Sum rule techniques for flavour physics

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Part 3: Various applications

- nonlocal effects in $b \to s\ell\ell$ exclusive transitions
- CP violation in charmed meson decays
- B meson decays into dark matter

□ $B \to K^{(*)} \ell^+\ell^-$, the effective Hamiltonian

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)\Big|_{\mu\sim m_b}$$

- "direct" $b \to s\ell\ell$, $b \to s\gamma$ operators:

$O_{9(10)} = \frac{\alpha_{em}}{4\pi}[\bar{s}_L\gamma_\mu b_L]\ell\gamma^\mu(\gamma_5)\ell$, $\quad C_9(m_b) \simeq 4.4$, $\quad C_{10}(m_b) \simeq = -4.7$,

$O_{7\gamma} = -\frac{em_b}{8\pi^2}[\bar{s}\sigma_{\mu\nu}(1+\gamma_5)b]F^{\mu\nu}$, $\quad C_7(m_b) \simeq -0.3$

- quark-gluon operators, combined with quark e.m. current :

$O_1^{(c)} = [\bar{s}_L\gamma_\rho c_L][\bar{c}_L\gamma^\rho b_L]$, $\quad C_1(m_b) \simeq 1.1$

$O_2^{(c)} = [\bar{c}_L\gamma_\rho c_L][\bar{s}_L\gamma^\rho b_L]$, $\quad C_2(m_b) \simeq -0.25$

$O_{8g} = -\frac{m_b}{8\pi^2}\bar{s}\sigma_{\mu\nu}(1+\gamma_5)bG^{\mu\nu}$, $\quad C_8(m_b) \simeq 0.2$

O_{3-6} - quark-penguin operators , $\quad C_{3,4,5,6} < 0.03$

- the $\sim V_{ub}V_{us}^*$ part neglected

□ $B \to K^{(*)} \ell^+ \ell^-$ decay amplitude

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i \langle K^{(*)}\ell^+\ell^- \mid O_i \mid B\rangle$$

- hadronic matrix elements:

$$A(B \to K^{(*)}\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \frac{\alpha_{em}}{2\pi} \Bigg[(\bar{\ell}\gamma^\rho\gamma_5\ell)\, C_{10} \langle K^{(*)}|\bar{s}\gamma_\rho(1-\gamma_5)b|B\rangle$$

$$+(\bar{\ell}\gamma^\rho\ell)\Big(C_9 \langle K^{(*)}|\bar{s}\gamma_\rho b|B\rangle + C_7 \frac{2m_b}{q^2} q^\nu \langle K^{(*)}|\bar{s}i\sigma_{\nu\rho}(1+\gamma_5)b|B\rangle$$

$$+\frac{8\pi^2}{q^2} \sum_{i=1,2,\dots,6,8} C_i\, \mathcal{H}_i^\rho \Big)\Bigg] \quad (1)$$

- include $B \to K^{(*)}$ form factors and nonlocal hadronic matrix elements

$$\boxed{\mathcal{H}_i^\rho(q,p) = \langle K^{(*)}(p)| i\int d^4x\, e^{iqx}\, T\{j^\rho_{em}(x), O_i(0)\}|B(p+q)\rangle},$$

- hereafter, consider the kaon final state, $B \to K\ell^+\ell^-$

□ Hadronic input in $B \to K\ell\ell$

$$A(B \to K\ell^+\ell^-) = \frac{G_F}{\sqrt{2}} \frac{\alpha_{em}}{\pi} V_{tb} V_{ts}^* \Bigg[\bar{\ell}\gamma_\mu \ell \, p^\mu \Bigg(C_9 f_{BK}^+(q^2)$$

$$+ \frac{2(m_b + m_s)}{m_B + m_K} C_7^{eff} f_{BK}^T(q^2) + \sum_{i=1,2,\ldots,6,8} C_i \, \mathcal{H}_i^{(BK)}(q^2) \Bigg) + \bar{\ell}\gamma_\mu\gamma_5 \ell \, p^\mu C_{10} f_{BK}^+(q^2) \Bigg]$$

