NNLO QCD corrections to the mixing of neutral B-mesons

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based on 2106.05979, 2202.12305, 2205.07907

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Implications of LHCb measurements and future prospects 2022

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Outline

- Neutral meson systems can oscillate between their flavor eigenstates

$$K^0 - \bar{K}^0, D^0 - \bar{D}^0 \quad \text{and} \quad B_q^0 - \bar{B}_q^0 \quad \text{with} \quad q = s, d.$$

- Loop-induced FCNC processes.
- B_q meson properties equally well accessible to theory and experiment.
- Flavor physics features multiple anomalies (LFU violation, muon $g-2$, $|V_{cb}|$, ...) challenging the SM.
- Precision physics in the flavor sector as a tool for addressing these challenges.
- Precise *measurements* and precise *theoretical predictions* equally important.

- $B_s^0 - \bar{B}_s^0$ oscillations between flavor eigenstates $|B_s^0\rangle$ and $|\bar{B}_s^0\rangle$

$$i \frac{d}{dt} \begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix} = \left(\hat{M} - \frac{i}{2} \hat{\Gamma} \right) \begin{pmatrix} |B_s^0(t)\rangle \\ |\bar{B}_s^0(t)\rangle \end{pmatrix},$$

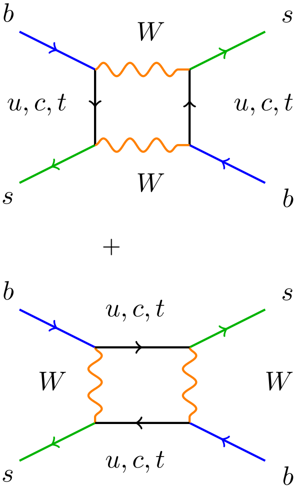
$$\hat{M} = \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{22} \end{pmatrix}, \quad \hat{\Gamma} = \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{22} \end{pmatrix}$$

- Diagonalize the matrices, introduce mass eigenstates

$$|B_{s,L}\rangle = p\,|B_s^0\rangle + q\,|\bar{B}_s^0\rangle, \quad |B_{s,H}\rangle = p\,|B_s^0\rangle - q\,|\bar{B}_s^0\rangle$$

- The complex coefficients obey $|p|^2 + |q|^2 = 1$

$$M_{12} - \frac{i}{2}\Gamma_{12} \propto$$



- Physical observables depend on: $|M_{12}|$, $|\Gamma_{12}|$, ϕ_s
- ΔM_s: $B_s^0 - \bar{B}_s^0$ oscillation frequency: t quark is dominant in SM, sensitivity to NP in the loops

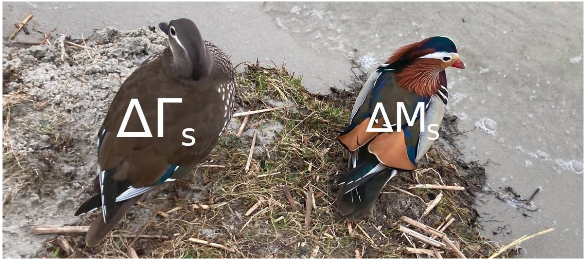
$$\Delta M_s = M_H - M_L \approx 2|M_{12}|$$

- $\Delta\Gamma_s$: $B_s^0 - \bar{B}_s^0$ width difference: only u and c contribute, precision probe of SM, little room for NP

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H \approx 2|\Gamma_{12}| \cos\phi_s$$

- ϕ_s: CP-asymmetry in the mixing

$$a_{\text{fs}} = \text{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right) = \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin\phi_s, \quad \phi_s = \arg\left(-\frac{M_{12}}{\Gamma_{12}}\right)$$



- Our interest: $\Delta\Gamma_s$ from $B_s^0 - \bar{B}_s^0$
- Experimental value (HFLAV 2021 average)

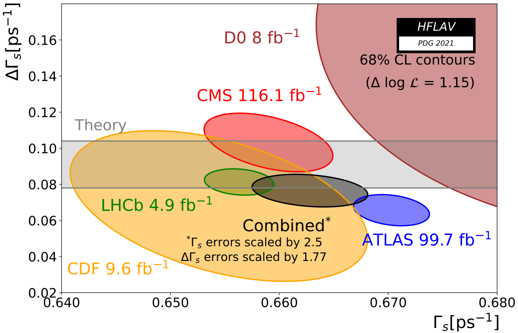
$$\Delta\Gamma^{\text{exp}} = (0.085 \pm 0.005)\ \text{ps}^{-1}$$

