Hadronic Resonances and Exotics from Lattice QCD

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LHCb implications, CERN

IMP

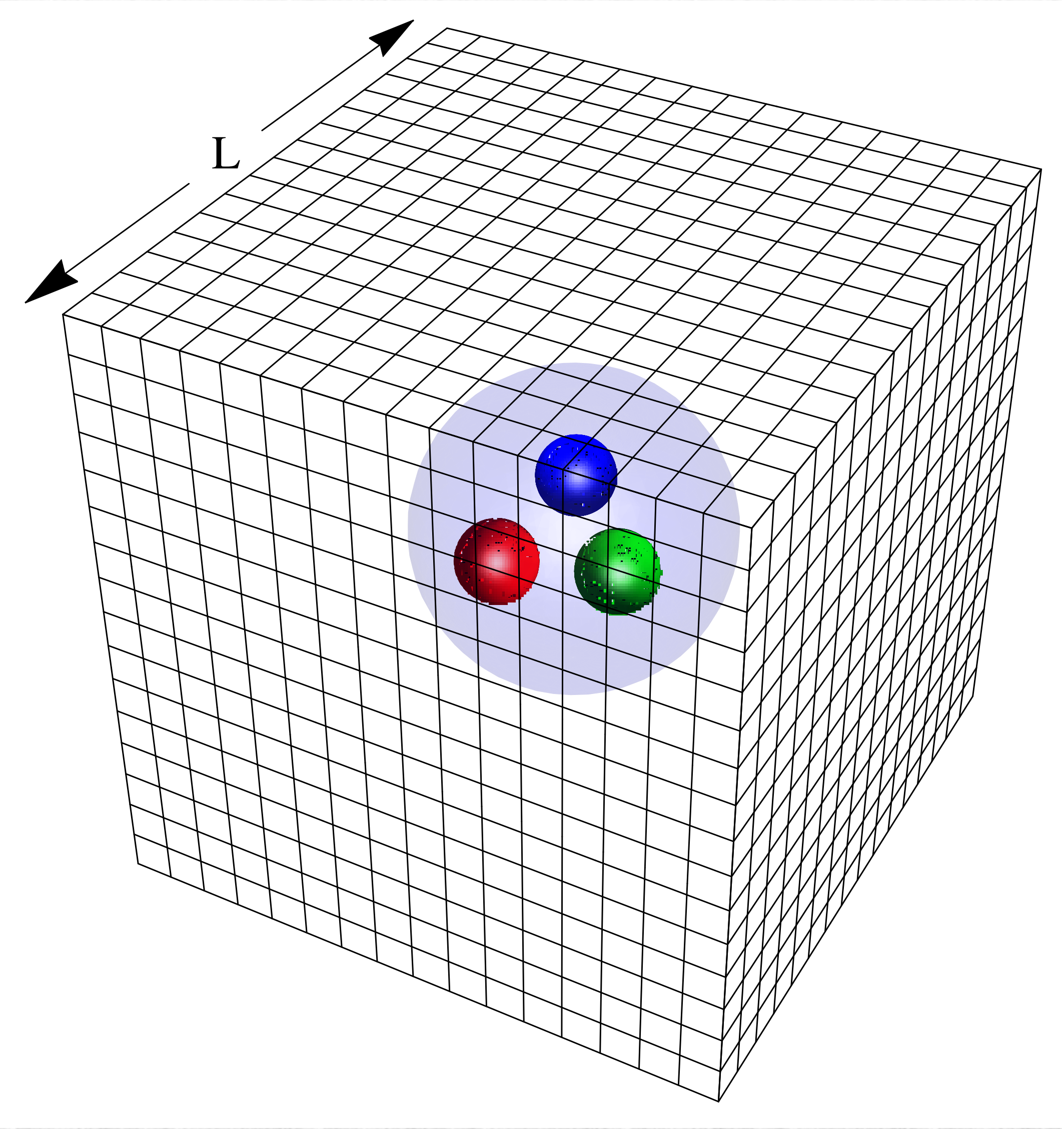
CLQCD

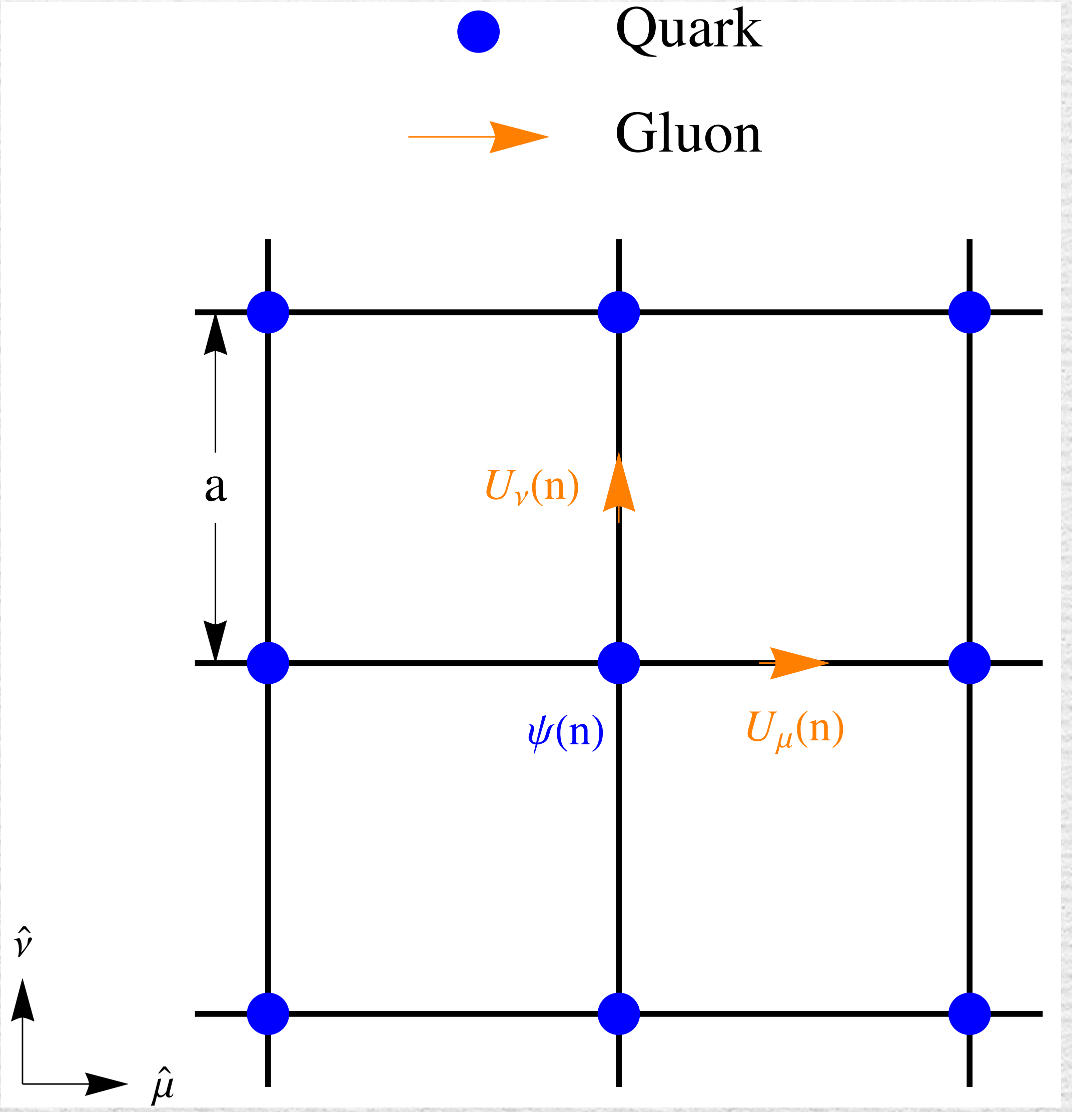
Outline

- ✦ Introduction
  - Lattice QCD
  - Spectroscopy in lattice QCD
- ✦ Some recent results
  - The P_c pentaquarks
  - The tetraquark T_{cc}

Lattice QCD

- QCD theory defined on finite discretized space → numerical calculation.
- First principle approach to non-perturbative QCD
- Controllable statistical and systematic uncertainties.





Spectroscopy on lattice

- Write down an interpolating operator $\mathcal{O}$ with certain quantum number, e.g. pion operator $\bar{u}\gamma_5 d$
- Compute the correlation function

$$< 0|\mathcal{O}(t)\mathcal{O}(0)^\dagger|0> = \sum_n \frac{< 0|\mathcal{O}|n > < n|\mathcal{O}|0 >}{2E_n} e^{-E_n t} \xrightarrow[t \to \infty]{} \propto e^{-E_0 t}$$

- At large t, fit the correlation function to an exponential.
- Usually only the ground state can be obtained.

