Mixing & $\cancel{CP}$ in the B-system 27.5.23

Literature:

$\boxed{\text{R1}}$ General mixing & $\cancel{CP}$ formalism

B physics at the Tevatron: Run II and beyond

#1

K. Anikeev (MIT), D. Atwood (Iowa State U.), F. Azfar (Oxford U.), S. Bailey (Harvard U.), C.W. Bauer (UC, San Diego) et al. (Dec, 2001)

Proceedings of: Workshop on B Physics at the Tevatron: Run II and Beyond, Workshop on B Physics at the Tevatron: Run II and Beyond • e-Print: hep-ph/0201071 [hep-ph]

$\boxed{\text{R2}}$ B_s-mixing; higher order corrections in diagonalisation for B-mixing (newer exp. results)

CP violation in the B_s^0 system

#33

Marina Artuso (Syracuse U.), Guennadi Borissov (Lancaster U.), Alexander Lenz (Durham U., IPPP) (Nov 30, 2015)

Published in: *Rev.Mod.Phys.* 88 (2016) 4, 045002, *Rev.Mod.Phys.* 91 (2019) 4, 049901 (addendum) • e-Print: 1511.09466 [hep-ph]

$\boxed{\text{R3}}$ D-mixing: exact diagonalisation

Mixing and CP Violation in the Charm System

#12

Alexander Lenz (Siegen U.), Guy Wilkinson (Oxford U.) (Nov 9, 2020)

Published in: *Ann.Rev.Nucl.Part.Sci.* 71 (2021) 59-85 • e-Print: 2011.04443 [hep-ph]

→ Mixing ←

Interaction Eigenstate ≠ Mass ES

e.g.
- $W_{1,2,3}; B \longleftrightarrow W^+, W^-, Z^0, A'$
- Neutrinos
- V_{CKM}
- ...

Mixing of neutral mesons:

(i) **Def:** meson defined via Quark content

$$B_s \equiv (\bar{b} s) \qquad \bar{B}_s = (b \bar{s})$$

(ii) Naive expectation time evolution

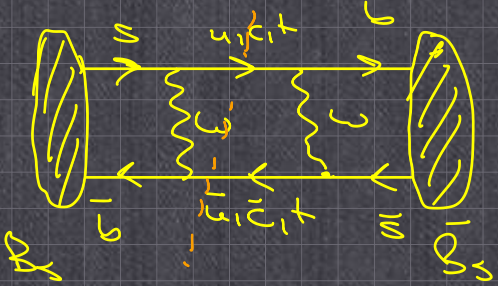
$$\overset{(-)}{B}_s(t) \overset{(*)}{=} \overset{(-)}{B}_s(0) \cdot \underbrace{e^{-iM_s t}}_{\text{mass}} \; \underbrace{e^{-\frac{1}{2}\Gamma_s t}}_{\text{decay rate}}$$

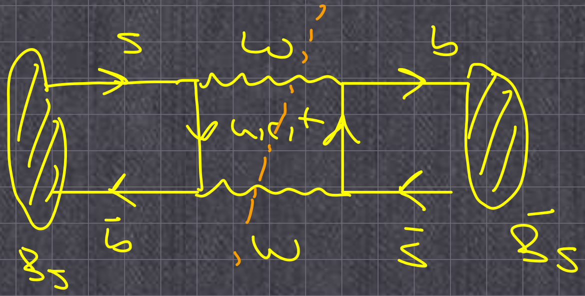
(*) is equivalent to

$$i\hbar \partial_t \begin{pmatrix} B_s \\ \bar{B}_s \end{pmatrix} \overset{(**)}{=} \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & \cancel{0} \\ \cancel{0} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} B_s \\ \bar{B}_s \end{pmatrix}$$

so far:
$M_{11} = M_{22} = M_s$
$\Gamma_{11} = \Gamma_{22} = \Gamma_s$

ⓘⓘⓘ But: weak interaction allows $B_s \leftrightarrow \bar{B}_s$





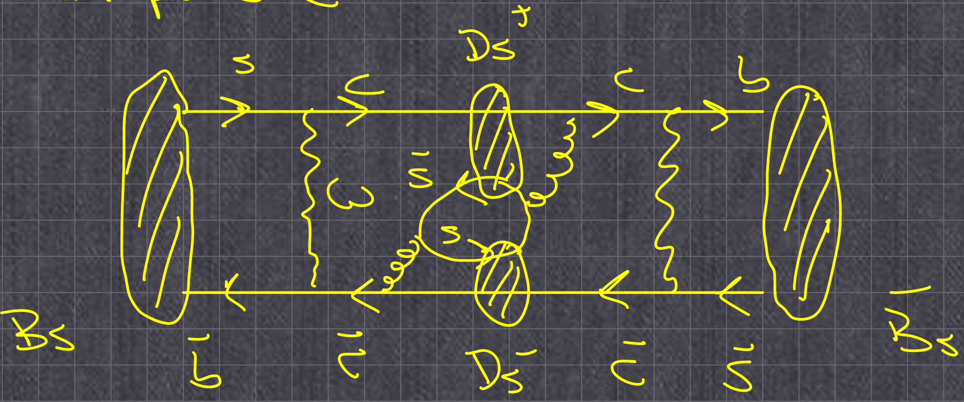
M_{12} = off-shell part of the Box-diagram

u, c, t, W

Γ_{12} = on-shell part of the Box-diagram

u, c

Example: $c - \bar{c}$



(iv) Most general form of mixing matrix:

$$\begin{pmatrix} M_{11}-\frac{i}{2}\Gamma_{11} & M_{12}-\frac{i}{2}\Gamma_{12} \\ M_{21}-\frac{i}{2}\Gamma_{21} & M_{22}-\frac{i}{2}\Gamma_{22} \end{pmatrix}$$

CPT

$M_{11}=M_{22}$

$\Gamma_{11}=\Gamma_{22}$

$M_{21}=M_{12}^{*}$

$\Gamma_{21}=\Gamma_{12}^{*}$

$\Longrightarrow$ non-diagonal

$\Longrightarrow$ B_s & $\bar{B}_s$ are not mass eigenstates

$\rightarrow$ diagonalise Mass matrix

$$\boxed{\begin{aligned} B_{s,H} &= p\,B_s + q\,\bar{B}_s \\ B_{s,L} &= p\,B_s - q\,\bar{B}_s \end{aligned}}$$

H ≡ Heavy

L ≡ Light

$\Downarrow$

Mass eigenstate

$\Downarrow$

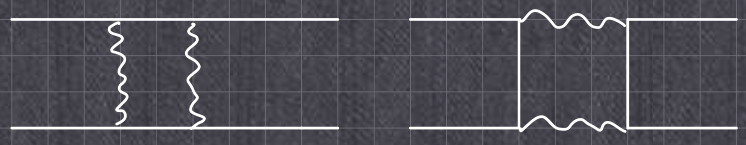
diagonal Mass & decay rate matrix

Quark/Flavour eigenstates

$$\begin{pmatrix} M_{s,H}-\frac{i}{2}\Gamma_{s,H} & 0 \\ 0 & M_{s,L}-\frac{i}{2}\Gamma_{s,L} \end{pmatrix}$$

Physical observables:

Boxdiagrams:



$$M_{12}, \Gamma_{12} \in \mathbb{C}$$

3 physical quantities: $|M_{12}|, |\Gamma_{12}|, \phi_{12} := \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$

$\Rightarrow$ 3 observables:

① Mass difference

$$\Delta M_s = M_{s,H} - M_{s,L} = \Delta M_s(M_{12}, \Gamma_{12})$$

② Decay rate difference

$$\Delta \Gamma_s \doteq \Gamma_{s,L} - \Gamma_{s,H} = \Delta \Gamma_s(M_{12}, \Gamma_{12})$$

③ Flavour-specific CP asymmetries

$$a_{fs} \equiv a_{se} = a_{fs}(M_{12}, \Gamma_{12})$$

$\equiv \cancel{CP}$ in mixing, see below

$\boxed{\text{I}}$ Diagonalise mass & decay rate matrix

$\boxed{\text{II}}$ in the B-system one finds: $|\Gamma_{12}| \ll |M_{12}|$

$\Rightarrow$ Taylor expansion in $\left|\frac{\Gamma_{12}}{M_{12}}\right| \simeq 5 \cdot 10^{-3}$

① $\Delta M_s = 2|M_{12}| + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right)$

② $\Delta \Gamma_s = 2|\Gamma_{12}^s| \cos(\phi_{12}^s) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right)$

③ $a_{fs}^s = \left|\frac{\Gamma_{12}^s}{M_{12}^s}\right| \sin(\phi_{12}^s) + \mathcal{O}\left(\left|\frac{\Gamma_{12}}{M_{12}}\right|^2\right)$

$5 \cdot 10^{-3}$ $\quad \hat{=}\ 250$

see R2 for exact form

$\Rightarrow a_{fs}^s \simeq 2 \cdot 10^{-5}$!

Comment: in D-system $|\Gamma_{12}| \simeq |M_{12}|$

$\Rightarrow$ no Taylor expansion in $|\Gamma_{12}/M_{12}|$ possible

see R3 for exact diagonalisation formulae

$\rightarrow$ however: exp. indicates ϕ_{12}^D is small

$\Rightarrow$ do Taylor in ϕ_{12}^D (see below)

How to measure these quantities?