- Theory prediction (NLO + n_f-piece of NNLO QCD corrections) as of 2020 [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999; Ciuchini, Franco, Lubicz, & Mescia, 2002; Ciuchini, Franco, Lubicz, Mescia, & Tarantino, 2003; Lenz & Nierste, 2007; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020; Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017]

$$\Delta\Gamma_s = (0.077 \pm 0.015_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.017_{\Lambda_{\text{QCD}}/m_b}) \times \text{ps}^{-1}\ \text{(pole)}$$

$$\Delta\Gamma_s = (0.088 \pm 0.011_{\text{pert.}} \pm 0.002_{B,\tilde{B}_S} \pm 0.014_{\Lambda_{\text{QCD}}/m_b}) \times \text{ps}^{-1}\ (\overline{\text{MS}})$$

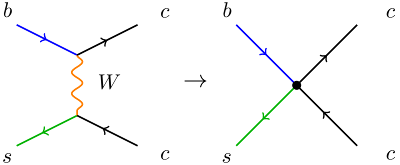
- Large perturbative uncertainty from the uncalculated NNLO corrections (pert.)
- Can be reduced by including relevant 2- and 3-loop QCD corrections
- Theory under pressure, full NNLO corrections highly desirable



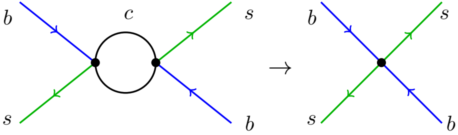
$$\Gamma_s = (\Gamma_L + \Gamma_H)/2$$

Overview of the matching calculation

- $|\Delta B| = 1$ EFT ($m_b \ll m_W, m_t$)

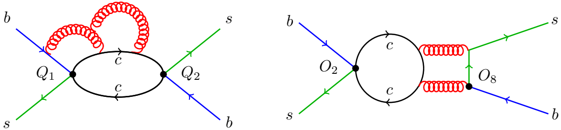


- $|\Delta B| = 2$ EFT (via HQE)

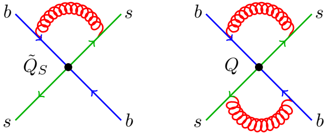


$$\Gamma_{12} \sim \frac{1}{m_b^3} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_3^{(i)} + \frac{1}{m_b^4} \sum_i \left(\frac{\alpha_s}{4\pi}\right)^j \Gamma_4^{(i)} + \ldots$$

- Calculation done using $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$ in the CMM operator basis for $b \to s c \bar{c}$ [Chetyrkin, Misiak, & Munz, 1998]
- Representative diagrams in the $|\Delta B| = 1$ EFT needed for the NNLO accuracy



matched to the $|\Delta B| = 2$ EFT



$|\Delta B| = 1$ side of the matching: operator basis

Effective Hamiltonian of the $|\Delta B| = 1$ theory in the CMM basis [Chetyrkin, Misiak, & Munz, 1998]

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = \frac{4G_F}{\sqrt{2}} \Bigg[-V_{ts}^* V_{tb} \left(\sum_{i=1}^{6} C_i Q_i + C_8 Q_8 \right) - V_{us}^* V_{ub} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + V_{us}^* V_{cb} \sum_{i=1}^{2} C_i Q_i^{cu} + V_{cs}^* V_{ub} \sum_{i=1}^{2} C_i Q_i^{uc} \Bigg] + \text{h.c.} \,,$$

- Dominant current-current (cc) operators

$$Q_1 \equiv Q_1^{cc} = \bar{s}_L \gamma_\mu T^a c_L \; \bar{c}_L \gamma^\mu T^a b_L \,,$$
$$Q_2 \equiv Q_1^{cc} = \bar{s}_L \gamma_\mu c_L \; \bar{c}_L \gamma^\mu b_L$$