- the leading short-distance contributions determined by $B \to K$ form factors calculable in QCD
- remaining nonlocal matrix elements:

$$\mathcal{H}_i^{(BK)}(q^2) \sim \langle K(p)| i \int d^4x \, e^{iqx} \, T\{ j_{em}^\rho(x), O_i(0)\} |B(p+q)\rangle$$

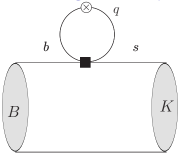
$j_{em}^\rho = \sum_{q=u,d,s,c,b} Q_q \bar{q}\gamma^\rho q$, the hierarchy $O_i = O_{1,2}^{(c)}, \; O_{8g}, O_{3,4,5,6}^{(q)}, O_{1,2}^{(u)}$

$\Rightarrow$ corrections to fundamental short-distance coeff.:

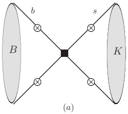
$C_9 \to C_9 + \sum_i \Delta C_9^{(BK,i)}(q^2)$ (q^2- and process-dependent)

- have to be estimated one by one

□ Diagrams (topologies) of nonlocal matrix elements at $q^2 \ll 4m_c^2$

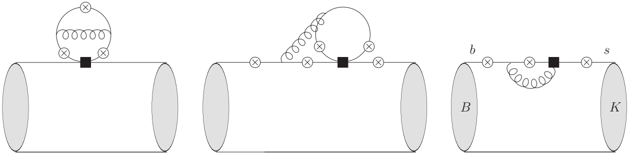


LO (factor.)

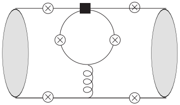


weak annihilation

$\otimes$ -virtual photon

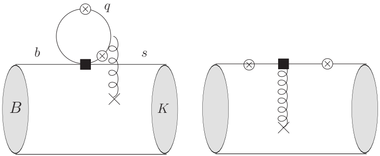


NLO (factor.) H. Asatryan, C. Greub st al. [hep-ph/0109140]



spectator (nonfactor.)

M.Beneke, Th.Feldmann, D.Seidel (2001)



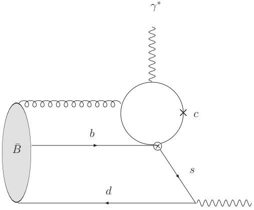
soft (low virtuality) gluons(nonfactor.)

□ LCSR for the soft-gluon hadronic matrix element

A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, 1006.4945 [hep-ph] (2010)

- soft gluon emission from charm loop reduced to an effective nonlocal operator

$$\widetilde{\mathcal{O}}_\mu(q) = \int d\omega \, I_{\mu\rho\alpha\beta}(q, m_c, \omega) \bar{s}_L \gamma^\rho \delta[\omega - \frac{(in_+\mathcal{D})}{2}] \widetilde{G}_{\alpha\beta} b_L \ ,$$



- the correlation function in terms of three-partilce B-meson DAs:

$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = i \int d^4y e^{ip\cdot y} \langle 0 | T\{ j^K_\nu(y) \widetilde{\mathcal{O}}_\mu(q)\} | B(p+q)\rangle \ ,$$

