$K \to \mu^+\mu^-$ as a probe for New Physics

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Implications of LHCb measurements and future prospects

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$K \to \mu^+\mu^-$ is exciting!
A lot of recent activity

Theory:

- D'Ambrosio Kitahara, 1707.06999
- Dery Ghosh Grossman StS, 2104.06427
- Buras Venturini, 2109.11032
- Dery Ghosh, 2112.05801
- Brod Stamou, 2209.07445
- Dery Ghosh Grossman Kitahara StS, 22xx.SOON

Experiment:

- LHCb, 2001.10354: Upper limit on $K_S \to \mu^+\mu^-$
- LHCb, KAON'22: Upper limits on $K_{S,L} \to 2(\mu^+\mu^-)$

$K \to \mu^+\mu^-$ is exciting!

[D'Ambrosio Kitahara 1707.06999, Dery Ghosh Grossman StS, 2104.06427]

The new idea

- We can very cleanly measure $\mathrm{Im}(V_{td}^* V_{ts})$ (or η) from $K \to \mu^+\mu^-$.
- We can do so employing time-dependent interference effects.

- Third golden channel alongside:
  $K^+ \to \pi^+ \nu\bar{\nu}$ gives $|V_{td}V_{ts}|$ **NA62**
  $K_L \to \pi^0 \nu\bar{\nu}$ gives $\mathrm{Im}(V_{td}^* V_{ts})$ (or η) **KOTO**

- Determine the unitarity triangle purely with kaon decays.
  ↳Crucial intergenerational consistency check of the SM.

- New ways to probe for new physics.

The three golden channels

$K^+ \to \pi^+ \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$

"Theoretically clean, experimentally hard"

$K \to \mu^+\mu^-$, common lore

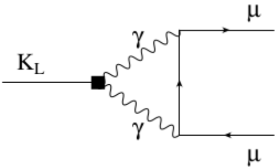
"Theoretically **not** clean, experimentally not hard."

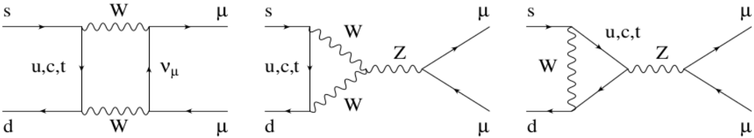
$K \to \mu^+\mu^-$, this talk

"Theoretically **clean**, experimentally hard."

- Only hadronic uncertainty from f_K.
- Challenging to measure time-dependent interference effects.

Long-Distance and Short-Distance Physics





[Isidori Unterdorfer hep-ph/0311084]

Can we overcome soft QCD?

- To get a theoretically clean method we need a theory error of $\lesssim 1\%$.
- We are currently not able to achieve theory precision of long distance (LD) effects in $K \to \mu^+\mu^-$ below $\sim 10\%$.
- We know short-distance (SD) physics at desired precision.
- How can we measure the SD physics?

Basics of $K \to \mu^+\mu^-$

Approximation

- In this talk we neglect CP-violating in mixing ε_K.
  ➥Can be incorporated into analysis [Brod Stamou 2209.07445].

Angular momentum conservation: Only $(\mu\mu)_{l=0}$ or $(\mu\mu)_{l=1}$

- CP-conserving decays

K_L	$\to$	$(\mu\mu)_{l=0}$		K_S	$\to$	$(\mu\mu)_{l=1}$
CP-odd		CP-odd		CP-even		CP-even

- CP-violating decays

K_S	$\to$	$(\mu\mu)_{l=0}$		K_L	$\to$	$(\mu\mu)_{l=1}$
CP-even		CP-odd		CP-odd		CP-even

CP of muons: $(-1)^{l+1}$.

$K \to \mu^+\mu^-$ in the Standard Model

To good approximation:

- LD effects are CP conserving.
  ↳CP violating amplitudes are purely SD.

