SIMULATING NEUTRINOS IN LARGE SCALE STRUCTURES

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CERN - GENEVE 17th June 2011 – PPC 2011

OUTLINE

- Quantifying the impact of neutrinos on cosmological observables

- Simulating neutrinos beyond linear theory: neutrinos and LSS

- Review of (tightest) constraints on neutrino masses

- Sterile neutrinos and the coldness of cold dark matter at small scales

EVOLUTION of COSMOLOGICAL LSS – I: methods

Linear theory -- use popular codes like CAMB http://camb.info/ or CLASS http://class-code.net

Non-linear evolution -- approximations (e.g. Lognormal modelling /Peacock & Dodds,PT etc.) or N-body/hydrodynamic/adaptive mesh refinement parallel codes

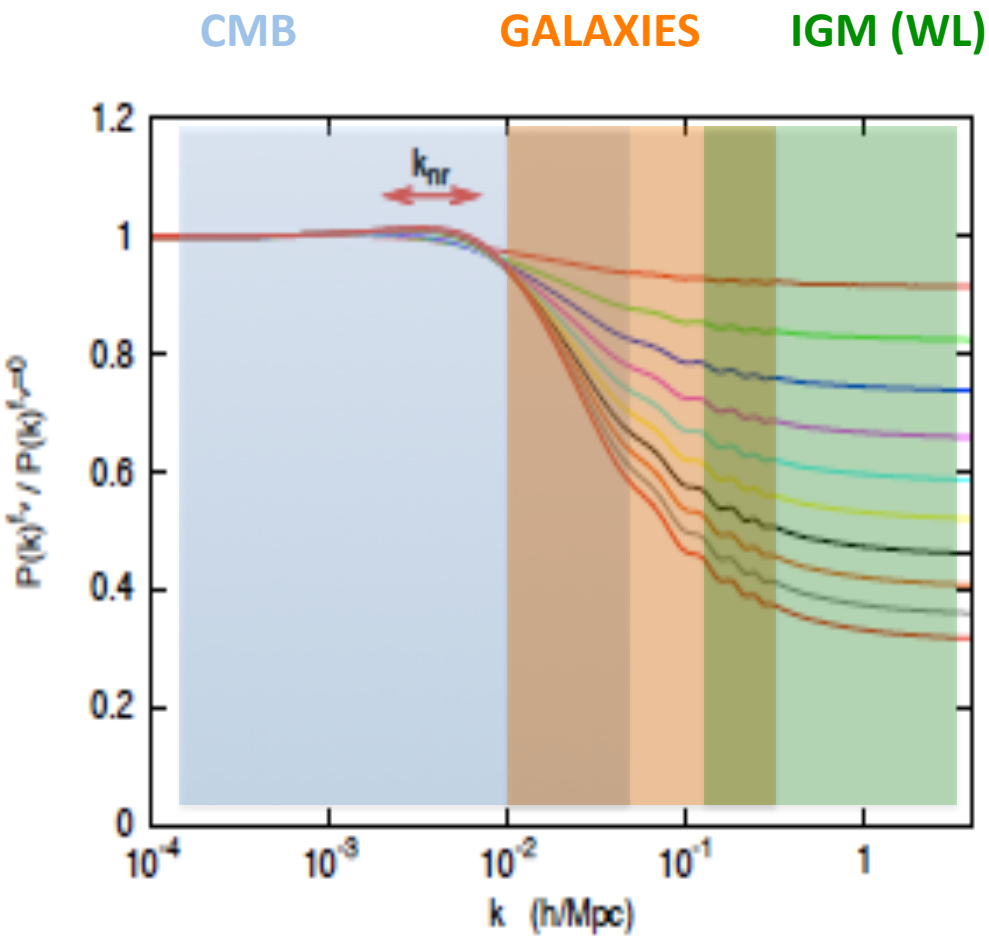
Early simulations: **direct summation** method for the gravitational N-body problem (still useful for stellar systems) Holmberg 1941, Aarseth 1979, Peebles, White etc.

Improvement made in the 90s to compute large scale force via Fourier/**mesh techniques** **Tree algorithms** arrange particles in groups and compute forces by summing over multipole expansions.

These two have been combined into **Tree+PM** codes, that could include hydrodynamic processes using for example the smoothed particle hydrodynamics (**SPH**, Lucy 1977) .

Hydrodynamic processes are important at small scales

EVOLUTION of LSS –II : dynamics in the linear regime



Effects in terms of matter clustering,
Hubble constant,
Energy density

(see Lesgourgues & Pastor 2006)

Different evolution in terms of **dynamics** and **geometry** as compared to massless neutrino universes

SIMULATION of LSS – I: basic equations

$f(\mathbf{r},\ v,\ t)$

DM

$$\frac{\mathrm{d}f}{\mathrm{d}t} \equiv \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0$$

Collisionless Boltzmann Equation

$$\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \int f(\mathbf{r}, \mathbf{v}, t)\, \mathrm{d}\mathbf{v}$$

Poisson equation

N-body problem: follow the Newton's equation of motions for a large number of particles under their own self-gravity

GAS

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \mathbf{v} = 0$$

Continuity

$$\frac{\mathrm{d}u}{\mathrm{d}t} = -\frac{P}{\rho}\nabla \cdot \mathbf{v} - \frac{\Lambda(u, \rho)}{\rho}$$

Energy

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{\nabla P}{\rho} - \nabla \Phi$$

Euler

$$P = (\gamma - 1)\rho u$$

Gas eq.of state

SIMULATION of LSS – III: historical background

1980: Bond et al. 1980 – linear theory (also Russian school with Zeldovich)
1983: Bond et al. – Evolution of Boltzmann-Einstein equations. **Clustering properties of galaxies not reproduced if the universe is dominated by neutrinos** (White et al. 1983) – numerical experiment

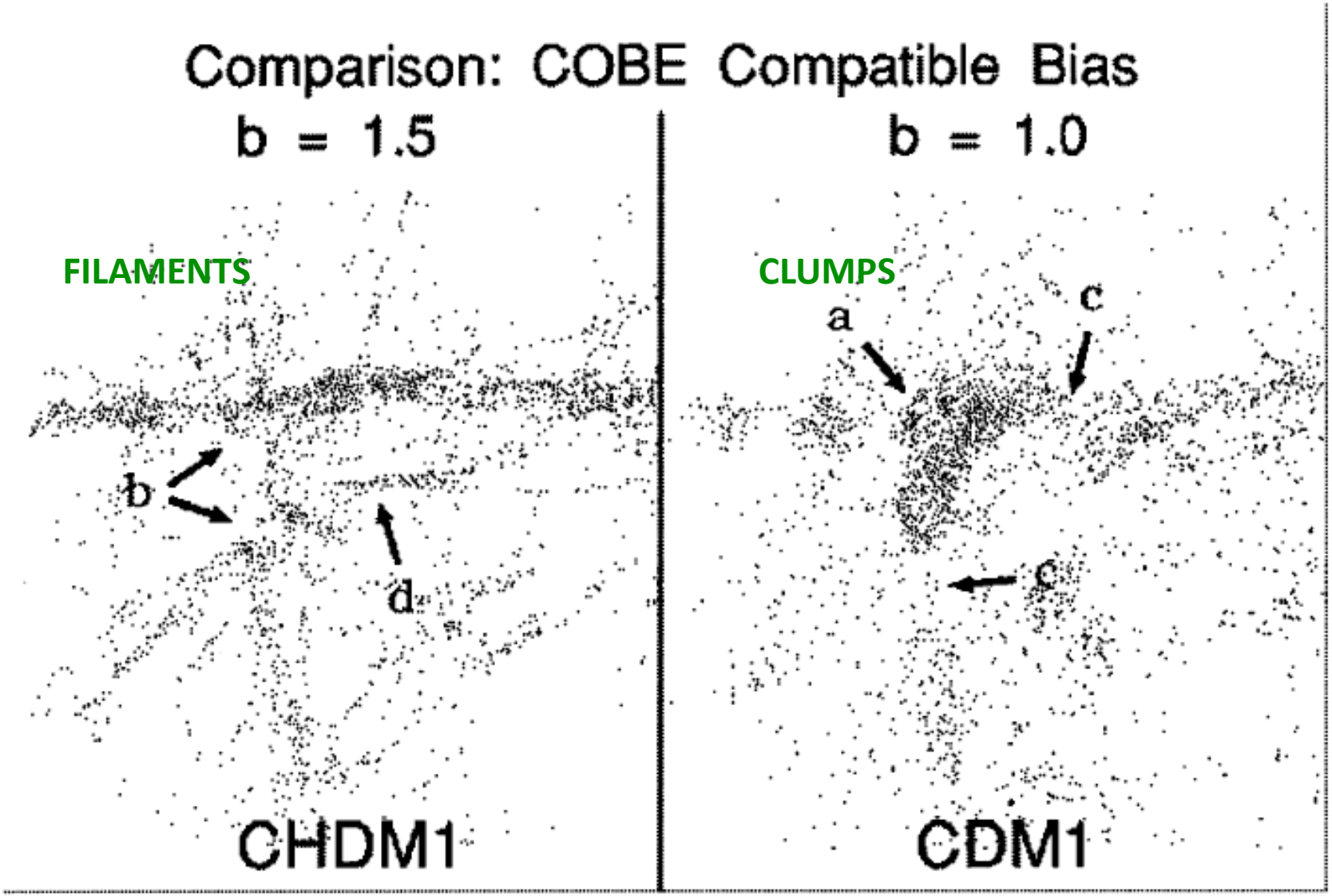
1992: Davis et al. HDM or CHDM (MDM) models P3M codes with neutrino particles placed as the dark matter ones (same CDM spectrum + velocities): 32^3 particles

1993: Klypin et al. 2 x 128^3 particles at z_IC=14 with the right power spectrum

1994: Ma & Bertschinger approximate linear scheme evolved at z=13 and after that pure N-body

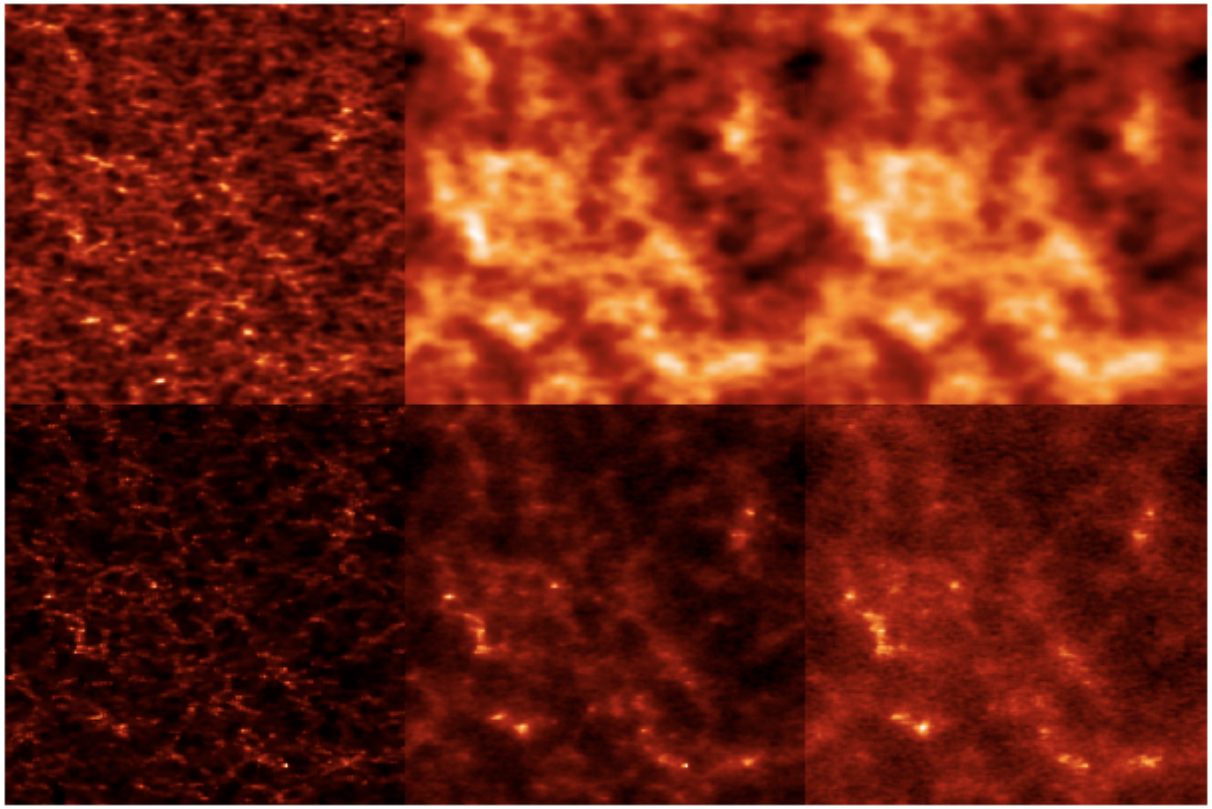
SIMULATION of LSS – IV: the distribution of matter

Pure HDM not allowed. However CHDM is still viable and impacts on the cosmic web



Brodbeck et al. 98

N-body simulations – I: particles



z=4

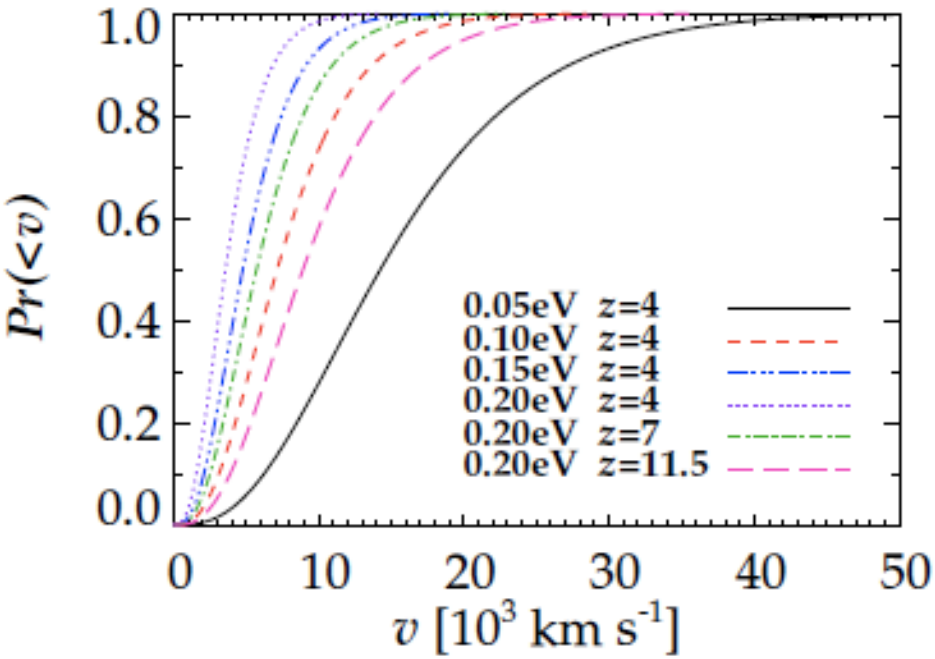
z=0

COLD DM NEUTRINOS 0.6 eV NEUTRINOS 0.3 eV

Brandbyge, Hannestad, Haugbolle, Thomsen 08

Simulation of neutrinos as an independent set of particles that interact gravitationally

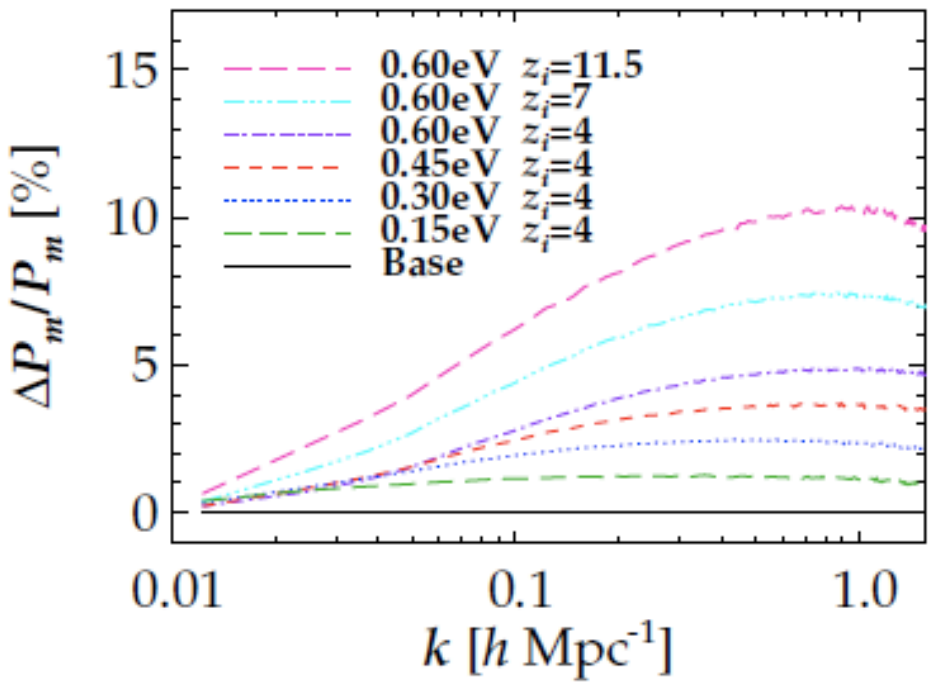
N-body simulations – II: neutrino velocities do matter!



