Methods to explore the nature of the new exotic resonances from data

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Implications of LHCb measurements
and future prospects
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Methods to explore the nature of the new exotic resonances from data

- New: Impressive work by LHCb in spectroscopy
- Nature & exotic:
  - Technically, "exotic" is something with quantum numbers different from $q\bar{q}$, qqq
  - More generally, something that does not "fit well" within **constituent quark models**
  - Here, exotic possibilities are molecule or compact tetraquark, …
- from data: avoid model/assumptions for dynamics
- Method: Weinberg's compositeness condition

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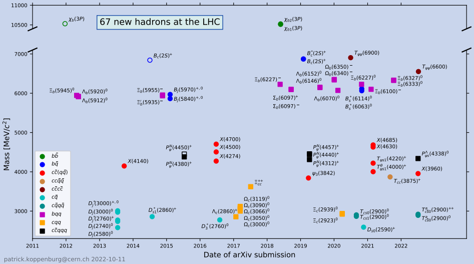
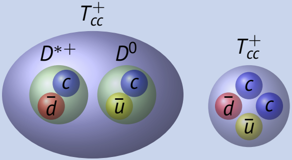
Evidence That the Deuteron Is Not an Elementary Particle*

STEVEN WEINBERG†
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received 30 September 1964)

- Elementary ∼ not composite ∼ compact
- not elementary ∼ composite ∼ molecule





Outline

1. **Weinberg's compositeness**
   - Standard compositeness criteria
   - Extension of criteria
2. **Examples**
   - Canonical example: the deuteron
   - $D_{s0}^*(2317)$ (exotic)
3. **Detailed example: T_{cc}^+**
4. **Summary**

Weinberg's compositeness

- Identity resolution:

$$\mathbb{1} = \sum_n |n\rangle\langle n| + \int d\alpha\, |\alpha\rangle\langle\alpha|$$

- $\hat{H} = \hat{H}_0 + \hat{V}$

[Weinberg, PR,137,B672('65)]

- Eigenstates of $\hat{H}_0$:
  - continuum $|\alpha\rangle$, $\hat{H}_0 |\alpha\rangle = E(\alpha) |\alpha\rangle$
  - bare elementary $|n\rangle$, $\hat{H}_0 |n\rangle = E_n |n\rangle$
- Eigenstates of $\hat{H}$:
  - $|d\rangle$ ("deuteron") $\hat{H} |d\rangle = E_B |d\rangle$
- Normalized:

$$\langle\alpha|\beta\rangle = \delta(\beta - \alpha) \qquad \langle\alpha|n\rangle = 0$$
$$\langle m|n\rangle = \delta_{mn} \qquad \langle d|d\rangle = 1$$

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- Sandwich $\langle d|\mathbb{1}|d\rangle = 1$

$$1 = Z + \int \mathrm{d}\alpha \frac{|\langle\alpha|V|d\rangle|^2}{(E(\alpha) - E_B)^2}$$

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Fundamental quantity $Z = \sum_n |\langle n|d\rangle|^2$

«(...) Z is the probability of finding the deuteron in a bare elementary-particle state.» [Weinberg, PR,137,B672]

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$$1 = Z + \int d\alpha \frac{|\langle\alpha|V|d\rangle|^2}{(E(\alpha) - E_B)^2}$$

- 1st crucial approximation: $\langle\alpha|V|d\rangle \simeq g/(2\pi)^{3/2}$

$$1 - Z = g^2 \frac{\mu}{2\pi^2} \int_{E_{\rm th}}^{\infty} dW \frac{k(W)}{(W - E_B)^2} = \frac{\mu^2 g^2}{2\pi\gamma_B}$$

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>
> *«(...) Z is the probability of finding the deuteron in a bare elementary-particle state.»* [Weinberg, PR,137,B672]

- Measure: $d\alpha = d^3\vec{k} = 4\pi\,\mu\,k(W)dW$

Weinberg's compositeness (II)

[Weinberg, PR,137,B672('65)]

