String Inflation

Higher-dimensional Inflation from brane back-reaction

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On the shoulders of giants

With thanks to Ulf Danielsson

Outline

- Inflation and the UV
- A simple model
  - Flux compactification
  - Brane back-reaction
- Higher-dimensional inflation
Inflation and the UV

- Cosmology provides rare access to UV physics
  - high energies directly accessed when generating primordial fluctuations
  - ingredients for successful cosmology (like light scalar fields) are difficult to embed into UV
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  - can find plausible low-energy 4D EFTs with interesting cosmology – *is this good enough?*
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  - Control over all approximations (semi-classical limit, low-energy EFT, consistency of ingredients)
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  eg in Higgs inflation: semiclassical approx controlled by $E/\Lambda$, where

  $\Lambda \sim \frac{M_p}{\xi}$ when $h \sim 0$

  and $\Lambda \sim \frac{M_p}{\sqrt{\xi}}$ when $h \sim M_p$

  For comparison $E \sim H \sim \frac{M_p}{\xi}$ during inflation ($h \sim M_p$)
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  - Control...
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eg: couplings to other heavy particles

\[ V(\phi, \psi) = V_0(\phi) + M^2 \psi^2 + g \phi^2 \psi^2 + \cdots \]

\[ \delta m_\phi \propto M \]

or: ‘Planck slop’

\[ V(\phi) = V_0 \left( 1 + \frac{\phi^2}{M_p^2} + \cdots \right) \]

\[ V_0 = H^2 M_p^2 \]
Simple model

- Few explicit extra-dimensional cosmologies
  - work in 4D effective theory, or
  - work with moving branes in static backgrounds
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  - work in 4D effective theory, or
  - work with moving branes in static backgrounds
- What do extra dimensions do during inflation?
- Require explicit solution of higher-dim eqs
  - Must stabilize moduli
  - Include brane back-reaction
Simple model

- 6D Einstein-Maxwell-scalar system

\[ L = \frac{1}{2\kappa^2} \left[ R + (\partial \phi)^2 \right] + e^{-\alpha \phi} F_{mn} F^{mn} + V(\phi) \]

6D supergravity: \( \alpha = 1 \) and \( V = \left( \frac{2g_R^2}{\kappa^4} \right) e^\phi \)

Scale invariance: \( g_{\mu\nu} \rightarrow \lambda g_{\mu\nu} \) and \( e^{-\phi} \rightarrow \lambda e^{-\phi} \)
Simple model

• Simple solution

\[ ds^2 = \hat{g}_{mn} dx^m dx^n + [dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2] e^{-\alpha \phi_0} \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) e^{-\alpha \phi_0} \quad \phi = \phi_0 \]

Flat direction: \( r^2 = L^2 e^{\phi_0} \)
Simple model

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Field equations

\[ \frac{2g_R^2}{\kappa^2} = \frac{\kappa^2 Q^2}{2} \]

\[ \kappa^2 Q^2 L^2 = 1 \quad \hat{R} = 0 \]

Flux quantization

\[ \frac{n}{g} = 2 \alpha L^2 Q = \frac{\alpha}{g_R} \]
Simple model

How does system respond to changes in brane tension: \( T \rightarrow T_0 + \delta T(\phi) ? \)

Flux quantization: \[
\frac{n}{g} = 2\alpha L^2 Q = \frac{\alpha}{g_{\beta}}
\]

Obstructs \( T \) to \( \delta T \)?

\[
1 - \alpha = \frac{\kappa^2 T}{2\pi}
\]
Simple model

- Brane back-reaction: branes dictate boundary conditions in the near-brane limit

\[ e.g. \quad r \frac{d\phi}{dr} \to \frac{\kappa^2}{2\pi} \left( \frac{\delta S_b}{\delta \phi} \right) \]

- \( S(\phi) \) stabilizes \( \phi \), giving exponentially large \( r^2 = L^2 e^{-\phi} \)
Simple model

- Subdominant effects in the brane action are important for flux quantization

\[
\text{if } L_b = T_b(\phi) + \Phi_b(\phi) \ast F + \ldots
\]

\[
\text{then } \frac{n}{g} = Q \int dr \ e^{B - 4W} + \frac{1}{2\pi} \sum_b \Phi_b
\]

- \( \Phi \) has interpretation as brane-localized flux
Higher-dimensional inflation

- Exact time-dependent solution: \( e^{-\phi} = (H_0 \tau)^{c+2} \)

\[
ds^2 = (H_0 \tau)^c \left[ g_{mn} dx^m dx^n + \tau^2 (g_{ij} dx^i dx^j) \right]
\]

- FRW time in 4D Einstein frame: \( dt = \mp (H_0 \tau)^{c+1} d\tau \)

- If \( c = -2 \) then \( a(t) = e^{H_0 t} \) and \( r \) constant

- If \( c \neq -2 \) then \( a(t) = (H_0 t)^p \) and \( r(t) = (H_0 t)^{1/2} \)
  with \( p = (c + 1)/(c + 2) \)
  
  acceleration if \( p > 1 \) and so \( c < -2 \)
Higher-dimensional inflation

- What is the source and how does it end?