$$T_\nu \simeq T_\gamma (4/11)^{1/3}$$

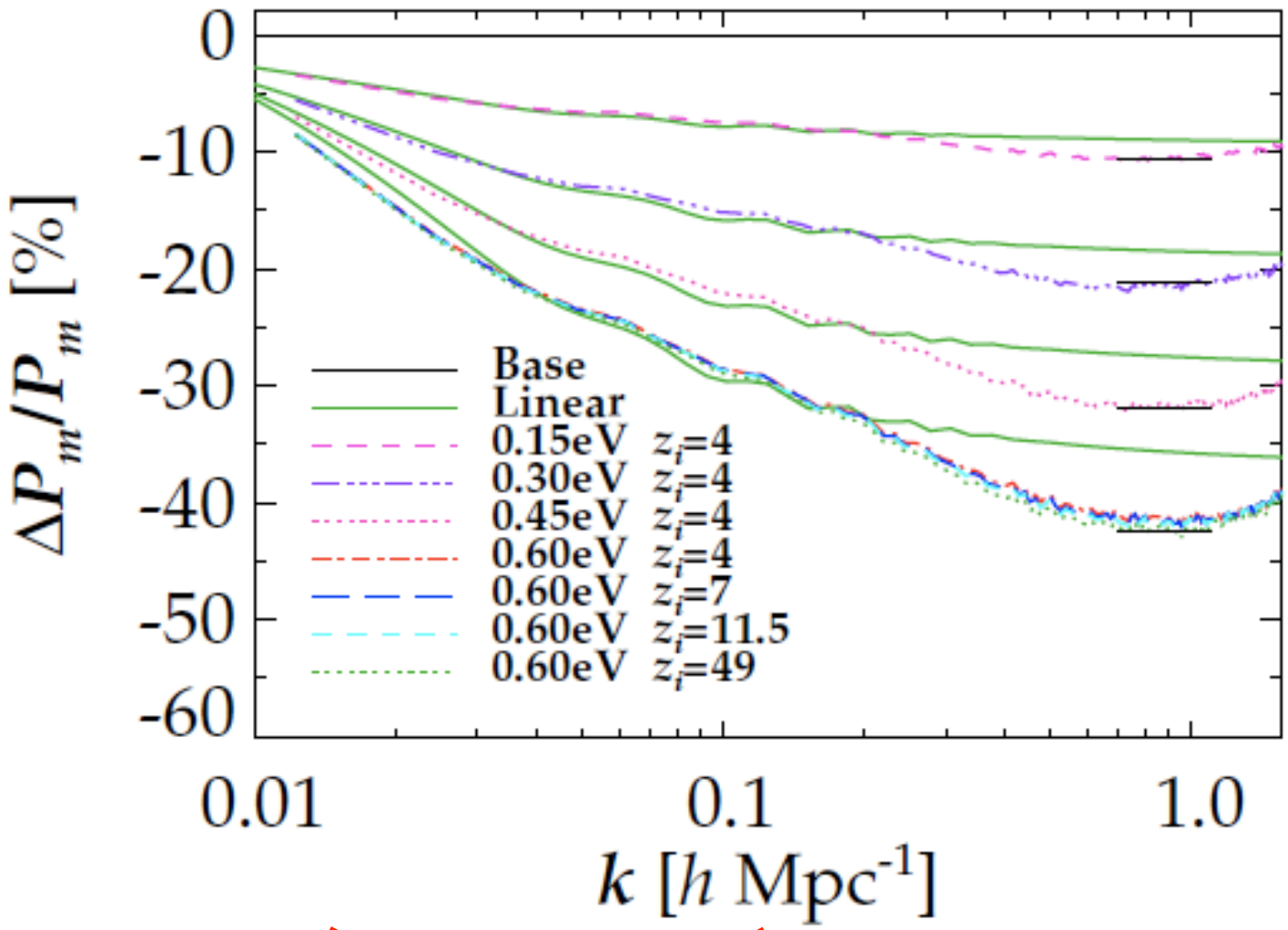
$$Pr(<p) = N \int_0^p \frac{p'^2}{e^{p'c/k_b T_\nu} + 1} dp'$$

Draw velocity from Fermi-Dirac distribution



Brandbyge et al 08

N-body simulations – III: effects in terms of non-linear power



Brandbyge et al 08

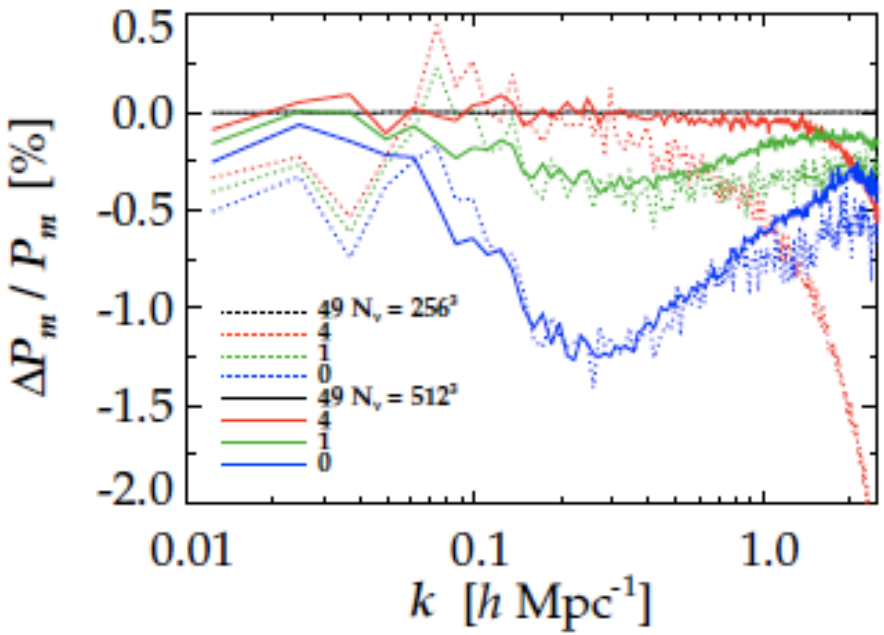
$$\cancel{\left.\frac{\Delta P}{P}\right|_{\max} \sim -8\frac{\Omega_\nu}{\Omega_m}} \rightarrow \left.\frac{\Delta P}{P}\right|_{\max} \sim -9.8\frac{\Omega_\nu}{\Omega_m}$$

N-body simulations – IV: mesh method

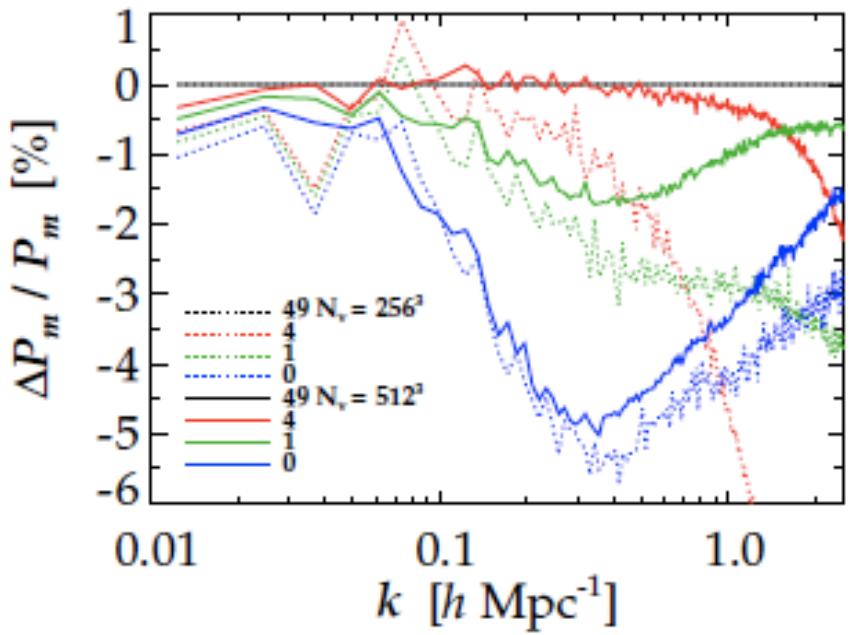
Computing the neutrino gravitational potential on the PM grid and summing up its contribution to the total matter gravitational potential – this is much faster!

COMPARISON GRID VS PARTICLES

M ν =0.6 eV



M ν = 1.2 eV



Brandbyge et al 08b

N-body simulations – V: a hybrid approach

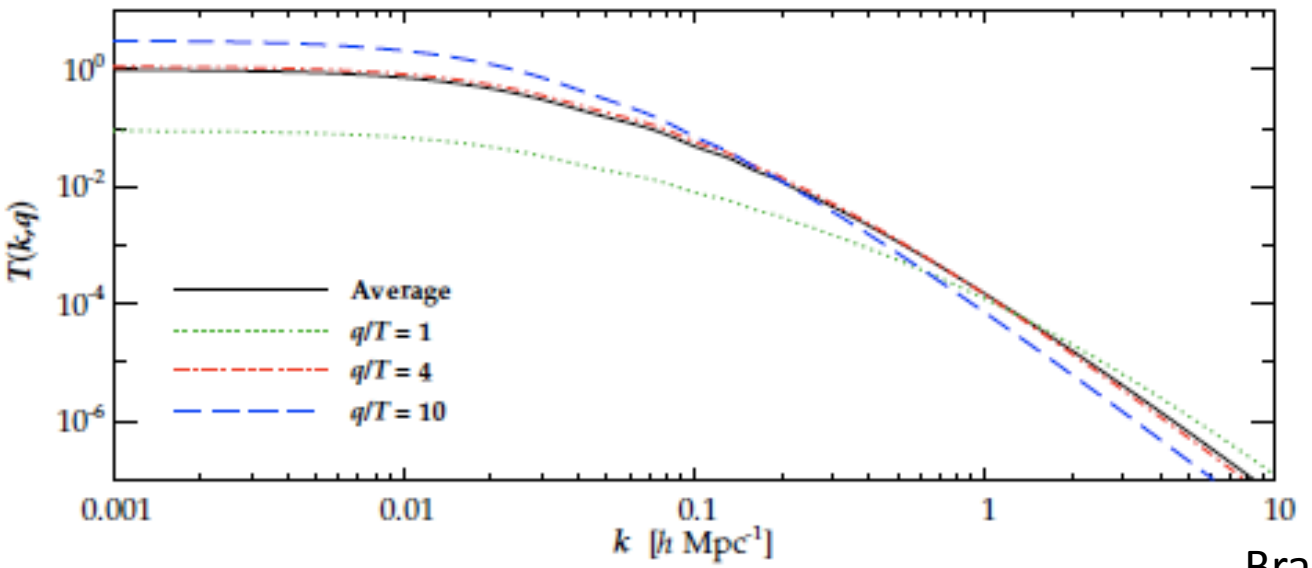
$$f = f_0 + \frac{\partial f_0}{\partial T}\delta T = f_0(1+\Psi) \qquad f_0(q) = \frac{1}{e^{q/T}+1}$$

After neutrino decoupling CBE

$$\frac{df}{d\tau} = \frac{\partial f}{\partial \tau} + \frac{dx^i}{d\tau}\frac{\partial f}{\partial x^i} + \frac{dq}{d\tau}\frac{\partial f}{\partial q} + \frac{dn_i}{d\tau}\frac{\partial f}{\partial n_i} = 0$$

$$\delta\rho_\nu(k) = 4\pi a^{-4}\int q^2 dq \epsilon f_0 \Psi_0 \qquad \epsilon = (q^2 + a^2 m^2)^{1/2}$$

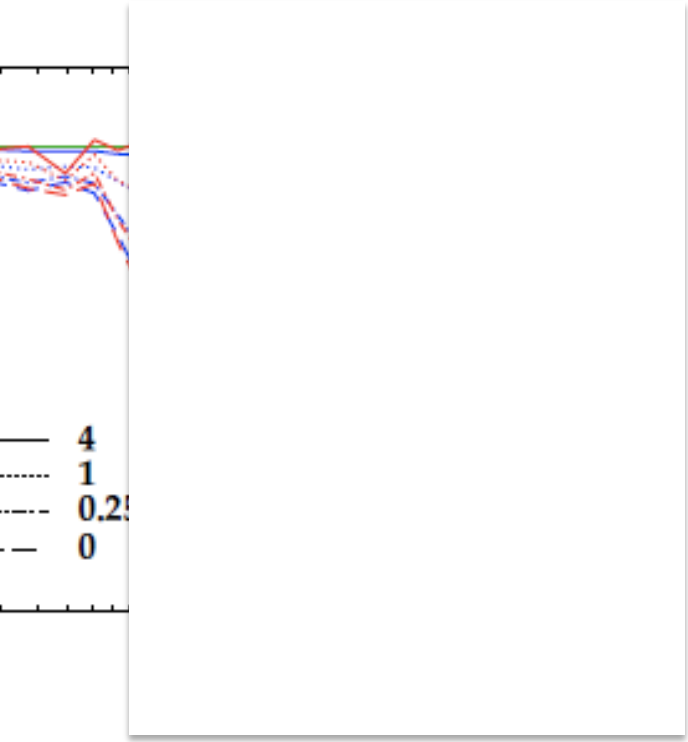
$$\dot{\Psi}_0 = -\frac{qk}{3\epsilon}\Psi_1 - \dot{\phi}\frac{d\ln f_0}{d\ln q},$$
$$\dot{\Psi}_1 = \frac{qk}{\epsilon}\left(\Psi_0 - \frac{2}{5}\Psi_2\right) - \frac{\epsilon k}{q}\psi\frac{d\ln f_0}{d\ln q},$$
$$\dot{\Psi}_l = \frac{qk}{\epsilon}\left(\frac{l}{2l-1}\Psi_{l-1} - \frac{l+1}{2l+3}\Psi_{l+1}\right),\ l \geq 2.$$

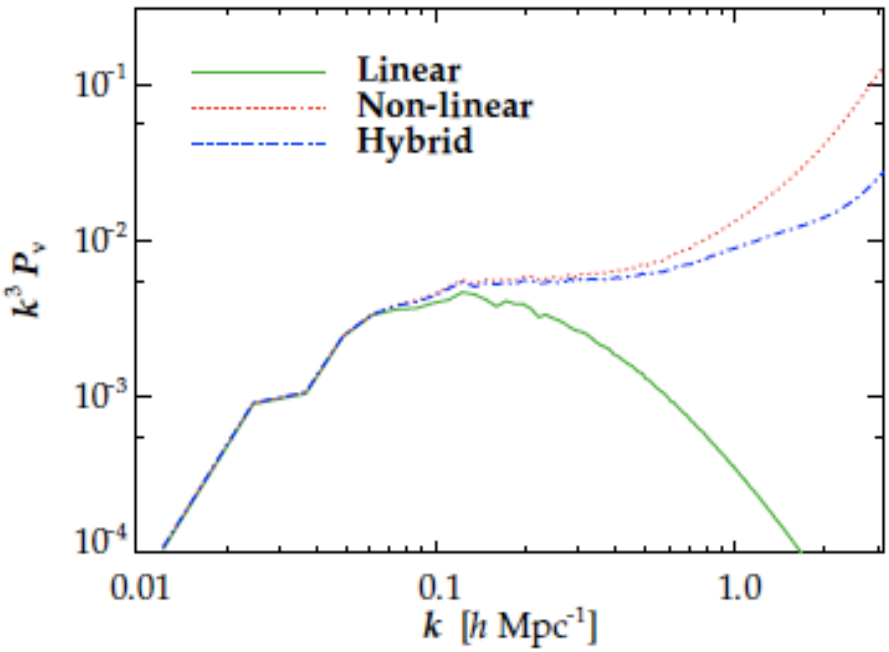


$$\Psi_0(k,q,z) = T(k,q,z)\Psi_0^I(k,q)$$

Brandbyge & Hannestad 09

N-body simulations – VI: comparison



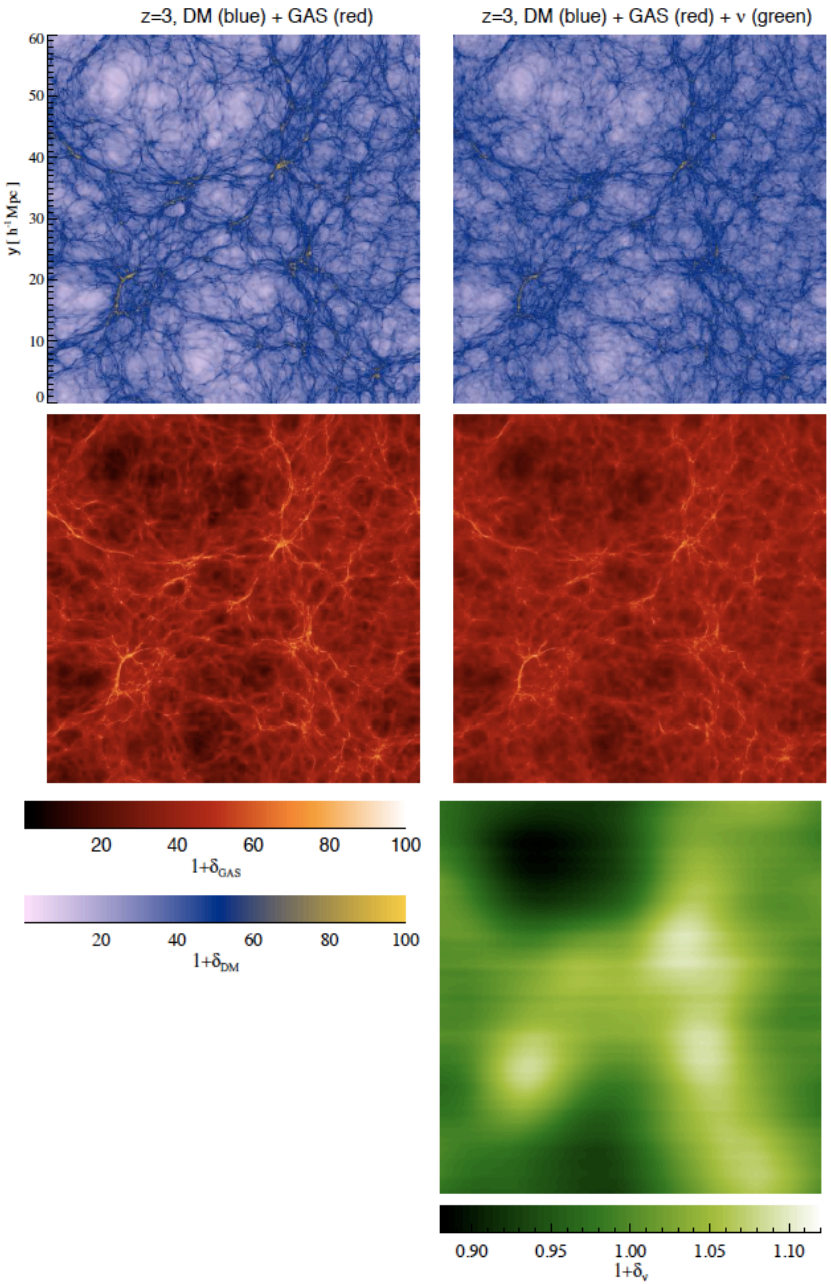


PARTICLES: accurate non-linear sampling but prone to shot-noise errors

GRID: fast and accurate but no phase mixing (i.e. non-linear regime suppression maybe it is less than it should be)

HYBRID: ideal for non-linear objects but memory demanding and prone to convergence issues

N-body + Hydro simulations – I: slices

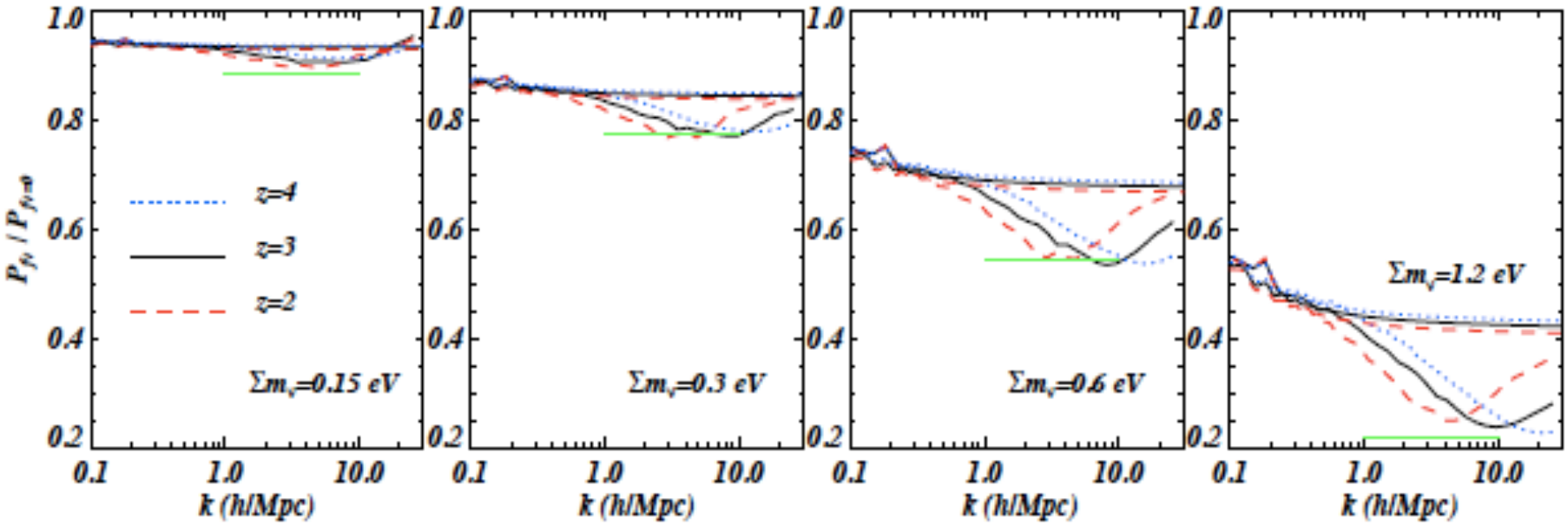


TreeSPH code Gadget-III
follows DM, neutrinos, gas and star particles in a cosmological volume

Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

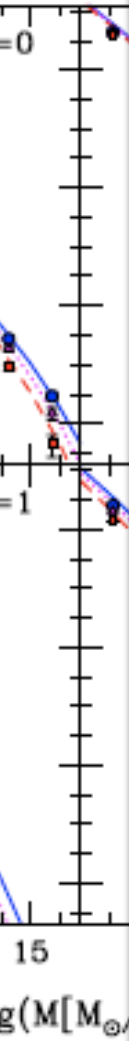
Hydro simulations – II: redshift/scale dependence of non-linear power

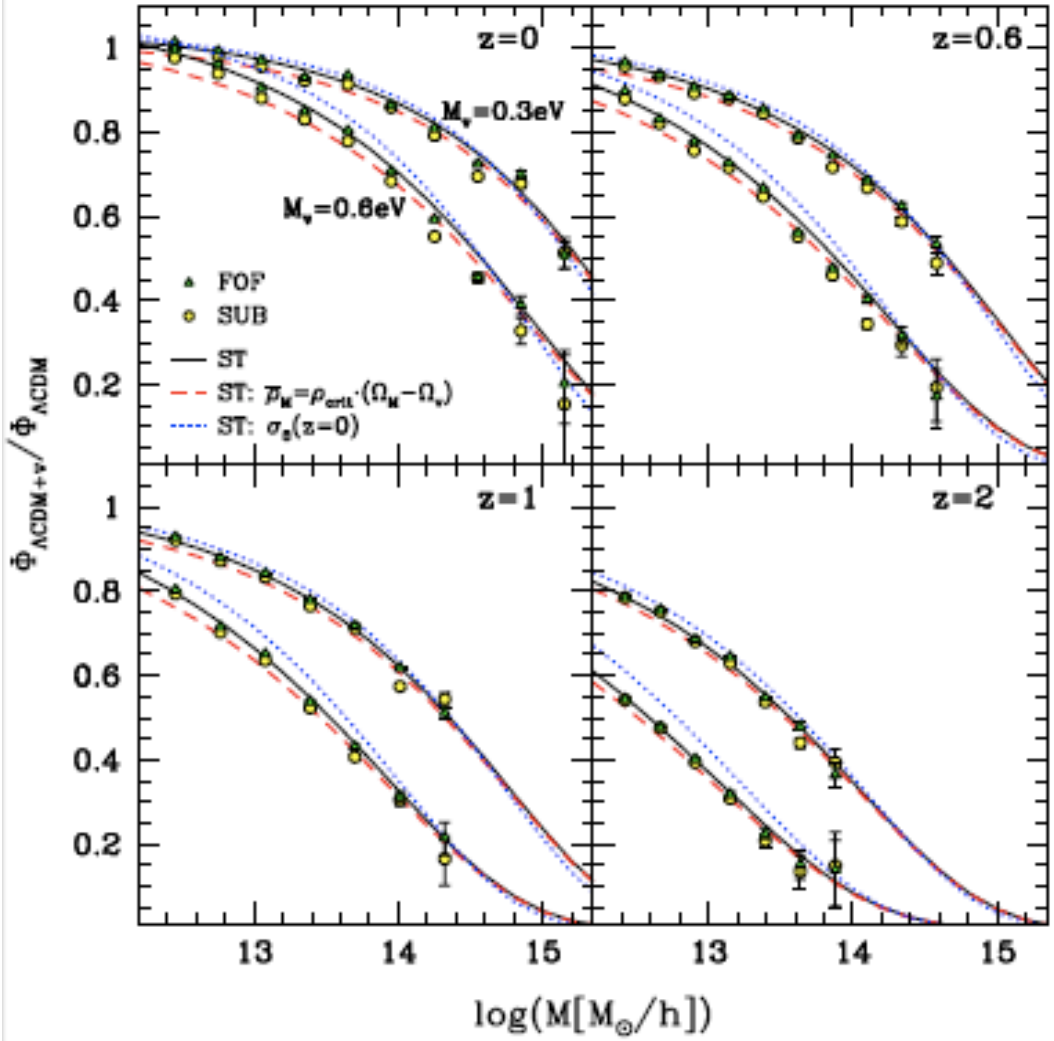
Full hydro simulations: gas physics does impact at the <10 % level at scales k < 10 h/Mpc



Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

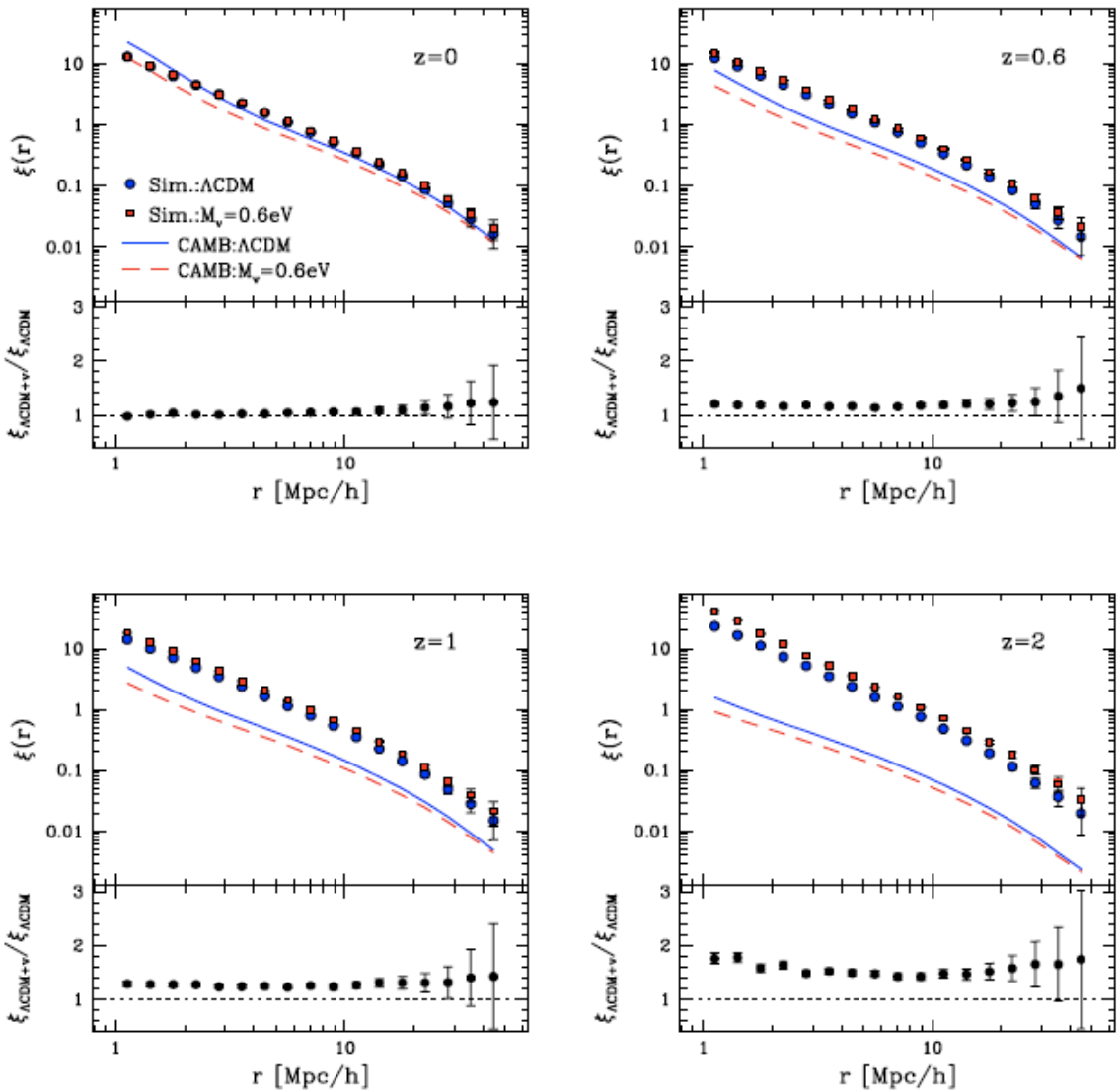
Hydro simulations – III: halo mass functions





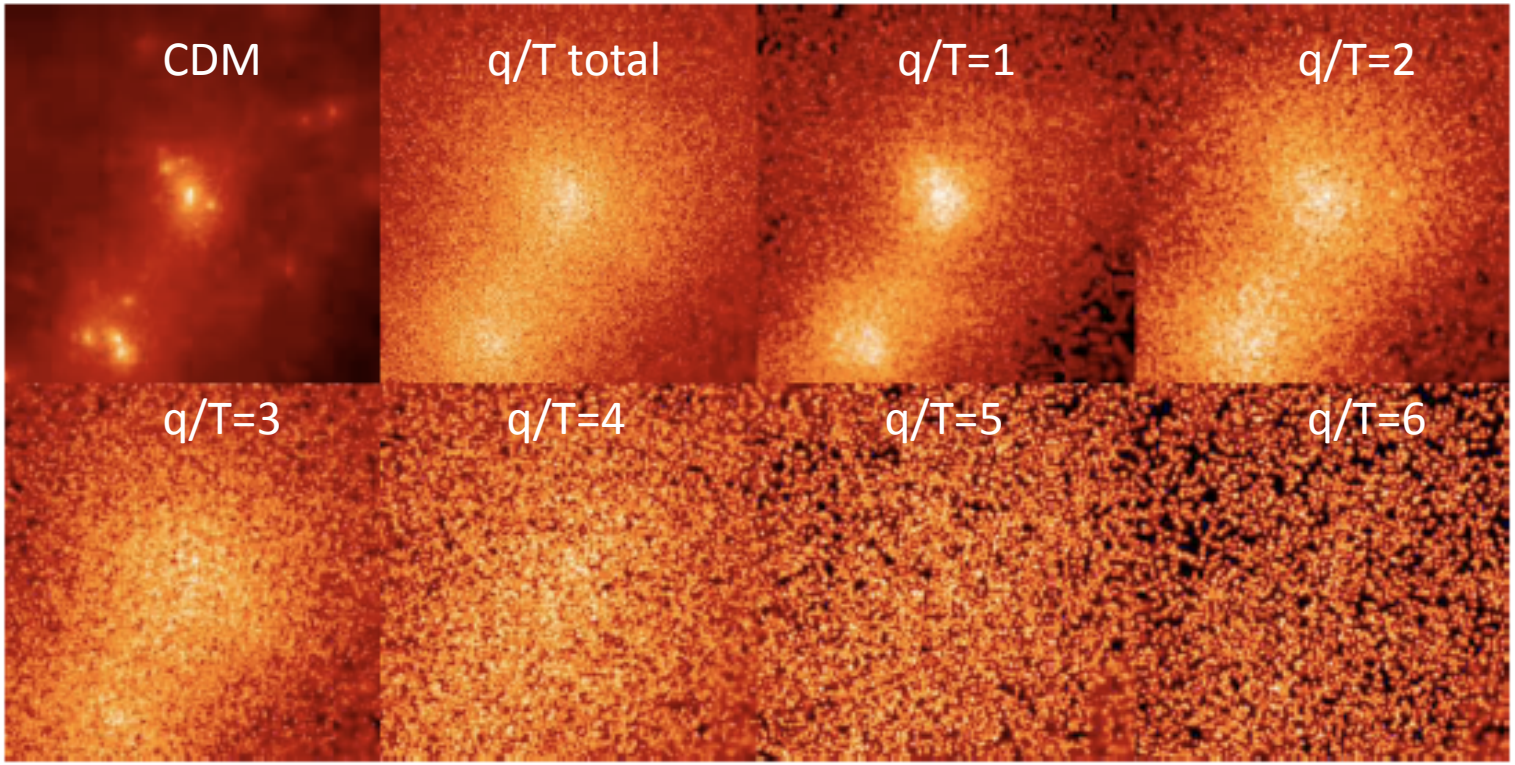
Marulli, Carbone, MV, Moscardini e Cimatti 2011 arxiv: 1103.0278

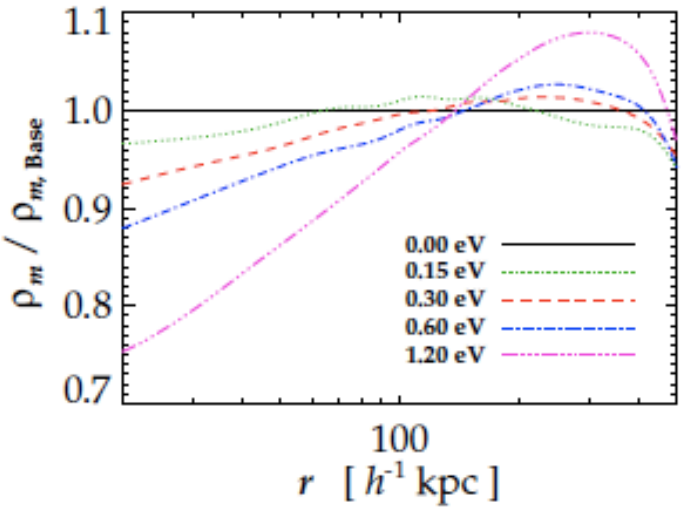
Hydro simulations – IV: matter and halo clustering



Marulli, Carbone, MV, Moscardini e Cimatti 2011 arxiv: 1103.0278

N-body simulations – V: halo density profile

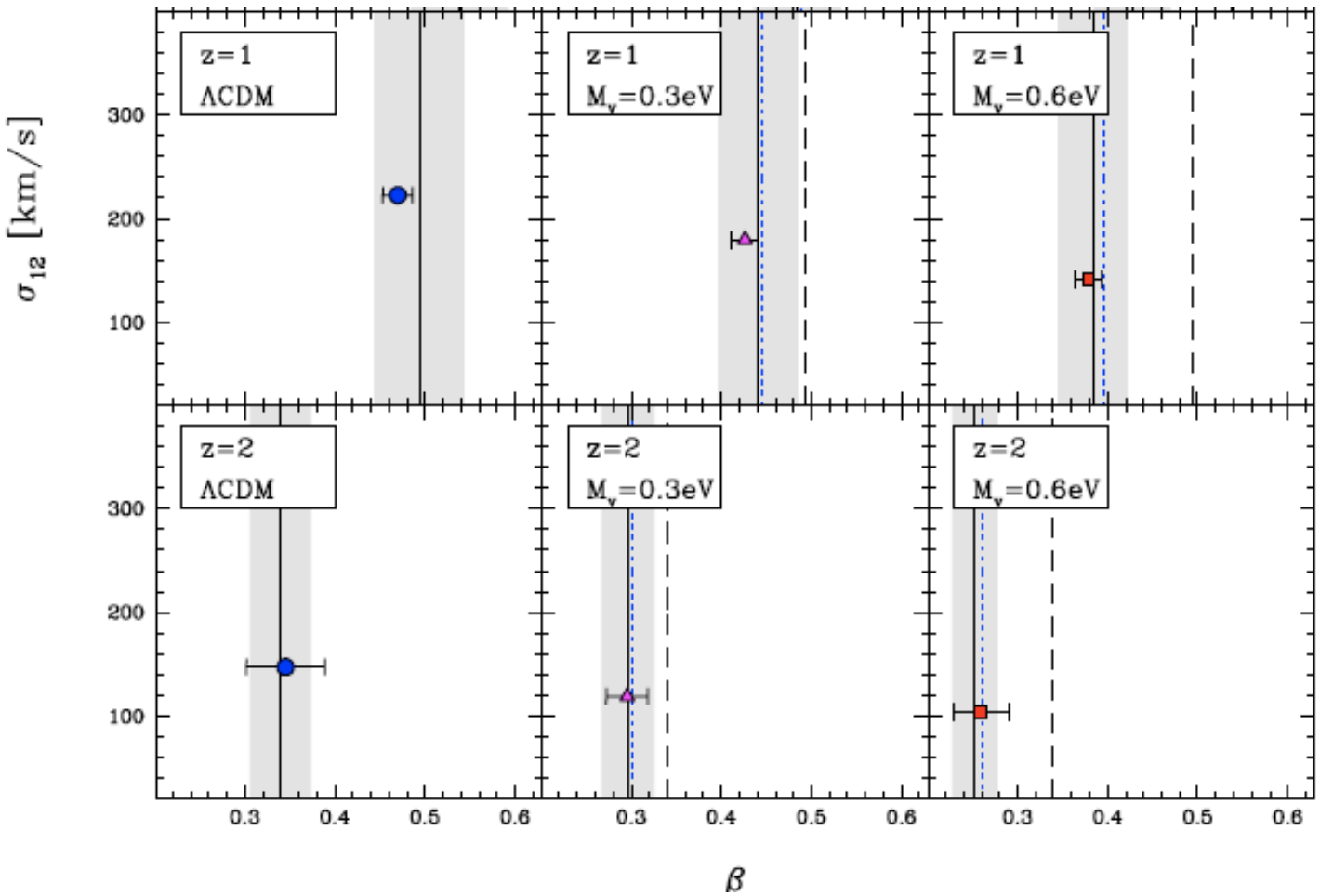




Brandbyge, Hannestad
Haugbolle, Wong 2010

Hydro simulations – VI: redshift space distortions

$$\xi(s_\perp, s_\parallel) = \int_{-\infty}^{\infty} dv f(v) \xi(s_\perp, s_\parallel - v/H(z)/a(z))$$

