A Standard Model Prediction of the muon g-2?

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OUTLINE

- Introduction

- general context
- why is there a question mark in the title?

- What are we talking about?

- defining the anomalous magnetic moment of a charged lepton (why g-2?)
- why the muon?

- How do we measure it?

- brief description of experimental aspects

- What is the SM prediction for it? I: QED

- What is the SM prediction for it? II: Weak interactions

- What is the SM prediction for it? III: strong interactions

- hadronic light-by-light (HLxL)
- hadronic vacuum polarization (HVP)

- Summary - Conclusion - Outlook

Introduction

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General context: looking for physics beyond the Standard Model

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- Probe the intensity frontier $\longrightarrow$ precision physics

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Direct searches at colliders (the energy frontier) have so far not provided evidence of physics beyond the SM

Indirect indications exist, either from the cosmic frontier (dark matter,...) or from the precision frontier (muon g-2,...)

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General context: looking for physics beyond the Standard Model

- Probe the energy frontier $\longrightarrow$ collider physics

- Probe the cosmic frontier $\longrightarrow$ astrophysical/cosmological observations

- Probe the intensity frontier $\longrightarrow$ precision physics

possible evidence of new physics by studying quite subtle quantum effects

Requires:

- high precision measurement
- equally precise prediction of the SM value

INTRODUCTION

$a_\mu \equiv (\mathrm{g}_\mu - 2)/2$ is experimentally measured to very high precision

$$\tau_\mu = (2.19703 \pm 0.00004) \times 10^{-6} \text{ s}$$

$$\gamma \sim 29.3,\ p \sim 3.094 \text{ GeV/c}$$

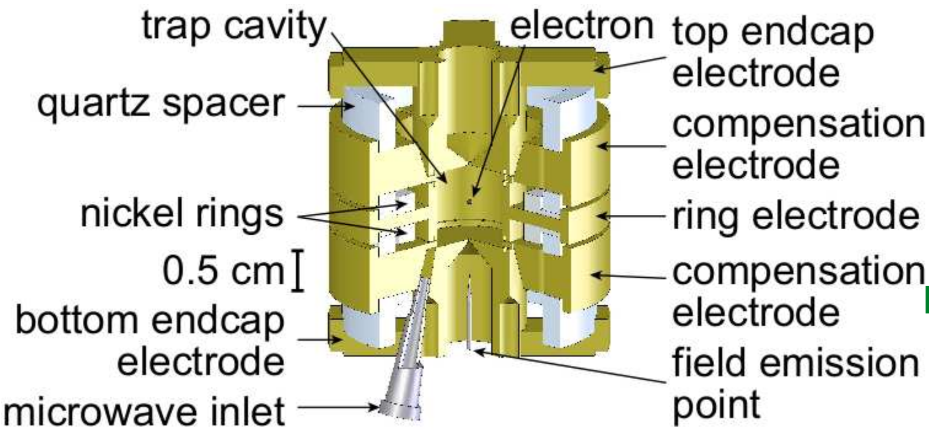
$$a_\mu^{\mathrm{exp;WA}} = 116\,592\,061(41) \cdot 10^{-11}$$

$$\Delta a_\mu^{\mathrm{exp;WA}} = 4.1 \cdot 10^{-10} \text{ [0.35ppm]}$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

INTRODUCTION

a_μ and a_e are experimentally measured to very high precision



$$a_e^{\text{exp}} = 1\,159\,652\,180.73(0.28) \cdot 10^{-12}$$

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D. Hanneke et al, Phys. Rev. Lett. 100, 120801 (2008)

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On April 7, 2021, the members of the FNAL-E989 experiment released their first result of a measurement of the anomalous magnetic moment of the muon a_μ, based on the Run-1 data collected during Spring 2018

$$a_{\mu^+}^{\mathrm{E989}} \quad = \quad 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

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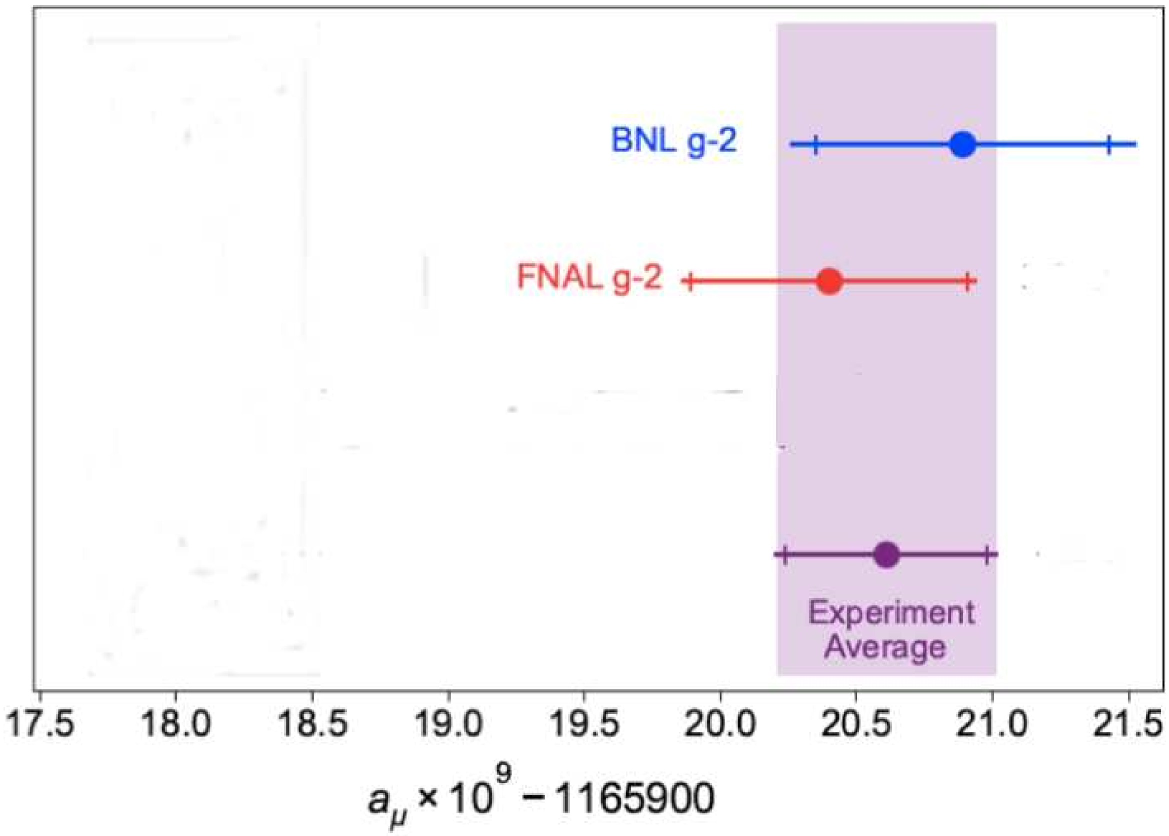
It confirms the value obtained almost 20 years ago by the BNL-E821 experiment with a comparable precision

$$a_{\mu}^{\mathrm{E821}} = 116\,592\,089(63) \cdot 10^{-11} \;\; [0.54\,\mathrm{ppm}]$$

G. W. Bennett et al. [Muon g-2 Coll.], PRD 73, 072003 (2006)

a_μ: Unblinding



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perfectly in line with decades of muon storage ring experiments

Experiment	Years	Polarity	$a_\mu \times 10^{10}$	Precision [ppm]
CERN I	1961	μ^+	11 450 000(220 000)	4300
CERN II	1962-1968	μ^+	11 661 600(3100)	270
CERN III	1974-1976	μ^+	11 659 100(110)	10
CERN III	1975-1976	μ^-	11 659 360(120)	10
BNL E821	1997	μ^+	11 659 251(150)	13
BNL E821	1998	μ^+	11 659 191(59)	5
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Uncertainty largely statistics dominated [434 ppb out of 462 ppb]

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$\longrightarrow$ room for improvement, not the end of the story

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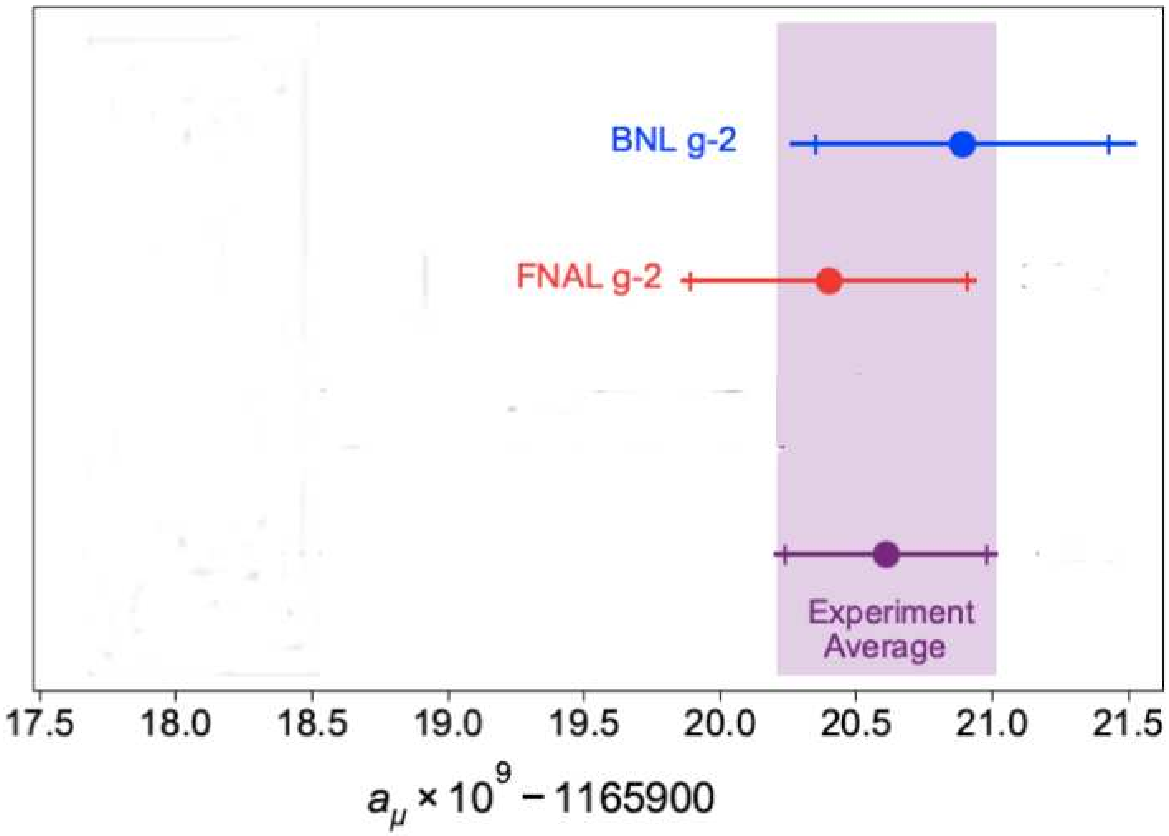
FNAL E989	2021	μ^+	11 659 2040(54)	0.46
FNAL E989	2023?	μ^+	???	$\sim$ 0.14
[J-PARC E34	2027?	μ^+	???	$\sim$ 0.45]

a_μ: Unblinding



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Theory situation?

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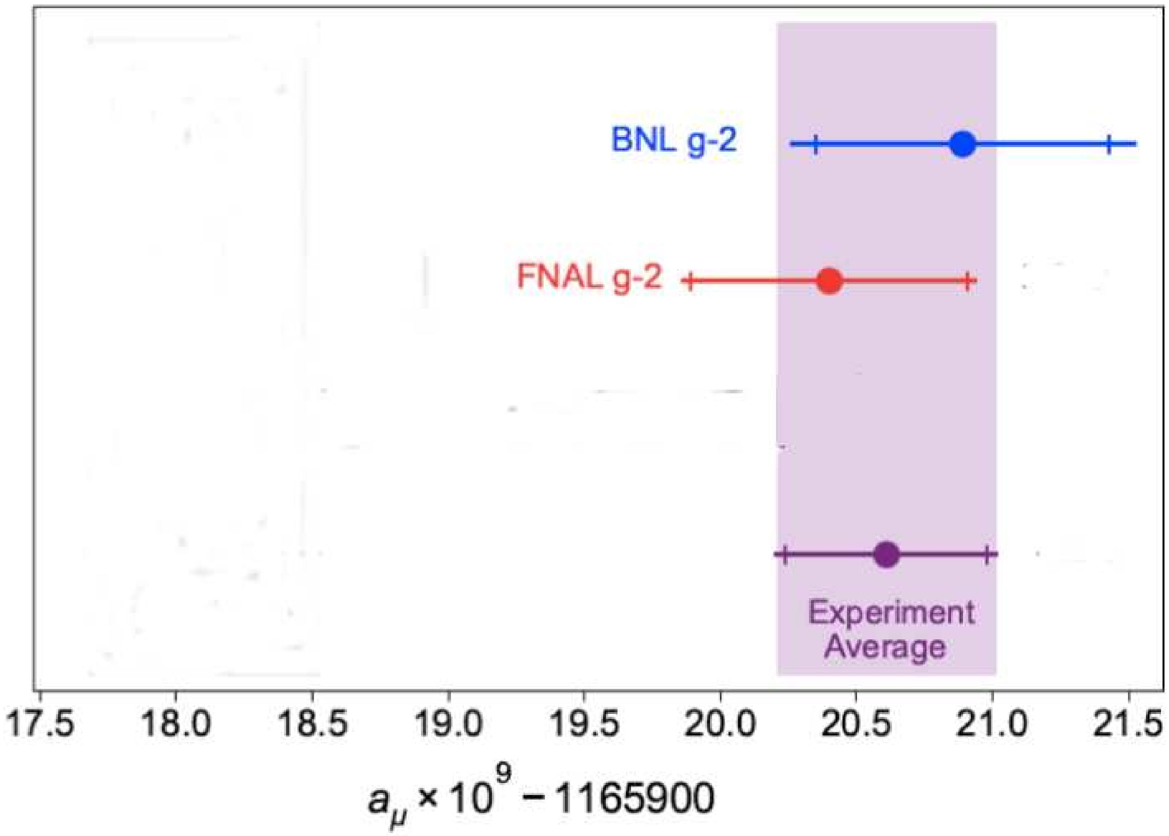

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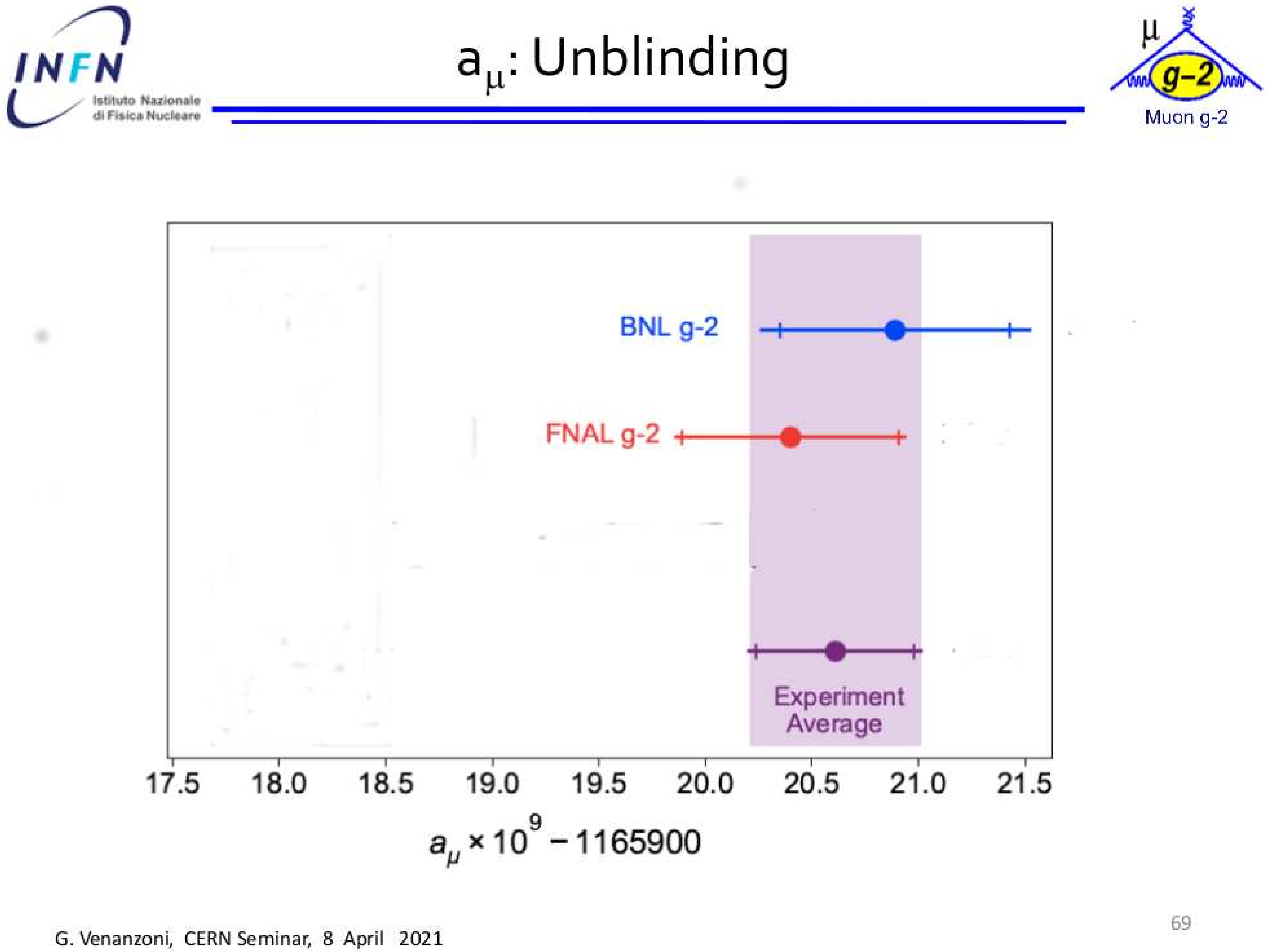
Theory situation? $\longrightarrow$ the main topic of these lectures



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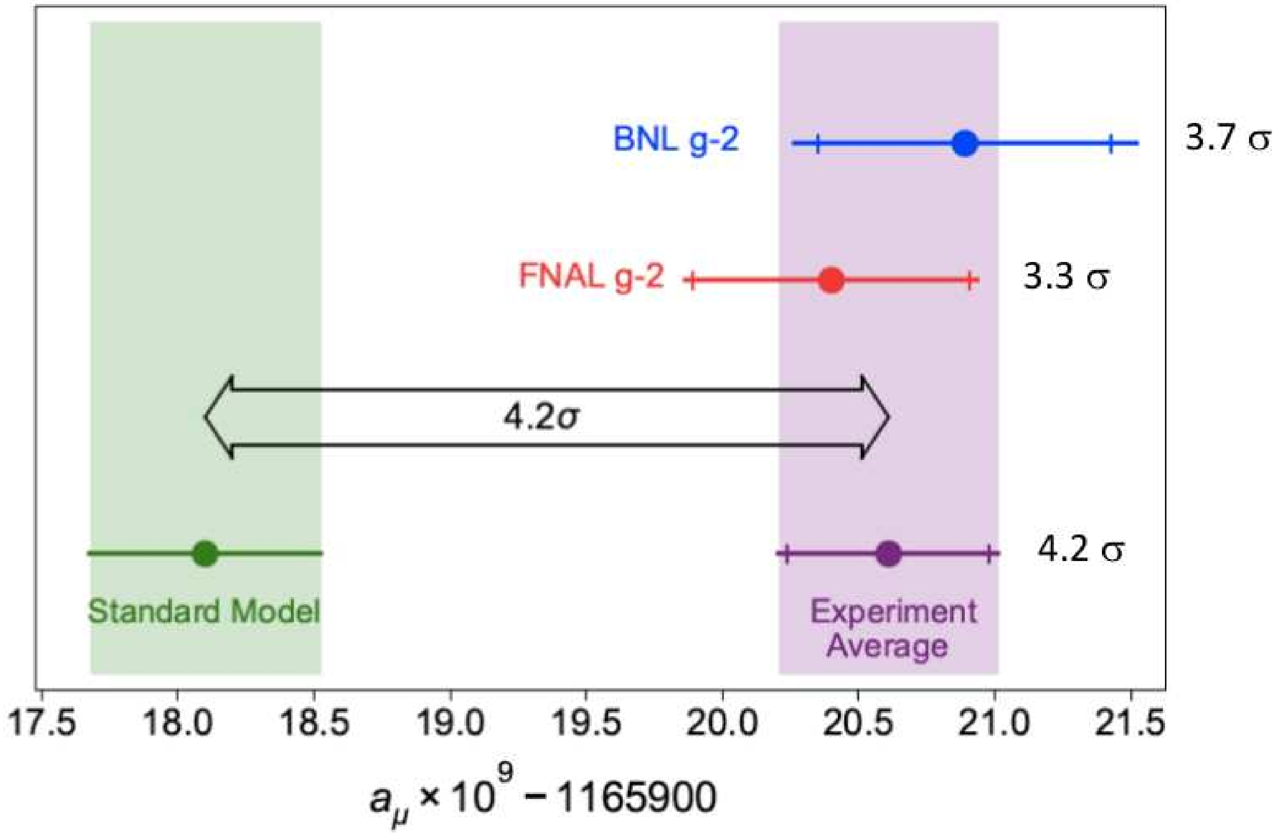
Theory situation?

