

# Cabibbo angle anomaly: statistical assessment & New Physics interpretations

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Implications of LHCb measurements and future prospects

CERN, Geneva, October 19 - 21, 2022



Based on *JHEP 07 (2020) 068*,  
*Y. Grossman, E.P. and S. Schacht*

# Outline

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1. Introduction and Motivation: Cabbibo angle anomaly
2. Statistical Assessment
3. New Physics Interpretations
4. Conclusion and Outlook

# 1. Introduction and Motivation

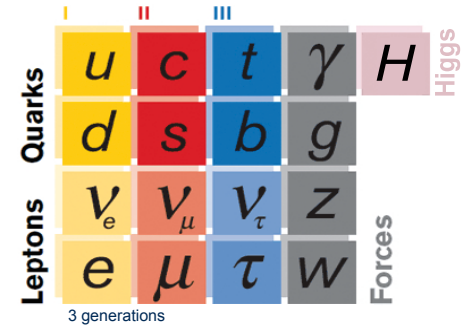
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# 1.1 Test of the Standard Model: $V_{us}$ and CKM unitarity

- Extraction of the Cabibbo-Kobayashi-Maskawa matrix element  $V_{us}$ 
  - Fundamental parameter of the Standard Model

Description of the **weak interactions**:

$$\mathcal{L}_{EW} = \frac{g}{\sqrt{2}} W_{\alpha}^{+} \left( \bar{D}_L V_{CKM} \gamma^{\alpha} U_L + \bar{e}_L \gamma^{\alpha} \nu_{eL} + \bar{\mu}_L \gamma^{\alpha} \nu_{\mu L} + \bar{\tau}_L \gamma^{\alpha} \nu_{\tau L} \right) + \text{h.c.}$$



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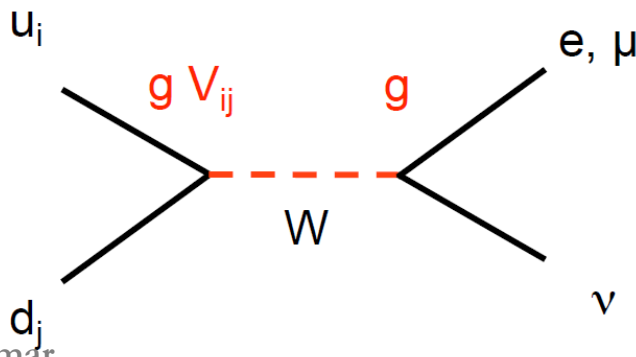
Unitary matrix

	I	II	III		
Quarks	u	c	t	$\gamma$	H
	d	s	b	g	
Leptons	$\nu_e$	$\nu_\mu$	$\nu_\tau$	Z	
	e	$\mu$	$\tau$	W	
	3 generations			Forces	

- Check unitarity of the first row of the CKM matrix:

➔ **Cabibbo Universality:**  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

Negligible  $\sim 2 \times 10^{-5}$   
(B decays)



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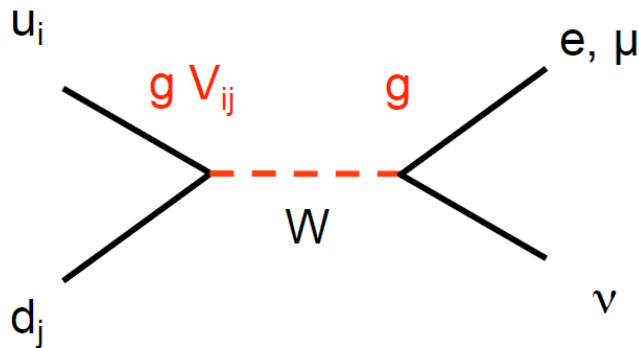
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Gauge coupling

- Universality: Is  $G_F$  from  $\mu$  decay equals to  $G_F$  from  $\pi$ , K, nuclear  $\beta$  decay?

$$G_{\mu}^2 = (g_{\mu} g_e)^2 / M_W^4 \stackrel{?}{=} G_{CKM}^2 = (g_q g_l)^2 (|V_{ud}|^2 + |V_{us}|^2) / M_W^4$$