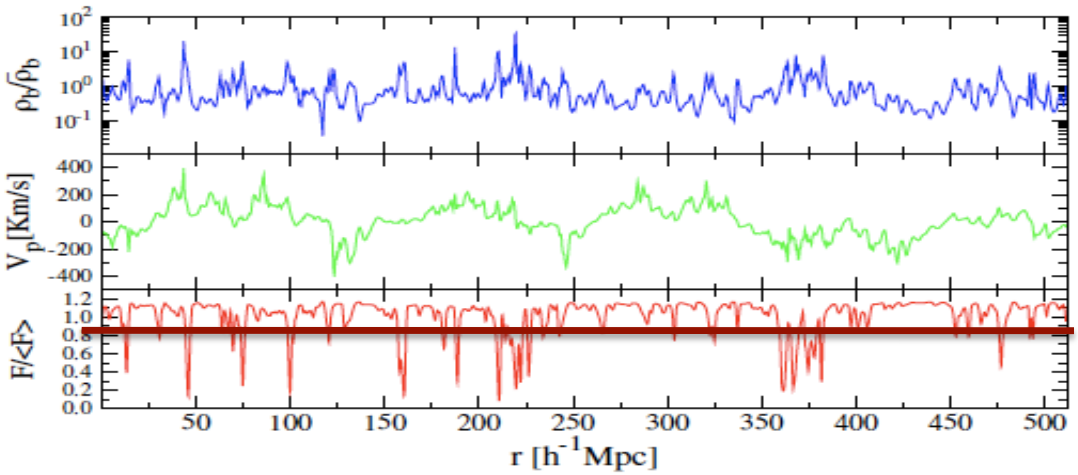


$$f_{\rm exp}(v) = \frac{1}{\sigma_{12}\sqrt{2}} \exp\left(-\frac{\sqrt{2}|v|}{\sigma_{12}}\right)$$

$$P(k) = (1 + \beta\mu^2)^2 P_{\rm lin}(k)$$

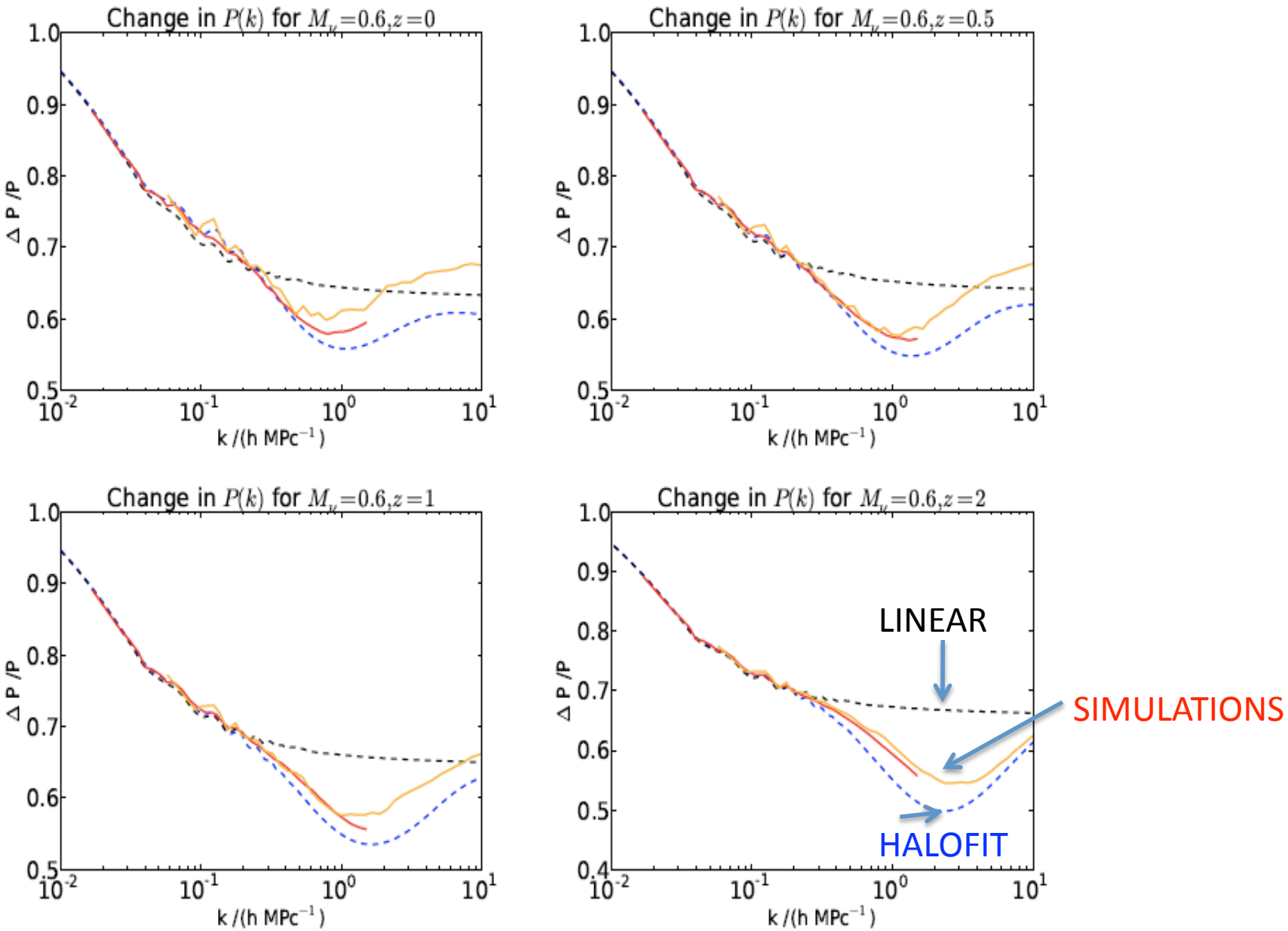
Marulli, Carbone, MV, Moscardini e Cimatti 2011 arxiv: 1103.0278

Hydro simulations – VII: the distribution of high-z voids



Navarro-Villaescusa, Vogelsberger, MV, Loeb 2011, arXiv: 1106.2543

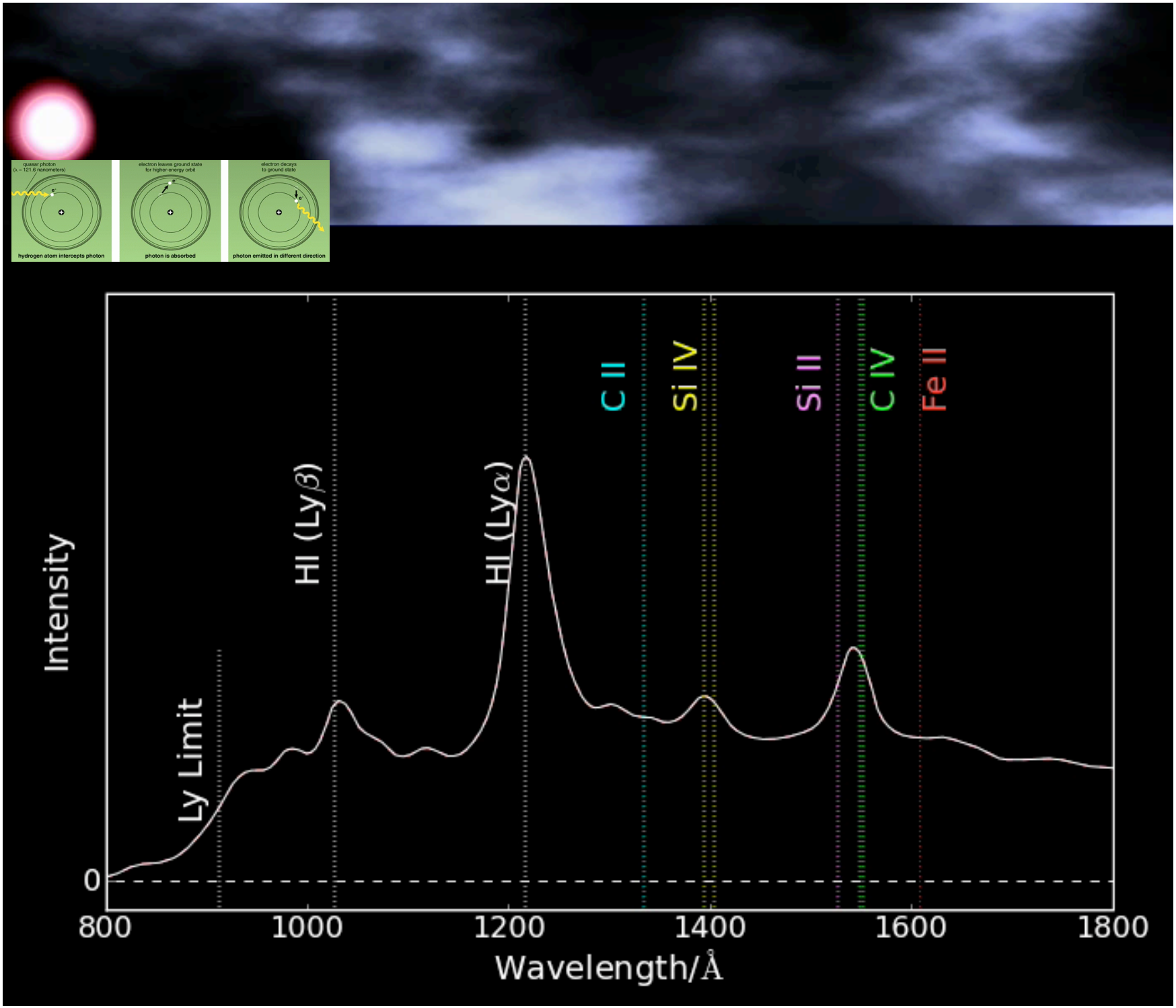
Hydro simulations – VIII: very non-linear regime



Bird, MV, Haehnelt 11 (in prep.)

IGM

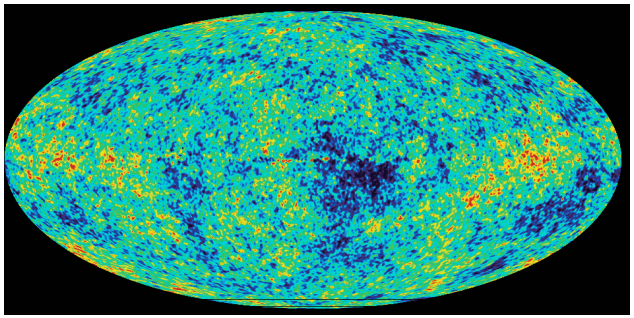
Ordinary baryonic matter that fills the space between galaxies


quasar photon
(λ = 121.6 nanometers)
hydrogen atom intercepts photon
electron leaves ground state
for higher-energy orbit
photon is absorbed
electron decays
to ground state
photon emitted in different direction
Intensity
0
Ly Limit
HI (Lyβ)
HI (Lyα)
C II
Si IV
Si II
C IV
Fe II
800
1000
1200
1400
1600
1800
Wavelength/Å

The interpretation: full grid of sims - I

SDSS power analysed by forward modelling motivated by the huge amount of data with small statistical errors

CMB: Spergel et al. (05) Galaxy P(k): Sanchez & Cole (07)



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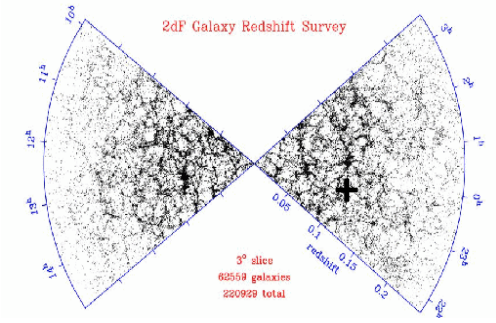
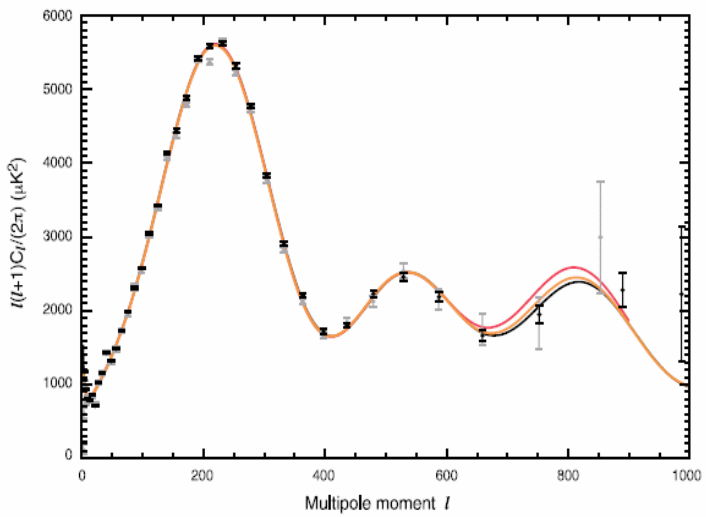
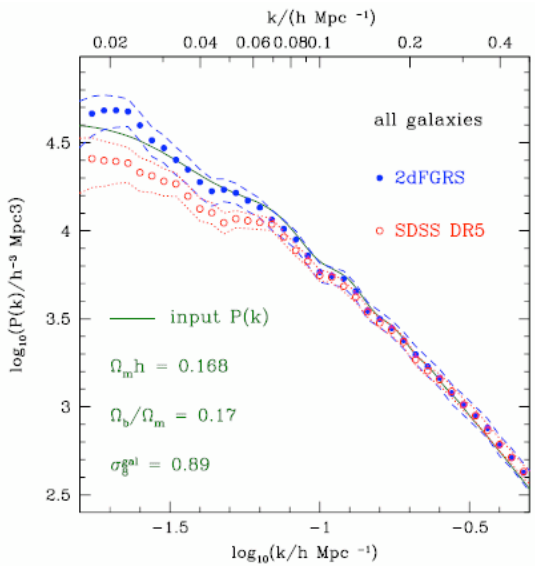
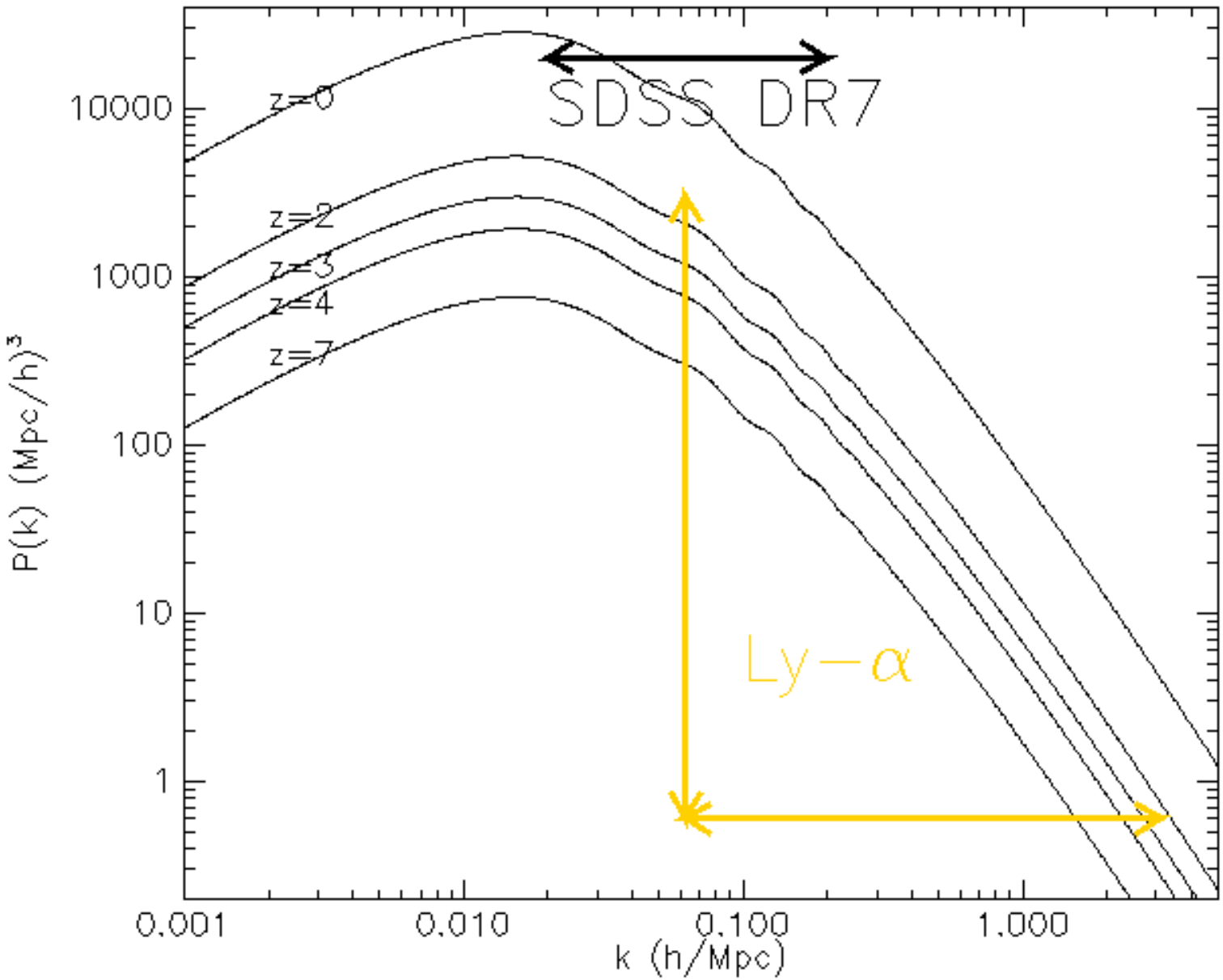
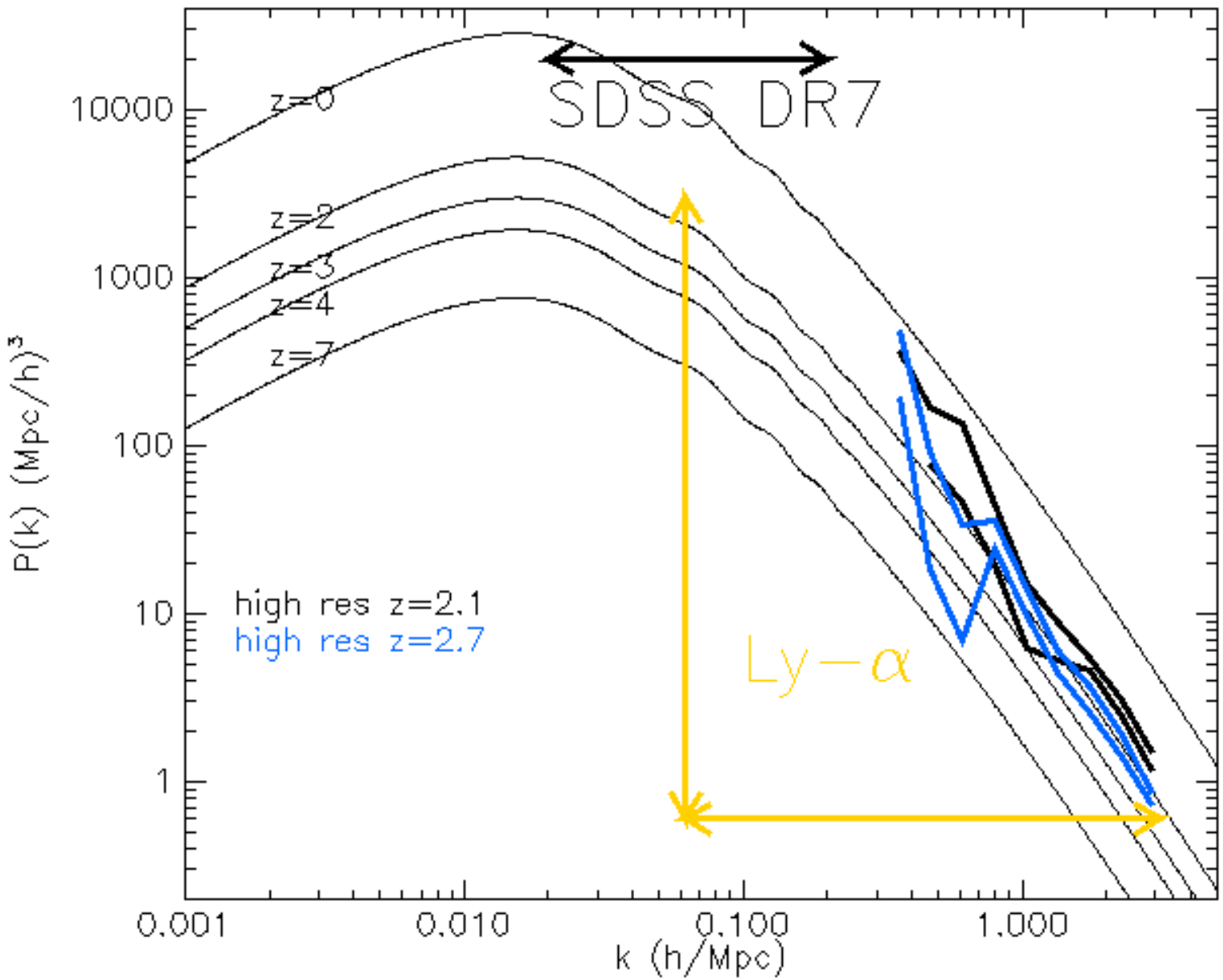


