Leptogenesis: CTP, CPT & Flavour

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Outline

I. Leptogenesis: Introduction & Standard Approach

II. The Closed-Time-Path (CTP) Formalism

III. Theory of Leptogenesis within the CTP Formalism

IV. Finite Density Corrections

V. Flavour Leptogenesis

I. Leptogenesis: Introduction & Standard Approach

Baryon Asymmetry of the Universe

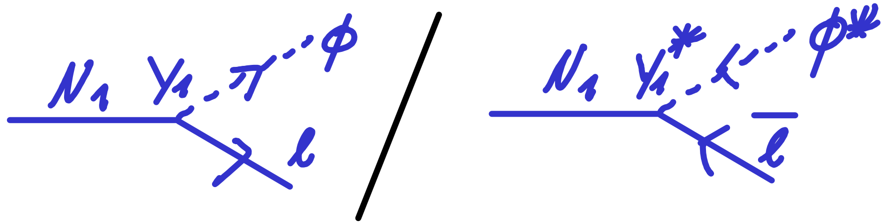
$$\frac{n_B}{s} = \begin{cases} (6{,}7 - 9{,}2) \cdot 10^{-11} & \text{Big Bang Nucleosynthesis} \\ (8{,}36 - 9{,}32) \cdot 10^{-11} & \text{Cosmic Microwave Background} \end{cases}$$

- Leptogenesis is a very plausible scenario for the dynamic generation of this asymmetry
- Theory challenge: Accuracy of predictions should match experimental precision

Leptogenesis

Fukugita & Yanagida (1986)

- A very plausible explanation for the matter-antimatter asymmetry of the Universe
- Requires heavy ($> 10^9$ GeV in many models) singlet Majorana neutrinos N_i
- Decay asymmetry



N_1 out-of equilibrium as required by CPT theorem

- Occurs when $M_{N_1} \sim T$ (largest interaction rates compared to H before Maxwell-suppression)
- (Inverse) decay rates $\left\{ \begin{matrix} \text{faster} \\ \text{slower} \end{matrix} \right\}$ than expansion H

  $\longrightarrow \left\{ \begin{matrix} \text{strong} \\ \text{weak} \end{matrix} \right\}$ washout
- Lepton asymmetry $\rightarrow \boxed{\text{sphalerons}} \rightarrow$ baryon asymmetry

Asymmetry from Kinetic Equations

$$\nabla_\mu j_\ell^\mu = \partial_t q_\ell - \underbrace{\vec{\nabla}\cdot\vec{j}_\ell}_{\equiv 0 \text{ for spatially homogeneous systems}} + 3Hq_\ell = \mathcal{C}$$

$\mathcal{C}$ → collision term

Boltzmann approach

- Quasi-particle picture
- Classical relativistic mechanics → continuity equation
- Scattering rates integrated over phase-space

$$\longrightarrow \mathcal{C} = \frac{1}{2E_\ell}\int\left\{\prod_i \frac{d^3p_i}{(2\pi)^3 2E_i}\right\}\delta^4(p_\ell \pm p_1 \pm p_2 \pm \ldots)\left\{|\mathcal{M}_{X\to\ell Y}|^2 - |\mathcal{M}_{X^*\to\bar{\ell}Y^*}|^2 - |\mathcal{M}_{\ell Y\to X}|^2 + |\mathcal{M}_{\bar{\ell}Y^*\to X^*}|^2\right\}$$

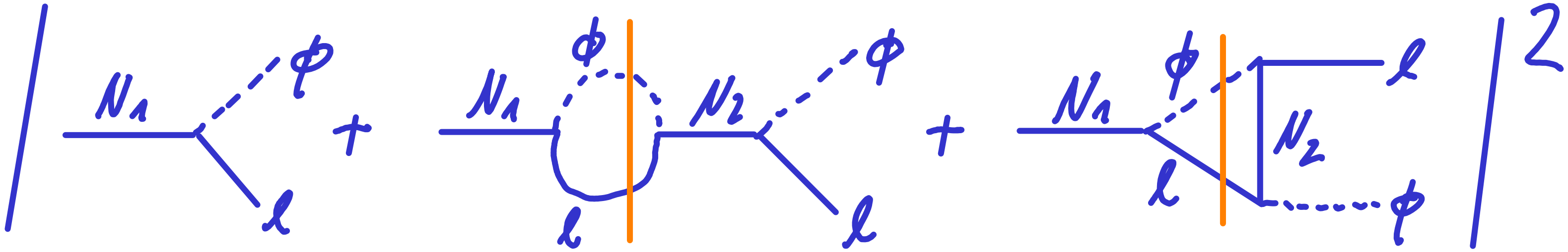
CTP approach:

- Schwinger-Dyson equations from first principles of QFT

Asymmetry in Boltzmann Approach

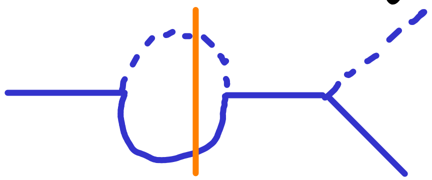
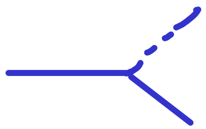
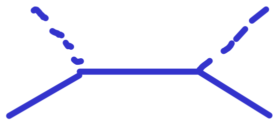
Fukugita & Yanagida (1986)
Covi, Roulet, Vissani (1996)
Buchmüller & Plümacher (1996)

- Interference of tree & loop amplitudes $\longrightarrow$ CP violation



(*)

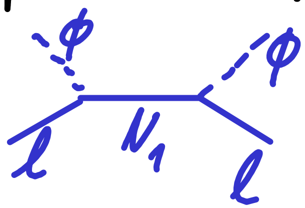
- CP violating contributions from disontinuities
  $\longrightarrow$ loop momenta where cut particles are on shell
  (Cutkosky rules)

- Is  an extra process or is it already accounted for by  and  ?

- Including (*) only $\longrightarrow$ CP asymmetry generated even in equilibrium

CPT, Unitarity & RIS

- Generation of CP asymmetry in equilibrium ↯ CPT theorem ↯
- N_1 unstable, not an asymptotic state of a unitary S-matrix
  $\longrightarrow$ Multiplication of matrix elements implicit in Boltzmann equations leads to non-unitary evolution
- Usual fix: subtract Real Intermediate States (RIS) from



Kolb & Wolfram (1980)

Why going beyond Boltzmann (and use CTP)?