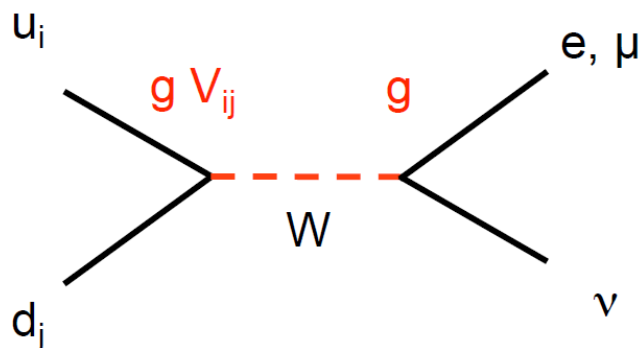
## 1.2 Constraining New Physics

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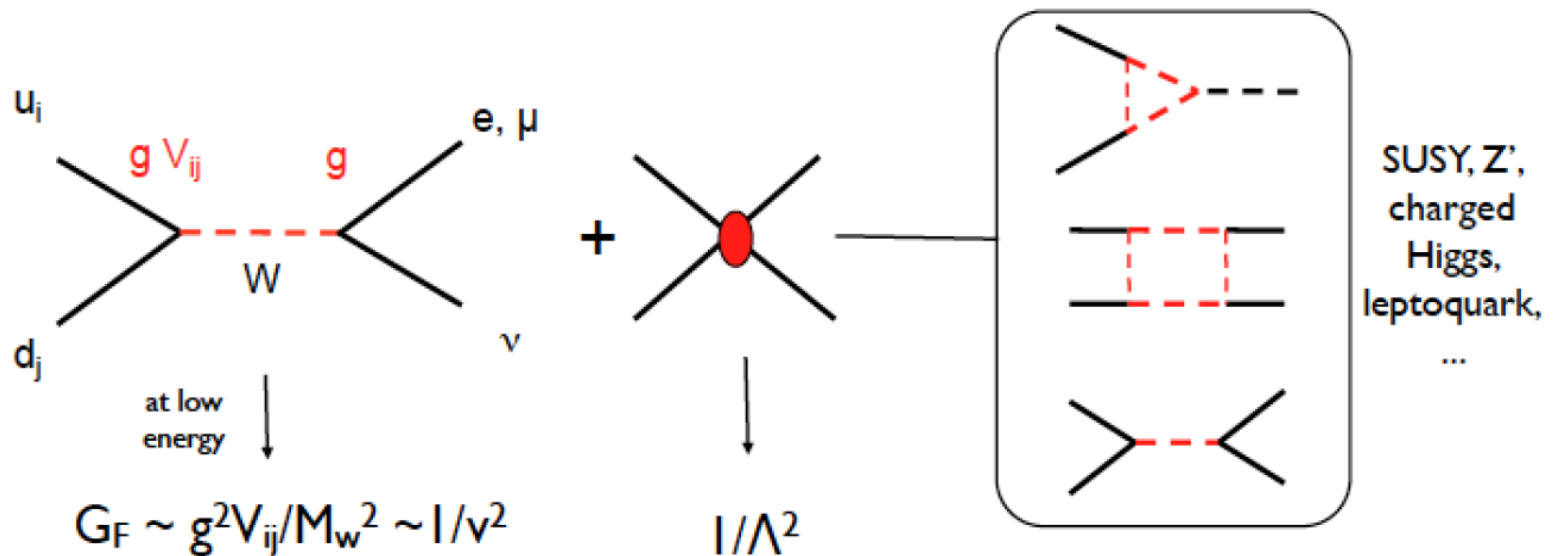
- Look for **new physics**
  - In the Standard Model : W exchange  $\Rightarrow$  only V-A structure



# 1.2 Constraining New Physics

- BSM: sensitive to tree-level and loop effects of a large class of models

➔  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$



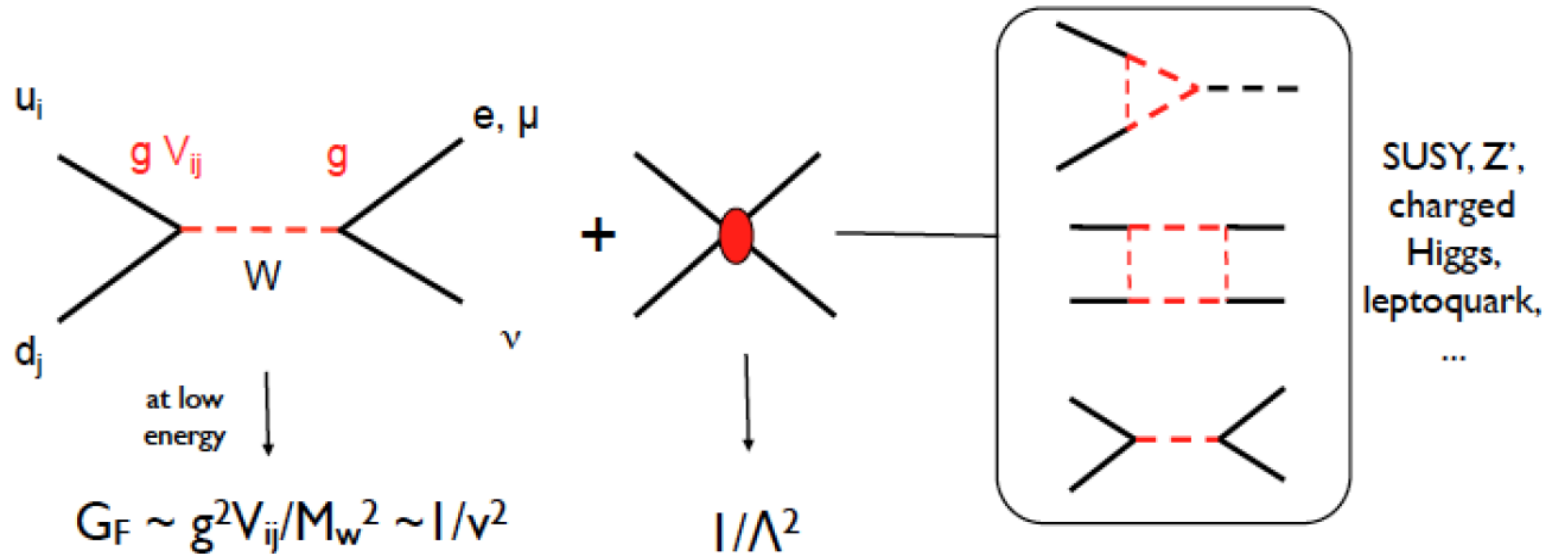
➔ BSM effects :  $\Delta \sim \frac{c_n}{g^2} \frac{M_W^2}{\Lambda^2} \leq 10^{-2} - 10^{-3} \leftrightarrow \Lambda \sim 1-10 \text{ TeV}$