Short-distance (SD) and long-distance (LD) physics

- CP-conserving decays: SD and LD

K_L	$\to$	$(\mu\mu)_{l=0}$		K_S	$\to$	$(\mu\mu)_{l=1}$
CP-odd		CP-odd		CP-even		CP-even

- CP-violating decays: Only SD

K_S	$\to$	$(\mu\mu)_{l=0}$		K_L	$\to$	$(\mu\mu)_{l=1}$
CP-even		CP-odd		CP-odd		CP-even

$K \to \mu^+\mu^-$ in the Standard Model

- SM: SD operator $(\bar{\mu}\gamma^\mu\mu_L)$ does **not** generate $(\mu\mu)_{l=1}$ state (CPT).

Short-distance (SD) and long-distance (LD) physics

- CP-conserving decays: SD and LD

$$K_L \to (\mu\mu)_{l=0} \qquad K_S \to (\mu\mu)_{l=1}$$

CP-odd CP-odd CP-even CP-even

- CP-violating decays: Only SD

$$K_S \to (\mu\mu)_{l=0} \qquad K_L \to (\mu\mu)_{l=1}$$

CP-even CP-odd CP-odd CP-even

$$= \mathbf{0}$$

Counting of Theory Parameters

- CP-conserving decays: SD and LD

$$|A(K_L \to (\mu\mu)_{l=0})| \qquad |A(K_S \to (\mu\mu)_{l=1})|$$

CP-odd CP-odd CP-even CP-even

- CP-violating decays: Only SD

$$|A(K_S \to (\mu\mu)_{l=0})| \qquad |A(K_L \to (\mu\mu)_{l=1})| = \mathbf{0}$$

CP-even CP-odd CP-odd CP-even

- Phases

$$\varphi_0 \equiv \arg\left(\mathcal{A}^*(K_S \to (\mu\mu)_{l=0})\mathcal{A}(K_L \to (\mu\mu)_{l=0})\right)$$
$$\varphi_1 \equiv \arg\left(\mathcal{A}^*(K_S \to (\mu\mu)_{l=1})\mathcal{A}(K_L \to (\mu\mu)_{l=1})\right) = \mathbf{0}$$

- A priori: 6 parameters: 4 magnitudes, 2 phases.
- In SM/large class of NP models: Reduction to 4.
- 1 of which is pure SD.

$K_S \to (\mu\mu)_{l=0}$ is the key to SD physics

- We can cleanly calculate it in the SM.

$$\mathcal{B}(K_S \to (\mu\mu)_{l=0}) = 1.9 \cdot 10^{-13} \times \left(\frac{A^2 \lambda^5 \eta}{1.3 \times 10^{-4}} \right)$$

[Isidori Unterdorfer hep-ph/0311084, Dumm Pich hep-ph/9801298]

- Hadronic uncertainties from $f_K < 1\%$.

- Way to extract η theoretically clean.

- We can also calculate $\mathcal{B}(K_S \to (\mu\mu)_{l=0})$ cleanly in NP models.

In practice we measure incoherent sum

- Muon states with specific angular momentum $(\mu\mu)_{l=0}$ and $(\mu\mu)_{l=1}$: Not available to us: We cannot separate $l = 0$ and $l = 1$.

- Instead, we measure the incoherent sum:

$$\Gamma(K_S \to \mu^+\mu^-)_{\text{meas.}} = \Gamma(K_S \to (\mu^+\mu^-)_{l=0}) + \Gamma(K_S \to (\mu^+\mu^-)_{l=1})$$

$$\Gamma(K_L \to \mu^+\mu^-)_{\text{meas.}} = \Gamma(K_S \to (\mu^+\mu^-)_{l=0}) + \Gamma(K_S \to (\mu^+\mu^-)_{l=1})$$

$\Rightarrow$ “So what are you talking about?”

Solution: Look at time dependence

- Generic time dependence of K decay:

$$\left(\frac{d\Gamma}{dt}\right) \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\left(C_{sin}\sin(\Delta m t) + C_{cos}\cos(\Delta m t)\right) e^{-\Gamma t}$$

- $\Gamma = (\Gamma_S + \Gamma_L)/2$. Δm: Kaon mass difference.