- Resolve RIS/unitarity problem (in particular including quantum statistical factors $(1-f_{N_1})$, $(1-f_\ell)$, $(1+f_\phi)$)
  $\longrightarrow$ important for weak washout
- Build platform to systematically include higher order corrections/quantum coherence effects:
  * thermal masses & widths, collinear enhancement
  * flavour leptogenesis (oscillations of ℓ_i)
  * resonant leptogenesis (oscillations of N_i) [cf. De Simone & Riotto (2007)]
- Develop techniques/gain experience for other applications such as EWBG

II. The Closed-Time-Path (CTP) Formalism

Purpose of the Closed-Time-Path (CTP) Formalism

- Scattering theory: S-matrix elements between free asymptotic in- and out-states
- S-matrix elements are obtained from time-ordered Green functions (LSZ reduction formula)
- Here, we are interested in the expectation value of an operator without time-ordering:

  $\langle \overline{\psi}_\ell \gamma^0 \psi_\ell \rangle$ lepton charge density

$\longrightarrow$ Calculate this using the CTP formalism

Functional Approach (in-out)

- In-out generating functional for time ordered expectation values:

$$Z[J] = \mathcal{N}^{-1} \langle vac_{out} | vac_{in} \rangle_J = \int \mathcal{D}\phi \, e^{i\int d^4x \left(\mathcal{L} + J(x)\phi(x)\right)}$$

$$\langle T[\phi(x)\phi(y)] \rangle = -\frac{\delta^2}{\delta J(x)\,\delta J(y)} \log Z[J] \Big|_{J=0}$$

Calzetta & Hu (1987, 1988)

The Closed-Time-Path

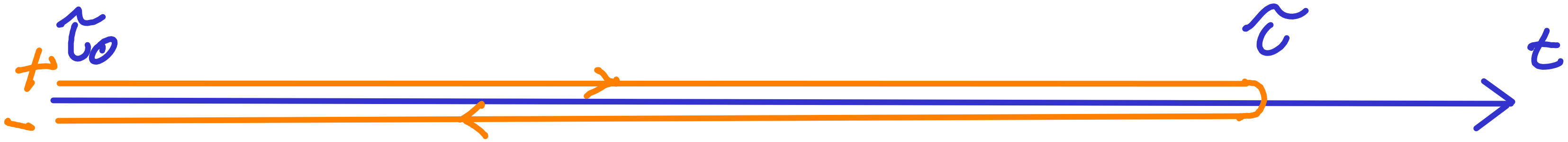
In-In generating functional:

$$\phi_{in}(\vec{x}) = \phi(\vec{x}, \tau_0)$$

$$Z[J_+, J_-] = \int \mathcal{D}\psi\, \mathcal{D}\phi^-_{in}\, \mathcal{D}\phi^+_{in} \langle \phi^-_{in} | \psi, \tau \rangle_{J_-} \langle \psi, \tau | \phi^+_{in} \rangle_{J_+} \langle \phi^-_{in} | \rho | \phi^+_{in} \rangle$$

$$= \int \mathcal{D}\phi^+ \mathcal{D}\phi^- e^{i\int d^4x \{ \mathcal{L}[\phi^+] + J_+\phi^+ - \mathcal{L}[\phi^-] - J_-\phi^- \}} \langle \phi^-_{in} | \rho | \phi^+_{in} \rangle$$

The Closed Time Path:



Path ordered Green functions:

$$i\Delta^{ab}_{\phi}(u,v) = -\frac{\delta^2}{\delta J_a(u)\, \delta J_b(v)} \log Z[J_+, J_-] \Big|_{J_\pm = 0}$$

$$= i \langle \mathcal{C}[\phi^a(u) \phi^b(v)] \rangle$$

Path Ordered Green Functions

$$i\Delta_\phi^{<}(u,v) = i\Delta_\phi^{+-}(u,v) = \langle \phi(v)\,\phi(u)\rangle$$

$$i\Delta_\phi^{>}(u,v) = i\Delta_\phi^{-+}(u,v) = \langle \phi(u)\,\phi(v)\rangle$$

$$i\Delta_\phi^{T}(u,v) = i\Delta_\phi^{++}(u,v) = \langle T[\phi(u)\,\phi(v)]\rangle$$

$$i\Delta_\phi^{\bar{T}}(u,v) = i\Delta_\phi^{--}(u,v) = \langle \bar{T}[\phi(u)\,\phi(v)]\rangle$$

(anti-) particle distributions

Free propagators in Minkowski-space:

$$i\Delta_\phi^{<}(p) = 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,f_\phi(\vec{p}) + \vartheta(-p_0)\,(1+\bar{f}_\phi(-\vec{p}))\right]$$

$$i\Delta_\phi^{>}(p) = 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,(1+f_\phi(\vec{p})) + \vartheta(-p_0)\,\bar{f}_\phi(-\vec{p})\right]$$

$$i\Delta_\phi^{T}(p) = \frac{i}{p^2-m_\phi^2+i\varepsilon} + 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,f_\phi(\vec{p}) + \vartheta(-p_0)\,\bar{f}_\phi(-\vec{p})\right]$$

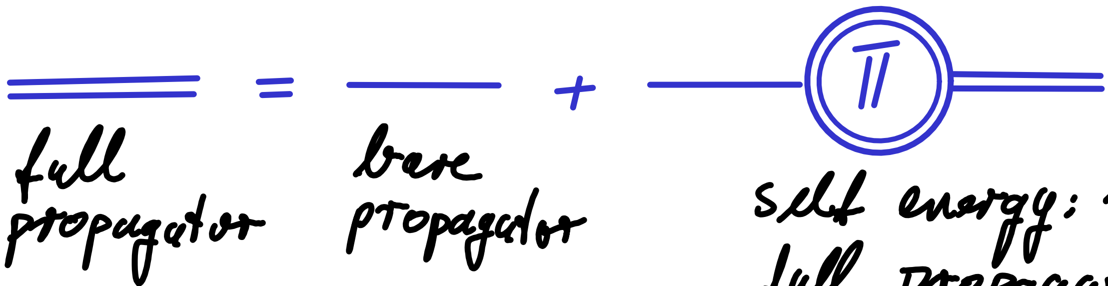
$$i\Delta_\phi^{\bar{T}}(p) = \frac{i}{p^2-m_\phi^2-i\varepsilon} + 2\pi\,\delta(p^2-m_\phi^2)\left[\vartheta(p_0)\,f_\phi(\vec{p}) + \vartheta(-p_0)\,\bar{f}_\phi(-\vec{p})\right]$$

Feynman Rules

- Vertices either $+$ or $-$
- Connect vertices $a = \pm$ and $b = \pm$ with $i\Delta^{ab}$
- Factor -1 for each $-$ vertex

Schwinger-Dyson Equations

- $$i\Delta^{ab} = i\Delta^{(0)ab} + cd\, i\Delta^{(0)ac} \odot \Pi^{cd} \odot \Delta^{db}$$



$$A(x,w) \odot B(w,y) = \int d^4w\, A(x,w)\, B(w,y)$$

Summary (CTP Formalism)

- Method for calculating complete set of Green functions.
- Useful for finite density systems out-of/in equilibrium and/or for space-time dependent backgrounds (QFT in curved space-time).
- The Schwinger-Dyson equations on the CTP describe <u>in principle</u> the full time evolution of the system.