- $1-Z$ written in terms of g^2: $\quad 1-Z = \dfrac{\mu^2 g^2}{2\pi\gamma_B}$

  Next step, move to *low energy n-p scattering*, and relate g^2 to a, r
- Start from *a version* of the Low equation:

$$T(E) = \frac{g^2}{E-E_B} + \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{\sqrt{\vec{k}^2}}{E - E_{\rm th} - \vec{k}^2/(2\mu)} \left|T(E(\vec{k}^2))\right|^2$$

- *A solution* proposed by Weinberg reads
  [2nd crucial assumption]:

$$T^{-1}(E) = \frac{E-E_B}{g^2} - \frac{\mu^2}{2\pi\gamma_B}(E+E_B-2E_{\rm th}) + i\frac{\mu k(E)}{2\pi}$$

- This solution for the amplitude **exactly satisfies the Effective Range Expansion** (ERE):

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$$a\gamma_B = -2\left(1 + \frac{2\pi\gamma_B}{\mu^2 g^2}\right)^{-1}$$

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Weinberg's compositeness condition(s)

$$a = -\frac{2}{\gamma_B}\frac{1 - Z}{2 - Z} \qquad r = -\frac{1}{\gamma_B}\frac{Z}{1 - Z}$$

Extension of Weinberg's compositeness

[MA, J. Nieves, EPJ,C82,8('22)]

(A) r, a are expansions in $\frac{1}{\beta}\left(\frac{\gamma_B}{\beta}\right)^{n-1}$, with β^{-1} an interaction range [in np, $\beta \sim m_\pi$]

$$r = \overbrace{-\frac{1}{\gamma_B}\frac{Z}{1-Z}}^{r_{\rm LO}(Z)\sim\mathcal{O}(\gamma_B^{-1})} + \mathcal{O}(\gamma_B^0\beta^{-1})$$

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- ***The trick*** is to combine A & B **to correlate δr and δa:**
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- Final equations, main result of [MA, J. Nieves, EPJ,C82,8('22)]:

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \delta r + \mathcal{O}(\gamma_B/\beta^2)$$

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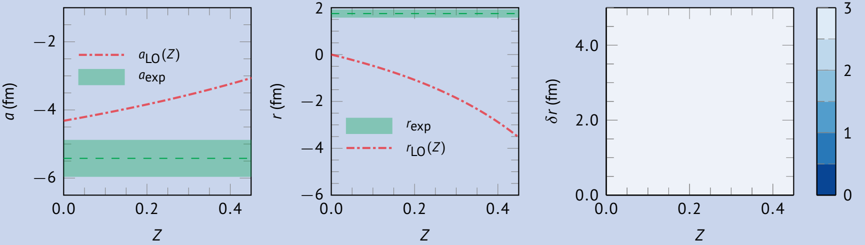
Canonical example: the deuteron

- Exp. data: $\left\{ \begin{array}{ll} a_{\rm exp} & = -5.42(1)\ \text{fm} \\ r_{\rm exp} & = +1.75(1)\ \text{fm} \\ \gamma_{B\,\rm exp} & = 45.7\ \text{MeV} \end{array} \right\}$

If Z is **naively evaluated** from these data, one gets $P = 1 - Z = 1.68$ [which makes no sense!]



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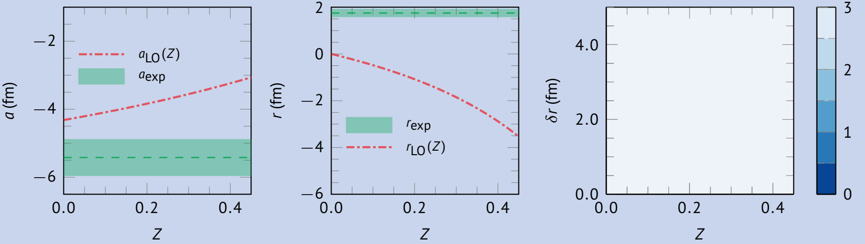
$$\mathcal{L}(Z,\delta r) = \frac{1}{3}\left[\left(\frac{a_{\rm exp} - a_{\rm NLO}}{\Delta a_{\rm exp}}\right)^2 + \left(\frac{r_{\rm exp} - r_{\rm NLO}}{\Delta r_{\rm exp}}\right)^2 + \left(\frac{\gamma_b^{\rm exp} - \gamma_b^{\rm NLO}}{\Delta \gamma_b^{\rm exp}}\right)^2\right]$$