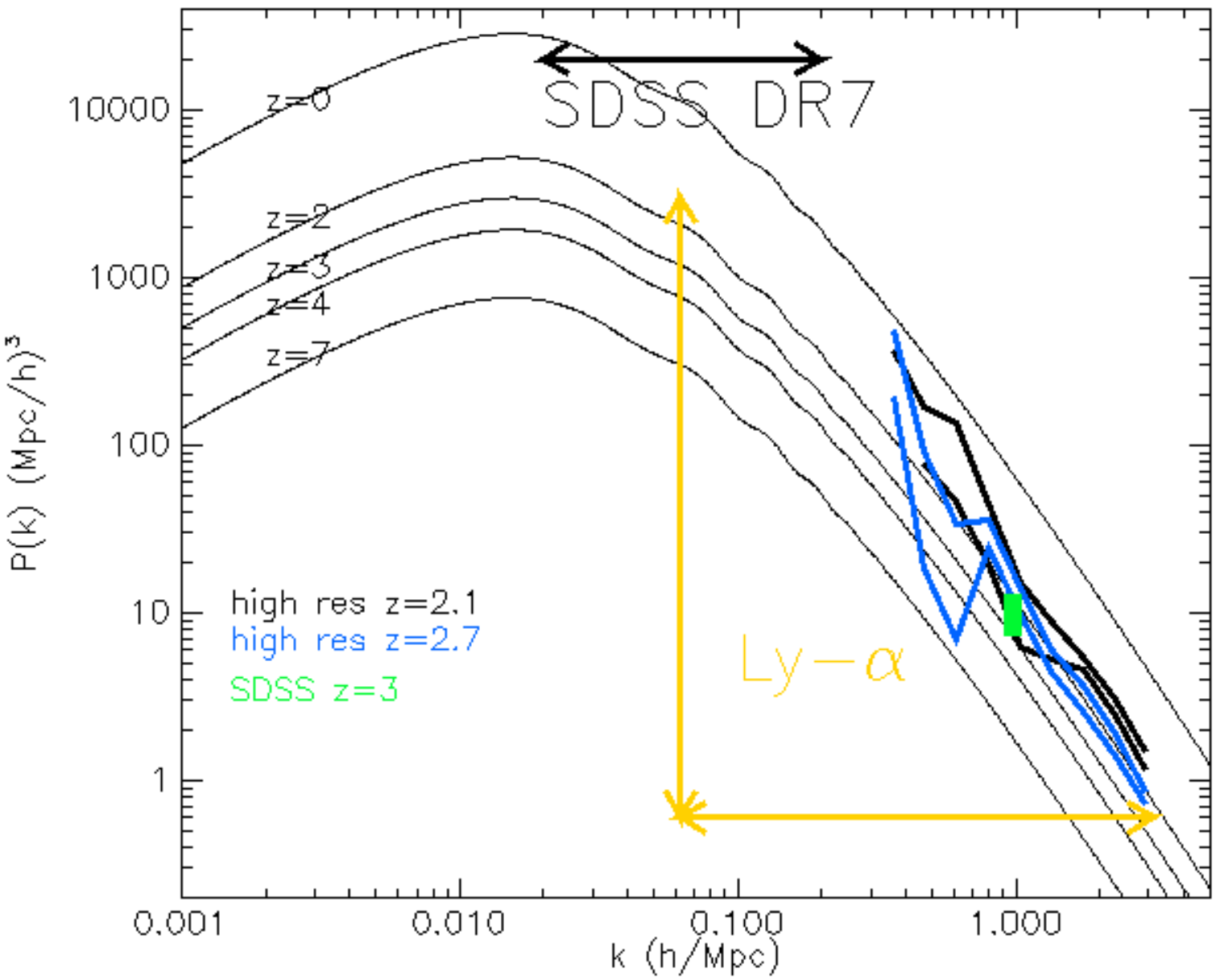


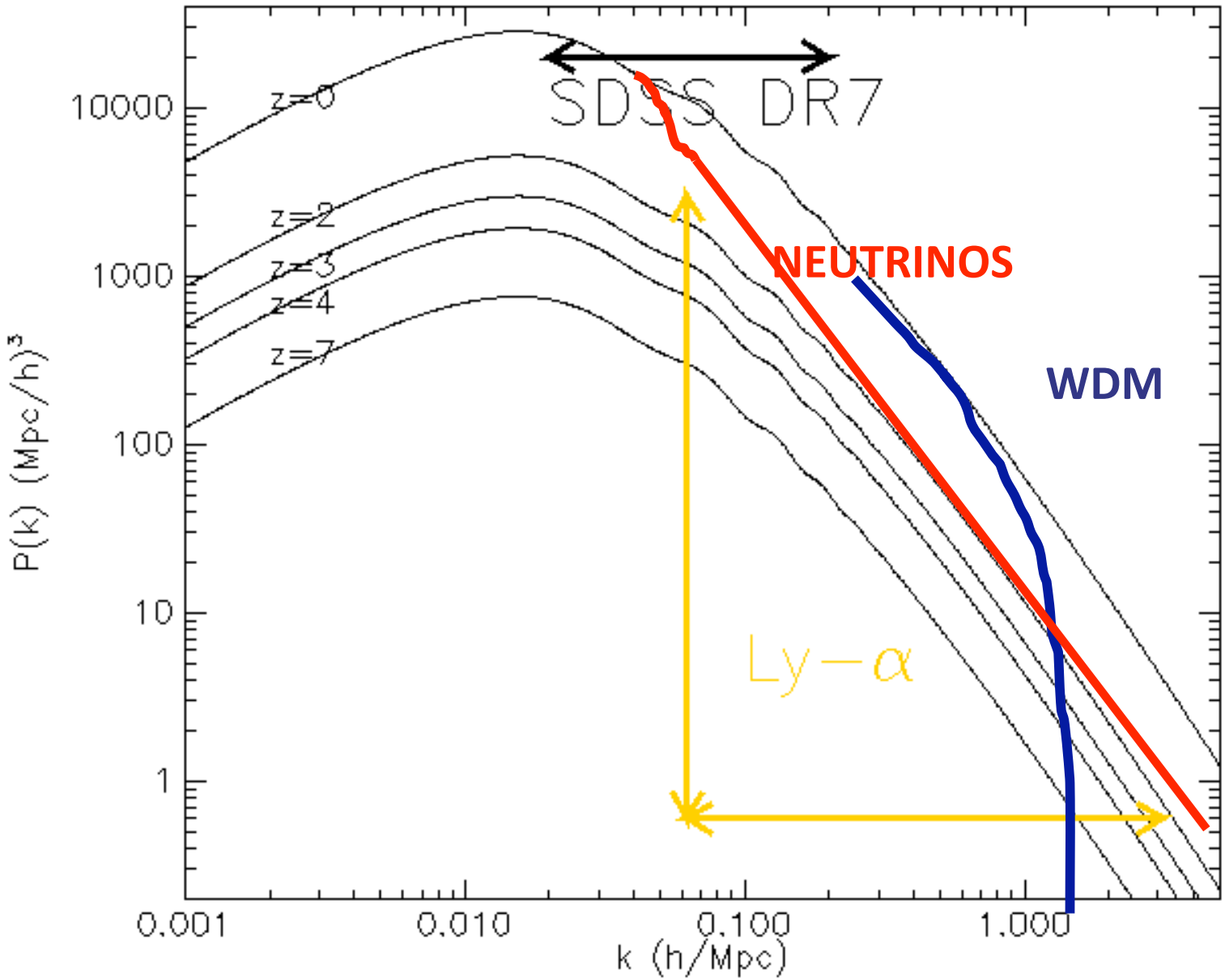


Cosmological parameters + e.g. bias


SDSS DR7
Ly−α
z=0
z=2
z=3
z=4
z=7
10000
1000
100
10
1
P(k) (Mpc/h)³
0.001
0.010
0.100
1.000
k (h/Mpc)


SDSS DR7
z=0
z=2
z=3
z=4
z=7
high res z=2.1
high res z=2.7
Ly−α
10000
1000
100
10
1
P(k) (Mpc/h)³
0.001
0.010
0.100
1.000
k (h/Mpc)


SDSS DR7
z=0
z=2
z=3
z=4
z=7
high res z=2.1
high res z=2.7
SDSS z=3
Ly−α
10000
1000
100
10
1
P(k) (Mpc/h)³
0.001
0.010
0.100
1.000
k (h/Mpc)


SDSS DR7
NEUTRINOS
WDM
Ly−α
z=0
z=2
z=3
z=4
z=7
10000
1000
100
10
1
0.001
0.010
0.100
1.000
k (h/Mpc)
P(k) (Mpc/h)³

Summary (highlights) of results from the high-res and low-res data

Why Lyman-α ? Small scales
High redshift
Most of the baryonic mass is in this form
Quasars sample 75% of the age of the universe

1. Tightest constraints to date on neutrino masses and running of the spectral index

   Seljak, Slosar, McDonald JCAP (2006) 10 014

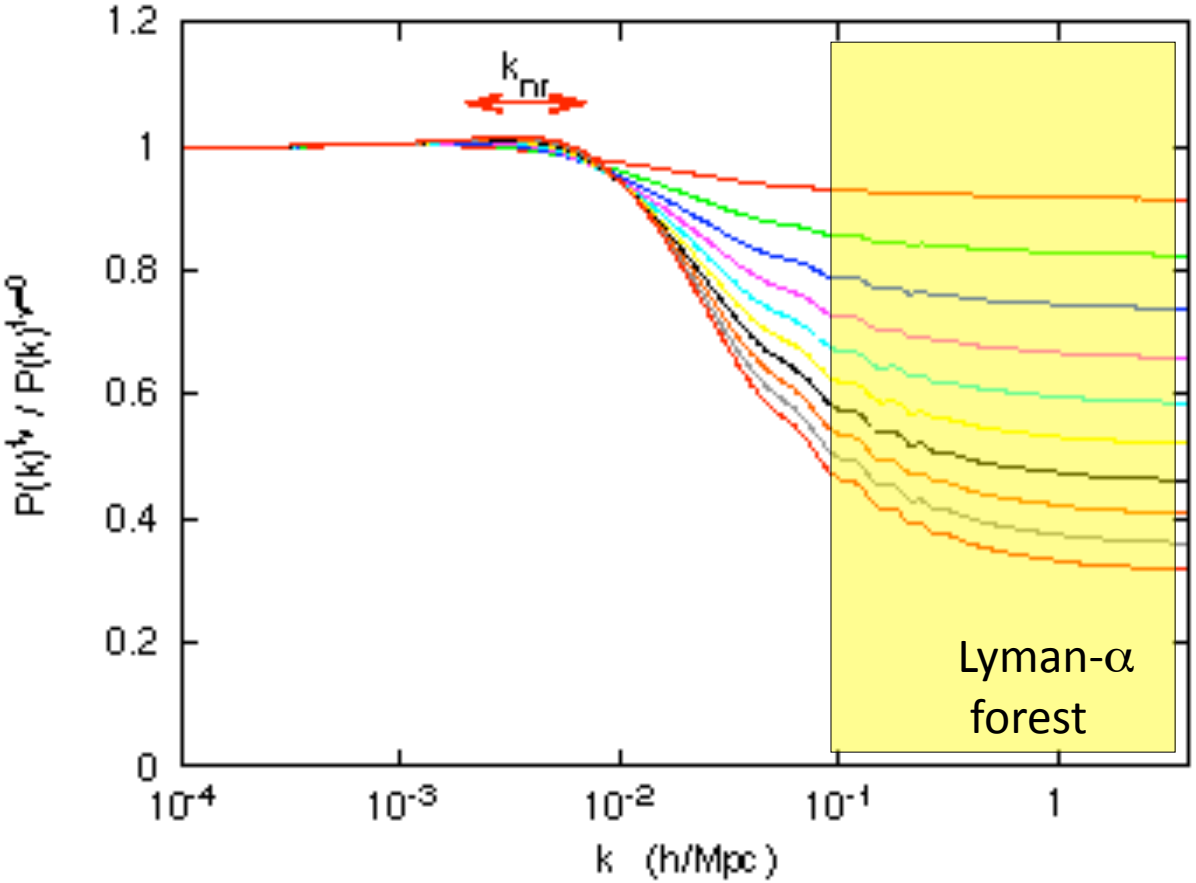
2. Tightest constraints to date on the coldness of cold dark matter

   MV et al., Phys.Rev.Lett. 100 (2008) 041304

Active neutrinos – I: the effect

$$k_{nr} \simeq 0.018\ \Omega_m^{1/2} \left(\frac{m}{1\,\mathrm{eV}}\right)^{1/2} h\,\mathrm{Mpc}^{-1}$$

Lesgourgues & Pastor Phys.Rept. 2006, 429, 307



$\Sigma m_\nu = 0.138$ eV

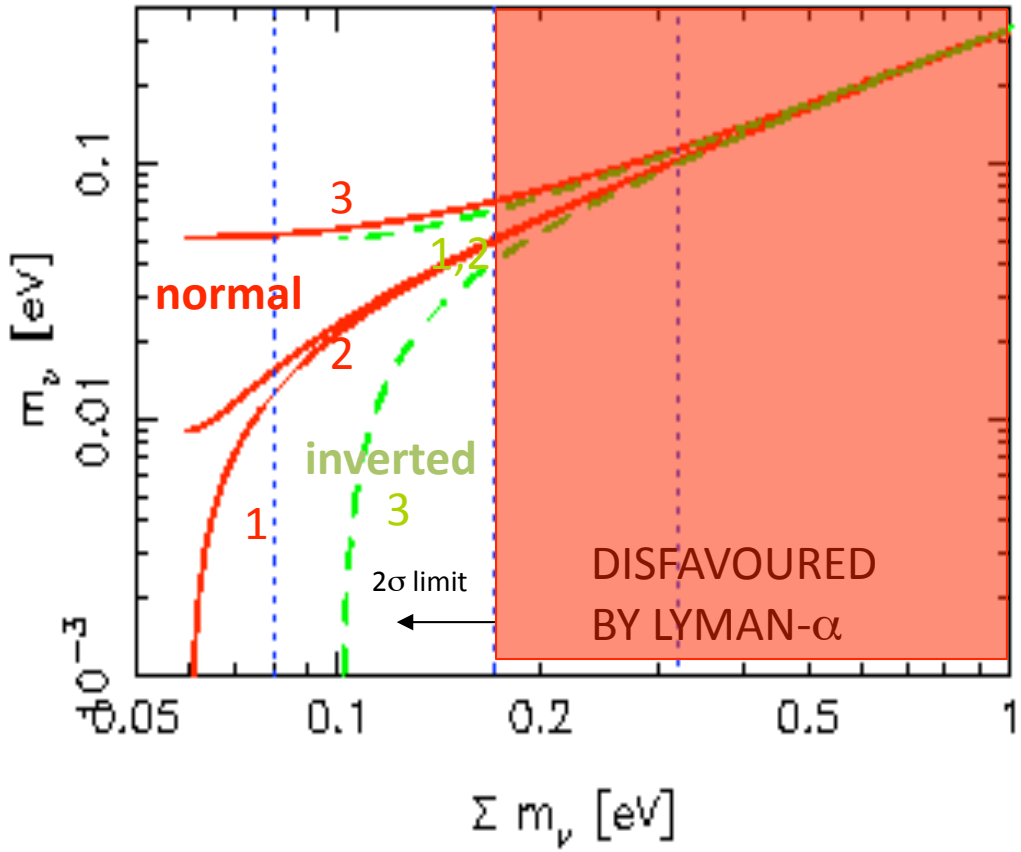
$\Sigma m_\nu = 1.38$ eV

$$v_{\rm th} \equiv \frac{\langle p \rangle}{m} \simeq \frac{3T_\nu}{m} = \frac{3T_\nu^0}{m}\left(\frac{a_0}{a}\right) \simeq 150(1+z)\left(\frac{1\,\mathrm{eV}}{m}\right)\mathrm{km\,s^{-1}}$$

$$k_{FS}(t) = \left(\frac{4\pi G\bar{\rho}(t)a^2(t)}{v_{\rm th}^2(t)}\right)^{1/2}, \qquad \lambda_{FS}(t) = 2\pi\frac{a(t)}{k_{FS}(t)} = 2\pi\sqrt{\frac{2}{3}}\frac{v_{\rm th}(t)}{H(t)}$$