III. Theory of Leptogenesis within the CTP Formalism

CTP approach to Leptogenesis

- More or less recent activities:

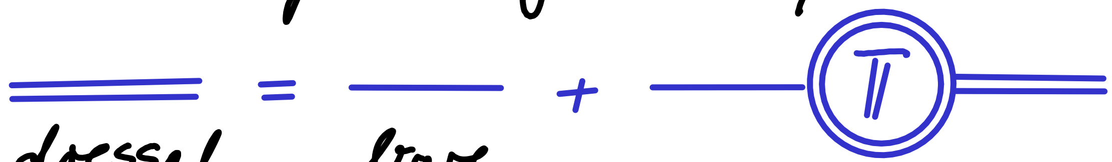
Buchmüller & Fredenhagen (2000)
De Simone & Riotto (2007)
Garny, Hohenegger, Kartatser & Lindner (2009, 2009)
Anisimov, Buchmüller, Drewes & Mendizabal (2010, 2010)
Garny, Hohenegger, Kartatser (2010)

Beneke, BG, Herranen, Schwaller (2010)
Beneke, BG, Fidler, Herranen, Schwaller (2010) This talk
BG (2010)

(10 papers hence far)

Kadanoff-Baym Equations

- Schwinger-Dyson Equations:



dressed propagator — bare propagator — self-energy

- The Kadanoff-Baym equations are the $<,>$ components:

$$(-\partial^2 - m^2)\Delta^{<,>} - \Pi^H \odot \Delta^{<,>} - \Pi^{<,>} \odot \Delta^H = \frac{1}{2}\left(\Pi^> \odot \Delta^< - \Pi^< \odot \Delta^>\right)$$

→ "continuity equation" — collision term

$\odot$: convolution → $[A \odot B](x,y) = \int d^4w \, A(x,w) B(w,y)$

- Remaining linear combination: mass-shell equations

$$(-\partial^2 - m^2) i\Delta^{R,A} - \Pi^{R,A} \odot i\Delta^{R,A} = i\delta^4$$

→ retarded/advanced propagator

Wigner Transformation

Calzetta & Hu (1988)

- Wigner transform:

$$A(k,x) = \int d^4r \, e^{ikr} A\left(x+\frac{r}{2}, x-\frac{r}{2}\right)$$

→ average coordinate — macroscopic evolution

→ relative coordinate — microscopic (quantum) properties

- For the convolutions, can show that:

$$\int d^4r \, e^{ikr} \int d^4w \, A\left(x+\frac{r}{2}, w\right) B\left(w, x-\frac{r}{2}\right) = e^{-i\diamond}\{A(k,x)\}\{B(k,x)\}$$

where $\diamond\{\cdot\}\{\cdot\} = \frac{1}{2}\left(\partial_x^{(1)}\cdot\partial_k^{(2)} - \partial_k^{(1)}\cdot\partial_x^{(2)}\right)$

⚠ Wigner & position space techniques yield different answers

cf. Anisimov, Buchmüller, Drewes & Mendizabal (2010, 2010)

- Position space result agrees when including damping of ℓ, ϕ ⟶ origin of discrepancy?

Gradient Expansion

Calzetta & Hu (1988)

- For slowly evolving system, expand in powers of

$$\partial_x \cdot \partial_k \sim H/T \sim T/m_{Pl} \ll 1$$

∂_k → typical momentum scale, i.e. T

∂_x → typical time scale, i.e. Hubble time H^{-1}

$\vec{\nabla}_x = 0$ for spatially homogeneous system

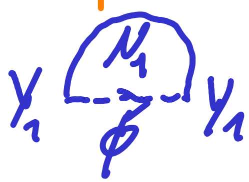
- Leptogenesis most efficient when

$$\Gamma_N = Y_1^2 \frac{1}{16\pi} M_1 \sim H \quad \text{and} \quad M_1 \sim T \implies Y_1^2 \frac{1}{16\pi} \sim H/T \ll 1$$

↷ Expand in $\partial_x \cdot \partial_k$ and Y_1^2 $(\sim T/m_{Pl})$

e.g. $$\not{\Sigma}_\ell \odot S_\ell \longrightarrow e^{-i\diamond}\{\not{\Sigma}_\ell(k,x)\}\{S_\ell(k,x)\} \approx \underbrace{\not{\Sigma}_\ell(k,x)\, S_\ell(k,x)}_{\text{local in time: "Markovian"}}$$

convolution
non-local in time

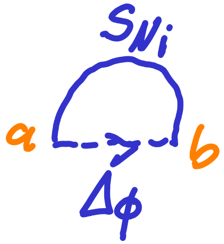


and $i\not{\partial} - m \longrightarrow i\gamma^0 \partial_t - \not{k} - m$

Tree Level Collision Terms

■ Leading order self-energies:

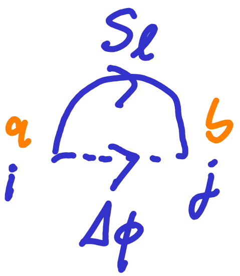
$$i\Sigma_\ell^{ab}(k) =$$



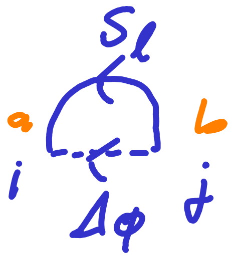
$$= |y_i|^2 \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k-k'-k'')$$

$$* P_R\, iS_{N_i}^{ab}(k')\, P_L\, i\Delta_\phi^{ba}(k'')$$

$$i\Sigma_{N_{ij}}^{ab}(k) =$$



$+$



2 from $SU(2)_L$

$$= g_w \int \frac{d^4k'}{(2\pi)^4}\frac{d^4k''}{(2\pi)^4}(2\pi)^4\delta^{(4)}(k-k'-k'')$$

$$* \Big\{ y_i y_j^* P_L\, iS_\ell^{ab}(k')\, P_R\, i\Delta_\phi^{ab}(k'')$$

$$+ y_i^* y_j\, C\left[P_L\, iS_\ell^{ab}(-k')\, P_R\right]^T C^\dagger\, i\Delta_\phi^{ba}(-k'')\Big\}$$