Spectroscopy on lattice

Excited states:

- ✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

- ✦ Solve the generalized eigenvalue problem(GEVP):

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

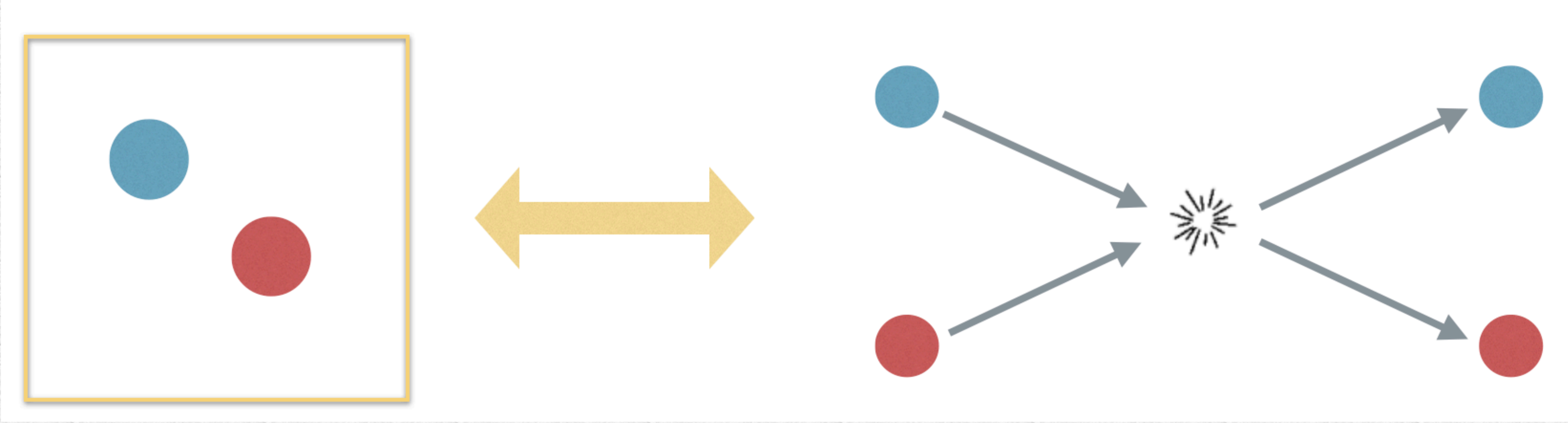
- ✦ Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

- ✦ Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

Scattering on lattice

Lüscher's finite volume method: M. Lüscher, Nucl. Phys. B354, 531(1991)

