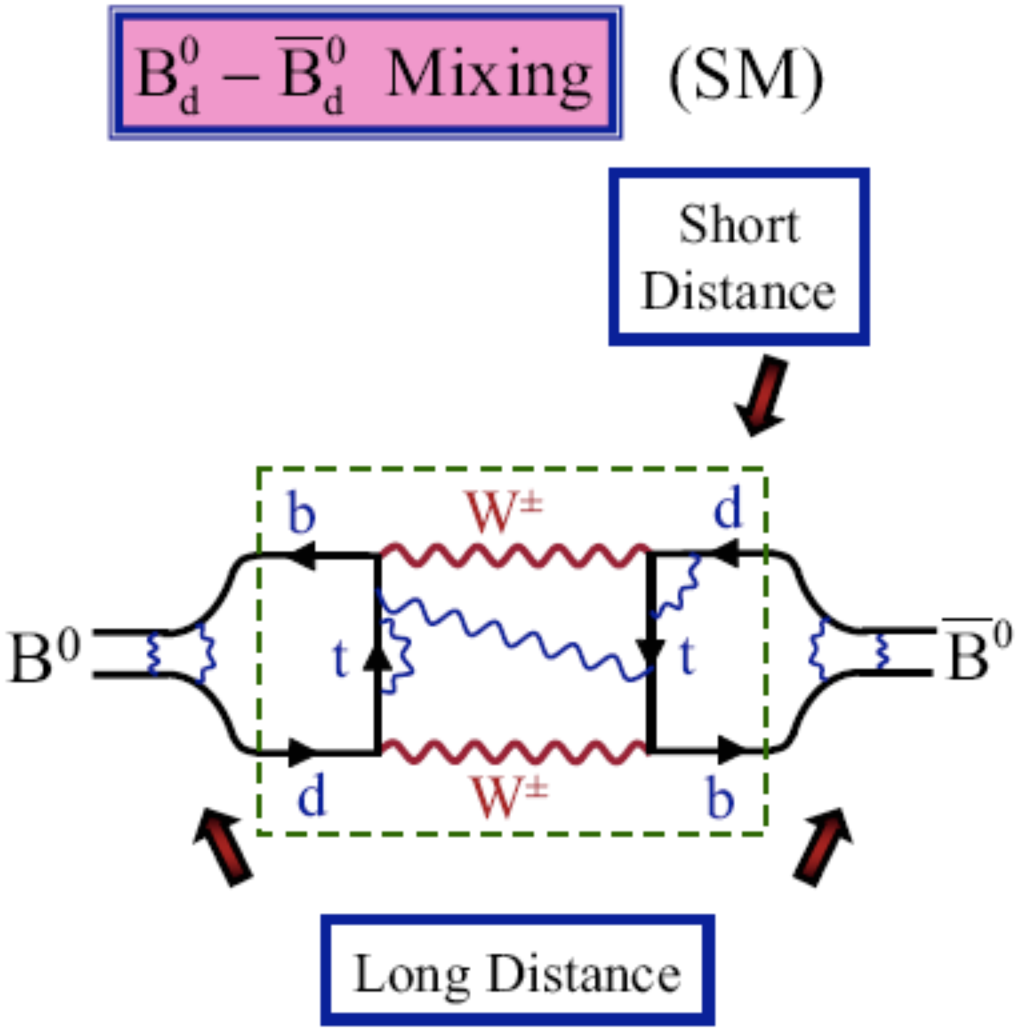
$$\frac{g^2}{q^2 - M_W^2} = \frac{-g^2}{M_W^2} + O(\frac{q^2}{M_W^4}) = -\frac{8G_F}{\sqrt{2}} + O(\frac{q^2}{M_W^4})$$

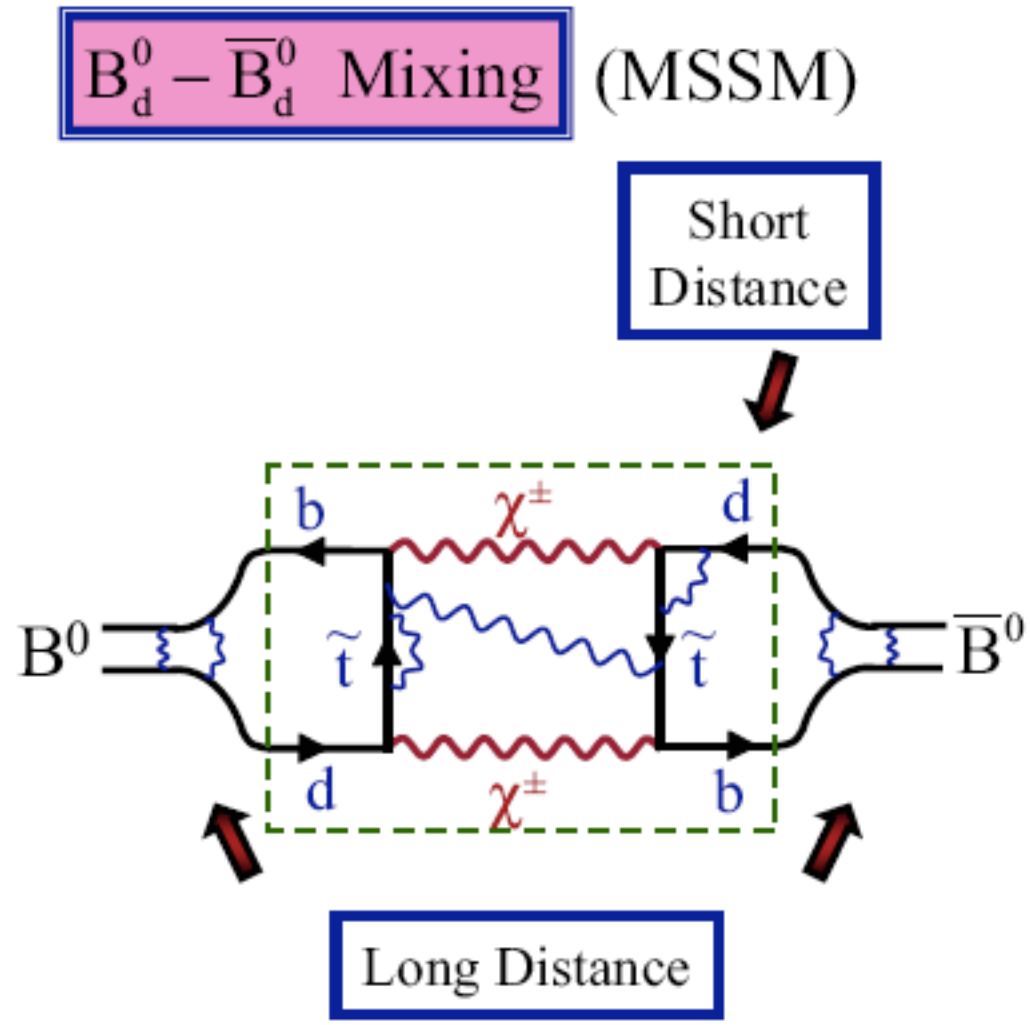
$$\mathscr{L}_{Fermi} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud}\, \bar{s}_L \gamma^\mu c_L \bar{u}_L \gamma_\mu d_L$$ **local dim 6 operator**

Wilson coefficient contains short distance info (M_W)
QCD, QED, higher order EW effects can all be considered.

A systematic expansion in powers of M_W
What do we do with loops?

The crucial problem





$\boxed{\text{SD}}$: Perturbative (Asymptotic Freedom)

$\boxed{\text{LD}}$: Non-Perturbative (Confinement)

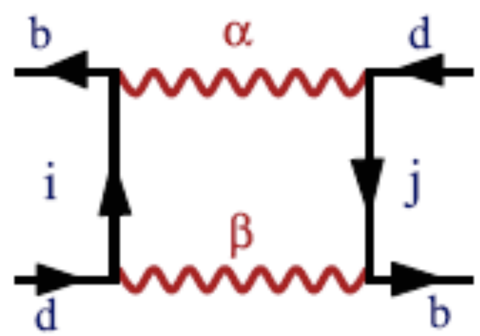
A. Buras

$$M_W \gg m_b \gg \Lambda_{QCD}$$

$$\Lambda_{NP} \gg m_b \gg \Lambda_{QCD}$$

B-$\bar{B}$ box calculation

$$\sum_{i,j=u,c,t} \; \sum_{\alpha,\beta=W^{\pm},\Phi^{\pm}}$$

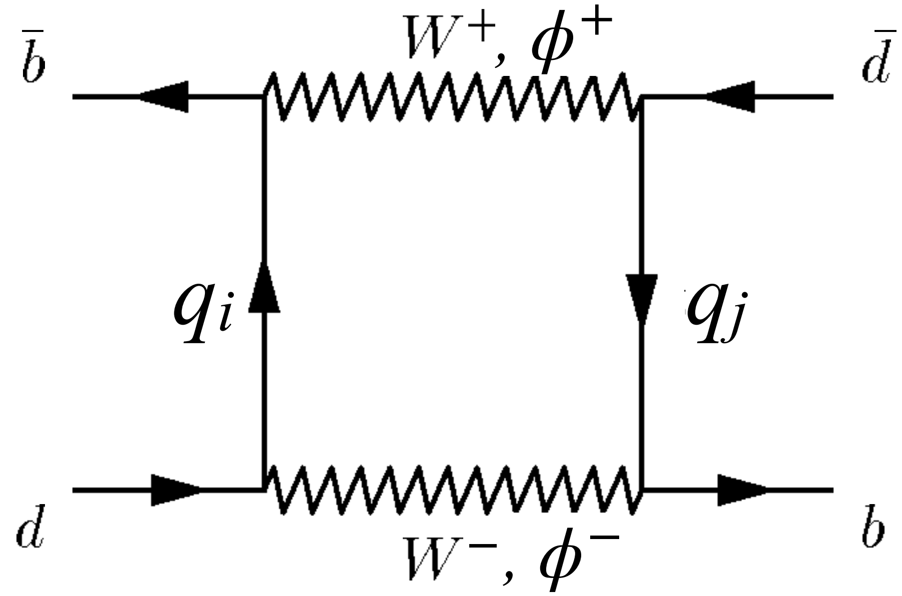


$$\equiv \sum_{i,j=u,c,t} F\left(x_i, x_j\right)\left(V_{ib}^{*} V_{id}\right)\left(V_{jb}^{*} V_{jd}\right) Q$$

$$Q(\Delta B = 2) = \left(\bar{b} d\right)_{V-A}\left(\bar{b} d\right)_{V-A}$$

Illustration of GIM mechanism, example of reduction to effective theory (calculation of Wilson coefficient at lowest order)