- $\Delta a_{\rm exp}$ (*et al.*): **relative error** taken as $(\gamma_B/m_\pi)^2 \simeq 0.1$ because exp. error is smaller



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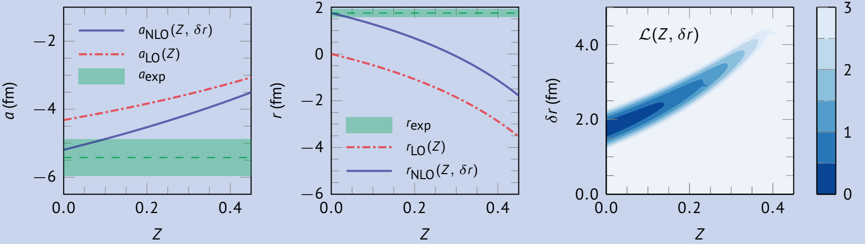
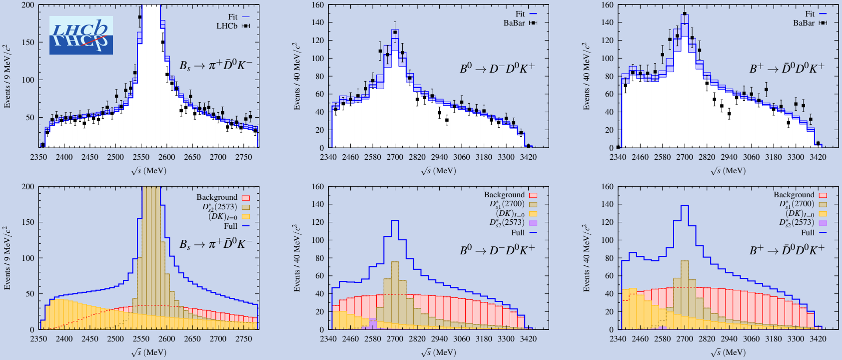
- $\Delta a_{\rm exp}$ (*et al.*): **relative error** taken as $(\gamma_B/m_\pi)^2 \simeq 0.1$ because exp. error is smaller
- The **NLO expressions** improve the agreement with data for $Z \simeq 0$ (molecular case), as expected
- The minimum is found for $\delta r \simeq r_{\rm exp} = 1.75$ fm ($\simeq m_\pi^{-1}$, as expected)



$D_{s0}^*(2317)$: a and r can be extracted from data... [MA, D. Jido, J. Nieves, E. Oset, EPJ,C76,6('16)]

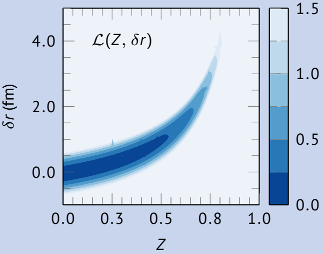
- DK ($I = 0$) interaction taken from Heavy Meson ChPT



Fit	$M_{D_{s0}^*}$ (MeV)	$P_{DK}(\%)$	a_0 (fm)
LHCb	$2326^{+16}_{-16}\,^{+1}_{-5}$	$74^{+7}_{-6}\,^{+9}_{-1}$	$-1.10^{+0.19}_{-0.39}\,^{+0.01}_{-0.02}$
BaBar	$2306^{+14}_{-23}\,^{+16}_{-9}$	$67^{+5}_{-7}\,^{+6}_{-10}$	$-0.87^{+0.15}_{-0.15}\,^{+0.11}_{-0.18}$
Combined	$2315^{+12}_{-17}\,^{+10}_{-5}$	$70^{+4}_{-6}\,^{+4}_{-8}$	$-0.95^{+0.15}_{-0.15}\,^{+0.08}_{-0.13}$