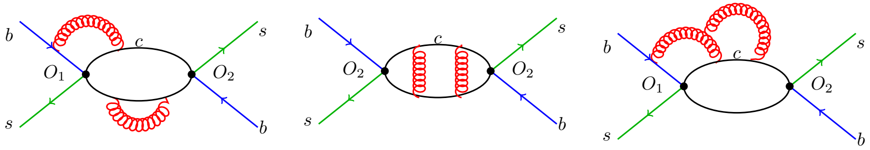
- $Q_{1,2}^u$, $Q_{1,2}^{cu}$ and $Q_{1,2}^{uc}$ follow the same pattern
- Also evanescent operators [Dugan & Grinstein, 1991; Herrlich & Nierste, 1995] (Dirac algebra in $d \neq 4$ dimensions)

- Penguin operators

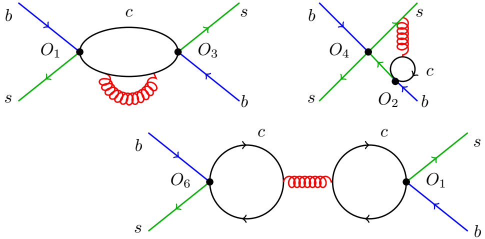
$$Q_3 = \bar{s}_L \gamma_\mu b_L \sum\nolimits_q \bar{q} \gamma^\mu q \,,$$
$$Q_4 = \bar{s}_L \gamma_\mu T^a b_L \sum\nolimits_q \bar{q} \gamma^\mu T^a q \,,$$
$$Q_5 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} b_L \sum\nolimits_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} q \,,$$
$$Q_6 = \bar{s}_L \gamma_{\mu_1} \gamma_{\mu_2} \gamma_{\mu_3} T^a b_L \sum\nolimits_q \bar{q} \gamma^{\mu_1} \gamma^{\mu_2} \gamma^{\mu_3} T^a q \,,$$
$$Q_8 = \frac{g_s}{16\pi^2} m_b \, \bar{s}_L \sigma^{\mu\nu} T^a b_R \, G_{\mu\nu}^a \,,$$

$|\Delta B| = 1$ side of the matching: representative diagrams

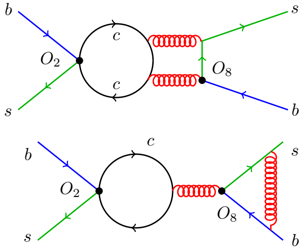
3-loop $O_{1,2} \times O_{1,2}$ correlators



2-loop $O_{1,2} \times O_{3-6}$ correlators



2-loop $O_{1,2} \times O_8$ correlators



$|\Delta B| = 2$ side of the matching: operator basis

- $\Delta\Gamma_s$ described by local $|\Delta B| = 2$ operators [Beneke, Buchalla, Greub, Lenz, & Nierste, 1999; Lenz & Nierste, 2007; Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017]
- Using Heavy Quark Expansion [Khoze & Shifman, 1983; Shifman & Voloshin, 1985; Khoze, Shifman, Uraltsev, & Voloshin, 1987; Chay, Georgi, & Grinstein, 1990; Bigi & Uraltsev, 1992; Bigi, Uraltsev, & Vainshtein, 1992; Bigi, Shifman, Uraltsev, & Vainshtein, 1993; Blok, Koyrakh, Shifman, & Vainshtein, 1994; Manohar & Wise, 1994] (expansion in Λ_{QCD}/m_b) one arrives at

$$\Gamma_{12} = -(\lambda_c^q)^2\Gamma_{12}^{cc} - 2\lambda_c^q\lambda_u^q\Gamma_{12}^{uc} - (\lambda_u^q)^2\Gamma_{12}^{uu}, \quad \lambda_{q'}^q \equiv V_{q'q}^* V_{q'b}$$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}}\left[H^{ab}(z)\langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(z)\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle\right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

- $H^{ab}(z)$ and $\widetilde{H}_S^{ab}(z)$: Wilson coefficients from the perturbative matching to the physical $|\Delta B| = 2$ operators, $z \equiv m_c^2/m_b^2$

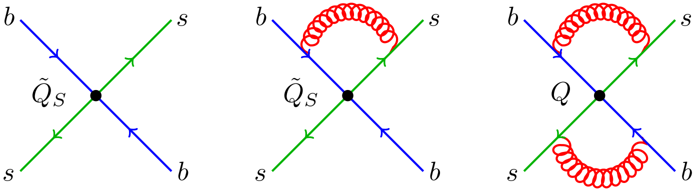
$$Q = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_i\;\bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_j \qquad \widetilde{Q}_S = \bar{s}_i\left(1+\gamma^5\right)b_j\;\bar{s}_j\left(1+\gamma^5\right)b_i$$