Effective theory allows for precise SM calculations and for straightforward parameterization of new physics



CKM factor: $\lambda_i = V_{ib}^* V_{id}$ *one for each quark line*

Unitarity implies $\sum_i V_{ib}^* V_{id} = \sum_i \lambda_i = \lambda_u + \lambda_c + \lambda_t = 0$

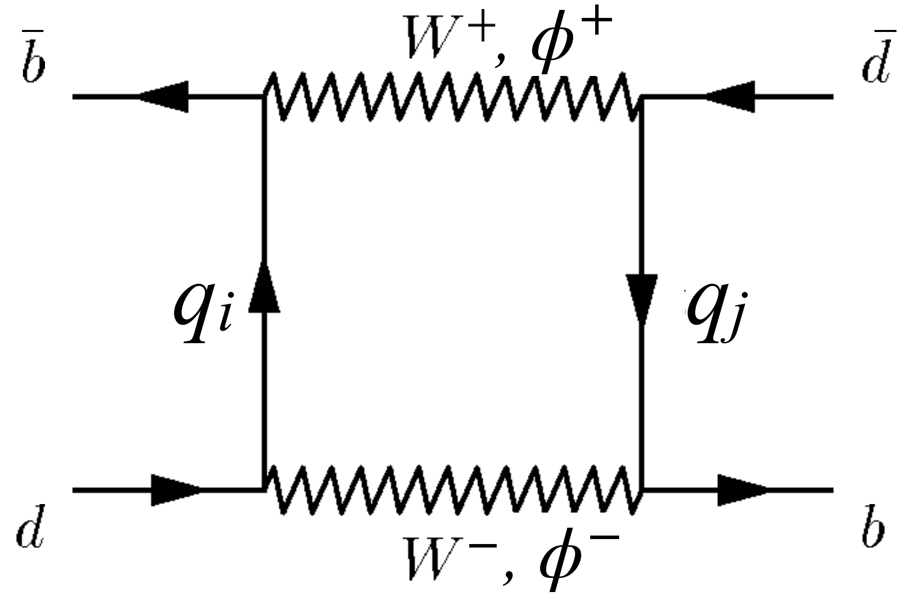
similarly $\sum_{i,j} \lambda_i \lambda_j = 0$*: for degenerate up quarks amplitude vanishes (GIM)*

Of course $m_u \neq m_c \neq m_t$ *hence loop function*

$$\mathscr{A}(i,j) = \mathscr{A}(p, m_i, m_j, M_W) \simeq G_F F(x_i, x_j)$$

where $x_i = \dfrac{m_i^2}{M_W^2}$ *and we neglected external momenta* $|p^\mu| \ll M_W$.

For up and charm $x_{u,c} \ll 1$, $F(u,u) \simeq F(c,c) \simeq F(u,c) \simeq F(0,0)$



$$\sum_{i,j} \lambda_i \lambda_j F(i,j) \simeq [\lambda_u^2 + 2\lambda_u\lambda_c + \lambda_c^2] F(0{,}0) + 2(\lambda_u + \lambda_c)\lambda_t F(0{,}t) + \lambda_t^2 F(t,t)$$

with $F(t{,}0) = F(0{,}t)$

$$\simeq \lambda_t^2 [F(t,t) - 2F(0{,}t) + F(0{,}0)]$$

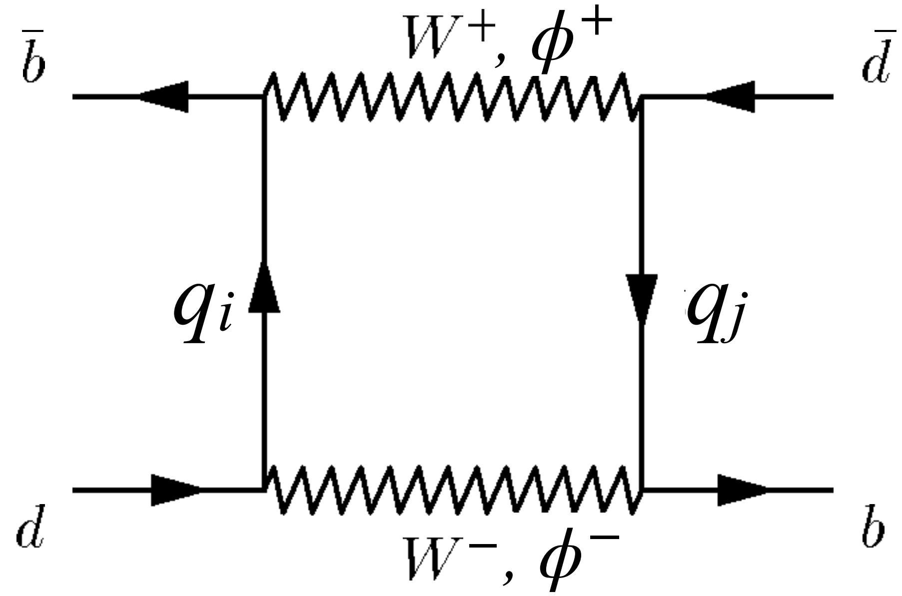
$Re\lambda_t^B = V_{tb}^* V_{td} = O(\lambda^3) \propto R_t$, $\quad Re\lambda_t^{B_s} = V_{tb}^* V_{ts} = O(\lambda^2)$, $\quad Re\lambda_t^K = V_{ts}^* V_{td} = O(\lambda^5)$

$$\mathscr{A}(t{,}0) \propto \int d^4k \left(\frac{-i}{k^2 - M_W^2} \right)^2 \bar{b}_L \gamma^\mu \frac{i(\not{k} + m_t)}{k^2 - m_t^2} \gamma^\nu d_L \otimes \bar{b}_L \gamma_\nu \frac{i\not{k}}{k^2} \gamma_\mu d_L$$

$$= \bar{b}_L \gamma^\mu \gamma^\alpha \gamma^\nu d_L \otimes \bar{b}_L \gamma_\nu \gamma^\delta \gamma_\mu d_L \int d^4k \frac{k_\alpha k_\delta}{k^2 (k^2 - m_t^2)(k^2 - M_W^2)^2}$$

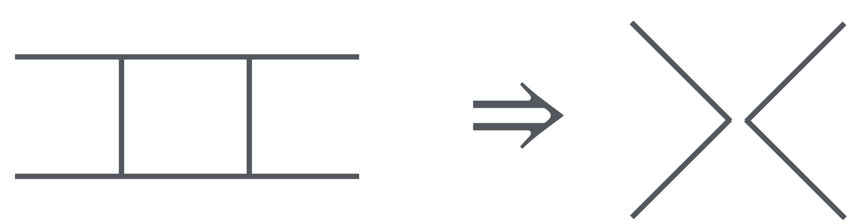
$$= 4\bar{b}_L \gamma^\nu d_L \otimes \bar{b}_L \gamma_\nu d_L \int d^4k \frac{1}{(k^2 - m_t^2)(k^2 - M_W^2)^2}$$

after using $\gamma^\mu \gamma^\alpha \gamma^\nu = g^{\mu\alpha}\gamma^\nu + g^{\nu\alpha}\gamma^\mu - g^{\mu\nu}\gamma^\alpha - i\epsilon^{\mu\nu\alpha\beta}\gamma_5\gamma_\beta$



$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_t^2)(k^2 - M_W^2)^2} = \frac{-i}{16\pi^2 M_W^2} \left(1 + \frac{x_t}{1 - x_t} + \frac{x_t \ln x_t}{(1 - x_t^2)^2} \right)$$

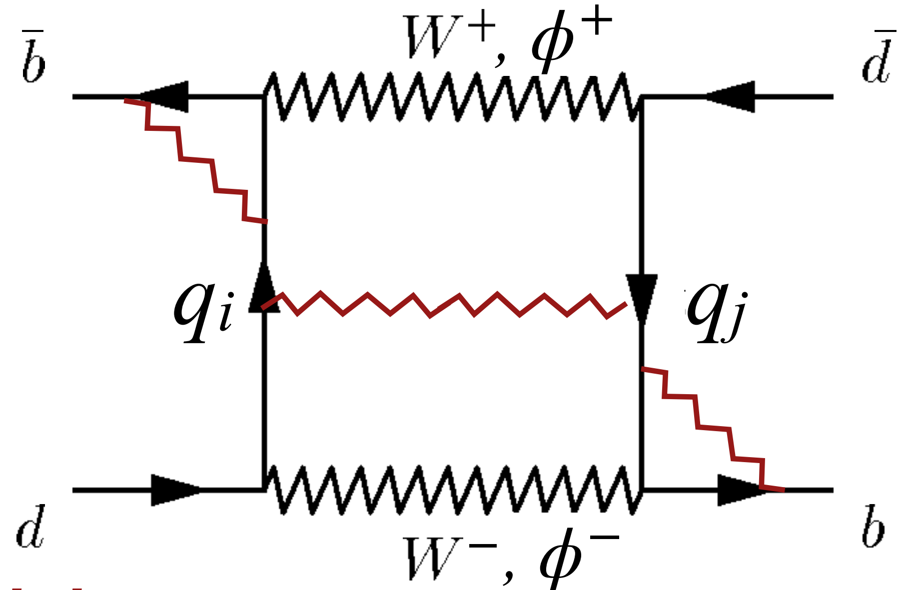
For $m_t \ll M_W$, $F(t,0) = F(0,0)$. $F(t,t)$ *similar calculation but prop to* x_t *for large* m_t *(diagrams with would-be Goldstone have four times the top Yukawa:* $y_t^4/m_t^2 \sim m_t^2/M_W^4$*).*