# 1.2 Constraining New Physics

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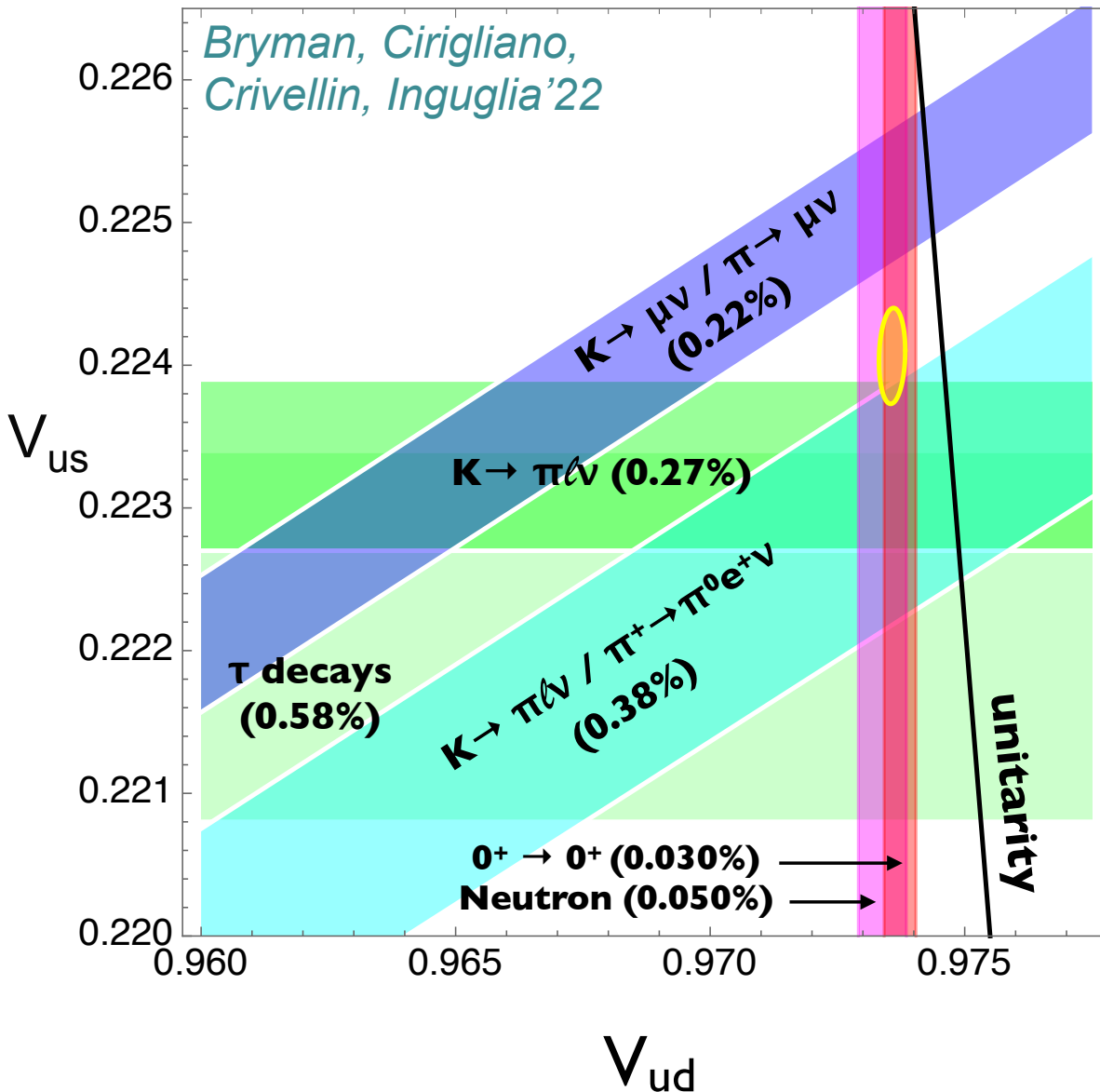
➔ 
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$$



- Look for new physics by comparing the extraction of  $V_{us}$  from different processes: helicity suppressed  $K_{\mu 2}$ , helicity allowed  $K_{l 3}$ , hadronic  $\tau$  decays

# 1.2 Cabibbo angle anomaly

Moulson &  
E.P.@CKM2021



$$|V_{ud}| = 0.97373(31)$$

$$|V_{us}| = 0.2231(6)$$

$$|V_{us}|/|V_{ud}| = 0.2311(5)$$

Fit results, no constraint

$$V_{ud} = 0.97365(30)$$

$$V_{us} = 0.22414(37)$$

$$\chi^2/\text{ndf} = 6.6/1 \text{ (1.0\%)}$$

$$\Delta_{\text{CKM}} = -0.0018(6)$$

**-2.7 $\sigma$**

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{\text{CKM}}$$

Negligible  $\sim 2 \times 10^{-5}$   
(B decays)

# Paths to $V_{ud}$ and $V_{us}$

- From kaon, pion, baryon and nuclear decays

$V_{ud}$	$0^+ \rightarrow 0^+$ $\pi^\pm \rightarrow \pi^0 e \nu_e$	$n \rightarrow p e \nu_e$	$\pi \rightarrow l \nu_l$
$V_{us}$	$K \rightarrow \pi l \nu_l$	$\Lambda \rightarrow p e \nu_e$	$K \rightarrow l \nu_l$

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{RC}) \times F_{\text{kin}}$$



Channel-dependent  
effective CKM element



Hadronic matrix  
element



Radiative corrections

- Recent progress on
- 1) Hadronic matrix elements from lattice QCD
  - 2) Radiative corrections from dispersive methods + Lattice QCD

*Seng, Gorchtein, Patel, Ramsey-Musolf'18,'19*

## 1.2 Cabibbo angle anomaly

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- At the moment discrepancy between:

- $V_{us}^{K_{\ell 3}}$ ,  $\left(V_{us}/V_{ud}\right)^{K_{\ell 2}}$ ,  $V_{ud}^{\beta}$  and CKM unitarity

- $V_{us}^{K_{\ell 3}}$  and  $V_{ud}^{\beta}$

- $V_{us}^{K_{\ell 3}}$  and  $\left(V_{us}/V_{ud}\right)^{K_{\ell 2}}$

## 2. Statistical Assessment

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## 2.1 CKM unitarity test

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- 2 types of tests:

- **Universality test of Cabibbo angle within the SM**

Assess the goodness of fit of the one-parameter null hypothesis

$$\theta_C = \theta_1 = \theta_2 = \dots = \theta_n$$

for  $n$  different experimental determinations of the Cabibbo angle with different observables



$$\nu_{\text{SM test}} = n - 1$$

- **CKM unitarity test:** use 2 parameters  $V_{us}$  and  $\Delta_{\text{CKM}}$

$$V_{ud} = \sqrt{1 - V_{us}^2 - \Delta_{\text{CKM}}}$$

Test the null-hypothesis  $\Delta_{\text{CKM}} = 0$  against the general case  $\Delta_{\text{CKM}} \neq 0$

$$\Delta\chi_{\text{unitarity test}}^2 \equiv \chi_{\text{min, unitary}}^2 - \chi_{\text{min, non-unitary}}^2$$