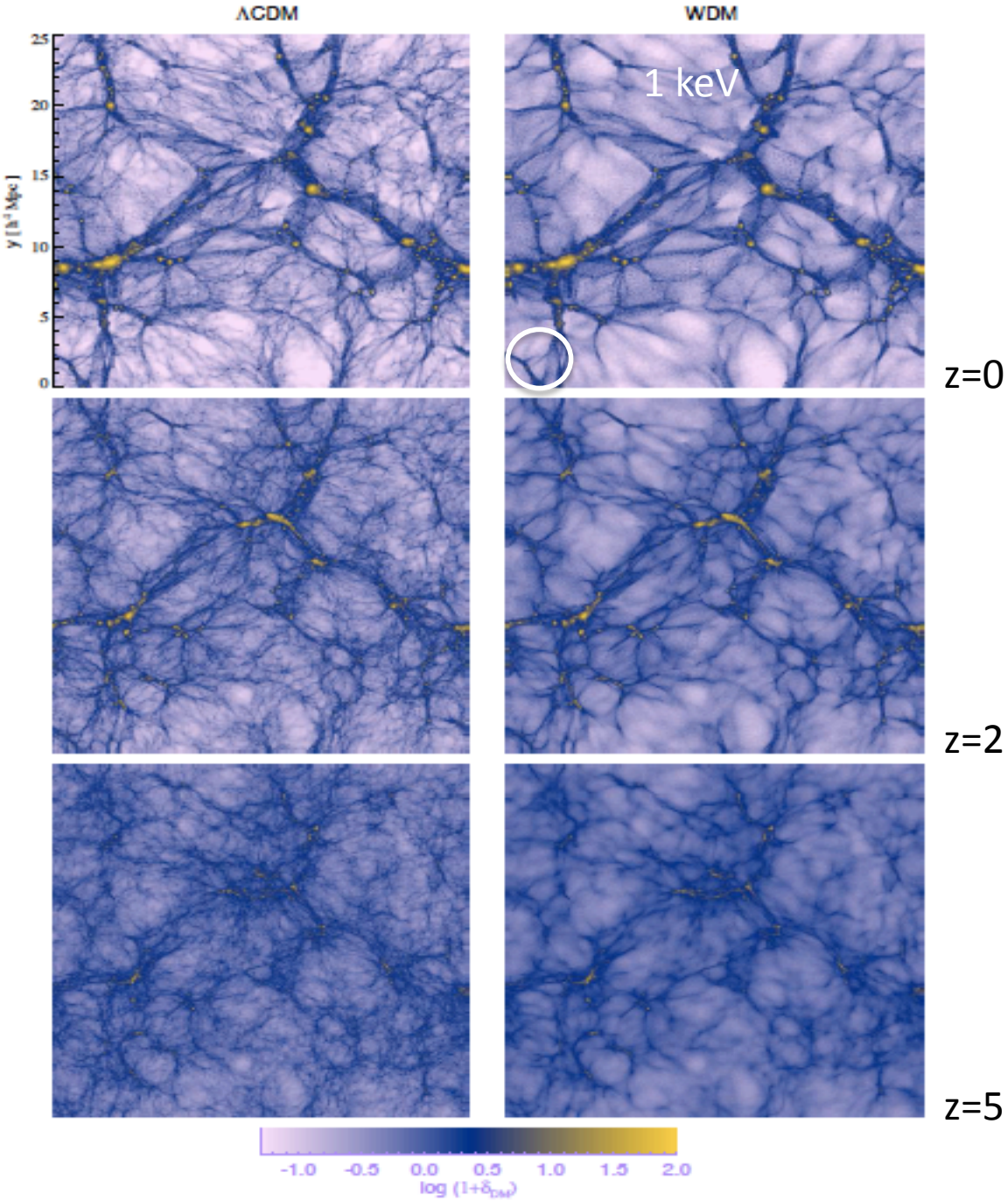
Active neutrinos – II: constraints

Seljak, Slosar, McDonald, 2006, JCAP, 0610, 014



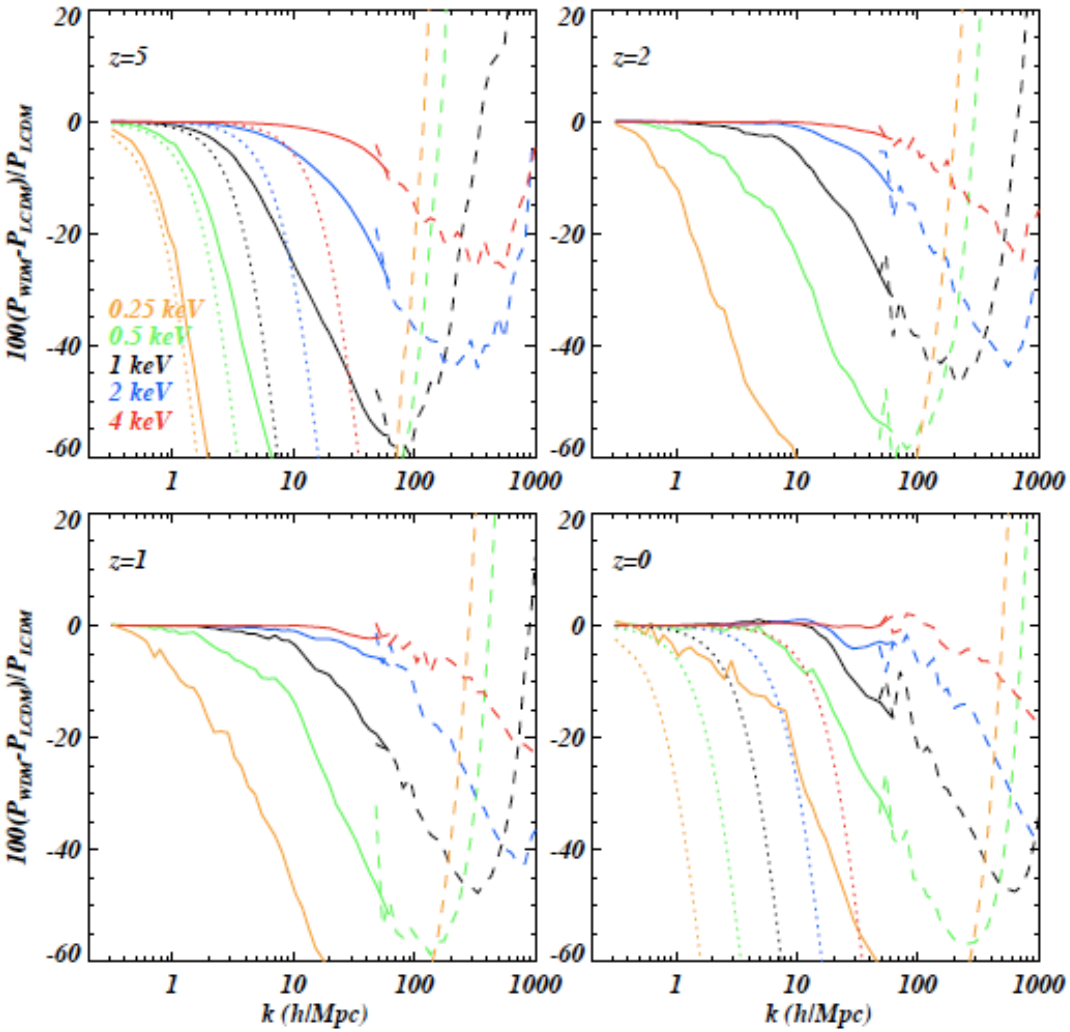
Warm Dark Matter and structure formation - I

$k_{FS} \sim 5\ T_v/T_x\ (m_x/1\text{keV})\ \text{Mpc}^{-1}$



See Bode, Ostriker, Turok 2001
Abazajian, Fuller, Patel 2001
Avila-Reese et al. 2001
Boyarsky et al. 2009
Colin et al. 2008
Wang & White 2007
Gao & Theuns 2007
Abazajian et al. 2007

Warm Dark Matter and non-linear power - II



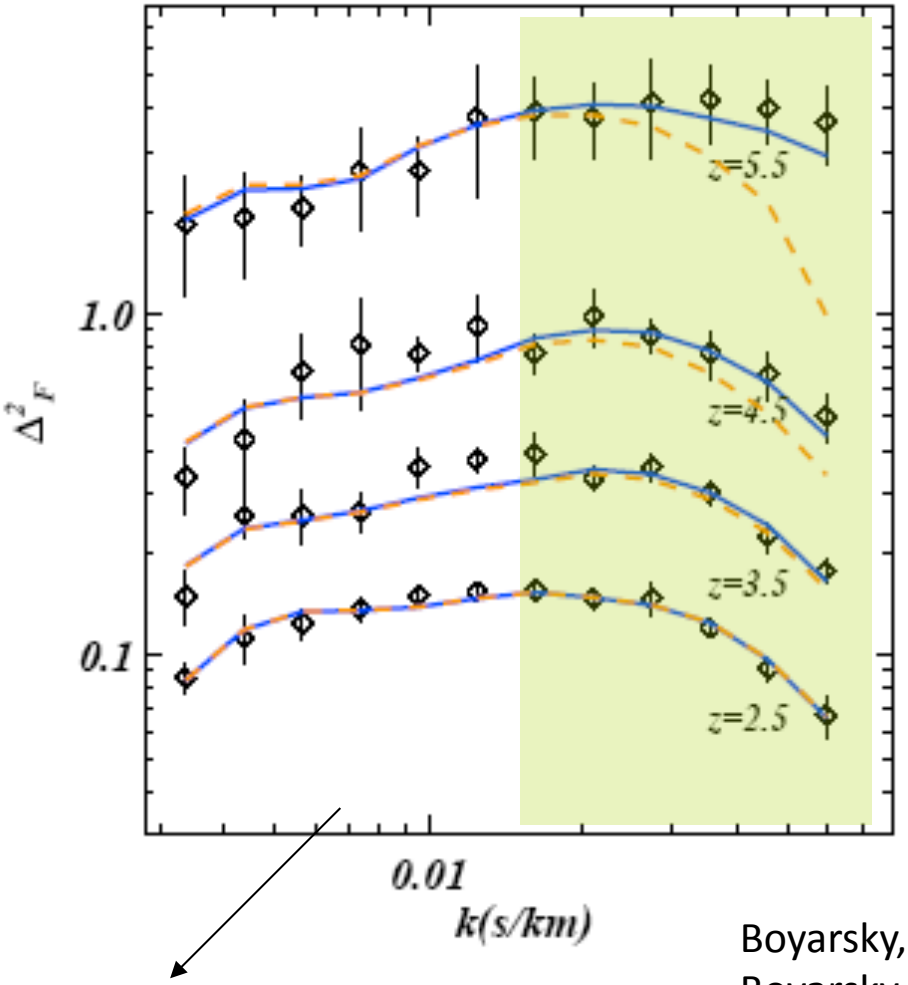
MV et al. 2011 (in prep.)

Range of wavenumbers important for weak lensing tomography , IGM and small scale clustering of galaxies!

Warm Dark Matter and Lyman-α - III

SDSS + HIRES(Keck) Quasar spectra

MV et al., PRL 100 (2008) 041304



SDSS range

Tightest constraints on mass of WDM particles to date:

m_{WDM} > 4 keV (early decoupled thermal relics)

$m_{sterile}$ > 28 keV (standard scenario)

Note that this limit becomes much weaker (~**2 keV** in RP mechanisms or mixed cold+warm scenarios)

Boyarsky, Lesgourgues. Ruchayskiy, MV, 2009, PRL, 102,201304
Boyarsky, Lesgourgues, Ruchayskiy, MV, 2009, JCAP, 05, 012

MV, Lesgourgues, Haehnelt, Matarrese, Riotto, PRD, 2005, 71, 063534 & PRL, 2006, 97, 071301

CONCLUSIONS

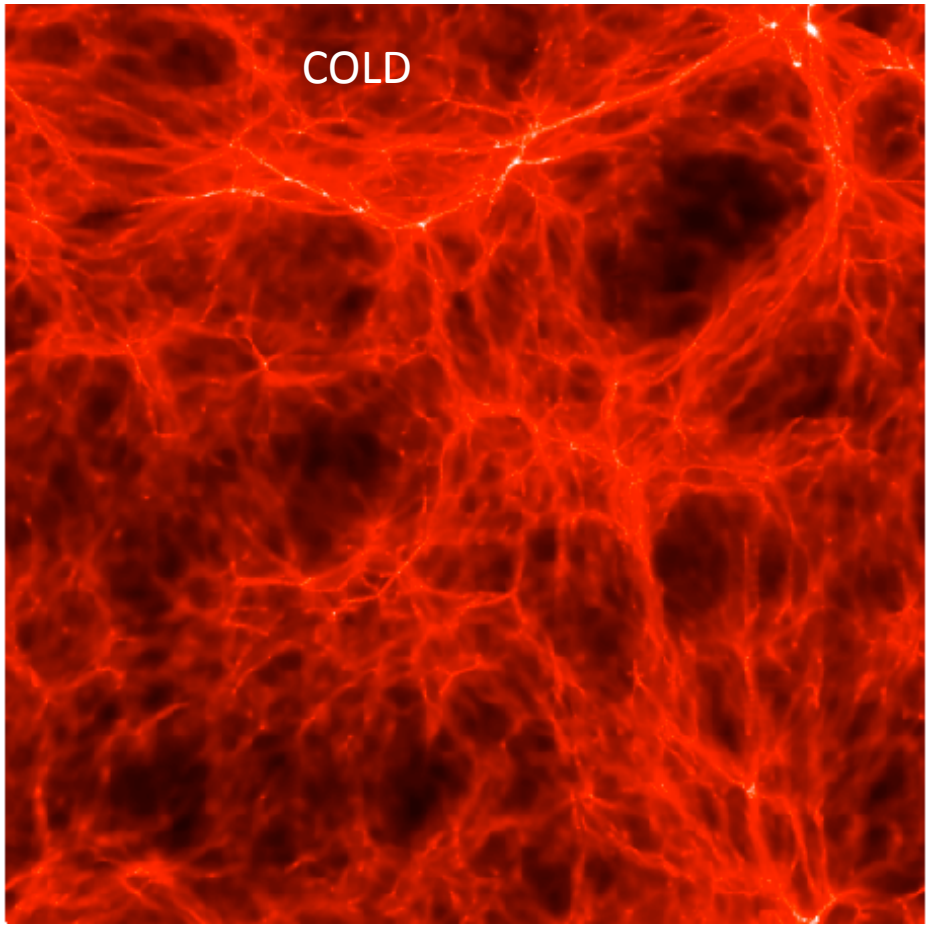
- **Neutrinos** do impact on the LSS at a level which is very much constrained by present data sets. The effect is small and **systematic effects** should be addressed at an unprecedented level of precision. Modelling the power spectrum at the 1 % level at small scales is difficult: **relevant physical processes and numerics** should be modelled and under control.

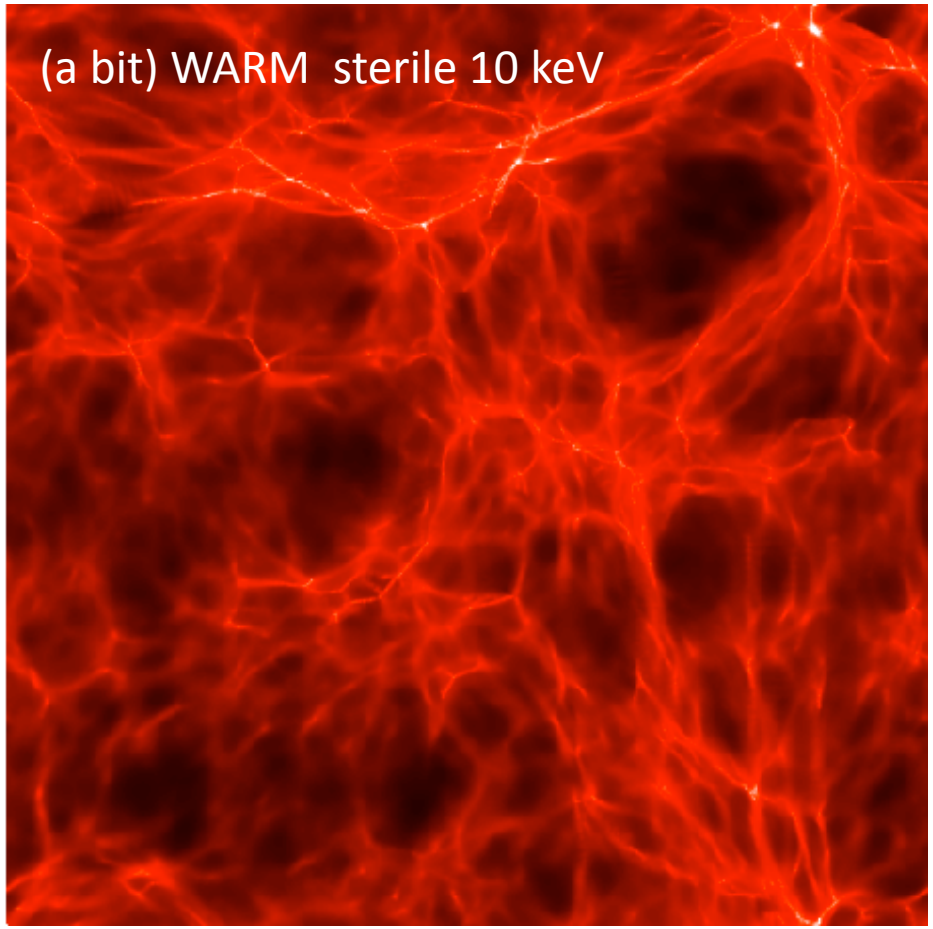
- Important role of the **IGM**, which is currently providing the tightest constraints on the mass (0.17 eV – 2σ upper limit); **weak lensing** and **galaxy redshift surveys** are likely to provide interesting results.

- **Coldness of cold dark matter** at small scales is a fundamental observable since possible deviations from the standard model can be measured or a candidate can show up. At present the constraints on the **sterile neutrinos** are tight (especially thanks to IGM data – but see also constraints from dwarf galaxies).