- The 4 Cs are the observables:
  - C_L is related to K_L decay rate.
  - C_S is related to K_S decay rate.
  - C_{sin} and C_{cos} are due to interference.

- We can calculate the 4 Cs in terms of the 4 theoretical parameters.

We can completely solve the system.

- For pure K^0 beam:

$$\begin{aligned} C_L &= |A(K_L)_{l=0}|^2 \\ C_S &= |A(K_S)_{l=0}|^2 + |A(K_S)_{l=1}|^2 \\ C_{cos} &= \mathrm{Re}\left(A(K_S)_{l=0} \times A^*(K_L)_{l=0}\right) \\ C_{sin} &= \mathrm{Im}\left(A(K_S)_{l=0} \times A^*(K_L)_{l=0}\right) \end{aligned}$$

- We can get the clean amplitude from the observable combination

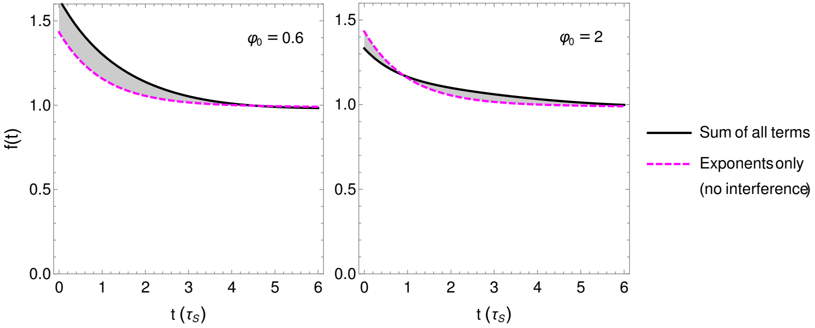
$$|A(K_S)_{l=0}|^2 = \frac{C_{cos}^2 + C_{sin}^2}{C_L}\,.$$

- We can rewrite this as:

$$\mathcal{B}(K_S \to (\mu^+\mu^-)_{l=0}) = \mathcal{B}(K_L \to \mu^+\mu^-) \times \frac{\tau_S}{\tau_L} \times \frac{C_{cos}^2 + C_{sin}^2}{C_L^2}$$

- Compare with calculation of $\mathcal{B}(K_S \to (\mu^+\mu^-)_{l=0}) \Rightarrow$ extract η.
- We need the interference terms!

Demonstration of Interference Effect



$$\left(\frac{d\Gamma}{dt}\right) \propto f(t) \equiv C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\left(C_{sin}\sin(\Delta m t) + C_{cos}\cos(\Delta m t)\right) e^{-\Gamma t}$$

- Using estimates, not showing large hadronic uncertainties for long-distance contributions.
- As examples, two ad-hoc values for the phase.
- All parameters can be determined from experiment.

Experimental Considerations

- Experimentally, not easy to have pure K^0 or $\overline{K}^0$ beam.

- NA62: charged kaons. KOTO: pure K_L. LHCb: almost equal mix.
  ↳In these limits no sensitivity to interference term.
  ↳Employ mixed beam. Need non-zero production asymmetry.
    - Regeneration of K_S in K_L beam through matter effects.
    - Charged exchange targets: turn charged K^+ beams into K^0 beams.
    - Post-selection using tagging (?)

- Interference terms are then diluted by dilution factor D:

$$D = \frac{N_{K^0} - N_{\overline{K}^0}}{N_{K^0} + N_{\overline{K}^0}} \quad C_{cos} \mapsto DC_{cos} \qquad C_{sin} \mapsto DC_{sin}$$

  N_K: Number of incoherent mixture of kaons/anti-kaons at $t = 0$.

- Pure $K^0/\overline{K}^0$: $D = \pm 1$.

