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FSI to enhance CP violation in charm decay

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Implications of LHCb measurements and future prospects

CERN 2022

motivation

- $\Delta A_{CP}^{\text{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$

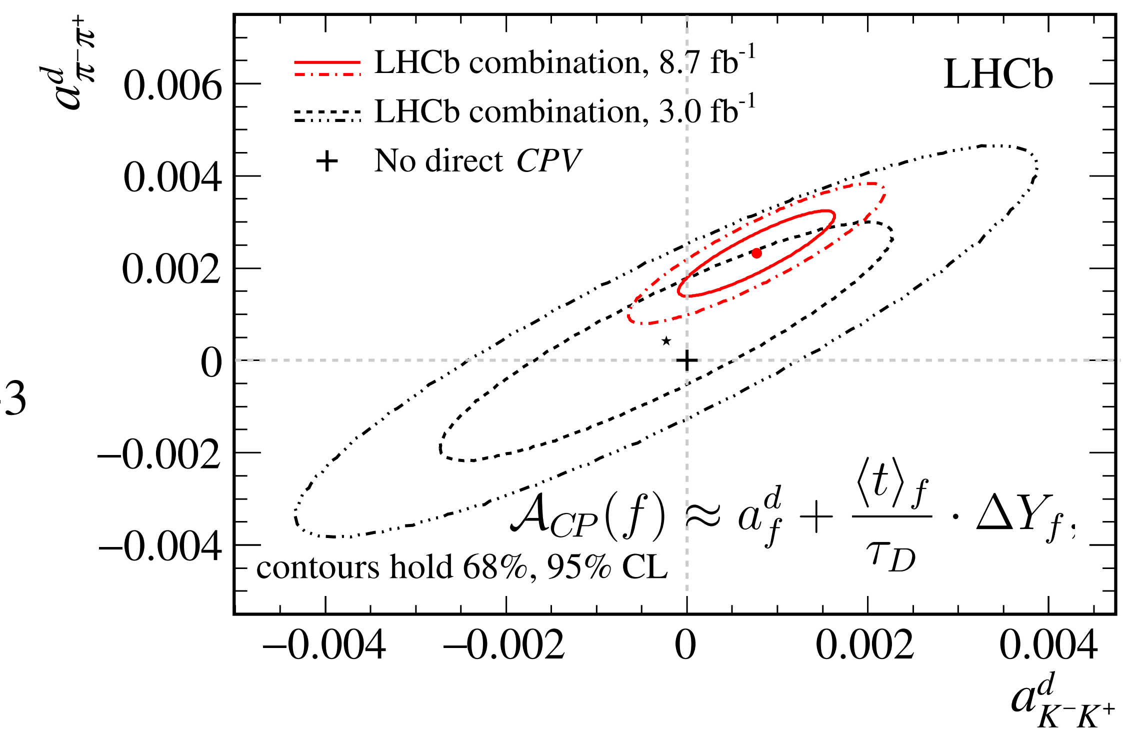
Phys. Rev. Lett.122, 211803 (2019)

→ direct CP asymmetry observation

- $A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$

↳ $A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$

arXiv:2209.03179



- QCD → LCSR predictions $A_{CP} \approx 10^{-4}$ (1 order magnitude bellow)

Khodjamirian, Petrov, Phys. Lett. B 774, 235 (2017)

↳ new physics? nonperturbative effects?!

→ CPV on $D \to hhh$?

→ searches in many process at LHCb, BESIII, BelelI

↳ is expected soon with LHCb run II

direct CP violation

- 2 amplitudes: SAME final state, $\neq$ strong (δ_i) and weak (ϕ_i) phases

$$\langle f\,|\,T\,|\,M\rangle = A_1\, e^{i(\delta_1+\phi_1)} + A_2\, e^{i(\delta_2+\phi_2)}$$

$$\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle = A_1\, e^{i(\delta_1-\phi_1)} + A_2\, e^{i(\delta_2-\phi_2)}$$

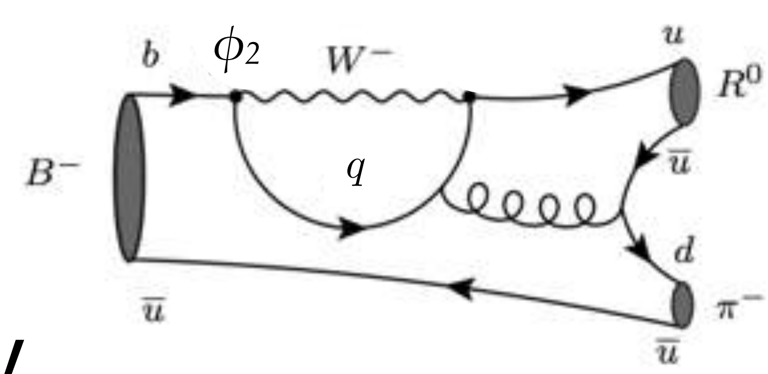
- weak phase → CKM
- strong phase → QCD

- $$\Delta\Gamma_{CP} = \Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f}) = |\langle f\,|\,T\,|\,M\rangle|^2 - |\langle \bar{f}\,|\,T\,|\,\bar{M}\rangle|^2 = -4A_1A_2 \sin(\delta_1-\delta_2)\,\sin(\phi_1-\phi_2)$$

- $$A_{CP} = \frac{\Gamma(M \to f) - \Gamma(\bar{M} \to \bar{f})}{\Gamma(M \to f) + \Gamma(\bar{M} \to \bar{f})}$$

- BSS model Bander Silverman & Soni PRL 43 (1979) 242

 + 



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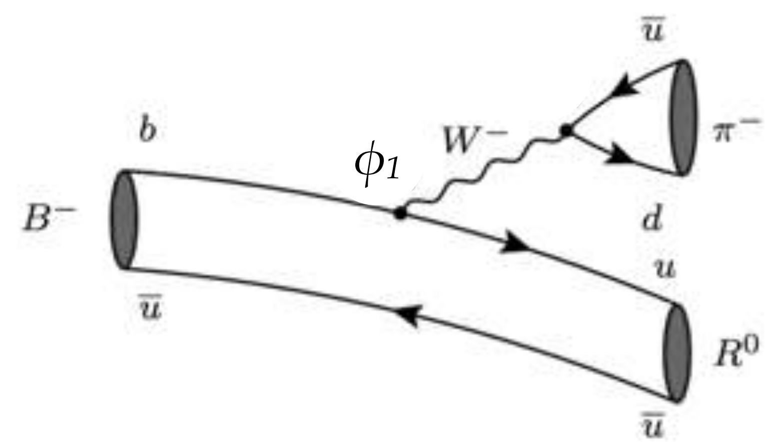


  - not enough for CPV

- hadronic interactions are natural sources of strong phase!