- hadronic dispersion relation in the kaon channel

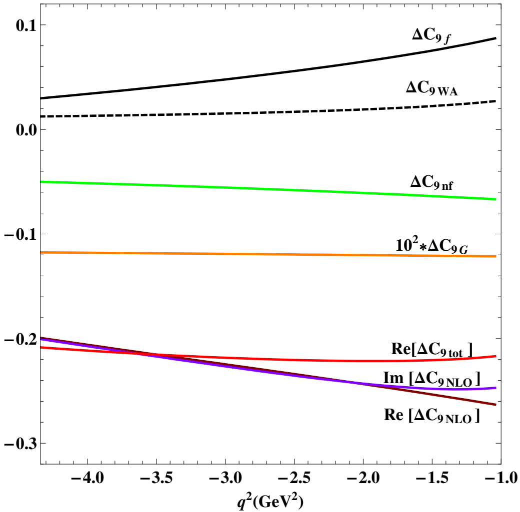
$$\mathcal{F}^{(B\to K)}_{\nu\mu}(p,q) = \frac{i f_K p_\nu}{m_K^2 - p^2}[(p\cdot q) q_\mu - q^2 p_\mu] \tilde{A}(q^2) + \int_{s_h}^{\infty} ds \ \frac{\tilde{\rho}_{\mu\nu}(s,q^2)}{s - p^2}$$

- new development: direct calculation, subtleties concerning LCDAs

M.-L. Piscopo, A.Rusov, work in progress

□ Influence of nonlocal effects in terms of $\Delta C_9(q^2 < 0)$

A.K., Th. Mannel and Yu-M. Wang, 1211.0234 [hep-ph] (2012)



□ Accessing the timelike q^2 region

- analyticity of the hadronic matrix element in $q^2 \oplus$ unitarity

$\Rightarrow$ hadronic dispersion relation:

$$\mathcal{H}_i^{(BK)}(q^2) = \mathcal{H}_i^{(BK)}(0) + q^2 \Big[\sum_{\psi=J/\psi,\psi(2S),..} \frac{f_\psi A_{B\psi K}}{m_\psi^2(m_\psi^2 - q^2 - im_\psi \Gamma_\psi^{tot})} + \int_{4m_D^2}^{\infty} ds \frac{\rho(s)}{s(s-q^2-i\epsilon)} \Big]$$

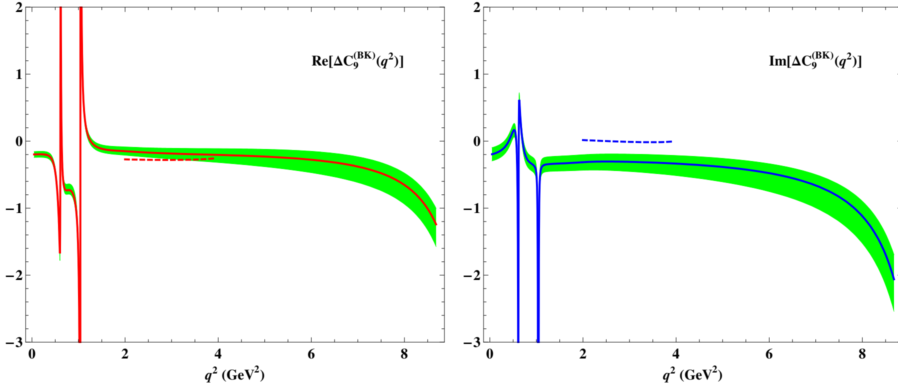
- the residues $|A_{B\psi K}|$ and $|f_\psi|$ from $BR(B \to \psi K)$, $BR(\psi \to \ell^+ \ell^-)$

- FSI phase in $A(B \to \psi K)$, (Im part in $(p+q)^2$) will appear , dual to perturbative gluon effects

- match the OPE calculation of $\mathcal{H}_i^{(BK)}(q^2)$ to the hadronic representation at small and negative q^2 and fit free parameters

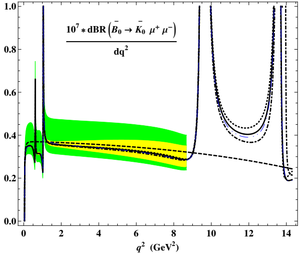
- use hadronic representation everywhere at $q^2 > 0$

□$\Delta C_9(q^2)$ below J/ψ region



the red (blue) solid curve corresponds to the Re (Im) part obtained from the hadronic dispersion relation, fitted to the QCD calculation at q2 < 0 (central input, default parametrization). The shaded areas indicate the uncertainties. The dashed curves correspond to the prediction of QCDF.