=> Look at time evolution of a neutral B-meson

Trivial for mass eigenstates

$$|B_{s,H/L}(t)\rangle = e^{-(iM^s_{H/L}+\frac{1}{2}\Gamma^s_{H/L})\cdot t}\,|B_{s,H/L}(0)\rangle$$

Rewrite mass eigenstates $|B_{s,H/L}(t,0)\rangle$ in terms of flavour eigenstates $|B_s(t,0)\rangle$

⋮

$$|B_s(t)\rangle = g_+(t)\,|B_s(0)\rangle + \frac{q}{p}\,g_-(t)\,|\bar{B}_s(0)\rangle$$

with $$g_+(t) = e^{-iM_{B_s}t}\,e^{-\frac{\Gamma_{B_s}t}{2}}\left[\cosh\frac{\Delta\Gamma_s t}{4}\cdot\cos\frac{\Delta M_s t}{2} - \sinh\frac{\Delta\Gamma_s t}{4}\cdot\sin\frac{\Delta M_s t}{2}\right]$$

here we used: $M_{B_s}=\frac{M_H+M_L}{2}$ $\quad\Gamma_{B_s}=\frac{\Gamma_H+\Gamma_L}{2}$

$\Delta M_{B_s} = M_H - M_L$ $\quad\Delta\Gamma_{B_s} = \Gamma_L - \Gamma_H$

$2M_{B_s} + \Delta M_{B_s} = 2M_H$ $\quad 2\cos x = e^{ix} + e^{-ix}$

$2M_{B_s} - \Delta M_{B_s} = 2M_L$ $\quad 2i\sin x = e^{ix} - e^{-ix}$

$2\cosh x = e^{x} + e^{-x}$

$2\sinh x = e^{x} - e^{-x}$

see R1, R2 for $g_-(t)$, $|\bar{B}_s(t)\rangle$ ...

- Now we have the time evolution of

$$|B_s(t)\rangle \quad \& \quad |\bar{B}_s(t)\rangle$$

- We are interested in measurable decay probabilities

$$* \; Br(B \to f) = \frac{\Gamma(B \to f)}{\Gamma_{tot}}$$

$$* \; \Gamma(B \to f) \sim \int_{PS} |\langle f | \mathcal{H}_{eff} | B \rangle|^2$$

- To determine $\Gamma(B_s(t) \to f)$ we have to square

$$|\langle f | \mathcal{H}_{eff} | B_s(t) \rangle|^2$$

$$\Downarrow$$

insert above time evolution and keep

$|\langle f | \mathcal{H}_{eff} | B_s \rangle|$ &

$|\langle f | \mathcal{H}_{eff} | \bar{B}_s \rangle|$

trivial, but tedious algebra

One gets: see R1, R2

time indep.

Normalisation factor (e.g. phase space effects)

$$\boxed{\begin{aligned}\Gamma\left(\bar{B}_s(t)\to f\right) = \overbrace{\mathcal{N}_f}\ |A_f|^2 \cdot \frac{1+|\lambda_f|^2}{2}\, e^{-\Gamma_s t} & \left\{\cosh\frac{\Delta\Gamma_s t}{2} - \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\cos\Delta M_s t\right. \\ & \left. - \sinh\frac{\Delta\Gamma_s t}{2}\,\frac{2\,\mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2} + \frac{2\,\mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2}\sin\Delta M_s t\right\} \\ & \left(1 + a_{fs}^{s}\right)\end{aligned}}$$

for
$$\left.\begin{aligned}&\Gamma\left(B_s(t)\to f\right)\\&\Gamma\left(\bar{B}_s(t)\to \bar{f}\right)\\&\Gamma\left(B_s(t)\to \bar{f}\right)\end{aligned}\right\}$$
see R1, R2

with the abbreviations

* $A_f = \langle f|\mathcal{H}_{eff}|B_q\rangle$ matrix element
$f \equiv$ hadron
$\Rightarrow$ very difficult to determine from first principles...

* $\bar{A}_f = \langle f|\mathcal{H}_{eff}|\bar{B}_q\rangle$

* $\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$

* $a_{fs}^{s} = \text{Im}\frac{\Gamma_{12}}{M_{12}} \simeq 2\cdot 10^{-5}$

  ↳ flavour-spezific

Definition: flavour specific decay B→f

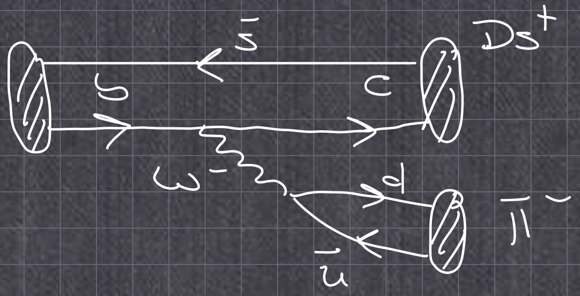
i) $B \not\to f$

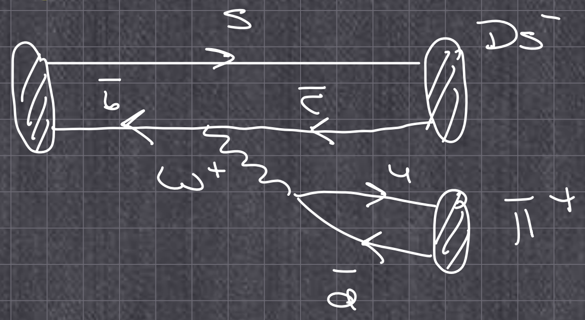
ii) $\bar{B} \not\to \bar{f}$

Examples

a) Semi-leptonic B-decays

b) $\bar{B}_s \to D_s^+ \pi^-$ $B_s \to D_s^- \pi^+$





=> for a flavour specific decay

$A_f \neq 0$; $\bar{A}_f = 0$ => $\lambda_f = 0$

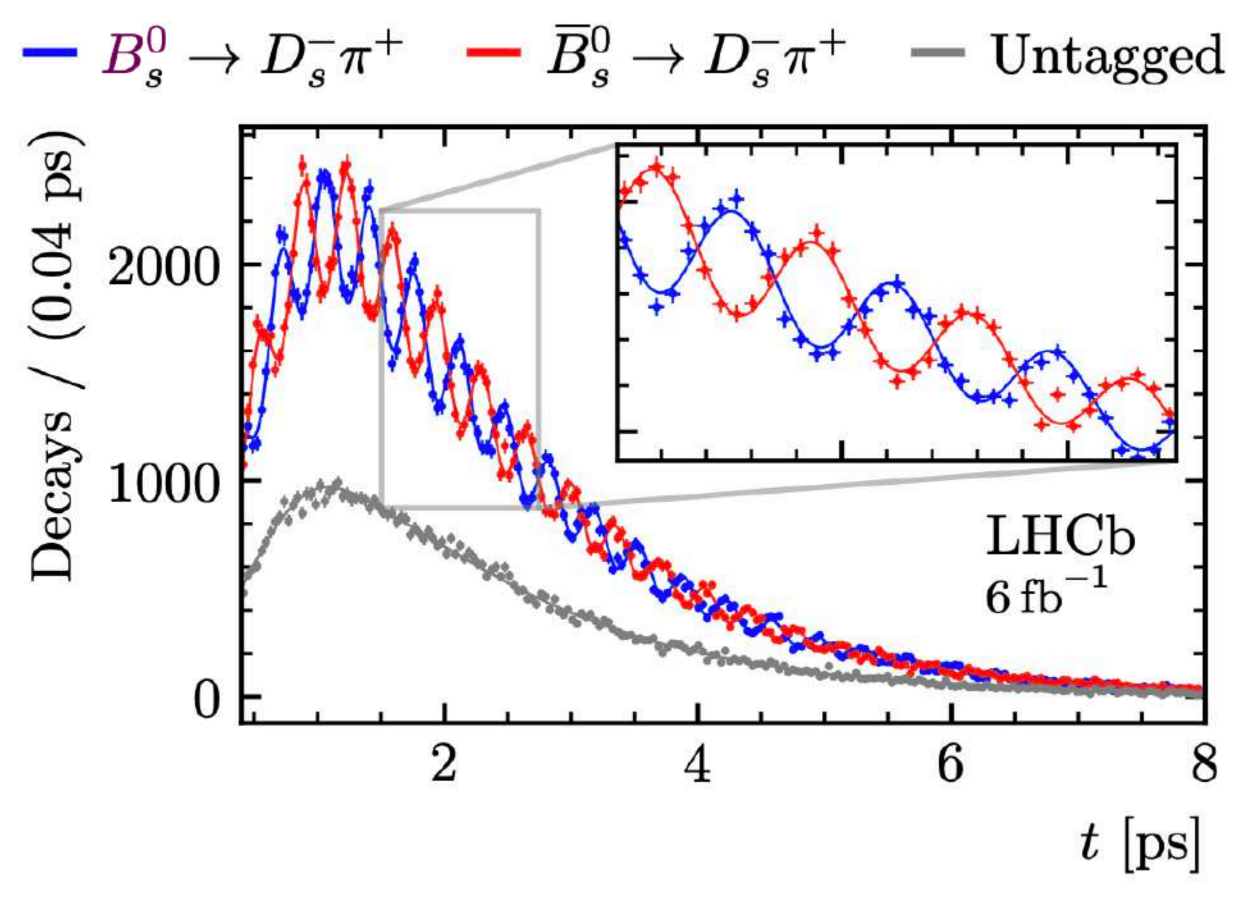
$$\Gamma\left(\bar{B}_s(t) \to f\right) = N_f \, |A_f|^2 \cdot \overset{1/2}{\cancel{\frac{1+|\lambda_f|^2}{2}}} \, e^{-\Gamma_s t}$$
$$\left\{ \cosh \frac{\Delta\Gamma_s t}{2} - \overset{1}{\cancel{\frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}}} \cos \Delta m_s t \right.$$
$$\left. - \overset{0}{\cancel{\sinh \frac{\Delta\Gamma_s t}{2} \frac{2 \mathrm{Re}(\lambda_f)}{1+|\lambda_f|^2}}} + \overset{0}{\cancel{\frac{2 \mathrm{Im}(\lambda_f)}{1+|\lambda_f|^2} \sin \Delta m_s t}} \right\}$$
$$\left(1 + a_{fs}^s\right)$$

=> simple, exact formula !

=> fit time evolution

=> $\cos \Delta m_s t$

Measurement of mixing observables



Results:

$$\Delta M_s = 17.765(6)\,\mathrm{ps}^{-1}$$

$$\Delta M_d = 0.5065(19)\,\mathrm{ps}^{-1}$$

$$\Delta \Gamma_s = 0.083(5)\,\mathrm{ps}^{-1}$$

super precise measurement !