CPV on heavy meson decays

- CPV in $B^{\pm} \to h^{\pm}h^{-}h^{+}$

LHCb Run II 5.9 fb^{-1}

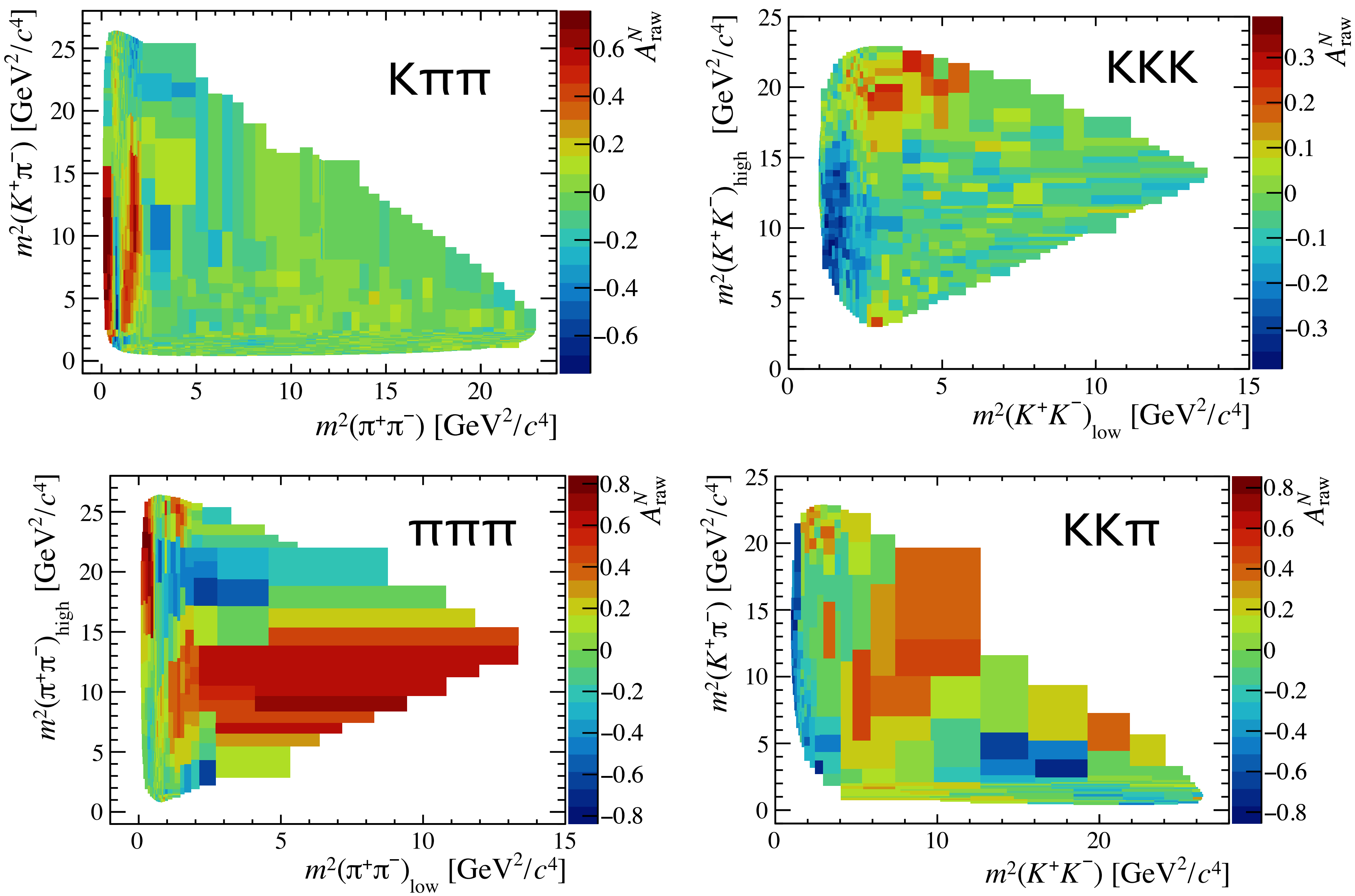
arXiv:2206.07622 PRD 2022 XX

- integrated

$$A_{CP}(B^{\pm} \to K^{\pm}\pi^{+}\pi^{-}) = +0.011 \pm 0.002,$$
$$A_{CP}(B^{\pm} \to K^{\pm}K^{+}K^{-}) = -0.037 \pm 0.002,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}\pi^{+}\pi^{-}) = +0.080 \pm 0.004,$$
$$A_{CP}(B^{\pm} \to \pi^{\pm}K^{+}K^{-}) = -0.114 \pm 0.007,$$

- massive localized Acp

- suggest dynamic effect



rescattering as a CPV mechanism

- CPT must be preserved

Lifetime $\tau = 1 / \Gamma_{total} = 1 / \overline{\Gamma}_{total}$

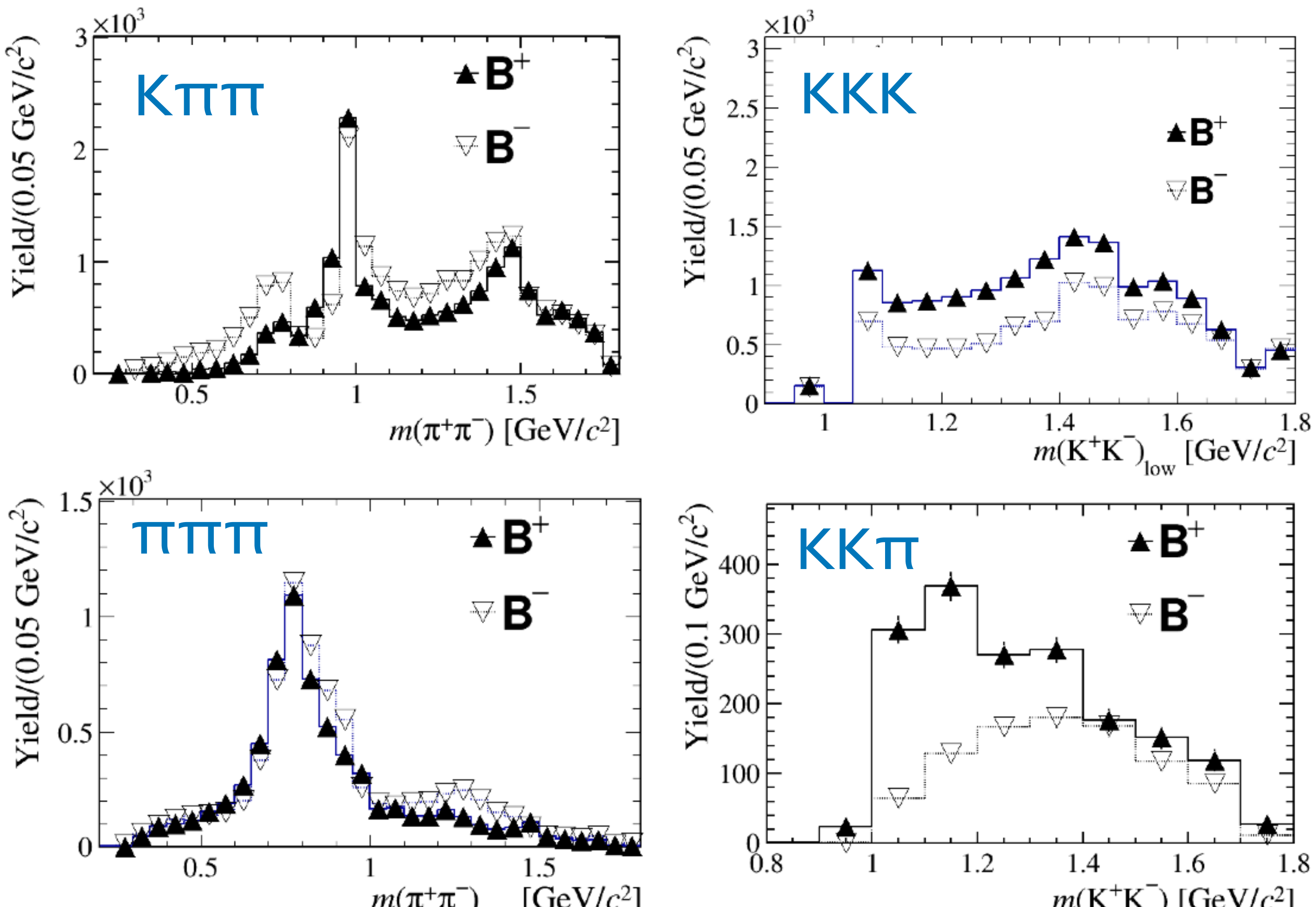
$$\Gamma_{total} = \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 + \Gamma_5 + \Gamma_6 + \ldots$$

$$\overline{\Gamma}_{total} = \overline{\Gamma}_1 + \overline{\Gamma}_2 + \overline{\Gamma}_3 + \overline{\Gamma}_4 + \overline{\Gamma}_5 + \overline{\Gamma}_6 + \ldots$$

→ $\sum \Delta\Gamma_{CP} = 0$

CPV in one channel should be compensated by another, same quantum #, with opposite sign

- LHCb run 1 projections



- rescattering $\pi\pi \to KK$
  - ↳ CPV at [1 -1.6] GeV

  Frederico, Bediaga, Lourenço PRD89(2014)094013

- implemented in LHCb amplitude analysis:
  - ↳ $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$ PRD101 (2020) 012006; PRL 124 (2020) 031801
  - ↳ $B^{\pm} \to \pi^{\pm}K^{-}K^{+}$ PRL 123 (2019) 231802

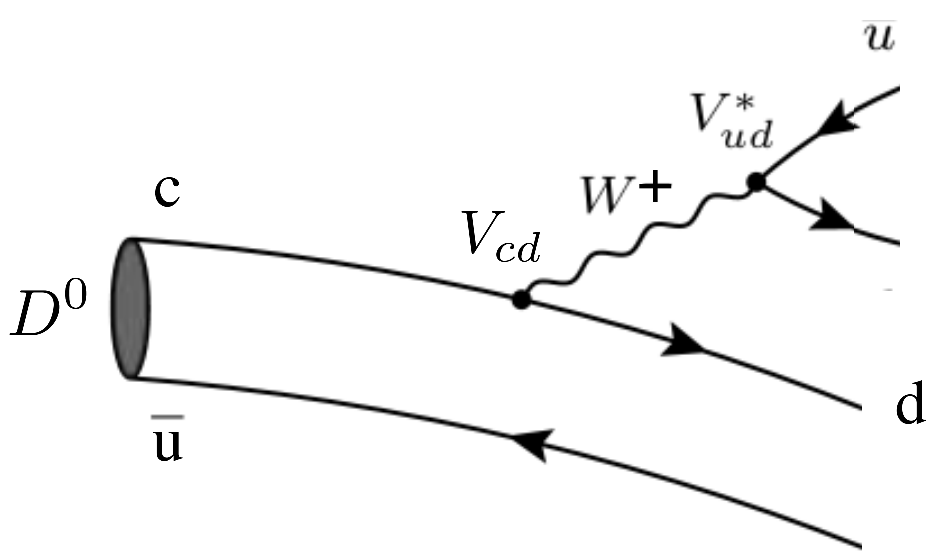
FSI as source of CP asymmetry in D decays

- how to explain the CPV in charm?

$$\Delta A_{CP}^{\mathrm{LHCb}} = A_{cp}(D^0 \to K^+K^-) - A_{cp}(D^0 \to \pi^+\pi^-) = -(1.54 \pm 0.29) \times 10^{-3}$$