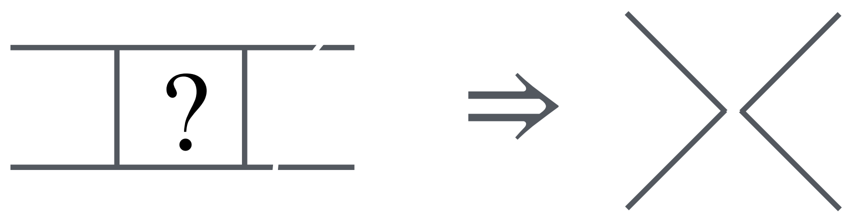
The $\bar{B} - B$ *mixing amplitude in the SM could have been computed from*

$$\mathscr{L}_{eff}^{\Delta B=2} = -\frac{G_F^2 M_W^2}{16\pi^2} \lambda_t^2 S(x_t)\, \bar{b}_L \gamma^\mu d_L \bar{b}_L \gamma_\mu d_L$$

$$\mathscr{L}^{SM} \to \mathscr{L}^{QED+QCD} + \mathscr{L}_{eff}^{\Delta B=2} \text{ up to } O(1/M_W^4)$$



In a generic NP model additional contributions, not necessarily suppressed by GIM



$$Q_1 = \bar{b}_L \gamma^\mu d_L \bar{b}_L \gamma_\mu d_L \qquad Q_2 = \bar{b}_R \gamma^\mu d_R \bar{b}_R \gamma_\mu d_R$$

$$Q_3 = \bar{b}_L d_R \bar{b}_L d_R \qquad Q_4 = \bar{b}_R d_L \bar{b}_R d_L$$

$$Q_5 = \bar{b}^\alpha_L d^\beta_R \bar{b}^\beta_L d^\alpha_R \qquad Q_6 = \bar{b}^\alpha_R d^\beta_L \bar{b}^\beta_R d^\alpha_L$$

$$Q_7 = \bar{b}_L d_R \bar{b}_R d_L \qquad Q_8 = \bar{b}^\alpha_L d^\beta_R \bar{b}^\beta_R d^\alpha_L$$

NP contribution to the Wilson coefficient of the SM operator + new operators
α, β color indices
Operator basis non-unique

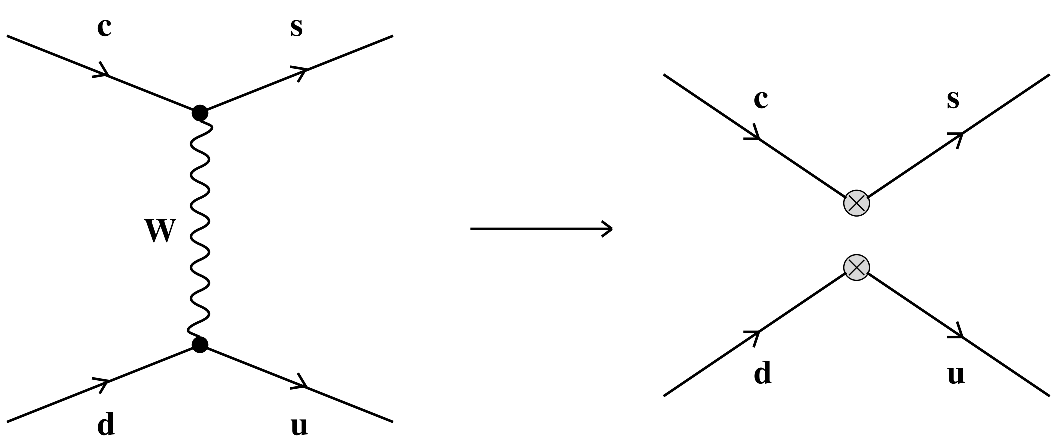
The $\bar{B} - B$ mixing amplitude could be computed from

$$\mathscr{L}^{\Delta B=2}_{eff,NP} = -\frac{1}{\Lambda^2_{NP}} \sum_i C_i Q_i \ + \text{higher dim}$$

but what about the matrix elements of these operators?
and how is all this modified by gluon exchanges?

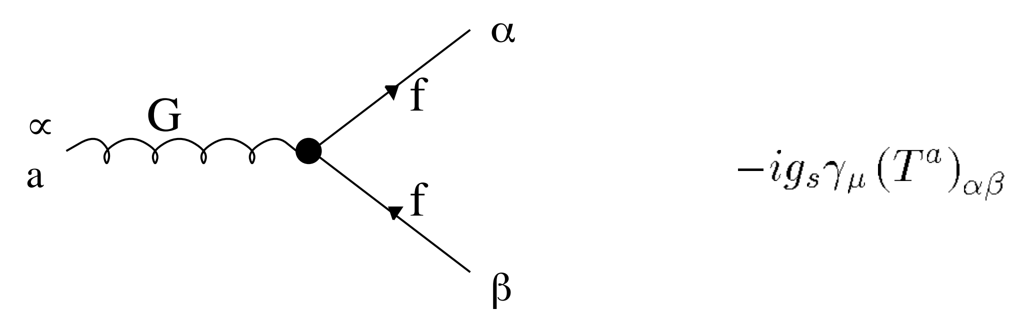
EFT with loops

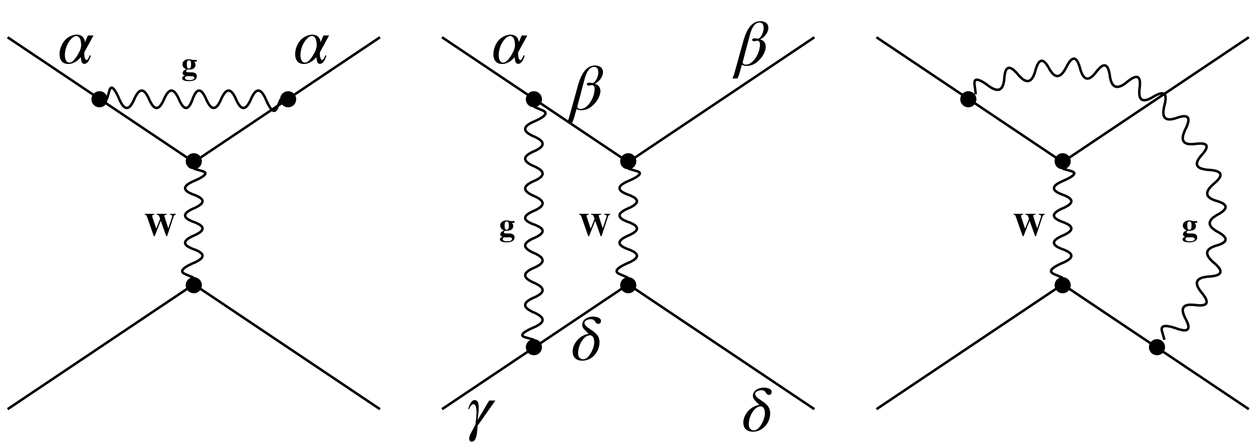
A ***simple but nontrivial example***: *nonleptonic c decay* $c \to s\bar{d}u$



$$A_{SM}^{(0)} \sim -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud}\, \bar{s}_L \gamma^\mu c_L \bar{u}_L \gamma_\mu d_L$$

with one gluon, α, β *color indices*





+ mirror diagrams

Boxes mix color indices: we need 2 dim6 ops

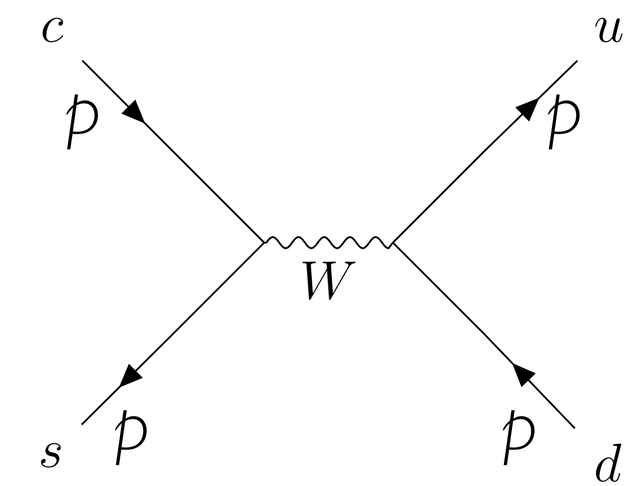
$$Q_1 = (\bar{s}_{L\alpha}\gamma^\mu c_{L\beta})(\bar{u}_{L\beta}\gamma_\mu d_{L\alpha})$$
$$Q_2 = (\bar{s}_{L\alpha}\gamma^\mu c_{L\alpha})(\bar{u}_{L\beta}\gamma_\mu d_{L\beta})$$

$$\mathscr{L}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[C_1 Q_1 + C_2 Q_2\right]$$

Wilson coeff $C_1 = 0$ *at LO in* α_s. Q_2 *mixes into* Q_1

Actually we have $T^a_{\alpha\beta}T^a_{\gamma\delta} = -\frac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \frac{1}{2}\delta_{\alpha\delta}\delta_{\beta\gamma}$ N=3 in QCD

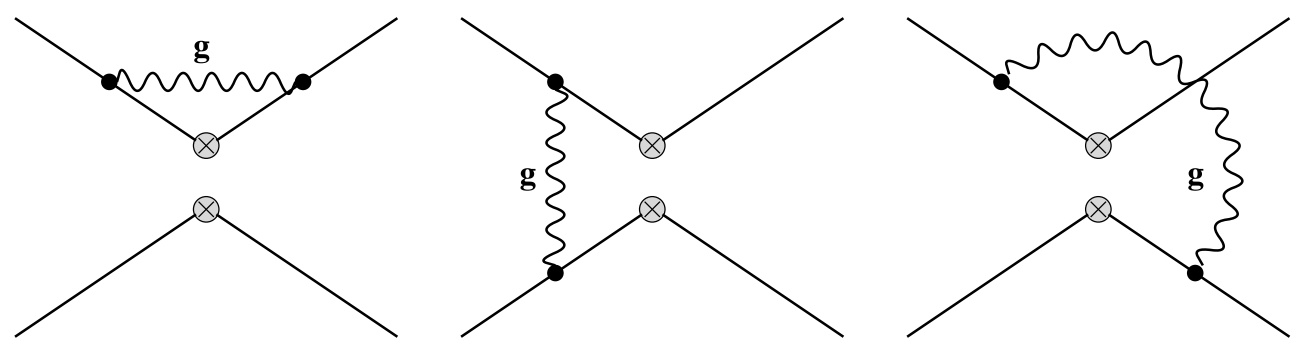
MATCHING: we make sure that $A_{SM} = A_{eff}$ up to $O(k^2/M_W^2), O(\alpha_s^2)$
We cannot neglect all external momenta and quark masses (IR sensitivity)