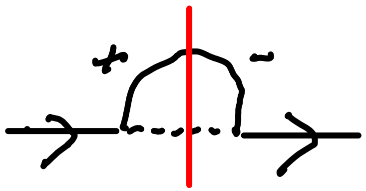
Tree Level Collision Terms (continued)

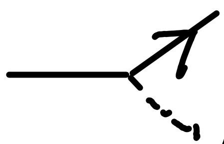
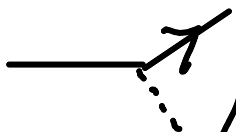
$$\frac{\partial}{\partial t}\int\frac{d^4k}{(2\pi)^4}\,\mathrm{tr}\,\gamma^0\, iS_\ell^{<,>}(k) = -\frac{\partial}{\partial t}\left(n_\ell-\bar n_\ell\right)$$

$$= -\int\frac{d^4k}{(2\pi)^4}\left[i\not{\Sigma}_\ell^{>}(k)P_L\, iS_\ell^{<}(k) - i\not{\Sigma}_\ell^{<}(k)P_L\, iS_\ell^{>}(k)\right]$$

$$= |Y_i|^2\int\frac{d^3k}{(2\pi)^3\,2|\vec k|}\,\frac{d^3k'}{(2\pi)^3\,2\sqrt{\vec k'^2+M_i^2}}\,\frac{d^3k''}{(2\pi)^3\,2|\vec k''|}\,(2\pi)^4\delta^4(k'-k-k'')\,2k\cdot k'$$

$$* \Big\{\left[1-f_{N_i}(\vec k')\right]*\left[f_\ell(\vec k)f_\phi(\vec k'')-\bar f_\ell(\vec k)\bar f_\phi(\vec k'')\right]$$
$$- f_{N_i}(\vec k')*\left[\left(1-f_\ell(\vec k)\right)\left(1+f_\phi(\vec k'')\right)-\left(1-\bar f_\ell(\vec k)\right)\left(1+\bar f_\phi(\vec k'')\right)\right]\Big\}$$



$\longrightarrow$ $\Big($  $\Big) * \Big($  $\Big)^* * f_N(1-f_\ell)(1+f_\phi)$

+ 3 additional combinations

Tree level in Boltzmann approach
$\leftrightarrow$ One loop in CTP approach

KMS* Relations

*Kubo-Martin-Schwinger

- Useful symmetry for simplifications close-to-equilibrium

$$iS_\ell^{eq>}(p) = -e^{\beta p_0}\, iS_\ell^{eq<}(p) \qquad iS_{N_i}^{eq>}(p) = -e^{\beta p_0}\, iS_{N_i}^{eq<}(p) \qquad i\Delta_\phi^{eq>}(p) = e^{\beta p_0}\, i\Delta_\phi^{eq<}(p)$$

- Generally holds for equilibrium Green functions, as can be shown in the imaginary time formalism
- Should also hold for the self-energies to all orders in perturbation theory:

$$i\Sigma_{\ell,N}^{eq>}(p) = -e^{\beta p_0}\, i\Sigma_{\ell,N}^{eq<}(p) \qquad i\Pi_\phi^{eq>}(p) = e^{\beta p_0}\, i\Pi_\phi^{eq<}(p)$$

KMS Relations (continued)

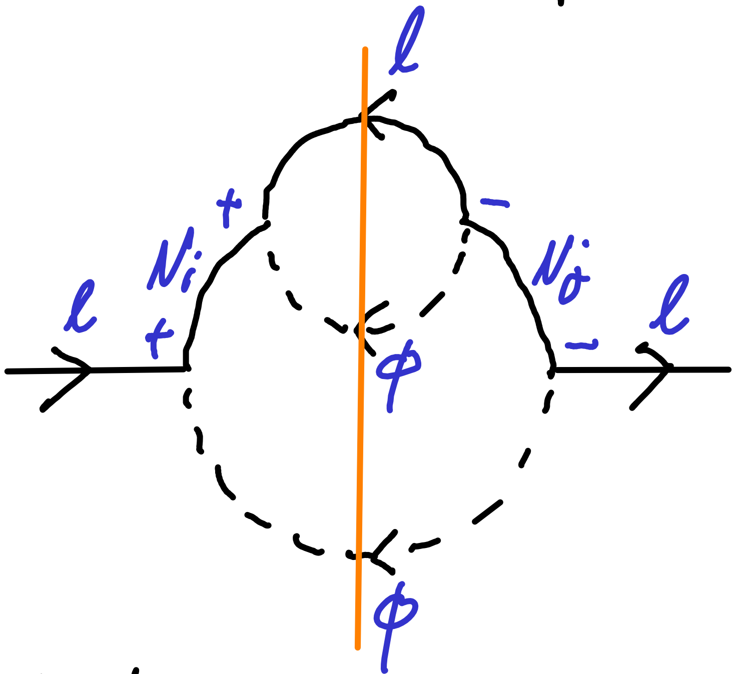
- KMS implies the vanishing of the collision term in equilibrium:

$$i\not{\Sigma}_\ell^{>}(k)P_L\, iS_\ell^{<}(k) - i\underbrace{\not{\Sigma}_\ell^{<}(k)}_{-e^{-\beta k^0}\not{\Sigma}_\ell^{>}(k)}P_L\, i\underbrace{S_\ell^{>}(k)}_{-e^{\beta k^0}S_\ell^{<}(k)} = 0$$

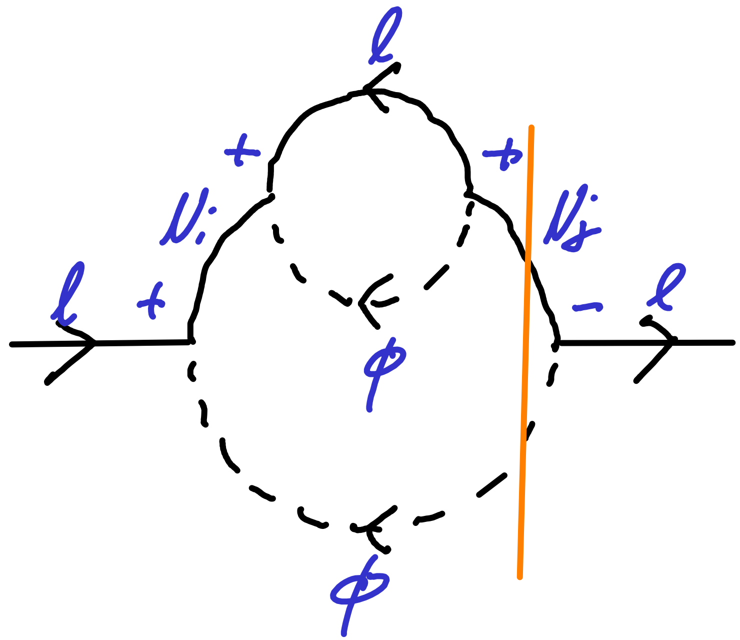
↷ Within the CTP formalism, get the vanishing of the asymmetry in equilibrium for free.