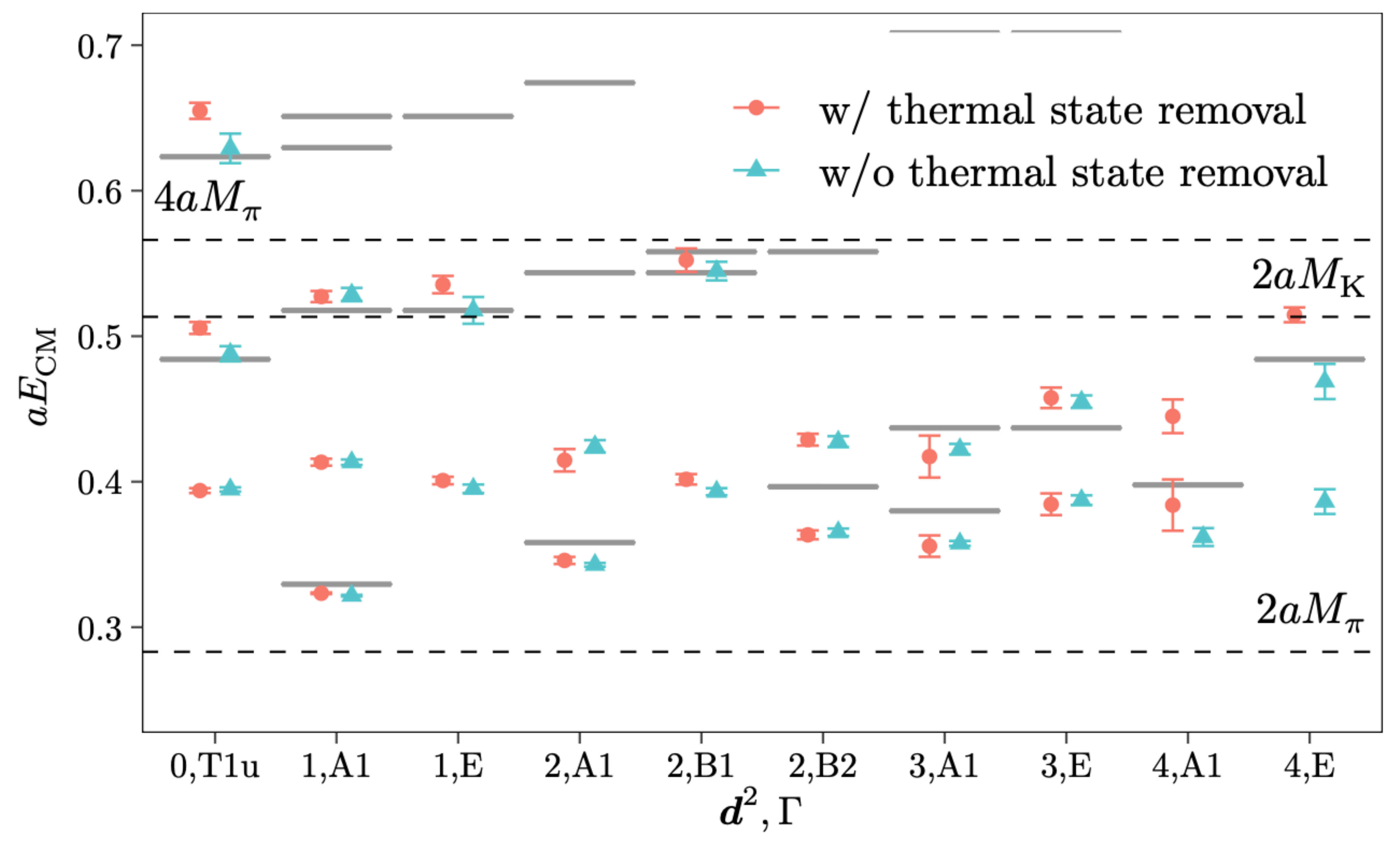


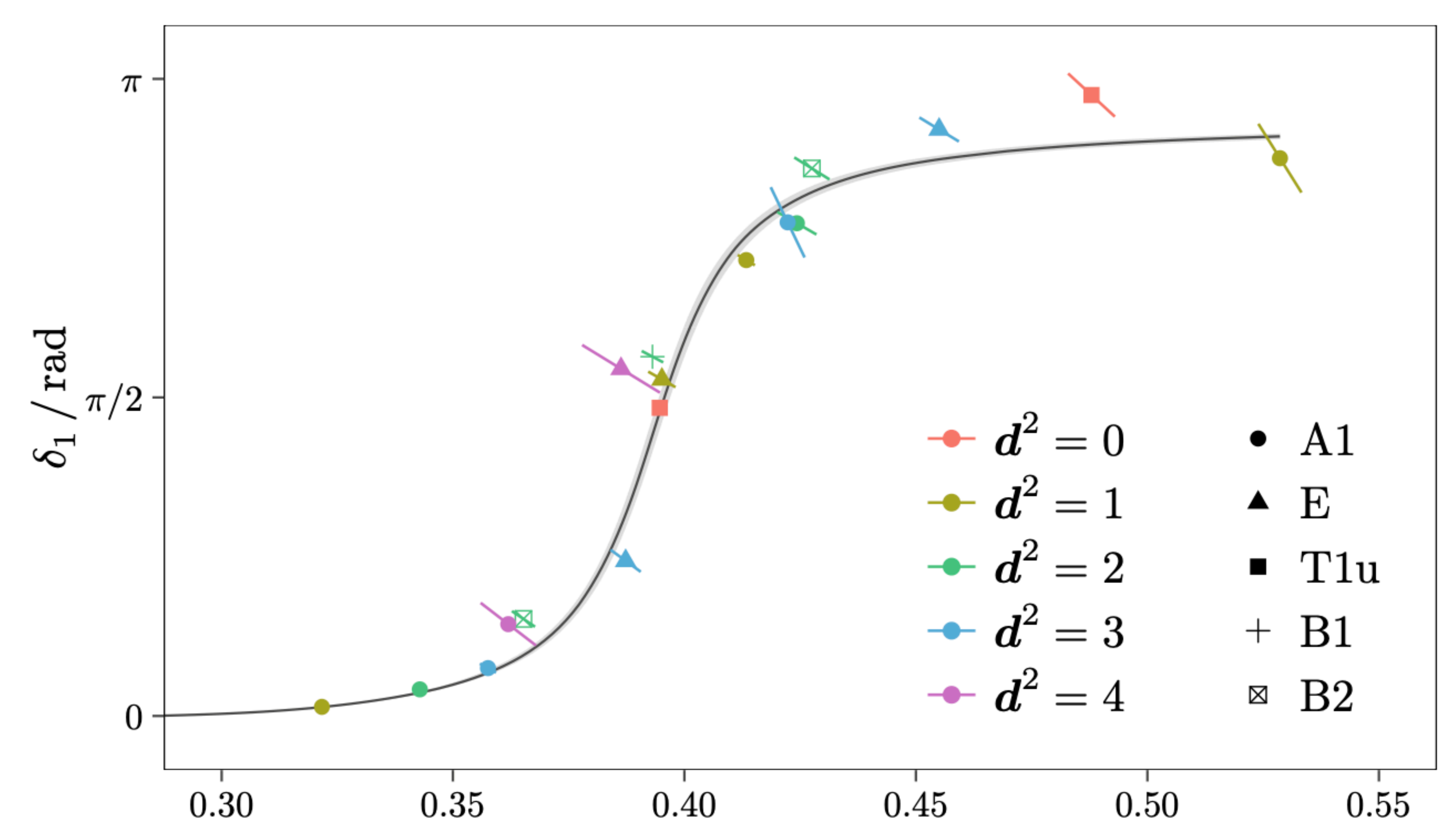
Finite volume spectrum

Infinite volume scattering parameters

Scattering on lattice

An example: ρ resonance $\rightarrow \pi\pi$ scattering





M. Werner et. al., Eur.Phys.J.A 56 (2020) 2, 61

Scattering on lattice

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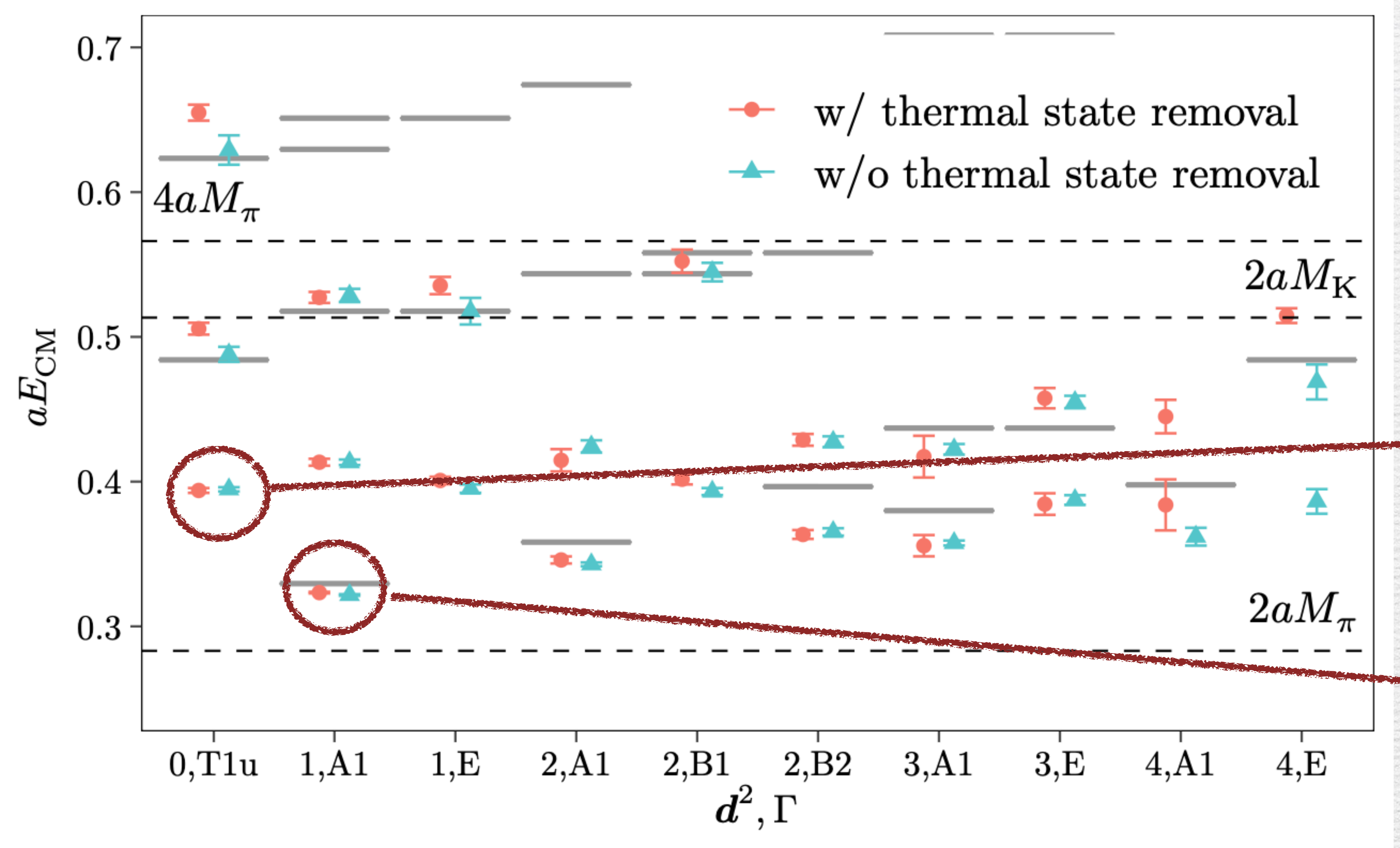
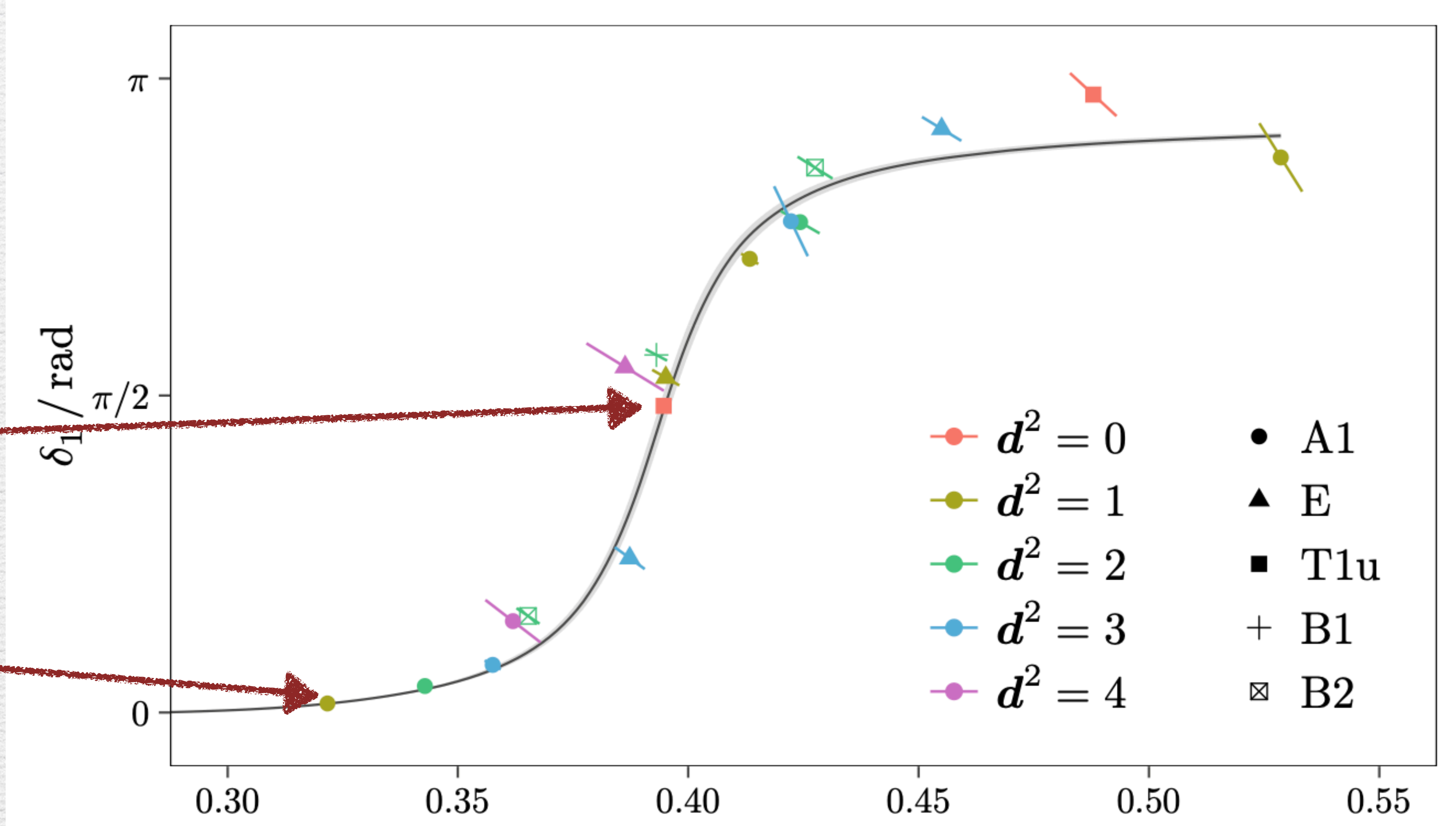
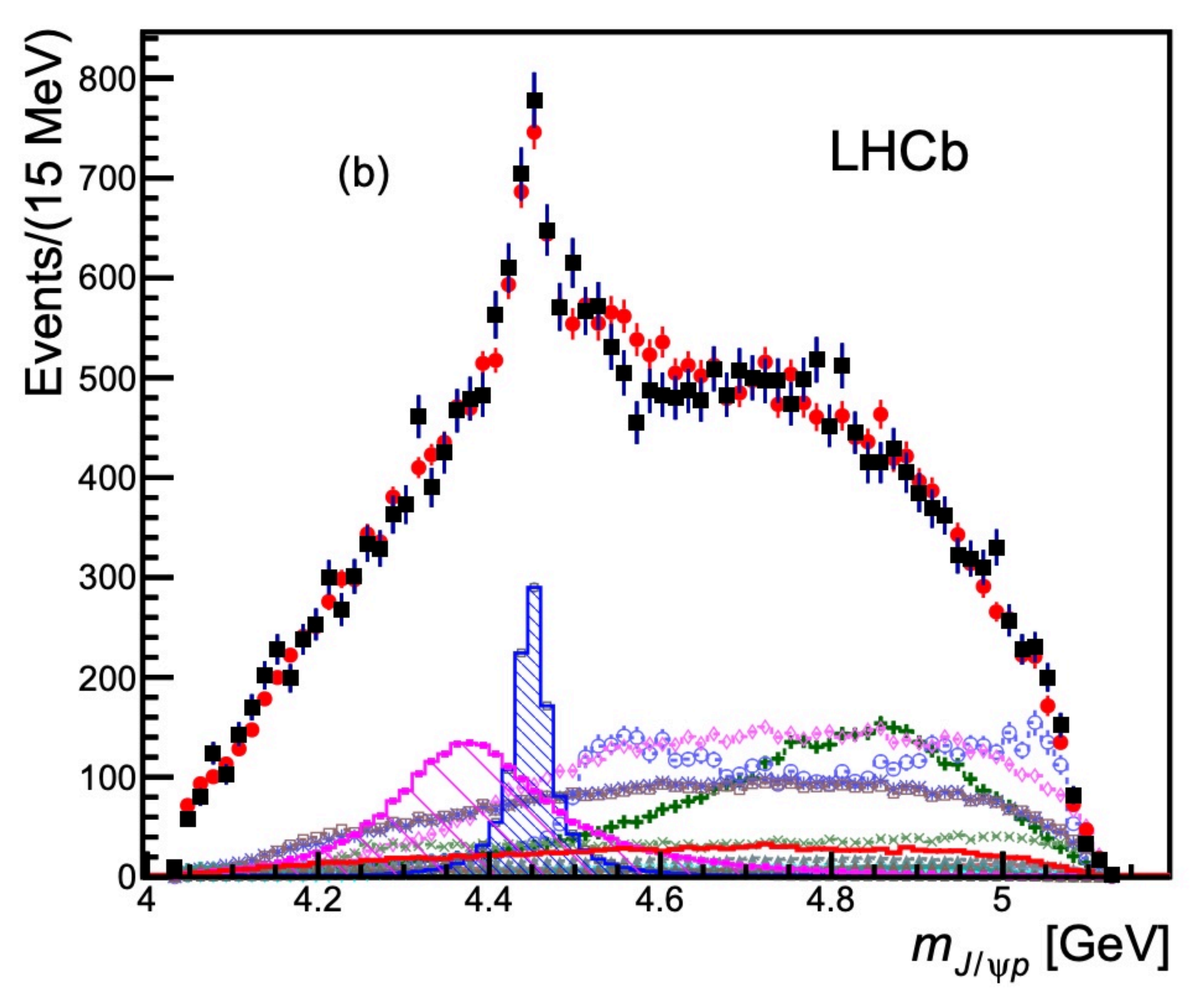
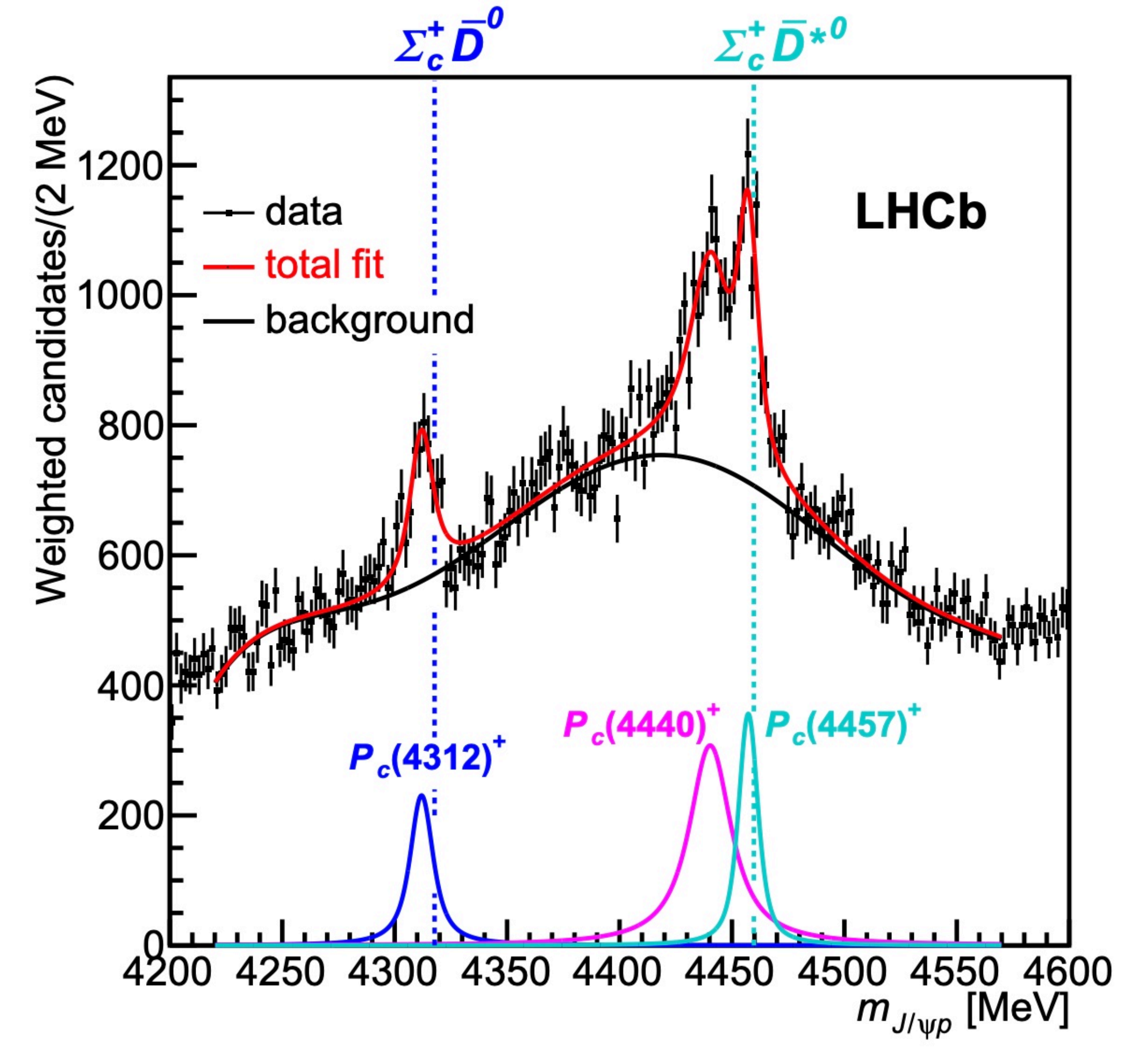