BACKUP SLIDES

Little room for warm dark matter…… at least in the standard DW scenario
…the cosmic web is likely to be quite “cold”





The interpretation: flux derivatives

Analysis of SDSS flux power

The flux power spectrum is a smooth function of k and z

Flux power

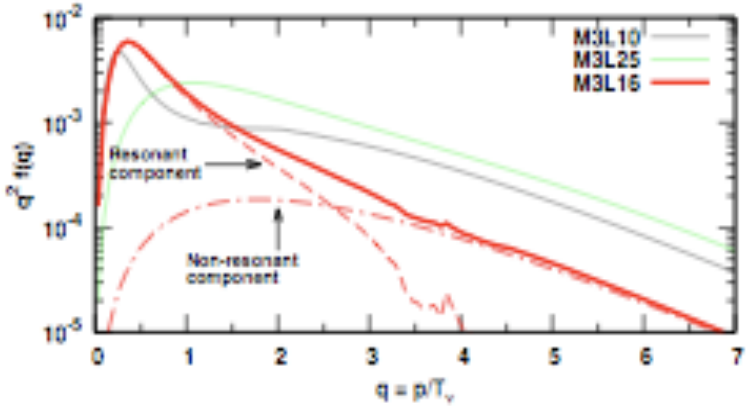
$$P_F(k, z; \mathbf{p}) = P_F(k, z; \mathbf{p}^0) + \sum_{i=1,N} \frac{\partial P_F(k, z; p_i)}{\partial p_i}\Bigg|_{\mathbf{p} = \mathbf{p}^0} (p_i - p_i^0)$$

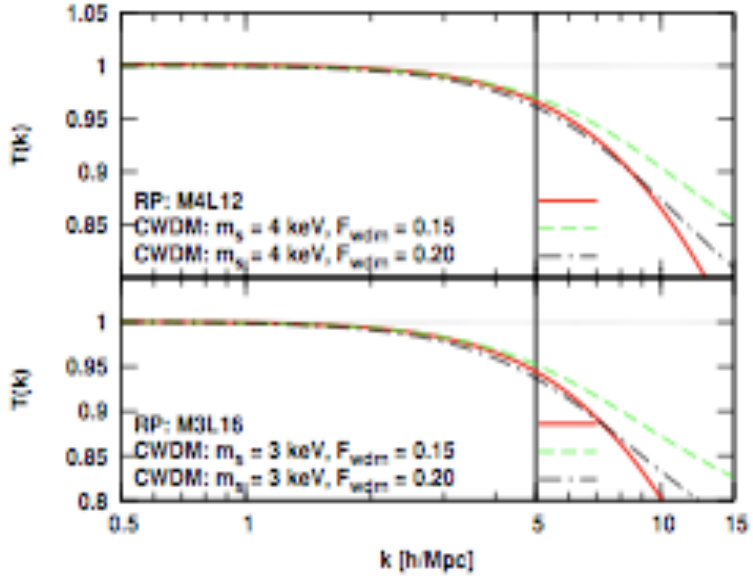
Best fit

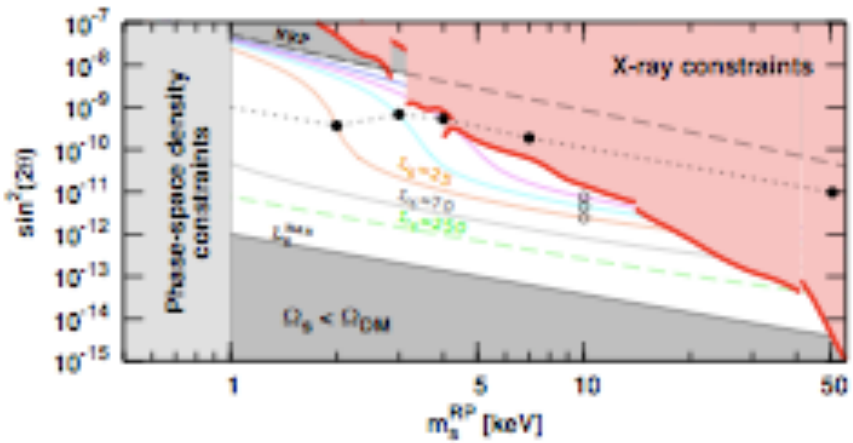
p: astrophysical and cosmological parameters

but even resolution and/or box size effects if you want to save CPU time

Lyman-α and resonantly produced sterile neutrinos -VI

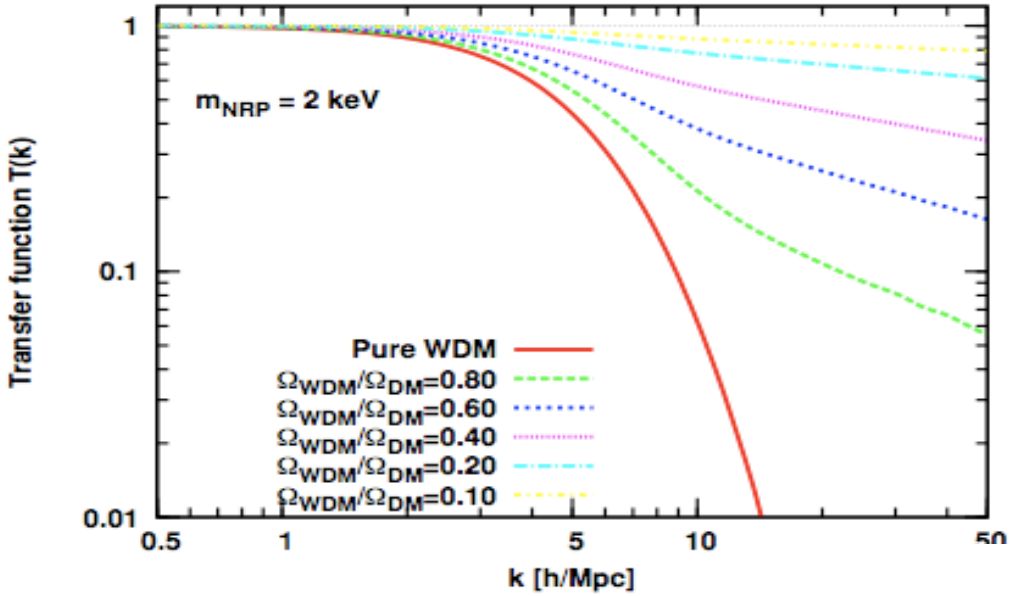




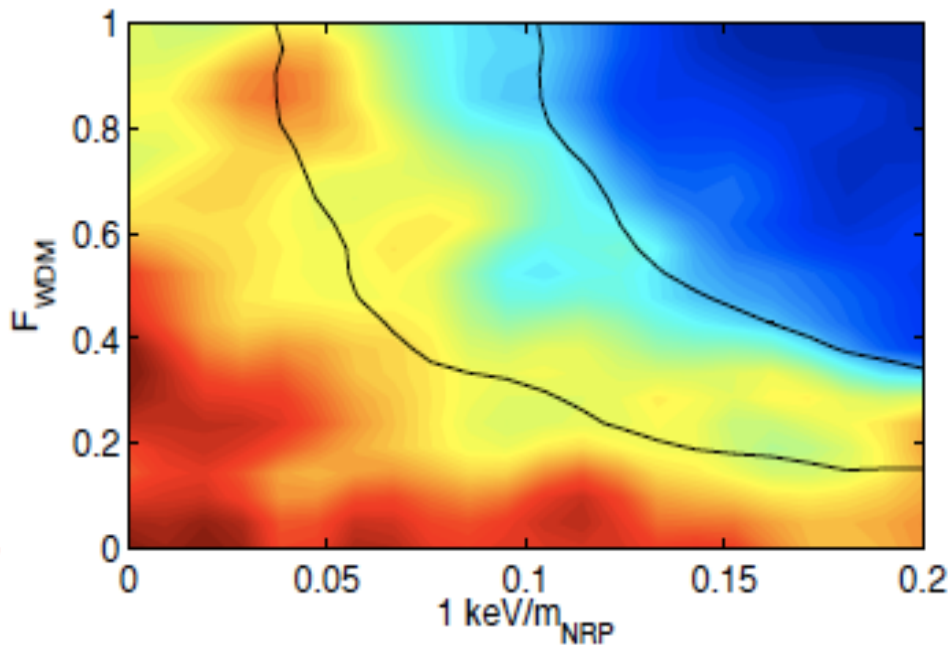


Boyarsky, Lesgourgues. Ruchayski, Viel, 2009, PRL, 102, 201304

Lyman-α and sterile neutrinos - V

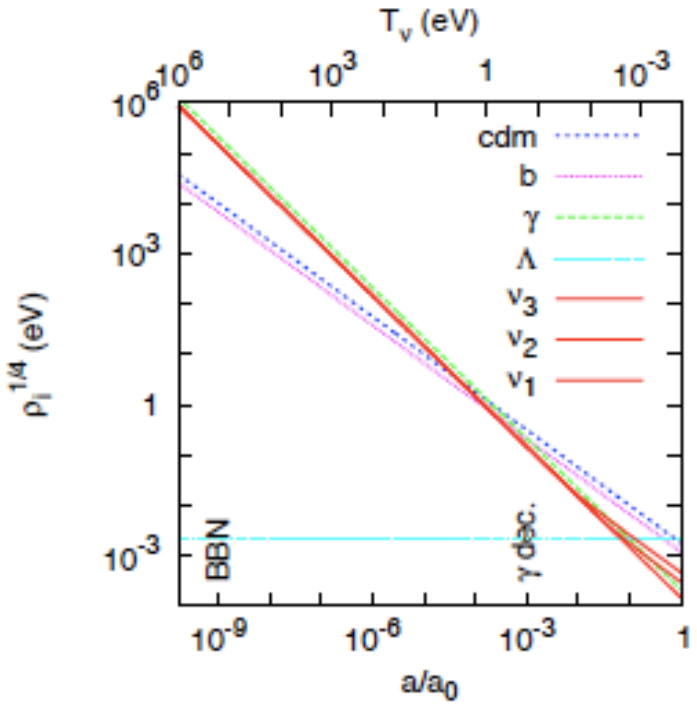


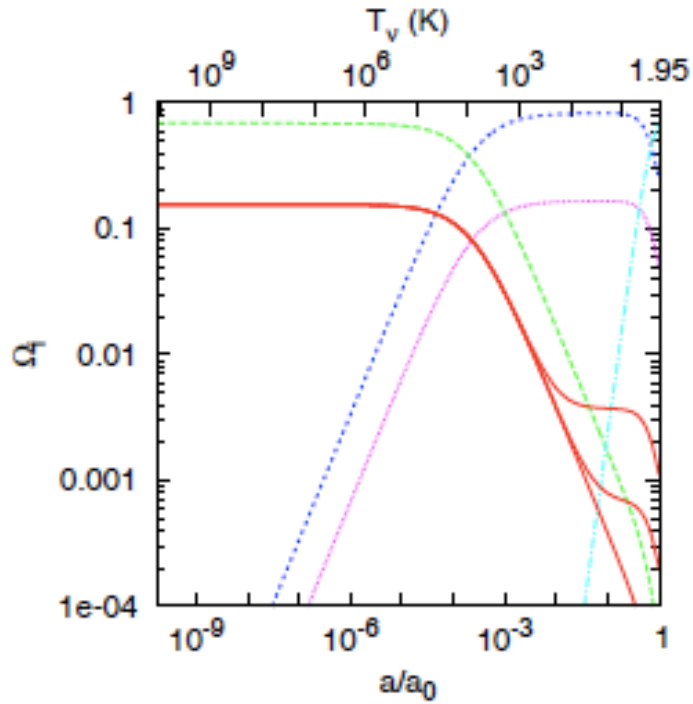
SDSS+WMAP5



Boyarsky, Lesgourgues, Ruchayskiy, Viel, 2009, JCAP, 05, 012– REVIEW!

EVOLUTION of LSS – III : background evolution





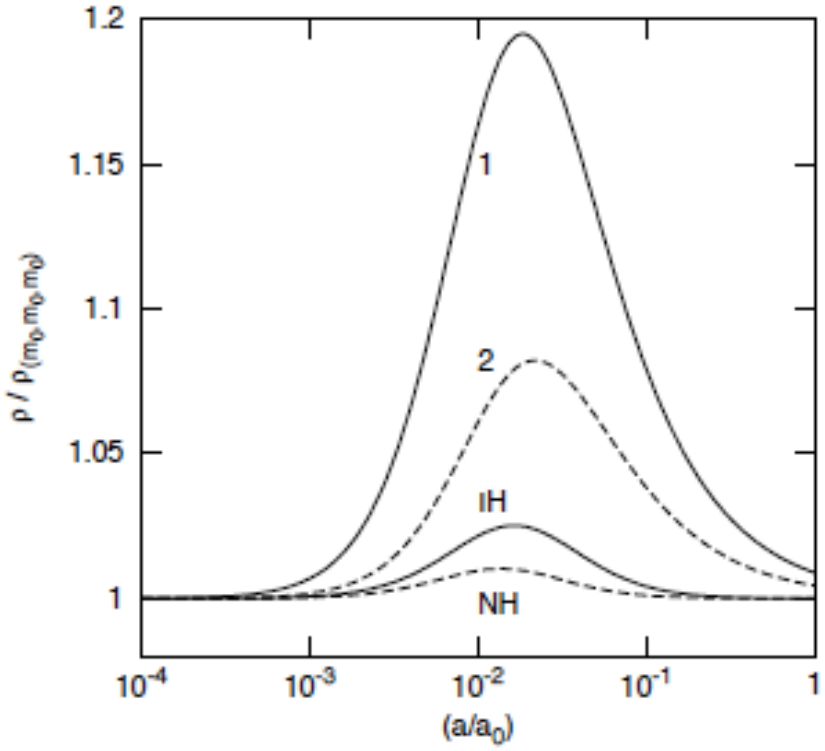
$$H(a) = H_0 \left[\frac{\Omega_m}{a^3} + \frac{\Omega_r}{a^4} + \frac{\Omega_k}{a^2} + \frac{\Omega_\Lambda}{a^{3(1+w_{\rm eff}(a))}} \right]^{1/2}$$

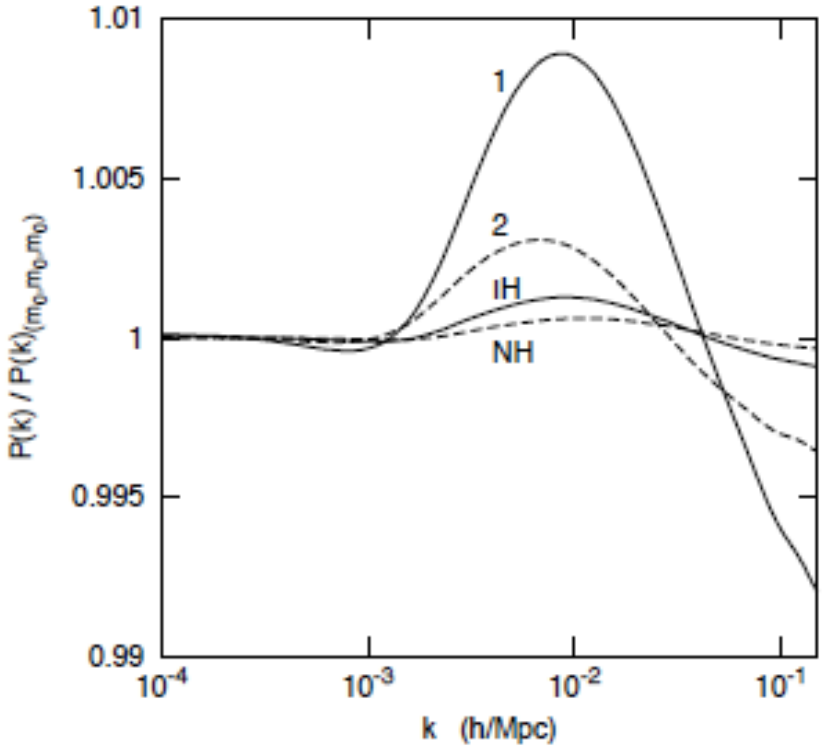
$$\Omega_r = \Omega_\gamma \left(1 + 0.2271 N_{\rm eff}\right)$$

Note that the equation above is not exact
but it is a good approximation (e.g. Komatsu et al 11)

EVOLUTION of LSS - IV: individual neutrino masses do matter

Lesgourgues & Pastor 2006



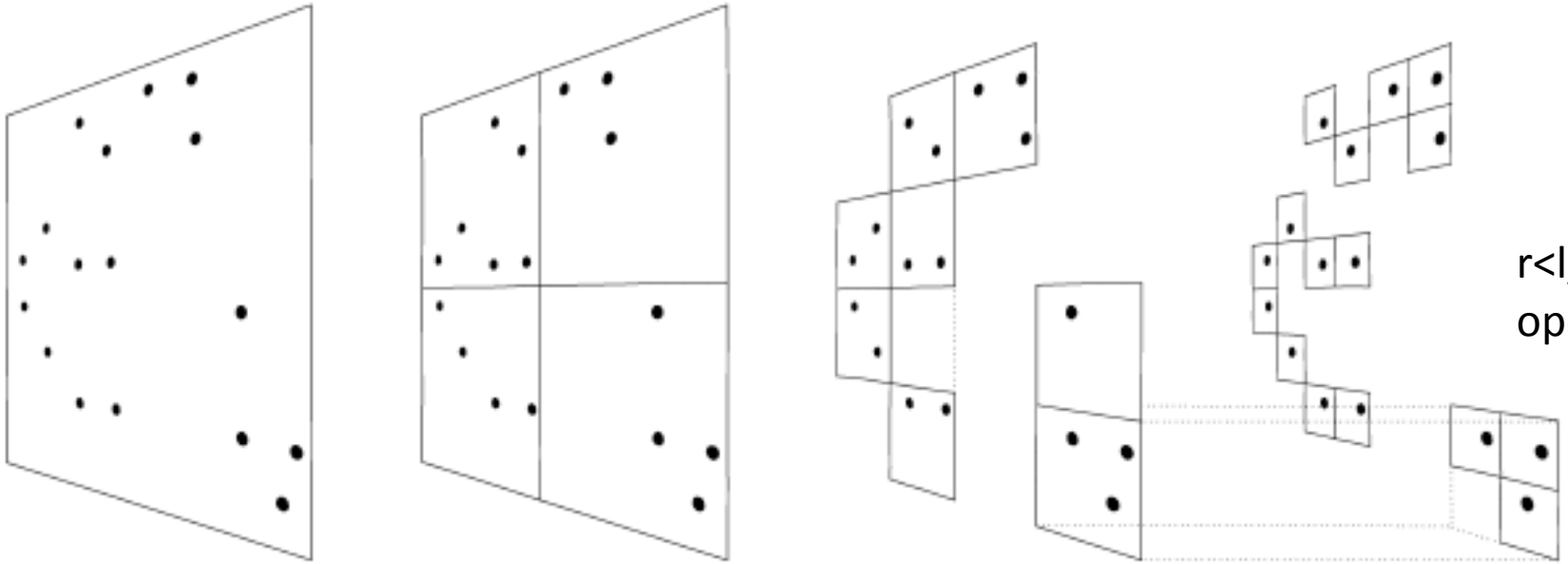


GEOMETRY

DYNAMICS

SIMULATION of LSS – II: basic equations for DM

Tree method – expansion of the gravitational potential in multipoles



r<l/θ then
open the node

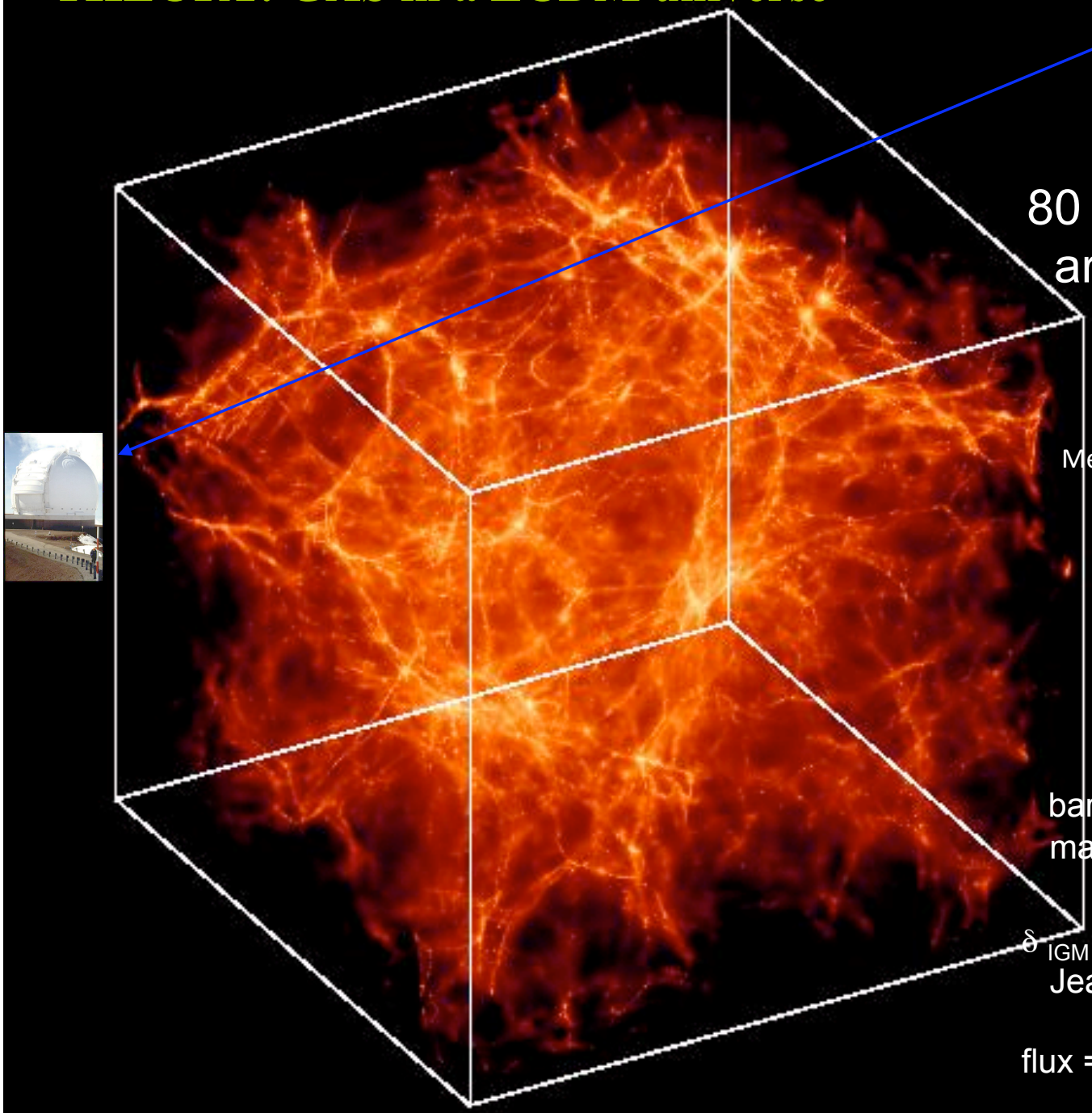
$$\Phi(r) = G\int_{V'} \frac{\rho(r')dV'}{|\vec{r}'-\vec{r}|}$$