□ $d\mathrm{BR}(B \to K\mu^+\mu^-)/dq^2$

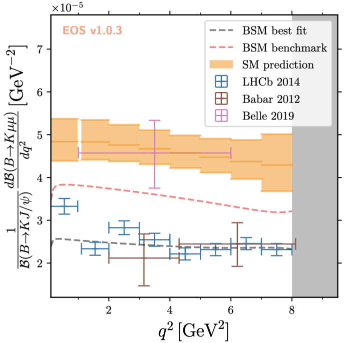


- from 1211.0234 [hep-ph]]

solid (dotted) lines - central input, default (alternative) parametrization for the dispersion integrals.

long-dashed line -the width calculated without nonlocal hadronic effects.

The green (yellow) shaded area indicates the uncertainties including (excluding) the one from the $B \to K$ FF normalization.



- from 2206.03797 [hep-ph]]
- another B -anomaly ??

Literature on nonlocal hadronic matrix elements

- charm-loop effects in $B \to K^{(*)}\ell\ell$

  A.K., Th. Mannel, A. Pivovarov and Yu-M. Wang, 1006.4945 [hep-ph] (2010)

- complete analysis of $B \to K\ell\ell$

  A.K., Th. Mannel and Yu-M. Wang, 1211.0234 [hep-ph] (2012)

- nonlocal effeects for $B \to \pi\ell\ell$, including CP violation

  C. Hambrock., A.K., A.Rusov, 1506.07760 [hep-ph] (2015)

- update of $B \to K\ell\ell$, also $B_s \to \bar{K}\ell\ell$, $B \to \pi\ell\ell$

  A.K., A.Rusov, 1703.04765 [hep-ph] (2017)

- most recent analysis of $B \to K^{(*)}\ell\ell$, $B_s \to \phi\ell\ell$

  N. Gubernari, M. Reboud, D. van Dyk and J. Virto, 2206.03797 [hep-ph].

□ CP asymmetry in $D^0 \to \pi^+\pi^-$, $D^0 \to K^+K^-$ decays

AK, A.Petrov, arXiv:1706.07780 [hep-ph].

- The direct *CP* asymmetry:

$$a_{CP}^{dir}(h^+h^-) = \frac{\Gamma(D^0 \to h^+h^-) - \Gamma(\bar{D}^0 \to h^-h^+)}{\Gamma(D^0 \to h^+h^-) + \Gamma(\bar{D}^0 \to h^-h^+)}$$

- to obtain $a_{CP}^{dir}(h^+h^-)$ in SM, it is necessary and sufficient to calculate a single hadronic matrix element ("penguin amplitude").

- penguin amplitudes from QCD Light-cone sum rules (LCSR)

□ Realization of direct $\not{CP}$ in $D^0 \to h^+ h^-$ decays

- Single Cabibbo-suppressed decays satisfy the conditions for direct $\not{C}$P:

$$A(D^0 \to h^+ h^-) = A_h^{(1)} e^{i\delta_1} e^{i\phi_1} + A_h^{(2)} e^{i\delta_2} e^{i\phi_2},$$
$$A(\bar{D}^0 \to h^- h^+) = A_h^{(1)} e^{i\delta_1} e^{-i\phi_1} + A_h^{(2)} e^{i\delta_2} e^{-i\phi_2},$$

the decay amplitude with two parts, weak $\phi_1 \neq \phi_2$ and strong $\delta_1 \neq \delta_2$ phases

- the asymmetry $$a_{CP}^{dir}(h^+ h^-) \sim \frac{A_h^{(1)}}{A_h^{(2)}} \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2).$$

- in more detail:

$$A(D^0 \to \pi^+ \pi^-) = \lambda_d \langle \pi^+ \pi^- | \mathcal{O}^d | D^0 \rangle + \lambda_s \langle \pi^+ \pi^- | \mathcal{O}^s | D^0 \rangle \,,$$
$$A(D^0 \to K^+ K^-) = \lambda_s \langle K^+ K^- | \mathcal{O}^s | D^0 \rangle + \lambda_d \langle K^+ K^- | \mathcal{O}^d | D^0 \rangle \,,$$