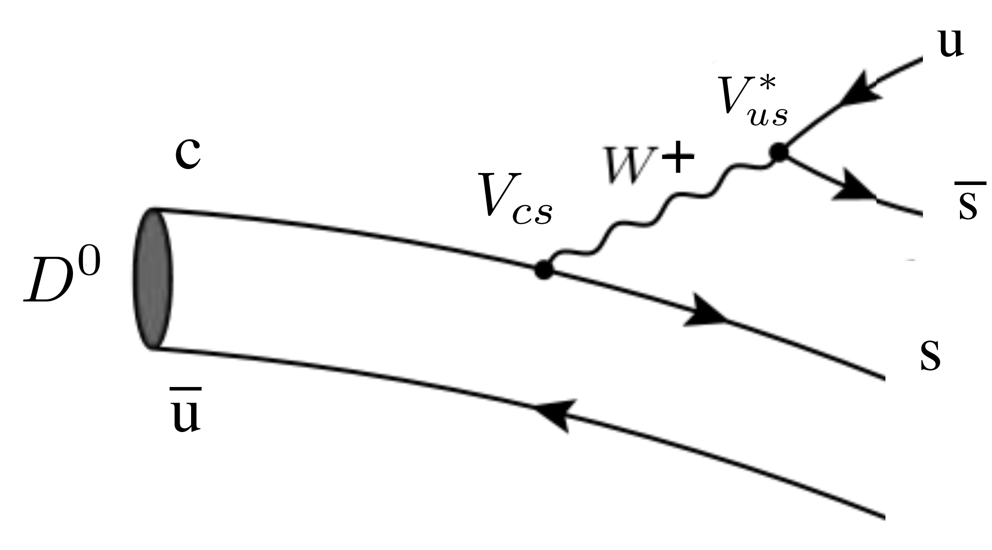
- single cabibbo suppressed decays

$D^0 \to \pi^+\pi^-$



$V_{cd}V^*_{ud} \approx \lambda(1 - \lambda^4\, e^{i\delta})$

$D^0 \to K^+K^-$



$V_{cs}V^*_{us} \approx \lambda(1 - \lambda^2)$

- weak phase in KK is 20 times smaller Lenz ,Wilkinson, Annu. Rev. Nucl. Part. Sci. 71, 59 (2021)

→ what about strong phases if not from penguin? hadronic FSI

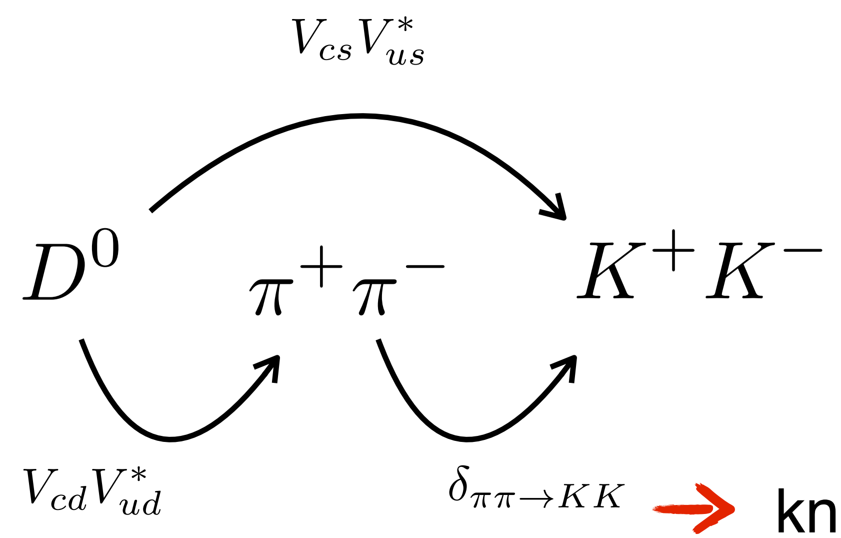
Grossman, Schacht JHEP07 20 (2019); Schacht, Soni PLB825 136855 (2022).

FSI to enhance CPV

- D and $\bar{D}$ can decay to $\pi\pi$ and KK

[arxiv 2203.04056v2](https://arxiv.org/abs/2203.04056v2)

Bediaga, Frederico, PCM



→ known from 80's experiment

- build amplitudes decays implying three constraints:
  - CPT invariance relates channels with same quantum numbers

    → $\sum \Delta\Gamma_{CP} = 0$

  - Watson theorem relates the strong phase from the rescattering process to the decay amplitudes
  - the unitarity of the strong S-matrix.

FSI and CPT

- FSI in $D^0 \to \pi^+\pi^-$ and $D^0 \to K^+K^-$ can include multiple mesons
  - general S-matrix can mix many FSI states $S = \begin{pmatrix} S_{2M,2M} & S_{2M,3M} & S_{2M,4M} & \cdots \\ S_{3M,2M} & S_{3M,3M} & S_{3M,4M} & \cdots \\ S_{4M,2M} & S_{4M,3M} & S_{4M,4M} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$

- assume only 2 coupled-channels to FSI: $\pi\pi$, $K\bar{K}$

$$\rightarrow \quad S_{2M,2M} = \begin{pmatrix} S_{\pi\pi,\pi\pi} & S_{\pi\pi,KK} \\ S_{KK,\pi\pi} & S_{KK,KK} \end{pmatrix}$$

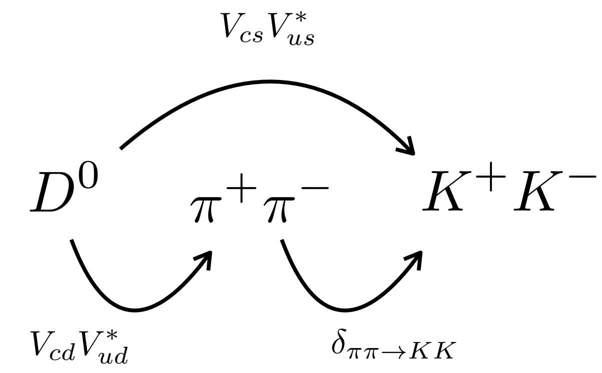
$$S_{\pi\pi,\pi\pi} = \eta\, e^{2i\delta_{\pi\pi}} \qquad S_{KK,KK} = \eta\, e^{2i\delta_{KK}}$$
$$S_{\pi\pi,KK} = S_{KK,\pi\pi} = \imath\sqrt{1-\eta^2}\, e^{\imath(\delta_{\pi\pi}+\delta_{KK})}$$

  - two pions cannot go to three pions due to G-parity
  - ignore four pion coupling to the 2M channel based on 1/Nc counting
  - ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.

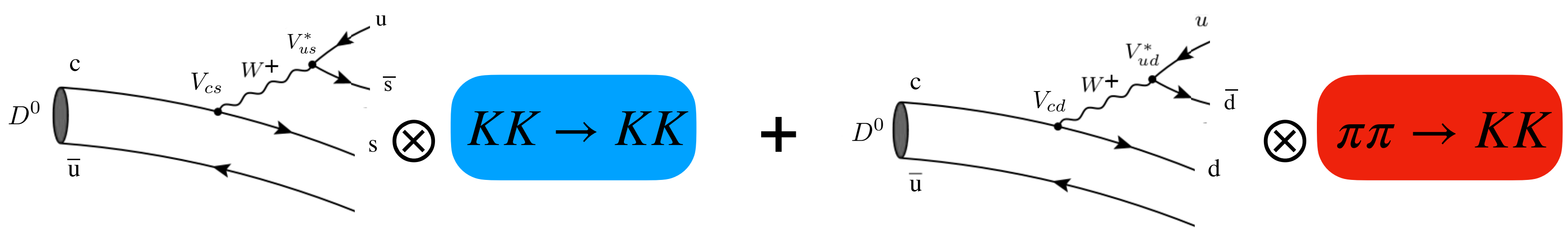
- CPT constraint restricted to the two-channels: $\sum_{f=(\pi\pi,KK)} \left(|\mathcal{A}_{D^0\to f}|^2 - |\mathcal{A}_{\bar{D}^0\to f}|^2\right) = 0$

Decay amplitudes

- dressing the weak tree topology with FSI
  - → penguin are suppressed



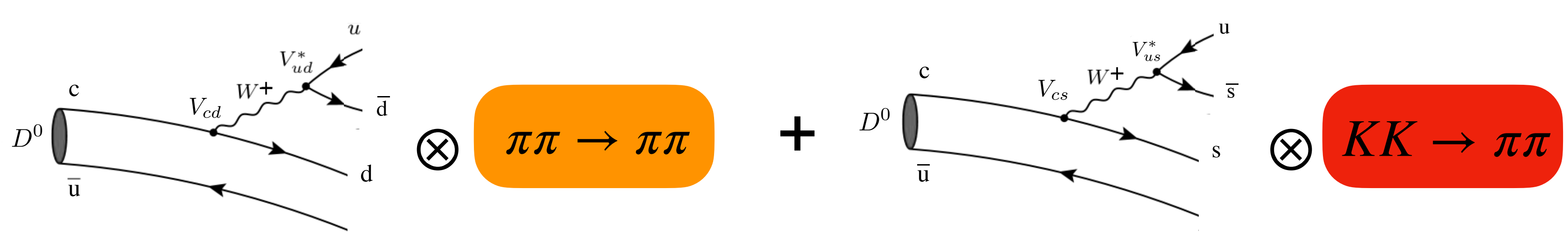
- $D^0 \to KK$



→ $\mathcal{A}_{D^0\to KK} = \eta\, \mathrm{e}^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$

→ $\mathcal{A}_{\bar{D}^0\to f}$ same with CKM cc.