## 2.2 Comparison of SM test and CKM unitarity test

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- $\chi_{\text{SM test}}^2 = \chi_{\text{min, unitary}}^2$  BUT
- For SM test  $\Rightarrow$  assess the goodness-of-fit of the Cabibbo angle universality hypothesis
- For CKM unitarity test  $\Rightarrow$  comparison of the hypotheses of unitary vs. non-unitary.

$$\Delta\chi_{\text{unitarity test}}^2 \equiv \chi_{\text{min, unitary}}^2 - \chi_{\text{min, non-unitary}}^2$$

$\Rightarrow$  *Different number of dofs*

$$\nu_{\text{unitarity test}} = 1$$

and

$$\nu_{\text{SM test}} = n - 1$$

## 2.2 Comparison of SM test and CKM unitarity test

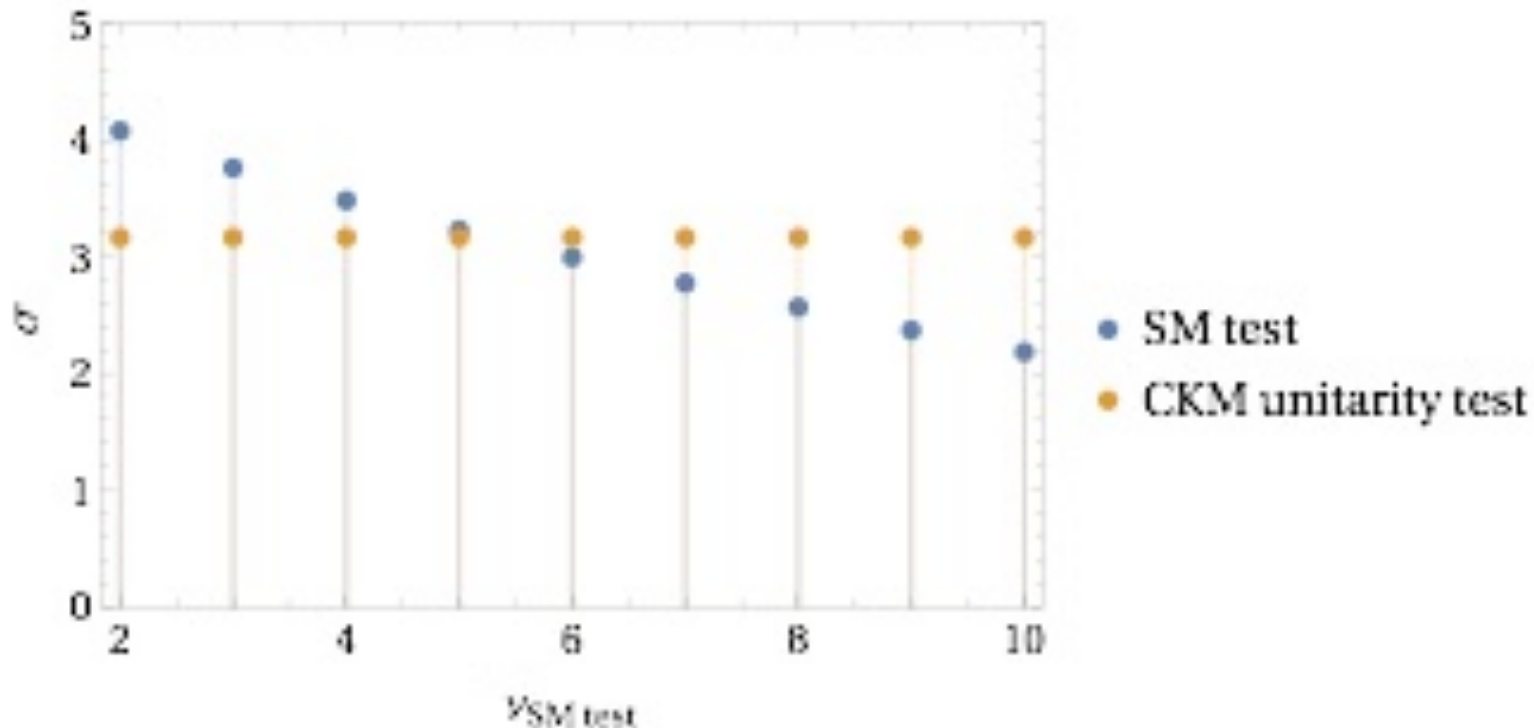
$n$	1	2	$\geq 3$
$\chi_{\text{SM test}}^2$	0	$\chi^2$	$\chi^2 > \Delta\chi_{\text{unitarity test}}^2$
$\nu_{\text{SM test}}$	0	1	$n - 1 \geq 2$
$p_{\text{SM test}}$	1	$p$	$\neq p_{\text{unitarity test}}$
$z_{\text{SM test}}$	0	$z$	$\neq z_{\text{unitarity test}}$
$\Delta\chi_{\text{unitarity test}}^2$	0	$\chi^2$	$< \chi_{\text{SM test}}^2$
$\chi_{\text{min, unitary}}^2$	0	$\chi^2$	$\chi^2$
$\chi_{\text{min, non-unitary}}^2$	0	0	$> 0$
$\nu_{\text{unitarity test}}$	1	1	1
$p_{\text{unitarity test}}$	1	$p$	$\neq p_{\text{SM test}}$
$z_{\text{unitarity test}}$	0	$z$	$\neq z_{\text{SM test}}$

➔ Test results are different starting from **3 observables**



## 2.2 Comparison of SM test and CKM unitarity test

- Two-sided p-value:  $p = 1 - P_{\nu/2}(\chi^2/2)$
- Significance of rejection of the SM:  $z = \sqrt{2} \text{Erf}^{-1}(1 - p)$
- Toy example for comparison of significances of the rejection of the SM and CKM unitarity for fixed  $\Delta\chi_{\text{SM test}}^2 = 20$  and  $\Delta\chi_{\text{unitarity test}}^2 = 10$