- SCS four-quark operators, a compact notation

$$H_{eff} = \underbrace{V_{ud} V_{cd}^*}_{\lambda_d} \underbrace{\frac{G_F}{\sqrt{2}} \Big[c_1 \, (\bar{u} \Gamma_\mu d) \, (\bar{d} \Gamma^\mu c) + c_2 \, (\bar{d} \Gamma_\mu d) \, (\bar{u} \Gamma^\mu c) \Big]}_{\mathcal{O}^d} + \{d \to s\}$$

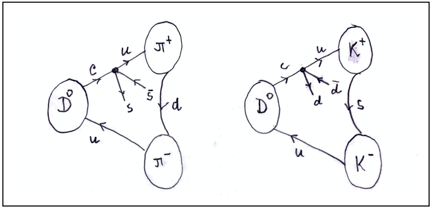
neglected $O_{i \geq 3}$ with $c_i \ll c_{1,2}$

“Penguin” amplitudes

- the "penguin" hadronic matrix elements:
$\mathcal{P}^s_{\pi\pi} = \langle \pi^+\pi^- | \mathcal{O}^s | D^0 \rangle \,, \quad \mathcal{P}^d_{KK} = \langle K^+K^- | \mathcal{O}^d | D^0 \rangle \,,$

- a generic definition: in a “pengiun” hadronic matrix element
  - there is a $\bar{q}q$ in the four-quark operator
  - no flavour q in the valence content of the hadrons,

  otherwise no relation to "topological (quark flow)" diagrams



- definition valid only if we use a method in which mesons or their intepolating currents have a definite valence content.

□ Penguins in the direct CP-asymmetry

- CKM unitarity in SM: $\lambda_d = -(\lambda_s + \lambda_b)$, $\lambda_b = (V_{ub}V^*_{cb}) \ll \lambda_{s,d}$,
- separating the $O(\lambda_b)$ contribution with CP-phase

$$A(D^0 \to \pi^+\pi^-) = -\lambda_s \mathcal{A}_{\pi\pi}\left\{1 + \frac{\lambda_b}{\lambda_s}\left(1 + r_\pi \exp(i\delta_\pi)\right)\right\},$$

$$A(D^0 \to K^+K^-) = \lambda_s \mathcal{A}_{KK}\left\{1 - \frac{\lambda_b}{\lambda_s} r_K \exp(i\delta_K)\right\},$$

the notation: $\frac{\lambda_b}{\lambda_s} \equiv r_b e^{-i\gamma}$, $r_b = \left|\frac{V_{ub}V^*_{cb}}{V_{us}V^*_{cs}}\right|$.

$$\mathcal{A}_{\pi\pi} = \langle\pi^+\pi^-|\mathcal{O}^d|D^0\rangle - \langle\pi^+\pi^-|\mathcal{O}^s|D^0\rangle\,, \quad \mathcal{A}_{KK} = \langle K^+K^-|\mathcal{O}^s|D^0\rangle - \langle K^+K^-|\mathcal{O}^d|D^0\rangle\,,$$

$$r_\pi = \left|\frac{\mathcal{P}^s_{\pi\pi}}{\mathcal{A}_{\pi\pi}}\right|, \quad r_K = \left|\frac{\mathcal{P}^d_{KK}}{\mathcal{A}_{KK}}\right|, \quad \delta_{\pi(K)} = arg[\mathcal{P}^{s(d)}_{\pi\pi(KK)}] - arg[\mathcal{A}_{\pi\pi(KK)}]$$

- a "clean" observable (after time-integration)

$$\Delta a^{dir}_{CP} = a^{dir}_{CP}(K^+K^-) - a^{dir}_{CP}(\pi^+\pi^-) = -2r_b \sin\gamma(r_K \sin\delta_K + r_\pi \sin\delta_\pi) + O(r_b^2)\,.$$