- Yet, it is instructive to demonstrate this explicitly in the present perturbative calculation for Leptogenesis

Wave-function Contribution



Interference between two s-channel scatterings.



Interference between loop and tree-level decays.

When neglecting quantum-statistical corrections (i.e. $(1-f_{\ell,N}) \longrightarrow 1$ and $(1+f_\phi) \longrightarrow 1$), and cutting as indicated, we recover the usual RIS-subtraction.

Result for Wave-Function Contribution

$$\int \frac{d^3k}{(2\pi)^3} \epsilon_\ell^{wf}(\vec{k}) = 4\, \mathrm{Im}\left[Y_1^2 Y_2^{*2}\right] \frac{M_1 M_2}{M_1^2 - M_2^2}$$

$$* \int \frac{d^3k'}{(2\pi)^3\, 2\sqrt{\vec{k}'^2 + M_1^2}}\, \delta f_N(\vec{k}') \frac{\Sigma_{N\mu}(\vec{k}') \Sigma_N^{\mu}(\vec{k}')}{g_w}$$

where we have the thermal decay rate

$$\Sigma_N^{\mu}(\vec{k}) = g_w \int \frac{d^3p}{(2\pi)^3\, 2|\vec{p}|} \frac{d^3q}{(2\pi)^3\, 2|\vec{q}|} (2\pi)^4 \delta^4(k-p-q)$$

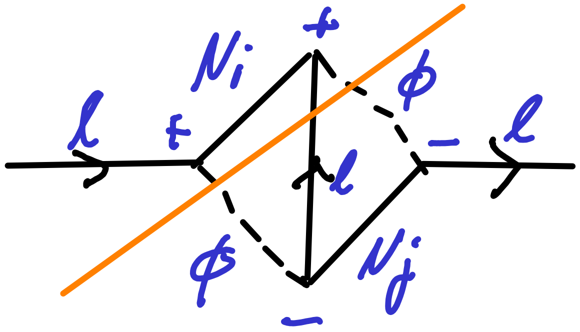
$$* p^{\mu}\left[1 - f_\ell(\vec{p}) + f_\phi(\vec{q})\right]$$

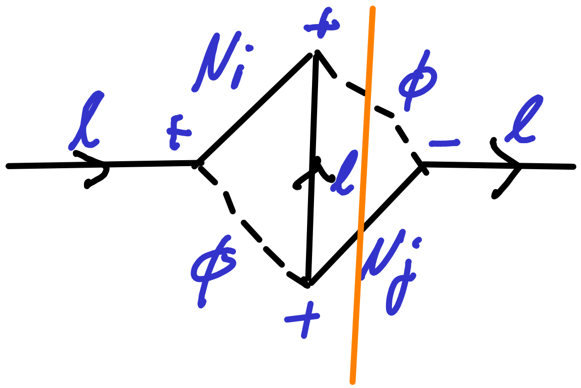
consistent derivation of quantum-statistical corrections

External phase-space & loop integral enter into the CP asymmetry at the same level.

$$\Sigma_N^{\mu}(\vec{k}) \xrightarrow[M_N \gg T]{} g_w \frac{k^{\mu}}{16\pi}$$ recover standard approximation

Vertex Contribution





Interference between s- and t-channel scatterings

Interference between loop and tree-level decays.

$$\int \frac{d^3p'}{(2\pi)^3} \epsilon^{V}(\vec{p}') = 4\,\mathrm{Im}\left[Y_1^2 Y_2^{*2}\right] \int \frac{d^3k}{(2\pi)^3\, 2\sqrt{\vec{k}^2+M_1^2}}\, \delta f_{N_1}(\vec{k})\, V(k)$$

$$V(k) = \int \frac{d^3p'}{(2\pi)^3 2|\vec{p}'|} \frac{d^3p''}{(2\pi)^3 2|\vec{p}''|} (2\pi)^4 \delta^4(k-p'-p'') p'^{\mu} \Gamma_\mu(k,p'') \left[1 - f_\ell(\vec{p}') + f_\phi(\vec{p}'')\right]$$

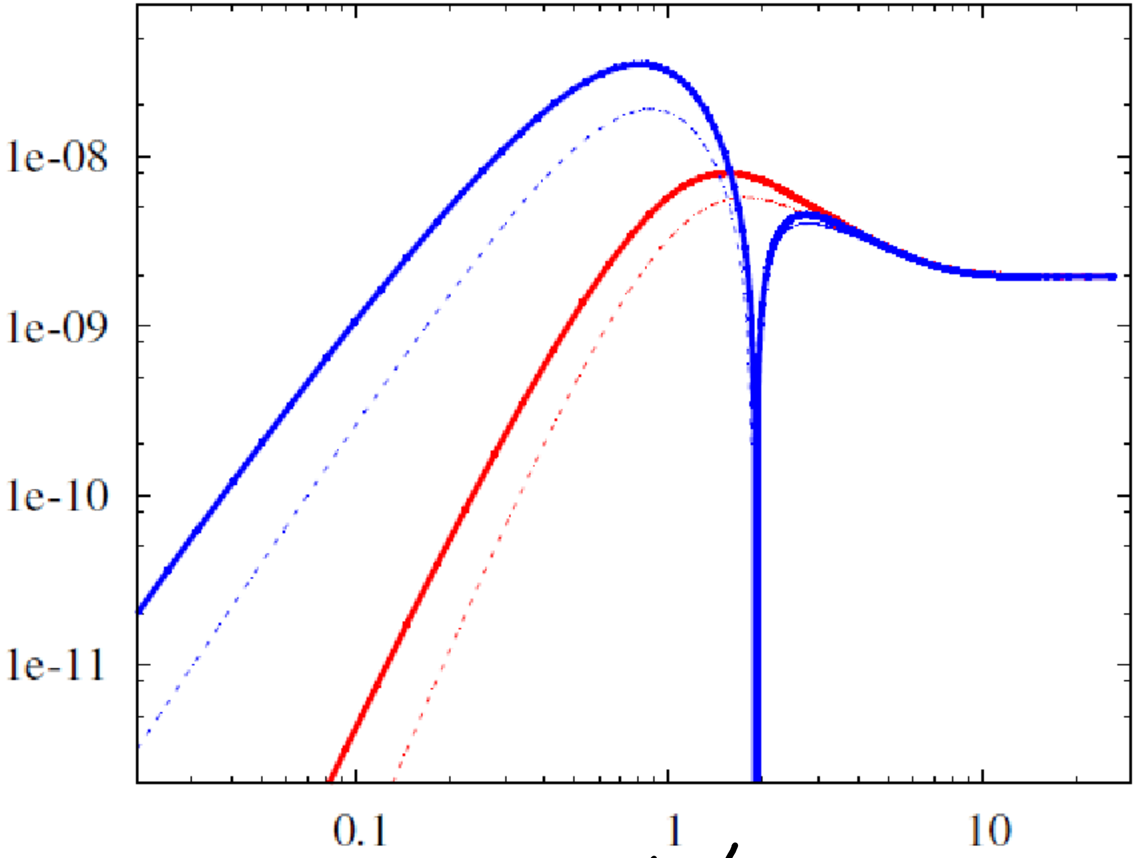
Thermal vertex function:

$$\Gamma_\mu(k,p'') = \int \frac{d^3k'}{(2\pi)^3 2|\vec{k}'|} \frac{d^3k''}{(2\pi)^3 2|\vec{k}''|} (2\pi)^4 \delta^4(k-k'-k'') k'_\mu \frac{M_1 M_2}{(k'-p'')^2 - M_2^2} \left[1 - f_\ell(\vec{k}') + f_\phi(\vec{k}'')\right]$$

IV. Finite Density Corrections

Numerical Results: Strong Washout

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



$z = M_1/T$

asymmetry first washed out & eventually freezes in in non-relativistic regime $\longrightarrow$ no quantum-statistical corrections

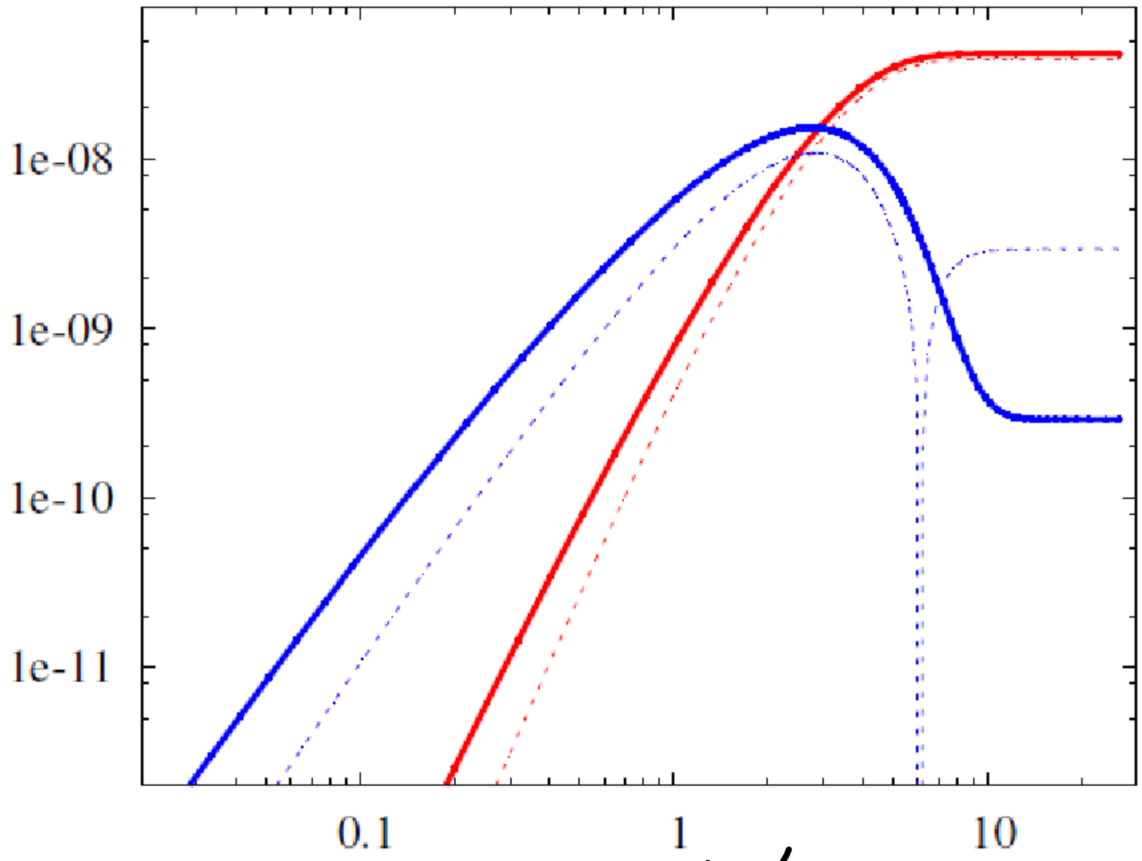
red: thermal initial f_{N_1} blue: zero initial f_{N_1}

solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 5 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

Numerical Results: Weak Washout

$$|Y_\ell| = \frac{n_\ell - \bar{n}_\ell}{s}$$



$z = M_1/T$

sign change for vanishing initial conditions

⚠ thermal corrections lead to $O(1)$ effects for $z \lesssim 0.5$

red: thermal initial f_{N_1} blue: zero initial f_{N_1}

solid: full solution dashed: no thermal corrections in loops

$M_1 = 10^{13}\,\mathrm{GeV}$ $M_2 = 10^{15}\,\mathrm{GeV}$ $Y_1 = 1 * 10^{-2}$ $Y_2 = 10^{-1}$ $\mathrm{Im}[Y_1 Y_2^*] = |Y_1 Y_2|$

V. Flavour Leptogenesis

Flavoured Leptogenesis

Abada, Davidson, Josse-Michaux, Losada, Riotto (2006)
Nardi, Nir, Roulet, Racker (2006)