We choose $m_i = 0$ and a small off-shell momentum p
p^2<0 for both SM and eff loops, dim reg for UV $d = 4 - 2\epsilon$

Define $S_{1,2} \equiv \langle Q_{1,2}\rangle^{(0)}$ tree level matrix el

$$A_{SM} = \frac{4G_F}{\sqrt{2}}V^*_{cs}V_{ud}\Big[\Big(1 + \frac{\alpha_s}{2\pi}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big)S_2 + \Big(\frac{3}{N}S_2 - 3S_1\Big)\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2} + \ldots\Big]$$



(a) (b) (c)

$$\langle Q_1^{(0)}\rangle = \Big[1 + \frac{\alpha_s}{2\pi}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\Big)\Big]S_1 + \Big(\frac{3}{N}S_1 - 3S_2\Big)\frac{\alpha_s}{4\pi}\Big(\frac{1}{\epsilon} + \ln\frac{\mu^2}{-p^2}\Big) + \ldots, \quad \langle Q_2^{(0)}\rangle = \langle Q_1^{(0)}\rangle_{1\leftrightarrow 2}$$

Actually we have $T^a_{\alpha\beta}T^a_{\gamma\delta} = -\dfrac{1}{2N}\delta_{\alpha\beta}\delta_{\gamma\delta} + \dfrac{1}{2}\delta_{\alpha\delta}\delta_{\beta\gamma}$ N=3 in QCD

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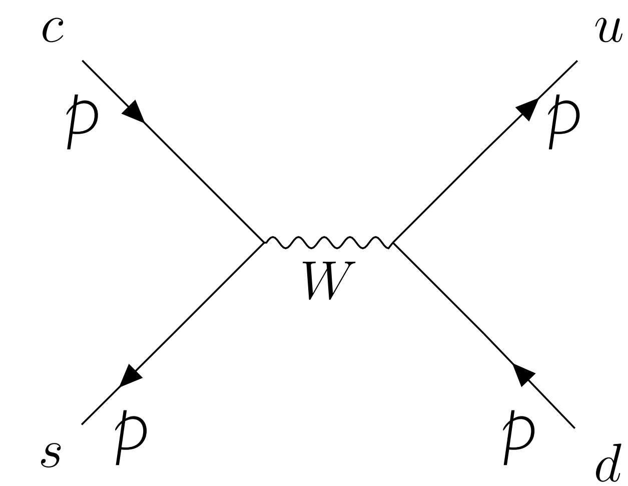

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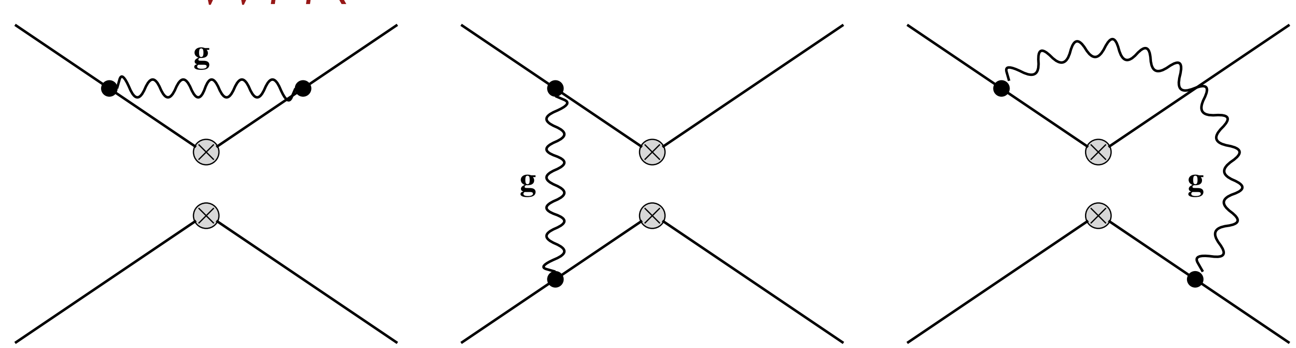
$$A_{SM} = \frac{4G_F}{\sqrt{2}}V^*_{cs}V_{ud}\left[\left(1+\frac{\alpha_s}{2\pi}\left(\cancel{\frac{1}{\epsilon}}+\ln\frac{\mu^2}{-p^2}\right)\right)S_2 + \left(\frac{3}{N}S_2 - 3S_1\right)\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{-p^2} + \ldots\right]$$

WFR



(a) (b) (c)

$$\langle Q_1^{(0)}\rangle = \left[1+\frac{\alpha_s}{2\pi}\left(\cancel{\frac{1}{\epsilon}}+\ln\frac{\mu^2}{-p^2}\right)\right]S_1 + \left(\frac{3}{N}S_1 - 3S_2\right)\frac{\alpha_s}{4\pi}\left(\cancel{\frac{1}{\epsilon}}+\ln\frac{\mu^2}{-p^2}\right) + \ldots, \quad \langle Q_2^{(0)}\rangle = \langle Q_1^{(0)}\rangle_{1\leftrightarrow 2}$$

WFR+operator renormalisation in MSbar

Operator renormalisation

bare $\langle Q_i^{(0)}\rangle = Z_q^{-n/2} Z_g^{n_g/2} Z_{ij}\langle Q_j\rangle$ for us $n=4, n_g=0$

in the $\overline{\mathbf{MS}}$ scheme $$Z_{ij} = \delta_{ij} + \frac{\alpha_s}{4\pi}\begin{pmatrix} 3/N & -3 \\ -3 & 3/N \end{pmatrix}\frac{1}{\epsilon} + O(\alpha_s^2)$$

Now we ask that $A_{SM} = A_{eff}$ and find

$$C_1 = -3\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}, \qquad C_2 = 1 + \frac{3}{N}\frac{\alpha_s}{4\pi}\ln\frac{M_W^2}{\mu^2}$$

with no dependence on p and on external states. WC depend only on M_W, μ

μ **cutoff between pert and nonpert** physics:

$$\underbrace{\ln\frac{M_W^2}{-p^2}}_{\text{SM}} = \underbrace{\ln\frac{M_W^2}{\mu^2}}_{\text{WC}} + \underbrace{\ln\frac{\mu^2}{-p^2}}_{\text{matrix elements}} \quad \text{or} \quad \int_{-p^2}^{M_W^2} d\kappa^2 = \int_{\mu^2}^{M_W^2} d\kappa^2 + \int_{-p^2}^{\mu^2} d\kappa^2$$

$\mu \sim m_c$ but cannot be too small! $\frac{\alpha_s}{\pi}\ln\frac{M_W^2}{m_c^2} \approx 2\frac{0.3}{\pi}\ln 80 \sim 0.9 = O(1)$

For $\mu \sim M_W$ convergent series: $\underbrace{C_i(M_W) \to C_i(\mu \sim m_c)}_{\text{RGE}}$ like for $\alpha_s(\mu)$

RGE improved pert series

Indeed, $C_i(\mu)$ *are* ***running effective couplings***

$$\mathcal{L}_{eff} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[C_1(\mu) Q_1(\mu) + C_2(\mu) Q_2(\mu) \right]$$

$$A_{phys} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ud} \sum_i^n C_i(\mu) \langle Q_i(\mu) \rangle_{hadr}$$ *must be* μ *indep*

if $\{Q_i\}$ *complete basis* $$\frac{d\langle Q_i \rangle}{d \ln \mu} = -\sum_{j=1}^n \gamma_{ij} \langle Q_j \rangle$$ *with* $\hat{\gamma}$ ***Anomalous Dimension Matrix***

series in α_s

follows from Z_{ij}

$$\frac{d}{d\mu} A_{phys} = 0 \;\Rightarrow\; \frac{dC_j}{d \ln \mu} = \sum_{i=1}^n \gamma_{ij}(\mu)\, C_i(\mu)$$ *or* $$\frac{d\vec{C}}{d \ln \mu} = \hat{\gamma}^T \vec{C}$$

similar to $\frac{dm(\mu)}{d \ln \mu} = -\gamma_m(\alpha_s)\, m(\mu)$ $\quad$ $\hat{\gamma}$ *at* $O(\alpha_s)$ *resums leading logs* $(\alpha_s \ln M/\mu)^n$

for instance $\quad C_1(m_c) \sim -0.5, \qquad C_2(m_c) \sim 1.2$

can be further improved with higher order corrections, but the difficult problem are the matrix elements