Full and detailed account [up to June 15, 2020] given in the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

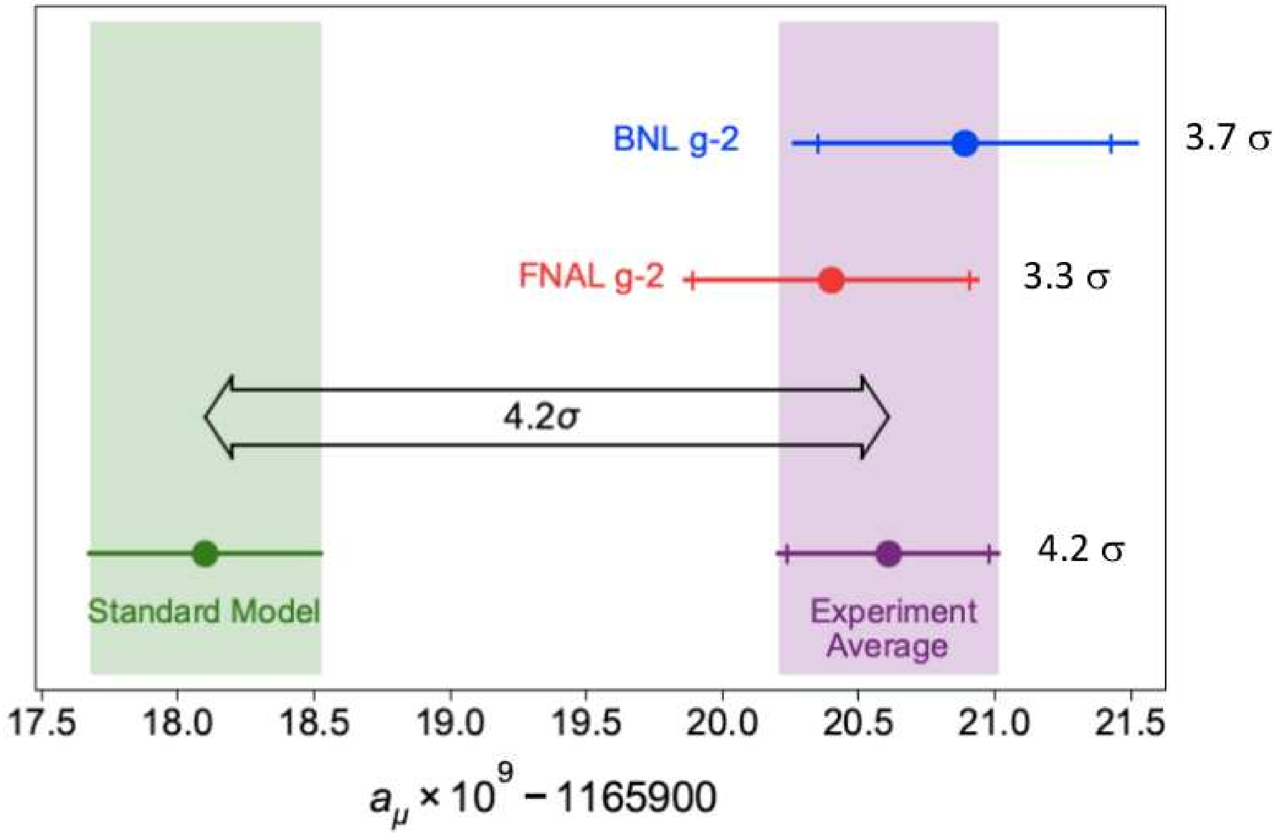
initiated by the Muon $g-2$ Theory Initiative



$$a_\mu^{\text{exp;WA}} = 116\,592\,061(41) \cdot 10^{-11} \; [0.35\text{ppm}]$$

Leads to a discrepancy th. vs. exp. at the level of 4.2σ

$$a_\mu^{\text{exp;WA}} - a_\mu^{\text{th;WP}} = 251(59) \cdot 10^{-11} \; [4.2\sigma]$$



$$a_\mu^{\exp;\mathrm{WA}} = 116\,592\,061(41) \cdot 10^{-11} \;\; [0.35\mathrm{ppm}]$$

Does $a_\mu^{\mathrm{th;WP}} = a_\mu^{\mathrm{th;SM}}$ still hold today?

Why *Anomalous* Magnetic Moment?

How is it defined?

Why the muon?

DEFINITION

Quantum mechanics of a charged particle (m_ℓ, q_ℓ) in an electro-magnetic field

$\longrightarrow$ Schrödinger equation with the minimal coupling precription

$$i\hbar \frac{\partial \varphi}{\partial t} = \left[\frac{(-i\hbar \boldsymbol{\nabla} - (q_\ell / c)\boldsymbol{\mathcal{A}})^2}{2m_\ell} + q_\ell \mathcal{A}_0 \right] \varphi$$

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May be appropriate descriptione for a spinless elementary particle but not for a particle with spin $1/2$

$\longrightarrow$ Schrödinger-Pauli equation

$$i\hbar \frac{\partial \varphi}{\partial t} = \left[\frac{(-i\hbar \boldsymbol{\nabla} - (q_\ell/c)\boldsymbol{\mathcal{A}})^2}{2m_\ell} - \underbrace{\frac{q_\ell \hbar}{2m_\ell c} \boldsymbol{\sigma} \cdot \boldsymbol{\mathcal{B}}}_{\boldsymbol{\mu}_\ell \cdot \boldsymbol{\mathcal{B}}} + q_\ell \mathcal{A}_0 \right] \varphi$$

with $\boldsymbol{\mu}_\ell = g_\ell \left(\frac{q_\ell}{2m_\ell c}\right) \mathbf{S}$, $\mathbf{S} = \hbar \frac{\boldsymbol{\sigma}}{2}$, i.e. $g_\ell^{\mathrm{Pauli}} = 2$ [g_ℓ = gyromagnetic factor]

DEFINITION

Relativistic effects?

$\longrightarrow$ Dirac equation with the minimal coupling precription

$$i\hbar\,\frac{\partial\psi}{\partial t} = \left[c\boldsymbol{\alpha}\cdot\left(-i\hbar\boldsymbol{\nabla} - \frac{q_\ell}{c}\boldsymbol{\mathcal{A}}\right) + \beta m_\ell c^2 + q_\ell\mathcal{A}_0\right]\psi$$

In the non relativistic limit, this reduces to the Pauli equation for the two-component spinor φ describing the large components of the Dirac spinor ψ,

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Misses an important aspect: ceaseless emission and absorption of virtual particles (SM or not!), cf. Lamb shift

$\longrightarrow$ requires QFT

DEFINITION

One wants to probe the response of a charged lepton to an external, static, and weak electromagnetic field $\longrightarrow$ linear response: $-e_\ell A^\rho J_\rho$

$$\langle \ell; p'|J_\rho(0)|\ell; p\rangle \equiv \bar{\mathrm{u}}(p')\Gamma_\rho(p',p)\mathrm{u}(p)$$

$$= \bar{\mathrm{u}}(p')\left[F_1(k^2)\gamma_\rho + \frac{i}{2m_\ell}F_2(k^2)\sigma_{\rho\nu}k^\nu - F_3(k^2)\gamma_5\sigma_{\rho\nu}k^\nu + F_4(k^2)(k^2\gamma_\rho - 2m_\ell k_\rho)\gamma_5\right]\mathrm{u}(p)$$

uses only the conservation of the electromagnetic current $J_\rho \equiv \bar{\psi}_\ell\gamma_\rho\psi_\ell$, $k_\mu \equiv p'_\mu - p_\mu$

$F_1(k^2) \rightarrow$ Dirac form factor , $F_1(0) = 1$

$F_2(k^2) \rightarrow$ Pauli form factor $\rightarrow F_2(0) = a_\ell$

$F_3(k^2) \rightarrow \not{P}, \not{T}$, electric dipole moment $\rightarrow F_3(0) = d_\ell/q_\ell$

$F_4(k^2) \rightarrow \not{P}$, anapole moment

$$G_E(k^2) = F_1(k^2) + \frac{k^2}{4m_\ell^2}F_2(k^2),\ G_M(k^2) = F_1(k^2) + F_2(k^2)$$

$$G_E(0) = 1 \quad G_M(0) = 1 + F_2(0) = g_\ell/2$$

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At tree level in the SM

$$F_1^{\text{tree}} = 1 \quad \text{and} \quad F_2^{\text{tree}} = F_3^{\text{tree}} = F_4^{\text{tree}} = 0$$

Tree-level contributions to $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ would require operators of dimensions > 4 $\longrightarrow$ no counterterms available in the SM!

in the SM, $F_2(k^2)$, $F_3(k^2)$, $F_4(k^2)$ are only induced by loops

$\longrightarrow$ calculable! (i.e. UV finite)

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At tree level in the SM, $g_\ell = g_\ell^{\mathrm{Dirac}} \equiv 2$.

The *anomalous* magnetic moment is induced at loop level:

$$a_\ell \equiv \frac{g_\ell - g_\ell^{\mathrm{Dirac}}}{g_\ell^{\mathrm{Dirac}}} \ (\equiv F_2(0))$$

a_ℓ probes all the degrees of freedom of the standard model

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and possibly beyond...

DEFINITION

Why the muon?

Leptonic sector of the three-family standard model

- charged leptons: $\ell = e^{\pm}, \mu^{\pm}, \tau^{\pm}$

$$q = \pm 1 \text{ (charge)} \qquad s = \frac{1}{2} \text{ (spin)}$$

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differ only through their couplings to the Higgs:

this is the only source of LFU violation in the SM!

$$\begin{aligned} m_e &= 0.510\,998\,946\,1(3\,1)\ \text{MeV} \\ m_\mu &= 105.658\,374\,5(2\,4)\ \text{MeV} \\ m_\tau &= 1\,776.86(12)\ \text{MeV} \end{aligned}$$

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- has dramatic consequences for the lifetimes:

$$\tau_e > 6.6 \cdot 10^{28}\ y, \quad \tau_\mu = 2.1969811(22) \cdot 10^{-6}\ s, \quad \tau_\tau = 290.3(5) \cdot 10^{-15}\ s$$

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and hence for experiment!

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The anomalous magnetic moment of a lepton ℓ is a dimensionless quantity

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$\longrightarrow$ caveat: assumes that NP is decoupling and couples in a LFU way!

How do we measure it?

Experimental aspects

- The electron case

1947: hf splitting in H and D (0.2% discrepancy with the value $g_e^{\rm Dirac} = 2$)

[J. Nafe, E. B. Nelson, I. I. Rabi, Phys. Rev. 71, 914 (1947)]

1958: first direct measurement of g_e for *free* electrons

[H. G. Dehmelt, Phys. Rev. 109, 381 (1958)]

1968 $\rightarrow$ 1987: Penning trap type experiments $\rightarrow$ single trapped electron (geonium)

$$
\begin{aligned}
a_{e^-}^{exp} &= 1\,159\,652\,188.4(4.3)\cdot 10^{-12} \quad [3.7\,\mathrm{ppb}]\\
a_{e^+}^{exp} &= 1\,159\,652\,187.9(4.3)\cdot 10^{-12} \quad [3.7\,\mathrm{ppb}]
\end{aligned}
$$

[R.S. van Dyck Jr. et al., PRL 59, 26 (1987)]

g_{e^-}/g_{e^+} = 1 + (0.5$\pm$2.1)$\times 10^{-12}$ probes CPT invariance

$\left(\rightarrow |M_{K^0} - M_{\overline{K}^0}|/M_{K^0} \leq 10^{-18}\right.$ (90% CL)$\left.\right)$

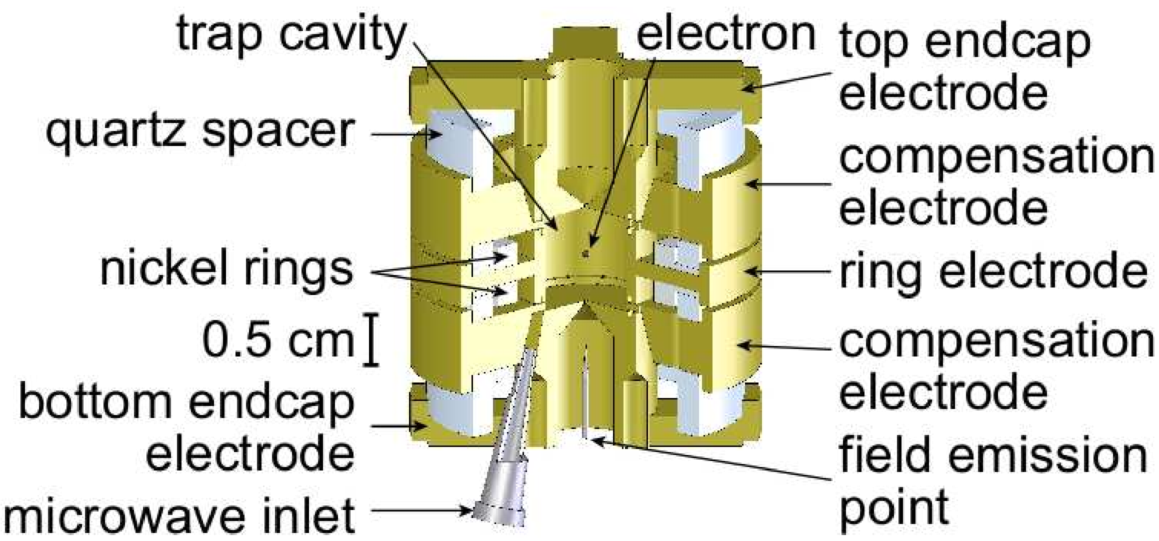
New series of high precision measurements conducted by the Harvard group (G. Gabrielse et al.)

$$a_e^{exp} \;=\; 1\,159\,652\,180.85(0.76)\cdot 10^{-12} \;\; [0.66\,\mathrm{ppb}]$$

[Odom et al., PRL 97, 030801 (2006)]

$$a_e^{exp} \;=\; 1\,159\,652\,180.73(0.28)\cdot 10^{-12} \;\; [0.24\,\mathrm{ppb}]$$

[D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)]



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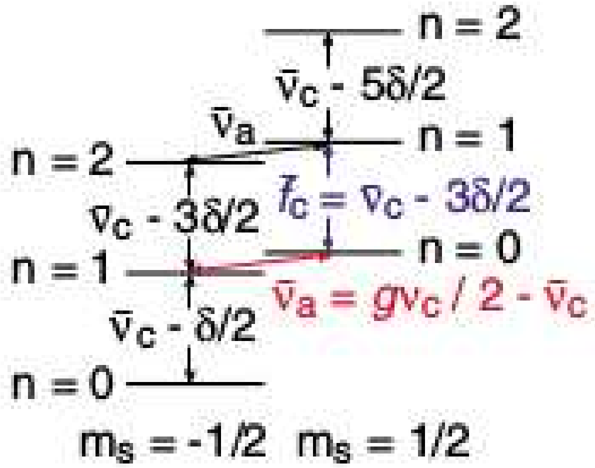
$$a_e^{exp} \;=\; 1\,159\,652\,180.85(0.76)\cdot 10^{-12}\;\;[0.66\,\mathrm{ppb}]$$

[Odom et al., PRL 97, 030801 (2006)]

$$a_e^{exp} \;=\; 1\,159\,652\,180.73(0.28)\cdot 10^{-12}\;\;[0.24\,\mathrm{ppb}]$$

[D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)]



- The muon case

Remember: $\tau_\mu = 2.1969811(22) \cdot 10^{-6}\, s$

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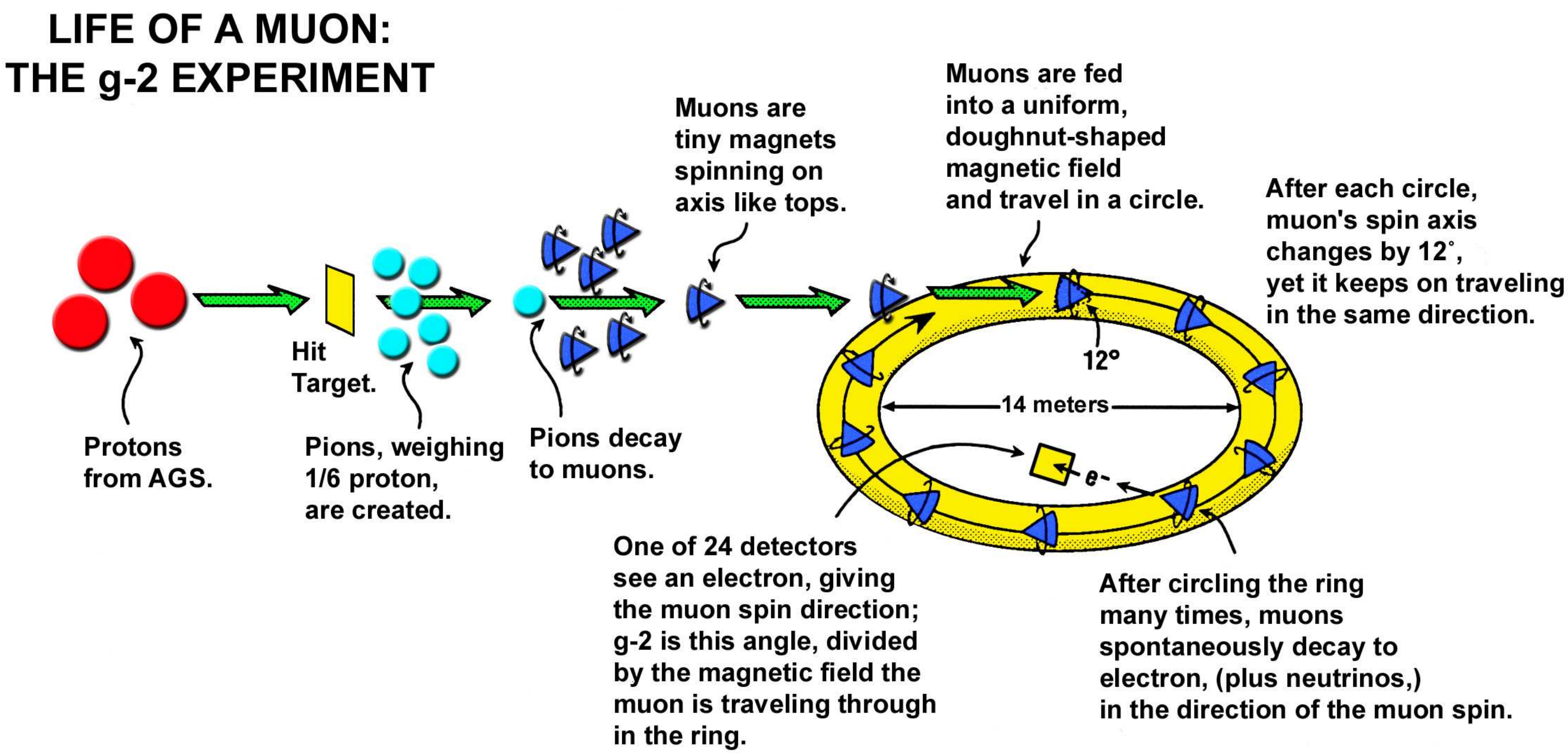

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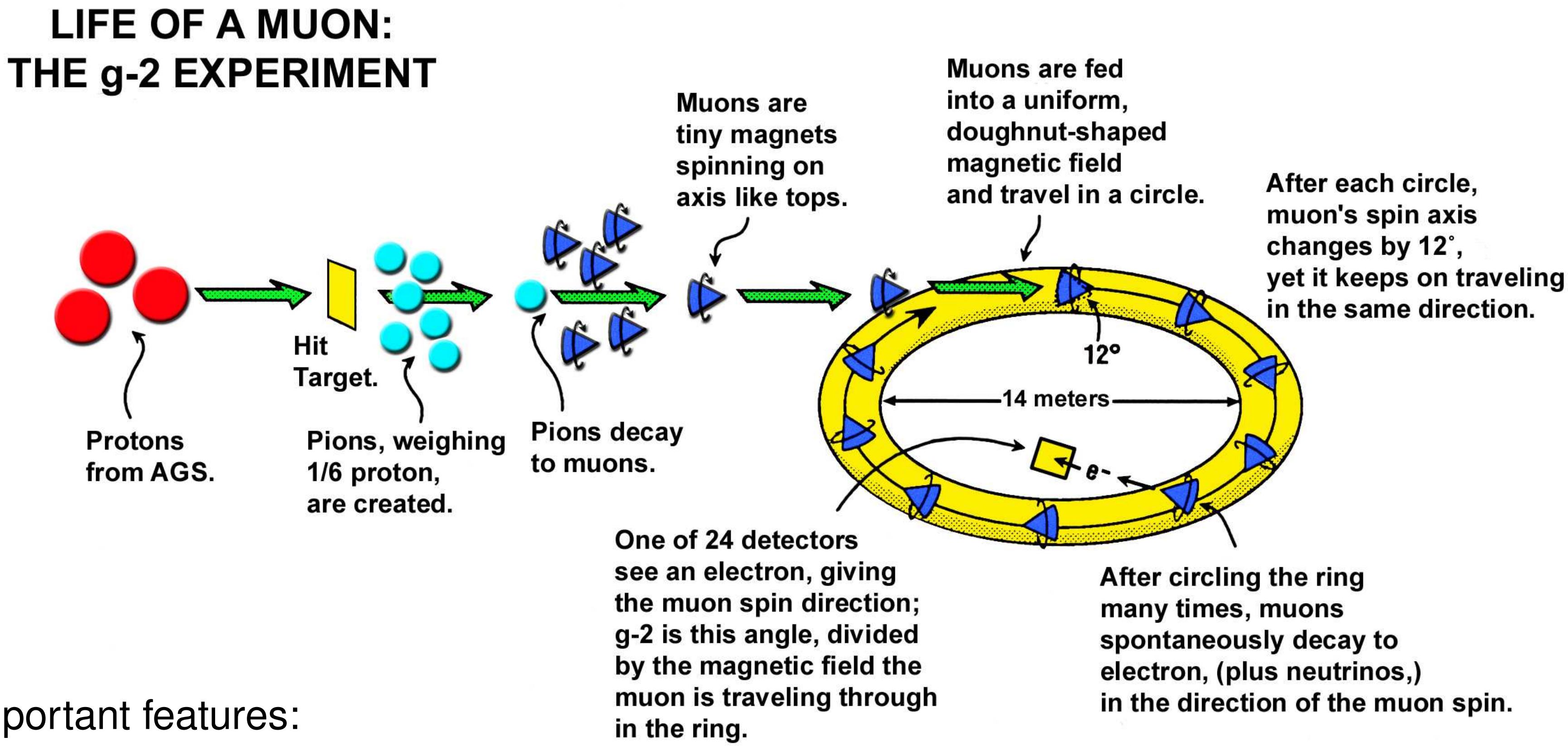

Two important features:

- the most energetic muons emitted in the decay of the pions are forwards polarized
- the most energetic positrons are emitted in the direction of the spin of the decaying μ^+

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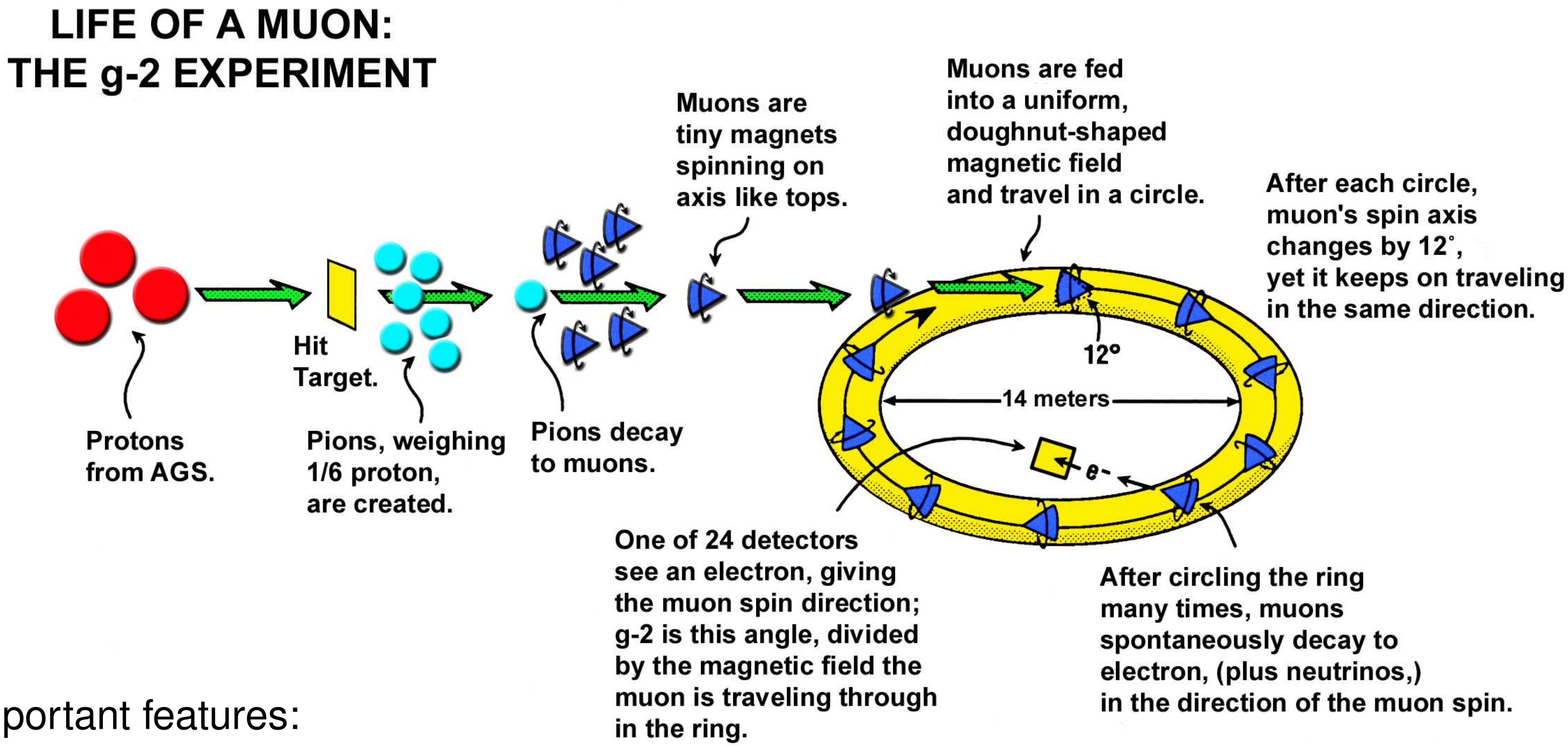

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- the most energetic muons emitted in the decay of the pions are forwards polarized
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These are experimental facts, no need to assume SM is valid!