M. Werner et. al., Eur.Phys.J.A 56 (2020) 2, 61

P_c Pentaquarks





$P_c(4380)$ | $P_c(4312)$

$P_c(4450)$ → $P_c(4440)$, $P_c(4457)$

R. Aaij et al. (LHCb), Phys. Rev. Lett. 115, 072001 (2015)

R. Aaij et al. (LHCb), Phys. Rev. Lett. 122, 222001 (2019)

Theory interpretations:

- Molecular bound states
- Compact pentaquark states
- Hadrocharmonium
- ……

$\Sigma_c^{(*)}\bar{D}^{(*)}$ moleculars:

$$\Sigma_c\bar{D}, J^P = \frac{1}{2}^-, P_c(4312)$$

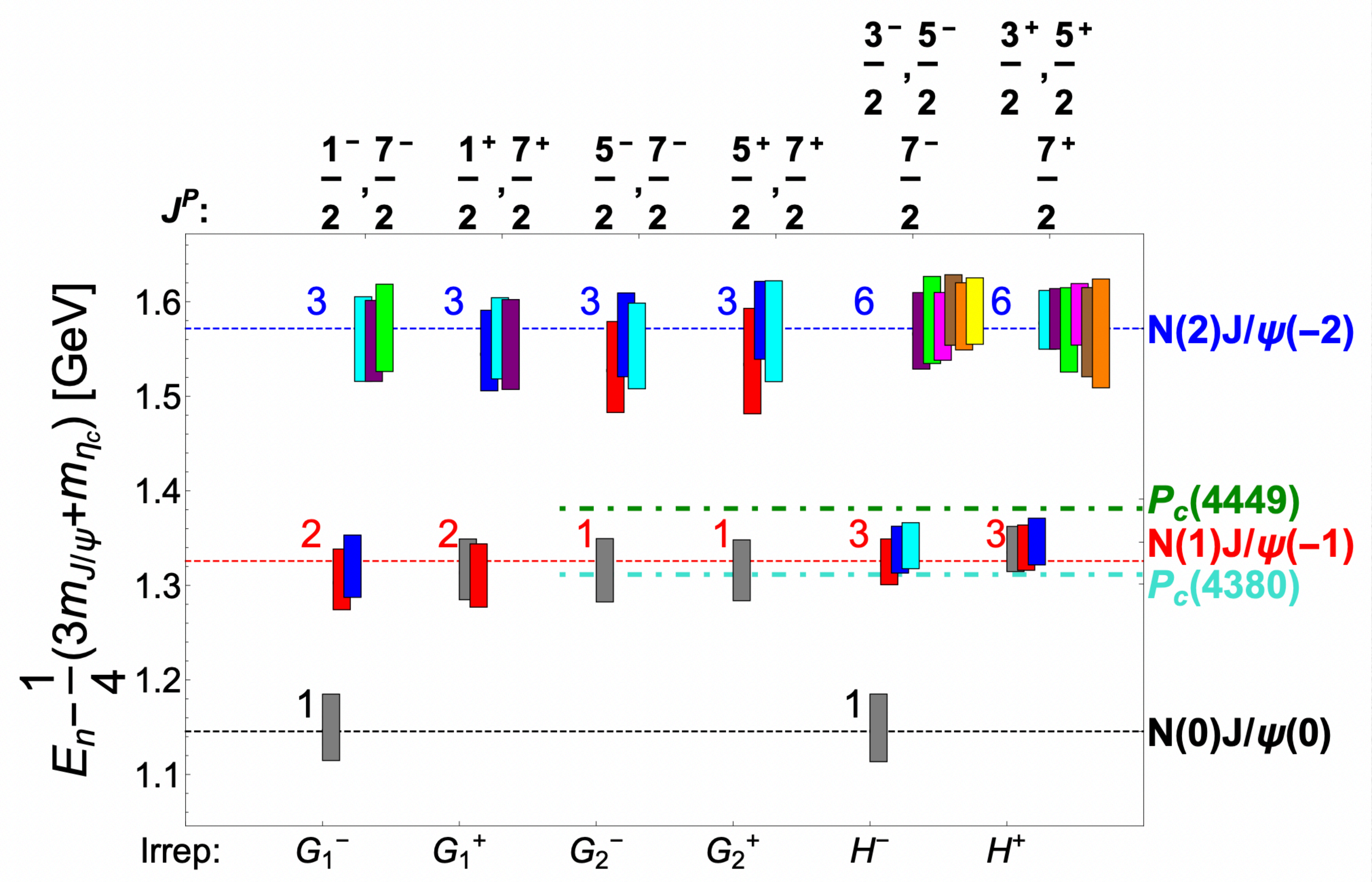
$$\Sigma_c\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-), P_c(4312)/P_c(4457)$$