Summary of Weak Effective Theory

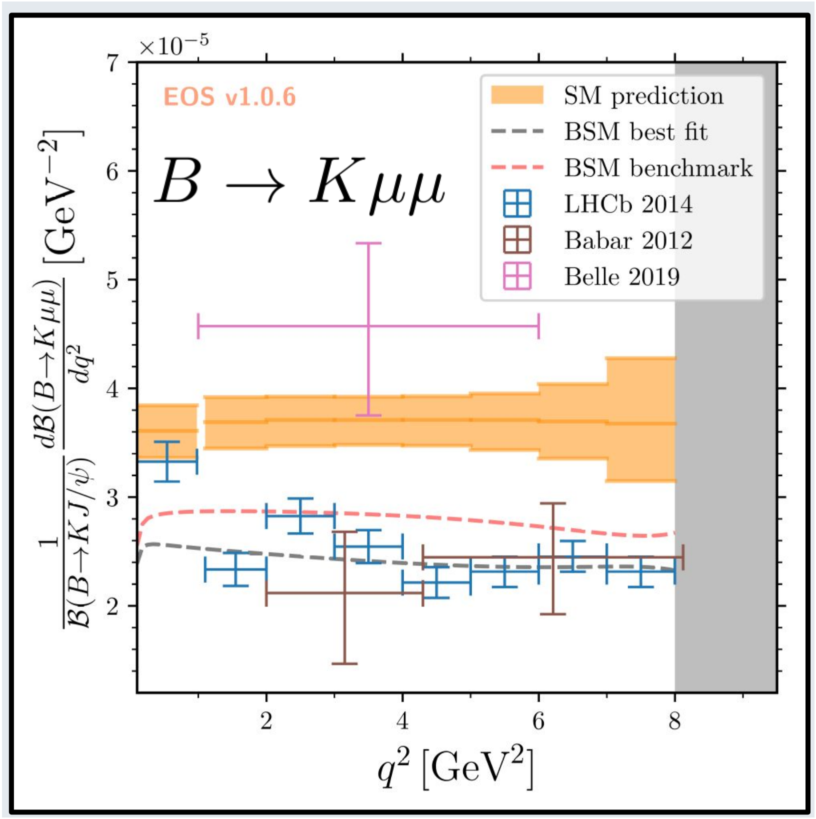
In the SM

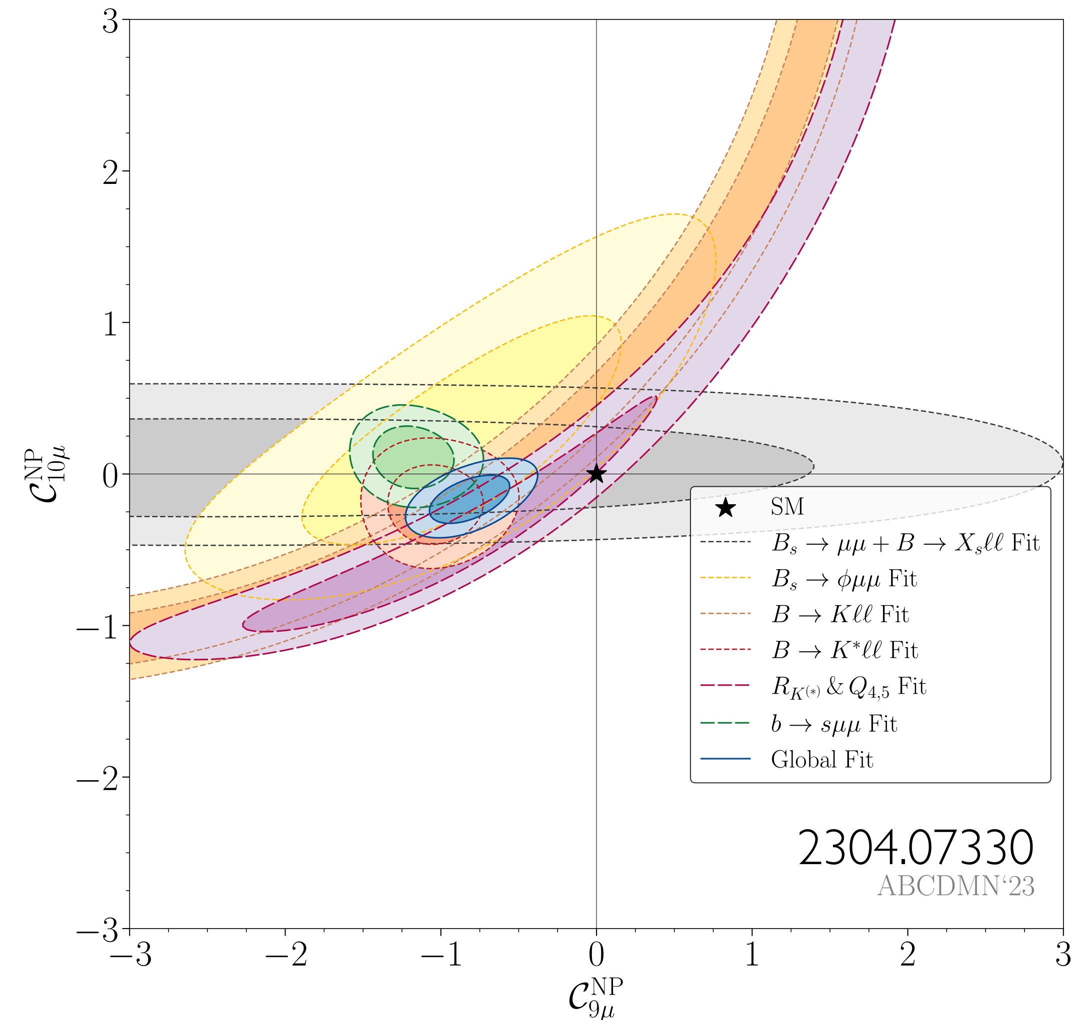
- identify all dim 6 operators relevant to process of interest (basis) containing only light fields
- compute their ADM at appropriate order
- match WC at scale $\mu_W \sim M_W, m_t$ and evolve them down to appropriate scale μ_{low} ($\sim m_b$ for B physics)
- compute matrix elements using nonpert methods

Beyond SM, model independently: extend operator basis if necessary, $C_i(\mu_{low}) = C_i^{SM}(\mu_{low}) + C_i^{NP}(\mu_{low})$, fit data for NP contributions, possibly evolve back to weak scale, match to SMEFT, identify main SMEFT operators responsible for deviations

Systematic method valid for other EFT as well.

Anomaly in $B \to K^{(*)}\ell^+\ell^-$

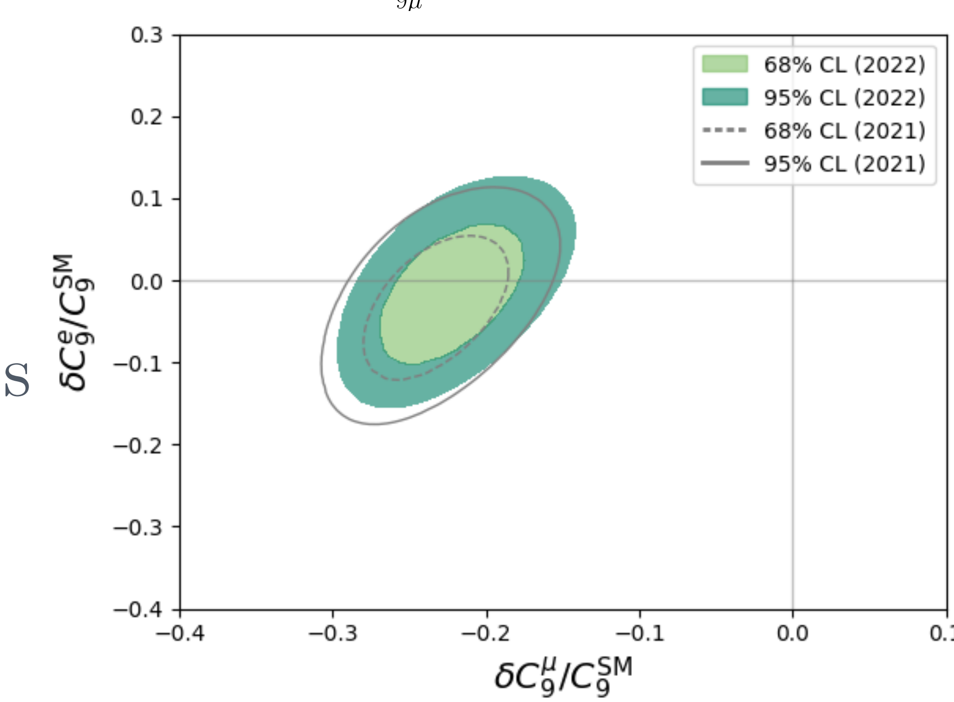




Assuming NP only in muon couplings

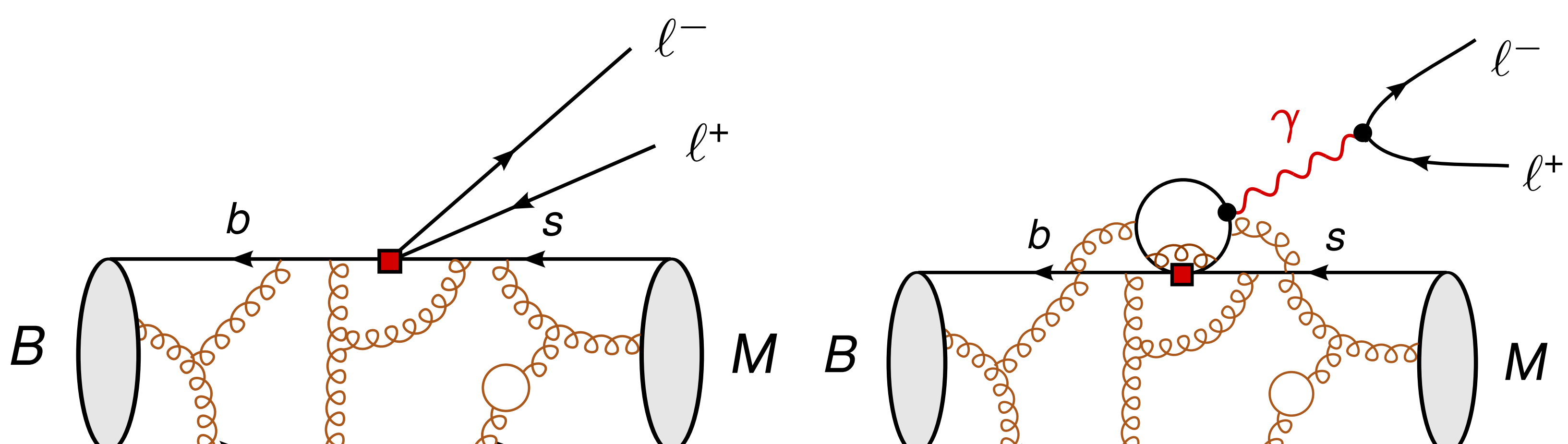
$$Q_{9\mu} = \bar{s}_L\gamma_\mu b_L \bar{\mu}\gamma^\mu\mu \qquad C_{9\mu}^{SM}(m_b) \sim 4.1$$

$$Q_{10\mu} = \bar{s}_L\gamma_\mu b_L \bar{\mu}\gamma^\mu\gamma_5\mu \qquad C_{10\mu}^{SM}(m_b) \sim -4.3$$

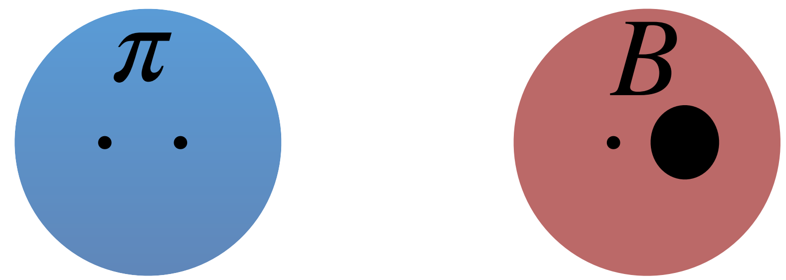


Local matrix elements

Non-local matrix elements



Heavy Quark Symmetry



similar size $\sim \Lambda_{QCD}^{-1} \sim 1\mathrm{fm}$

$$m_b \gg \Lambda_{QCD}$$

Can we disentangle these scales using EFT and learn on matrix elements?