- approximation: $-\lambda_s\mathcal{A}_{\pi\pi} \simeq A(D^0 \to \pi^+\pi^-)$, $\lambda_s\mathcal{A}_{KK} \simeq A(D^0 \to K^+K^-)$
- a calculation of $\mathcal{P}^s_{\pi\pi}$ and $\mathcal{P}^d_{KK}$ is necessary and sufficient, combined with $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$ extracted from experiment

□ Calculation of the “penguin” matrix elements

AK, A.Petrov, arXiv:1706.07780 [hep-ph].

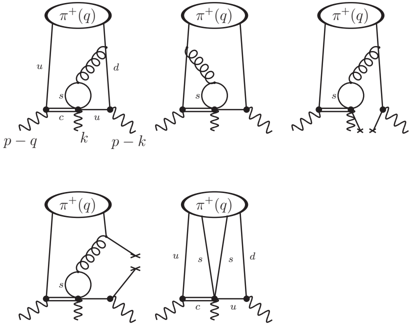
- the method formulated and used earlier for the $B \to \pi\pi$ decays

  AK, Nucl.Phys. **B605**, 558, (2001),

  AK, T. Mannel and B. Melic, Phys. Lett. B **571** (2003) 75

- correlation function for $D \to \pi^+\pi^-$ ($\pi \to K$, $s \leftrightarrow d$ for $D \to K^+K^-$)

- OPE diagrams
  in terms of pion LCDAs:

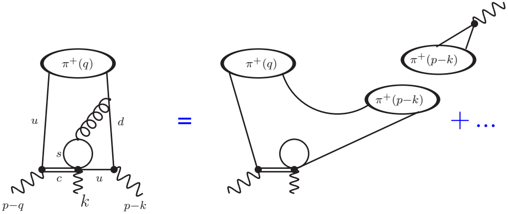


- some details:
  - finite quark masses m_c, m_s
  - $SU(3)$ not used, only isospin
  - tw 2,3 accuracy, fact. tw 5,6
  - selection of diagrams
    (see eariler $B \to \pi\pi$ papers)
  - LCSR's for $D \to \pi, K$ form factors

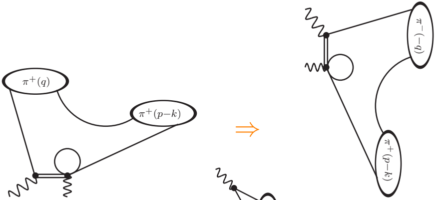
- LCSR input: quark masses, pion, kaon DAs, parameters used in the LCSR calculation of $D \to \pi$, $D \to K$ and pion e.m. form factor

□ Obtaining LCSR

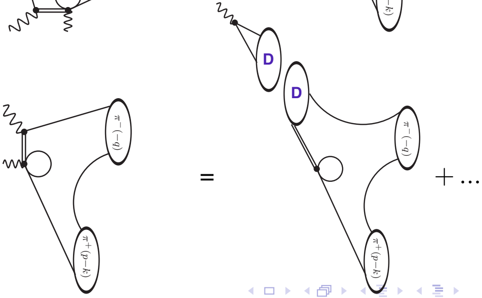
- **step 1:**
  Dispersion relation in the pion channel ⊕ duality



- **step 2:**
  Analytic continuation
  $P^2=(p-q-k)^2<0 \Rightarrow \quad P^2=m_D^2$



- **step 3:**
  Dispersion relation in the D channel ⊕ duality



Results for direct CP asymmetry

- numerical results obtained from LCSRs:

$$|\mathcal{P}^s_{\pi\pi}| = (1.96 \pm 0.23) \times 10^{-7}\text{GeV}\,, \quad |\mathcal{P}^d_{KK}| = (2.86 \pm 0.56) \times 10^{-7}\text{GeV}\,,$$