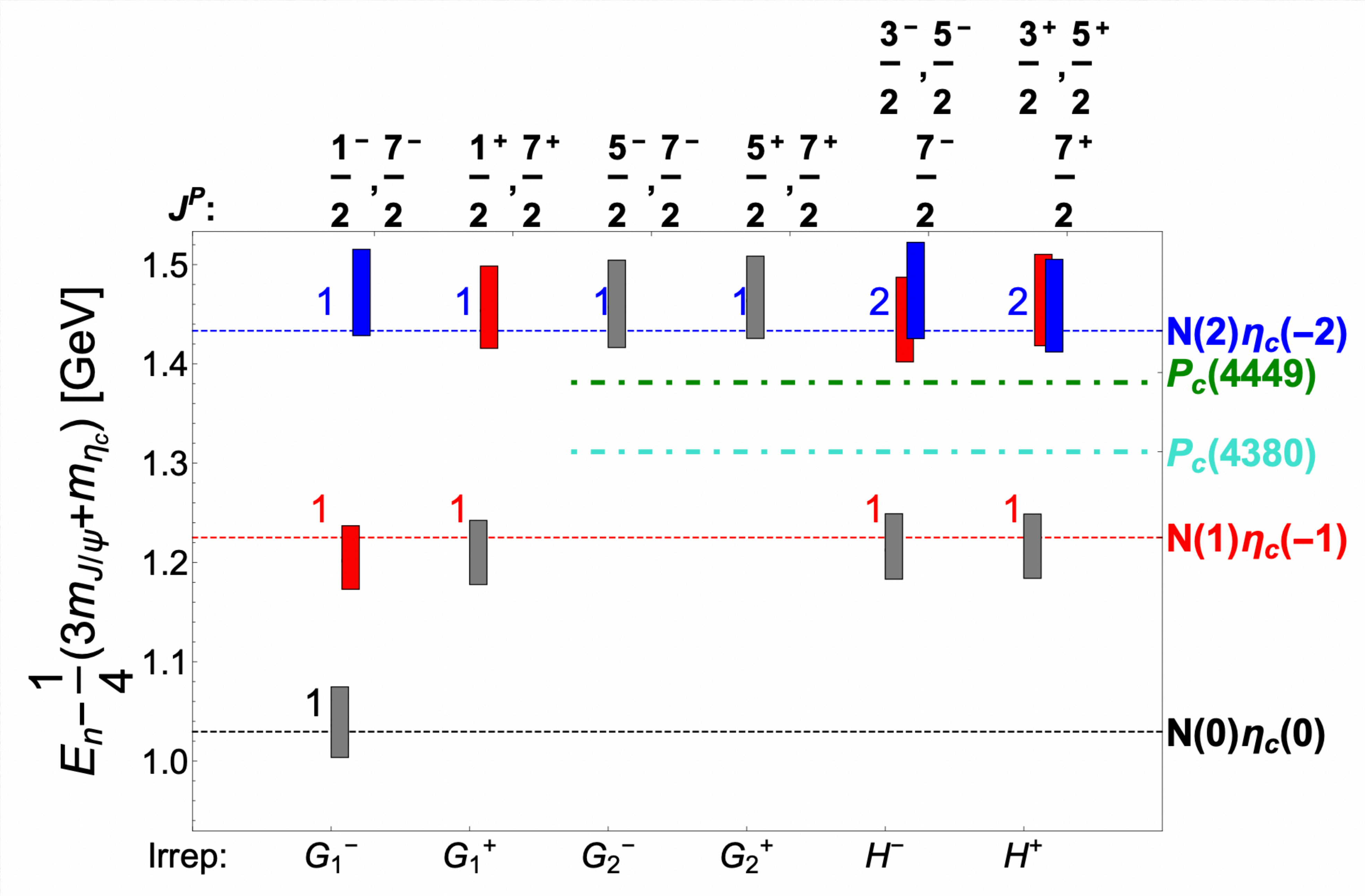
$$\Sigma_c^*\bar{D}, J^P = \frac{3}{2}^-, \quad \Sigma_c^*\bar{D}^*, J^P = (\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-),$$

P_c Pentaquarks on lattice

U. Skerbis and S. Prelovsek, Phys. Rev. D 99, 094505(2019)

$N - J/\psi$ and $N - \eta_c$ scattering:





★ Found no strong evidence for the P_c states.

P_c Pentaquarks on lattice

H. Xing, J. Liang, L. Liu, P. Sun and Y.-B. Yang, arXiv:2210.08555

$\Sigma_c \bar{D}$ and $\Sigma_c \bar{D}^*$ scattering$(J^P = \frac{1}{2}^-)$:

- $a = 0.08\,fm,\ m_\pi \sim 294$ MeV
- Five operators:

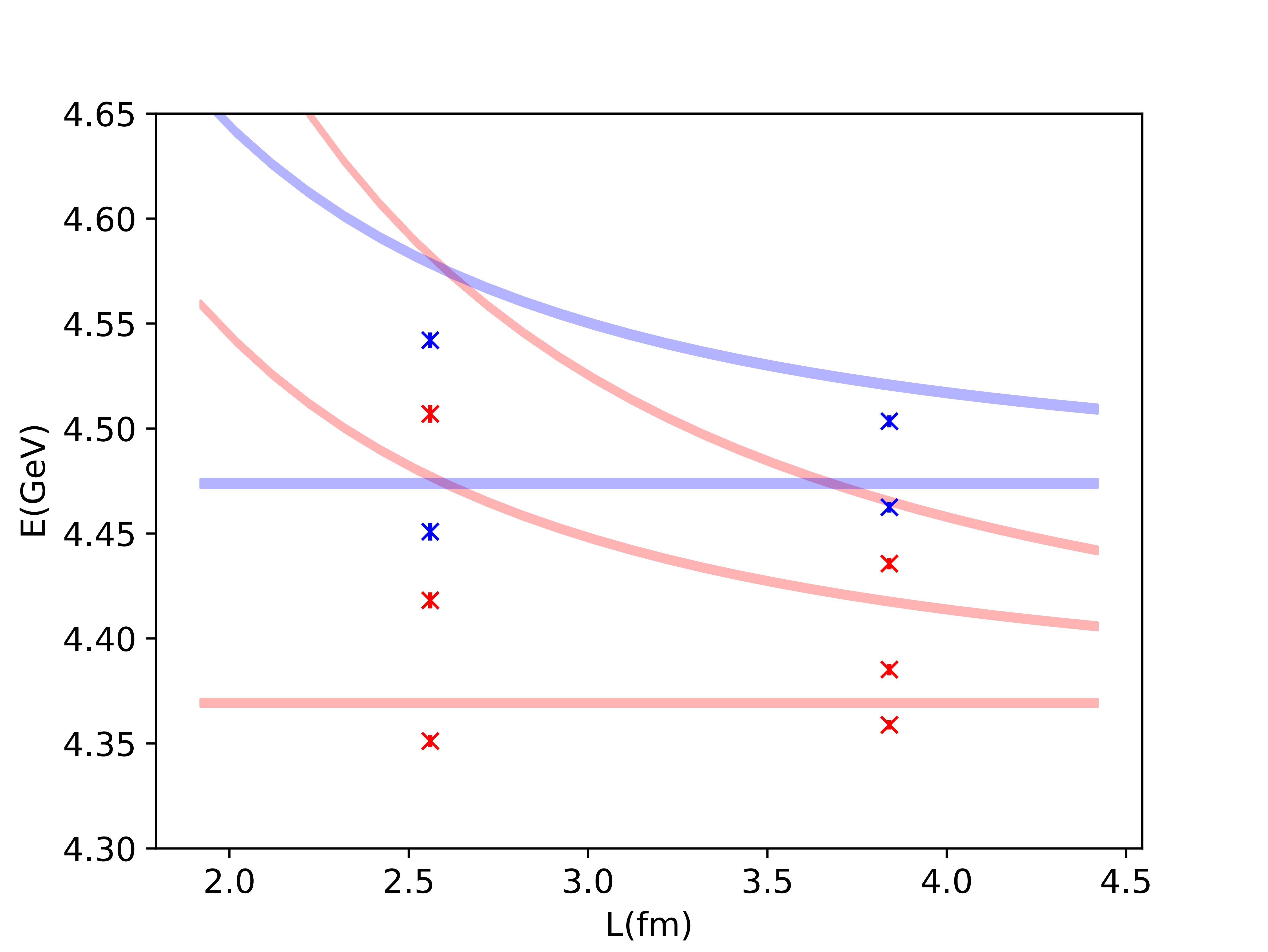
  $\Sigma_c \bar{D}(|p| = 0)$

  $\Sigma_c \bar{D}(|p| = 1)$

  $\Sigma_c \bar{D}(|p| = \sqrt{2})$

  $\Sigma_c \bar{D}^*(|p| = 0)$

  $\Sigma_c \bar{D}^*(|p| = 1)$



P_c Pentaquarks on lattice

Scattering amplitude:

$$T \sim \frac{1}{p\cot\delta - ip}$$

Bound state pole:

$$p = i|p_B|$$

Effective range expansion:

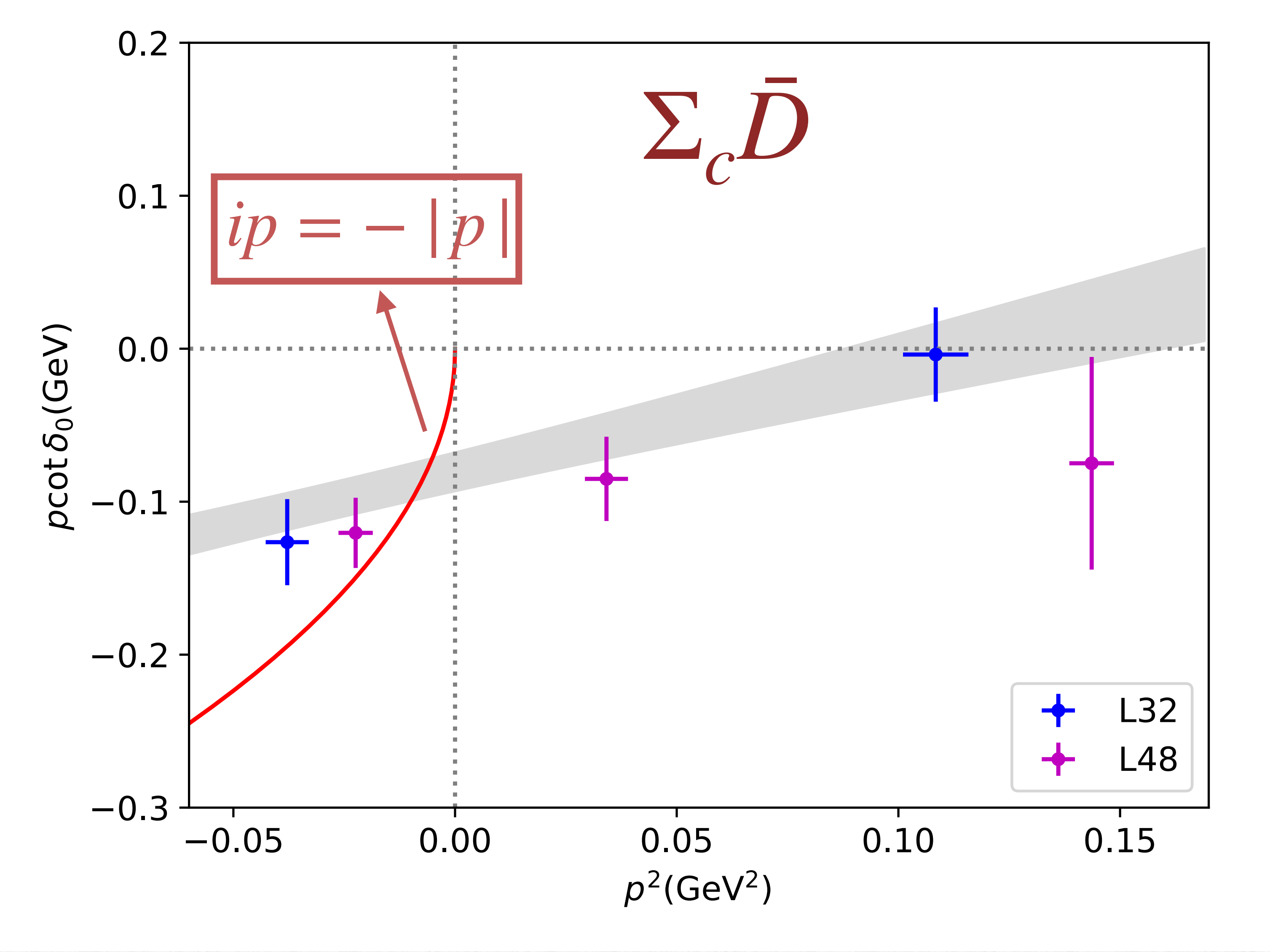
$$p\cot\delta(p) = \frac{1}{a_0} + \frac{1}{2} r_0 p^2 + \cdots$$