- Additional operators needed at intermediate stages (e. g. basis changes, def. of evanescent operators)

$$\widetilde{Q} = \bar{s}_i\gamma^\mu\left(1-\gamma^5\right)b_j\;\bar{s}_j\gamma_\mu\left(1-\gamma^5\right)b_i\,, \quad Q_S = \bar{s}_i\left(1+\gamma^5\right)b_i\;\bar{s}_j\left(1+\gamma^5\right)b_j\,,$$

- Not shown here: evanescent $|\Delta B| = 2$ operators and $1/m_b$ suppressed operators

$|\Delta B| = 2$ side of the matching: representative diagrams



- Wilson coefficients of the $|\Delta B| = 2$ theory determined in the matching to $|\Delta B| = 1$, $z \equiv m_c^2/m_b^2$

$$\Gamma_{12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \langle B_s|Q|\bar{B}_s\rangle + \widetilde{H}_S^{ab}(z) \langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle \right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

- Nonperturbative ME $\langle B_s|Q|\bar{B}_s\rangle$ and $\langle B_s|\widetilde{Q}_S|\bar{B}_s\rangle$ (also for B_d mesons) from QCD/HQET sum rules [Ovchinnikov & Pivovarov, 1988; Reinders & Yazaki, 1988; Korner, Onishchenko, Petrov, & Pivovarov, 2003; Mannel, Pecjak, & Pivovarov, 2011; Grozin, Klein, Mannel, & Pivovarov, 2016; Kirk, Lenz, & Rauh, 2017; King, Lenz, & Rauh, 2019, 2022], lattice QCD [Bazavov et al., 2016; Dowdall, Davies, Horgan, Lepage, Monahan, et al., 2019] or combined [Di Luzio, Kirk, Lenz, & Rauh, 2019]

$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

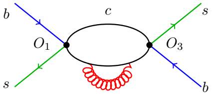
$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

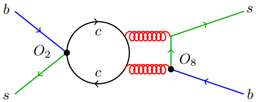
- Important scale: $z \equiv m_c^2/m_b^2$
- Can expand in z, good convergence already for $\mathcal{O}(z)$
- The final result incorporates various $O_{i-j} \times O_{k-l}$ contributions at 1, 2 or 3 loops (with $i,j,k,l \in \{1-6,8\}$)
- At 3 loops we consider only $O_{1-2} \times O_{1-2}$
- Available **literature** results: mostly z-exact but **often** concern **only the fermionic n_f-piece**
- **Our calculation: full results (n_f and non-n_f)** but expanded up to $\mathcal{O}(z)$
- ✓ Many cross checks through comparisons to the existing results

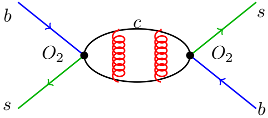
$|\Delta B| = 1$ contributions needed for NNLO (always 2 insertions from $\mathcal{H}_{\text{eff}}^{|\Delta B|=1}$)

$$C_i O_i \sim \begin{cases} 1 & \text{for } i = 1,2 \\ \alpha_s & \text{for } i = 3,4,5,6 \quad (C_{3-6} \text{ numerically small}) \\ \alpha_s & \text{for } i = 8 \quad (\text{explicit strong coupling in the definition of } O_8 \end{cases}$$

- NNLO (i. e. $\mathcal{O}(\alpha_s^2)$) contributions to $\Delta\Gamma_s$
  - 3-loop $O_{1-2} \times O_{1-2}$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, $\mathcal{O}(\sqrt{z})$)
  - 2-loop $O_{1-2} \times O_8$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)
  - 1-loop $O_8 \times O_8$ correlators [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] (n_f piece only, z-exact)
- This work
  - Full (n_f + non-n_f) results for all 2-loop correlators at $\mathcal{O}(z)$ (including $O_8 \times O_8 \Rightarrow$ N^{3}LO)
  - Full (n_f + non-n_f) results for the 3-loop $O_{1-2} \times O_{1-2}$ at $\mathcal{O}(z)$
  - Renormalization matrix Z_{ij} for the $|\Delta B| = 2$ theory at $\mathcal{O}(\alpha_s^2)$