- "Leptogenesis basis" in which Y_{ia} as in $Y_{ia} N_i^\dagger \varphi \ell_a$ is lower triangular generally different from lepton flavour basis where $h_{\tau a}$ as in $h_{\tau a} R_\tau^\dagger \varphi^\dagger \ell_a$ is diagonal
- For $T \lesssim 10^{12}\,\mathrm{GeV}$ ($10^9\,\mathrm{GeV}$, $10^4\,\mathrm{GeV}$) $h_{\tau\tau}$ ($h_{\mu\mu}$, h_{ee}) is in equilibrium (interactions faster than expansion H)
- Lepton charge densities projected on flavour basis (decoherence of flavour off-diagonal correlations)
  $\longrightarrow$ suppression of washout (because of "hidden" asymmetry)
- So far: either fully flavoured or unflavoured description; intermediate regime in heuristic Boltzmann/density matrix approach

Flavoured Leptogenesis in the CTP approach

- Schwinger-Dyson equations, Green functions straightforwardly decorated with flavour indices
- Need systematic approximations — account for flavour sensitive & flavour blind interactions & dispersion relations
- Flavour blind interactions through $W^{0,\pm}, B$ impose $\delta n^{+}_{\ell ab} = -\delta n^{-}_{\ell ab}$ [deviation of (anti-)lepton density from equilibrium]
- Flavour oscillations: $\delta n^{\pm}_{\ell ab} \sim \exp\left[\mp i \,\#\, \dfrac{m_a^{th\,2} - m_b^{th\,2}}{T} t\right]$

thermal masses like to induce flavour oscillations in opposite directions

$\delta n^{+}_{\ell ab}$

$\delta n^{-}_{\ell ab}$

$W^{0,\pm}, B$ like to keep these aligned

Gauge interactions win tug-of-war: oscillations overdamped

Suppression of Flavour Oscillations

Essential dynamics is captured by the toy system

$$\frac{d}{dt}\delta g^+(t) = -i\Delta\omega\,\delta g^+(t) - \Gamma^{bl}\left[\delta g^+(t) + \delta g^-(t)\right] \qquad \Gamma^{bl} \sim g_2^4 T$$

$$\frac{d}{dt}\delta g^-(t) = +i\Delta\omega\,\delta g^-(t) - \Gamma^{bl}\left[\delta g^+(t) + \delta g^-(t)\right] \qquad \Delta\omega \sim h_t^2 T \ll \Gamma^{bl}$$

$\longrightarrow$ short & long modes: $\delta g_{s,l} \simeq \delta g^+ \pm \left(1 \mp i\frac{\Delta\omega}{\Gamma}\right)\delta g^-$

$$\tau_{s,l}^{-1} = \Gamma^{bl} \pm \sqrt{\Gamma^{bl\,2} - \Delta\omega^2}$$

* identify long mode with q_l
* constrain $\delta g^+ + \delta g^- = 0$

$$\tau_s \simeq \frac{1}{2\Gamma^{bl}} \quad \text{pair creation/annihilation}$$

$$\tau_l \simeq \frac{2\Gamma^{bl}}{\Delta\omega^2} \sim \frac{g^4}{h_t^4 T} \gg \tau^{bl} \sim \frac{1}{g^2 h_t^2 T}$$

flavour oscillations over-damped because of fast pair creation/annihilation

$\longrightarrow$ Flavour sensitive damping dominates the dynamics of off-diagonal densities.

Flavoured Kinetic Equations

$$\boxed{\begin{aligned} \frac{\partial q_{\ell ab}}{\partial \eta} &= -\sum_c \left[W_{ac}\, q_{\ell cb} + q_{\ell ac}\, W_{cb} \right] + 2 S_{ab} - \Gamma^{fl}_{\ell ab} \\ \frac{\partial q_{Rab}}{\partial \eta} &= -\Gamma^{fl}_{Rab} \end{aligned}}$$

washout

source

Can work in fixed basis, since oscillations are frozen in.

Take $h = \begin{pmatrix} h_{\tilde{\tau}} & 0 \\ 0 & 0 \end{pmatrix}$

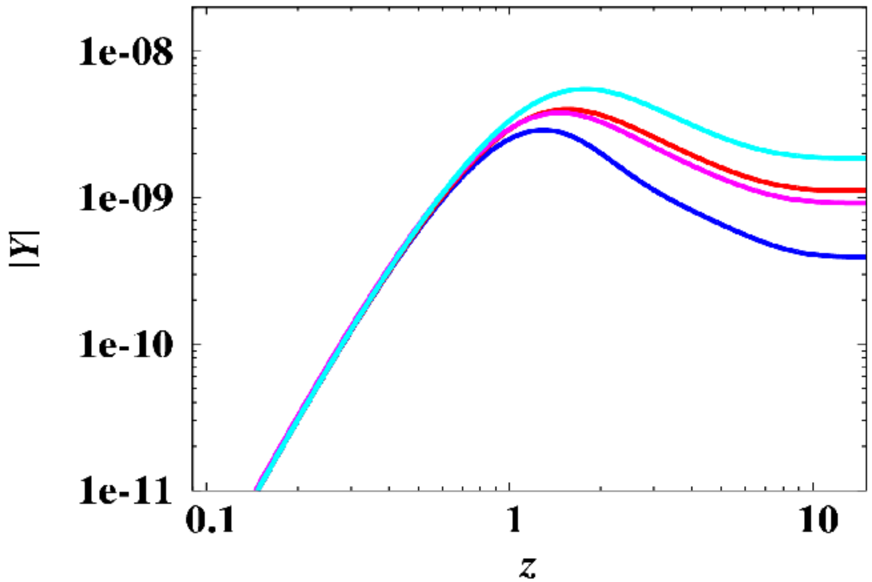
$$\Gamma^{fl}_{\ell} \sim h_{\tilde{\tau}}^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_\ell + q_\ell \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{R11} & 0 \\ 0 & 0 \end{pmatrix} \right]$$

$$\Gamma^{fl}_{R} \sim h_{\tilde{\tau}}^2 \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} q_R + q_R \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} - 2 \begin{pmatrix} q_{\ell 11} & 0 \\ 0 & 0 \end{pmatrix} \right]$$