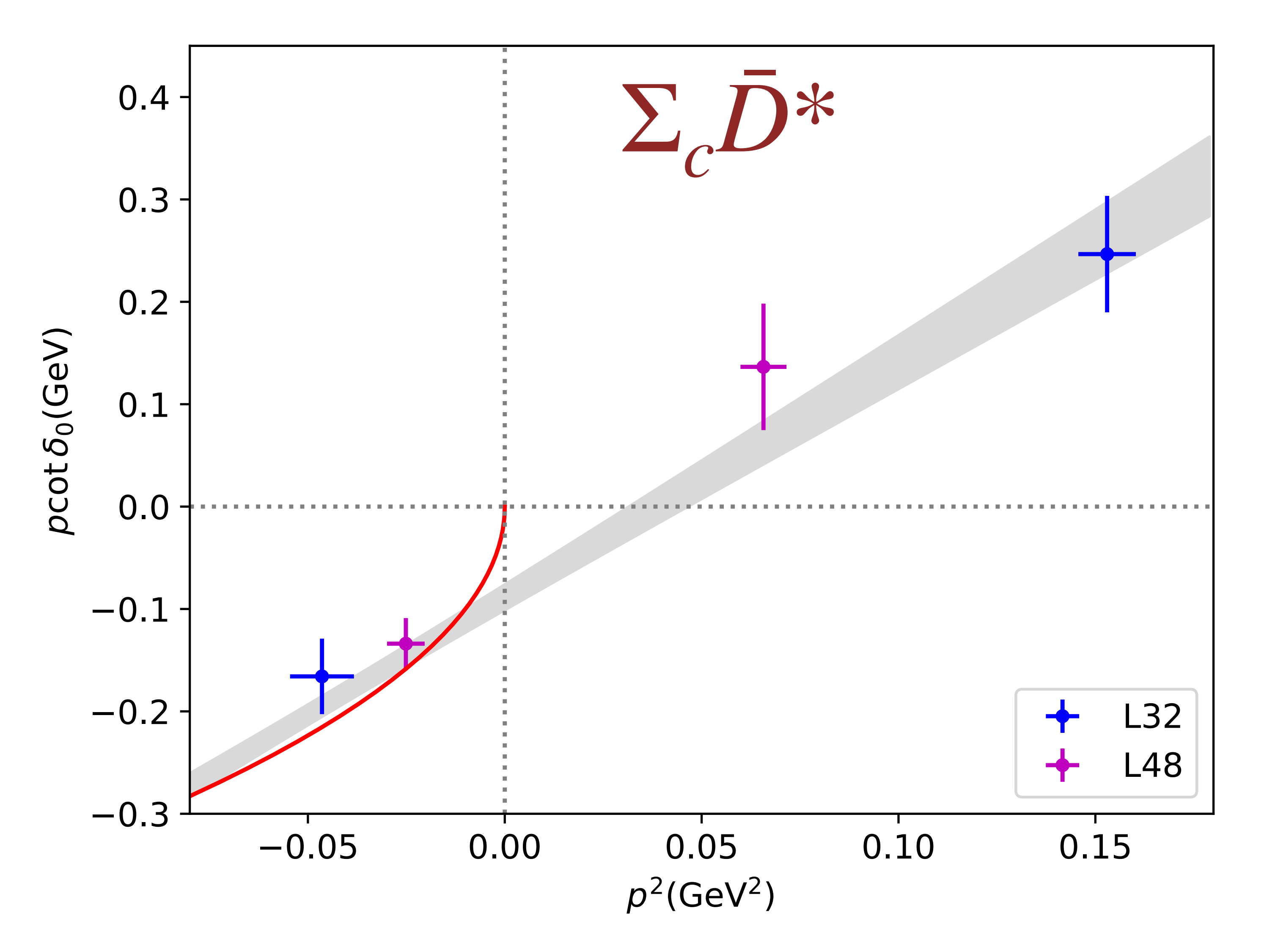
$\Sigma_c\bar{D}$: $P_c(4312)$?
$a_0 = -2.0(3)(5)\text{fm}$
$E_B = 6(2)(2)\text{MeV}$

Luscher's formula:

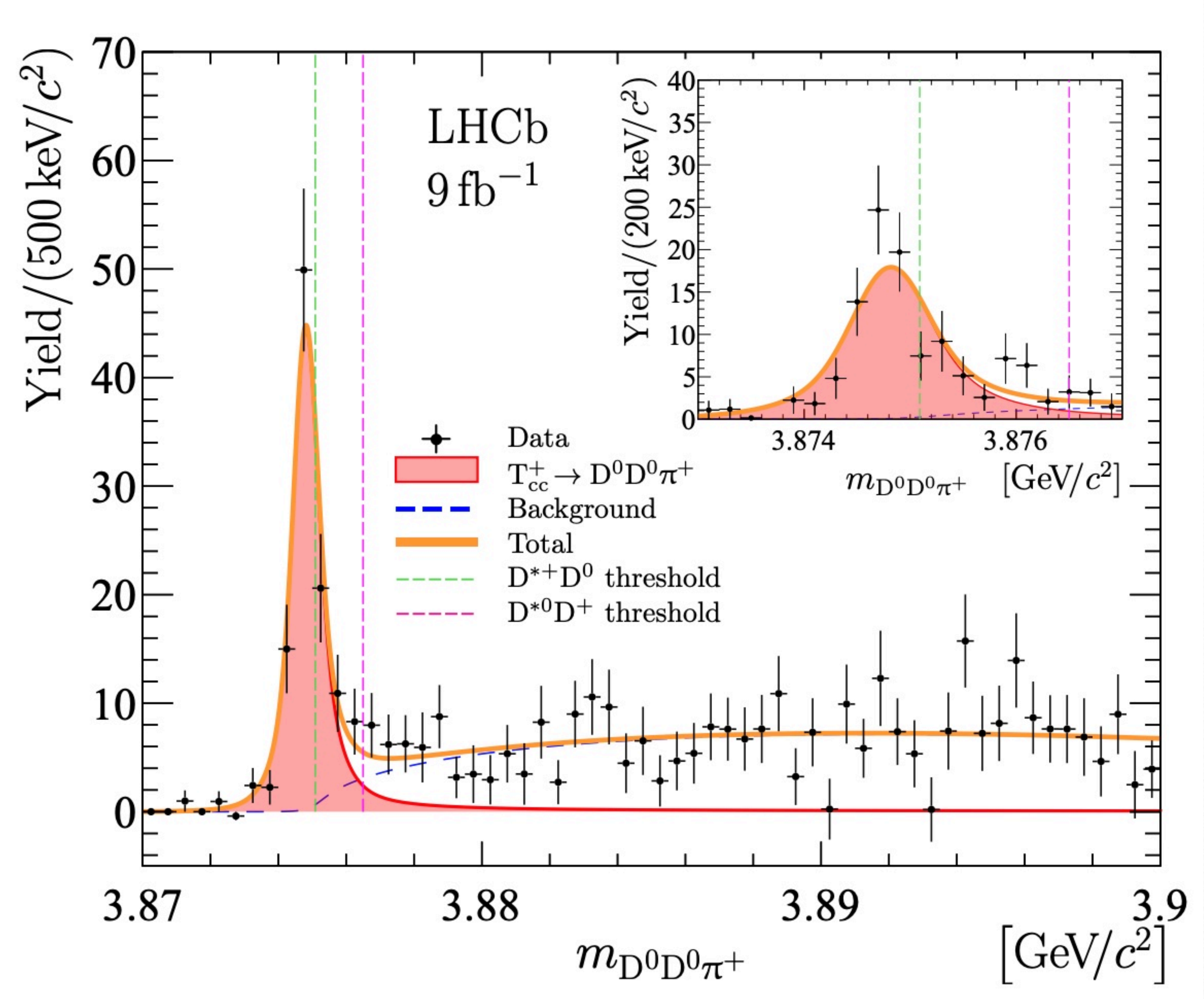
$$p\cot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$\Sigma_c\bar{D}^*$: $P_c(4440)/P_c(4457)$?
$a_0 = -2.3(5)(1)\text{fm}$
$E_B = 7(3)(1)\text{MeV}$