For $m_b \to \infty$ *the b is a* ***static color source*** *interacting softly with light d.o.f. in the hadron, similar to p in H atom. It cannot be integrated out. The scale separation implies the light d.o.f. (brown muck) cannot resolve the value of* m_b*. The velocity v of the HQ (*$p = m_b v$*) is unchanged because* $\Delta v = \Delta p / m_b \to 0$*.*

In the HQ limit: ***HQ Symmetry*** *(b and c equivalent), and HQ* ***Spin Symmetry*** *(chromomagnetic moment suppressed by masses).*

The HQ propagator $$\frac{i}{\not p - m + i\epsilon} = \frac{i[m(1+\not v) + \not k]}{k^2 + 2mv\cdot k + i\epsilon} \to \frac{1+\not v}{2}\frac{i}{v\cdot k + i\epsilon}$$

where $(1+\not v)/2$ *projects on the positive frequency components of the Dirac field. They are the only ones to propagate in the HQ limit.*

Heavy Quark Effective Theory - HQET (I)

In QCD $\mathcal{L} = \bar{\Psi} i \not{D} \Psi - m \bar{\Psi} \Psi \qquad D_\mu = \partial_\mu - i g T^a A^a_\mu$

Decompose the momentum of a heavy quark

$$v^\mu = (1,0,0,0)$$
$$p = mv + k \qquad k = \mathcal{O}(\Lambda_{QCD}) \ll m \quad \textit{non-relativistic}$$

Define

$$h_v(x) \equiv e^{imv\cdot x} \frac{1+\not{v}}{2} \Psi(x)$$
$$H_v(x) \equiv e^{imv\cdot x} \frac{1-\not{v}}{2} \Psi(x)$$

h_v and H_v are large and small Dirac components

$$D^\mu_\perp \equiv D^\mu - v^\mu v \cdot D$$

$$\left.\begin{aligned} iv\cdot D h_v &= -i \not{D}_\perp H_v \\ (iv\cdot D + 2m) H_v &= i \not{D}_\perp h_v \end{aligned}\right\} \textit{EOM}$$

Decoupling H_v

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \bar{h}_v i \not{D}_\perp \frac{1}{iv\cdot D + 2m} i \not{D}_\perp h_v$$

$$\mathcal{L} = \bar{h}_v iv\cdot D h_v + \frac{1}{2m} \bar{h}_v (iD_\perp)^2 h_v + \frac{g}{4m} \bar{h}_v \sigma^{\mu\nu} G_{\mu\nu} h_v$$

HQET Lagrangian: Residual QCD dynamics after decoupling high frequency modes: Flavour & Spin symmetric

Power corrections: non-relativistic expansion kinetic and chromomagnetic terms Pauli non-relativistic Hamiltonian

$$H_{Pauli} = \frac{(\vec{p} - q\vec{A})^2}{2m} + qA^0 - q\frac{\vec{\sigma}\cdot\vec{B}}{2m}$$

HQET (II)

<u>Spectroscopy</u>

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} \qquad m_D = m_c + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_c}$$

$$m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} \qquad m_{D^*} = m_c + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_c}$$

+ *higher orders*

$$\lambda_1 \equiv \frac{\langle B|\bar{h}(iD)^2 h|B\rangle}{2m_B} \qquad \lambda_2 \equiv \frac{1}{6}\frac{\langle B|\bar{h}g\sigma\cdot Gh|B\rangle}{2m_B}$$

$$\lambda_1 = -\mu_\pi^2 \simeq -0.4$$
$$\lambda_2 = \mu_G^2/3 \simeq 0.12$$

<u>Semileptonic decay</u>

$$\frac{d\Gamma(\bar{B}\to D^* l\bar{\nu})}{dw} = |V_{cb}|^2\,\mathcal{K}(w)\,\mathcal{F}^2(w) \qquad w = v\cdot v'$$

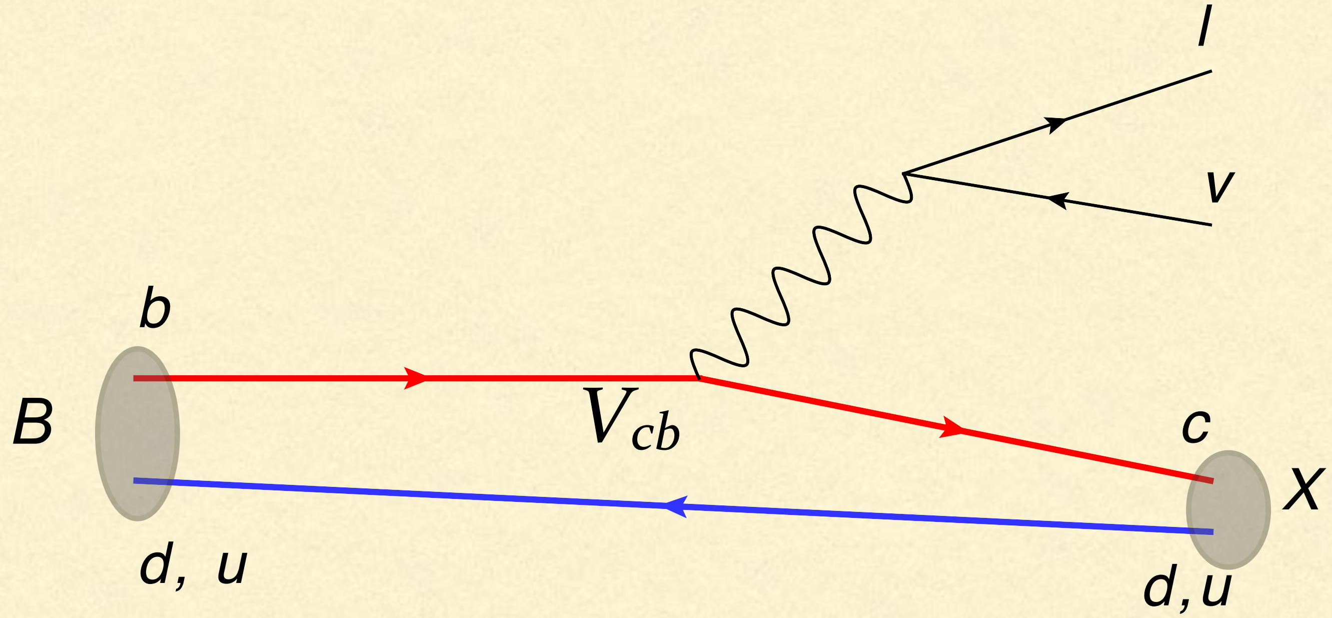
*v and v' four-velocities of B and D**

Isgur-Wise function $\mathcal{F}(w) = \xi(w) \qquad \xi(1) = 1$ *at leading order*

$$\boxed{\mathcal{F}(1) = \eta_A(1 + \delta_{1/m^2})}$$

At zero recoil the b quark at rest decays into a D^* at rest. The light hadronic cloud does not notice the flavour change. Power corrections quadratic for B→D (Ademollo-Gatto, a.k.a. Luke's theorem) but $1/m_c^2$ cannot be neglected for present studies:*
LQCD $\mathcal{F}(1) \sim 0.90$, pert factor $\eta_A \simeq 0.96$

EXCLUSIVE DECAYS



$= D, D^*, \ldots$

There are 1(2) and 3(4) FFs for D and D* for light (heavy) leptons, for instance

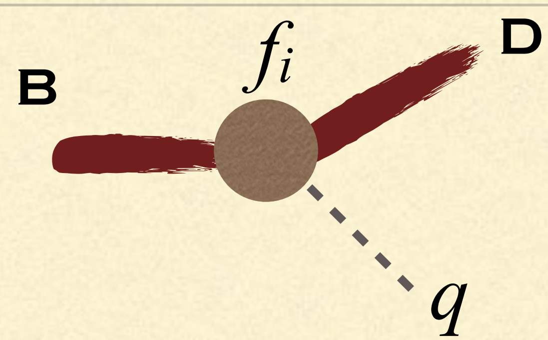
$$\langle D(k)|\, \bar{c}\gamma^{\mu} b\, |\bar{B}(p)\rangle = \left[(p+k)^{\mu} - \frac{M_B^2 - M_D^2}{q^2} q^{\mu}\right] f_+^{B\to D}(q^2) + \frac{M_B^2 - M_D^2}{q^2} q^{\mu} f_0^{B\to D}(q^2)$$

This is a conserved current: no RGE QCD effect. Information on FFs from LQCD (at high q^2), LCSR (at low q^2), HQE, exp, extrapolation, …