- muon storage ring experiment $[\vec{\beta}\cdot\vec{E} = \vec{\beta}\cdot\vec{B} = 0]$

$$\vec{\omega}_a \equiv \vec{\omega}_s - \vec{\omega}_c = -\frac{e}{m_\mu c}[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}] - 2\,\vec{d}_\mu \cdot [\vec{\beta}\times\vec{B} + \vec{E}]$$

- $\gamma \sim 29.3$ [electrostatic focusing will not affect the spin]

Muon g-2 Coll., H. N. Brown et al., Phys. Rev. Lett. 86, 2227 (2001)

- $|d_\mu| < 1.9\cdot 10^{-19}$ [95% CL] Muon g-2 Coll., G. W. Bennett et al., Phys. Rev. D 80 (2009)

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- Still need to measure the magnetic field $\longrightarrow$ NMR probes $\longrightarrow \omega_{\rm p} = -g_{\rm p}\frac{e}{2m_{\rm p}}B$

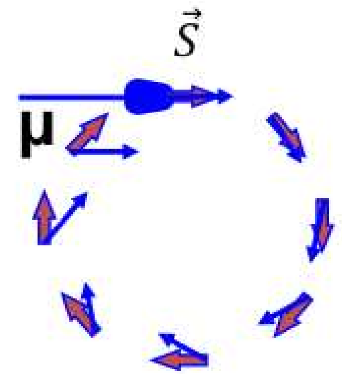
$$a_\mu = \frac{g_{\rm p}}{2}\frac{\omega_a}{\omega_{\rm p}}\frac{m_\mu}{m_{\rm p}} = \frac{g_e}{2}\frac{\omega_a}{\omega_{\rm p}}\frac{m_\mu}{m_e}\frac{\mu_{\rm p}}{\mu_e}$$

$$\frac{\Delta g_e}{g_e} = 0.26\text{ppt},\quad \frac{\Delta(m_\mu/m_e)}{m_\mu/m_e} = 22\text{ppb},\quad \frac{\Delta\mu_{\rm p}/\mu_e}{\mu_{\rm p}/\mu_e} = 3\text{ppb},\quad \frac{\Delta\omega_{\rm p}}{\omega_{\rm p}} = 70\text{ppb}$$

Key ingredients

1) Polarized muons

~95% polarized for forward decay



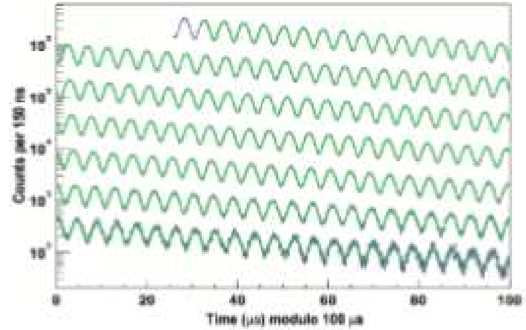
2) Precession proportional to (g-2)

$$\omega_a = \omega_{spin} - \omega_{cyclotron} = \left(\frac{g-2}{2}\right)\frac{eB}{mc}$$

$a_\mu = (g-2)/2$

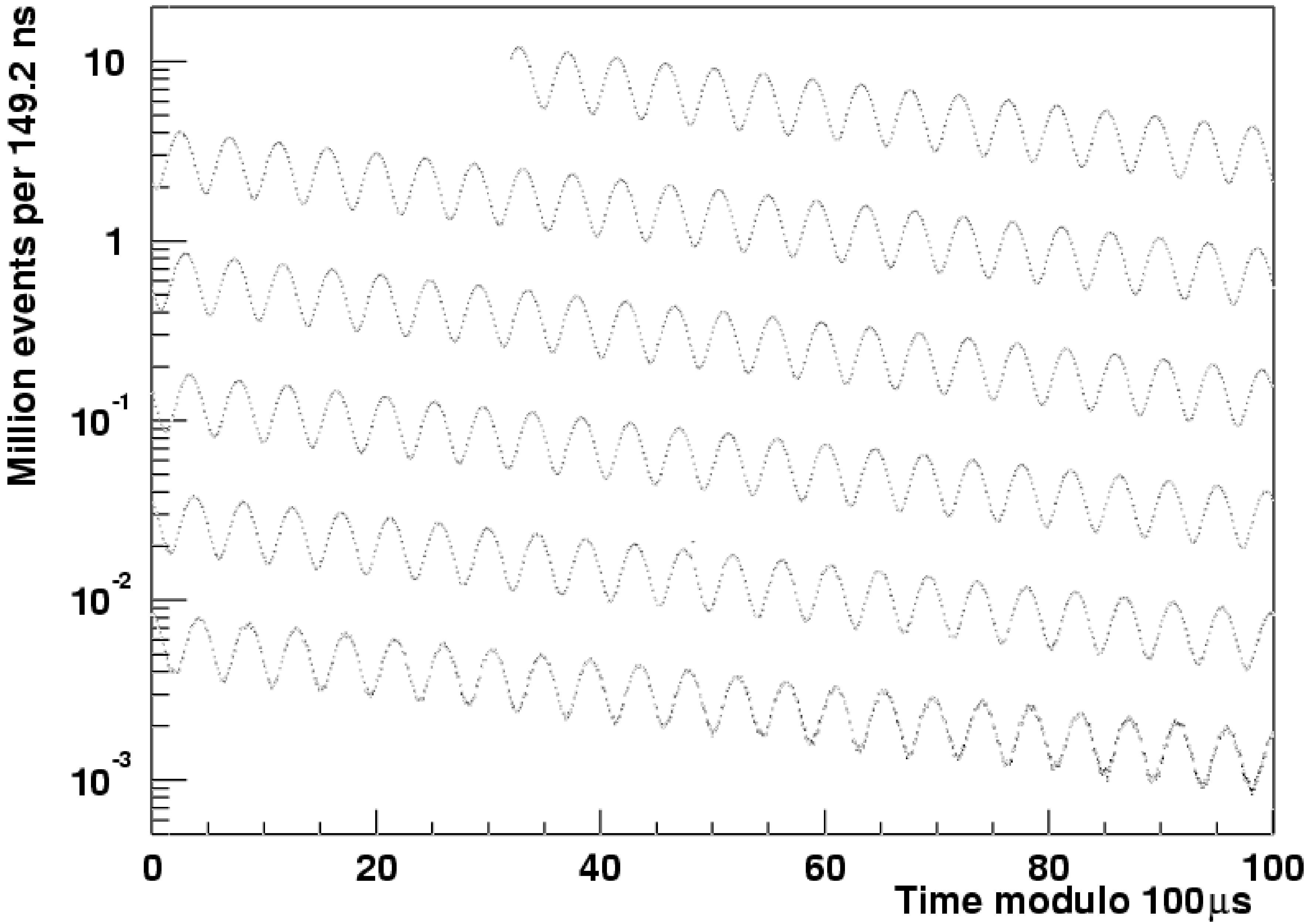
3) P_μ magic momentum = 3.09 GeV/c

$$\vec{\omega}_a = \frac{e}{mc}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1}\right)\vec{\beta}\times\vec{E}\right]$$



4) Decay e^+ emitted preferably in spin direction of the muon

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$


Million events per 149.2 ns
10
1
10^{-1}
10^{-2}
10^{-3}
0
20
40
60
80
100
Time modulo 100μs

BNL-E821

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$-0.052 < a_\tau^{exp} < +0.058$ (L3, 1998, 95% CL) Phys. Lett. B 434, 169 (1998)

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- $e^+e^- \to e^+e^-\tau^+\tau^-$

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Eur. Phys. J. C 35, 159 (2004)

- Reanalysis of experiments $\longrightarrow$ $-0.007 < a_\tau^{exp} < +0.005$

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$\longrightarrow$ Various proposals to improve the situation exist: arXiv:0707.2496, arXiv:0807.2366 arXiv:1601.07987, arXiv:1803.00501, arXiv:1810.06699, arXiv:1908.05180, arXiv:2002.05503, arXiv:2111.10378

Standard Model Prediction

SM prediction

Considering SM contributions only, one has, by order of importance

$$a_\mu = a_\mu^{\mathsf{QED}} + a_\mu^{\rm had} + a_\mu^{\rm weak}$$

a_μ^{QED}: loops with only photons and leptons

$a_\mu^{\rm had}$: loops with photons and leptons and at least one quark loop dressed with gluons

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For a full and detailed account [up to June 15, 2020], see the White Paper

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

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$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

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$$(\alpha/\pi)^4 = 2.91\ldots \cdot 10^{-11} \qquad (\alpha/\pi)^5 = 6.76\ldots \cdot 10^{-14}$$

$\longrightarrow$ becomes technically challenging $(1, 6, 72, 891, 12\,672, \ldots)$

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)} \left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)} \left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)} \left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)} \left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)} \left(\frac{\alpha}{\pi}\right)^5 + \ldots$$

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$A_1^{(2n)} \longrightarrow$ mass-independent (universal) contributions (one-flavour QED)

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mass-dependent (non-universal) contributions (multi-flavour QED)

– a_ℓ is finite (no renormalization needed) and dimensionless

– QED is decoupling

— Massive fermions with $m_{\ell'} \gg m_\ell$ contribute to a_ℓ through powers of $m_\ell^2/m_{\ell'}^2$, times logarithms

— Light degrees of freedom with $m_{\ell'} \ll m_\ell$ give logarithmic contributions to a_ℓ, e.g. $\ln(m_\ell^2/m_{\ell'}^2)$

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mass-dependent (non-universal) contributions (multi-flavour QED)

For the electron, $A_1^{(2n)}$ matter most, whereas $A_2^{(2n)}$ and $A_3^{(2n)}$ are suppressed by powers of m_e^2/m_μ^2 and m_e^2/m_τ^2 (times logarithms)

QED contribution : loops with only photons and leptons

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mass-dependent (non-universal) contributions (multi-flavour QED)

For the muon, $A_1^{(2n)}$ are negligible, whereas $A_2^{(2n)}(m_\mu/m_e)$ are enhanced by powers of $\ln(m_\mu/m_e)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$ $\left(\pi^2 \ln \frac{m_\mu}{m_e} \sim 50\right)$

Even more true for the tau...

QED contribution : loops with only photons and leptons

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Expressions for $A_1^{(2)}$, $A_1^{(4)}$, $A_2^{(4)}$, $A_1^{(6)}$, $A_2^{(6)}$, $A_3^{(6)}$ known analytically

J. Schwinger, Phys. Rev. 73, 416L (1948)

C. M. Sommerfield, Phys. Rev. 107, 328 (1957); Ann. Phys. 5, 26 (1958)

A. Petermann, Helv. Phys. Acta 30, 407 (1957)

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M. Passera, Phys. Rev. D 75, 013002 (2007)

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$\longrightarrow$ no uncertainties in $A_1^{(2)}$, $A_1^{(4)}$, $A_1^{(6)}$

$\longrightarrow$ precision of $A_2^{(4)}$, $A_2^{(6)}$, $A_3^{(6)}$ only limited by precision in $m_\ell/m_{\ell'}$

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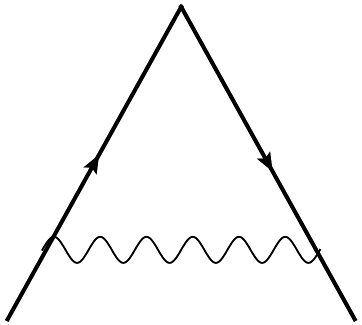

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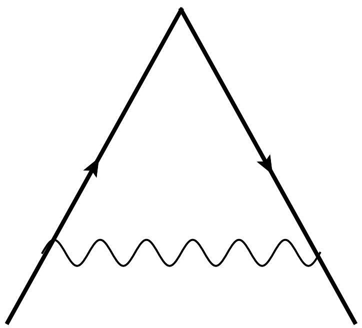

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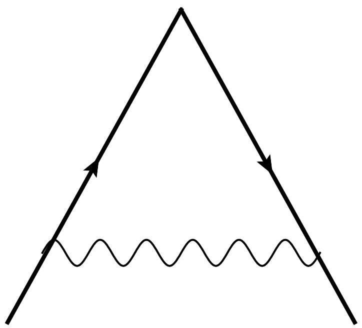

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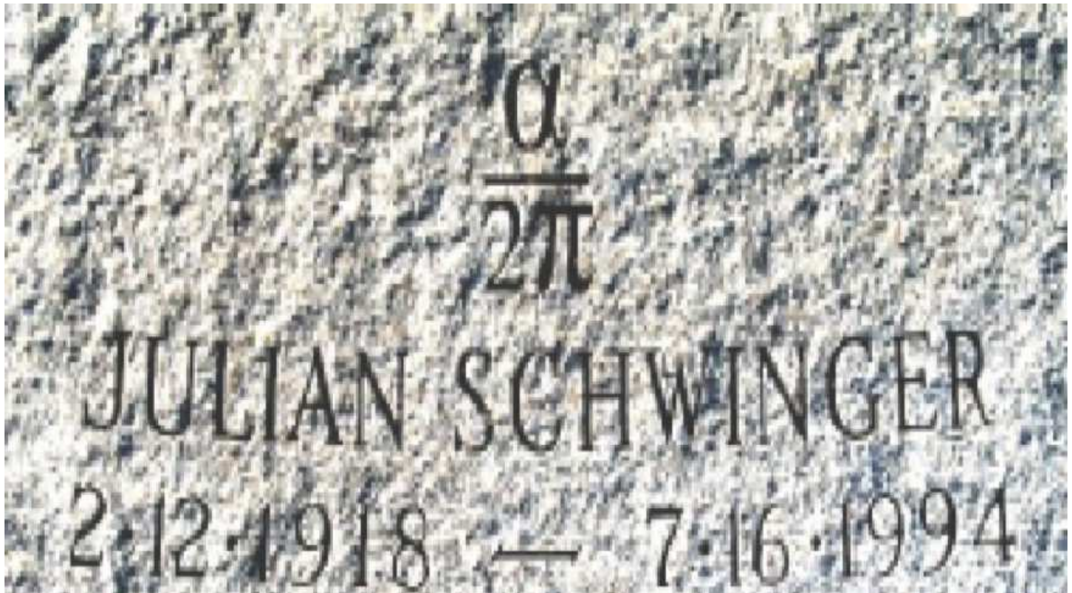

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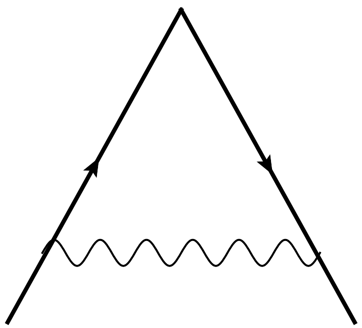

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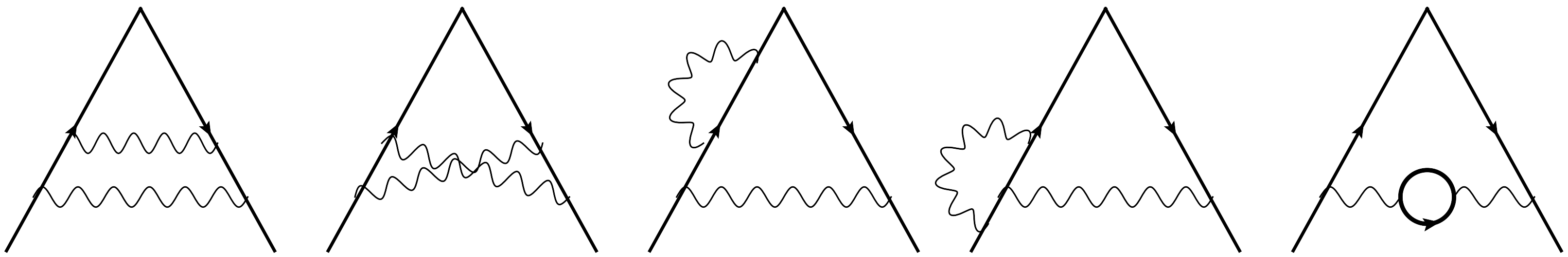

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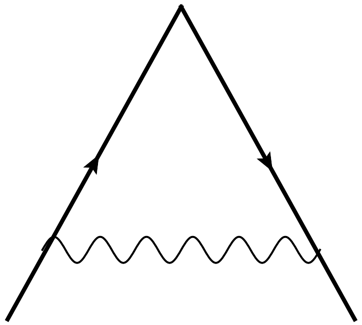

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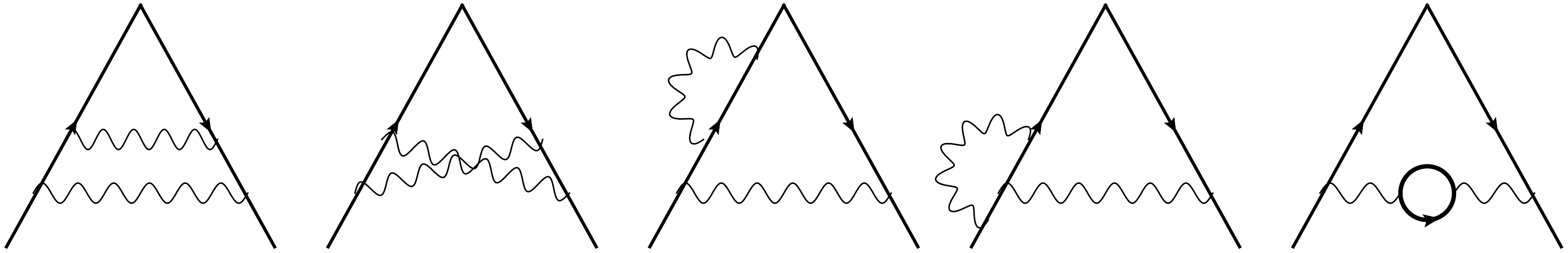

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The last diagram gives also gives $A_2^{(4)}$

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$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} ds\,\sqrt{1-\frac{4m_{\ell'}^2}{s}}\,\frac{s+2m_{\ell'}^2}{s^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{s}{m_\ell^2}}$$

Cross section for the scattering of a charged-lepton pair $\ell^+\ell^-$ into another charged-lepton pair $\ell'^+\ell'^-$ at lowest order in QED

$$\sigma_{\mathrm{LO}}^{(\ell^+\ell^-\to\ell'^+\ell'^-)}(s) = \frac{4\pi}{3}\frac{\alpha^2}{s^2}\sqrt{1-\frac{4m_{\ell'}^2}{s}}(s+2m_{\ell'}^2)$$

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$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} ds K(s)\, R^{(\ell')}(s)$$

$$R^{(\ell')}(s) = \frac{\sigma_{\mathrm{LO}}^{(\ell^+\ell^-\to\ell'^+\ell'^-)}(s)}{\sigma_\infty^{e^+e^-\to\mu^+\mu^-}(s)} \qquad \sigma_\infty^{e^+e^-\to\mu^+\mu^-}(s) = \frac{4\pi}{3}\frac{\alpha^2}{s}$$

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H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{3}\ln\left(\frac{m_\ell}{m_{\ell'}}\right) - \frac{25}{36} + \frac{\pi^2}{4}\frac{m_{\ell'}}{m_\ell} - 4\left(\frac{m_{\ell'}}{m_\ell}\right)^2 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &\quad +3\left(\frac{m_{\ell'}}{m_\ell}\right)^2 + \mathcal{O}\left[\left(\frac{m_{\ell'}}{m_\ell}\right)^3\right], \quad m_\ell \gg m_{\ell'} \end{aligned}$$