T_{cc}



$$\delta m = M_{T_{cc}^+} - (M_{D^{*+}} + M_{D^0})$$

$$= -361 \pm 40(\text{keV})$$

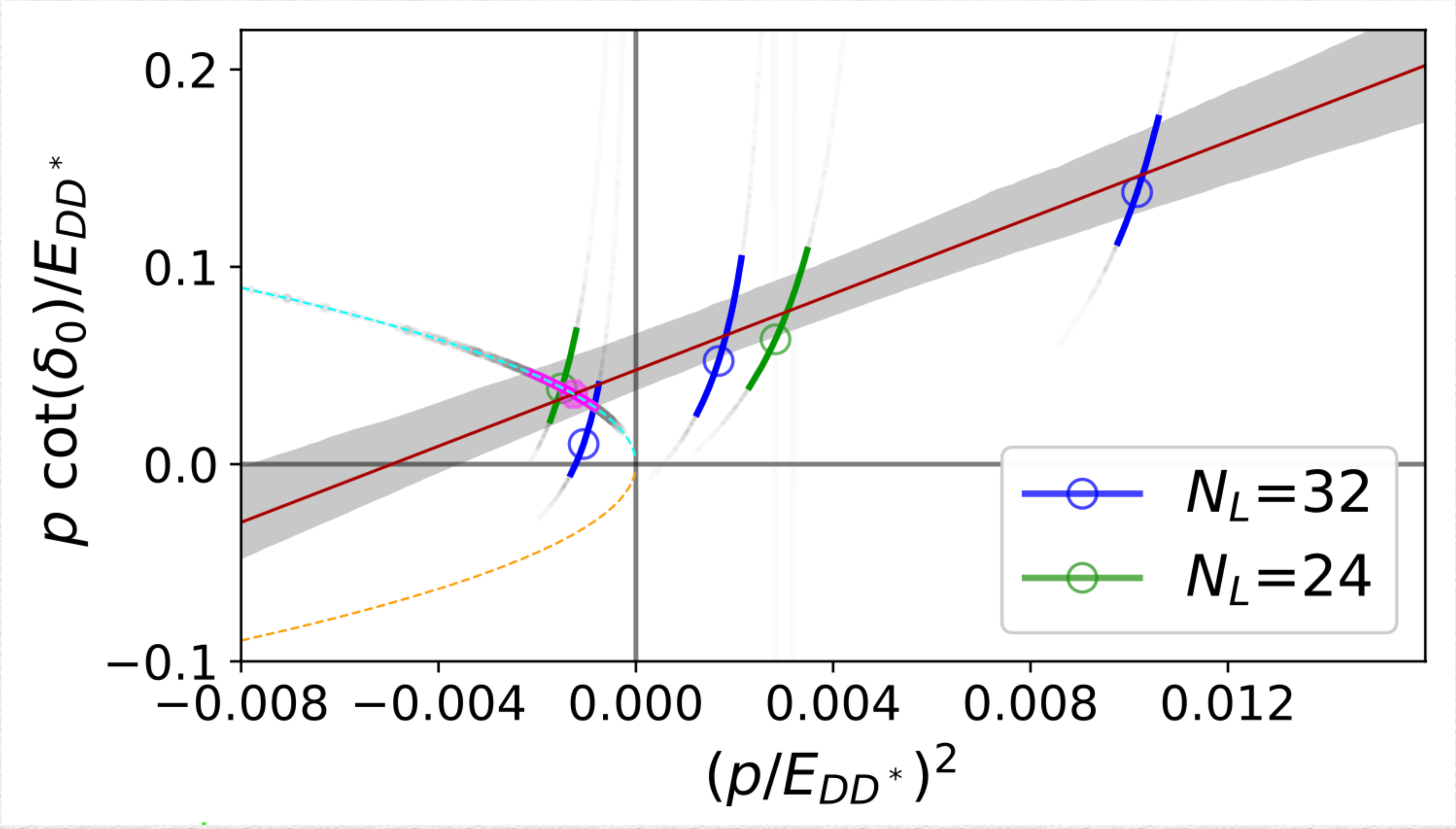
$$\Gamma = 47.8 \pm 1.9(\text{keV})$$

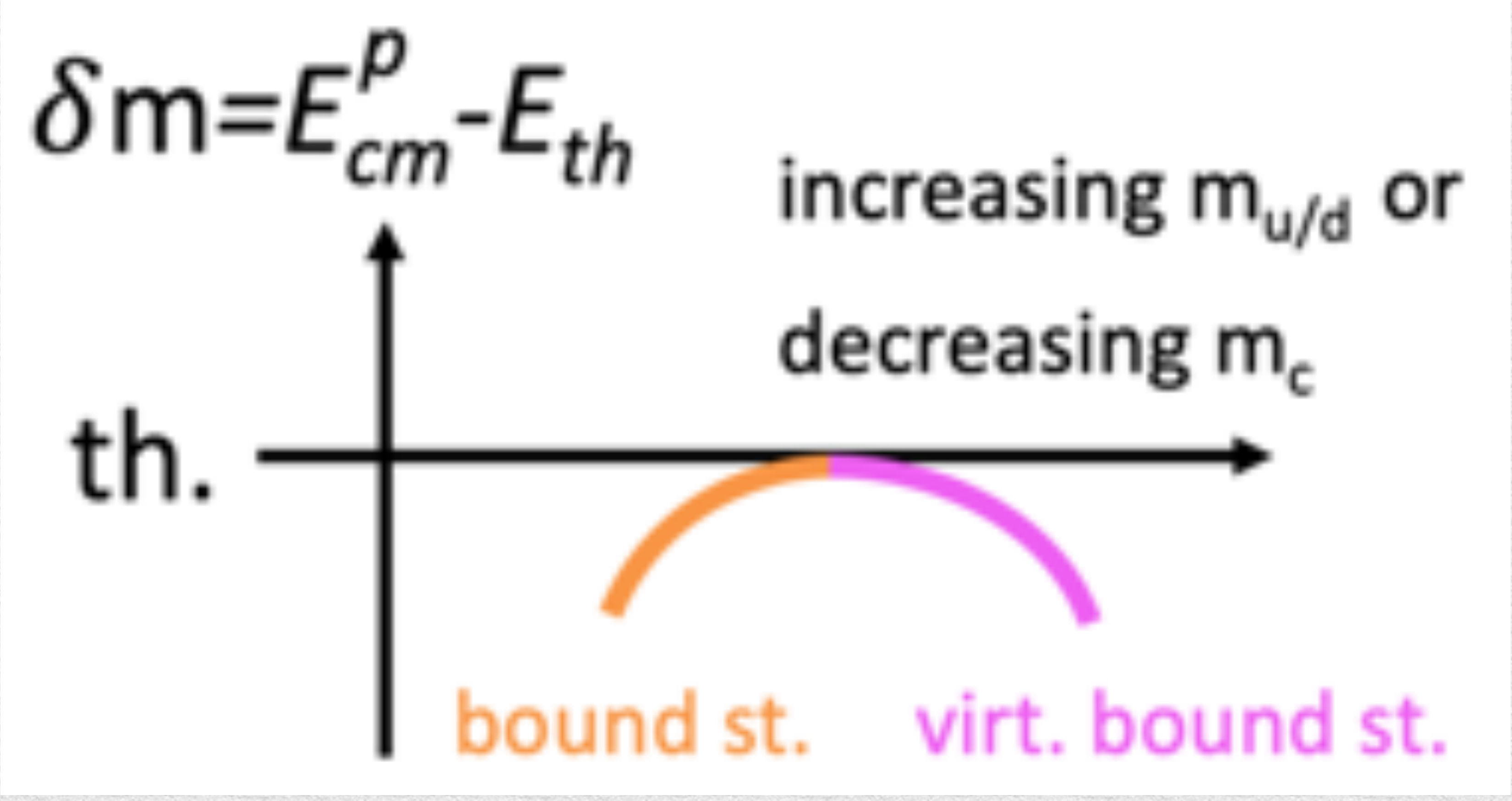
LHCb collaboration, R. Aaij et al., *Nature Phys*. 18 (2022) 7, 751-754.

LHCb collaboration, R. Aaij et al., Nature Communicatitions, 13, 3351 (2022)