## 2.3 Flaw of unitarity tests

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- When using CKM unitarity tests  $\Rightarrow$  only 2 parameters  $V_{us}$  and  $\Delta_{CKM}$

Comparison of a one-parameter fit to a two-parameter fit only  
No matter how many measurements #d.o.f of the CKM unitarity test is always fixed

- When tension in 2 measurements among 3  $\Rightarrow$  no significant difference
- When tension in 3 measurements  $\Rightarrow$  the significances for the rejection of the SM via the Cabibbo angle and CKM unitarity are different

*Antonelli et al.'09,'11*

*Gonzalez-Alonso & Martin Camalich'16*

*Grossman, E.P., Schacht'20*

## 2.4 Application to $K_{l3}$ , $K_{l2}$ and superallowed $\beta$ decays

Fit	$n$	$\chi_{\text{SM test}}^2$	$\nu_{\text{SM test}}$	$p_{\text{SM test}}$	$z_{\text{SM test}}$	$\Delta\chi_{\text{unitarity test}}^2$	$p_{\text{unitarity test}}$	$z_{\text{unitarity test}}$
$K_{l3} + K_{l2}$	2	8.5	1	0.0036	$2.9 \sigma$	8.5	0.0036	$2.9 \sigma$
$K_{l3} + K_{l2} + \beta$ (SGPRM)	3	30.0	2	$3.1 \cdot 10^{-7}$	$5.1 \sigma$	22.8	$1.8 \cdot 10^{-6}$	$4.8 \sigma$
$K_{l2} + \beta$ (SGPRM)	2	11.6	1	0.00065	$3.4 \sigma$	11.6	0.00065	$3.4 \sigma$
$K_{l3} + \beta$ (SGPRM)	2	30.0	1	$4.4 \cdot 10^{-8}$	$5.5 \sigma$	30.0	$4.4 \cdot 10^{-8}$	$5.5 \sigma$
$K_{l3} + K_{l2} + \beta$ (CMS)	3	16.5	2	0.00027	$3.6 \sigma$	9.0	0.0027	$3.0 \sigma$
$K_{l2} + \beta$ (CMS)	2	3.6	1	0.056	$1.9 \sigma$	3.6	0.056	$1.9 \sigma$
$K_{l3} + \beta$ (CMS)	2	15.1	1	0.00010	$3.9 \sigma$	15.1	0.00010	$3.9 \sigma$

SGPRM: Seng, Gorchtein,  
Patel, Ramsey-Musolf'18,'19  
CMS: Czarnecki, Marciano,  
Sirlin'19

Sign of rejection of  
Cabibbo angle universality

Sign of CKM  
unitarity rejection

➡ Test results are different starting from  $n \geq 3$

### 3. New Physics Interpretations

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## 3.1 Right-handed currents

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*Bernard, Oertel, E.P., Stern'08*

$$\mathcal{L}_W = \frac{e(1 - \xi^2 \rho_L)}{\sqrt{2}s} \left\{ \bar{N}_L V_{MNS} \gamma^\mu L_L + (1 + \delta) \bar{U}_L V_L \gamma^\mu D_L + \epsilon \bar{U}_R V_R \gamma^\mu D_R \right\} W_\mu^+ + \text{h.c.}$$

- See also *Antonelli et al.'09*  
*Alioli, Cirigliano, Dekens, de Vries, Mereghetti'17*  
*T. Kitahara@HC2NP 2019*

### 3.1 Right-handed Currents

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$$V_{us}^{K_{l3}} = |\sin \theta_C + \epsilon_s| , \quad \leftarrow \text{Vector s quark}$$

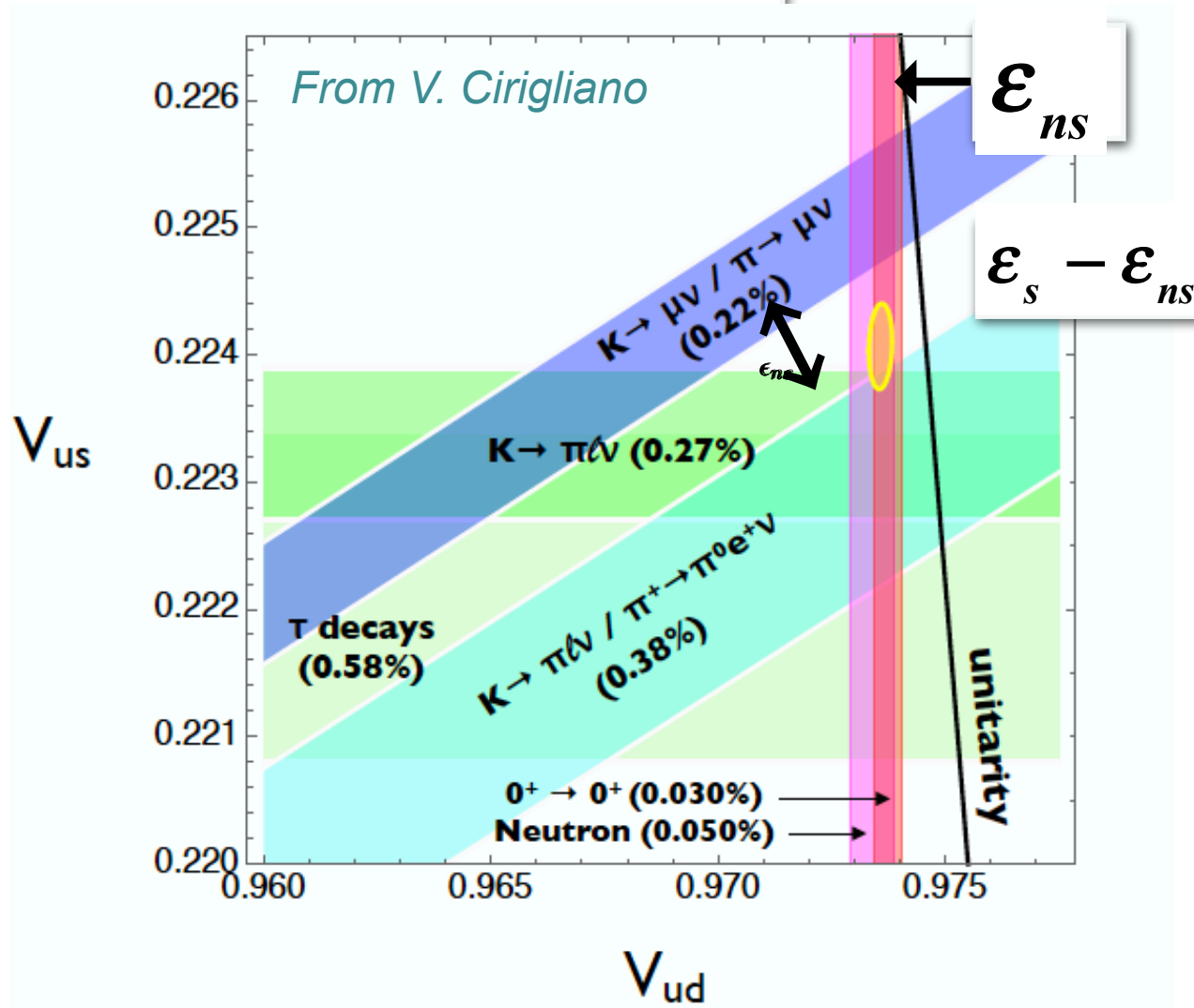
$$\left( \frac{V_{us}}{V_{ud}} \right)^{K_{l2}} = \left| \frac{\sin \theta_C - \epsilon_s}{\cos \theta_C - \epsilon_{ns}} \right| \quad \leftarrow \text{Axial}$$