M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)

$$A_2^{(4)}(m_\mu/m_e) = 1.094\,258\,309\,3(76)$$

$$m_\mu/m_e = 206.768\,2843(52)$$

P. J. Mohr, B. N. Taylor, D. B. Newell, CODATA 2010, arXiv:1203.5425v1[physics.atom-ph]

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$A_2^{(4)}(m_\ell/m_{\ell'}) = \frac{1}{3}\int_{4m_{\ell'}^2}^{\infty} ds\,\sqrt{1-\frac{4m_{\ell'}^2}{s}}\,\frac{s+2m_{\ell'}^2}{s^2}\int_0^1 dx\,\frac{x^2(1-x)}{x^2+(1-x)\,\frac{s}{m_\ell^2}}$$

H. Suura and E. Wichmann, Phys. Rev. 105, 1930 (1955)

A. Petermann, Phys. Rev. 105, 1931 (1955)

H. H. Elend, Phys. Lett. 20, 682 (1966); Err. Ibid. 21, 720 (1966)

M. Passera, Phys. Rev. D 75, 013002 (2007)

$$\begin{aligned} A_2^{(4)}(m_\ell/m_{\ell'}) &= \frac{1}{45}\left(\frac{m_\ell}{m_{\ell'}}\right)^2 + \frac{1}{70}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 \ln\left(\frac{m_\ell}{m_{\ell'}}\right) \\ &\quad + \frac{9}{19600}\left(\frac{m_\ell}{m_{\ell'}}\right)^4 + \mathcal{O}\left[\left(\frac{m_\ell}{m_{\ell'}}\right)^3 \ln\left(\frac{m_\ell}{m_{\ell'}}\right)\right], \quad m_{\ell'} \gg m_\ell \end{aligned}$$

B.E. Lautrup and E. de Rafael, Phys. Rev. 174, 1835 (1965)

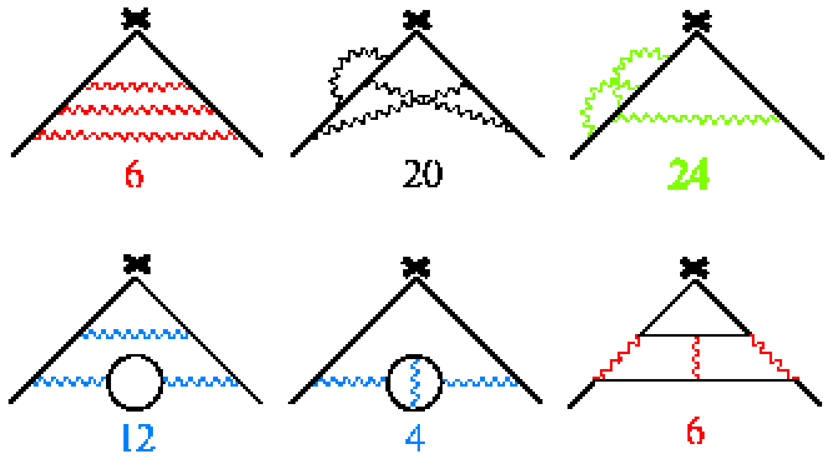
M. A. Samuel and G. Li, Phys. Rev. D 44, 3935 (1991)

$$\begin{aligned} A_2^{(4)}(m_e/m_\mu) &= 5.197\,386\,676(24)\cdot 10^{-7} \\ A_2^{(4)}(m_e/m_\tau) &= 1.837\,90(25)\cdot 10^{-9} \\ A_2^{(4)}(m_\mu/m_\tau) &= 7.8076(11)\cdot 10^{-5} \end{aligned}$$

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

order $(\alpha/\pi)^3$: 72 diagrams



$$A_1^{(6)} = \frac{87}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(a_4 + \frac{1}{24}\ln^4 2\right) - \frac{1}{24}\pi^2\ln^2 2\right] - \frac{239}{2160}\pi^4$$
$$+\frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184} \qquad \left[a_p = \sum_1^\infty 1/(2^n n^p)\right]$$

S. Laporta, E. Remiddi, Phys. Lett. B265, 182 (1991); B356, 390 (1995); B379, 283 (1996)

S. Laporta, Phys. Rev. D 47, 4793 (1993); Phys. Lett. B343, 421 (1995)

$$A_1^{(6)} = 1.181\,241\,456...$$

numerical evaluations: $A_1^{(6)}(\text{num}) = 1.181\,259(40)...$

[T. Kinoshita, Phys. Rev. Lett. 75, 4728 (1995)]

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

order $(\alpha/\pi)^4$: 891 diagrams

$A_1^{(8)}$ has also been evaluated! (a_e) S. Laporta, Phys. Lett. B 772, 232 (2017)

$$A_1^{(8)} = -1.912\,245\,764\,926\,445\,574\,152\,647\,167\,439\,830\,054\,060\,873\,390\,658\,725\,345\,171\,329\ldots$$

Good agreement with earlier numerical evaluations

$$A_1^{(8)} = -1.912\,98(84)$$

T. Aoyama et al.,, Phys. Rev. D 91, 033006 (2015)

Mass-dependent contributions (a_μ)

only a few diagrams are known analytically $\longrightarrow$ numerical evaluation

Automated generation of diagrams, systematic numerical evaluation of multi-dimensional integrals over Feynman-parameter space

$$A_2^{(8)}(m_e/m_\mu) = 9.161\,970\,703(373)\cdot 10^{-4} \quad A_2^{(8)}(m_e/m_\tau) = 7.429\,24(118)\cdot 10^{-6}$$

$$A_3^{(8)}(m_e/m_\mu, m_e/m_\tau) = 7.468\,7(28)\cdot 10^{-7}$$

$$A_2^{(8)}(m_\mu/m_e) = 132.685\,2(60) \quad A_2^{(8)}(m_\mu/m_\tau) = 0.042\,494\,1(53)$$

$$A_3^{(8)}(m_\mu/m_e, m_\mu/m_\tau) = 0.062\,722(10)$$

Independent check of mass-dependent contributions A. Kataev, Phys. Rev. D 86, 013019 (2012)

A. Kurz, T. Liu, P. Marquard, M. Steinhauser, Nucl. Phys. B 879, 1 (2014)

A. Kurz, T. Liu, P. Marquard, A. V. Smirnov, V. A. Smirnov, M. Steinhauser, Phys. Rev. D 92, 073019 (2015)]

Agreement at the level of accuracy required by present and future experiments for a_μ

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

order $(\alpha/\pi)^5$: 12 672 diagrams

6 classes, 32 gauge invariant subsets

Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

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No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ (a_e) is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

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$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

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Five of these subsets are known analytically

S. Laporta, Phys. Lett. B 328, 522 (1994)

J.-P. Aguilar, D. Greynat, E. de Rafael, Phys. Rev. D 77, 093010 (2008)

Complete numerical results have been published

T. Kinoshita and M. Nio, Phys. Rev. D 73, 053007 (2006); T. Aoyama et al., Phys. Rev. D 78, 053005 (2008); D 78, 113006 (2008); D 81, 053009 (2010); D 82, 113004 (2010); D 83, 053002 (2011); D 83, 053003 (2011); D 84, 053003 (2011); D 85, 033007 (2012); Phys. Rev. Lett. 109, 111807 (2012); Phys. Rev. Lett. 109, 111808 (2012)

No systematic cross-checks even for mass-dependent contributions

An independent numerical evaluation of $A_1^{(10)}$ (a_e) is in progress

S. Volkov, Phys. Rev. D 98, 076018 (2018); arXiv:1905.08007; Phys.Rev.D 100, 096004 (2019)

$\longrightarrow$ discrepancy [4.8σ] found in the contribution of graphs without fermion loops

$\longrightarrow$ semi-analytical evaluation by S. Laporta?

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

	$\ell = e$	$\ell = \mu$
$C_\ell^{(2)}$	0.5	0.5
$C_\ell^{(4)}$	$-0.328\,478\,444\,00\ldots$	$0.765\,857\,425(17)$
$C_\ell^{(6)}$	$1.181\,234\,017\ldots$	$24.050\,509\,96(32)$
$C_\ell^{(8)}$	$-1.911\,321\,390\ldots$	$130.878\,0(61)$
$C_\ell^{(10)}$	$6.733(159)$	$750.72(93)$

n	1	2	3	4	5
$(\alpha/\pi)^n$	$2.32\ldots\cdot 10^{-3}$	$5.39\ldots\cdot 10^{-6}$	$1.25\ldots\cdot 10^{-8}$	$2.91\ldots\cdot 10^{-11}$	$6.76\ldots\cdot 10^{-14}$

$$\Delta C_e^{(10)}\cdot(\alpha/\pi)^5 \sim 0.15\cdot 10^{-13} \qquad \Delta a_e^{\text{exp}} = 2.8\cdot 10^{-13}$$

$$[\Delta C_e^{(8)}\cdot(\alpha/\pi)^4 \text{ was } \sim 0.2\cdot 10^{-13} \text{ before Laporta's calculation}]$$

A few comments about the QED contributions

- Uncertainties on the coefficients $C_\mu^{(2n)}$ not relevant for a_μ at the present (and future) level of precision

$$\Delta C_\mu^{(4)} \cdot (\alpha/\pi)^2 \sim 0.9 \cdot 10^{-13} \qquad \Delta C_\mu^{(6)} \cdot (\alpha/\pi)^3 \sim 0.04 \cdot 10^{-13}$$

$$\Delta C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 1.8 \cdot 10^{-13} \qquad \Delta C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 0.7 \cdot 10^{-13} \qquad \Delta a_\mu^{\text{exp}} = 41 \cdot 10^{-11}$$

- Order $\mathcal{O}(\alpha^4)$ and even order $\mathcal{O}(\alpha^5)$ relevant for a_μ at the present (and future) level of precision

$$C_\mu^{(8)} \cdot (\alpha/\pi)^4 \sim 380 \cdot 10^{-11} \qquad C_\mu^{(10)} \cdot (\alpha/\pi)^5 \sim 5 \cdot 10^{-11}$$

- Drastic increase with n in the coefficients $C_\mu^{(2n)}$ [$\pi^2 \ln(m_\mu/m_e) \sim 50$!]
- Estimate of $\mathcal{O}(\alpha^6)$ contributions with these enhancement factors

$$\delta a_\mu \sim A_2^{(6)}(m_\mu/m_e; \mathrm{LxL}) \left[\frac{2}{3} \ln \frac{m_\mu}{m_e} - \frac{5}{9}\right]^3 \cdot 10 \left(\frac{\alpha}{\pi}\right)^6 \sim 0.54 \cdot 10^4 \cdot \left(\frac{\alpha}{\pi}\right)^6 \sim 0.08 \cdot 10^{-11}$$

- No sign of substantial contribution to a_μ from higher order QED

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

In order to have an accurate evalustion of a_ℓ one needs a determination of α with precision

$$\frac{\Delta\alpha}{\alpha} \sim \frac{\Delta a_\ell}{a_\ell} \sim \begin{cases} 0.35 \text{ ppm} \longrightarrow \sim 0.14 \text{ ppm} & \text{for } \ell = \mu \\ 0.24 \text{ ppb} \longrightarrow \sim 0.025 \text{ ppb} & \text{for } \ell = e \longrightarrow \Delta\alpha \lesssim 10^{-12} - 10^{-13} \end{cases}$$

QED contribution : loops with only photons and leptons

$\longrightarrow$ can be computed in perturbation theory:

$$a_\ell^{\text{QED}} = C_\ell^{(2)}\left(\frac{\alpha}{\pi}\right) + C_\ell^{(4)}\left(\frac{\alpha}{\pi}\right)^2 + C_\ell^{(6)}\left(\frac{\alpha}{\pi}\right)^3 + C_\ell^{(8)}\left(\frac{\alpha}{\pi}\right)^4 + C_\ell^{(10)}\left(\frac{\alpha}{\pi}\right)^5$$

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- quantum Hall effect

$$\alpha^{-1}[qH] = 137.036\,00300(270) \qquad [19.7\text{ppb}]$$

[P. J. Mohr, B. N. Taylor, D. B. Newell, Rev. Mod. Phys. 80, 633 (2008)]

- atomic recoil velocity through photon absorption

$$\alpha^2 = \frac{2R_\infty}{c} \cdot \frac{M_{\text{atom}}}{m_e} \cdot \frac{h}{M_{\text{atom}}}$$

$$\frac{\Delta R_\infty}{R_\infty} = 1.9 \cdot 10^{-12} \qquad \Delta\left(\frac{M_{\text{Rb}}}{m_e}\right) = 2.9 \cdot 10^{-11}$$

$$\alpha^{-1}[Cs\,02] = 137.036\,0001(11) \qquad [7.7\ \mathrm{ppb}]$$

A. Wicht, J. M. Hensley, E. Sarajilic, S. Chu, Phys. Scr. T102, 82 (2002)

$$\alpha^{-1}[Rb\,06] = 137.035\,998\,84(91) \qquad [6.7\ \mathrm{ppb}]$$

P. Cladé et al, Phys. Rev. A 74, 052109 (2006)

$$\alpha^{-1}[Rb\,08] = 137.035\,999\,45(62) \qquad [4.6\ \mathrm{ppb}]$$

M. Cadoret et al, Phys. Rev. Lett. 101, 230801 (2008)

$$\alpha^{-1}[Rb\,11] = 137.035\,999\,037(91) \qquad [0.66\ \mathrm{ppb}]$$

R. Bouchendira, P. Cladé, S. Ghelladi-Khélifa, F. Nez, F. Biraben, Phys. Rev. Lett. 106, 080801 (2011)

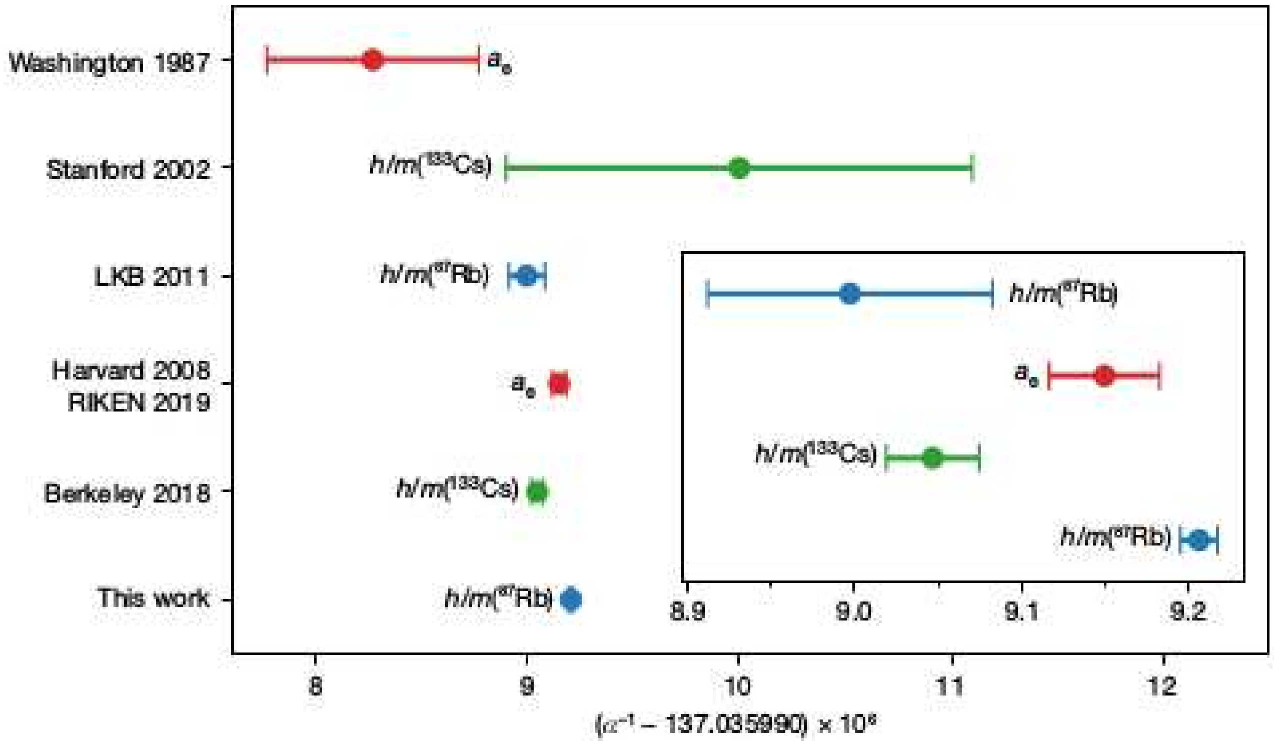
$$\alpha^{-1}[Rb\,11] = 137.035\,998\,995(85) \qquad [0.62\ \mathrm{ppb}]$$ [shift in R_∞]

$$\alpha^{-1}[Cs\,18] = 137.035\,999\,046(27) \qquad [0.20\ \mathrm{ppb}]$$

R. H. Parker, C. Yu, W. Zhong, B. Estey, H. Müller, Science 360, 191 (2018)

$$\alpha^{-1}[Rb\,20] = 137.035\,999\,206(11) \qquad [81\ \mathrm{ppt}]$$

L. Morel, Z. Yao, P. Cladé, S. Ghelladi-Khélifa, Nature 588, 61 (2020)



$\longrightarrow$ need to understand discrepancy between $\alpha(Cs\,18)$ and $\alpha(Rb\,20)$, but also between $\alpha(Rb11)$ and $\alpha(Rb20)$

$\longrightarrow$ particularly important in view of the possiblity to improve the accuracy on $a_e^{\rm exp}$ by an order of magnitude!

G. Gabrielse, S. E. Fayer, T. G. Myers, X. Fan, Atoms 7, 45 (2019)

- $$a_e^{\mathrm{QED}}(Rb\,11) = 1\,159\,652\,180.311(720)_{\alpha(Rb11)}(11)_{\alpha^5} \cdot 10^{-12}$$

$$a_e^{\mathrm{QED}}(Cs\,19) = 1\,159\,652\,179.880(229)_{\alpha(Cs19)}(11)_{\alpha^5} \cdot 10^{-12}$$

$$a_e^{\mathrm{QED}}(Rb\,20) = 1\,159\,652\,178.525(093)_{\alpha(Rb20)}(11)_{\alpha^5} \cdot 10^{-12}$$

$$a_e^{\mathrm{exp}} - a_e^{\mathrm{QED}}(Rb\,20) = 2.20(28) \cdot 10^{-12}$$

- $$a_e^{\mathrm{QED}}(Rb\,11) = 1\,159\,652\,180.311(720)_{\alpha(Rb11)}(11)_{\alpha^5} \cdot 10^{-12}$$

  $$a_e^{\mathrm{QED}}(Cs\,19) = 1\,159\,652\,179.880(229)_{\alpha(Cs19)}(11)_{\alpha^5} \cdot 10^{-12}$$

  $$a_e^{\mathrm{QED}}(Rb\,20) = 1\,159\,652\,178.525(093)_{\alpha(Rb20)}(11)_{\alpha^5} \cdot 10^{-12}$$

  $$a_e^{\exp} - a_e^{\mathrm{QED}}(Rb\,20) = 2.20(28) \cdot 10^{-12}$$

- $$a_\mu^{\mathrm{QED}}(Cs\,19) = 116\,584\,718.931(7)_{\mathrm{mass}}(17)_{\alpha^4}(6)_{\alpha^5}(100)_{\alpha^6}(23)_{\alpha(Cs19)} \cdot 10^{-11}$$

  $$a_\mu^{\exp;\mathrm{WA}} - a_\mu^{\mathrm{QED}}(Cs\,19) = 7342(41) \cdot 10^{-11}$$

- QED provides more than 99.99% of the total value, without uncertainties at this level of experimental precision

- The missing part has to be provided by weak and strong interactions (or else, new physics...)

A Standard Model Prediction of the muon g-2?

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Pre-FPCP2023 School, IP2I-Lyon University, May 26-27, 2023

2nd part

OUTLINE

- Introduction

- general context
- why is there a question mark in the title?

- What are we talking about?

- defining the anomalous magnetic moment of a charged lepton (why g-2?)
- why the muon?

- How do we measure it?