- using measured branching fractions of $D \to \pi^+\pi^-$, and $D \to K^+K^-$ for $\mathcal{A}_{\pi\pi}$ and $\mathcal{A}_{KK}$:

$$r_\pi = \frac{|\mathcal{P}^s_{\pi\pi}|}{|\mathcal{A}_{\pi\pi}|} = 0.093 \pm 0.011\,, \qquad r_K = \frac{|\mathcal{P}^d_{KK}|}{|\mathcal{A}_{KK}|} = 0.075 \pm 0.015\,.$$

- the difference of asymmetries:

$$\Delta a^{dir}_{CP} = -2 r_b \sin\gamma (r_K \sin\delta_K + r_\pi \sin\delta_\pi)\,, \quad r_b \sin\gamma = 0.64 \times 10^{-3}$$

- the resulting upper limits: (independent of strong phases)

$$\left| a^{dir}_{CP}(\pi^+\pi^-) \right| < 0.012 \pm 0.001\%, \quad \left| a^{dir}_{CP}(K^+K^-) \right| < 0.009 \pm 0.002\%,$$

$$\left| \Delta a^{dir}_{CP} \right| < 0.020 \pm 0.003\%\,.$$

- was much smaller than the 2019 LHCb result:

$\Delta a^{dir}_{CP} = (-0.154 \pm 0.029)\%$ R. Aaij *et al.* [LHCb], 1903.08726 [hep-ex] (2019)

- however, in 2022 LHCb measured:

$a^{dir}_{CP}(K^+K^-) = (0.077 \pm 0.057)\%, \; a^{dir}_{CP}(\pi^+\pi^-) = (0.231 \pm 0.061)\%$

□ *B*-meson decay into proton and dark antibaryon from LCSRs

[A. Khodjamirian and M. Wald, arXiv:2206.11601 [hep-ph]].

- The B-mesogenesis scenario of baryon asymmetry and relic DM predicts decays such as $B \to p + \Psi$, (Ψ - a dark antibaryon)

[G.Elor, M.Escudero, A.Nelson (2019)]

- the decay can be observed as $B \to p +$ invisible, accessible at Belle-II

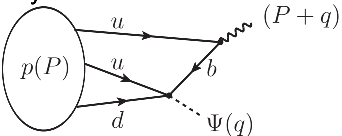
- generated by an effective three-quark -Ψ interaction, the typical “model (d)” :

$$\mathcal{H}^{(d)} = G_{(d)}\bar{\mathcal{O}}_{(d)}\Psi^c + h.c., \;\; \mathcal{O}_{(d)} = i\epsilon_{ijk} d_R^i \left(\bar{b}_R^{cj} u_R^k\right)$$

$G_{(d)}$ - Fermi-type coupling; switch to “model (b)” by ($d \leftrightarrow b$)

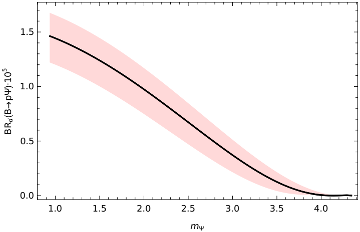
- calculated the $B \to p$ hadronic matrix element of $\mathcal{O}_{(d)}$ and $\mathcal{O}_{(b)}$, from LCSRs with proton distribution amplitudes and B-meson interpolating current

- the correlator in LO with twist-3 accuracy for nucleon DAs

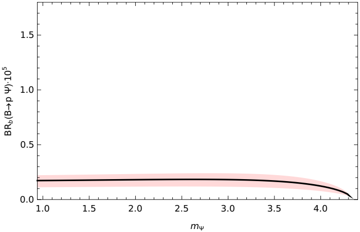


□ Results

- Branching fractions of $B \to p\Psi$ vs m_Ψ



model (d)



model (b)

- nontrivial sensitivity to the structure of the effective operator
- perspectives: other B and Λ_b exclusive decay modes in b-mesogenesis and similar flavoured DM models