…but we take it from LQCD (apologies)

[MA, J. Nieves, EPJ,C82,8('22)]



- a and r: [Martínez-Torres *et al.*, JHEP,05,153('15)] [Mohler *et al.*, PRL,111,222001('13); PR,D90,034510('14)]

$$a = -1.3(5)\,\text{fm}$$
$$r = -0.1(3)\,\text{fm}$$

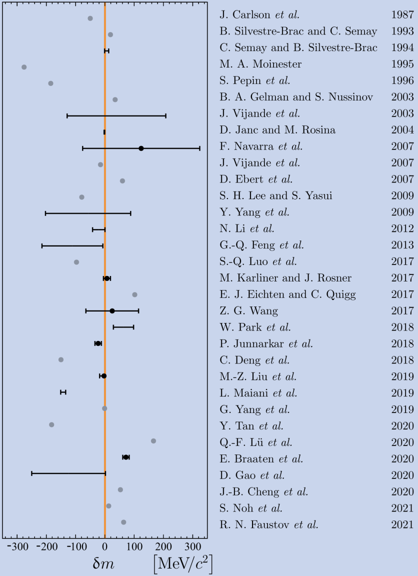
[Values compatible with those in the previous slide]

- $E_B = -45(4)$ MeV [from PDG compilation]

- Molecular probabilities $P = 1 - Z \gtrsim 0.5$
- We are not as specific as in other cases:
  - Larger uncertainties on the input (a, r, E_B)
  - Formalism pushed to (or beyond?) the limits: $\gamma_B/\beta \sim 0.6$ $[(\gamma_B/\beta)^2 = 36\%$

T_{cc}^{+} and previous predictions

- T_{cc}^{+} is a **tetraquark** with constituent $cc\bar{u}\bar{d}$
- Models give broad range of predictions.
- Not observed until now (only Ξ_{cc}^{++} [LHCb])
  [PRL,119,112001('17)]
- LQCD: not conclusive in the charm sector; more agreement in the bottom sector.
  [Leskovec *et al.*,PR,D100,014503('19)]
  [Bicudo *et al.*,PR,D103,114506('21)]



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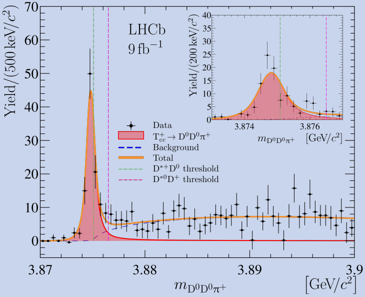
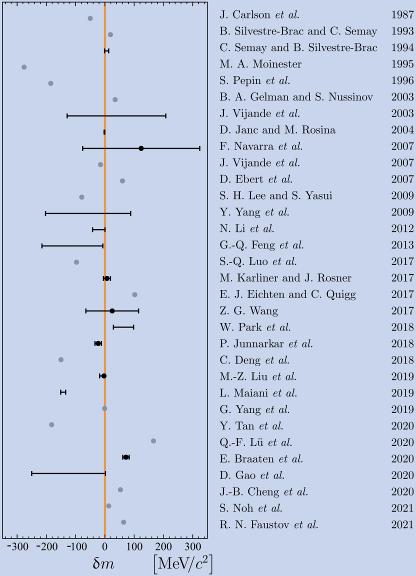
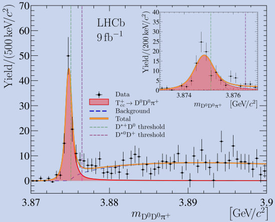
- Then comes LHCb… [2109.01038;2109.01056]

