Nature’s shortest length: UV-Completion by Classicalization.

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with:
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Fundamental physics is about understanding nature on various length-scales.

Some important examples are:

- Hubble length ($10^{28}$ cm),
- QCD-length ($10^{-14}$ cm),
- Weak-length ($10^{-16}$ cm), ...

and the shortest length scale of nature, $L_*$
At any given length-scale we describe nature in terms of an effective (field) theory valid at distances larger than a certain cutoff length, $L$. This length marks a size of a black box beyond which a given theory has to be replaced by a more powerful microscopic theory capable of resolving sub-structure at distances shorter than $L$. 

![Diagram of a black box with a line indicating the length $L$.]
Theories, describing nature at different length-scales, are embedded in one another like Russian dolls.
Usual (Wilsonian) picture of UV-completion assumes that such embedding has no end:
With enough energy, one can resolve nature at arbitrarily-short length-scales.

In this talk we are going to suggest an alternative possibility:

Certain theories incorporate the notion of a shortest length \( L_* \), beyond which no Wilsonian UV-completion exists.

Instead, at high energies theory self-completes itself by classicalization.
In such a theory a very high energy scattering is dominated by formation of big classical object, Classicalons.

So, a high-energy physics becomes a long-distance physics.

We suggest that one example of theory self-completed in this way is Einstein’s gravity. Gravity provides a shortest length of nature

\[ L_* = L_p = \frac{1}{M_p}, \]

beyond which length-scales cannot be resolved in principle.
The reason is simple. To resolve a distance $L$, one needs a microscope. Any microscope puts at least energy $E = 1/L$, within that distance. But energy gravitates, and when gravitational Schwarzschild radius exceeds $L$,

$$R = \frac{L^2_p}{L} > L,$$

the entire microscope collapses into a classical black hole!

Which becomes larger with the growing energy. Thus, harder we try to improve resolution of our microscope, worse it becomes.
Usual (Wilsonian) approach to UV-completion of non-renormaizable theories, with a cutoff length $L_*$, is to integrate-in some weakly-coupled new physics.
Our idea is to suggest an alternative, in which a theory “refuses” to get localized down to short distances, and instead of becoming short and quantum, becomes large and classical.

This can happen in theories that are (self)-sourced by energy (e.g., containing gravity or Nambu-Goldstone bosons).

In this case, the high-energy scattering is dominated by production of classical configurations (classicalons) of a certain bosonic field (classicalizer):

Theory classicalizes

In remaining 15 min, let me give you a general sense of this idea.
In a generic theory cross section can be written as a geometric cross section

\[ \sigma \sim r_*(E)^2 \]

where \( r_*(E) \) is a classical radius, not containing any dependence on \( \hbar \).

\( r_*(E) \) can be defined as a distance down to which, at given center of mass energy \( E \), particles propagate freely, without experiencing any significant interaction.

But, if a classical \( r_* \)-radius can be defined in any theory, what determines classicality of a given theory?
In a generic weakly-coupled theory, \( r_* \) is a classical radius, but it is much shorter than the other relevant quantum length-scales in the problem.

For example, in Thomson scattering the role of \( r_* \) is played by the classical radius of electron, \( r_* = r_e = e^2/m_e \),

but system is quantum because it is much shorter than relevant quantum length in the problem, the Compton wave-length of electron.
Another example is the gravitational Schwarzschild radius of electron,

\[ r_S = m_e L_p^2 , \]

where \( L_p = 10^{-33}\text{cm} \) is the Planck length.

Although \( r_S \) is a classical radius, electron is not a classical gravitating system, because \( r_S \) is much shorter than the Compton wave-length of electron,

\[ r_S \ll \frac{1}{m_e} , \]
The crucial point is, that in classicalizing theories $r_*(E)$-radius grows with $E$, and becomes much larger than any relevant quantum scale in the problem.