- Calculational strategy
  - Matching done **on-shell**: $p_b^2 = m_b^2$
  - The s-quark mass is neglected $\Rightarrow p_s = 0$
  - **Asymptotic expansion** in $z \equiv m_c^2/m_b^2$ (at first up to $\mathcal{O}(z)$)
  - Only the **imaginary part** of the $|\Delta B| = 1$ diagrams enters the matching
- Regularization
  - Dimensional regularization used **both for UV- and IR-divergences**
  - Cross-check: **massive gluons** in IR-divergent diagrams at 2 loops
  - $\varepsilon_{\text{UV}} + m_g$: renormalized amplitudes manifestly finite $\Rightarrow$ the limit $d \to 4$ is safe
  - $\varepsilon = \varepsilon_{\text{UV}} = \varepsilon_{\text{IR}}$: products of $1/\varepsilon_{\text{IR}}$ and evanescent ME are of $\mathcal{O}(\varepsilon^0)$

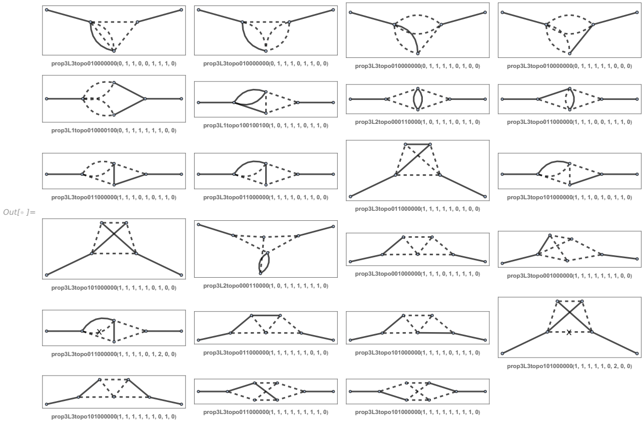
- All computations done using our well-tested automatic setup
  - Diagram generation: **QGRAF** [Nogueira, 1993]
  - Feynman rules and topology identification: **Q2E/EXP** [Seidensticker, 1999; Harlander, Seidensticker, & Steinhauser, 1998] or **TAPIR** [Gerlach, Herren, & Lang, 2023]
  - Amplitude evaluation: in-house **FORM**-based [Ruijl, Ueda, & Vermaseren, 2017] **CALC** setup
  - IBPs: **FIRE** 6 [Smirnov & Chuharev, 2020] + **LiteRed** [Lee, 2014]
  - Analytic MI evaluation: **HyperInt** [Panzer, 2015], **HyperlogProcedures** [Schnetz], **PolyLogTools** [Duhr & Dulat, 2019]
  - Numerical MI evaluation: **FIESTA** [Smirnov, 2016] and **pySecDec** [Borowka, Heinrich, Jahn, Jones, Kerner, et al., 2018]
- Cross-checks of selected intermediate results: **FeynArts** [Hahn, 2001], **FeynRules** [Christensen & Duhr, 2009; Alloul, Christensen, Degrande, Duhr, & Fuks, 2014] and **FeynCalc** [Mertig, Böhm and Denner, 1991, VS, Mertig & Orellana, 2016, 2020]
- Two complementary approaches to tensor integrals in **FORM**
  - Explicit decomposition formulas (1 ext. momentum, max. rank 10), calculated using **FeynCalc** and **FERMAT** [Lewis]
  - Projections to the occurring color and 4-fermion Dirac structures

- New on-shell 3-loop integrals with massive (solid) lines
- Only imaginary parts are relevant and turn out to be very simple
- Appearing constants

$$\pi, \ln(2), \zeta_2, \zeta_3, \zeta_4, \mathrm{Cl}_2(\pi/3), \sqrt{3},$$
$$\mathrm{Li}_4(1/2), \ln\left((1+\sqrt{5})/2\right)$$

$$\mathrm{Cl}_2(x) = \frac{i}{2}\left(\mathrm{Li}_2(e^{-ix}) - \mathrm{Li}_2(e^{ix})\right)$$

- Real parts (obtained as a byproduct) more complicated but irrelevant for $\Delta\Gamma_s$
- We can directly integrate the Feynman parameter integrals using **HyperInt** [Panzer, 2015]