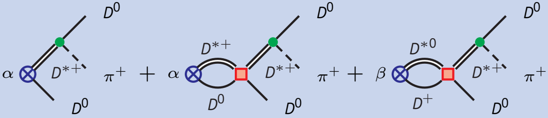




Production Model



- LHCb spectrum is essentially T_{cc}^+ signal and a D^*D phase space background
- Reasonable to assume that all $DD\pi$ events are produced through D^*D
- Small range ($\sim 30\,\mathrm{MeV}$) $DD\pi$ invariant mass: assume D^*D in S-wave



$$\mathcal{N}_{\rm ev}(Q^2) = \mathcal{N}_0 \left(\frac{Q^2_{\rm th}}{Q^2}\right)^{\frac{3}{2}} \int_{s_{\rm th}}^{s_{\rm max}(Q^2)} \mathrm{d}s \int_{t_-(s,Q^2)}^{t_+(s,Q^2)} \mathrm{d}t \sum_\lambda \left|\mathcal{M}_\lambda(Q^2,s,t,u)\right|^2 ,$$

$$\mathcal{M}_\lambda(Q^2,s,t,u) = g_{D^*D\pi}\, p_\pi^\nu \epsilon_S^\mu(\lambda) \left[\frac{K_t(Q^2)}{t-m^2_{D^*_{(t)}}}\left(-g_{\mu\nu}+\frac{k_\mu^{(t)}k_\nu^{(t)}}{t}\right) + \frac{K_u(Q^2)}{u-m^2_{D^*_{(u)}}}\left(-g_{\mu\nu}+\frac{k_\mu^{(u)}k_\nu^{(u)}}{u}\right)\right]$$

$$K_t(Q^2) = \alpha\left(1+G_1(Q^2)T_{11}(Q^2)\right) C_{D^{*+}\to D^0\pi^+} + \beta G_2(Q^2) T_{12}(Q^2)\, C_{D^{*+}\to D^0\pi^+} .$$

D^*D scattering amplitude

- Coupled T-matrix for the $D^{*+}D^0$, $D^{*0}D^+$ channels:

$$T^{-1}(E) = V^{-1}(E) - \mathcal{G}(E) \ ,$$

- $I_z = 0$: the isospin decomposition reads:

$$\left|D^{*+}D^0\right\rangle = -\frac{1}{\sqrt{2}}\left(|D^*D, I=1\rangle + |D^*D, I=0\rangle\right) \ ,$$
$$\left|D^{*0}D^+\right\rangle = -\frac{1}{\sqrt{2}}\left(|D^*D, I=1\rangle - |D^*D, I=0\rangle\right) \ ,$$

$V(E)$: **interaction** kernels written in terms of $C_{I=0,1}$ (constants):

$$V(E) = \frac{1}{2}\begin{pmatrix} C_0 + C_1 & C_1 - C_0 \\ C_1 - C_0 & C_0 + C_1 \end{pmatrix}$$

$\mathcal{G}(E)$: **loop functions** of the $D^{*+}D^0$, $D^{*0}D^+$ channels:

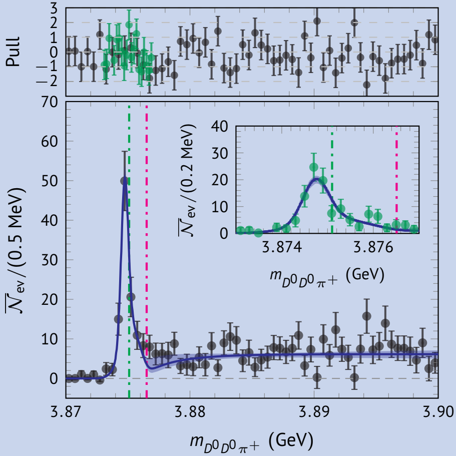
$$G_i(E) = \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3} \frac{e^{-\frac{2\vec{k}^2}{\Lambda^2}}}{E - E_{\rm th}^i - \frac{\vec{k}^2}{2\mu_i}}$$

- Width of the D^*: the loop functions are analytically continued to complex values of the D^* mass, $m_{D^*} \to m_{D^*} - i\Gamma_{D^*}/2$.
- Two values for the cutoff, $\Lambda = 0.5$ GeV and $\Lambda = 1.0$ GeV.
- The V-matrix elements depend now on the cutoff, $C_I(\Lambda)$.