A ***model independent parametrization*** is necessary

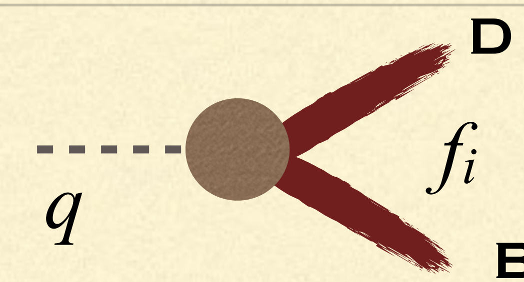
MODEL INDEPENDENT FF PARAMETRIZATION

crossing + analitycity



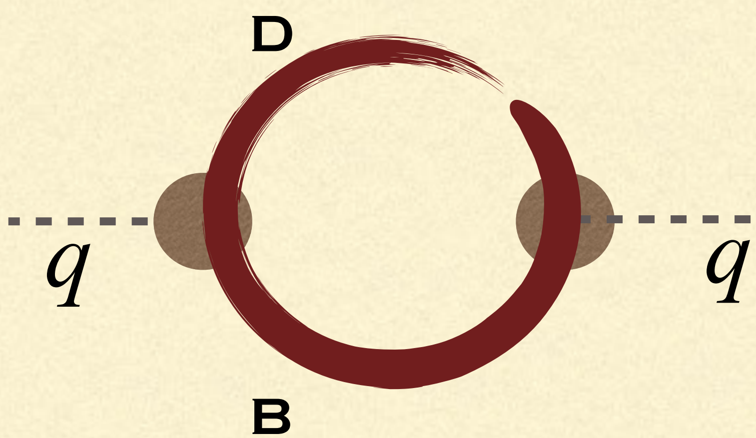
physical semileptonic region
$m_\ell^2 \leq q^2 \leq (m_B - m_D)^2$

$\longleftrightarrow$



2-point correlator cuts
$q^2 \geq (m_B + m_D)^2$

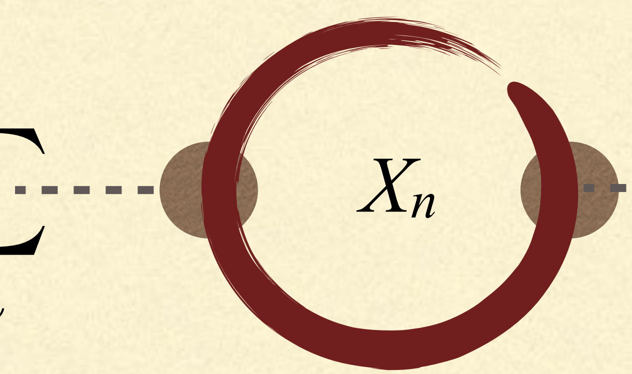
Im



$$\propto |f_i(q^2)|^2 < \sum_n \mathrm{cuts}(X_n)$$

also poles at $q^2 = M_{B_c}^2$ *etc*

$\sum_n$



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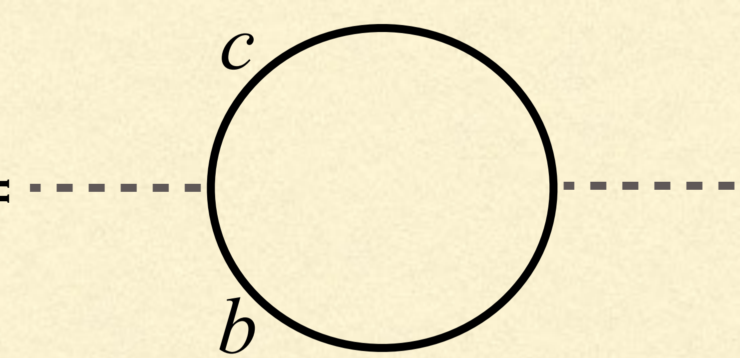


+ *pert corr* + *condensates*

using quark-hadron duality (OPE) + dispersion relations

PARAMETRIZATIONS

- ***Boyd-Grinstein-Lebed*** (***BGL*** 1995) based on crossing & analyticity, unitarity constraints

  based on OPE $$F(q^2) = \bar{F}(q^2) \sum_{k=0}^{\infty} a_k\, z(q^2)^n \qquad \text{with} \qquad \sum_k a_k^2 \le 1,$$

  $0 < z < \sim 0.06$ in the physical region. Series must be truncated in a controlled way.

- HQET for $B^{(*)} \to D^{(*)}$ form factors:

$$F_i(w) = \xi(w)\left[1 + c_\alpha^i \frac{\alpha_s}{\pi} + c_b^i \frac{\overline{\Lambda}}{2m_b} + c_c^i \frac{\overline{\Lambda}}{2m_c} + \dots\right]$$

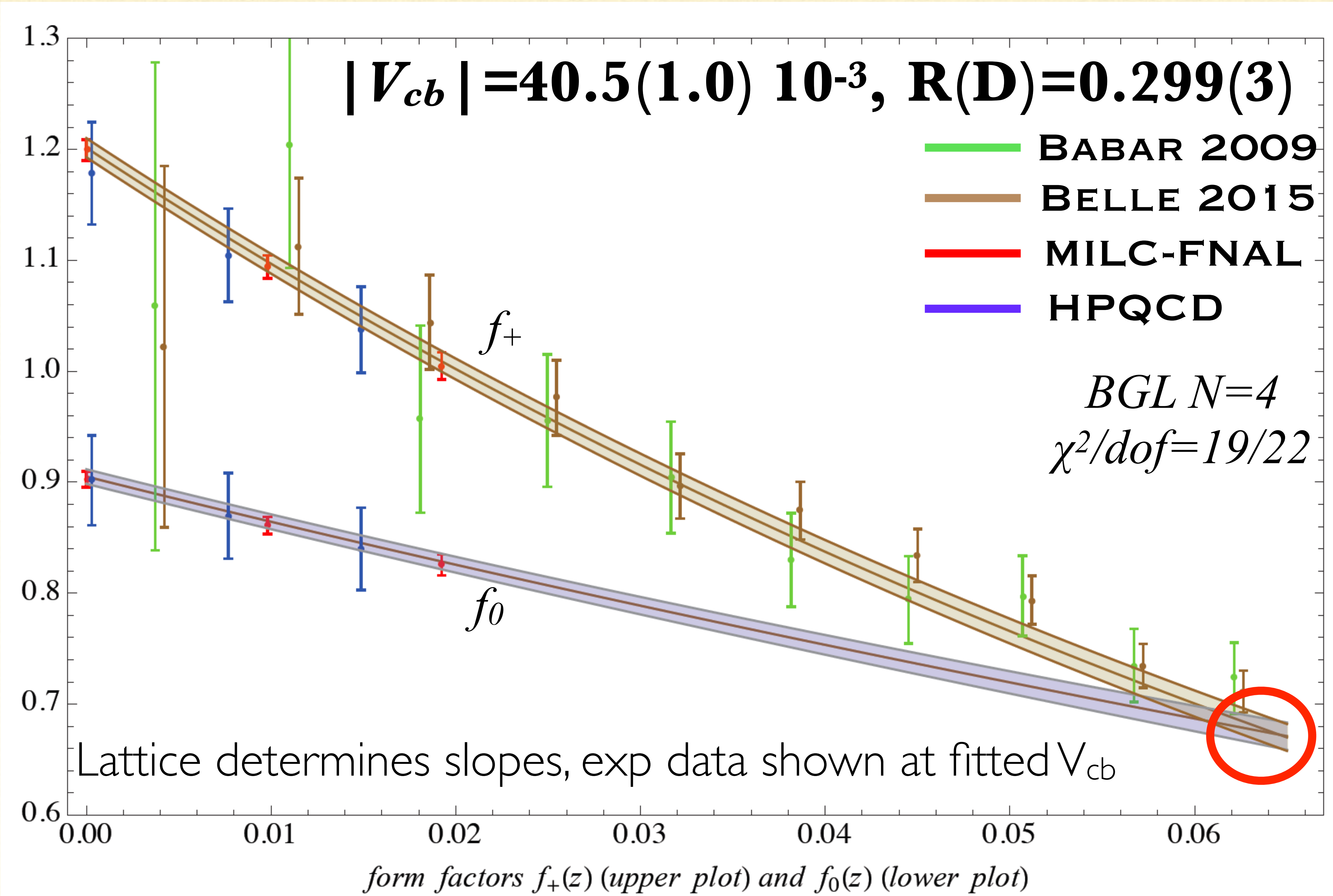
- $c_{b,c}^i$ can be computed using subleading IW functions from QCD sumrules Neubert, Ligeti, Nir 1992-93, Bernlochner et al 1703.05330

- *Ratios free of Isgur-Wise function: can use to get **strong unitarity bounds** but $1/m_c^2$ corrections can be significant as shown by lattice calculations*

- ***Caprini-Lellouch-Neubert*** (***CLN*** 1998) parametrization is simpler with fewer parameters, but relies on NLO HQET. All exp analyses before 2017 were based on CLN, did not include uncertainty.

LATTICE + EXP BGL FIT for $B \to Dl\nu$

Bigi, PG 1606.08030



R(D)=0.299(3)

1.9σ from exp

FLAG has very similar results

CLN cannot fit both ff

kinematic constraint at $q^2=0$

$|V_{cb}|$ from $B \to D^* l\nu$

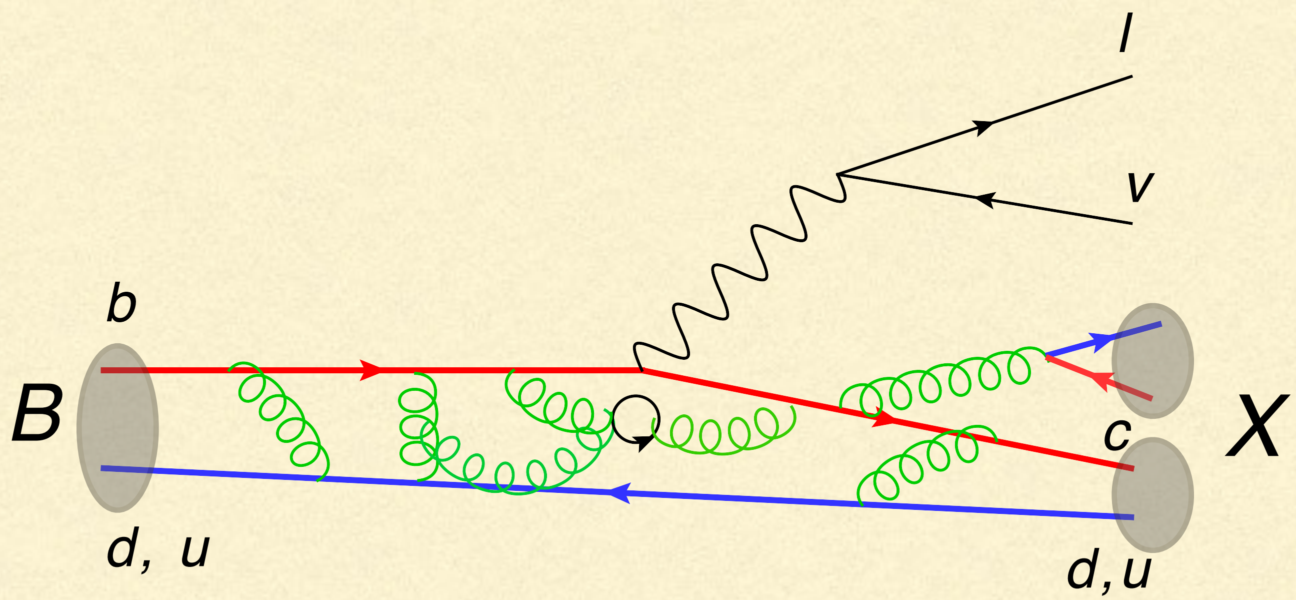
More complicated: 4 FFs, angular spectra, D^* unstable. ***Present status unclear.***