T_{cc} on lattice

M. Padmanath and S. Prelovsek, Phys.Rev.Lett. 129 (2022) 3, 032002

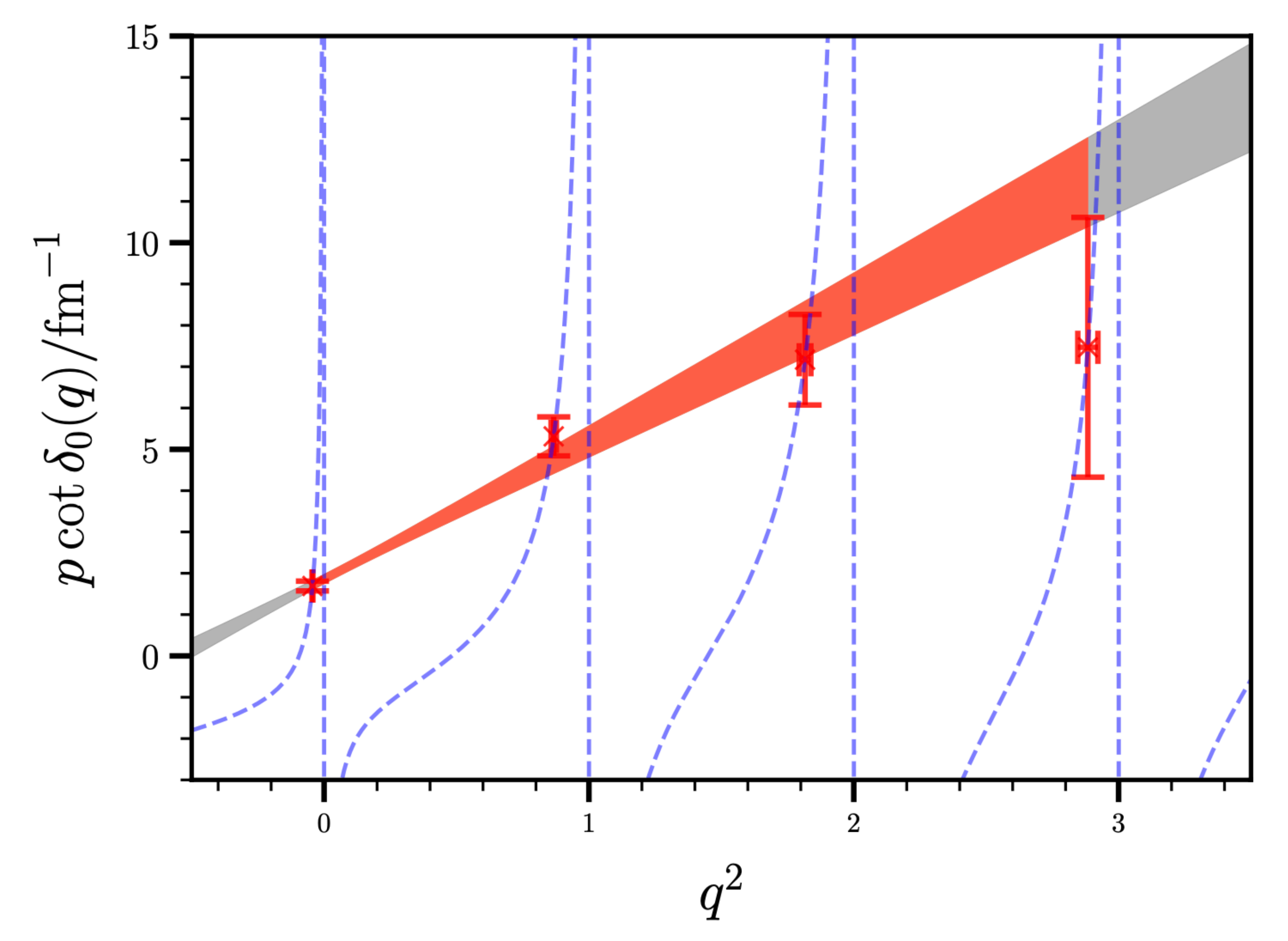




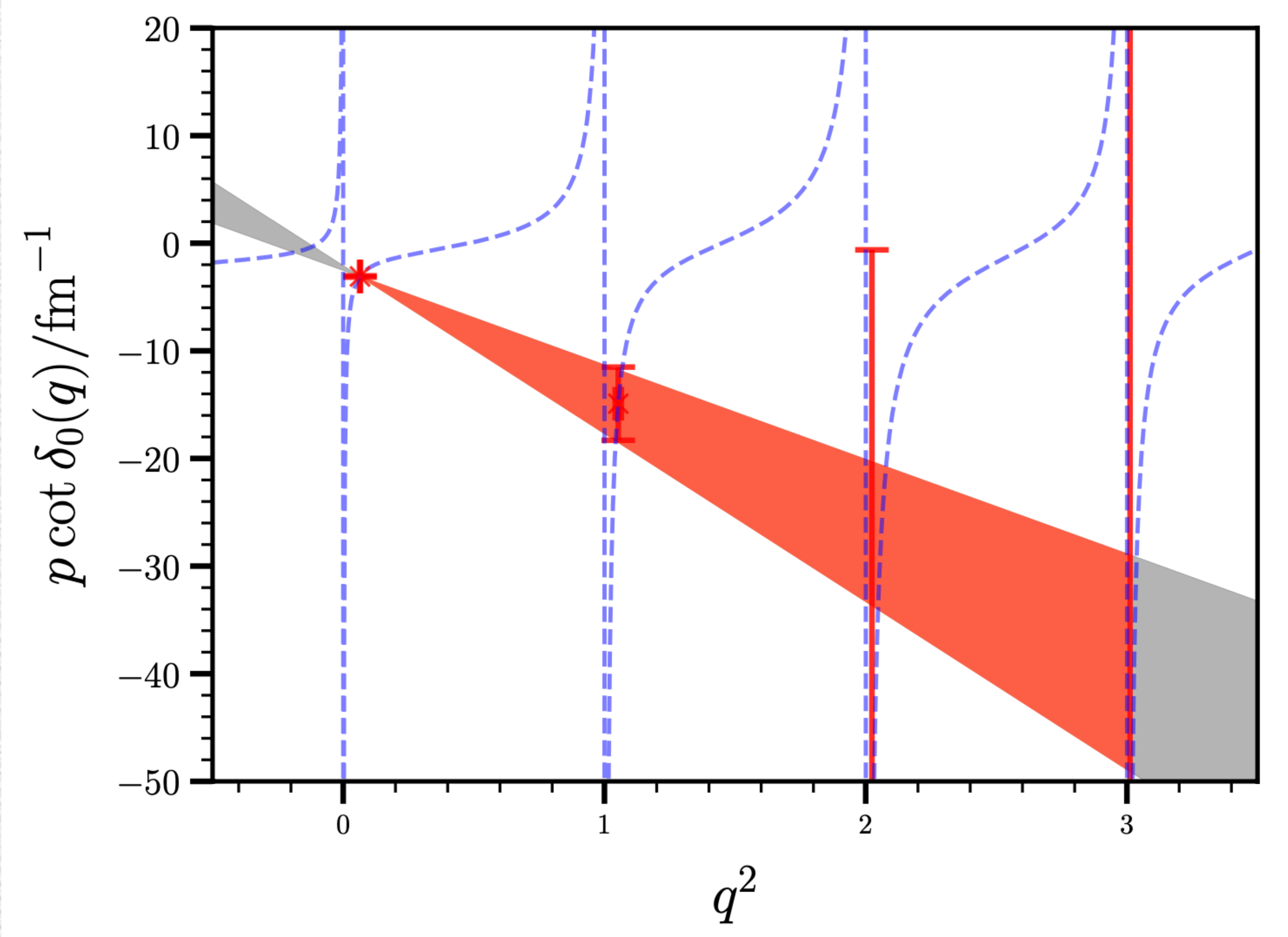
- $a = 0.086\, fm,\ m_\pi \sim 280$ MeV
- Finite-volume energy eigenvalues are extracted using *DD** interpolating operatros in rest and moving frames.
- A virtual bound state pole is found in the scattering amplitude.
- Quark mass dependence of the binding energy and pole position.

T_{cc} on lattice

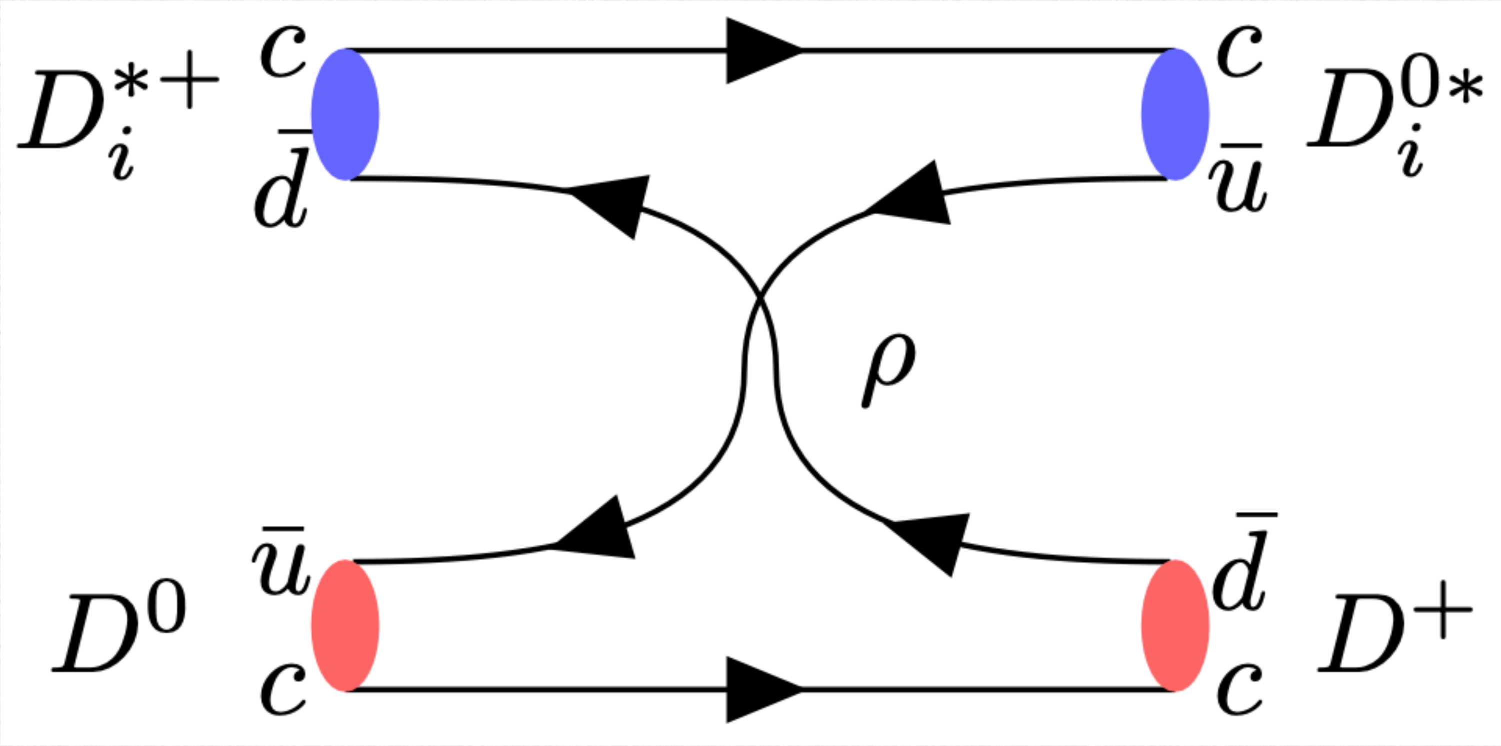
Siyang Chen, et.al (CLQCD), arXiv:2206.06185



I=0
Attractive



I=1
Repulsive



The attraction in I=0 channel mainly comes from the charged ρ exchange.

Summary

- Lattice QCD can provide information on the hadronic resonance and exotics from first principles.
- Preliminary lattice results supports the interpretation of the P_c states as $\Sigma_c \bar{D}^{(*)}$ bound states.
- Lattice results indicate a (virtual) bound state in DD^* scattering, which might corresponds to the tetraquark T_{cc} .
- Further improvements are needed: including couple channel effects, smaller lattice spacing, physical light quark mass…