At this point the system Classicalizes!
To be short, consider (a tree-level) scattering in two theories,

\[ L = (\partial \phi)^2 - \lambda \phi^4 \]

and

\[ L = (\partial \phi)^2 + G (\partial \phi)^4 \]
In both theories cross section can be written as

$$\sigma \sim r_*^2,$$

where, $r_*$ is distance at which the correction to a free-wave becomes significant:

$$\phi = \phi_0 + \phi_1$$

where $\phi_0$ is a free wave and $\phi_1$ is a correction due to scattering.

In $\phi^4$-theory we have $r_* = \lambda / E$, and

$$\sigma = (\lambda / E)^2$$
Story of

\[ L = (\partial \phi)^2 + L_\ast^4 (\partial \phi)^4, \]

is very different.

First of all, notice that the problem contains two quantum lengths:

\[ L_\ast = (\hbar G)^{1/4} \text{ a length where quantum fluctuations become important} \]

and

\[ L = \hbar/E \]

Out of this two quantum scales we can make up a unique classical length:

\[ r_\ast = L_\ast (L_\ast /L)^{1/3} \]

What is its meaning?
\[ L = (\partial \phi)^2 + L_*^4 (\partial \phi)^4, \]

Now, \( \phi_1 \) is sourced by the energy of \( \phi_0 \).

Let at \( t=\infty \) and \( r = \infty \), \( \phi_0 \) be a free collapsing in-wave of a huge center of mass energy \( E \) and very small occupation number.

Because of energy self-sourcing,

\[
\partial^2 \phi_1 = -L_*^4 \partial (\partial \phi_0 (\partial \phi_0)^2)
\]

scattering takes place at a macroscopic classical distance

\[
r_* = L_* (E L_* )^{1/3}
\]
For example, take
\[ \phi_0 = A \frac{\exp((t+r)/a)^2}{r}, \]
where \( E = \frac{A^2}{a} \).

Then, for \( r \gg a \),
\[ \phi_1 = A \Theta(t-r) \frac{r^3}{(r(t-r)^3)} \]

In other words, the energy of the initial few hard quanta is converted into many soft quanta of momentum \( 1/r_\ast \)!
the scattering took place at a **macroscopic classical** distance,

\[ r_* = L_* (E L_*)^{1/3} \gg L_* \gg 1/E \]
Let us summarize what happened. By scattering very energetic particles, we thought that we could probe very short distances, such as $1/E$ (or at least, $L_*$).

Instead, the scattering took place at a macroscopic classical distance,

$$r_* = L_* (EL_*)^{1/3} >> L_* >> 1/E$$

At this distance $\phi_1$ becomes same order as $\phi_0$, and we simply cannot deny, that scattering took place.

The free-waves refuse to get localized:

Theory classicalizes!
An immediate consequence is, that $2 \rightarrow 2$ scattering takes place via very low momentum-transfer:

$$A_{2 \rightarrow 2} \sim (L*/r*)^{4/3}$$

The dominant process is

$$2 \rightarrow \text{classicalon} \rightarrow \text{many}$$

The total cross section becomes geometric

$$\sigma = r_* (E)^2 = L_*^2 (EL_*)^{2/3}$$
What does classicalization teach us about gravity?

Systems considered here share the bare essentials with gravity: existence of a classical length that grows with energy. In this sense, gravity is an universal classicalizer:

\[ 2 \rightarrow \text{Black Hole} \rightarrow \text{many} \]

From the point of view of unitarization, this is no different than

\[ 2 \rightarrow \text{Classicalon} \rightarrow \text{many} \]

Other peculiarities of the Black hole physics, such as entropy and (approximate) thermality, can be understood as accompanying properties of classicaliation.
To see this, do exactly the same scattering with
\[ \phi_0 = A \left[ \exp \left( \frac{(t+r)/a^2}{r} \right) \right]/r, \]
where again \( E = A^2/a \), but instead of self-coupling now couple \( \phi \) to gravity.