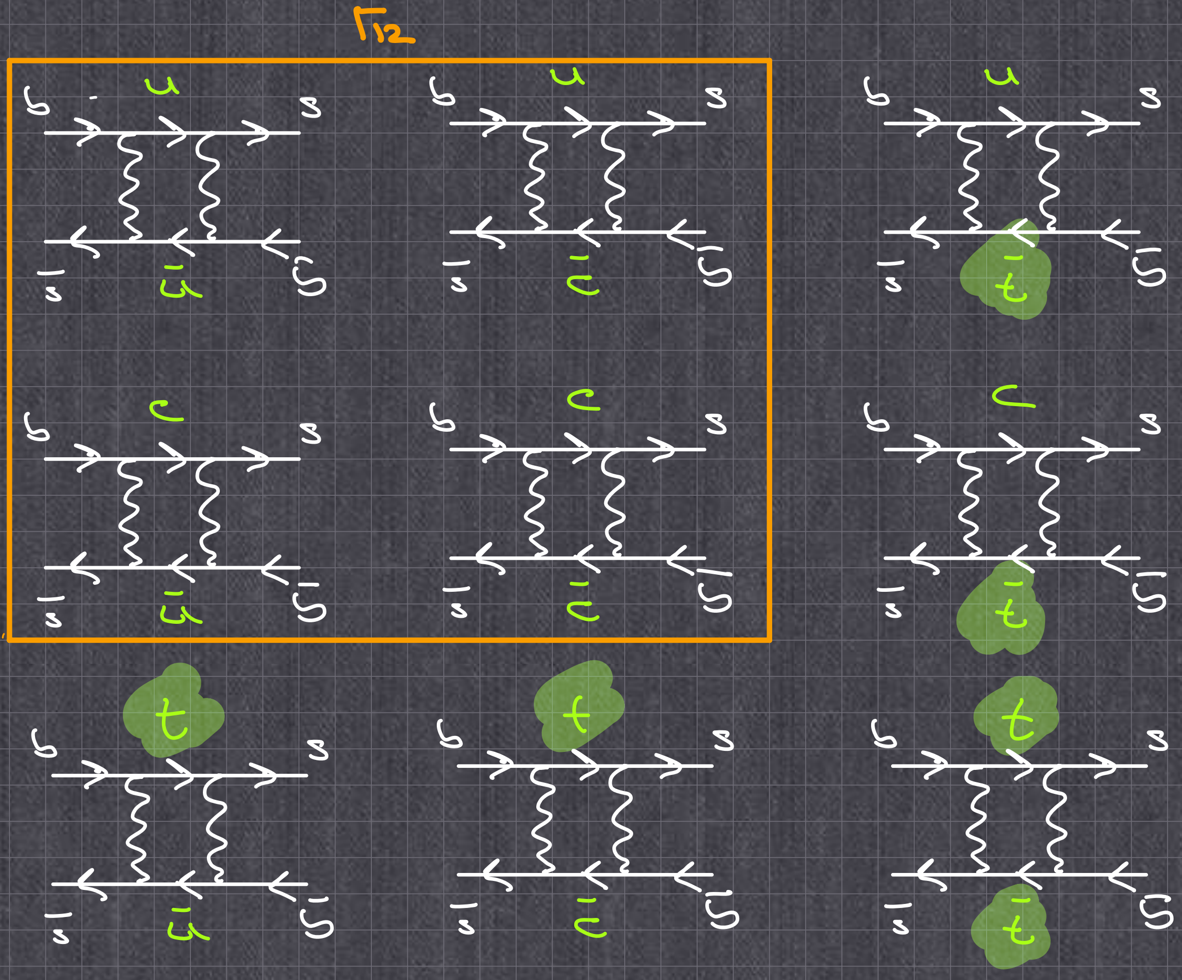
$$\Delta\Gamma_d / \Gamma_d = 0.001(10)$$

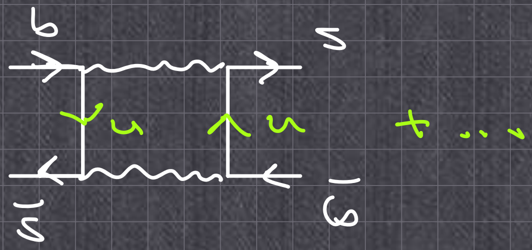
$$a_{fs}^{d} = (-21 \pm 17) \cdot 10^{-4}$$

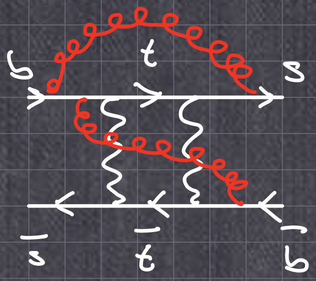
$$a_{fs}^{s} = (-60 \pm 280) \cdot 10^{-5}$$

experimental measurement still missing

How to calculate mixing?







$\lambda_u = V_{ub} V_{us}^*$ $\lambda_s = V_{cb} V_{cs}^*$ $\lambda_t = V_{tb} V_{ts}^*$

$$\Gamma_{12} = \lambda_u^2 F(u,u) + \lambda_u \lambda_c F(u,c) + \lambda_u \lambda_t F(u,t)$$
$$+ \lambda_c \lambda_u F(c,u) + \lambda_c^2 F(c,c) + \lambda_c \lambda_t F(c,t)$$
$$+ \lambda_t \lambda_u F(t,u) + \lambda_t \lambda_c F(t,c) + \lambda_t^2 F(t,t)$$

		wolfenstein	vgl. Bd
CKM:	$\lambda_u = V_{us}^* V_{ub} \sim$	$\lambda^{4.8}$	$\lambda^{3.8}$
	$\lambda_c = V_{cs}^* V_{cb} \sim$	λ^2	λ^3
	$\lambda_t = V_{ts}^* V_{tb} \sim$	λ^2	λ^3

<u>Im SM:</u> V_{CKM} is unitary $\Rightarrow \lambda_u + \lambda_c + \lambda_t = 0$

$$\lambda_c = -\lambda_u - \lambda_t$$

$$\Rightarrow \Gamma_{12} = \lambda_u^2 \left[F(c,c) - 2F(u,c) + F(u,u) \right]_1$$
$$+ 2\lambda_u \lambda_t \left[F(c,c) - F(u,c) + F(u,t) - F(c,t) \right]_2$$
$$+ \lambda_t^2 \left[F(c,c) - 2F(c,t) + F(t,t) \right]_3$$

CKM- leading

observations:

- $m_u = m_c = m_t \Rightarrow [\dots]_1 = [\dots]_2 = [\dots]_3 = 0$

  GIM - Mechanism

  Glashow - Illiopoulos - Maiani

- Loop - integration:

  $x = \frac{m^2}{M_W^2}$

  $$F(p,q) = \underbrace{f_0}_{\text{const.}} + f(x_q, x_p)$$

  GIM: f_0 cancels exactly

- Quarkmasses:

  $$x_u = \frac{m_u^2}{m_W^2} \simeq 0$$

  $$x_c = \frac{m_c^2}{m_W^2} \simeq 2.5 \cdot 10^{-4} \simeq 0$$

  $$x_t = \frac{m_t^2}{m_W^2} \simeq 4.5$$

$$\Rightarrow M_{12} = \lambda_u^2 \cdot 0$$

$$+ 2\lambda_u\lambda_t \cdot 0$$

$$+ \lambda_t^2 \underbrace{\left[f(0,0) - 2f(0,x_t) + f(x_t,x_t)\right]}_{S(x_t)}$$

Inami Lim-function

with

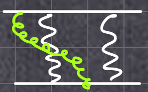
$$S(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x \ln x}{2(1-x)^2}$$

Performing the full SM calculation taking also hadronic bound states of B_s & $\bar{B}_s$ into account

one gets

$$M_{12}^s = \frac{G_F^2 M_W^2}{12\pi^2} \lambda_t^2 \overset{\checkmark}{\overset{\uparrow}{S(x_t)}} \overset{?}{\overset{\uparrow}{f_{B_s}^2 B_{B_s}}} \cdot \hat{\eta}_B$$

pert. QCD corrections



Buras, Jamin, Weisz 1990

$$M_{12} = \langle B_s | \mathcal{H} | \bar{B}_s \rangle$$

$$= \dots \langle B_s | \underbrace{(\bar{s}b)_{V-A} (\bar{s}b)_{V-A}}_{Q_1} | \bar{B}_s \rangle$$

Define: $\langle B_s | Q_1 | \bar{B}_s \rangle = \frac{8}{3} f_{B_s}^2 B_{B_s} M_{B_s}^2$

f_{B_s} → decay constant
⇒ Lattice
(sum rules)

B_{B_s} → Bag parameter
⇒ sum rules, lattice

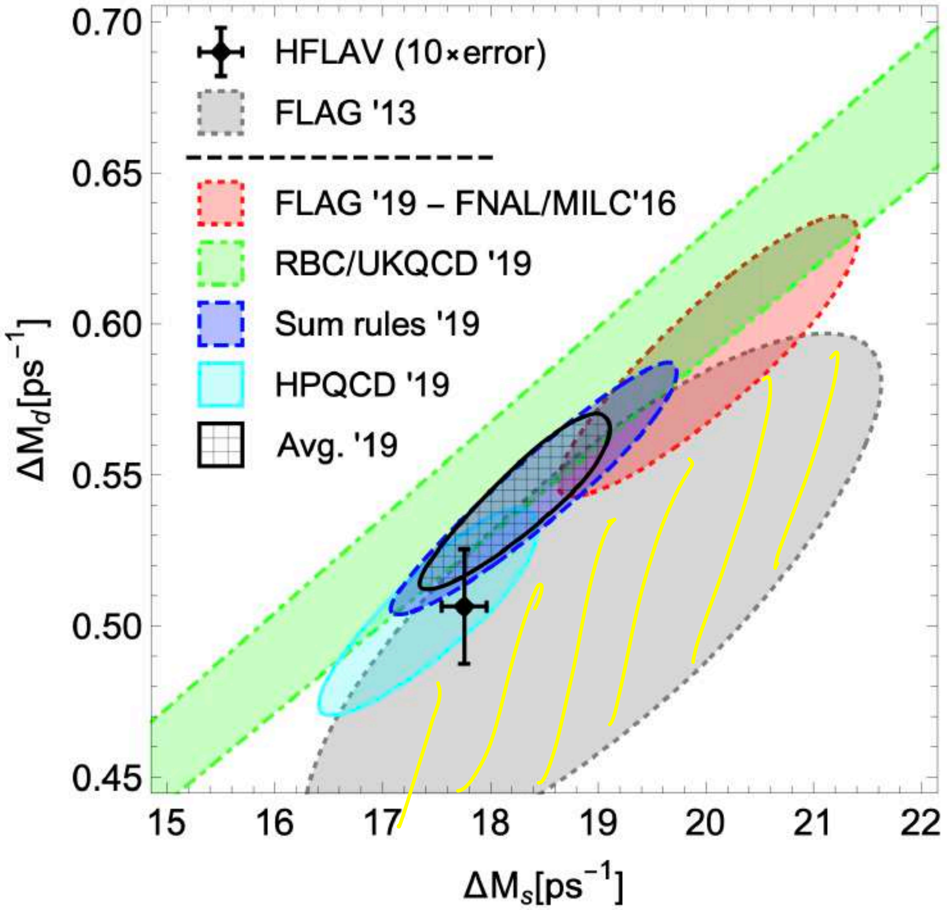
ΔM^{SM} only Q_1 arises
ΔM^{BSM} $Q_1 \dots Q_5$ (see Paolo Gambino)
$\Delta\Gamma^{SM}$ $Q_1 \dots Q_5$ arise
$\Delta\Gamma^{BSM}$ $Q_1 \dots Q_5$ arise