$$V_{ud}^\beta = |\cos \theta_C + \epsilon_{ns}| \quad \leftarrow \text{Vector no s quark}$$

$$\left( \frac{V_{us}}{V_{ud}} \right)^{K_{l3}} = \left| \frac{\sin \theta_C + \epsilon_s}{\cos \theta_C + \epsilon_{ns}} \right| \quad \leftarrow \text{Vector}$$

- The SM is obtained in the limit  $\epsilon_s = \epsilon_{ns} = 0$ .
- Perfect fit to data  $\chi_{\min, \text{RH}}^2 = 0$
- Not obvious how to define CKM unitarity test in this case

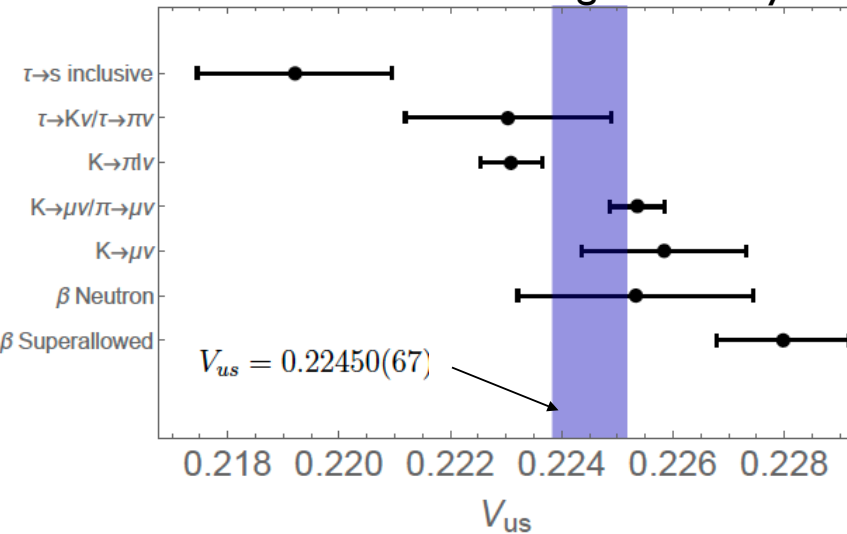
# 3.1 Right-handed Currents



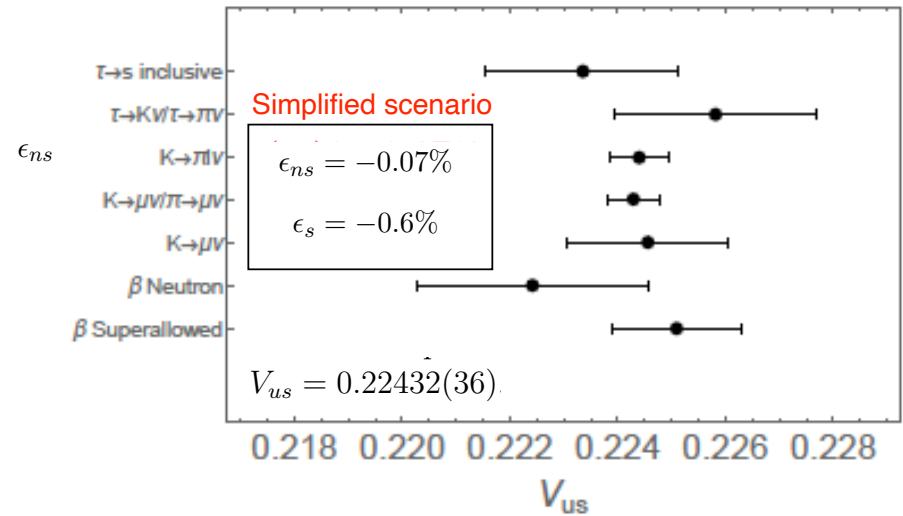
# 4.1 Right-handed Currents

- Global fit to CC processes involving light quarks and all lepton families
- SM hypothesis ( $\epsilon_s = \epsilon_{ns} = 0$ ) disfavored (p-value 0.3%)