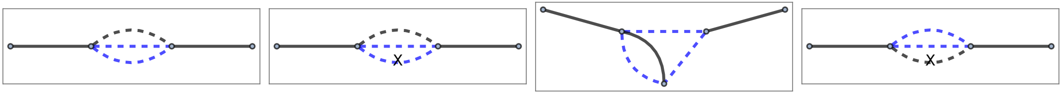


- New contributions to Γ_{12}^s computed in the course of this project ($z = m_c^2/m_b^2$)

Contribution	Most recent literature result	**This work**
$Q_{1,2} \times Q_{3-6}$	2 loops, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{1,2} \times Q_8$	2 loops, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{3-6} \times Q_{3-6}$	2 loop, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{3-6} \times Q_8$	1 loop, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_8 \times Q_8$	1 loop, z-exact, n_f-part only [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020]	2 loops, $\mathcal{O}(z)$, full
$Q_{1,2} \times Q_{1,2}$	3 loops, $\mathcal{O}(\sqrt{z})$, n_f-part only [Asatrian, Hovhannisyan, Nierste, & Yeghiazaryan, 2017]	3 loops, $\mathcal{O}(z)$, full

- All building blocks required for the NNLO prediction are available.

- What about higher orders in z? Can we go beyond $\mathcal{O}(z)$?
- Very easy at 2 loops, only 4 nontrivial master integrals with massive charm lines



- Asymptotic expansion in m_c straightforward.
- Can be automatized using **FEYNCALC 10**, **ASY.M** [Jantzen, Smirnov, & Smirnov, 2012], **FEYNHELPERS** [VS, 2016] and **FIRE**
- $\mathcal{O}(z^2)$ results for all 2-loop matching coefficients already computed, cross-checks still pending
- Work in progress: z-exact results using differential equations. [Kotikov, 1991a, 1991b, 1991c; Bern, Dixon, & Kosower, 1994; Remiddi, 1997; Gehrmann & Remiddi, 2000; Henn, 2013] and **CANONICA** [Meyer, 2018]
- At 3 loops we have several hundreds master integrals with m_c, but $\mathcal{O}(z^2)$ corrections are clearly doable!

- Ingredients for the theory prediction

$$\Gamma_{12}^s = -(\lambda_t^s)^2 \left[\Gamma_{12}^{s,cc} + 2\frac{\lambda_u^s}{\lambda_t^s} (\Gamma_{12}^{s,cc} - \Gamma_{12}^{s,uc}) + \left(\frac{\lambda_u^s}{\lambda_t^s}\right)^2 (\Gamma_{12}^{s,uu} + \Gamma_{12}^{s,cc} - 2\Gamma_{12}^{s,uc}) \right]$$

$$\Gamma_{s,12}^{ab} = \frac{G_F^2 m_b^2}{24\pi M_{B_s}} \left[H^{ab}(z) \underbrace{\langle B_s | Q | \bar{B}_s \rangle}_{\frac{8}{3} M_{B_s}^2 f_{B_s}^2 B_{B_s}} + \widetilde{H}_S^{ab}(z) \underbrace{\langle B_s | \widetilde{Q}_S | \bar{B}_s \rangle}_{\frac{1}{3} M_{B_s}^2 f_{B_s}^2 \widetilde{B}'_{S,B_s}} \right] + \mathcal{O}\left(\Lambda_{\text{QCD}}/m_b\right)$$

$$M_{12} = (\lambda_t^s)^2 \frac{G_F^2 M_{B_s}}{12\pi^2} M_W^2 \hat{\eta}_B S_0 \left(\frac{m_t^2}{M_W^2}\right) f_{B_s}^2 B_{B_s}$$

- Cancellation of $(\lambda_t^s)^2 = (V_{ts}^* V_{tb})^2$, f_{B_s}, M_{B_s} and bag parameters (to large extent) in the ratio Γ_{12}^s / M_{12}^s
- Following [Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020] we calculate

$$\frac{\Delta\Gamma_s}{\Delta M_s} = -\text{Re}\left(\frac{\Gamma_{12}^s}{M_{12}^s}\right), \quad \Delta\Gamma_s = \left(\frac{\Delta\Gamma_s}{\Delta M_s}\right) \Delta M_s^{\text{exp}}$$

- $|V_{cb}|$ controversy (exclusive vs. inclusive determinations) irrelevant!