- brief description of experimental aspects

- What is the SM prediction for it? I: QED

- What is the SM prediction for it? II: Weak interactions

- What is the SM prediction for it? III: strong interactions

- hadronic light-by-light (HLxL)
- hadronic vacuum polarization (HVP)

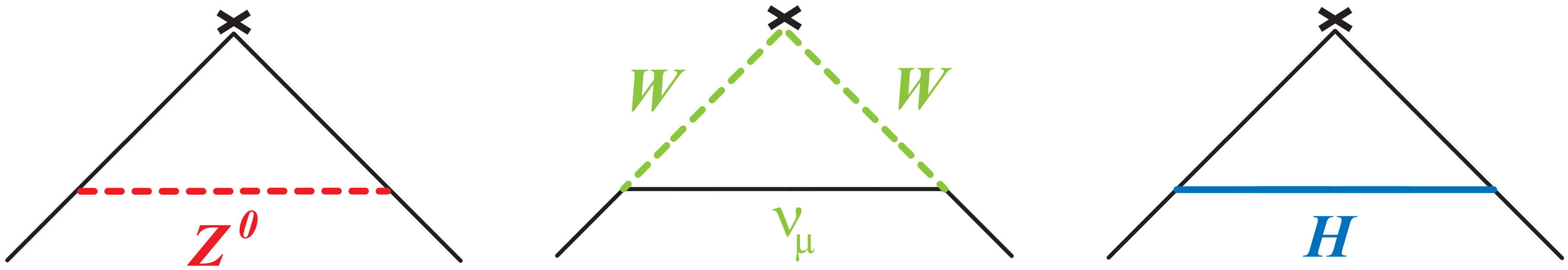
- Summary - Conclusion - Outlook

Standard Model Prediction

II Weak interactions

Weak contributions : W, Z,... loops

One-loop contributions



$$a_\mu^{\text{weak(1)}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left[\frac{5}{3}+\frac{1}{3}\left(1-4\sin^2\theta_W\right)^2+\mathcal{O}\left(\frac{m_\mu^2}{M_Z^2}\log\frac{M_Z^2}{m_\mu^2}\right)+\mathcal{O}\left(\frac{m_\mu^2}{M_H^2}\log\frac{M_H^2}{m_\mu^2}\right)\right]$$
$$= 194.8\cdot 10^{-11}$$

W.A. Bardeen, R. Gastmans and B.E. Lautrup, Nucl. Phys. B46, 315 (1972)

G. Altarelli, N. Cabbibo and L. Maiani, Phys. Lett. 40B, 415 (1972)

R. Jackiw and S. Weinberg, Phys. Rev. D 5, 2473 (1972)

I. Bars and M. Yoshimura, Phys. Rev. D 6, 374 (1972)

M. Fujikawa, B.W. Lee and A.I. Sanda, Phys. Rev. D 6, 2923 (1972)

Weak contributions : W, Z,... loops

Two-loop bosonic contributions

$$a_\mu^{\text{weak(2);bos}} = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\frac{\alpha}{\pi}\cdot\left[-5.96\ln\frac{M_W^2}{m_\mu^2}+0.19\right] = \frac{G_F}{\sqrt{2}}\frac{m_\mu^2}{8\pi^2}\left(\frac{\alpha}{\pi}\right)\cdot(-79.3)$$

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. Lett. 76, 3267 (1996)

Two-loop fermionic contributions

A. Czarnecki, B. Krause, W. J. Marciano, Phys. Rev. D 52, R2619 (1995)
M. K., S. Peris, M. Perrottet, E. de Rafael, JHEP11, 003 (2002)
A. Czarnecki, W.J. Marciano, A. Vainshtein, Phys. Rev. D 67, 073006 (2003). Err.-ibid. D 73, 119901 (2006)

Complete three-loop short-distance leading logarithms

G. Degrassi and G. F. Giudice, Phys. Rev. D 58, 053007 (1998)

$$\begin{aligned} a_\mu^{\text{weak}} &= (154\pm 1)\cdot 10^{-11} \\ a_e^{\text{weak}} &= (0.0297\pm 0.0005)\cdot 10^{-12} \end{aligned}$$

Updated a few years ago: $a_\mu^{\text{weak}} = (153.6\pm 1.0)\cdot 10^{-11}$

C. Gnendiger, D. Stöckinger, H. Stöckinger-Kim, Phys. Rev. D 88, 053005 (2013)

Recent numerical evaluation: $a_\mu^{\text{weak}} = (152.9\pm 1.0)\cdot 10^{-11}$

T. Ishikawa, N. Nakazawa and Y. Yasui, Phys. Rev. D 99, 073004 (2019)

Weak contributions : W, Z,... loops

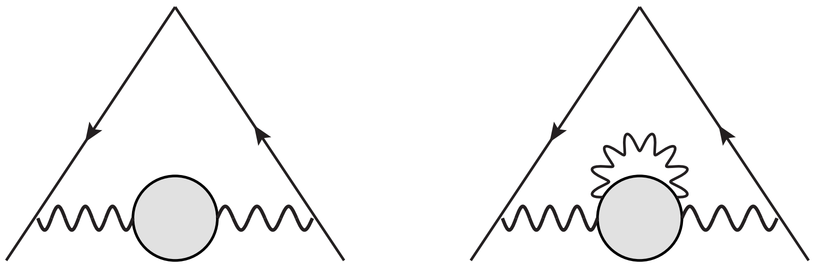
$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}} - a_\mu^{\mathsf{weak}} = 7188(41) \cdot 10^{-11}$$

Standard Model Prediction

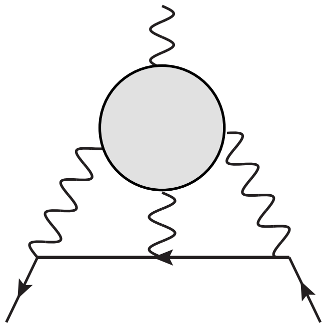
III Strong interactions

Contributions from strong interactions

- hadronic vacuum polarization



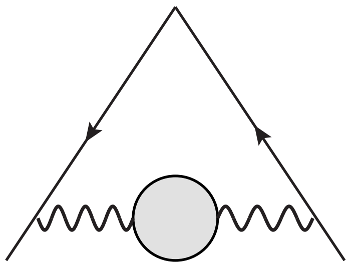
- (virtual) hadronic light-by-light (HLxL)

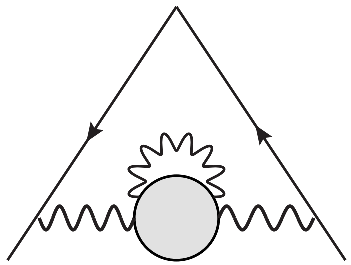


$\longrightarrow$ non-perturbative regime of QCD

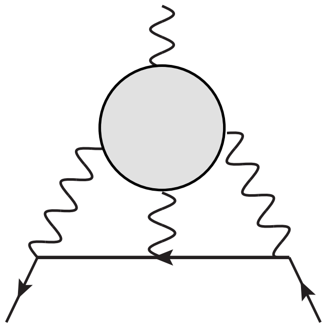
Contributions from strong interactions

- hadronic vacuum polarization





- (virtual) hadronic light-by-light (HLxL)



$\longrightarrow$ non-perturbative regime of QCD

$$a_\ell^{\text{had}} = a_\ell^{\text{HVP-LO}} + a_\ell^{\text{HVP-NLO}} + a_\ell^{\text{HVP-NNLO}} + a_\ell^{\text{HLxL}}$$

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$

$$a_\ell^{\text{HVP-LO}} = 4\alpha^2 \int_{4M_\pi^2}^{\infty} \frac{ds}{s} K(s) \frac{1}{\pi}\text{Im}\Pi(s) \quad K(s) = \int_0^1 dx \, \frac{x^2(1-x)}{x^2 + (1-x)\,\frac{s}{m_\ell^2}}$$

$$= 2\alpha^2 \int_0^1 dx \, (1-x)(2-x)\mathcal{A}\left(\frac{x^2}{1-x} m_\ell^2\right)$$

B. E. Lautrup, A. Peterman, E. de Rafael, Phys. Rep. 3, 193 (1972)

- Involves the vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{j_\mu j_\nu\}|0\rangle = i(q^2\eta_{\mu\nu} - q_\mu q_\nu)\Pi(q^2)$$

or the Adler function

$$\mathcal{A}(Q^2) = -Q^2 \frac{\partial\Pi(Q^2)}{\partial Q^2} = \int_0^\infty ds \frac{Q^2}{(s+Q^2)^2} \frac{1}{\pi}\text{Im}\Pi(s) \quad [Q^2 \equiv -q^2]$$

Hadronic vacuum polarization

- Occurs first at order $\mathcal{O}(\alpha^2)$
- Can be expressed as

$$a_\ell^{\text{HVP-LO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{4M_\pi^2}^{\infty} \frac{ds}{s}\, K(s) R^{\text{had}}(s) \quad K(s) = \int_0^1 dx\, \frac{x^2(1-x)}{x^2 + (1-x)\frac{s}{m_\ell^2}}$$

C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

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C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
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- $K(s) > 0$ and $R^{\text{had}}(s) > 0 \Longrightarrow a_\ell^{\text{HVP-LO}} > 0$
- $K(s) \sim m_\ell^2/(3s)$ as $s \to \infty \Longrightarrow$ the (non perturbative) low-energy region dominates

Hadronic vacuum polarization

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- Can be expressed as

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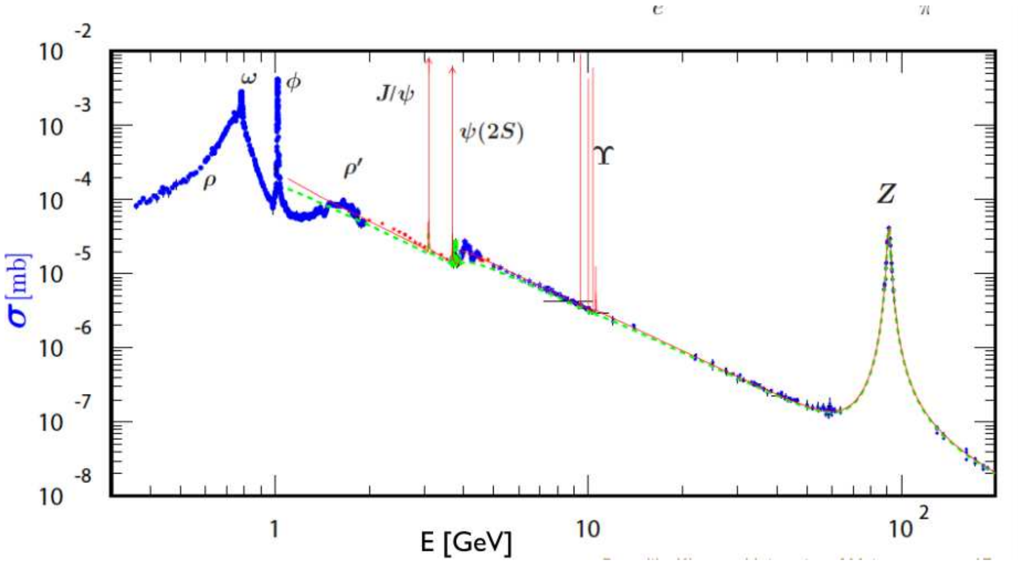
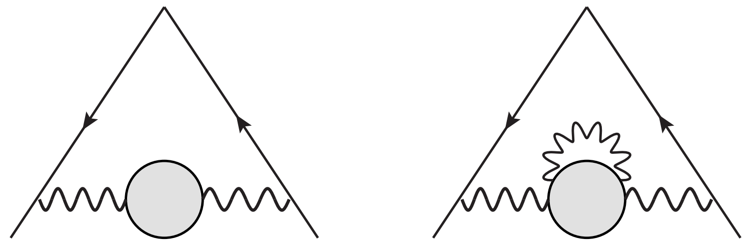
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- Can be evaluated using available experimental data





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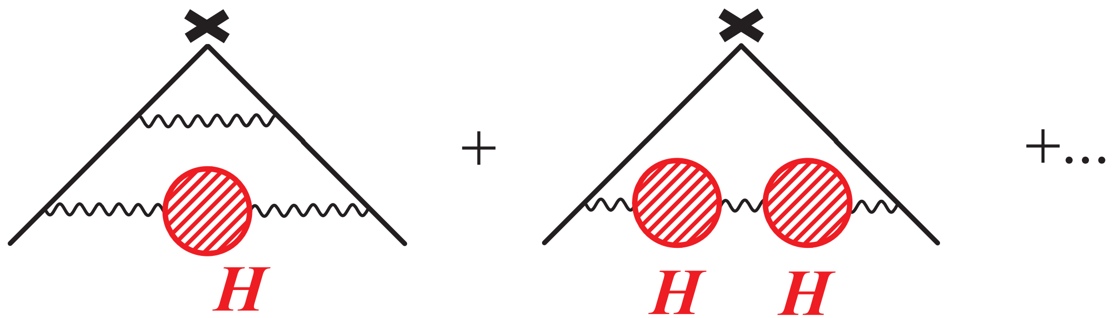
C. Bouchiat, L. Michel, J. Phys. Radium 22, 121 (1961)
L. Durand, Phys. Rev. 128, 441 (1962); Err.-ibid. 129, 2835 (1963)
M. Gourdin, E. de Rafael, Nucl. Phys. B 10, 667 (1969)

- Can be evaluated using available experimental data
- Full NLO $\mathcal{O}(\alpha^3)$, and even NNLO $\mathcal{O}(\alpha^4)$ corrections are also available

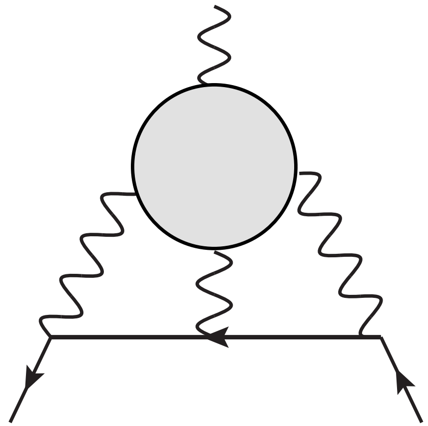
$$a_\mu^{\text{HVP-NLO}} = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^3 \int_{4M_\pi^2}^{\infty} \frac{ds}{s} K^{(2)}(s) R^{\text{had}}(s)$$

J. Calmet, S. Narison, M. Perrottet, E. de Rafael, Phys. Lett. B 61, 283 (1976)
B. Krause, Phys. Lett. B 390, 392 (1997)



Hadronic light-by-light

- Occurs at order $\mathcal{O}(\alpha^3)$
- Not related, as a whole, to an experimental observable...

? $\longrightarrow$ 

- Involves the fourth-rank vacuum polarization tensor

$$\text{F.T.}\ \langle 0|T\{j_\mu j_\nu j_\rho j_\sigma\}|0\rangle \longrightarrow \Pi_{\mu\nu\rho\sigma}(q_1,q_2,q_3,q_4) \quad q_1+q_2+q_3+q_4=0$$

- QCD short-distance constraints have been worked out

K. Melnikov, A. Vainshtein, Phys.Rev.D 70, 113006 (2004)
J. Bijnens, N. Hermansson-Truedsson, A. Rodríguez-Sánchez, Phys.Lett. B 798, 134994 (2019)
J. Bijnens, N. Hermansson-Truedsson, L. Laub and A. Rodríguez-Sánchez, JHEP 10, 203 (2020); JHEP 04, 240 (2021)
J. Bijnens, N. Hermansson-Truedsson and A. Rodríguez-Sánchez, JHEP 02, 167 (2023)

Hadronic light-by-light: phenomenological approaches

- Identify individual contributions (π^0, η, η' or scalar, axial-vector poles, $\pi^\pm$ and $K^\pm$ loops,...)

- Need some organizing principle: ChPT, large-N_c (turns out to be most relevant in practice)

E. de Rafael, Phys. Lett. B 322, 239 (1994)

$$a_\mu^{\mathrm{HLxL}} = +N_c \left(\frac{\alpha}{\pi}\right)^3 \frac{N_c}{F_\pi^2} \frac{m_\mu^2}{48\pi^2} \left[\ln^2 \frac{M_\rho}{M_\pi} + c_\chi \ln \frac{M_\rho}{M_\pi} + \kappa\right] + \mathcal{O}(N_c^0)$$

M. Knecht, A. Nyffeler, Phys. Rev. D 65, 073034 (2002)
M. Knecht, A. Nyffeler, M. Perrottet, E. de Rafael, Phys. Rev. Lett. 88, 071802 (2002)
M. J. Ramsey-Musolf, M. B. Wise, Phys. Rev. Lett. 89, 041601 (2002)

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85 ± 13	82.7 ± 6.4	83 ± 12	114 ± 10	–	114 ± 13	99 ± 16
π, K loops	-19 ± 13	-4.5 ± 8.1	–	–	–	-19 ± 19	-19 ± 13
π, K l. + subl. in Nc	–	–	–	0 ± 10	–	–	–
axial vectors	2.5 ± 1.0	1.7 ± 1.7	–	22 ± 5	–	15 ± 10	22 ± 5
scalars	-6.8 ± 2.0	–	–	–	–	-7 ± 7	-7 ± 2
quark loops	21 ± 3	9.7 ± 11.1	–	–	–	2.3	21 ± 3
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

BPP: J. Bijnens, E. Pallante, J. Prades, Phys. Rev. Lett. 75 (1995) 1447 [Err.-ibid. 75 (1995) 3781]; Nucl. Phys. B 474 (1996) 379; [Err,-ibid. 626 (2002) 410]
HKS: M. Hayakawa, T. Kinoshita, A. I. Sanda, Phys. Rev. Lett. 75 (1995) 790; Phys. Rev. D 54 (1996) 3137
KN: M. Knecht, A. Nyffeler, Phys. Rev. D 65 (2002) 073034
MV: K. Melnikov, A. Vainshtein, Phys. Rev. D 70 (2004) 113006
BP: J. Bijnens, J. Prades, Acta Phys. Polon. B 38 (2007) 2819; Phys. Proc. Suppl. 181-182 (2008) 15; Mod. Phys. Lett. A 22 (2007) 767

Hadronic light-by-light: phenomenological approaches

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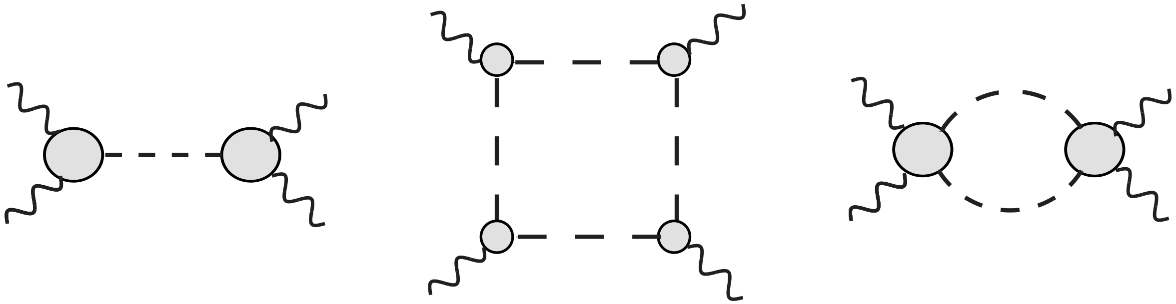
total	83 ± 32	89.6 ± 15.4	80 ± 40	136 ± 25	110 ± 40	105 ± 26	116 ± 39

- Highly model-dependent, uncertainties large and difficult to quantify
- At least the sign (positive) was eventually fixed

Hadronic light-by-light: dispersive approach

- Set up dispersion relations for the invariant functions into which $\Pi_{\mu\nu\rho\sigma}$ decomposes
- Identify specific contributions through the singularities they produce



$$\Pi = \Pi^{\pi^0,\eta,\eta' \text{ poles}} + \Pi^{\pi^\pm,K^\pm \text{ box}} + \Pi^{\pi\pi} + \Pi^{\text{residual}}$$

G. Colangelo, M. Hoferichter, M. Procura, P. Stoffer, JHEP09, 091 (2014); JHEP09, 074 (2015)

- Identify corresponding form factors

G. Colangelo, M. Hoferichter, B. Kubis, M. Procura, P. Stoffer, Phys. Lett. B 738, 6 (2014)
A. Nyffeler, arXiv:1602.03398 [hep-ph]

- Use input from data (when available) for these form factors
- Implement QCD short-distance constraints

G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub, P. Stoffer, JHEP03, 101 (2020)
J. Lüdtke, M. Procura, Eur. Phys. J. C 80, 1108 (2020)
J. G. Colangelo, F. Hagelstein, M. Hoferichter, L. Laub and P. Stoffer, Eur. Phys. J. C 81, 702 (2021)

Hadronic light-by-light: lattice QCD

- Several lattice-QCD results were already available at the time of the White Paper (WP)

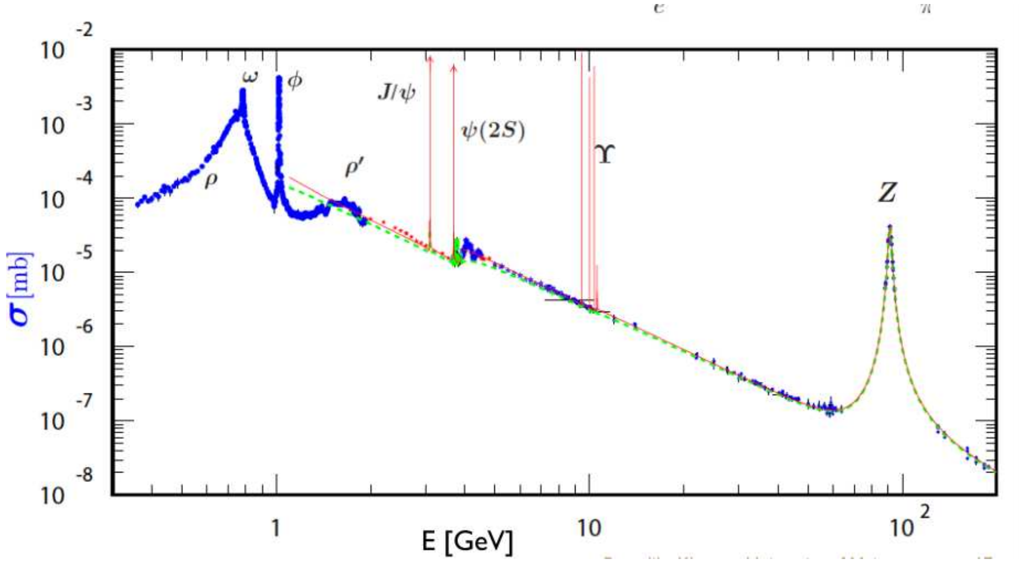
A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
D. Giusti *et al.*, Phys. Rev. D 99, 114502 (2019)
T. Blum *et al.*, Phys. Rev. Lett. 121, 022003 (2018)
S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

Hadronic light-by-light: White Paper Summary

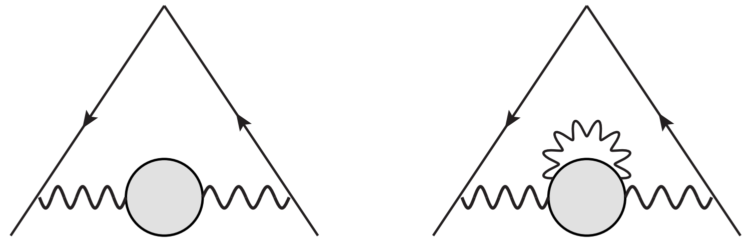
T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

$$a_\mu^{\mathrm{HLxL}} = 92(19) \cdot 10^{-11}$$

Hadronic vacuum polarization: data-driven determination



$\longrightarrow$



- Presently: combination of ~ 39 exclusive channels

$\longrightarrow$ Scan experiments (e.g. @ VEPP)

$\longrightarrow$ ISR experiments (e.g. @ DAΦNE, B-factories, BEPC)

- Precision on $a_\ell^{\text{HVP-LO}}$ has reached $\sim 0.5\%$

Hadronic vacuum polarization: data-driven determination

$a_\mu^{\text{HVP-LO}} \cdot 10^{10}$, e^+e^-

$692.3(4.2)$	M. Davier et al., Eur. Phys. J. C 71, 1515 (2011)
$694.9(4.3)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$690.75(4.72)$	F. Jegerlehner, R. Szafron, Eur. Phys. J. C 71, 1632 (2011)
$688.07(4.14)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$693.1(3.4)$	M. Davier et al., Eur. Phys. J. C 77, 827 (2017)
$693.26(2.46)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$694.0(4.0)$	M. Davier et al., Eur. Phys. J. C 80, 341 (2020); Err. C 80, 410 (2020)
$692.78(2.42)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NLO}} \cdot 10^{10}$, e^+e^-

$-9.84(7)$	K. Hagiwara et al., J. Phys. G 38, 085003 (2011)
$-9.93(7)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)
$-9.82(4)$	A. Keshavarzi et al., Phys. Rev. D 97, 114025 (2018)
$-9.83(4)$	A. Keshavarzi et al., Phys. Rev. D 101, 014029 (2020)