- $D^0 \to \pi\pi$



→ $\mathcal{A}_{D^0\to \pi\pi} = \eta\, \mathrm{e}^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, \mathrm{e}^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$

- a_{KK} and $a_{\pi\pi}$ do not carry any strong phases (real)

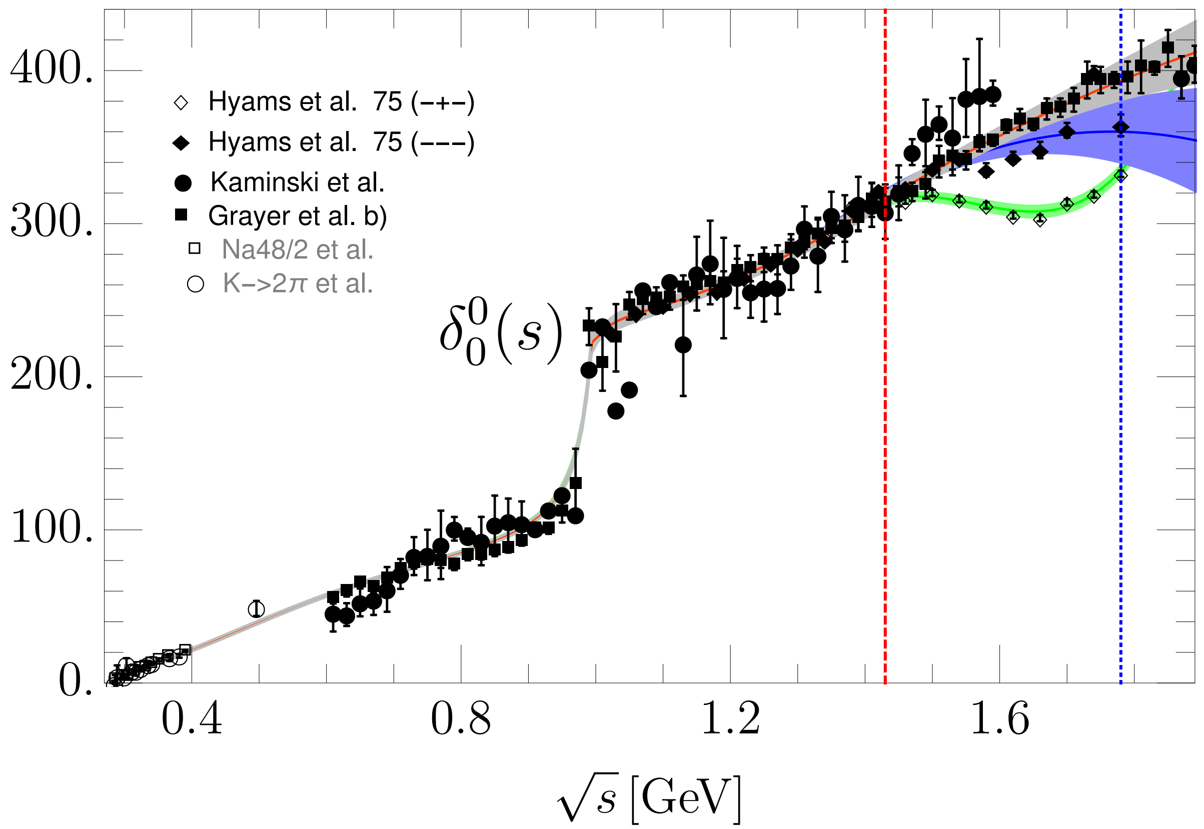
Watson theorem

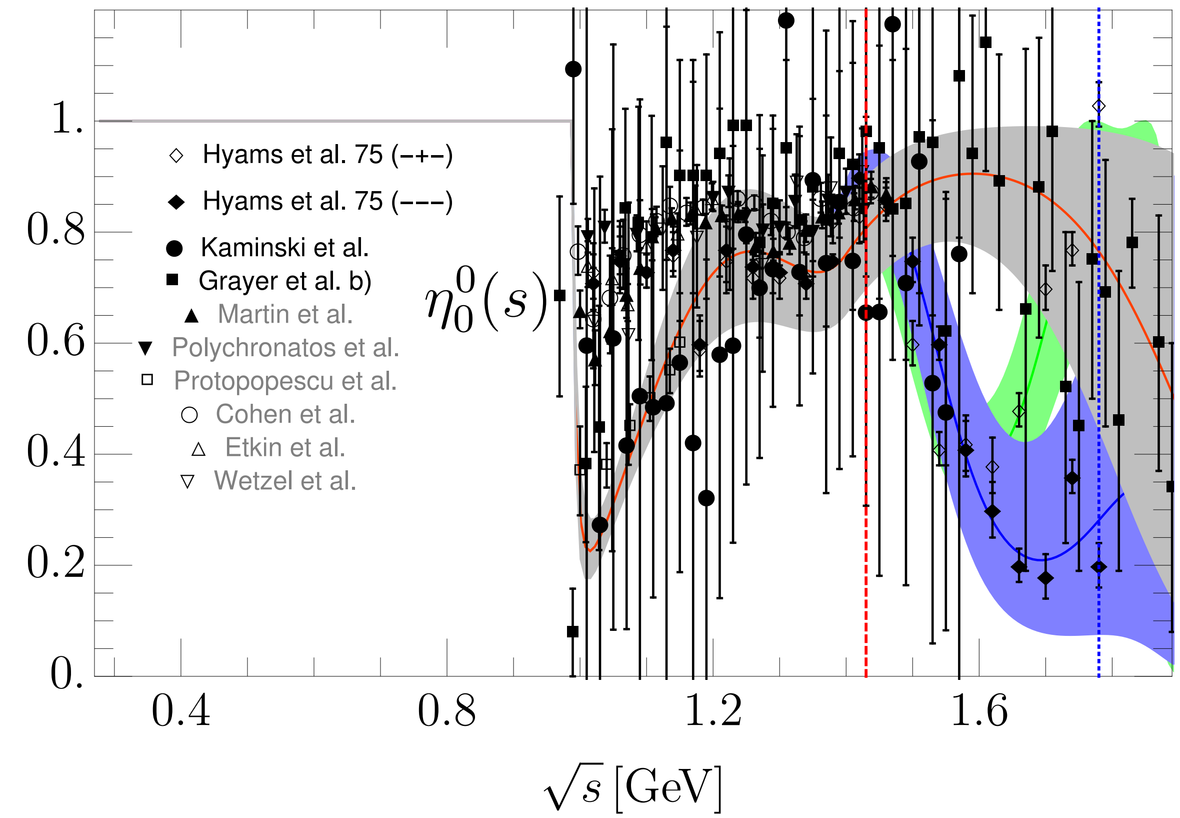
- $\delta_{\pi\pi}$, δ_{KK} and $\delta_{\pi\pi\to KK}$ are the same independent of the initial process

  → we can use CERN-Munich data from 80's

Longacre et al., Phys. Lett. B 177, 223 (1986)
Hyams et al., Nucl. Phys. B 100, 205 (1975)
Ochs, J. Phys. G 40, 043001 (2013)

- $\pi\pi \to \pi\pi$





Pelaez, Rodas, Elvira *Eur.Phys.J.C* 79 (2019) 12, 1008

amplitude $\hat{f}_l(s) = \left[\frac{\eta_l e^{2i\delta_l} - 1}{2i}\right].$

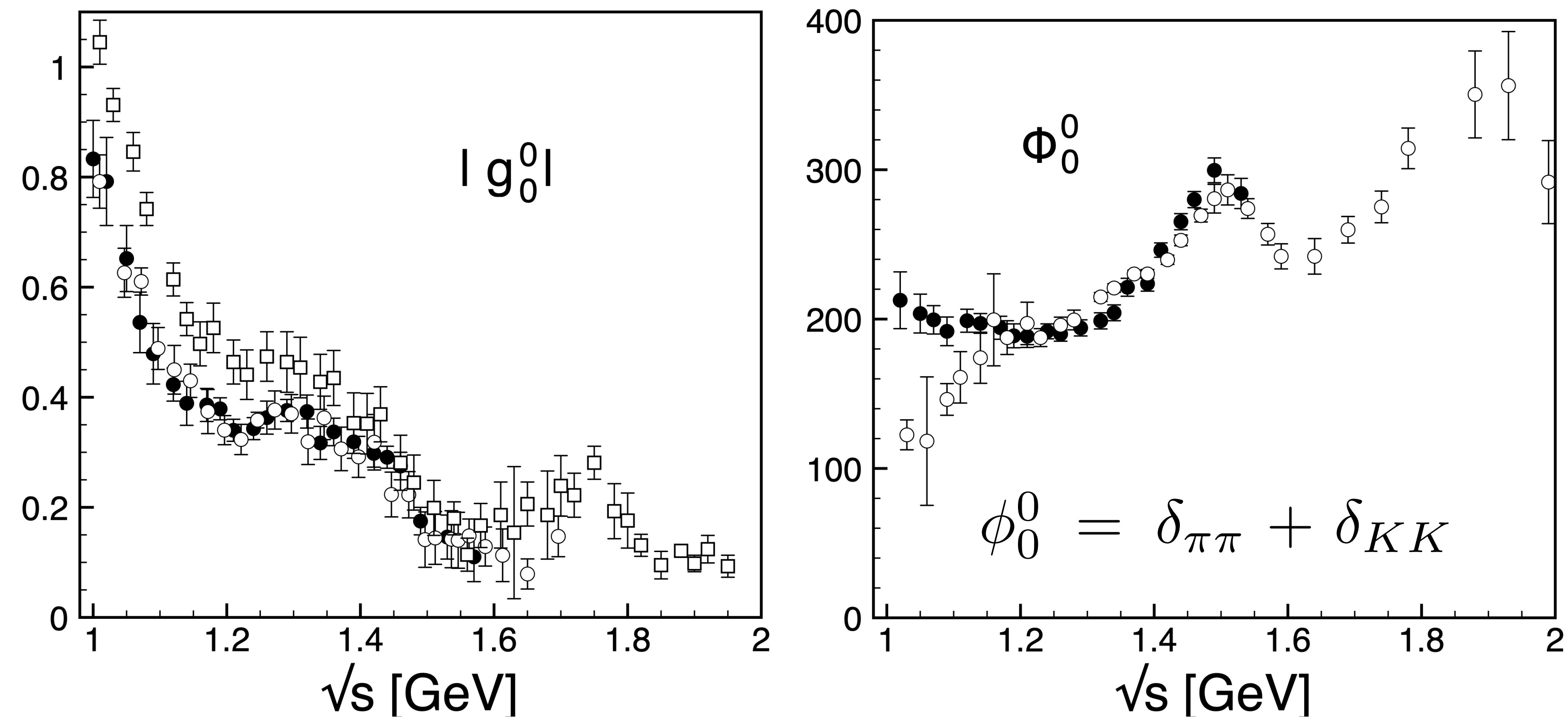
→ elasticity drops dramatically near $K\bar{K}$ → strongly couple