- In addition to the leading power result for $\Delta\Gamma_s$ (our work) we also need the $1/m_b$-correction (known at LO only) [Beneke, Buchalla, & Dunietz, 1996]
- On the scheme choice: m_b and m_c in the matching coefficients in the $\overline{\text{MS}}$ scheme at $\mu_b = \mu_c = m_b$, i. e. $\bar{z} = (m_c^{\text{MS}}(m_b)/m_b^{\text{MS}}(m_b))^2$
- Freedom to choose the scheme for m_b^2 in the prefactor of Γ_{12}
- We use $\overline{\text{MS}}$, potential-subtracted (PS) [Beneke, 1998] and pole schemes in the leading power term
- m_b^2 in the subleading $1/m_b$ term (LO only) is converted to the PS scheme
- **RunDec** [Herren & Steinhauser, 2018] for the running and decoupling of quark masses and α_s.
- Numerical input [Zyla et al., 2020; Chetyrkin, Kuhn, Maier, Maierhofer, Marquard, et al., 2017; Dowdall, Davies, Horgan, Lepage, Monahan, et al., 2019; Bazavov et al., 2018; Asatrian, Asatryan, Hovhannisyan, Nierste, Tumasyan, & Yeghiazaryan, 2020; Amhis et al., 2021]

$$
\begin{aligned}
\alpha_s(M_Z) &= 0.1179 \pm 0.001, \quad m_c(3\text{ GeV}) = 0.993 \pm 0.008 \;\text{ GeV}, \\
m_b(m_b) &= 4.163 \pm 0.016 \;\text{ GeV}, \quad m_t^{\text{pole}} = 172.9 \pm 0.4 \;\text{ GeV}, \\
M_{B_s} &= 5366.88 \;\text{ MeV} \quad f_{B_s} = (0.2307 \pm 0.0013) \;\text{ GeV}, \\
B_{B_s} &= 0.813 \pm 0.034, \quad \widetilde{B}'_{S,B_s} = 1.31 \pm 0.09, \\
\lambda_u^s/\lambda_t^s &= -(0.00865 \pm 0.00042) + (0.01832 \pm 0.00039)i \\
\Delta M_s^{\text{exp}} &= (17.749 \pm 0.020) \;\text{ ps}^{-1}
\end{aligned}
$$

- Our published results for $\Delta\Gamma_s/\Delta M_s$ and $\Delta\Gamma_s$ in the 3 schemes [Gerlach, Nierste, VS, Steinhauser, 2022]

$$\frac{\Delta\Gamma_s}{\Delta M_s} = (3.79^{+0.53}_{-0.58}{}_{\text{LP scale}}\,{}^{+0.09}_{-0.19}{}_{\text{NLP scale}} \pm 0.11_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad \text{(pole)},$$
$$\frac{\Delta\Gamma_s}{\Delta M_s} = (4.33^{+0.23}_{-0.44}{}_{\text{LP scale}}\,{}^{+0.09}_{-0.19}{}_{\text{NLP scale}} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad (\overline{\text{MS}}),$$
$$\frac{\Delta\Gamma_s}{\Delta M_s} = (4.20^{+0.36}_{-0.39}{}_{\text{LP scale}}\,{}^{+0.09}_{-0.19}{}_{\text{NLP scale}} \pm 0.12_{B\tilde{B}_S} \pm 0.78_{1/m_b} \pm 0.05_{\text{input}}) \times 10^{-3} \quad \text{(PS)},$$

- LP: scale variation in the leading power piece; NLP: scale variation in the $1/m_b$ term
- μ_1 ($|\Delta B| = 1$ theory), and μ_b, μ_c (quark masses) simultaneously varied between 2.1GeV and 8.4GeV.
- Combined prediction ($\overline{\text{MS}}$ + PS, uncertainties added in quadrature)