Status Quo:

Lattice:
2016 FNAL / MILC
2018 RBC / UKQCD → on going + Siegen + JLQCD
2019 HPQCD

HQET sum rules
2016 Siegen ΔM_d^{SM}
2017 Durham ΔM_d^{BSM}
2019 Durham ΔM_s

Average lattice & sum rules : 1909.11087



SM predictions: 1912.0762

$\Delta M_s = 18.77(86)\ \mathrm{ps}^{-1}$ $\delta^{\mathrm{The}}/\delta^{\mathrm{Exp}} \simeq 140$

$\Delta M_d = 0.543(29)\ \mathrm{ps}^{-1}$ $\delta^{\mathrm{The}}/\delta^{\mathrm{Exp}} \simeq 15$

$\Delta \Gamma_s = 0.091(13)\ \mathrm{ps}^{-1}$ $\delta^{\mathrm{The}}/\delta^{\mathrm{Exp}} \simeq 2.6$

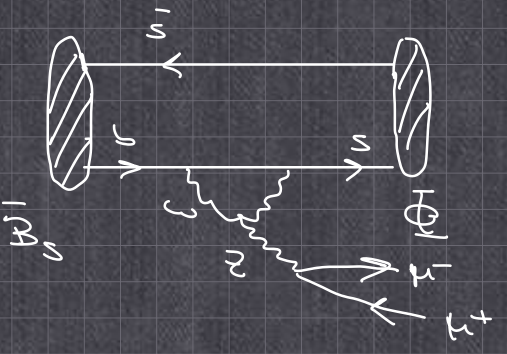
$\Delta \Gamma_d = 2.6(4)\cdot 10^{-3}\ \mathrm{ps}^{-1}$ no exp value

$a_{s\ell}^{s} = 2.06(18)\cdot 10^{-5}$ no exp. value

$a_{s\ell}^{d} = -4.73(42)\cdot 10^{-4}$ no exp. value

→ This already gives a strong constraint on many BSM models!

Example look at $b \to s \mu\mu$ anomalies

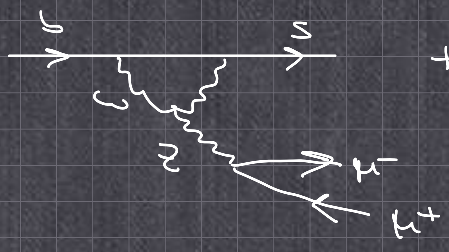


Still:

$Br^{Exp} \neq Br^{Theo}$

maybe ∃ BSM, e.g. new Z'

Exp =



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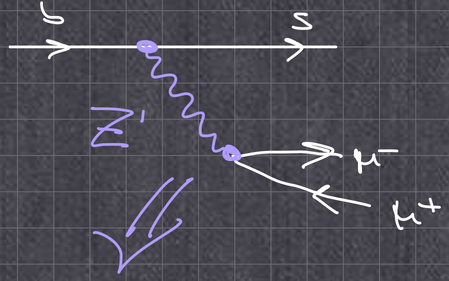
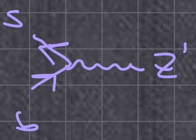


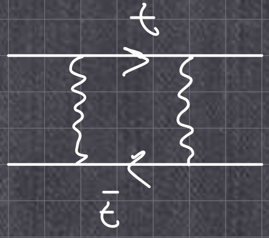
$$\sim \frac{g'_{bs} \cdot g'_{\mu\mu}}{k^2 - M_{Z'}^2}$$

But: a



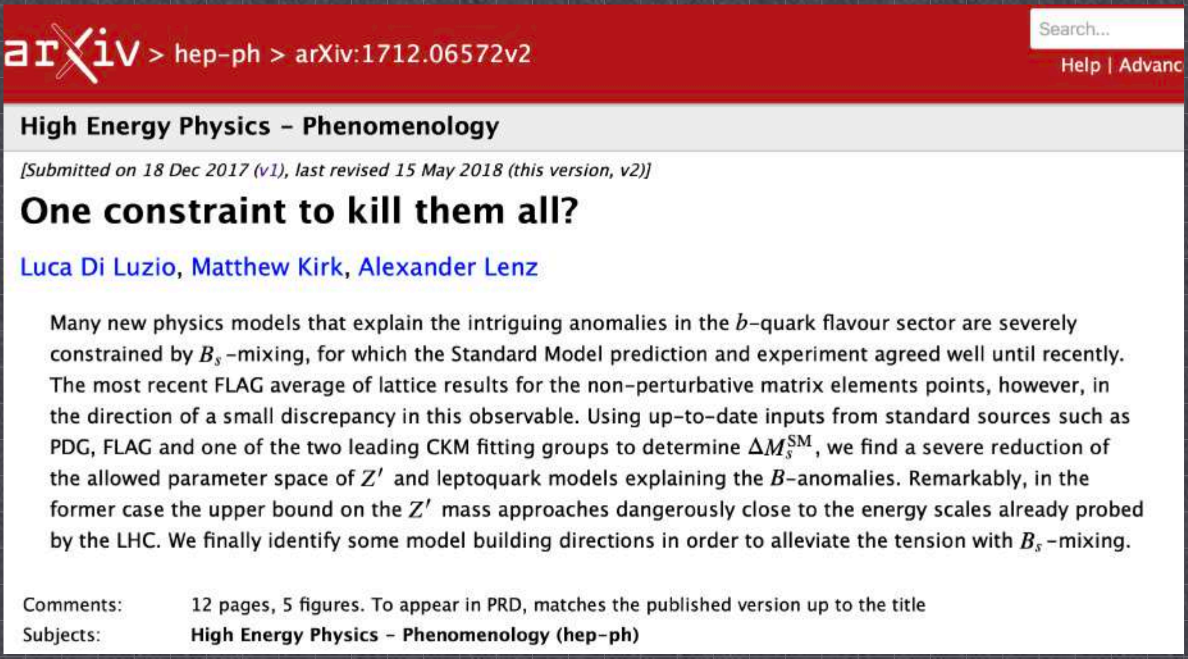
vertex contributes to M_{12}^s!

Exp (ΔM_s) =



+

$$\sim \frac{g'^2_{bs}}{k^2 - M_{Z'}^2}$$



arXiv > hep-ph > arXiv:1712.06572v2

High Energy Physics – Phenomenology

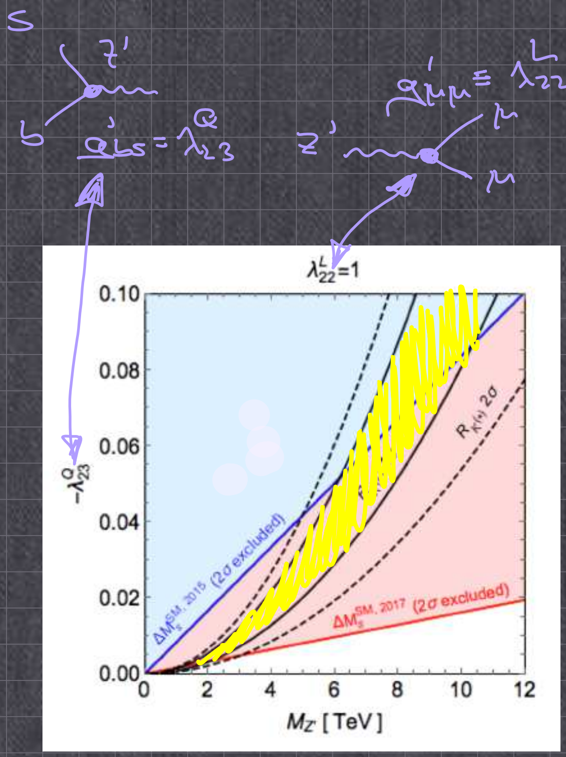
[Submitted on 18 Dec 2017 (v1), last revised 15 May 2018 (this version, v2)]

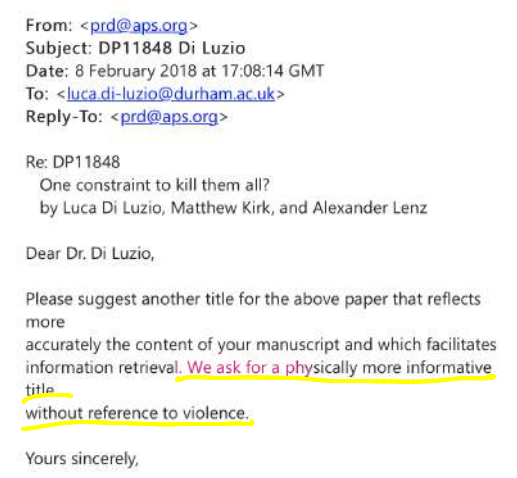
One constraint to kill them all?