Watson theorem

- $\pi\pi \to KK$

$$\rightarrow \quad S_{\pi\pi,KK}(s) = \imath\sqrt{1-\eta^2}\, e^{\imath(\delta_{\pi\pi}+\delta_{KK})} = i\,4\sqrt{\frac{q_\pi q_K}{s}}|g_0^0(s)|\, e^{i\phi_0^0(s)}\,\Theta(s-4m_K^2)$$

Pelaez and Rodas, Eur. Phys. J. C 78, 897 (2018)



Cohen et al., Phys. Rev. D 22, 2595 (1980)
Etkin et al., Phys. Rev. D 25, 1786 (1982)

- Pelaez parametrization @ M_D^2:

$$|g_0^0(M_D^2)| \approx 0.125 \pm 0.025 \quad \rightarrow \quad \sqrt{1-\eta^2} \approx 0.229 \pm 0.046 \quad \rightarrow \quad \eta \approx 0.973$$

$$\phi_0^0 = \delta_{\pi\pi} + \delta_{KK} \approx 343^o \pm 8^o$$

Partial decay widths

- $\Delta\Gamma_f = \Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)$

$$\mathcal{A}_{D^0\to\pi\pi} = \eta\, e^{2i\delta_{\pi\pi}}\, V_{cd}^* V_{ud}\, a_{\pi\pi} + i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cs}^* V_{us}\, a_{KK}$$

$$\mathcal{A}_{D^0\to KK} = \eta\, e^{2i\delta_{KK}}\, V_{cs}^* V_{us}\, a_{KK} + i\sqrt{1-\eta^2}\, e^{i(\delta_{\pi\pi}+\delta_{KK})}\, V_{cd}^* V_{ud}\, a_{\pi\pi}$$

→ $\Delta\Gamma_{\pi\pi} = -\Delta\Gamma_{KK} = 4\,\mathrm{Im}[V_{cs}V_{us}^*V_{cd}^*V_{ud}]\, \underline{a_{\pi\pi}\, a_{KK}}\, \eta\sqrt{1-\eta^2}\cos\phi$

  - $\phi = \delta_{KK} - \delta_{\pi\pi}$
  - the sign of $\Delta\Gamma_f$ is determined by the CKM elements and the S-wave phase-shifts
  - need to quantify $a_{\pi\pi}$ and a_{KK}:

at D^0 mass $\sqrt{1-\eta^2} << 1$ → $\Gamma_{\pi\pi} \approx \eta^2|V_{cd}^*V_{ud}|^2\, a_{\pi\pi}^2$, $\Gamma_{KK} \approx \eta^2|V_{cs}^*V_{us}|^2 a_{KK}^2$

$Br[D \to f] = \Gamma_f/\Gamma_{total}$

we can use experimental input

- $A_{CP}(f) = \dfrac{\Gamma\left(D^0 \to f\right) - \Gamma(\bar{D}^0 \to f)}{\Gamma\left(D^0 \to f\right) + \Gamma(\bar{D}^0 \to f)}$ $= \Delta\Gamma_f/2\Gamma_f$

Final values for A_{CP}

- $$A_{CP}(f) \approx \pm 2 \frac{\mathrm{Im}[V_{cs}V_{us}^* V_{cd}^* V_{ud}]}{|V_{cs}V_{us}^* V_{cd}^* V_{ud}|} \eta^{-1} \sqrt{1-\eta^2} \cos\phi \left[\frac{\mathrm{Br}(D^0 \to K^+K^-)}{\mathrm{Br}(D^0 \to \pi^+\pi^-)} \right]^{\pm\frac{1}{2}}$$

  $+$ $\pi\pi$
  $-$ KK

  - $\mathrm{Br}(D^0 \to \pi^+\pi^-) = (1.455 \pm 0.024) \times 10^{-3}$
    $\mathrm{Br}(D^0 \to K^+K^-) = (4.08 \pm 0.06) \times 10^{-3}$ PDG
  - $$\frac{\mathrm{Im}[V_{cs}V_{us}^* V_{cd}^* V_{ud}]}{|V_{cs}V_{us}^* V_{cd}^* V_{ud}|} = (6.02 \pm 0.32) \times 10^{-4}$$ PDG
  - $\cos\phi$: $\phi = \delta_{KK} - \delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$ (ϕ_0^0)

  from $\pi\pi$ and $\pi\pi \to KK$ data: $\cos\phi = 0.99 \pm 0.18$

- $$A_{CP}(\pi\pi) = (1.99 \pm 0.37) \times 10^{-3} \sqrt{\eta^{-2} - 1}$$
  $$A_{CP}(KK) = -(0.71 \pm 0.13) \times 10^{-3} \sqrt{\eta^{-2} - 1}$$

as a function of inelasticity

Predictions for ΔA_{CP}

$$\Delta A_{CP}^{\mathrm{LHCb}} = -(1.54 \pm 0.29) \times 10^{-3}$$

- $\Delta A_{CP}^{th} = -(2.70 \pm 0.50) \times 10^{-3} \sqrt{\eta^{-2} - 1}$

- from $\pi\pi \rightarrow KK$ data (only one set) → $\eta \approx 0.973 \pm 0.011$

$$\Delta A_{CP}^{th} = -(0.64 \pm 0.18) \times 10^{-3}$$ 3σ

→ largest theoretical prediction within SM without relying on fitting parameters

→ systematic uncertainties are unknown in η → error is underestimated

- Alternatively one can assume all inelasticity in $\pi\pi \rightarrow \pi\pi$ is due to KK