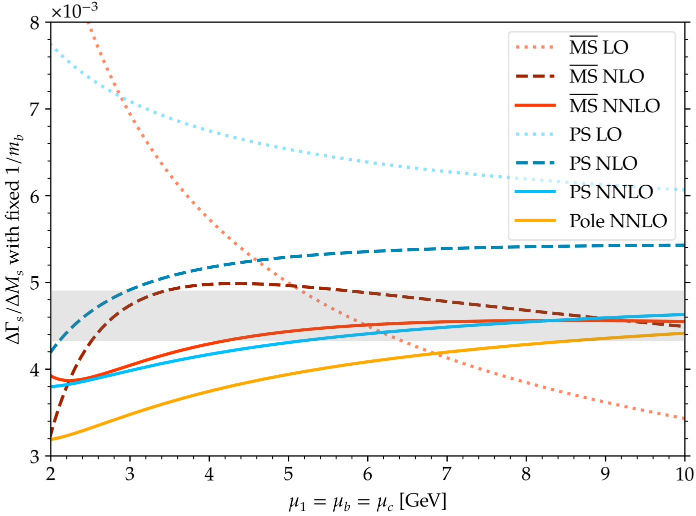
$$\Delta\Gamma_s^{\text{theo}} = (0.076 \pm 0.017) \;\times \text{ps}^{-1}$$

- Experiment:

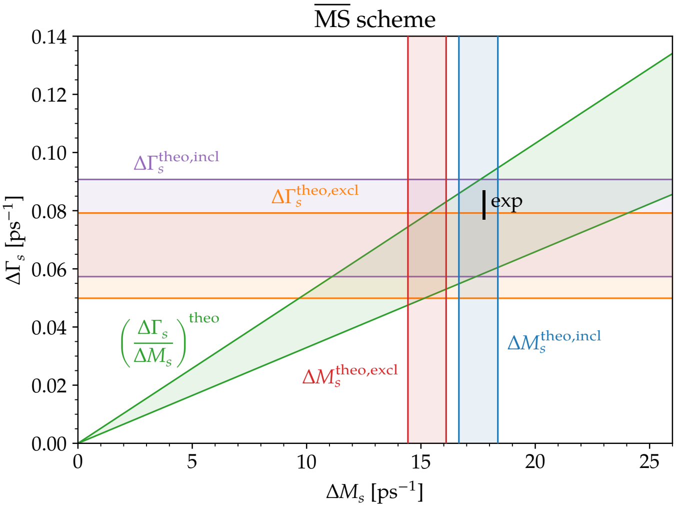
$$\Delta\Gamma_s^{\text{exp}} = (0.082 \pm 0.005) \;\times \text{ps}^{-1}$$

- Theory uncertainties only factor 3 larger than experimental error, now dominated by $1/m_b$ corrections!
- The pole scheme seems to be inadequate for the B-meson mixing observables!

- Renormalization scale dependence at LO, NLO and NNLO (no μ-variations in the subleading $1/m_b$-terms)
- μ_b, μ_c: renormalization scales of the quark masses
- μ-dependence at NNLO better than at NLO for $\overline{\text{MS}}$ and PS
- $\overline{\text{MS}}$- and PS-predictions close together: reduction of scheme uncertainty
- For $\mu \approx 9$ GeV the NNLO correction vanishes



- $\overline{\text{MS}}$-prediction for $\Delta\Gamma_s/\Delta M_s$ against individual predictions
- Individual predictions dominated by the uncertainties in $|V_{cb}|$
- Uncertainties from $\langle B_S|Q|\bar{B}_S\rangle$ less important in the ratio
- Currently cannot distinguish between $|V_{cb}|^{\text{excl.}}$ and $|V_{cb}|^{\text{incl.}}$



Summary

- Experimental precision of $\Delta\Gamma_s$ justifies and necessitates the NNLO calculation!
- We calculated all building blocks needed to obtain the NNLO correction to $B_s^0 - \bar{B}_s^0$ mixing
- New NNLO predictions for $\Delta\Gamma_s$ already available [Gerlach, Nierste, VS, Steinhauser, 2022]
- The scale uncertainty of leading power results reduces from 29% to 11% in the $\overline{\mathrm{MS}}$ and from 18% to 13% in the PS schemes

Outlook

- Analytic results for the 3-loop current-current matching coefficients to be published soon
- Higher order asymptotic expansions in $z \equiv m_c^2/m_b^2$ (already WIP)
- Could 3-loop penguin contributions help reducing the NNLO scale dependence even further?
- The theoretical precision of the $1/m_b$-term must be increased to the NLO accuracy