Luca Di Luzio, Matthew Kirk, Alexander Lenz

Many new physics models that explain the intriguing anomalies in the b-quark flavour sector are severely constrained by B_s-mixing, for which the Standard Model prediction and experiment agreed well until recently. The most recent FLAG average of lattice results for the non-perturbative matrix elements points, however, in the direction of a small discrepancy in this observable. Using up-to-date inputs from standard sources such as PDG, FLAG and one of the two leading CKM fitting groups to determine $\Delta M_s^{\rm SM}$, we find a severe reduction of the allowed parameter space of Z' and leptoquark models explaining the B-anomalies. Remarkably, in the former case the upper bound on the Z' mass approaches dangerously close to the energy scales already probed by the LHC. We finally identify some model building directions in order to alleviate the tension with B_s-mixing.

Comments: 12 pages, 5 figures. To appear in PRD, matches the published version up to the title
Subjects: **High Energy Physics – Phenomenology (hep-ph)**





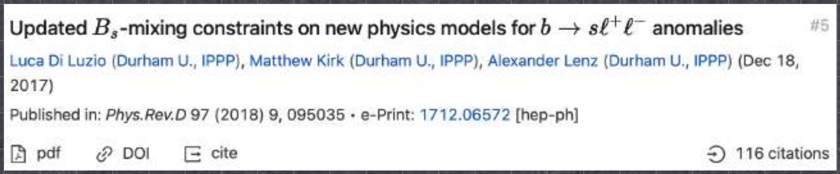
From: <prd@aps.org>
Subject: DP11848 Di Luzio
Date: 8 February 2018 at 17:08:14 GMT
To: <luca.di-luzio@durham.ac.uk>
Reply-To: <prd@aps.org>

Re: DP11848
One constraint to kill them all?
by Luca Di Luzio, Matthew Kirk, and Alexander Lenz

Dear Dr. Di Luzio,

Please suggest another title for the above paper that reflects more
accurately the content of your manuscript and which facilitates information retrieval. We ask for a physically more informative title
without reference to violence.

Yours sincerely,



Updated B_s-mixing constraints on new physics models for $b \to s\ell^+\ell^-$ anomalies

Luca Di Luzio (Durham U., IPPP), Matthew Kirk (Durham U., IPPP), Alexander Lenz (Durham U., IPPP) (Dec 18, 2017)

Published in: *Phys.Rev.D* 97 (2018) 9, 095035 • e-Print: 1712.06572 [hep-ph]

Can we do even more with
our mixing formalism?

$\Longrightarrow$ Search for $\cancel{CP}$

$\exists$ 3 kinds of $\cancel{CP}$

① $\cancel{CP}$ in mixing. (Theory ☺ Exp ☹)

Use flavour-specific decay, with no direct $\cancel{CP}$ i.e. $\bar{A}_{\bar{f}} = A_f$

Example: * $\bar{B}_s \to D_s^+ \pi^-$: $\bar{B}_s \not\to D_s^- \pi^+$

$B_s \not\to D_s^+ \pi^-$

* $\bar{B} \to D e^- \bar{\nu}$

Define: $$A^s_{fs} = \frac{\Gamma(\bar{B}_s(t) \to f) - \Gamma(B_s(t) \to \bar{f})}{\Gamma(\bar{B}_s(t) \to f) + \Gamma(B_s(t) \to \bar{f})}$$

$$= \dots = \frac{|\Gamma_{12}^s|}{|M_{12}^s|} \cdot \sin \phi_{12}^s \simeq 2 \cdot 10^{-5}$$

insert time evolution

Still huge discovery potential for BSM

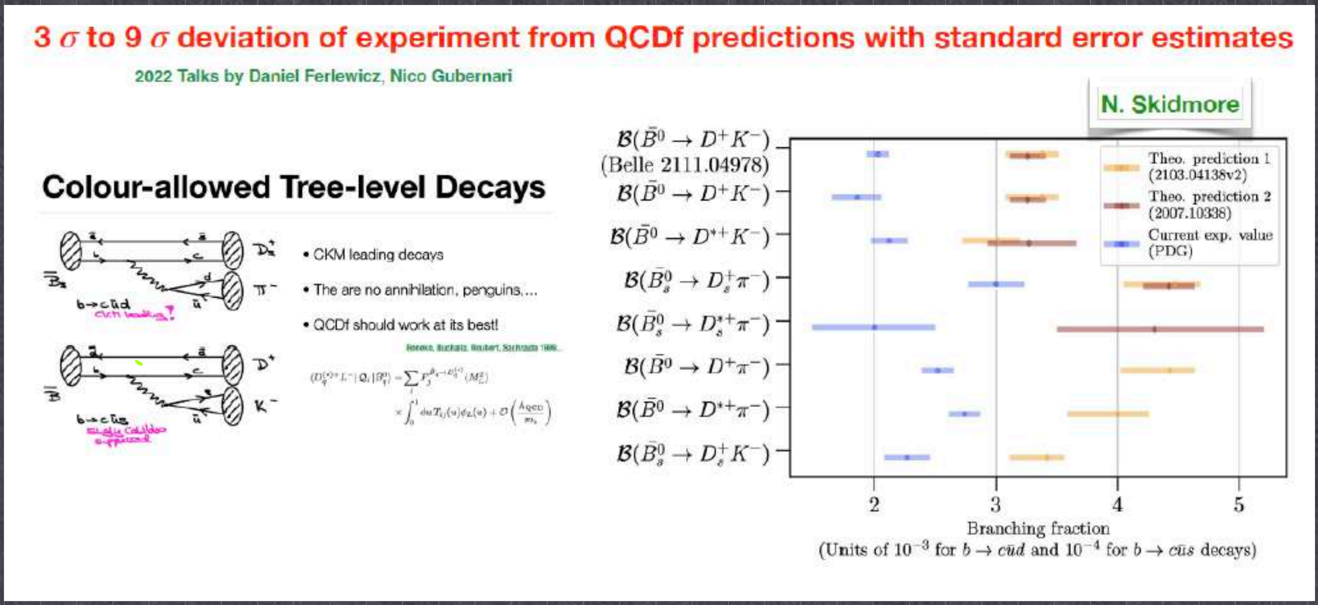
* $\delta \exp = \pm 280 \cdot 10^{-5}$; $a^s_{sl} = 2 \cdot 10^{-5}$!

* in the SM: 2111.04478

$$a_{fs}(\bar B_s \to D_s^+ \pi^-) \overset{\text{exact}}{\overset{\downarrow}{=}} \underbrace{a_{fs}(\bar B \to D \ell \nu)}_{\substack{\text{semi-lept. CP} \\ \text{asym.} = a_{sl}}}$$

Measuring: $a_{fs}(\bar B_s \to D_s^+ \pi^-) \neq a_{sl}$
would be an unambiguous sign
for new, $\cancel{CP}$ physics!

Are there any other indications for
BSM effects, e.g. in $\bar B_s \to D_s^+ \pi^-$?



3 σ to 9 σ deviation of experiment from QCDf predictions with standard error estimates

2022 Talks by Daniel Ferlewicz, Nico Gubernari

N. Skidmore

Colour-allowed Tree-level Decays

- CKM leading decays
- The are no annihilation, penguins,...
- QCDf should work at its best!

Beneke, Buchalla, Neubert, Sachrajda 1999...

$$\langle D_q^{(*)+} L^- | Q_i | \bar B_q^0 \rangle = \sum_j F_j^{\bar B_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u) \phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{QCD}}{m_b}\right)$$

② Indirect $\cancel{CP}$ or $\cancel{CP}$ in interference of mixing & decay

(theory: rough estimate ☺ golden-plated modes

high precision ☹ penguin pollution

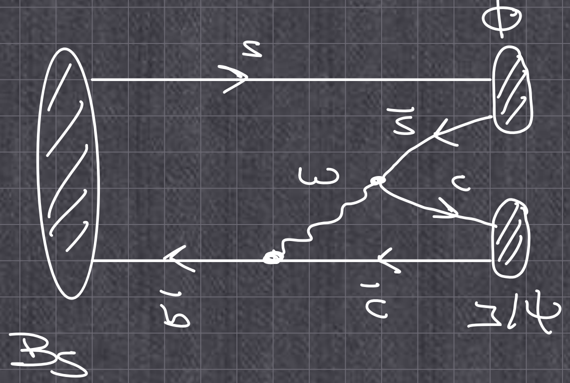
exp: ☺ new measurement from LHCb

Look at decays ⇒

$$B \to f \qquad \text{e.g.} \nearrow B_d \to J/\psi\, K_s$$
$$\bar{B} \to f \qquad\quad \searrow B_s \to J/\psi\, \phi$$

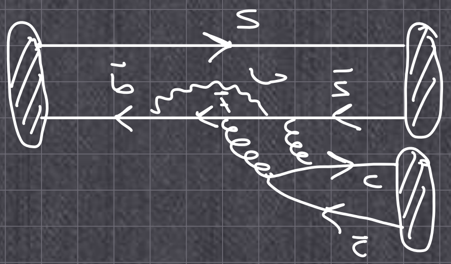
Diagrams:

Tree



+

Penguin



Define:

$$A^{s}_{ind.} \equiv \frac{\Gamma(\bar{B}_q(t) \to f) - \Gamma(B_q(t) \to f)}{\Gamma(\bar{B}_q(t) \to f) + \Gamma(B_q(t) \to f)}$$

⓶a Neglect penguins

only one (tree) decay amplitude

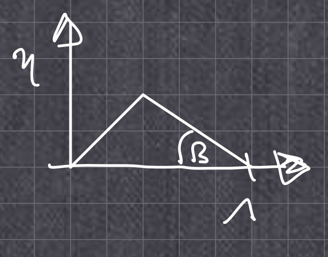
$$A_f = |A_f|\, e^{i\phi}\, e^{i\varphi}$$

$$\bar{A}_{\bar{f}} = |A_f|\, \underset{\text{QCD}}{\underset{\Downarrow}{e^{i\phi}}}\, \underset{\text{CKM weak}}{\underset{\Downarrow}{e^{-i\varphi}}} = \bar{A}_f$$

$$\Rightarrow \lambda_f = e^{-2i\varphi} + \text{tiny corrections}$$

↳ all hadronic uncert. cancel exactly

$$\Rightarrow A^{d}_{ind} \sim \sin(2\beta)$$



⓶b Include Penguins

$$A_f = |A_T|\, e^{i\phi_T} e^{i\varphi_T} + |A_P|\, e^{i\phi_P} e^{i\varphi_P}$$

$$A_{\bar{f}} = |A_T|\, e^{i\phi_T} e^{-i\varphi_T} + |A_P|\, e^{i\phi_P} e^{-i\varphi_P}$$

def: $r = \frac{|A_P|}{|A_T|}$; expect. to be small, how small?

$$\Rightarrow \lambda_f = \frac{1 + r\, e^{i\phi_P - \phi_T}\, e^{-i(\varphi_P - \varphi_T)}}{1 + r\, e^{i\phi_P - \phi_T}\, e^{i(\varphi_P - \varphi_T)}}$$

$$\Rightarrow A^{d}_{ind} \sim \sin(2\beta)\left[1 + r \ldots\right]$$

When will this be relevant?