$a_\mu^{\text{HVP-NNLO}} \cdot 10^{10}$, e^+e^-

$1.24(1)$	A. Kurz et al., Phys. Lett. B 734, 144 (2014)
$1.22(1)$	F. Jegerlehner, EPJ Web Conf. 166, 00022 (2018)

Hadronic vacuum polarization: lattice-QCD determination

- Several lattice-QCD results available at the time of the WP

A. Gérardin *et al.*, Phys. Rev. D 100, 014510 (2019)
C. T. H. Davies *et al.*, arXiv:1902.04223 [hep-lat]
E. Shintani and Y. Kuramashi, arXiv:1902.00885 [hep-lat]
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S. Borsanyi *et al.*, Phys. Rev. Lett. 121, 022002 (2018)
M. Della Morte *et al.*, JHEP 10, 020 (2017)

- Not competitive with data-driven determinations at the time of the WP

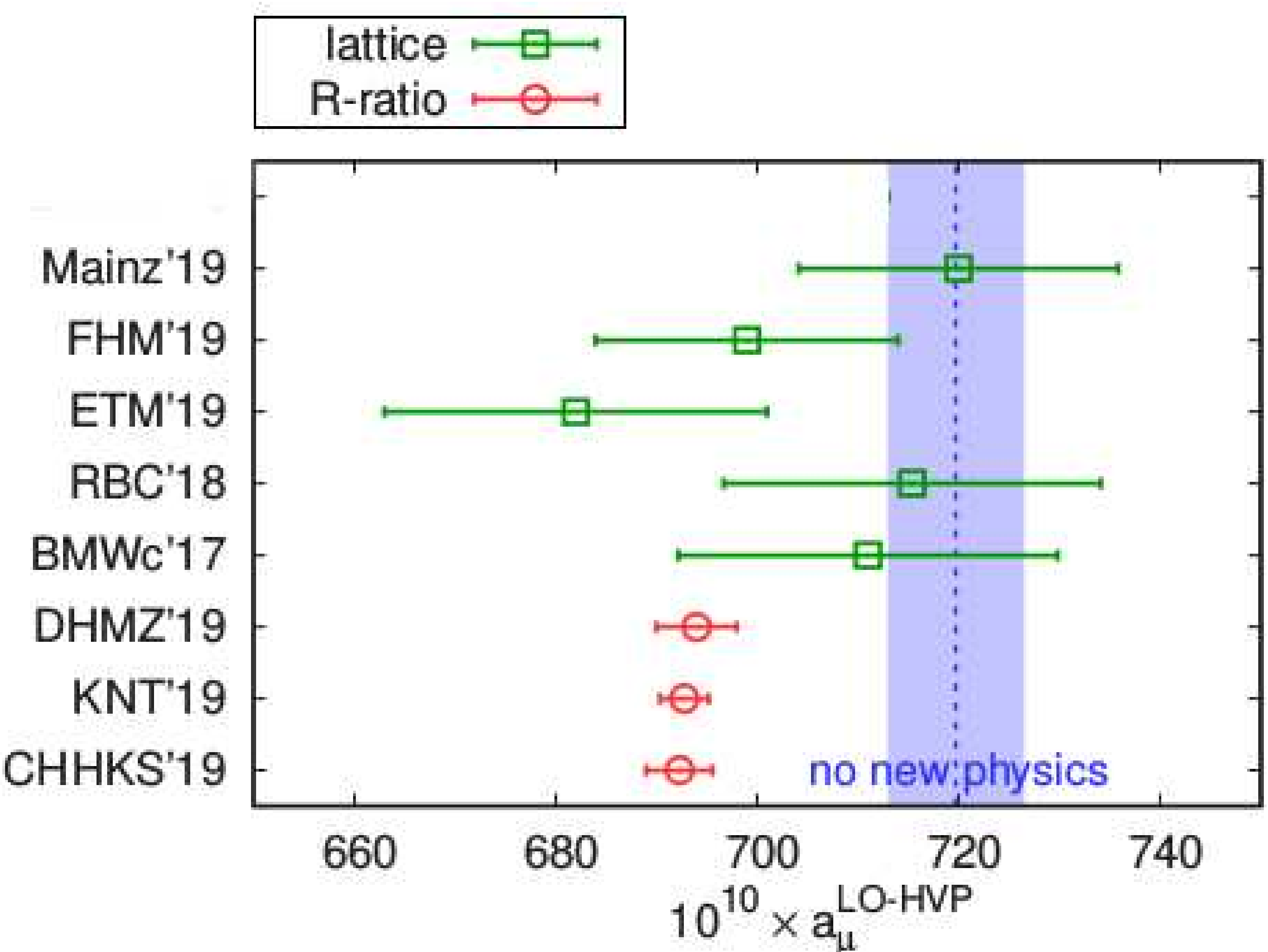
Hadronic vacuum polarization: White Paper Summary

T. Aoyama et al., Phys. Rep. 887, 1 - 166 (2020)

- Data evaluation:

$$a_\mu^{\mathrm{HVP;LO}} = 6931(40)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;NLO}} = -98.3(7)\cdot 10^{-11} \quad a_\mu^{\mathrm{HVP;LO}} = 12.4(1)\cdot 10^{-11}$$

- Lattice WA: $a_\mu^{\mathrm{HVP;LO}} = 7043(150)\cdot 10^{-11}$



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- Remember:

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}} - a_\mu^{\text{weak}} = 7188(41)\cdot 10^{-11}$$

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{th;WP}} \quad = \quad 251(59)\cdot 10^{-11} \;\; [4.2\sigma]$$

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$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{QED}} - a_\mu^{\text{weak}} = 7188(41) \cdot 10^{-11}$$

$$a_\mu^{\mathrm{exp;WA}} - a_\mu^{\mathrm{th;WP}} \quad = \quad 251(59) \cdot 10^{-11} \ [4.2\sigma]$$

- Does $a_\mu^{\mathrm{th;WP}} = a_\mu^{\mathrm{th;SM}}$ still hold today?

Standard Model Prediction

III Strong interactions

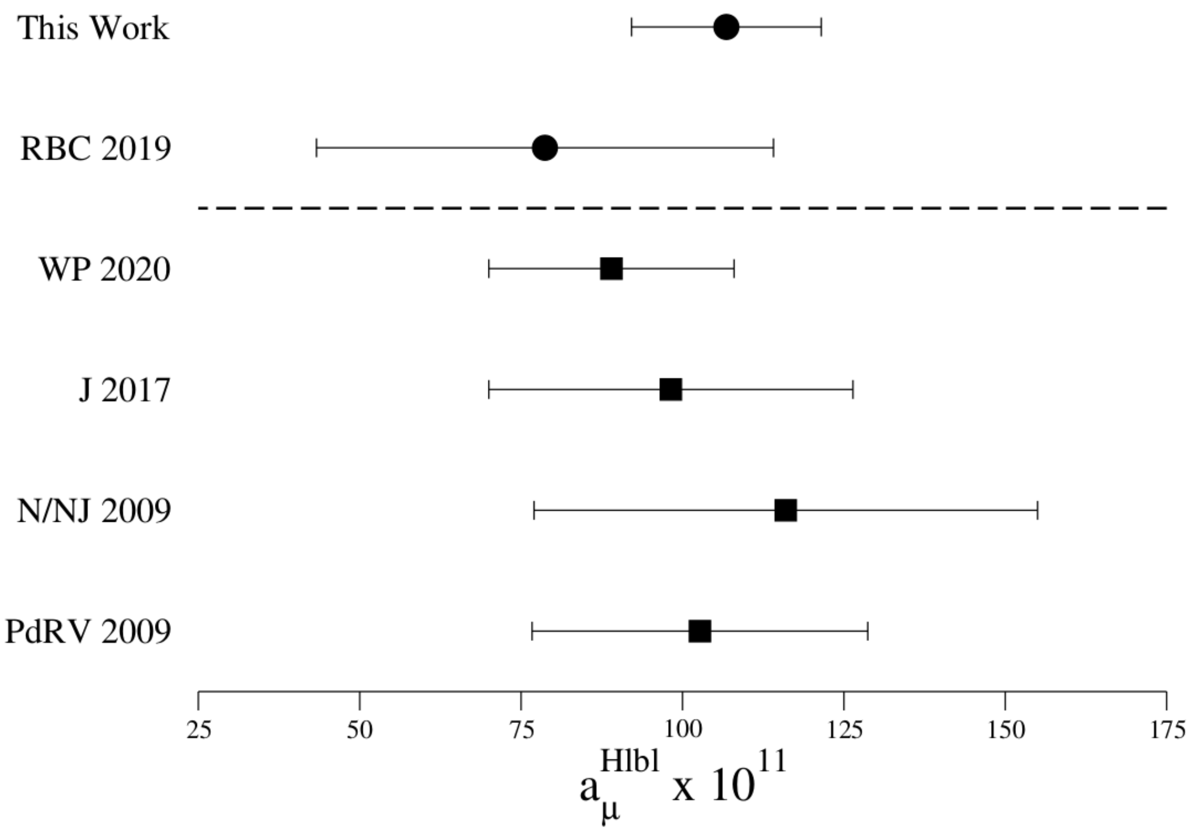
The post-WP era

Post-WP hadronic light-by-light

- New lattice-QCD result with 15% precision

$$a_\mu^{\mathrm{HLxL}} = 107.4(11.3)(9.2)\cdot 10^{-11}$$

E.-H. Chao et al., Eur. Phys. J. C 81, 651 (2021)



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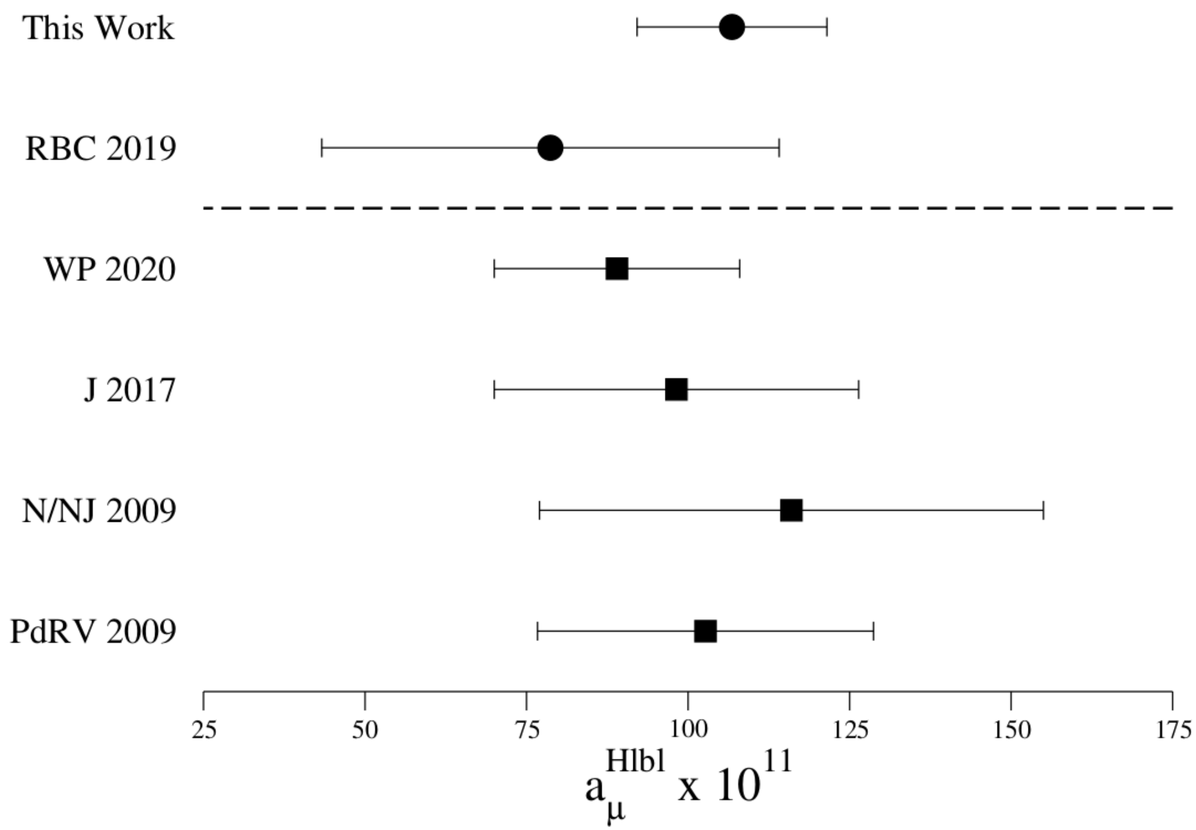


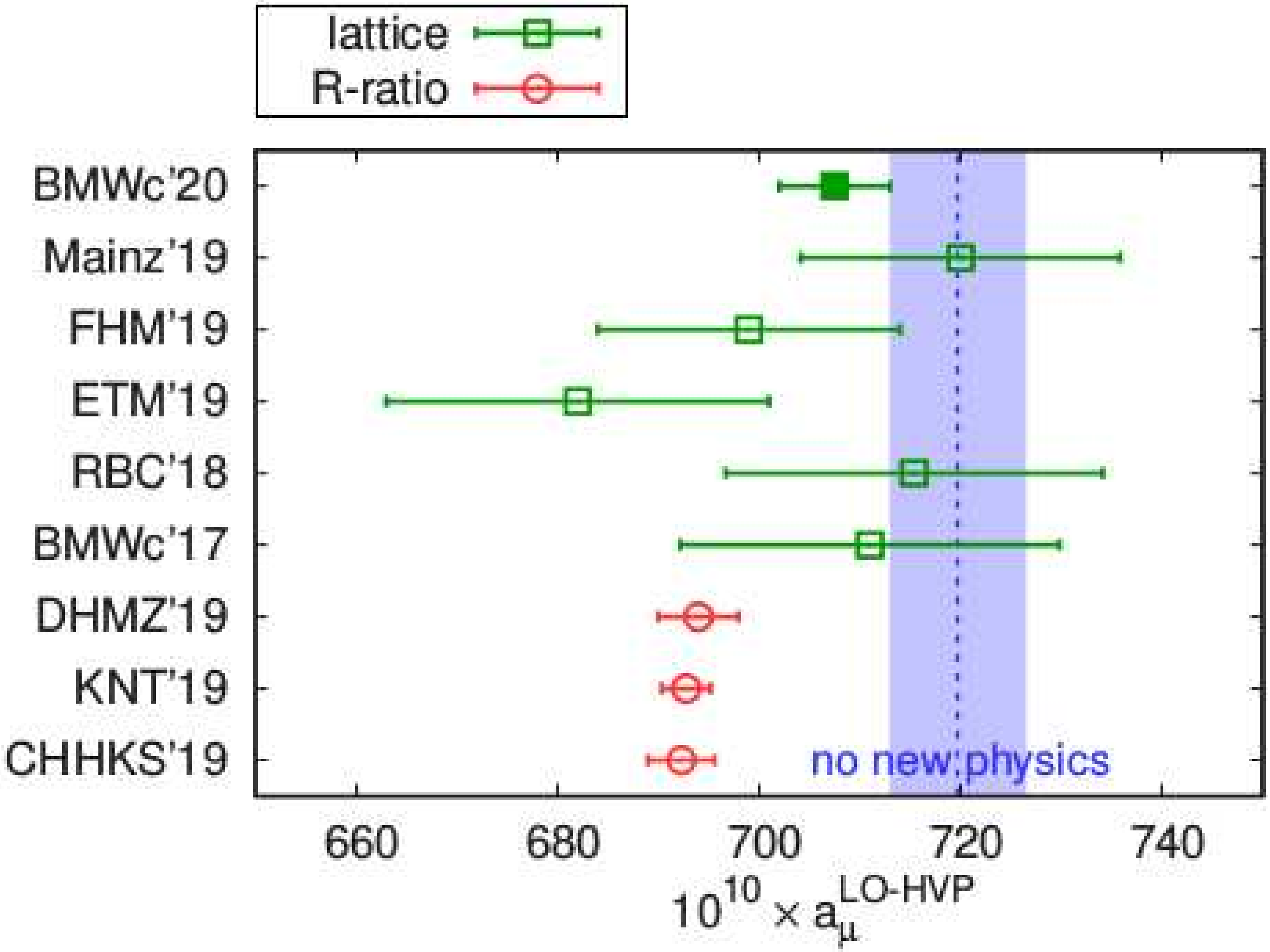
∼10% accuracy goal seems within reach

Post-WP hadronic vacuum polarization

- New lattice-QCD result with 0.8% precision

$$a_\mu^{\mathrm{HVP;LO}} = 7075(55) \cdot 10^{-11}$$

S. Borsanyi et al., Nature 593, 7857 (2021)



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S. Borsanyi et al., Nature 593, 7857 (2021)

Systematic effets (finite size, discretization,...) need to be scrutinized

Requires independent confirmation

Post-WP hadronic vacuum polarization

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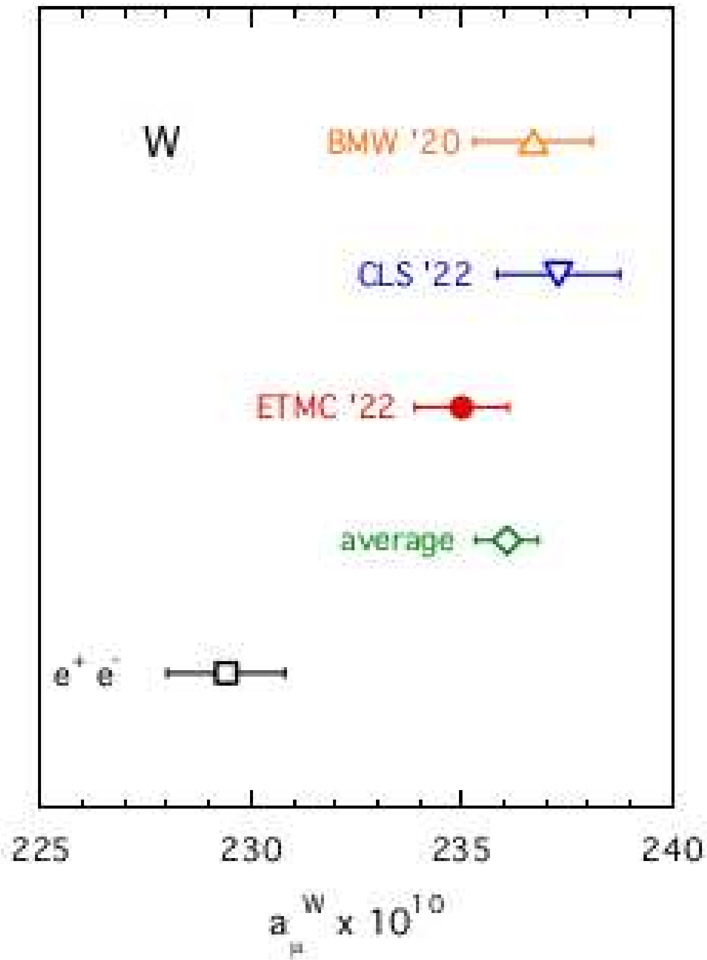
S. Borsanyi et al., Nature 593, 7857 (2021)



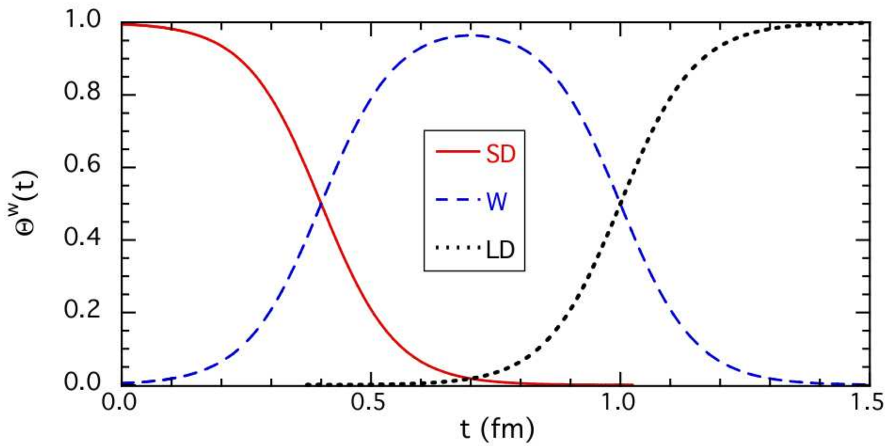
S. Borsanyi et al., Nature 593, 7857 (2021)

M. Cè et al., arXiv:2206.06582 [hep-lat]

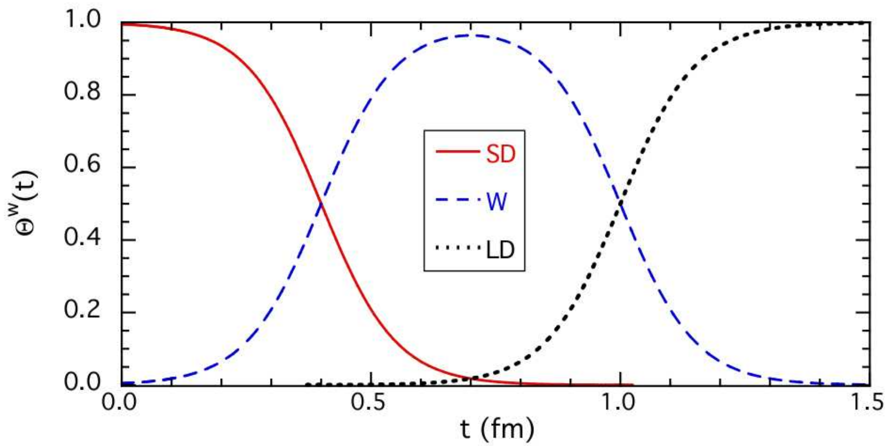
C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