SM limit: Cabibbo angle anomaly



Anomaly removed by turning on the  $\epsilon_R$  couplings




*Cirigliano, Diaz-Calderon, Falkowski, Gonzalez-Alonso, Rodriguez-Sanchez'21*

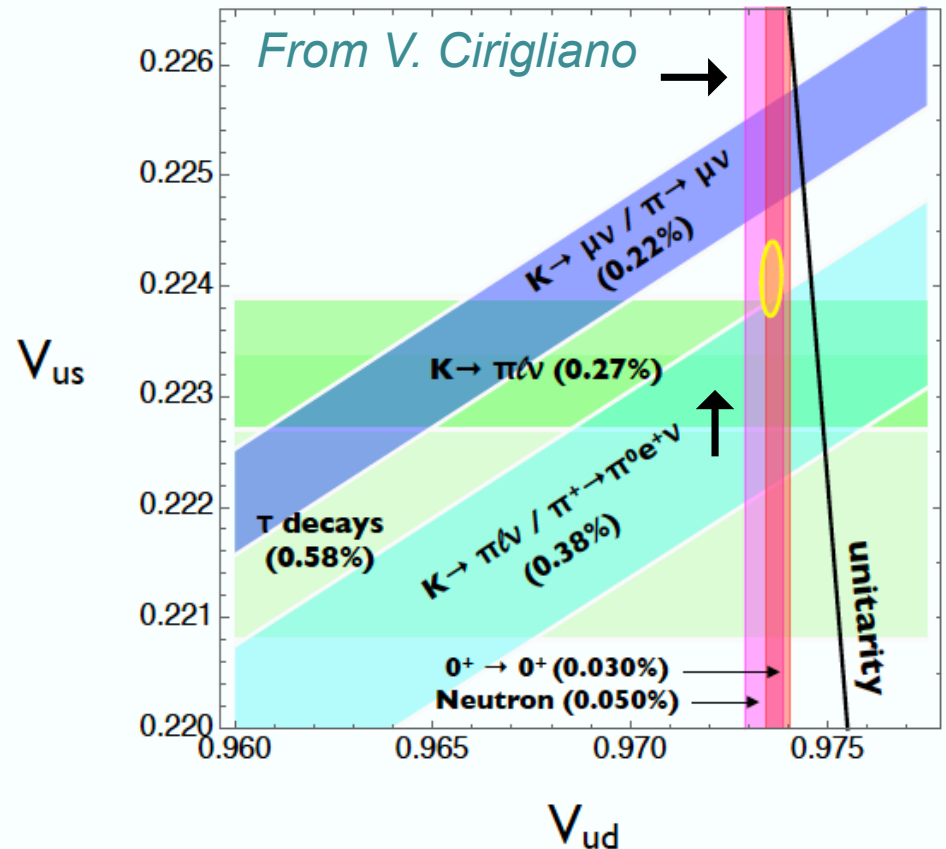


## 3.2 Other New Physics Models

- 4th quark  $b'$  *Belfatto, Beradze, Berezhiani'19*
- Gauge horizontal family symmetry
- Turn on only vertex corrections to leptons *Crivellin & Hoferichter'21*

 Shift the location of the  $V_{ud}, V_{us}$  bands but do not solve the tension between ratios

And many more....



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Connection with  $\pi \rightarrow e\nu / \pi \rightarrow \mu\nu$

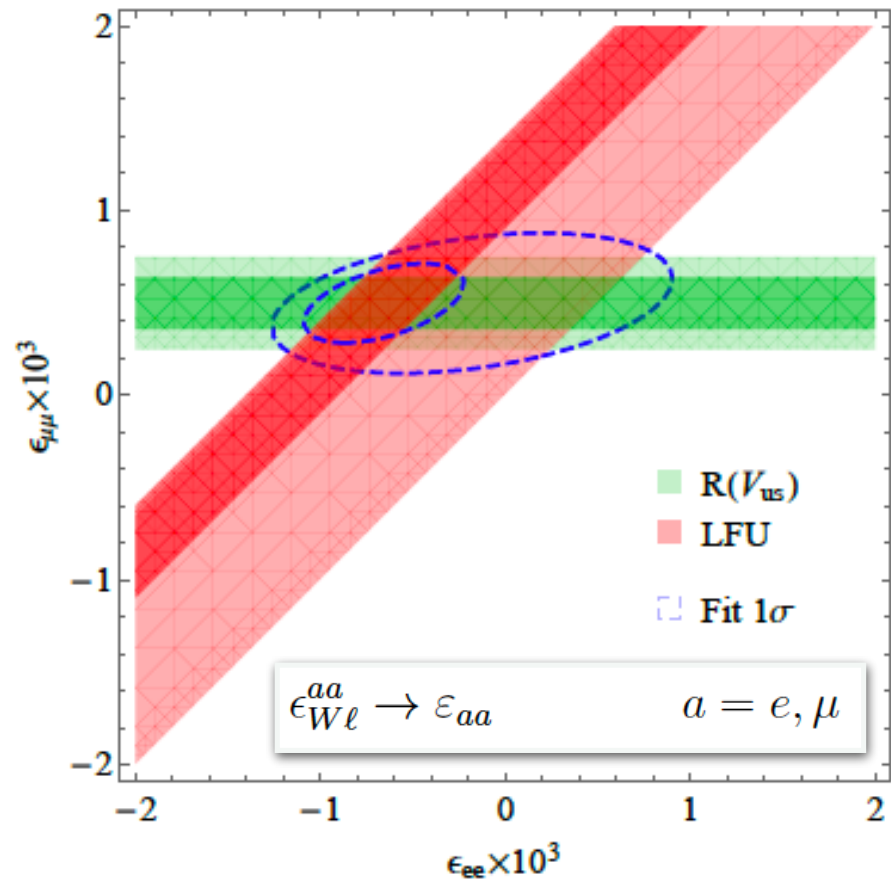
$$r_\pi = 1 + 2(\epsilon_{W\ell}^{ee} - \epsilon_{W\ell}^{\mu\mu})$$

(and other LFU probes)

And many more....