The role of \( \phi_1 \) is now played by graviton \( h_{\mu\nu} \).
Now, for \( r >> a \), we have
\[ h_{00} = \Theta(t+r) \ln(r-t) \left[ \frac{r_\ast}{r} \right] \]
Where \( r_\ast = E L_p^2 \)

Thus, black hole formation is nothing but classicalization with the radius given by \textbf{Schwarzschild}!
Demystification of black hole entropy.
Why (classical) black holes must carry entropy? Because Einsteinian Black Holes carry no hair.

Such Black Holes can only be distinguished by exactly conserved quantum numbers measurable at infinity. For example, such numbers are the mass of a Black Hole and its electric charge.

Other quantum numbers, e.g., such as quark and lepton flavors in the Standard Model are not exactly-conserved quantum numbers of nature, and are impossible to measure outside the Black Hole horizon.
One of the implications of no-memory is flavor-democracy of (semi-classical) Black Holes that can be visualized by the following thought experiment.

We can produce a large classical black hole by colliding particles of a given flavor, e.g., electron-positron. If the Hawking Temperature of this Black Hole is sufficiently high, it will evaporate in all three lepton generations (and in other possible species) fully democratically,

\[ e^+ + e^- \rightarrow \tau^+ \]

\[ \mu^- \]

\[ e^+ \]

\[ e^- \]

Thus, macroscopic Black Holes have no memory and they must carry entropy.
As one of the results of this absolute democracy, black holes must carry Bekenstein entropy:

\[ S = \frac{A}{L_p^2} = L_p^2 E^2 \]

But, what is the true (quantum) origin of the black hole entropy?

We understand very well, that eternal classical black hole is idealization. The production of real black holes as we have seen can be viewed as classicalization.

Then, \( S \) must be simply given by the log of number of micro-states that we cannot resolve classically.

What does classicalization tell us about this number?
This result demystifies the origin of the black hole entropy, and tells us, that it is simply given by the number of soft quanta, which make up the classicalon configuration.

Number of such soft quanta is:

\[ N = (r_*E) \]

which for a black hole case, \( r_* = E L_p^2 \), gives Bekenstein Entropy.

This is not surprising, since the number of states is exponentially sensitive to the number of particles making up a given classicalon.
By now, we understand, that black holes have no choice, they must have Bekenstein entropy, not just because they have horizon, but because they are classicalons made out of many soft quanta

\[ N = (r \ast E) . \]

This notion of entropy is equally applicable to any classicalon!

So, what is essential for unitarization is classicalization: Any consistent theory that in high energy scattering is dominated by

\[ 2 \rightarrow \text{Anything Classical} \rightarrow \text{many} \]

can (should) unitarize by classicalization.
This notion of entropy becomes less and less useful for small $N$. E.g., micro Black Holes (e.g., the ones that may be produced at LHC), will not have time to forget their origin, and therefore cannot carry a well-defined entropy.

Classicalon masses must be quantized.

Micro black holes will be qualitatively indistinguishable from classicalon resonances predicted by other classicalizing theories.
An example of a prediction:
Applying idea of classicalization to longitudinal WW-scattering:

\[ A_{2\rightarrow 2} \sim (\sqrt{s}/V)^{4/9} \]

and

\[ \sigma = V^{-2} (\sqrt{s} 250/V)^{2/3} \]

where \( V = 250 \) GeV

Above \( V \), scattering is dominated by production of \textit{W-classicalon} resonances, with quantized masses.
Cosmological implications?
After all, the Universe is a largest classicalon!

Number of soft gravitons in it:

\[ N = (r \ast E) = (H L_p)^{-2} \]

Can cosmological singularities be avoided by classicalization?

Some other ideas on cosmic classicalons

Berkhahn, Dietrich, Hofmann
Of course, there are many questions:

1) Does it work?
2) Is GR self-UV-completed by classicalization?
3) What does it mean from conventional point of view (in terms of degrees of freedom) to be classicalized?
4) What are the pheno-applications: Higgless SM? Yet another solution to the Hierarchy Problem?.....

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