1. ***Parametrisations matter and the related uncertainties require careful consideration.*** Belle 2017 dataset analysed with BGL or CLN leads to 6-8% difference in $|V_{cb}|$. Bigi, PG, Schacht, Grinstein, Kobach
   Discard old exp results obtained with CLN and provide data in a parametrisation independent way.

2. Despite recent progress, ***lattice calculations*** are indecisive. Tension between ***Fermilab/MILC*** 2021 and HPQCD 2023 results at non-zero recoil and ***Belle*** untagged 2018 data, while ***JLQCD*** preliminary results give a consistent picture.

3. ***Problems*** in Belle 2018 analysis (D'Agostini bias, μ/e 4σ tension in the FB asymmetry) PG, Jung, Schacht & Bobeth, Bordone, van Dyk, Gubernari, Jung
   other experimental analyses make conflicting claims but data not yet available for independent fits

INCLUSIVE DECAYS: BASICS



- ***Simple idea:*** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows us to express it in terms of B meson matrix elements of local operators.

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in*** α_s, Λ/m_b

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, two parameters at $O(\Lambda^2/m_b^2)$, 2 more at $O(\Lambda^3/m_b^3)$ ... Many higher order effects have been computed.

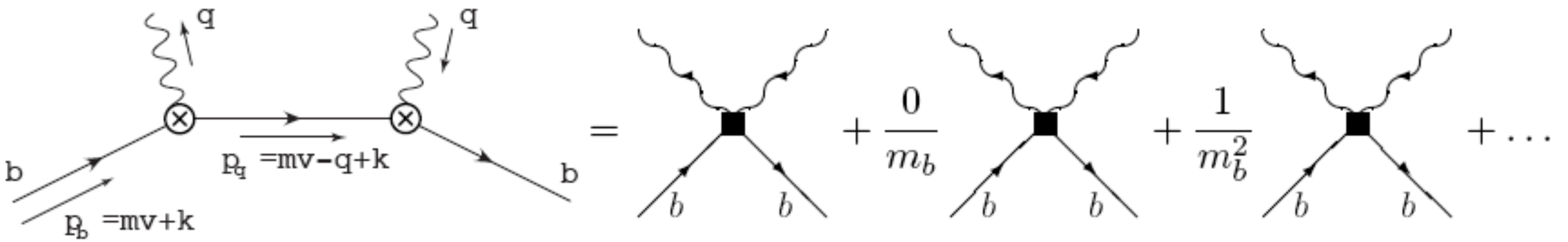
The OPE for semileptonic B decays

$$d\Gamma_H \sim \langle H|\mathcal{H}_{eff}|X\rangle\langle X|\mathcal{H}_{eff}|H\rangle \quad = L_{\alpha\beta} W^{\alpha\beta}$$

$$\begin{aligned} W^{\alpha\beta} &\sim \sum_{X_c} \delta^4(p_B - q - p_{X_c})\,|\langle \mathrm{B}|\mathrm{J}^{\alpha\dagger}_{\mathrm{bc}}|\mathrm{X_c}\rangle\,\langle \mathrm{X_c}|\mathrm{J}^{\beta}_{\mathrm{bc}}|\mathrm{B}\rangle|^2 \\ &\sim \mathrm{Im}\int \mathrm{d}x\, e^{-iq\cdot x}\,\langle \mathrm{B}|\,\mathrm{T}\{\mathrm{J}^{\alpha\dagger}_{\mathrm{bc}}(\mathrm{x})\,\mathrm{J}^{\beta}_{\mathrm{bc}}(0)\}\,|\mathrm{B}\rangle\,, \end{aligned}$$

HQET EOM

$$iv\cdot Dh_v = O\left(\frac{1}{m}\right)$$



Lowest order (tree level free quark decay)
ie k=0 and no gluon:

$$-\frac{1}{\pi}\mathrm{Im}\frac{i}{(mv-q)^2 - m_c^2} = \delta\left[(mv-q)^2 - m_c^2\right]$$

Higher orders in k/m$_b$ expansion:

$$-\frac{1}{\pi}\mathrm{Im}\left(\frac{i}{(mv-q)^2 - m_c^2}\right)^{n+1} = \frac{(-1)^n}{n!}\delta^{(n)}\left[(mv-q)^2 - m_c^2\right]$$

Power corrections appear in the triple differential rate as more and more singular divergencies at the end point. The OPE gives predictions only for global (integrated) quantities insensitive to local details

INCLUSIVE SEMILEPTONIC B DECAYS

Inclusive observables are double series in Λ_{QCD}/m_b and α_s

$$M_i = M_i^{(0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)}\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)}\right) \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} + \ldots$$

Global shape parameters (first moments of the distributions, with various lower cuts on E_l) tell us about m_b, m_c and the B structure, total rate about $|V_{cb}|$

OPE parameters describe universal properties of the B meson and of the quarks: they are useful in many applications (rare decays, V_{ub}, inclusive nonleptonic,…)

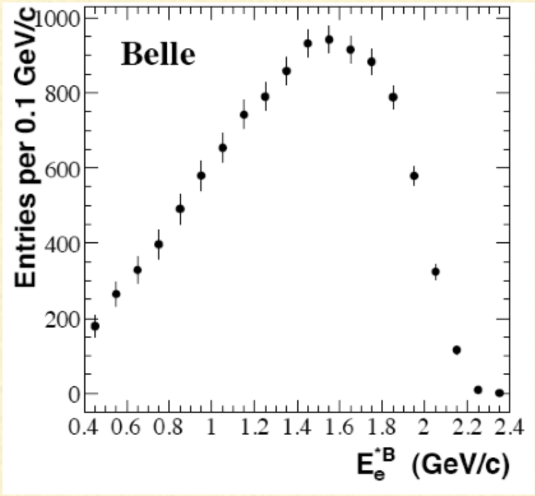
Reliability of the method depends on our control of ***higher order effects***. Quark-hadron duality violation would manifest itself as inconsistency in the fit.

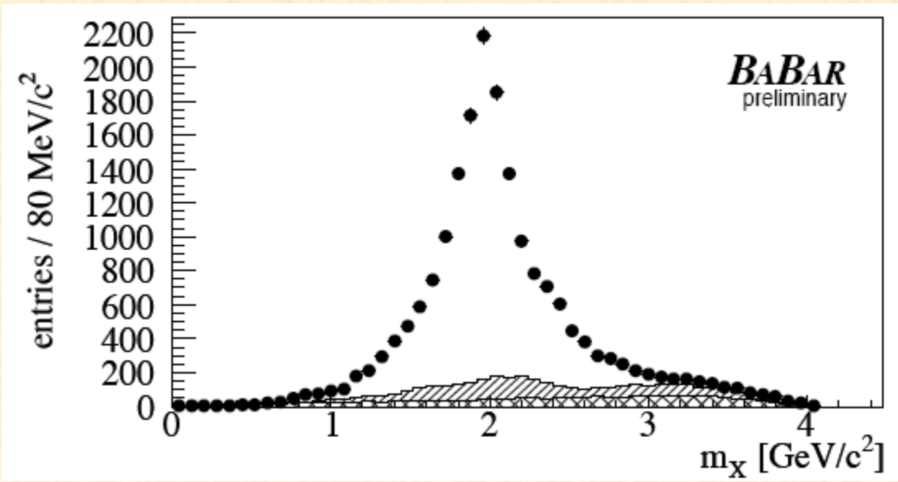
Kinetic scheme provides short distance definition of m_b and OPE parameters with hard cutoff $\mu_{kin} \sim 1\mathrm{GeV}$. Fit includes all corrections $O(\alpha_s^2, \alpha_s/m_b^2, 1/m_b^3)$, m_c constraint from sum rules/lattice, and recent $O(\alpha_s^3)$ contribution to width.

INCLUSIVE SEMILEPTONIC FITS

Bordone, Capdevila, PG, 2107.00604

Electron energy and invariant hadronic mass spectra





m_b^{kin}	$\overline{m}_c(2\text{GeV})$	μ_π^2	ρ_D^3	$\mu_G^2(m_b)$	ρ_{LS}^3	$\text{BR}_{c\ell\nu}$	$10^3\lvert V_{cb}\rvert$
4.573	1.092	0.477	0.185	0.306	-0.130	10.66	42.16
0.012	0.008	0.056	0.031	0.050	0.092	0.15	0.51

Higher power corrections see a proliferation of parameters but Wilson coefficients are known at LO. We use the Lowest Lying State Saturation Approximation (Mannel, Turczyk, Uraltsev 1009.4622) as loose constraint or priors (60% gaussian uncertainty, dimensional estimate for vanishing matrix elements) in a fit including higher powers.

$$\boxed{|V_{cb}| = 42.00(53) \times 10^{-3}}$$

Update of 1606.06174
similar results in 1S scheme Bauer et al.