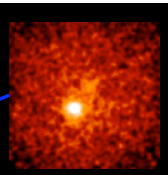
$$\frac{1}{|r'-r|} \approx \frac{1}{r} + \sum_i r_i' \frac{\partial(1/r)}{\partial r_i} + \frac{1}{2}\sum_i\sum_j r_i' r_j' \frac{\partial^2(1/r)}{\partial r_i \partial r_j} - \frac{1}{6}\sum_i\sum_j\sum_k r_i' r_j' r_k' \frac{\partial^3(1/r)}{\partial r_i \partial r_j \partial r_k}$$

$$\Phi(r) = \left\{ M(1/r) - \vec{P} \bullet \nabla(1/r) + \frac{1}{2}\mathbf{Q} : \nabla\nabla(1/r) - \frac{1}{6}\overline{\mathbf{S}} \cdot\cdot \nabla\nabla\nabla(1/r) + \cdots + \right\}$$

P dipole moment, Q tensor quadropole moment, S usually not considered

THEORY: GAS in a LCDM universe





80 % of the baryons at z=3 are in the Lyman-α forest

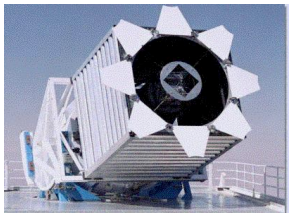
Meiksin's review (2007)

baryons as tracer of the dark matter density field

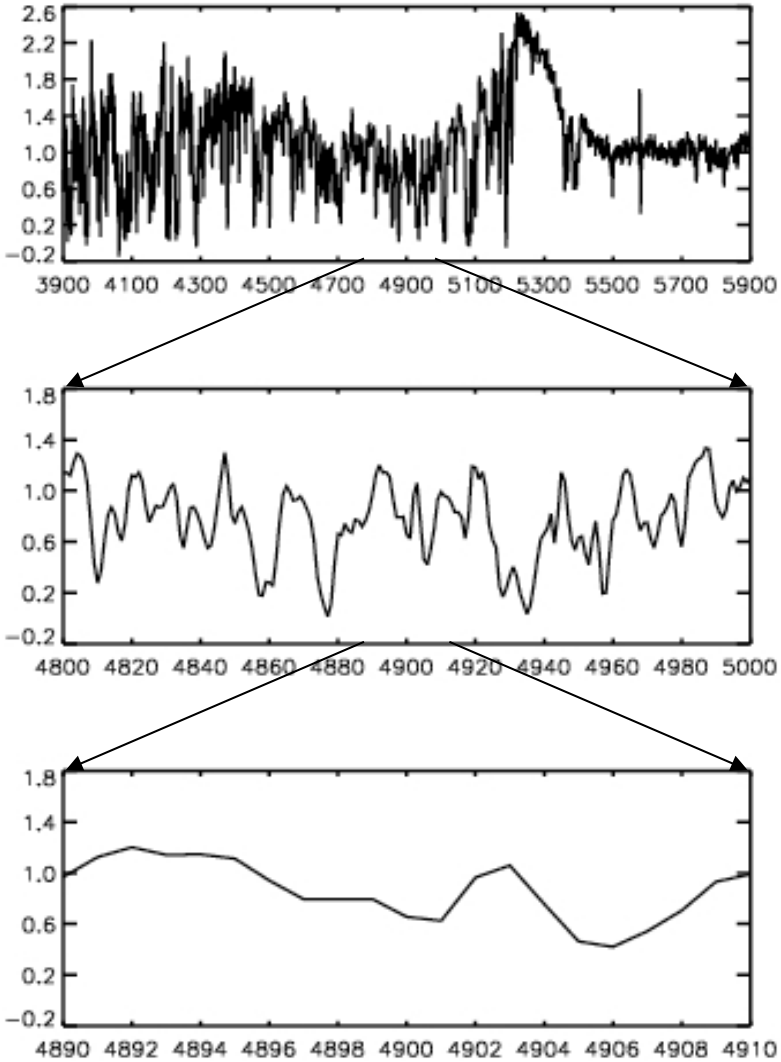
$\delta_{IGM} \sim \delta_{DM}$ at scales larger than the Jeans length ~ 1 com Mpc

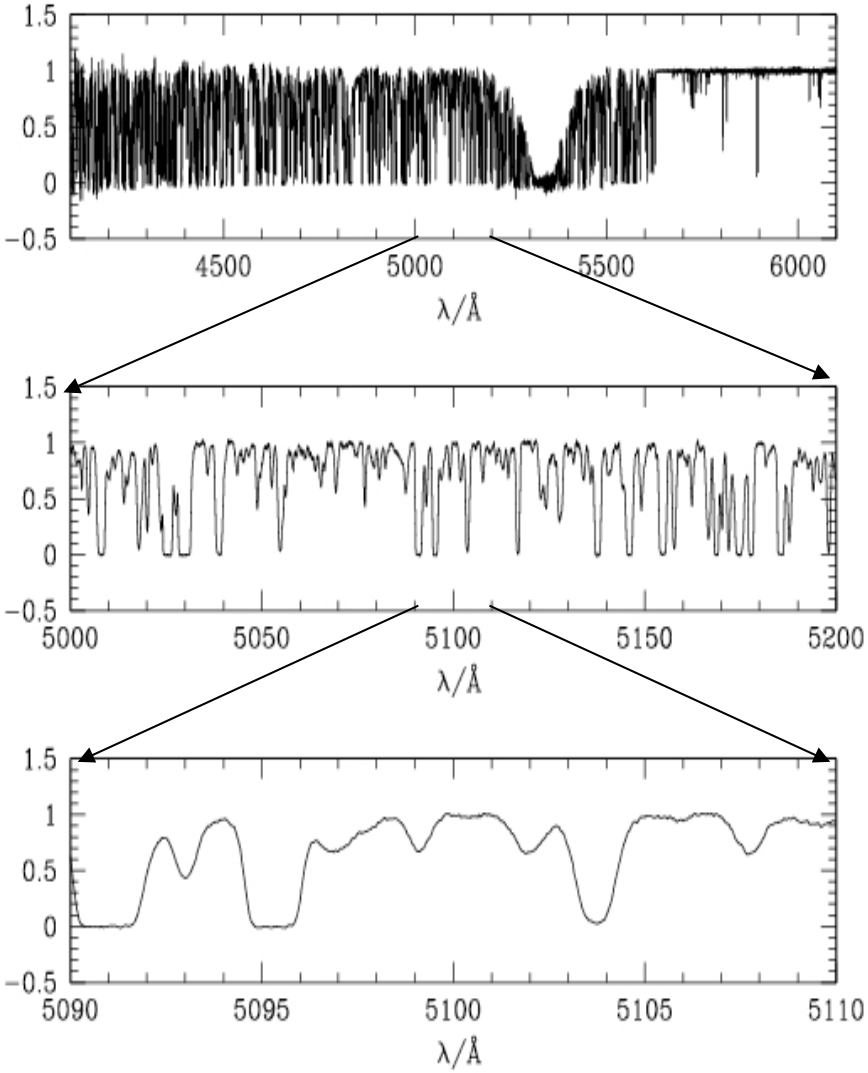
$\text{flux} = \exp(-\tau) \sim \exp(-(\delta_{IGM})^{1.6}\, T^{-0.7})$

The data sets



SDSS vs UVES





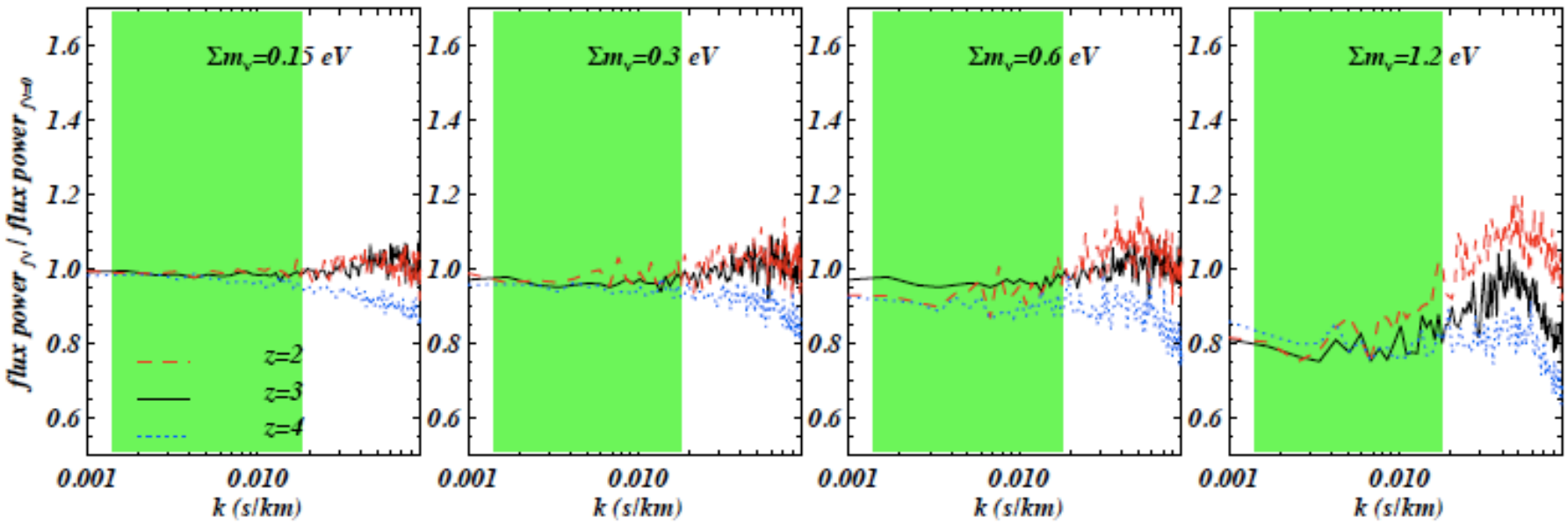
SDSS
3035 LOW RESOLUTION LOW S/N

vs

LUQAS
30 HIGH RESOLUTION HIGH S/N

Hydro simulations – VI: redshift/scale dependence of flux power

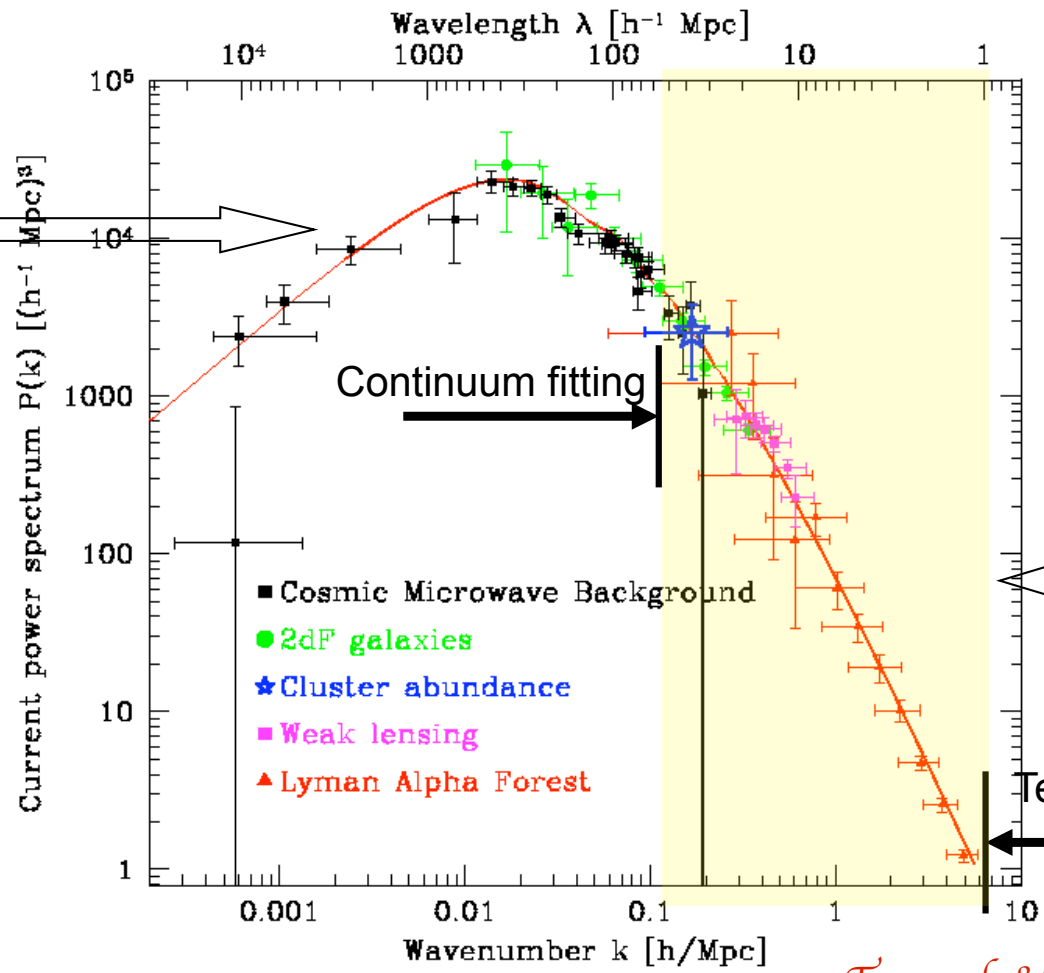
Effect on flux power observables is smaller than matter power



Viel, Haehnelt & Springel 2010, JCAP, 06 ,15

GOAL: the primordial dark matter power spectrum from the observed flux spectrum (filaments)

CMB physics
z = 1100
dynamics



Lya physics
z < 6
dynamics
+
termodynamics

Temperature, metals, noise

Tegmark & Zaldarriaga 2002

CMB + Lyman a → Long lever arm

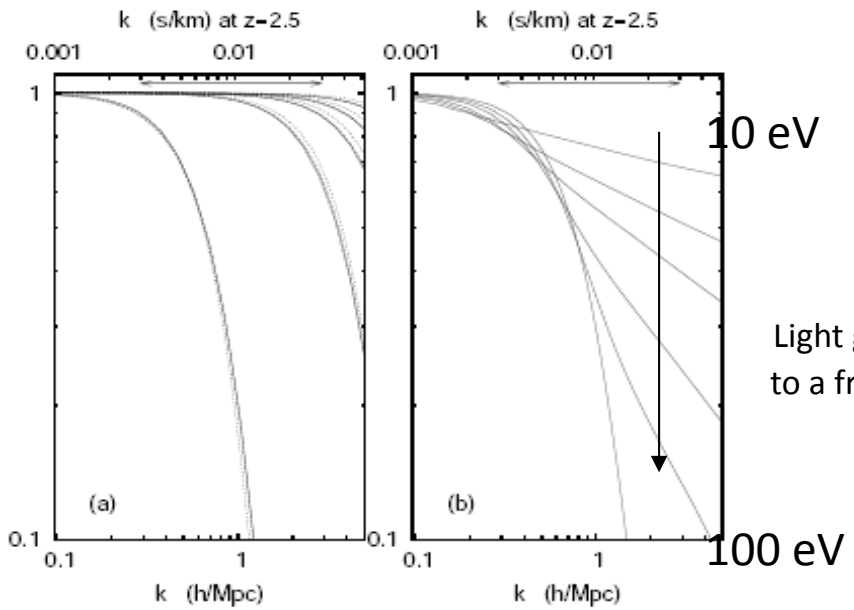
Constrain spectral index and shape

Relation: $P_{FLUX}(k) - P_{MATTER}(k)$??

Lyman-α and Warm Dark Matter - II

$P(k) = A\, k^n\, T^2(k)$

$[P(k)_{WDM}/P(k)_{CDM}]^{1/2}$

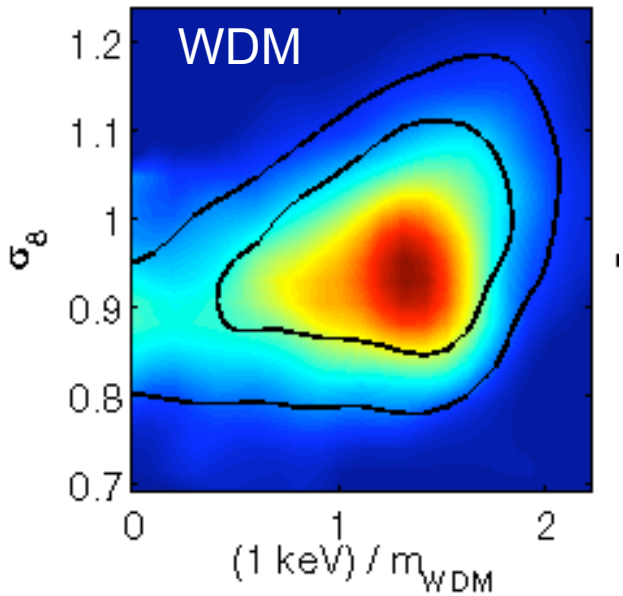


Light gravitino contributing to a fraction of dark matter

$$\frac{T_x}{T_\nu} = \left(\frac{10.75}{g(T_D)}\right)^{1/3}$$

MV, Lesgourgues, Haehnelt, Matarrese, Riotto, PRD, 2005, 71, 063534

Lyman-α and Warm Dark Matter - III



m_{WDM} > 550 eV thermal
> 2keV sterile neutrino
< 16 eV gravitino

Viel et al. (2005)from high-res

$$\Lambda_{\rm susy} \simeq \left(\sqrt{3} m_{3/2} M_p\right)^{1/2} \lesssim 260\,{\rm TeV}$$

Seljak, Makarov, McDonald, Trac, PhysRevLett, 2006, 97, 191303

MV, Lesgourgues, Haehnelt, Matarrese, Riotto, PhysRevLett, 2006, 97, 071301