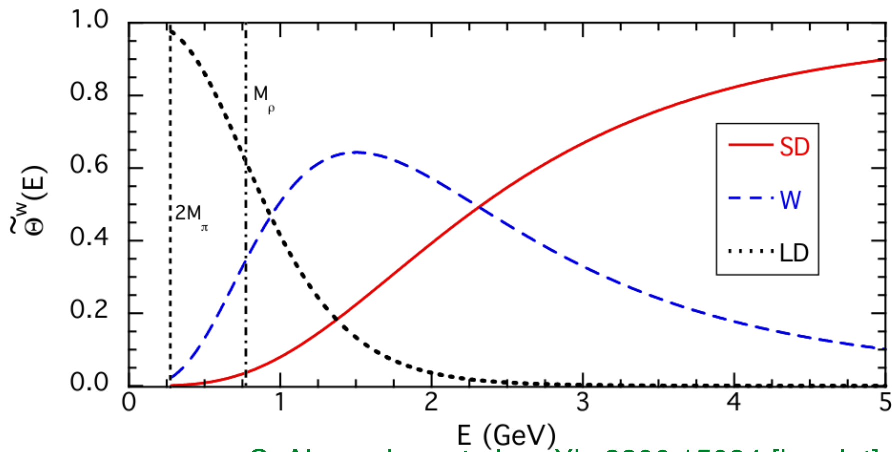
Several independent cross-checks for the intermediate window

a_μ^{IW}: $0.4\mathrm{fm} \leq t_{\mathrm{E}} \leq 1.0\mathrm{fm}$

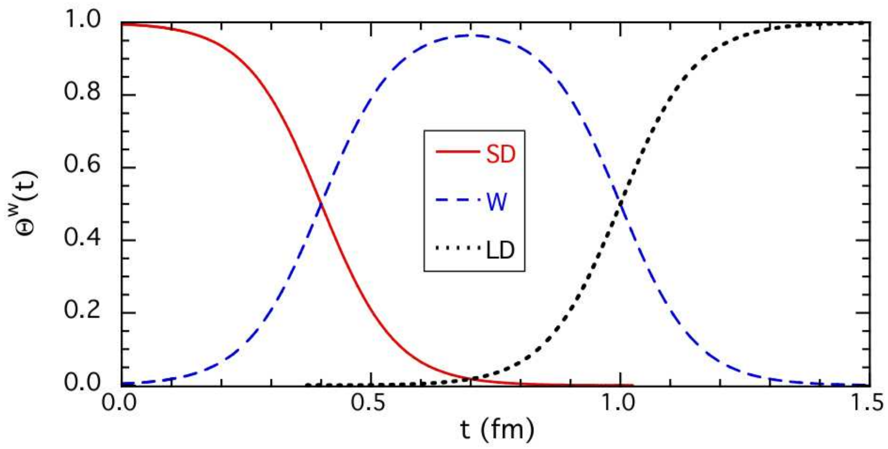


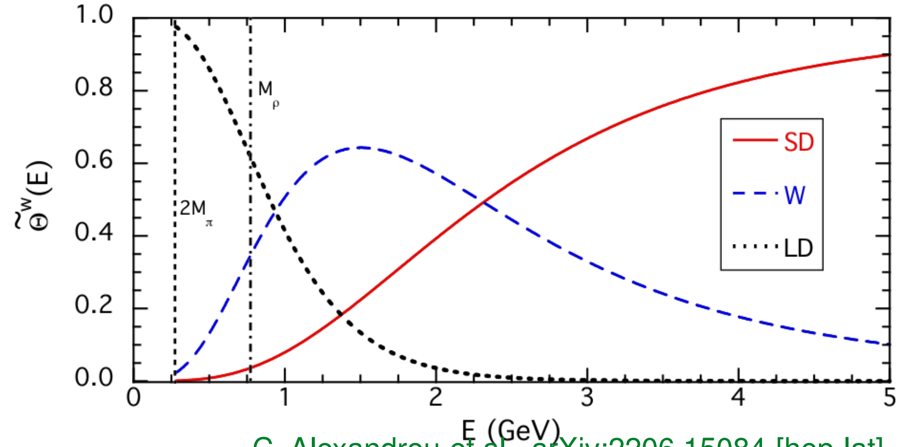
C. Alexandrou et al., arXiv:2206.15084 [hep-lat]





C. Alexandrou et al., arXiv:2206.15084 [hep-lat]





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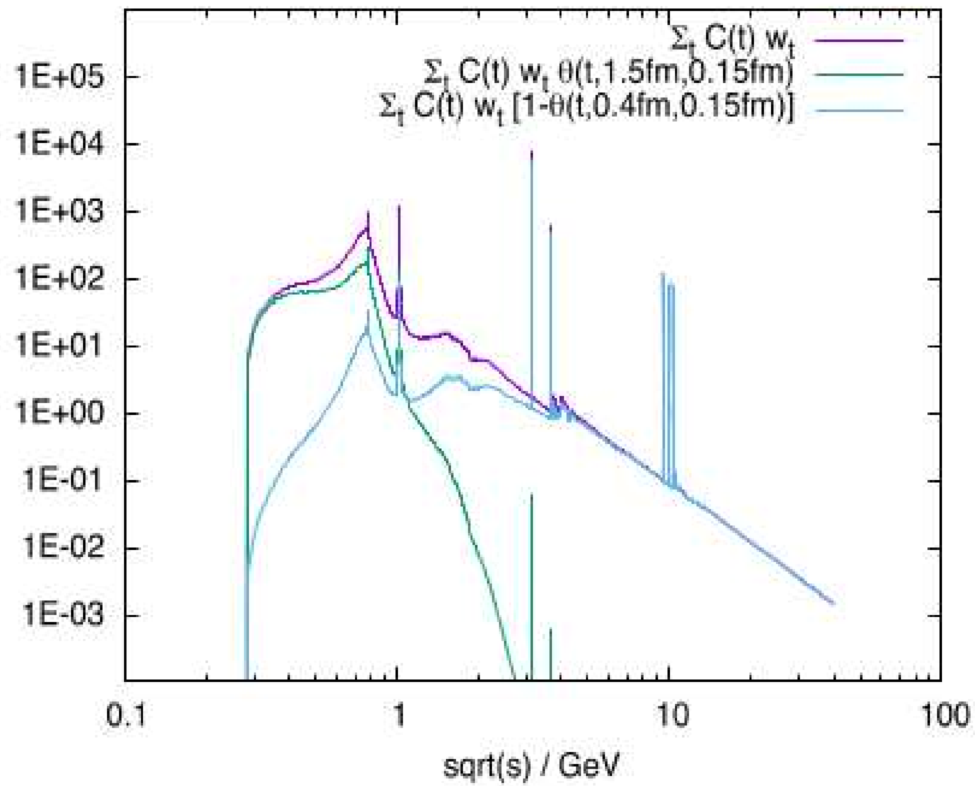
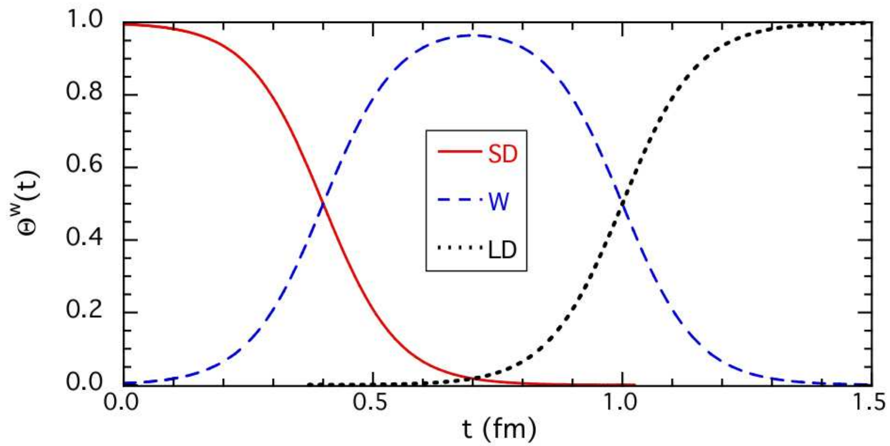
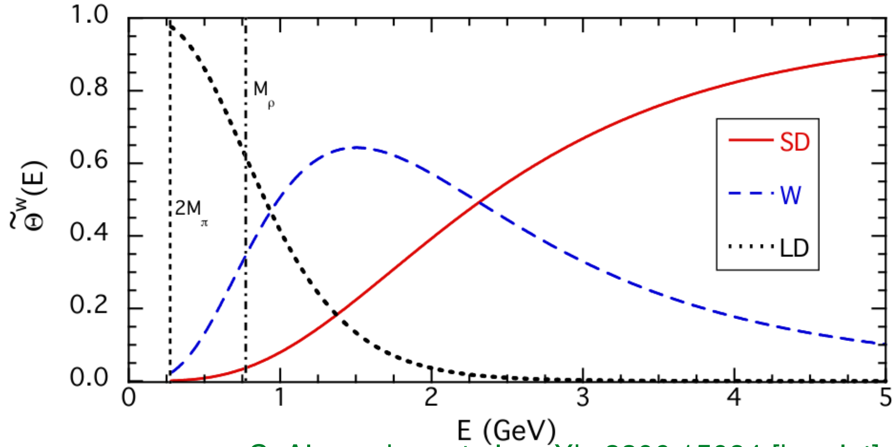






C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

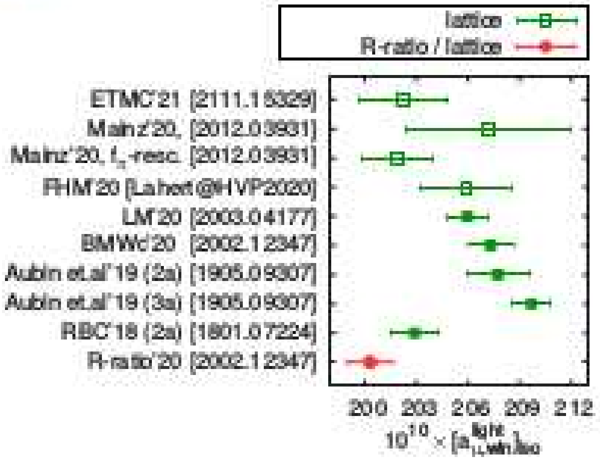
$$a_\mu^{\mathrm{SDW}} \sim 10\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

$$a_\mu^{\mathrm{IW}} \sim 30\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

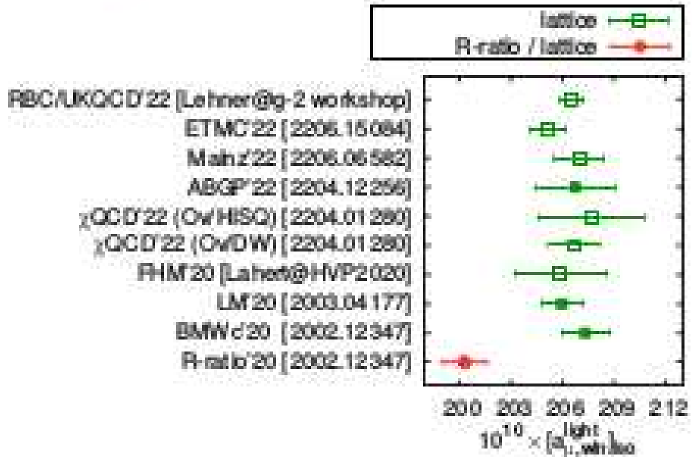
$$a_\mu^{\mathrm{LDW}} \sim 60\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

Window observable

- Status in 2021:

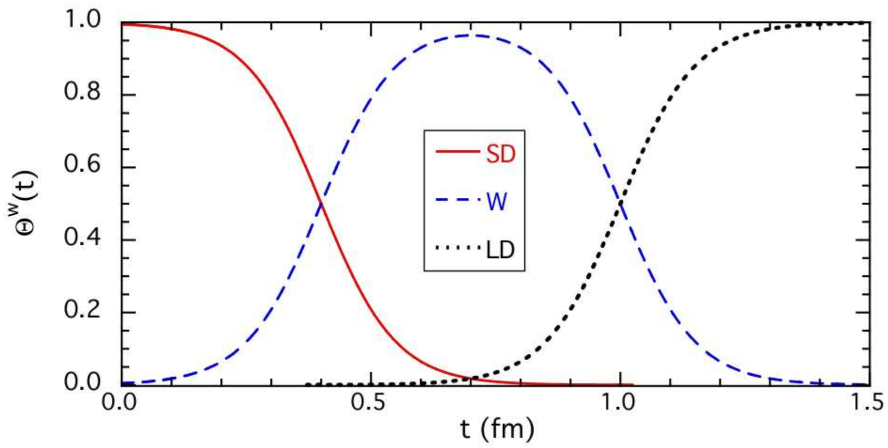


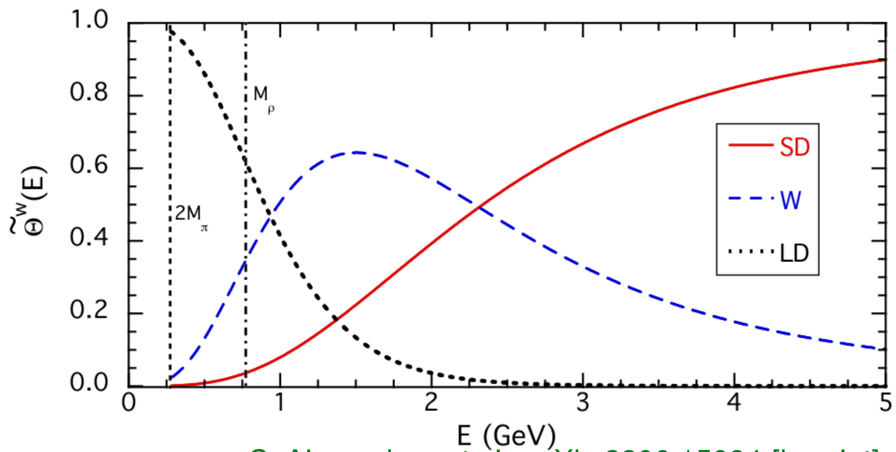
- Status now:



- Latest result from each group $\longrightarrow$ consensus within lattice community
- R-ratio vs lattice discrepancy has to be understood

B. Thot, FCCP 2022





C. Alexandrou et al., arXiv:2206.15084 [hep-lat]

$$a_\mu^{\mathrm{SDW}} \sim 10\% \text{ of } a_\mu^{\mathrm{HVP;LO}}$$

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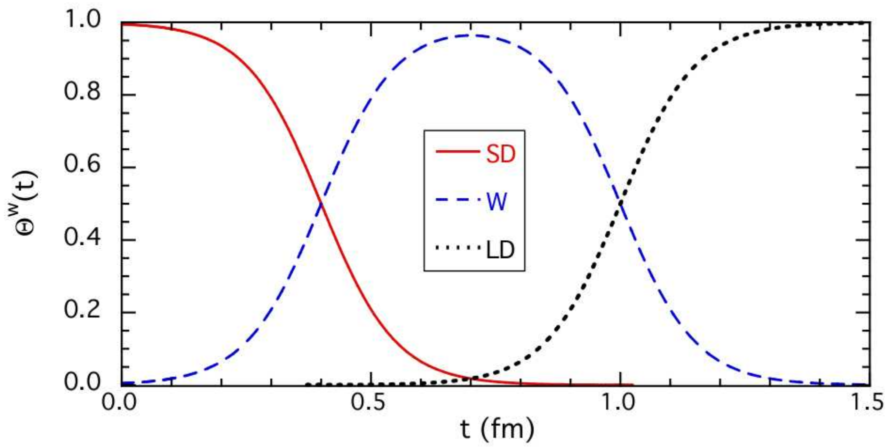
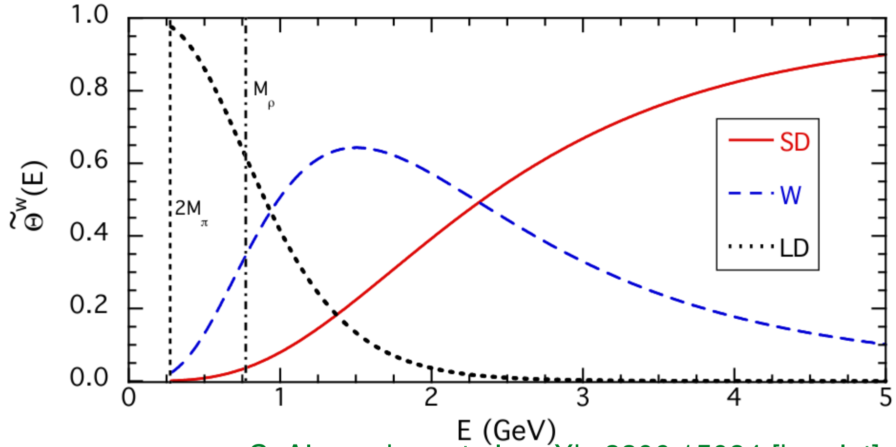
More windows can allow for finer studies of the discrepancy between LQCD and data-based determination

G. Colangelo, A. X. El-Khadra, M. Hoferichter, A. Keshavarzi, C. Lehner, P. Stoffer, T. Teubner, Phys. Lett. B 833, 137313 (2022)

Work in progress





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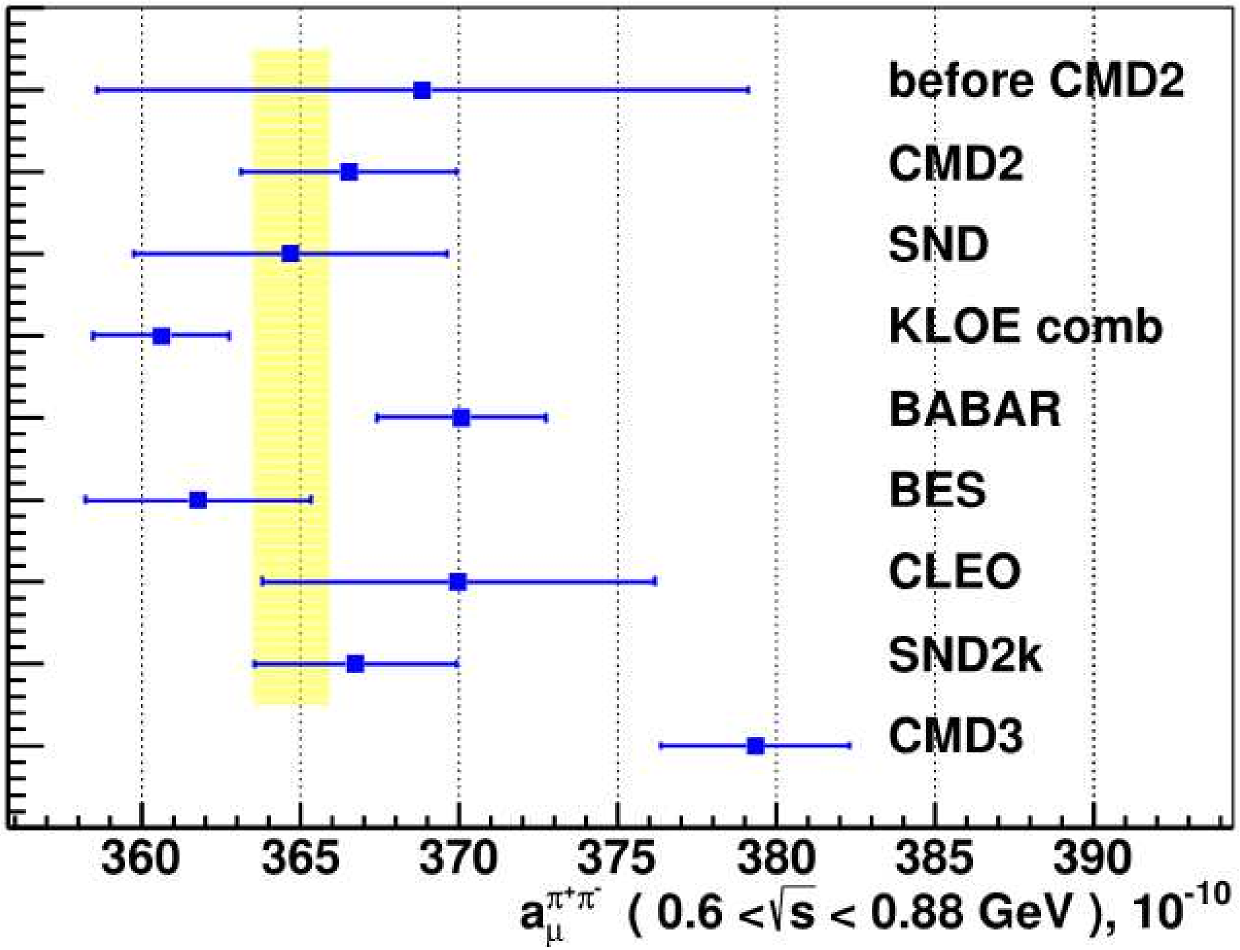
So far, no complete LQCD cross-check of BMWc result available

Post-WP hadronic vacuum polarization

- New experimental result from CMD-3 for $e^+e^- \to \pi^+\pi^-$ cross-section up to 1.2 GeV

F. V. Ignatov et al., [arXiv:2302.08834 [hep-ex]]



Summary - Conclusion - Outlook

- FNAL-E989 seems to work fine, BNL-E821 result confirmed

$$a_{\mu^+}^{\mathrm{E989}} = 116\,592\,040(54) \cdot 10^{-11} \quad [0.46\,\mathrm{ppm}]$$

B. Abi et al. [Muon g-2 Coll.], PRL 126, 120801 (2021)

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- Theory situation (as to June 2020) described in detail in the WP

$$a_{\mu}^{\mathrm{th;WP}} = 116\,591\,810(43) \cdot 10^{-11} \quad [0.35\,\mathrm{ppm}]$$

R. Aoyama et al., Phys. Rep. 887, 1 (2020)

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R. Aoyama et al., Phys. Rep. 887, 1 (2020)

- Discrepancy between the SM value given in the WP and the world-average experimental value

$$a_{\mu}^{\mathrm{exp;WA}} - a_{\mu}^{\mathrm{th;WP}} = 251(59) \cdot 10^{-11} \quad [4.2\sigma]$$

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R. Aoyama et al., Phys. Rep. 887, 1 (2020)

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$$a_{\mu}^{\mathrm{exp;WA}} - a_{\mu}^{\mathrm{th;WP}} = 251(59) \cdot 10^{-11} \quad [4.2\sigma]$$

- No obvious explanation within the SM for such a discrepancy

$$a_{\mu}^{\text{exp;WA}} - a_{\mu}^{\text{th;WP}} \sim \begin{cases} a_{\mu}^{\text{QED}}(\alpha^4) \\ 60 \cdot a_{\mu}^{\text{QED}}(\alpha^5) \\ 5 \cdot a_{\mu}^{\text{weak(2)}} \\ 3 \cdot a_{\mu}^{\text{HLxL}} \end{cases}$$

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R. Aoyama et al., Phys. Rep. 887, 1 (2020)

-

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- Recent (post-WP) determinations (lattice, exp) of $a_{\mu}^{\mathrm{HVP-LO}}$ make the situation quite confusing

Does $a_{\mu}^{\mathrm{th;WP}} = a_{\mu}^{\mathrm{SM}}$ still hold today?

- Possibility to measure HVP in the space-like region from μe scattering?