Results: Fit

- Exp. resolution taken from LHCb ($\delta \simeq 400$ keV):

$$\overline{\mathcal{N}}_{\rm ev}(E) = \int {\rm d}E'\, R_{\rm LHCb}\,(E,E')\,\mathcal{N}_{\rm ev}(E')$$



Parameter	$\Lambda = 1.0$ GeV	$\Lambda = 0.5$ GeV
$C_0(\Lambda)$ [fm^2]	$-0.7008(22)$	$-1.5417(121)$
$C_1(\Lambda)$ [fm^2]	$-0.440(79)$	$-0.71(27)$
β/α	0.228(108)	0.093(79)
χ^2/dof	0.95	0.92

- Good agreement (χ^2/dof $= \{0.92, 0.95\}$)
- Check: pull of the data seems randomly distributed.
- Statistical uncertainties obtained by MC bootstrap of the data

Spectroscopy

- Bound state pole in T-matrix, $\det(\mathbb{1} - V\,G) = 0$:

$$T_{ij}(E) = \frac{\widetilde{g}_i \widetilde{g}_j}{E^2 - \left(M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2\right)^2} + \cdots$$

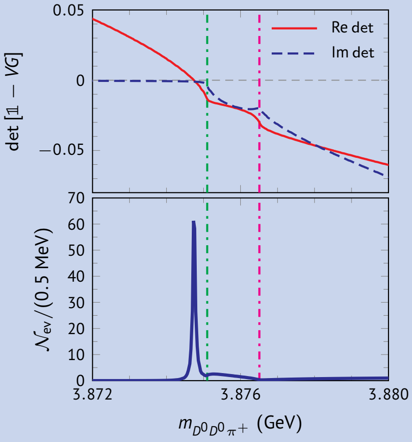
- Width: $m_{D^*} - i\,\Gamma_{D^*}/2 \Rightarrow M_{T_{cc}^+} - i\,\Gamma_{T_{cc}^+}/2$
- Pole position (wrt $D^{*+}D^0$ threshold):

Λ (GeV)	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
1.0	$-357(29)$	$77(1)$
0.5	$-356(29)$	$78(1)$

- Good agreement with LHCb determination:

	$\delta M_{T_{cc}^+}$ (keV)	$\Gamma_{T_{cc}^+}$ (keV)
[2109.01038]	$-273(61)$	$410(165)$
[2109.01056]	$-360(40)$	$48(2)$

- Our width is somewhat larger than the ~ 50 keV obtained by LHCb and [Feijoo *et al.*, 2108.02730], [Ling *et al.*, 2108.00947].
- [Du *et al.*, 2110.13765]: $\Gamma_{T_{cc}^+}$ depending on the model used.



- Results similar to [LHCb, 2109.0156] (top) and [Feijoo *et al.*, 2108.02730; Du *et al.*, 2110.13765] (bottom).

Molecular state?

- Weinberg compositeness [Weinberg, PR,137,B672('65)]: $P = 1 - Z \simeq \frac{\mu^2 g^2}{2\pi\gamma_B} = -g^2 G'(E_B)$
- We get $P_{D^{*+}D^0} = 0.78(5)(2)$, $P_{D^{*0}D^+} = 0.22(5)(2) \to P_{I=0} = 1$ **purely molecular state (model built-in!)**
- Relation to ERE parameters a, r [Weinberg,PR,137,B672('65)]

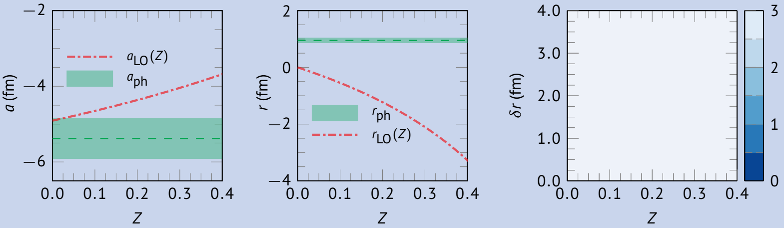
$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} + \cdots ,$$

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \cdots .$$

- **Single channel & isospin limit**:

Λ (GeV)	0.5	1.0
E_B (keV)	833(67)	856(53)
$a_{I=0}$ (fm)	$-5.57(25)$	$-5.18(16)$
$r_{I=0}$ (fm)	0.63	1.26

- Average values: $a_{\rm ph} = -5.38(30)$ fm, $r_{\rm ph} = 0.95(32)$ fm, $\gamma_{B\,\rm ph} = 40.4(1.7)$ MeV.