→ more precise data (Grayer) → $\eta = 0.78 \pm 0.08$

$$\Delta A_{CP}^{th} = -(2.17 \pm 0.70) \times 10^{-3}$$ 1σ

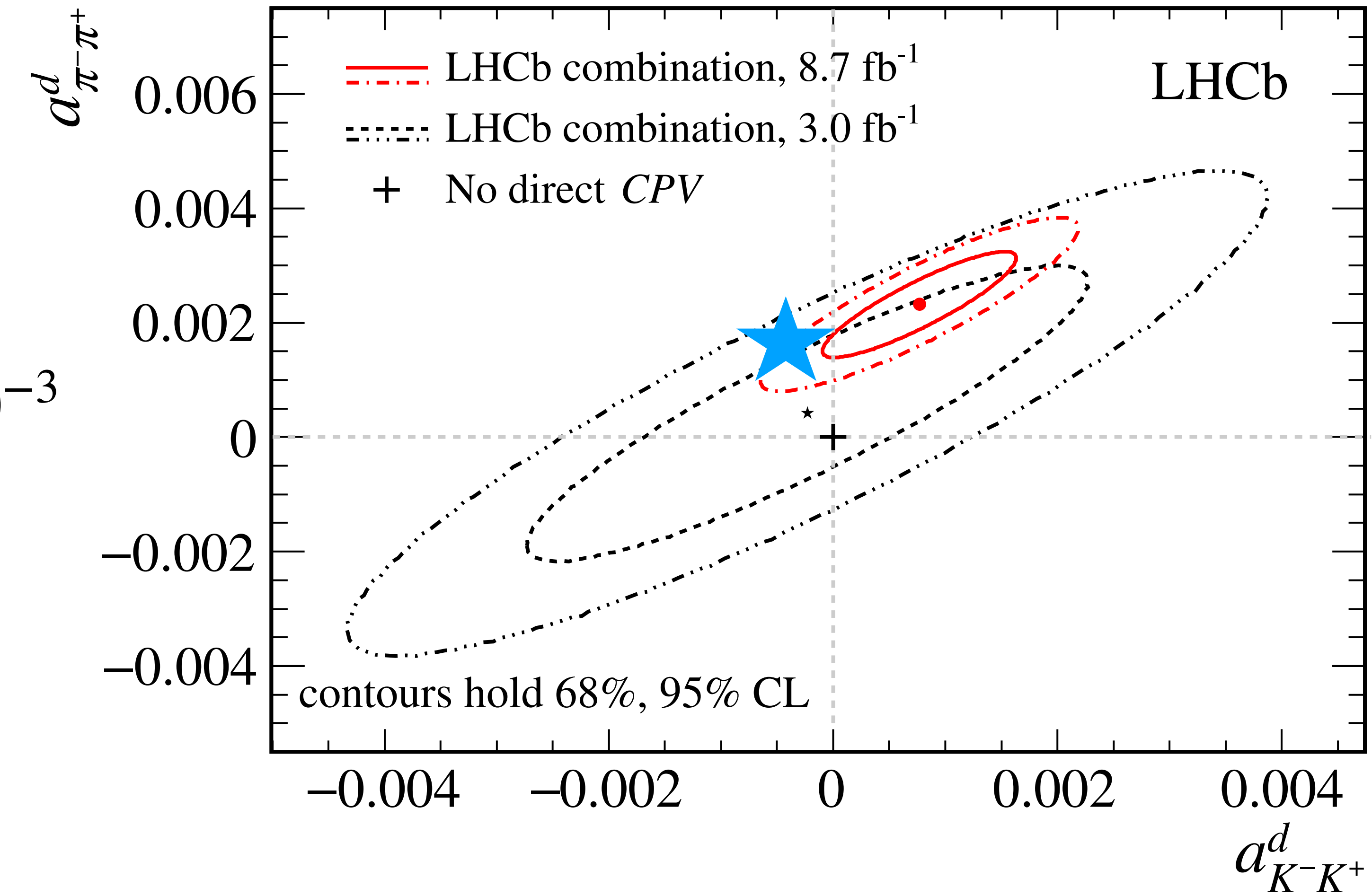
Predictions for $A_{CP}(hh)$

- direct CP asymmetry observation

$$A_{CP}^{LHCb}(KK) = (0.77 \pm 0.57) \times 10^{-3}$$

$$\hookrightarrow A_{CP}^{LHCb}(\pi\pi) = (2.32 \pm 0.61) \times 10^{-3}$$

arXiv:2209.03179



- with $\eta = 0.78 \pm 0.08$

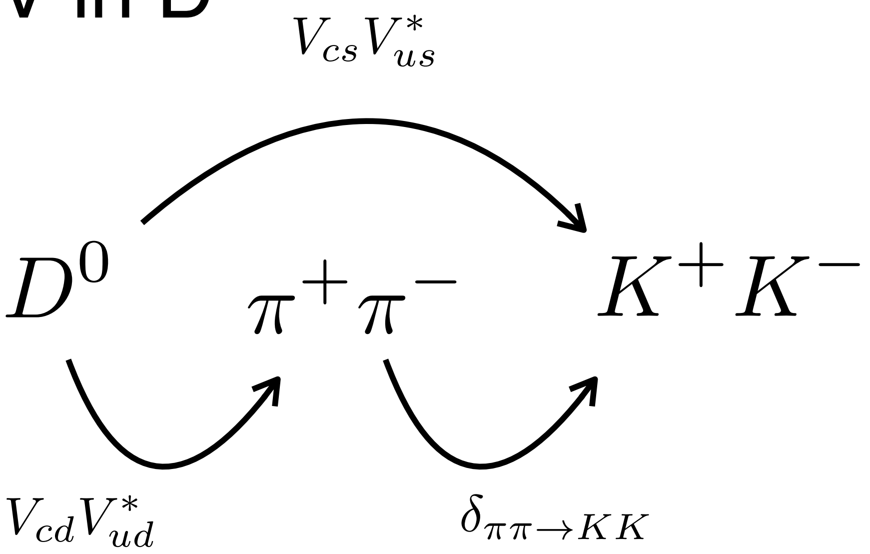
$$A_{CP}(KK) = -(0.57 \pm 0.18) \times 10^{-3} \quad 2\sigma$$

$$A_{CP}(\pi\pi) = (1.60 \pm 0.51) \times 10^{-3} \quad 1\sigma$$

Final remarks

- hadronic FSI (and their strong phases)are crucial to explain CP violation in B and D decays

- we proposed a mechanism that can explain CPV in D
  - coupling $\pi\pi \leftrightarrow K\bar{K}$ in a CPT invariant framework
  - still room to add 2nd order effects



→ predicted ΔA_{CP} which is compatible with LHCb

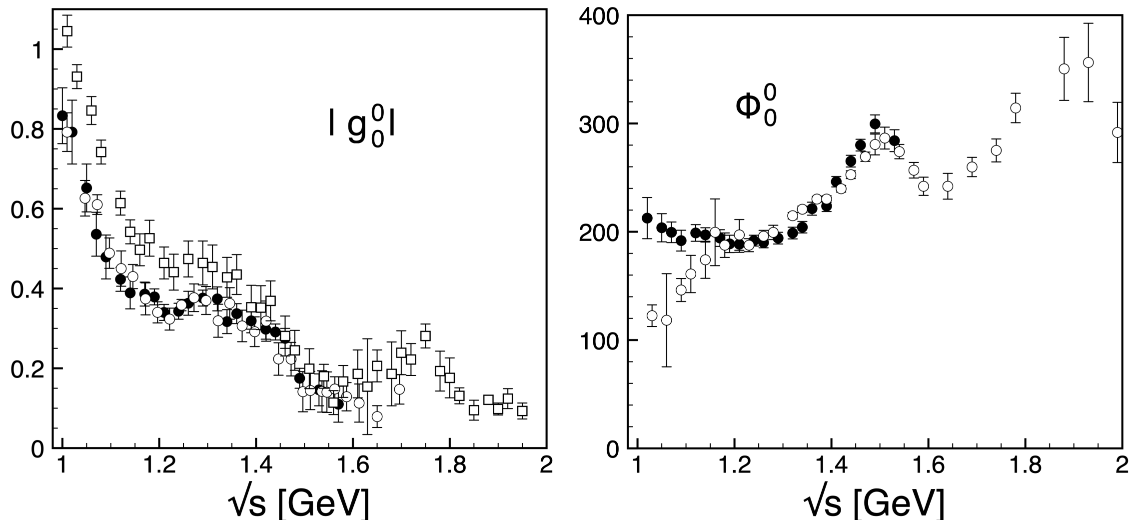
- new measurement for $A_{CP}(hh)$ from LHCb
  - agrees with our predictions with 2σ

  ↳ we still need more data to fully understood it

Final remarks

- In 3-body decays this effect will be bigger and phase-space distributed

↪ $D^+ \to \pi^+\pi^-\pi^+$ and $D^+ \to \pi^+K^-K^+$ have exactly the same Weak vertex



- expected CPV in run II analysis

stay tuned!

thank you!

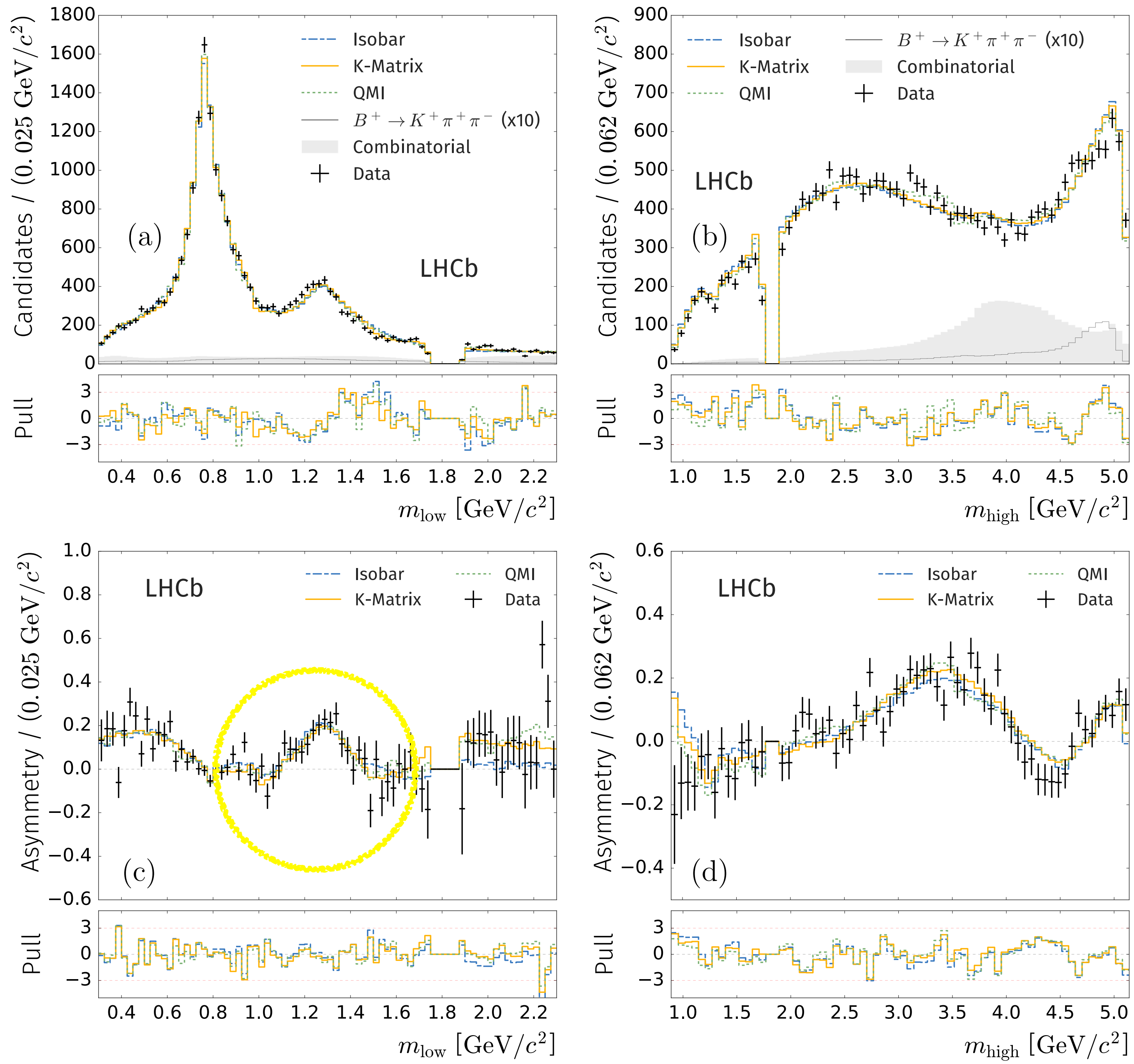
obrigada!!