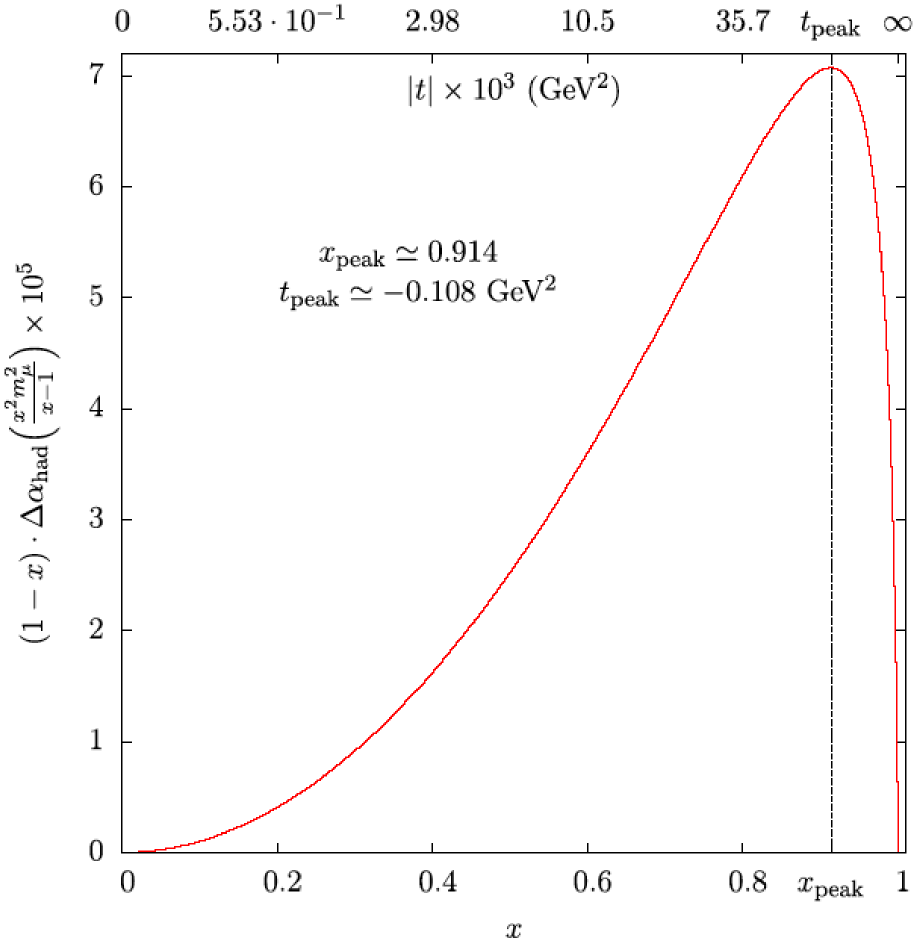
C. M. Carloni-Calame, M. Passera, L. Trentadue, G.Venanzoni, Phys. Lett. B 476, 325 (2015)

G. Abbiendi et al., Eur. Phys. J. C 77, 139 (2017)

- $a_\mu^{\rm HVP} - \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{\rm had}(-\frac{x^2}{1-x}m_\mu^2)$

$t = \frac{x^2 m_\mu^2}{x-1}\ \ 0 \le -t < +\infty\ 0 \le x < 1$

- $a_\mu^{\rm HVP}$ given by the integral
- measurement of $\Delta\alpha_{\rm had}$ in the space-like region
- contribution at small t enhanced
- a 0.3% error can be achieved in 2y of data taking with 1.3x10^7 $\mu/{\rm s}$ (CERN)



$\longrightarrow$ challenging (systematics)

$\longrightarrow$ inclusive measurement (more like lattice QCD)

$\longrightarrow$ MUonE coll. LoI CERN-LHCC-2017-009/CMS-TDR-014

$\longrightarrow$ test run (proof of concept, assessment of systematics,...) scheduled for the end of 2021

$\longrightarrow$ postponed

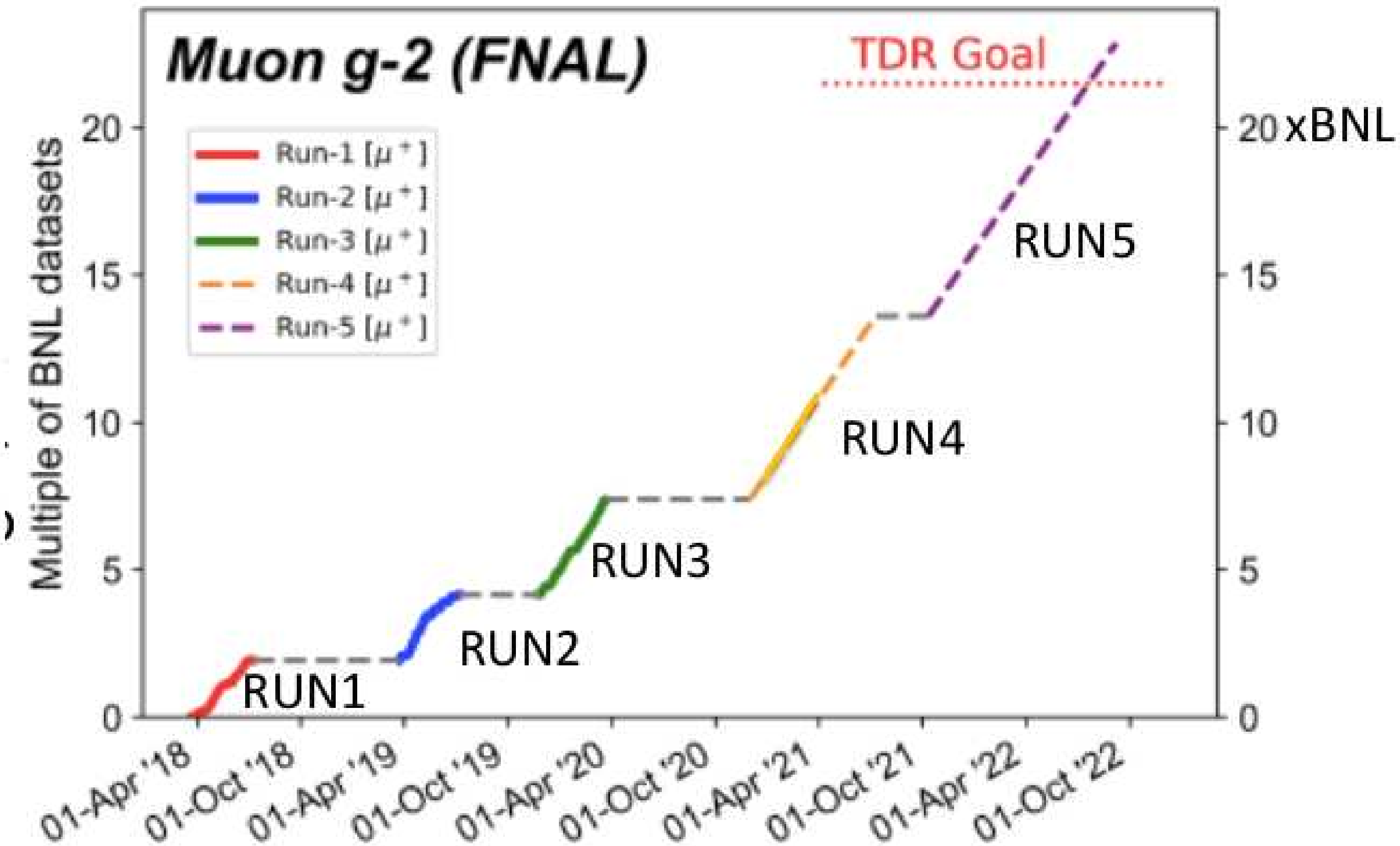
$\longrightarrow$ project starting in 202?, running time during LHC-Run3

- could the experiment be wrong?

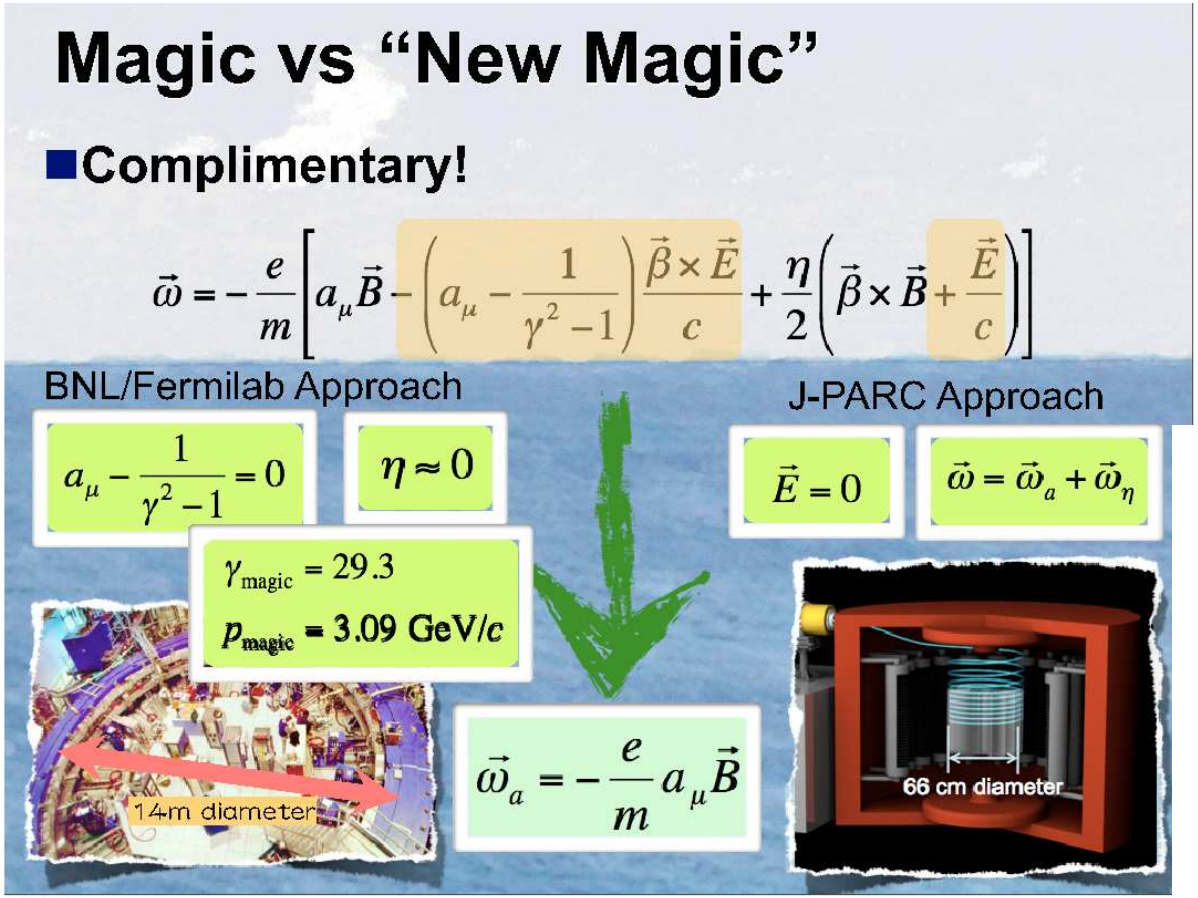
- could the experiment be wrong? We'll know more soon

$\longrightarrow$ only part of the data collected so far has been analysed

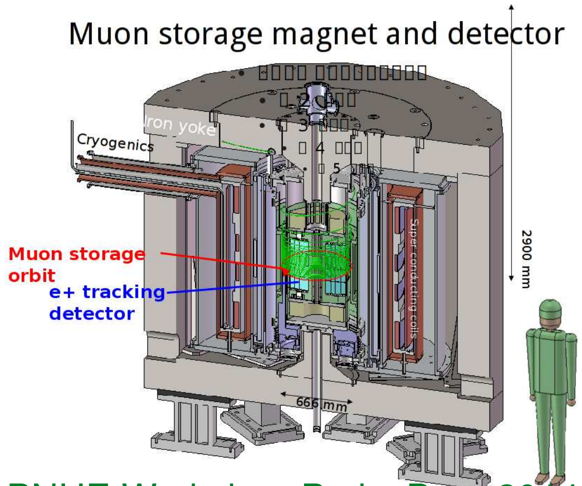
$\longrightarrow$ more habe been taken, to reach the accuracy goal of ~ 0.14ppm



• could the experiment be wrong? $\longrightarrow$ project to measure a_μ at J-PARC (E34)



N. Saito, LPNHE Workshop Paris, May 2012



T. Mibe, LPNHE Workshop Paris, Dec. 2014

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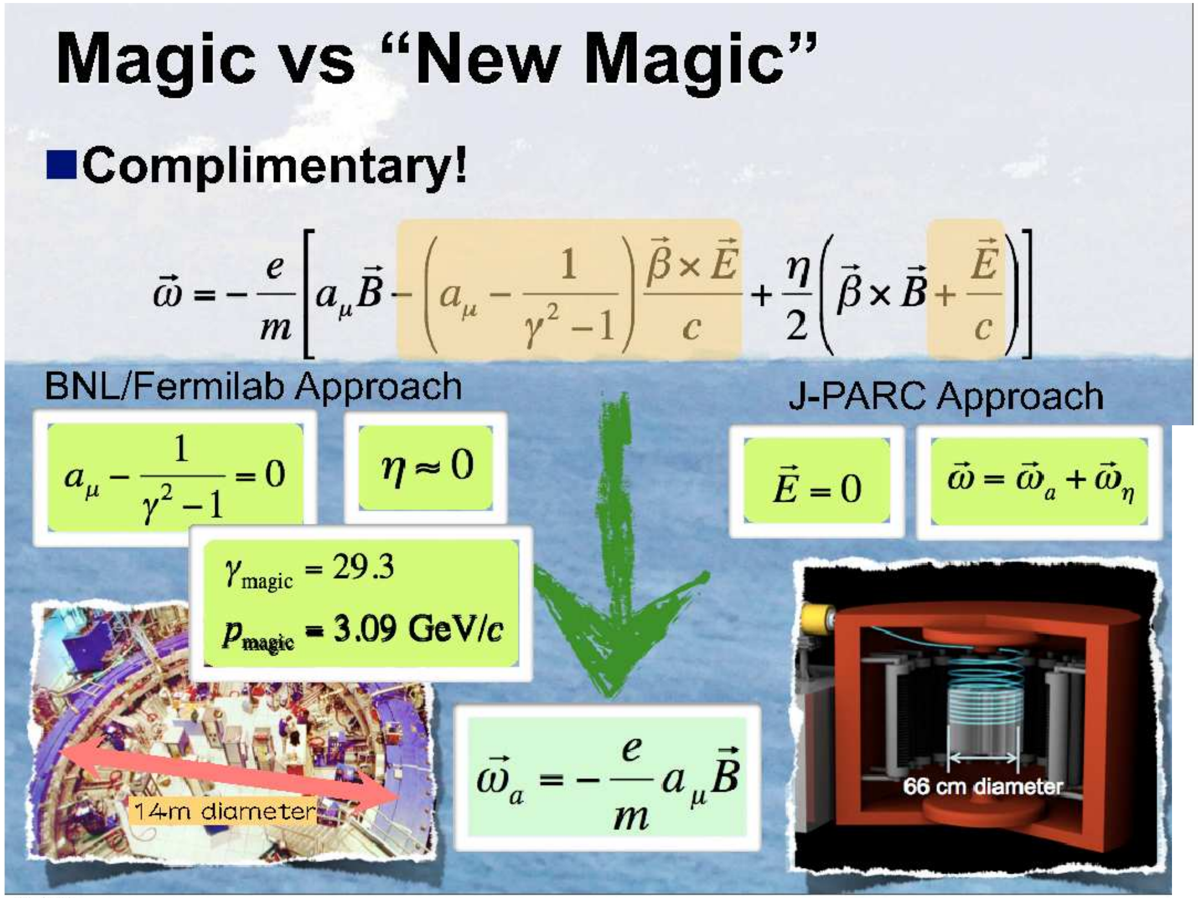

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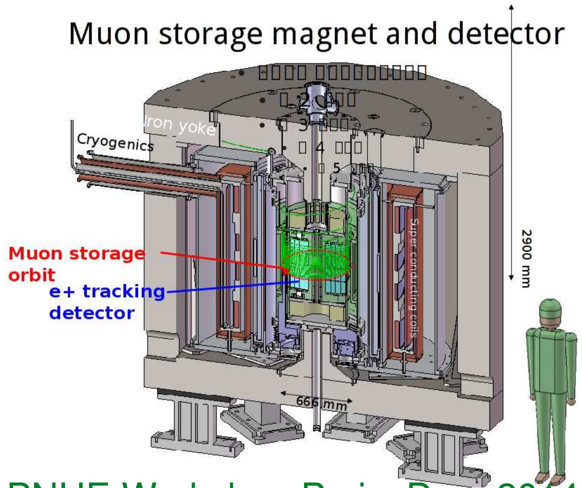


T. Mibe, LPNHE Workshop Paris, Dec. 2014

$\longrightarrow$ completely different set-up (uses slow muons)

$\longrightarrow$ never been tested before

$\longrightarrow$ data taking might start in 2025, accuracy goal 0.45ppm

A. Abe et al., PTEP 2019, 053C02 (2019)

- Testing the SM with a_e?

G. F. Giudice, P. Paradisi, M. Passera, JHEP 1211, 113 (2012)

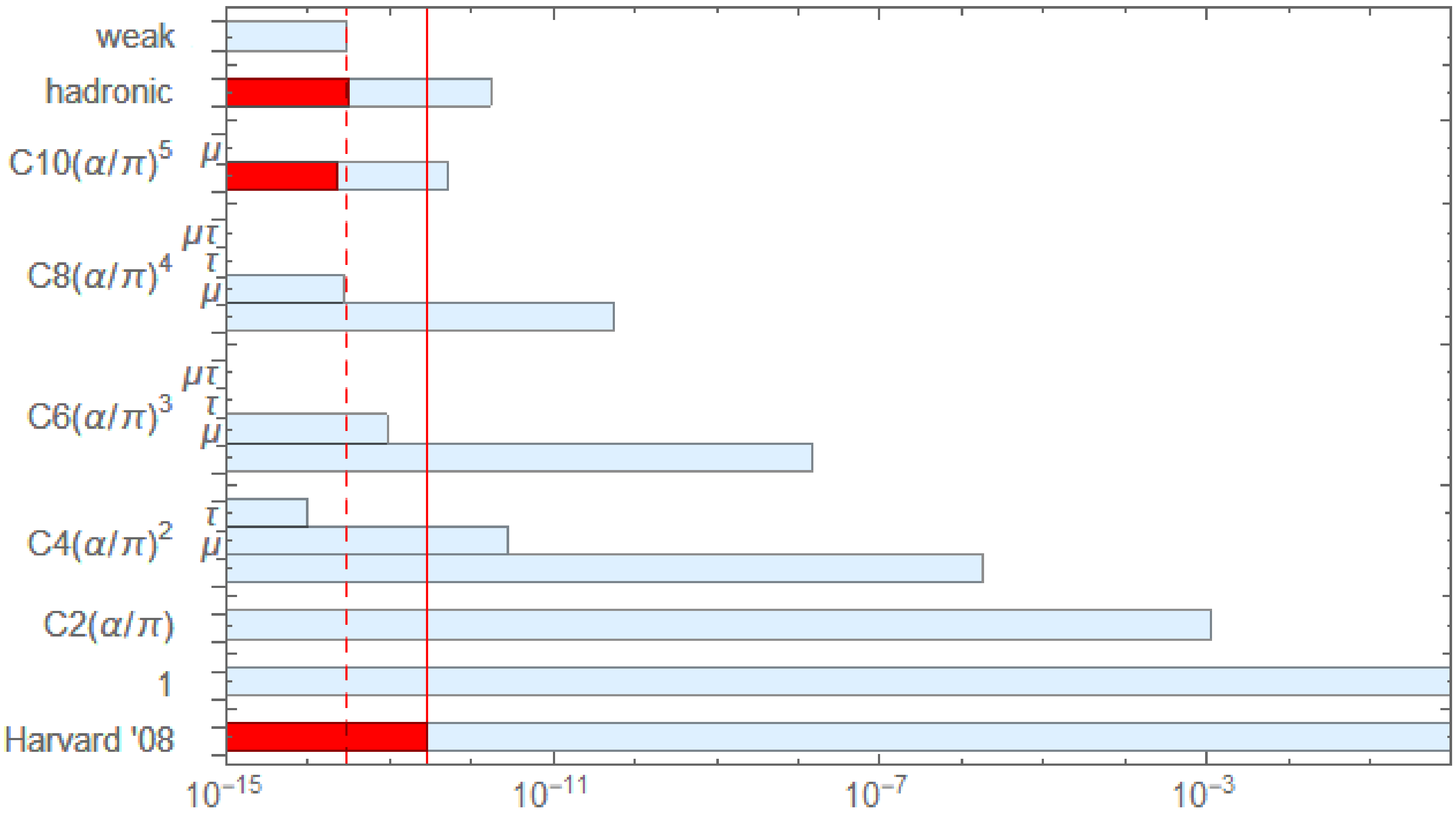
$\longrightarrow$ a_e one of the most precisely measured observable in particle physics

D. Hanneke, S. Forgwell, G. Gabrielse, PRL 100, 120801 (2008)

$\longrightarrow$ accuracy goal: from 0.24ppb to 0.02ppb (vs. 0.14ppm for a_μ)

G. Gabrielse, S. E. Fayer, T. G. Myers, X. Fan, Atoms 7, 45 (2019)

$\longrightarrow$ need to determine the fine structure constant at the same level of accuracy! (at least)



G. Gabrielse, S. E. Fayer, T. G. Myers, X. Fan, Atoms 7, 45 (2019)

Bibliography

M. Knecht, *The Anomalous magnetic moment of the muon: A Theoretical introduction*, Schladming Lectures 2003 Lect. Notes Phys. **629**, 37-84 (2004) [arXiv:hep-ph/0307239 [hep-ph]].

F. Jegerlehner and A. Nyffeler, *The Muon g-2*, Phys. Rept. **477**, 1-110 (2009) [arXiv:0902.3360 [hep-ph]].

K. Melnikov and A. Vainshtein, *Theory of the muon anomalous magnetic moment*, Springer Tracts Mod. Phys. **216**, 1-176 (2006).

F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon*, Springer Tracts Mod. Phys. **274**, pp.1-693 (2017), Springer (2017).

T. Aoyama *et al.*, *The anomalous magnetic moment of the muon in the Standard Model*, Phys. Rept. **887**, 1-166 (2020) [arXiv:2006.04822 [hep-ph]].

G. Colangelo *et al.*, *Prospects for precise predictions of a_μ in the Standard Model*, contribution to Snowmass 2021 [arXiv:2203.15810 [hep-ph]].