- Add inflaton $\chi$ to one of the two source branes

$$L_b = T + e^{-\phi}[ (\partial \chi)^2 + V_0 + V_1 e^{\lambda \chi} + \cdots ]$$

$$\chi = \chi_0 + \chi_1 \ln(H_0 \tau)$$

- Then $c + 2 = -\lambda \chi_1$ and $H_o^2 = \lambda V_1 / [\chi_1 (3 + 2\lambda \chi_1)]$
Conclusions

- Inflation not easy (but not impossible) to plausibly embed into UV physics
- Many pitfalls
  - Control of approximations
  - Necessity to control potential for all light scalars
  - The danger of those fields not explicitly written: naturalness and decoupling of heavy states
Conclusions

- Relatively little is known about explicitly higher-dimensional cosmology
- Branes and brane back-reaction can have important implications for low-energy theory
  - Little explored beyond codimension 1
Setup

- Simple solution (non-SUSY case)

\[ ds^2 = \hat{g}_{m n} dx^m dx^n + dr^2 + \alpha^2 L^2 \sin^2 \left( \frac{r}{L} \right) d\theta^2 \]

\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) \]

\[ \phi = \phi_0 \]

Field equations

\[ \frac{2}{L^2} = \kappa^2 \left( \frac{3Q^2}{2} + \Lambda \right) \]

\[ \hat{R} = \kappa^2 (Q^2 - 2\Lambda) \]

Flux quantization

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\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) \]

\[ \phi = \phi_0 \]

\[ Q = \frac{n}{2\alpha g L^2} \]

\[ \hat{R} = \kappa^2 (Q^2 - 2\Lambda) \]

\[ \frac{1}{L^2} = \frac{8\alpha^2 g^2}{3n^2 \kappa^2} \left[ 1 + \sqrt{1 - \left( \frac{3n^2 \kappa^4 \Lambda}{8\alpha^2 g^2} \right)} \right] \]
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\[ F_{r\theta} = Q\alpha L \sin \left( \frac{r}{L} \right) \quad \phi = \phi_0 \]

Tune \( \Lambda = \frac{Q^2}{2} \) so \( \hat{R} = 0 \)

If \( T \to T + \delta T \) then \( \hat{R} \to -\frac{\kappa^2 \rho}{\pi \alpha L^2} \) where \( \rho = 2 \delta T \)
**Calculation**

- **More general solutions**

\[ ds^2 = e^{2W} \hat{g}_{mn} dx^m \ dx^n + dr^2 + e^{2B} d\theta^2 \]

\[ F_{r\theta} = Q e^{B-4W} \quad \phi = \phi(r) \]
Calculation

- Perturb brane properties
  \[ T \rightarrow T + \delta T(\phi) \]
- To evade time-dependence add current
  \[ \Delta L_{bulk} = J\phi \quad \text{or} \quad \Delta L_{bulk} = J \]
- Find general solution to linearized equations
  \[ \kappa^2 JL^2 \ll 1 \]
Calculation

- Sample solutions

\[ \delta W = W_0 + W_1 \cos \left( \frac{r}{L} \right) \]

\[ \delta \phi = \phi_0 + \phi_1 \ln \left( \frac{1 - \cos(r/L)}{\sin(r/L)} \right) - \kappa^2 J L^2 \ln \left[ \sin \left( \frac{r}{L} \right) \right] \]

and so on
**Calculation**

- **Brane-bulk boundary conditions:**

\[
(e^B \phi')_b = \frac{\kappa^2}{2\pi} \left( \frac{\partial L_b}{\partial \phi} \right)
\]

\[
(e^B W')_b = \frac{\kappa^2}{4\pi} \left( \frac{\partial L_b}{\partial g_{\theta\theta}} \right) = U_b
\]

\[
(e^B B' - 1)_b = -\frac{\kappa^2}{2\pi} \left[ \left( \frac{\partial L_b}{\partial \phi} + \frac{3}{2} \frac{\partial L_b}{\partial g_{\theta\theta}} \right) \right]
\]

**Constraint:** \( 4U_b [2 - 2L_b - 3U_b] - \left( \frac{\partial L_b}{\partial \phi} \right)^2 = 0 \)
Calculation

• **Non-SUSY result:**

\[ V_{\text{eff}}(\phi) = \phi \int \frac{d\phi}{\phi^2} \left[ \frac{\pi \alpha L^2 \hat{R}(\phi)}{\kappa^2} \right] \]

\[ \rho = \sum_b \delta T_b(\phi_*) - 2Q \delta \Phi_b(\phi_*) \]

\[ \left[ \frac{\partial}{\partial \phi} \sum_b \delta T_b(\phi) \right]_{\phi_*} = 0 \]
Calculation

- SUSY result:

\[
\left[ \sum_b \delta T_b - 2Q\delta \Phi_b + \frac{1}{2} \frac{\partial}{\partial \phi} \delta T_b(\phi) \right]_{\phi_*} = 0
\]

ie Einstein frame potential: \( V = U(\phi)e^{2\phi} \)
Calculation

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\]

\[
\rho = \sum_b \delta T_b(\Phi_*) - 2Q \delta \Phi_b(\Phi_*) = \frac{1}{2} \sum_b \left( \frac{\partial}{\partial \Phi} \delta T_b \right)
\]
Calculation

**Intriguing choice:**

\[ \delta T_b = A + B \phi \]

with \( B \ll A \) since

\[ \rho = B \ll \delta T_b \]