$r \longleftrightarrow \pm 1^\circ$ in $\sin 2\beta$

LHCb last week $\quad \sin 2\beta = 0.716(13)(8)$

$\Rightarrow \delta\beta = \pm 0.6^\circ$!

The SM does not predict

$$A^{d}_{ind} = \sin(2\beta)$$

!

It predicts

$$A^{d}_{ind} = \sin(2\beta)\left[1 + \ldots r\right]$$

③ $\cancel{CP}$ in decay ≡ direct $\cancel{CP}$

Def: $$A_{dir} = \frac{\Gamma(\bar{B}(t) \to \bar{f}) - \Gamma(B(t) \to f)}{\Gamma(\bar{B}(t) \to \bar{f}) + \Gamma(B(t) \to f)}$$

→ works also for non-neutral mesons (no time dependence) ; $B^+ \to \dots$

→ gives only a contribution if the are at least 2 amplitudes

$$A_f = |A_T| e^{i\phi_T} e^{i\varphi_T} + |A_P| e^{i\phi_P} \cdot e^{i\varphi_P}$$

$$\Rightarrow A_{dir} = \frac{2|r| \sin \Delta\phi \cdot \sin \Delta\varphi}{1 + \dots}$$

⇒ no fully dependent on r

this is the leading term

⇒ Theory ☹

Exp ☺

SM??

Direct $\cancel{CP}$

$\Delta A_{CP} =$

$A_{CP}(D^0 \to K^+K^-)$

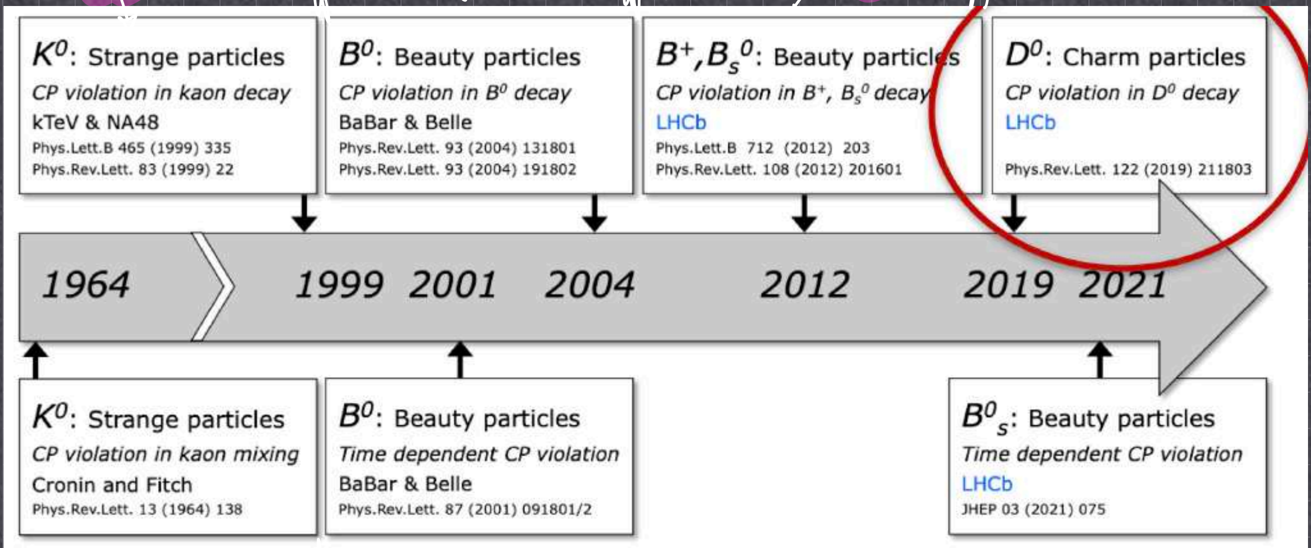
$- A_{CP}(D^0 \to \pi^+\pi^-)$

$\simeq 10^{-3}$

ε'/ε

direct $\cancel{CP}$

direct $\cancel{CP}$



K^0: Strange particles
CP violation in kaon decay
kTeV & NA48
Phys.Lett.B 465 (1999) 335
Phys.Rev.Lett. 83 (1999) 22

B^0: Beauty particles
CP violation in B^0 decay
BaBar & Belle
Phys.Rev.Lett. 93 (2004) 131801
Phys.Rev.Lett. 93 (2004) 191802

B^+, B_s^0: Beauty particles
CP violation in B^+, B_s^0 decay
LHCb
Phys.Lett.B 712 (2012) 203
Phys.Rev.Lett. 108 (2012) 201601

D^0: Charm particles
CP violation in D^0 decay
LHCb
Phys.Rev.Lett. 122 (2019) 211803

K^0: Strange particles
CP violation in kaon mixing
Cronin and Fitch
Phys.Rev.Lett. 13 (1964) 138

B^0: Beauty particles
Time dependent CP violation
BaBar & Belle
Phys.Rev.Lett. 87 (2001) 091801/2

B^0_s: Beauty particles
Time dependent CP violation
LHCb
JHEP 03 (2021) 075

$\cdot\varepsilon_K$

NP

10^{-3}

$B_d \to J/\psi K_s$

$\sin 2\beta$

$\simeq 50\%$

indirect $\cancel{CP}$

$B_s \to J/\psi\, \phi_s$

ϕ_s

Charm physics:

differences compared to B-system

1) stronger QCD coupling

$$\alpha_s(m_c) \simeq 0.30 \ldots 0.35 \longleftrightarrow \alpha_s(m_b) \simeq \underline{\underline{0.2}}$$

2) Charm quark is not really heavy ‽

$$m_c^{pole} = (1.67 \pm 0.07)\ \text{GeV}$$

$$\overline{m}_c(\overline{m}_c) = (1.27 \pm 0.02)\ \text{GeV}$$

$$\frac{m_b}{m_c} \simeq 3 ‽$$

3) There is almost no $\cancel{CP}$ $\longleftarrow \boxed{\text{Null tests}}$

CKMfitter:

$$V_{cd} = -0.2247 - 1.4 \cdot 10^{-4} i$$

$$V_{cs} = 0.97354 - 2.1 \cdot 10^{-5} i$$

$$V_{cb} = 0.0416$$

4) There are extremely pronounced GIM cancellations

$$\text{Observable} = A_1^{theo} - 2 A_2^{theo} + A_3^{theo}$$

- $A_i \gg$ Observable
- $A_1 - 2A_2 + A_3 \ll A_i$

Cancellations in the Charm system

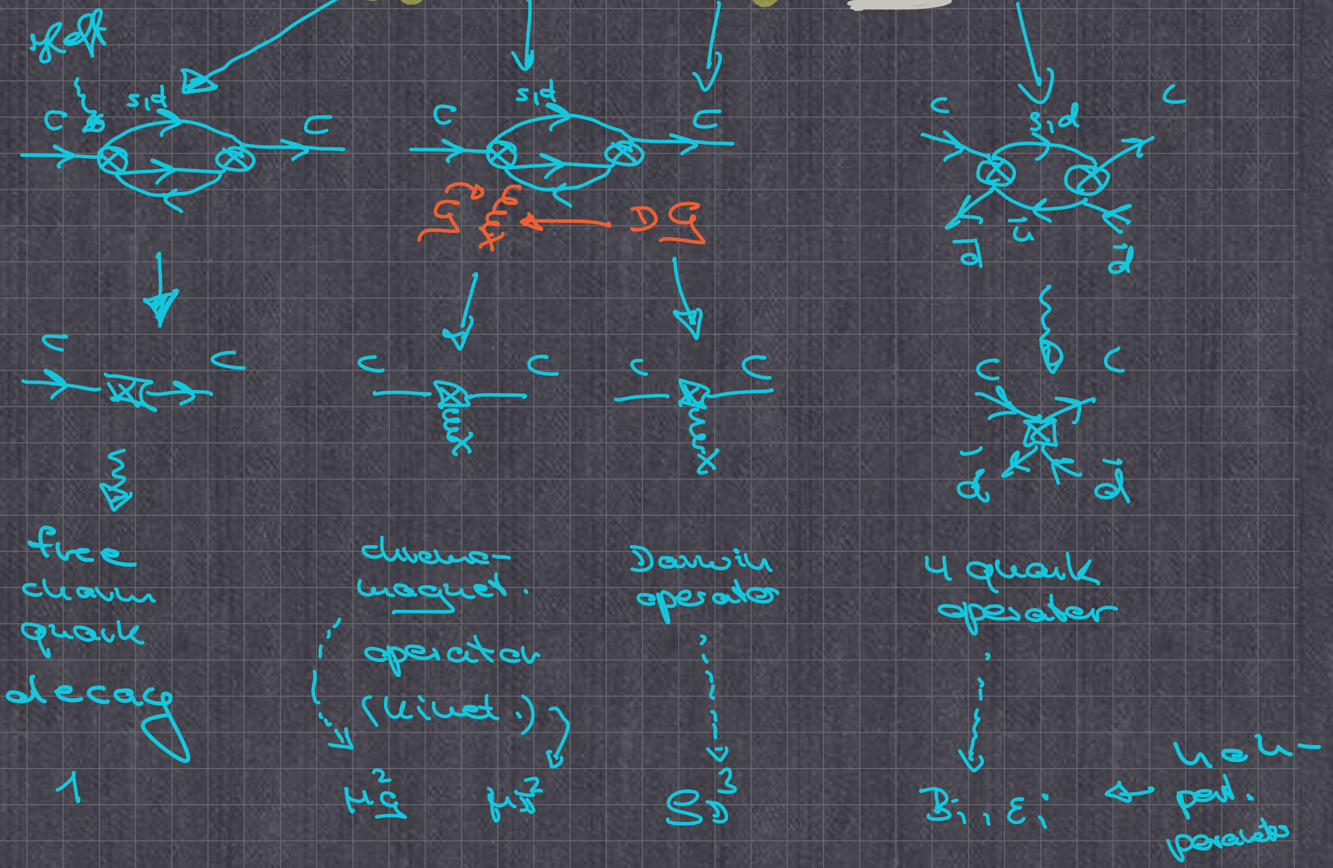
A) No Cancellations $\Gamma(D^0)$

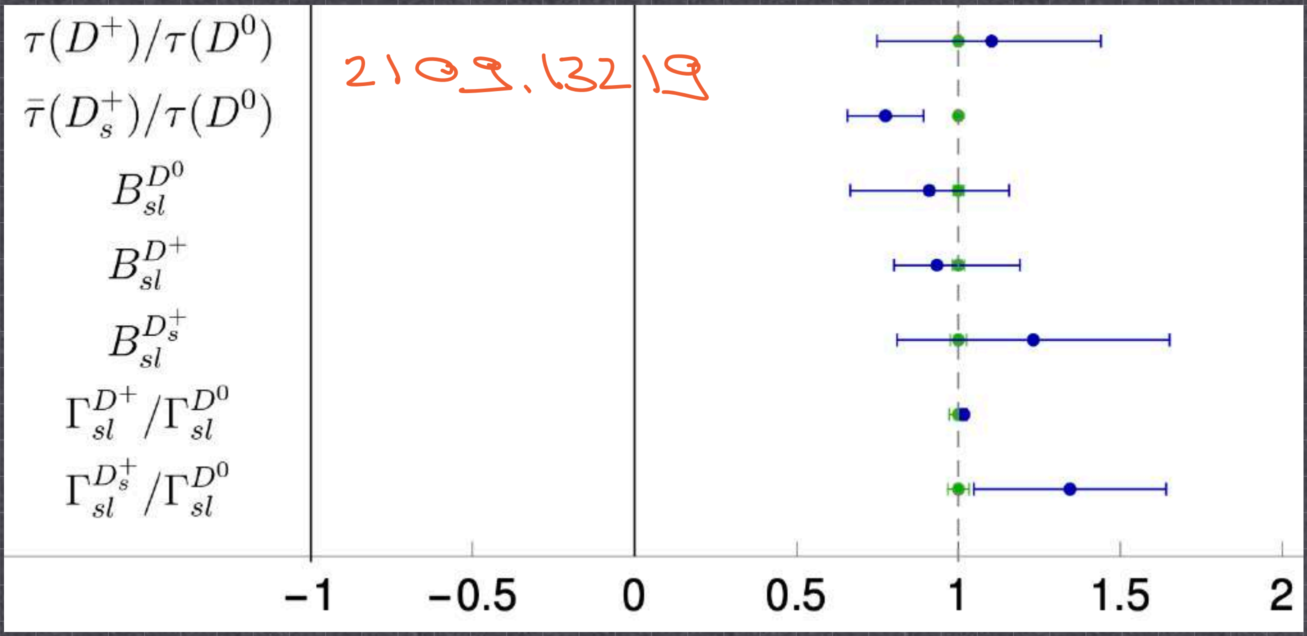
B) Strong Cancellations $\Gamma(D^+)$

C) Extreme Cancellations D-mixing

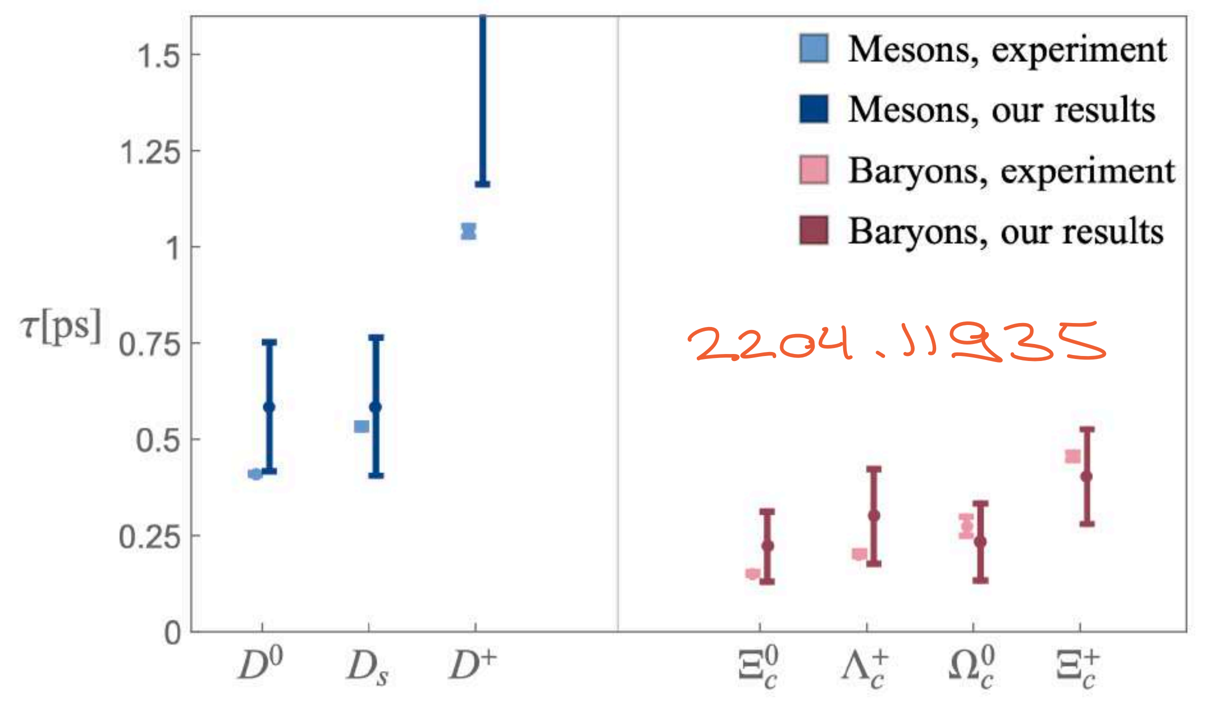
Does is make sense to expand in $1/m_c$?

$$\boxed{A}\ \Gamma(D^0) \overset{HQE}{=} \Gamma_3 + \Gamma_5 \frac{\langle O_5 \rangle}{m_c^2} + \Gamma_6 \frac{\langle O_6 \rangle}{m_c^3} + \ldots + \underline{16\pi^2} \left[\tilde{\Gamma}_6 \frac{\langle \tilde{O}_6 \rangle}{m_c^3} + \ldots \right]$$





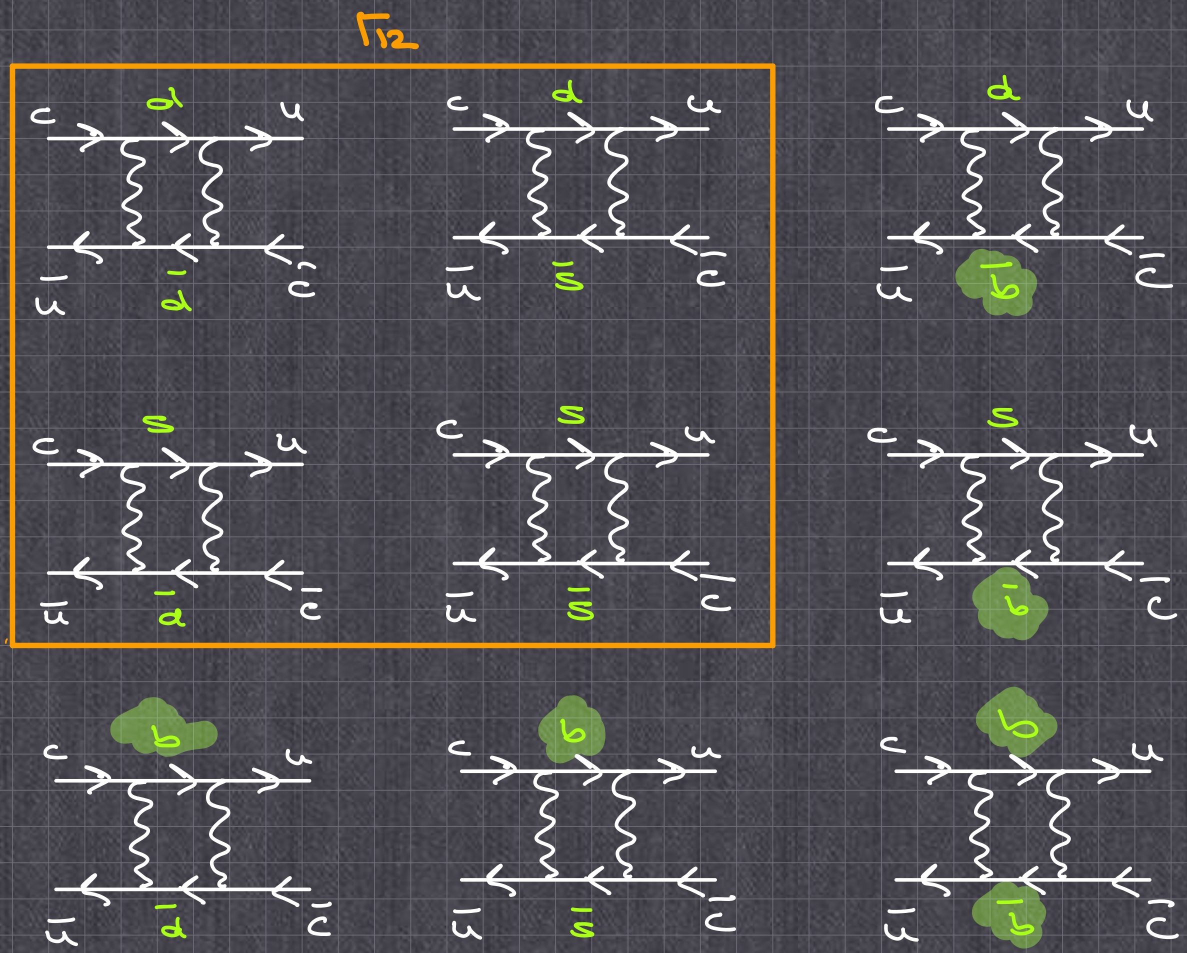
2109.13219

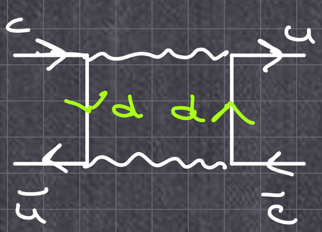


2204.11935

Looks promising!

so what about D-mixing?