*Belfatto, Beradze, Berezhiani'19*

*Crivellin & Hoferichter'21*



## 4. Conclusion and Outlook

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# 4.1 Conclusion


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- Recent precision determinations of  $V_{us}$  and  $V_{ud}$  enable unprecedented tests of the SM and constraints on possible NP models like right-handed currents.
- A SM test via the test of the Cabibbo angle universality goes beyond just a test of CKM unitarity and gives different test results if more than two observables are taken into account.
- In a CKM unitarity test one compares a constrained fit with a fit of free floating  $V_{us}$  and  $V_{ud}$ . The latter can not necessarily describe the data as well as a BSM model, in case the patterns go beyond just violating unitarity. This matters starting from three independent observables being taken into account.
  - ➡ Test the SM by testing for the universality of the Cabibbo angle
- If the anomaly persists ➡ RHCs could explain it

## 4.2 Experimental Prospects for $V_{us}$ (Snowmass)

On Kaon side

*Cirigliano et al'22*

- *NA62* could measure **several BRs**:  $K_{\mu 3}/K_{\mu 2}$ ,  $K \rightarrow 3\pi$ ,  $K_{\mu 2}/K \rightarrow \pi\pi$
- Note that the high precision measurement of  $\text{BR}(K_{\mu 2})$  (0.3%) comes only from a single experiment: KLOE. It would be good to have another measurement at the same level of accuracy
- *LHCb* : could measure  $\text{BR}(K_S \rightarrow \pi\mu\nu)$  at the  $< 1\%$  level?  
 $K_S \rightarrow \pi\mu\nu$  measured by KLOE-II but not competitive  
 $\tau_S$  known to 0.04% (vs 0.41% for  $\tau_L$ , 0.12% for  $\tau_{\pm}$ )
- $V_{us}$  from Tau decays at *Belle II*:  
Belle II with  $50 \text{ ab}^{-1}$  and  $\sim 4.6 \times 10^{10}$   $\tau$  pairs will improve  $V_{us}$  extraction from  $\tau$  decays  
Inclusive measurement is an opportunity to have a complete independent extraction of  $V_{us}$   not easy as you have to measure many channels



$$|V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}$$

To be competitive theory error will have to be improved as well

# $V_{us}$ from Hyperon decays

$V_{us}$  can be measured from Hyperon decays:

- $\Lambda \rightarrow p e \nu_e$  Possible measurement at *BESIII, Super  $\tau$ -Charm factory?*

- Possibilities at *LHCb?*

*Talk by Dettori@FPCP20*

Channel	$\mathcal{R}$	$\epsilon_L$	$\epsilon_D$	$\sigma_L(\text{MeV}/c^2)$	$\sigma_D(\text{MeV}/c^2)$	$\mathcal{R} = \text{ratio of production}$ $\epsilon = \text{ratio of efficiencies}$
$K_S^0 \rightarrow \mu^+ \mu^-$	1	1.0 (1.0)	1.8 (1.8)	$\sim 3.0$	$\sim 8.0$	
$K_S^0 \rightarrow \pi^+ \pi^-$	1	1.1 (0.30)	1.9 (0.91)	$\sim 2.5$	$\sim 7.0$	
$K_S^0 \rightarrow \pi^0 \mu^+ \mu^-$	1	0.93 (0.93)	1.5 (1.5)	$\sim 35$	$\sim 45$	
$K_S^0 \rightarrow \gamma \mu^+ \mu^-$	1	0.85 (0.85)	1.4 (1.4)	$\sim 60$	$\sim 60$	
$K_S^0 \rightarrow \mu^+ \mu^- \mu^+ \mu^-$	1	0.37 (0.37)	1.1 (1.1)	$\sim 1.0$	$\sim 6.0$	
$K_L^0 \rightarrow \mu^+ \mu^-$	$\sim 1$	$2.7 (2.7) \times 10^{-3}$	0.014 (0.014)	$\sim 3.0$	$\sim 7.0$	
$K^+ \rightarrow \pi^+ \pi^+ \pi^-$	$\sim 2$	$9.0 (0.75) \times 10^{-3}$	$41 (8.6) \times 10^{-3}$	$\sim 1.0$	$\sim 4.0$	
$K^+ \rightarrow \pi^+ \mu^+ \mu^-$	$\sim 2$	$6.3 (2.3) \times 10^{-3}$	0.030 (0.014)	$\sim 1.5$	$\sim 4.5$	
$\Sigma^+ \rightarrow p \mu^+ \mu^-$	$\sim 0.13$	0.28 (0.28)	0.64 (0.64)	$\sim 1.0$	$\sim 3.0$	
$\Lambda \rightarrow p \pi^-$	$\sim 0.45$	0.41 (0.075)	1.3 (0.39)	$\sim 1.5$	$\sim 5.0$	
$\Lambda \rightarrow p \mu^- \bar{\nu}_\mu$	$\sim 0.45$	0.32 (0.31)	0.88 (0.86)	—	—	
$\Xi^- \rightarrow \Lambda \mu^- \bar{\nu}_\mu$	$\sim 0.04$	$39 (5.7) \times 10^{-3}$	0.27 (0.09)	—	—	
$\Xi^- \rightarrow \Sigma^0 \mu^- \bar{\nu}_\mu$	$\sim 0.03$	$24 (4.9) \times 10^{-3}$	0.21 (0.068)	—	—	
$\Xi^- \rightarrow p \pi^- \pi^-$	$\sim 0.03$	0.41(0.05)	0.94 (0.20)	$\sim 3.0$	$\sim 9.0$	
$\Xi^0 \rightarrow p \pi^-$	$\sim 0.03$	1.0 (0.48)	2.0 (1.3)	$\sim 5.0$	$\sim 10$	
$\Omega^- \rightarrow \Lambda \pi^-$	$\sim 0.001$	$95 (6.7) \times 10^{-3}$	0.32 (0.10)	$\sim 7.0$	$\sim 20$	

- To be able to extract  $V_{us}$  one needs to compute form factors precisely

➡ Lattice effort from *RBC/UKQCD*

## 5. Back-up

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