#forabolsonaro

Backup slides

CPV: amplitude analysis

- $B^{\pm} \to \pi^{-}\pi^{+}\pi^{\pm}$
  - $(\pi^{-}\pi^{+})_{S-Wave}$ 3 different model:
    - ↪ σ as BW (!) + rescattering;
    - ↪ P-vector K-Matrix;
    - ↪ binned freed lineshape (QMI);



PRD101 (2020) 012006; PRL 124 (2020) 031801

Contribution	Fit fraction (10^{-2})	A_{CP} (10^{-2})	B^+ phase (°)	B^- phase (°)
Isobar model				
$\rho(770)^0$	$55.5 \pm 0.6 \pm 2.5$	$+0.7 \pm 1.1 \pm 1.6$	—	—
$\omega(782)$	$0.50 \pm 0.03 \pm 0.05$	$-4.8 \pm 6.5 \pm 3.8$	$-19 \pm 6 \pm 1$	$+8 \pm 6 \pm 1$
$f_2(1270)$	$9.0 \pm 0.3 \pm 1.5$	$+46.8 \pm 6.1 \pm 4.7$	$+5 \pm 3 \pm 12$	$+53 \pm 2 \pm 12$
$\rho(1450)^0$	$5.2 \pm 0.3 \pm 1.9$	$-12.9 \pm 3.3 \pm 35.9$	$+127 \pm 4 \pm 21$	$+154 \pm 4 \pm 6$
$\rho_3(1690)^0$	$0.5 \pm 0.1 \pm 0.3$	$-80.1 \pm 11.4 \pm 25.3$	$-26 \pm 7 \pm 14$	$-47 \pm 18 \pm 25$
S-wave	$25.4 \pm 0.5 \pm 3.6$	$+14.4 \pm 1.8 \pm 2.1$	—	—
Rescattering	$1.4 \pm 0.1 \pm 0.5$	$+44.7 \pm 8.6 \pm 17.3$	$-35 \pm 6 \pm 10$	$-4 \pm 4 \pm 25$
σ	$25.2 \pm 0.5 \pm 5.0$	$+16.0 \pm 1.7 \pm 2.2$	$+115 \pm 2 \pm 14$	$+179 \pm 1 \pm 95$
K-matrix				
$\rho(770)^0$	$56.5 \pm 0.7 \pm 3.4$	$+4.2 \pm 1.5 \pm 6.4$	—	—
$\omega(782)$	$0.47 \pm 0.04 \pm 0.03$	$-6.2 \pm 8.4 \pm 9.8$	$-15 \pm 6 \pm 4$	$+8 \pm 7 \pm 4$
$f_2(1270)$	$9.3 \pm 0.4 \pm 2.5$	$+42.8 \pm 4.1 \pm 9.1$	$+19 \pm 4 \pm 18$	$+80 \pm 3 \pm 17$
$\rho(1450)^0$	$10.5 \pm 0.7 \pm 4.6$	$+9.0 \pm 6.0 \pm 47.0$	$+155 \pm 5 \pm 29$	$-166 \pm 4 \pm 51$
$\rho_3(1690)^0$	$1.5 \pm 0.1 \pm 0.4$	$-35.7 \pm 10.8 \pm 36.9$	$+19 \pm 8 \pm 34$	$+5 \pm 8 \pm 46$
S-wave	$25.7 \pm 0.6 \pm 3.0$	$+15.8 \pm 2.6 \pm 7.2$	—	—
QMI				
$\rho(770)^0$	$54.8 \pm 1.0 \pm 2.2$	$+4.4 \pm 1.7 \pm 2.8$	—	—
$\omega(782)$	$0.57 \pm 0.10 \pm 0.17$	$-7.9 \pm 16.5 \pm 15.8$	$-25 \pm 6 \pm 27$	$-2 \pm 7 \pm 11$
$f_2(1270)$	$9.6 \pm 0.4 \pm 4.0$	$+37.6 \pm 4.4 \pm 8.0$	$+13 \pm 5 \pm 21$	$+68 \pm 3 \pm 66$
$\rho(1450)^0$	$7.4 \pm 0.5 \pm 4.0$	$-15.5 \pm 7.3 \pm 35.2$	$+147 \pm 7 \pm 152$	$-175 \pm 5 \pm 171$
$\rho_3(1690)^0$	$1.0 \pm 0.1 \pm 0.5$	$-93.2 \pm 6.8 \pm 38.9$	$+8 \pm 10 \pm 24$	$+36 \pm 26 \pm 46$
S-wave	$26.8 \pm 0.7 \pm 2.2$	$+15.0 \pm 2.7 \pm 8.1$	—	—

- $B^{\pm} \to \pi^{\pm}K^{-}K^{+}$ PRL 123 (2019) 231802

Contribution	Fit Fraction(%)	A_{CP}(%)	Magnitude (B^+/B^-)	Phase[°] (B^+/B^-)
$K^*(892)^0$	$7.5 \pm 0.6 \pm 0.5$	$+12.3 \pm 8.7 \pm 4.5$	$0.94 \pm 0.04 \pm 0.02$	0 (fixed)
			$1.06 \pm 0.04 \pm 0.02$	0 (fixed)
$K_0^*(1430)^0$	$4.5 \pm 0.7 \pm 1.2$	$+10.4 \pm 14.9 \pm 8.8$	$0.74 \pm 0.09 \pm 0.09$	$-176 \pm 10 \pm 16$
			$0.82 \pm 0.09 \pm 0.10$	$136 \pm 11 \pm 21$
Single pole	$32.3 \pm 1.5 \pm 4.1$	$-10.7 \pm 5.3 \pm 3.5$	$2.19 \pm 0.13 \pm 0.17$	$-138 \pm 7 \pm 5$
			$1.97 \pm 0.12 \pm 0.20$	$166 \pm 6 \pm 5$
$\rho(1450)^0$	$30.7 \pm 1.2 \pm 0.9$	$-10.9 \pm 4.4 \pm 2.4$	$2.14 \pm 0.11 \pm 0.07$	$-175 \pm 10 \pm 15$
			$1.92 \pm 0.10 \pm 0.07$	$140 \pm 13 \pm 20$
$f_2(1270)$	$7.5 \pm 0.8 \pm 0.7$	$+26.7 \pm 10.2 \pm 4.8$	$0.86 \pm 0.09 \pm 0.07$	$-106 \pm 11 \pm 10$
			$1.13 \pm 0.08 \pm 0.05$	$-128 \pm 11 \pm 14$
Rescattering	$16.4 \pm 0.8 \pm 1.0$	$-66.4 \pm 3.8 \pm 1.9$	$1.91 \pm 0.09 \pm 0.06$	$-56 \pm 12 \pm 18$
			$0.86 \pm 0.07 \pm 0.04$	$-81 \pm 14 \pm 15$
$\phi(1020)$	$0.3 \pm 0.1 \pm 0.1$	$+9.8 \pm 43.6 \pm 26.6$	$0.20 \pm 0.07 \pm 0.02$	$-52 \pm 23 \pm 32$
			$0.22 \pm 0.06 \pm 0.04$	$107 \pm 33 \pm 41$

Theoretical approaches to CPV on charm

QCD short-distance

- QCDF → how to calculate penguin contributions? call BSM effects

Chala, Lenz, Rusov, Scholtz, JHEP 07, 161 (2019).

- LCSR → QCD, model independent but predictions are 1 order magnitude bellow

Khodjamirian, Petrov, Phys. Lett. B 774, 235 (2017)

long distance effect:

- topological and group symmetry approach
  - with SU(3) breaking through FSI (fit agrees)

    H.-Y. Cheng and C.-W. Chiang, PRD 100, 093002 (2019).
    F. Buccella, A. Paul and P. Santorelli, PRD 99, 113001 (2019)

  - with resonances (fit agrees)