+ ...

$$\begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ & & 1 \end{pmatrix}$$

V_{ub}

V_{cb}

CKM faktoren:

$$\lambda_d = \overset{\lambda^1}{\overset{\nearrow}{V_{cd}}} \overset{\lambda^0}{\overset{\nearrow}{V_{ud}^*}} \sim \lambda^1$$

$$\lambda_s = V_{cs} V_{us}^* \sim \lambda^1$$

$$\lambda_b = \underset{\lambda^2}{\underset{\downarrow}{V_{cb}}} \underset{\lambda^{3.8}}{\underset{\downarrow}{V_{ub}^*}} \sim \lambda^{5.8}$$

$$M_{12} = \lambda_d^2 F(d,d) + \lambda_d\lambda_s F(d,s) + \lambda_d\lambda_b F(d,b)$$
$$\lambda_s\lambda_d F(s,d) + \lambda_s^2 F(s,s) + \lambda_s\lambda_b F(s,b)$$
$$\lambda_b\lambda_d (F b,d) + \lambda_s\lambda_b F(s,b) + \lambda_b^2 F(b,b)$$

$$\lambda_d + \lambda_s + \lambda_b = 0 \Rightarrow \lambda_s = -\lambda_d - \lambda_b$$

$$= \lambda_d^2 \quad [F(d,d) - 2F(d,s) + F(s,s)] \rightarrow \text{extrem}^2 \text{ GIM supp}$$
$$+ 2\lambda_d\lambda_b [F(s,s) - F(d,s) + F(d,b) - F(s,b)] \leftarrow -{''}-$$
$$+ \lambda_b^2 \quad [F(s,s) - 2F(s,b) + F(b,b)]$$

$$\underbrace{}_{\text{strong CKM hierarchy?}} \qquad \underbrace{}_{\text{GIM suppressed}}$$

$$m_d = m_s = m_b \Rightarrow \underline{\underline{0}}$$

$\Rightarrow$ <u>orthogonal</u> situation to B-mix!

$$F(x,y) = f_0 + f\left(\frac{m_x^2}{m_W^2}, \frac{m_y^2}{m_W^2}\right)$$

$$\frac{m_d^2}{m_W^2} \simeq 0.$$

$$\frac{m_s^2}{m_W^2} \simeq 1.3 \cdot 10^{-6} \qquad \simeq 0$$

$$\frac{m_b^2}{m_W^2} \simeq 2.8 \cdot 10^{-3} \qquad \simeq 0$$

$$\overline{\underline{\underline{0}}}$$

$$\underbrace{F(0,0) - 2F(0,b) + F(b,b)}_{\text{strongly GIM suppressed}}$$

$$x = \frac{\Delta M_D}{\Gamma_D} = \left(4.09^{+0.48}_{-0.49}\right) \cdot 10^{-3}$$

$$y = \frac{\Delta\Gamma_D}{2\Gamma_D} = \left(6.15^{+0.56}_{-0.55}\right) \cdot 10^{-3}$$

<u>Naive</u> $\frac{\Delta\Gamma_D^{HQE}}{\Delta\Gamma_D^{Exp}} \simeq \underline{\underline{10^{-5} \ldots 10^{-4}}}$

$$y^{HQE} \simeq \lambda_s^2 \left[F(d,d) - 2F(s,d) + F(s,s) \right] \simeq 10^{-5} y^{Exp}$$

$$\lambda_s^2 F(d,d) \simeq 5 \cdot y^{Exp}$$

Potential Theory solutions:

ⓐ HQE breaks down for D-mixing

ⓑ Intrinsic uncertainty of about 20% for each $F(x,y)$

$\Rightarrow$ intrinsic uncertainty in y

$\sim 20\% \cdot 5 y^{Exp} \sim y^{Exp}$:)

$\Rightarrow$ we cannot do better

- Renormalisation scale

$$\underset{\substack{\downarrow \\ \mu_{dd}}}{F(d,d)} - 2\underset{\substack{\downarrow \\ \mu_{sd}}}{F(s,d)} + \underset{\substack{\downarrow \\ \mu_{ss}}}{F(s,s)}$$

Naive: $\mu_{dd} = \mu_{sd} = \mu_{ss}$

better: $\mu_{xy} \in [1\,\text{GeV}, 2m_c]$

$\mu_{dd}, \mu_{sd}, \mu_{ss}$: vary independently

$$\Rightarrow y^{HQE} \in [10^{-5} \dots 1.5]\, y^{Exp}$$

Outlook for D-mixing:

ⓐ direct lattice studies

~20a ?

ⓑ higher orders in the HQE

GIM cancellations less pronounced

ⓒ Exclusive Approach

* so far no first principle prediction
* simple estimate of phase space effects indicates

$y \sim 1\%$, $x \sim 1\%$

realistic

* progress needed also

for $\Delta A_{CP} = A(D^0 \to KK) - A(D^0 \to \pi\pi)$

but keep in mind

$\bar{B}_s \to D_s^+ \pi^-$

- V_{cb} is large ☺
- only 1 topology ☺
- only 1 leading term ☺

$\Rightarrow 5\,\sigma$ deviation

of Exp $\Leftrightarrow$ QCDf !

now $D^0 \to K^+K^-, \pi^+\pi^-$

- m_c is smaller ☹
- several topologies ☹
- penguins might be large ☹

on the positive side:

a lot of things to do in theory & experiment in the next years ☺

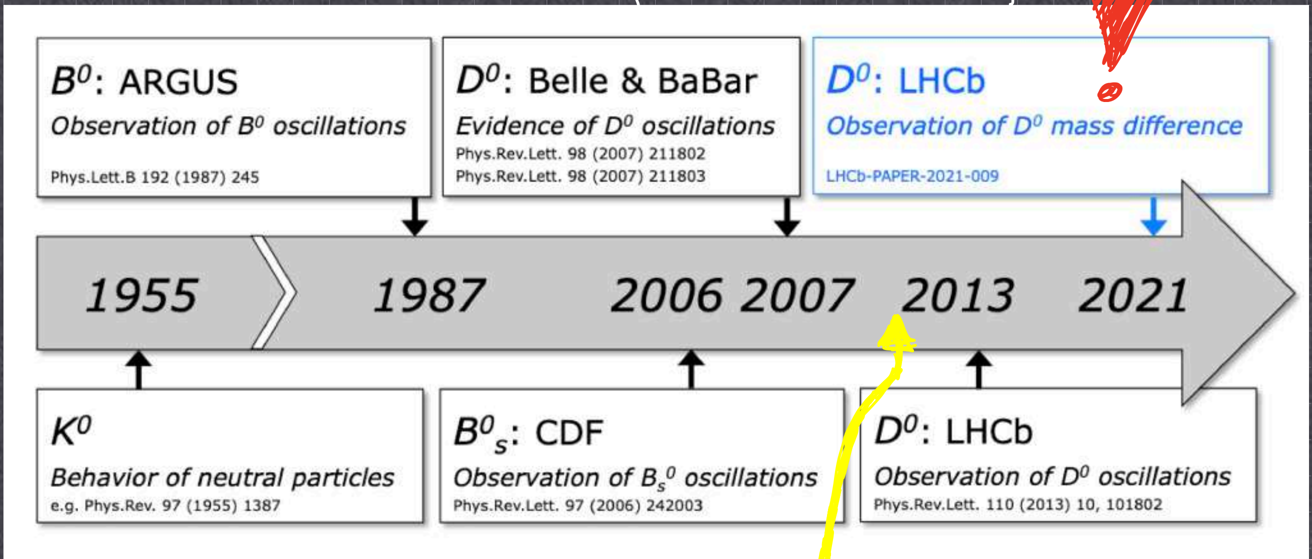
Mixing News: Charm 2021 !

→ 0.5 ps^{-1}

ΔM_d => wait !

3σ $x \to \Delta M/\Gamma$, $y \to \Delta\Gamma/2\Gamma$

$x = \frac{\Delta M}{\Gamma}$ 7σ



B^0: ARGUS
Observation of B^0 oscillations
Phys.Lett.B 192 (1987) 245

D^0: Belle & BaBar
Evidence of D^0 oscillations
Phys.Rev.Lett. 98 (2007) 211802
Phys.Rev.Lett. 98 (2007) 211803

D^0: LHCb
Observation of D^0 mass difference
LHCb-PAPER-2021-009

1955 1987 2006 2007 2013 2021

K^0
Behavior of neutral particles
e.g. Phys.Rev. 97 (1955) 1387

B^0_s: CDF
Observation of B_s^0 oscillations
Phys.Rev.Lett. 97 (2006) 242003

D^0: LHCb
Observation of D^0 oscillations
Phys.Rev.Lett. 110 (2013) 10, 101802

ΔM_s: 18 ps^{-1}

8σ $y = \frac{\Delta\Gamma}{2\Gamma}$

2012

Measurement of $\Delta\Gamma_s$ by LHCb

$\Delta\Gamma_s$: at least as interesting as ϕ_s & ΔM_s !

QCD: quite tricky → interesting BSM options

penguin pollution

QCD: relatively straight-forward → BSM test