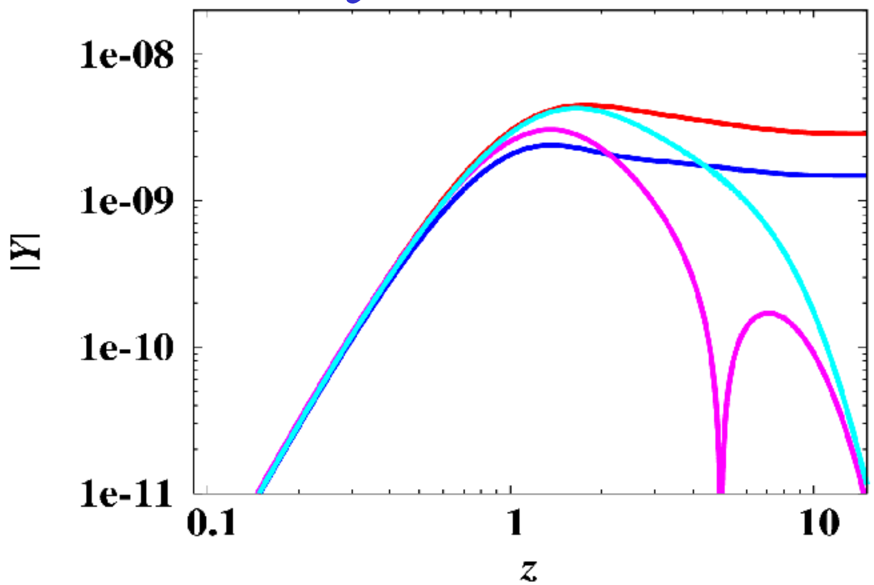
$\longrightarrow$ off-diagonals suppressed for $\Gamma^{fl}_{\ell,R} \gg H, \Gamma^{ID} = \Gamma_{\ell\phi \to N_1}$

Suppression of the off-Diagonals

$h_\tau = 0$



$h_\tau = 7 * 10^{-3}$



In flavour basis: $\begin{pmatrix} Y_{\ell 11} & Y_{\ell 12} \\ Y_{\ell 21} & Y_{\ell 22} \end{pmatrix}$ lepton number to entropy ratio

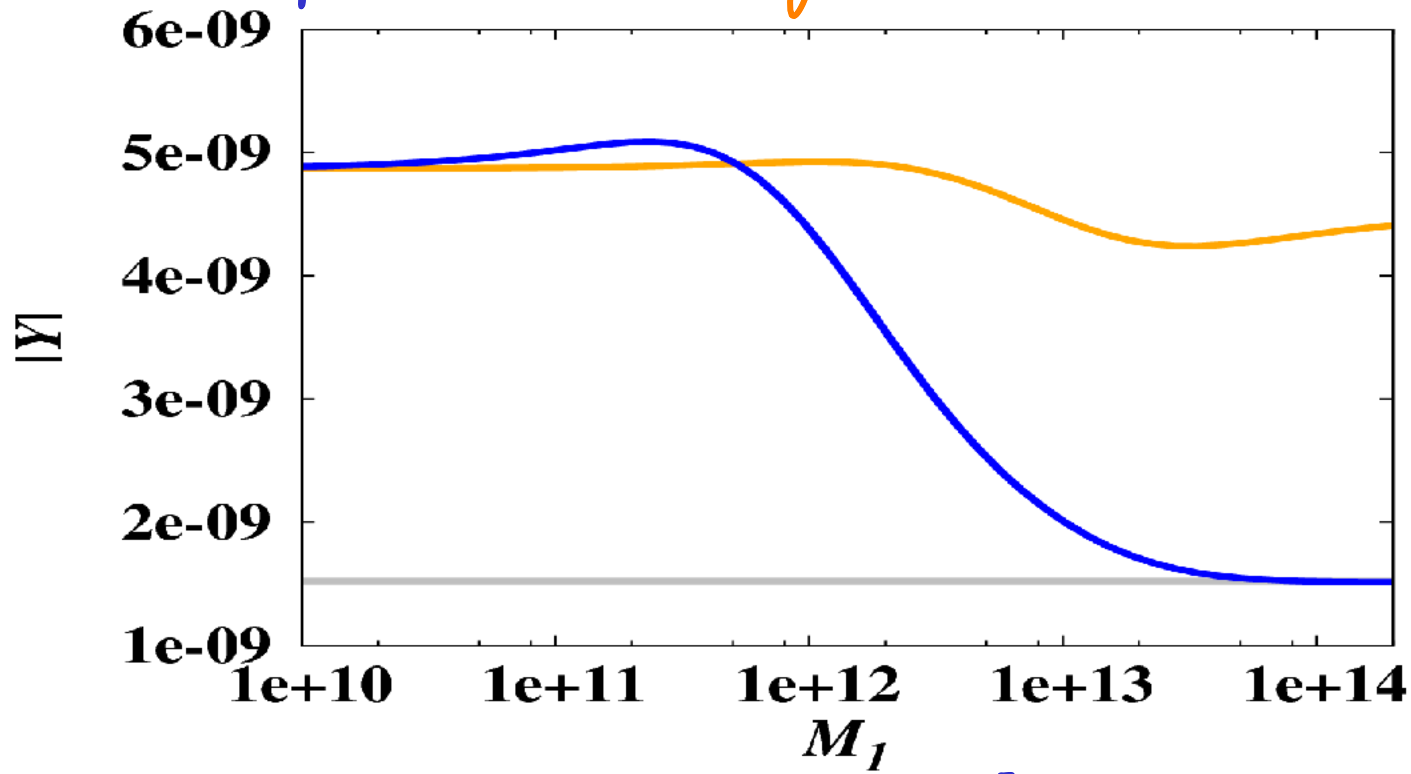
$$\begin{array}{c} Y = \left.\begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix}\right\} \text{r.h. neutrino} \\ \qquad l_\tau \equiv l_1 \qquad\qquad l_{\mu,e} \equiv l_2 \end{array}$$

$M_1 = 10^{12}$ GeV

$M_2 = 10^{14}$ GeV

Full Result Interpolates Between Flavoured/Unflavoured Limits

full result / fully flavoured / unflavoured



$h_\tau = 7 * 10^{-3}$

$$\left.\begin{aligned} M_1 &\longrightarrow \alpha M_1 \\ Y_{11} &\longrightarrow \alpha Y_{11} \\ Y_{12} &\longrightarrow \alpha Y_{12} \end{aligned}\right\} \text{fixed } Y_\ell \text{ in the unflavoured limit}$$

$$Y = \begin{pmatrix} 1.4 * 10^{-2} & 1 * 10^{-2} \\ i * 10^{-1} & 10^{-1} \end{pmatrix} \begin{matrix} \text{r.h.} \\ \text{neutrino} \end{matrix}$$

$\ell_\tau \equiv \ell_1 \qquad \ell_{\mu,e} \equiv \ell_2$

Conclusions

- Work of various groups now agrees on the unflavoured scenario and finite density corrections, but debate on approximations to get there
- First consistent description of flavoured Leptogenesis in the intermediate regime
- Potential impact on the phenomenology of some areas (flavour Leptogenesis, weak washout, N_2-Leptogenesis, resonant Leptogenesis) $\longrightarrow$ Will explore!
- First principle description of an important process in the Early Universe