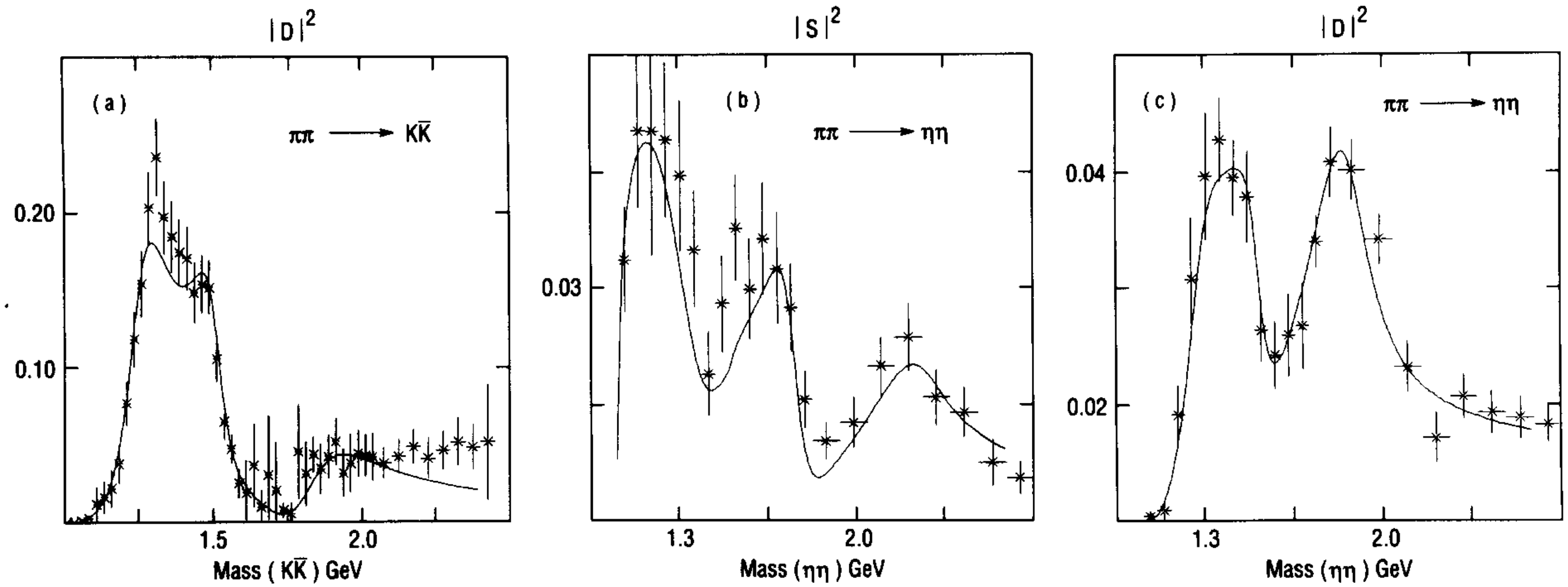
    Schacht and A. Soni, Phys. Lett. B 825, 136855 (2022).
    Y. Grossman and S. Schacht, JHEP 07, 20 (2019)

- FSI with CPT (prediction agrees)

bediaga, Frederico, PCM
arxiv 2203.04056v2

$\eta\eta$ coupling to ππ

- coupling of $\pi\pi \to KK$ in D wave is bigger than $\eta\eta$ in S-wave



- $\sim M_D$ (1.864) mass

Coupled channel analysis of $J^{PC}=0^{++}$ and 2^{++} isoscalar mesons with masses below 2.0 GeV ☆

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Physics Letters B 274 (1992) 492–497

- ignore $\eta\eta$ channel once their coupling to the $\pi\pi$ channel are suppressed with respect to $K\bar{K}$.

4π coupling to ππ

- although the $D^0 \to 4\pi$ decays have a large branching fraction, there is no compelling experimental evidence that 4π is strongly coupled to $\pi\pi$ at M_{D_0}

- $f_0(1500)$ decays in bot channels

$f_0(1500)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)	Scale factor
Γ_1	$\pi\pi$	(34.5 ± 2.2) %	1.2
Γ_2	$\pi^+\pi^-$	seen	
Γ_3	$2\pi^0$	seen	
Γ_4	4π	(48.9 ± 3.3) %	1.2
Γ_5	$4\pi^0$	seen	
Γ_6	$2\pi^+2\pi^-$	seen	
Γ_7	$2(\pi\pi)_{S\text{-wave}}$	seen	
Γ_8	$\rho\rho$	seen	
Γ_9	$\pi(1300)\pi$	seen	
Γ_{10}	$a_1(1260)\pi$	seen	
Γ_{11}	$\eta\eta$	(6.0 ± 0.9) %	1.1
Γ_{12}	$\eta\eta'(958)$	(2.2 ± 0.8) %	1.4
Γ_{13}	$K\overline{K}$	(8.5 ± 1.0) %	1.1
Γ_{14}	$\gamma\gamma$	not seen	

PDG

- The nearest $f_0(1710)$ resonance have no observation of four pions reported.

$f_0(1710)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$K\overline{K}$	seen
Γ_2	$\eta\eta$	seen
Γ_3	$\pi\pi$	seen
Γ_4	$\gamma\gamma$	seen
Γ_5	$\omega\omega$	seen

- we don't have data from KK scattering !
  - we can use $\pi\pi$ and $KK \to \pi\pi$ data: $\delta_{KK} - \delta_{\pi\pi} = \phi_0^0 - 2\delta_{\pi\pi} = (\delta_{KK} + \delta_{\pi\pi}) - 2\delta_{\pi\pi}$
  - CERN-Munich data (revised Ochs)

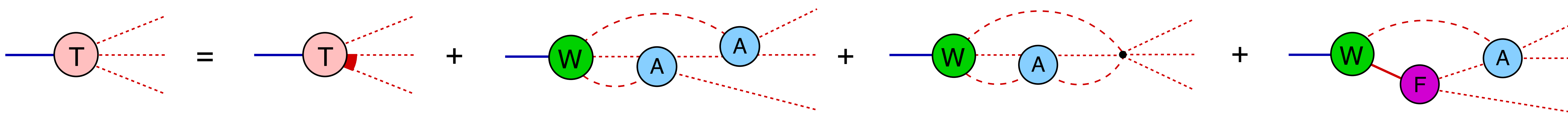
$\sqrt{s}$ [GeV]	$\cos\phi$
1.58	0.989 ± 0.149
1.62	0.994 ± 0.105
1.66	0.999 ± 0.040
1.70	0.987 ± 0.160
1.74	0.999 ± 0.048
1.78	0.999 ± 0.037
m_D^2 → 1.846	0.987 ± 0.175

→ $\cos(\delta_{KK} - \delta_{\pi\pi}) \lesssim 1$

→ Pelaez parametrization

Full story in 3-body decay

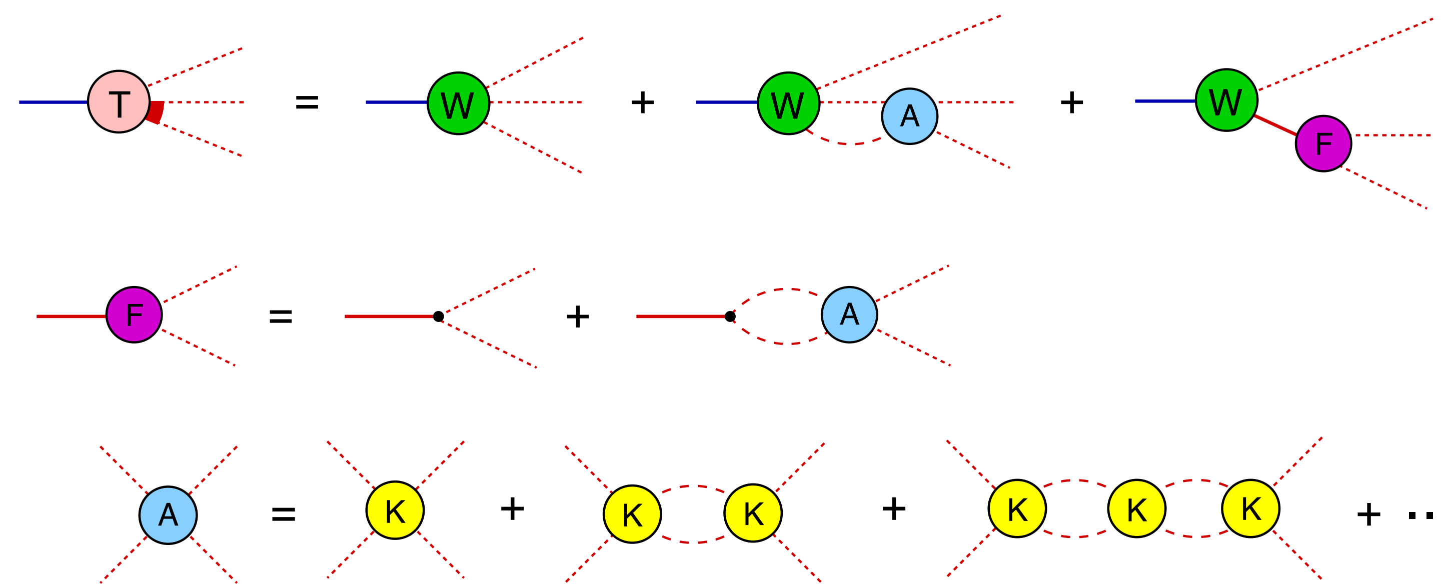
- Any 3-body decay amplitude



(2+1) approach

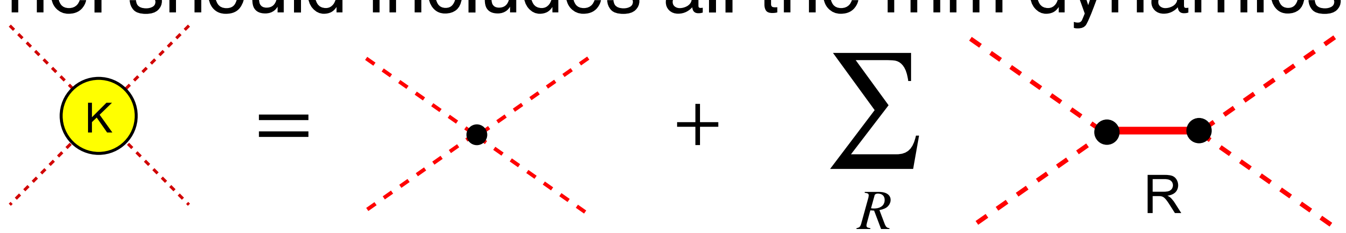
Form factor

meson-meson



MAGALHAES, A.dos Reis, Robilotta
PRD 102, 076012 (2020)

- kernel should includes all the mm dynamics



- Unitarized amplitude should includes all channels with the same (J,I)