The values obtained clearly support a molecular picture for T_{cc}^+

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- Relation to ERE parameters a, r
  [Weinberg,PR,137,B672('65)] + [MA, J. Nieves, EPJ,C82,8('22)]

$$a = -\frac{2}{\gamma_B}\frac{1-Z}{2-Z} - 2\delta r\left(\frac{1-Z}{2-Z}\right)^2 + \cdots ,$$

$$r = -\frac{1}{\gamma_B}\frac{Z}{1-Z} + \delta r + \cdots .$$

- **Single channel & isospin limit**:

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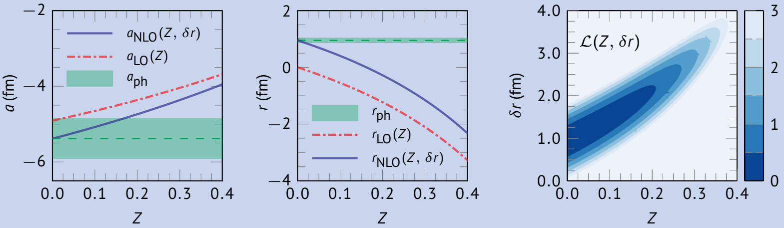
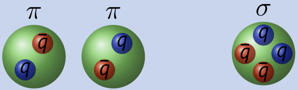
- Average values: $a_{\rm ph} = -5.38(30)$ fm, $r_{\rm ph} = 0.95(32)$ fm, $\gamma_{B\,\rm ph} = 40.4(1.7)$ MeV. Minimum at $\delta r \simeq r_{\rm ph} \simeq 1$ fm

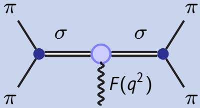


The values obtained clearly support a molecular picture for T_{cc}^+

Size

- Can we address the question of $4q$, $q\bar{q}$, molecule based on the size of the object?





- For $\pi\pi$ scattering, σ meson: MA, Oller, PR,D86,034003('12)
  - $\sqrt{\langle r^2\rangle_\sigma^S} \simeq 0.44$ fm vs $\sqrt{\langle r^2\rangle_\pi^S} \simeq 0.81$ fm
- Perhaps only theoretical? Future lattice QCD calculations?

Briceño *et al.*, PR,D103,114512('21) [and refs. therein]

Summary

- Hadron spectroscopy keeps living exciting times, as shown by the **LHCb discovery** of the T_{cc}^{+} state: a tetraquark with double charm

- **Weinberg's compositeness condition** is a fundamental tool to study of the nature (compact vs molecule) of the newly discovered states

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Article

Weinberg's Compositeness †

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[2] Department of Physics, University of Arizona, Tucson, AZ 85721, USA
† Dedicated to the memory of Steven Weinberg, who always chose the right degrees of freedom.

- We have proposed a **NLO extension** to Weinberg's compositeness condition, that rely on the same assumptions and do not assume any underlying dynamics [MA, J. Nieves, EPJ,C82,8('22)]

- The method has been applied to **deuteron** and to $D_{s0}^{*}(2317)$

- A coupled channel T-matrix allows a **good description of the T_{cc}^{+} data** with few parameters, and to compute the **scattering length** and the **effective range** in the isospin (single-channel) limit [MA, PL,B829,137052('22)]

- Applying the new method to these parameters, the T_{cc}^{+} state is found to be **largely molecular**

Methods to explore the nature of the new exotic resonances from data

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Miguel Albaladejo (IFIC)

Implications of LHCb measurements
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