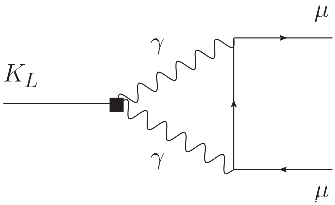
A Precision relation between $\Gamma(K \to \mu^+\mu^-)(t)$ and $\mathcal{B}(K_L \to \gamma\gamma)/\mathcal{B}(K_L \to \mu^+\mu^-)$

[Dery Ghosh Grossman Kitahara StS 22xx.SOON: preliminary results]

- We know more about $\varphi_0 \equiv \arg\left(\mathcal{A}^*(K_S \to (\mu\mu)_{l=0})\mathcal{A}(K_L \to (\mu\mu)_{l=0})\right)$

We find the precision relation

$$\cos^2\varphi_0 = (\text{known QED factor}) \times \frac{\mathcal{B}(K_L \to \gamma\gamma)}{\mathcal{B}(K_L \to \mu^+\mu^-)}$$



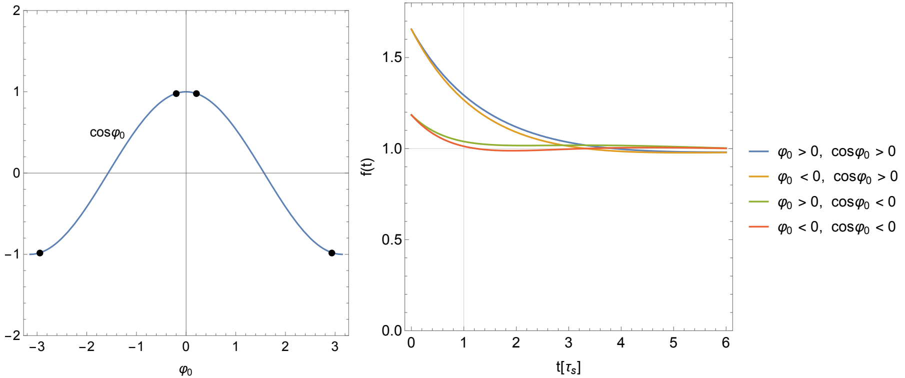
- Assume other intermediate on-shell contributions $(3\pi, \pi\pi\gamma)$ negligible.

[Martin De Rafael Smith 1970]

- Preliminary result: $\cos^2\phi_0 = 0.96 \pm 0.02$.

Discrete Ambiguities

[Dery Ghosh Grossman Kitahara StS 22xx.SOON: preliminary results]



$$\left(\frac{d\Gamma}{dt}\right) \propto f(t) \equiv C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + 2\left(C_{sin}\sin(\Delta m t) + C_{cos}\cos(\Delta m t)\right) e^{-\Gamma t}$$

Resolving discrete ambiguities: Work in progress.

↳ Need sign of $A(K_L \to \gamma\gamma)$ either from theory or experiment.

How much room is there for NP?

[Dery Ghosh 2112.05801]

- 2020 measurement of LHCb [LHCb, 2001.10354]

$$\mathcal{B}(K_S \to \mu^+\mu^-) < 2.1 \cdot 10^{-10}$$

- Sum of contributions with different CP (no interference):

$$\mathcal{B}(K_S \to \mu^+\mu^-) = \mathcal{B}(K_S \to \mu^+\mu^-)^{(\mathrm{LD})}_{\mathrm{CPC}} + \mathcal{B}(K_S \to \mu^+\mu^-)^{(\mathrm{SD})}_{\mathrm{CPV}}$$

- Conservative interpretation: set LD contribution = 0
  ⇔ i.e. interpret bound as bound on SD alone.
- ⇒ A lot of room for NP in the SD amplitude:

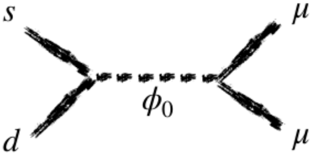
$$R(K_S \to \mu^+\mu^-)_{l=0} \equiv \frac{\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}}{\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}^{\mathrm{SM}}} \leq 1280\,.$$

How much room is there for NP?

[Dery Ghosh 2112.05801]

- Scalar Leptoquarks:

- 2HDM:



- Both can saturate bound, consistent with existing constraints.
- Updated bounds from LHCb important to constrain the model space further.

[Diagrams courtesy Avital Dery]

Another very clean SM test

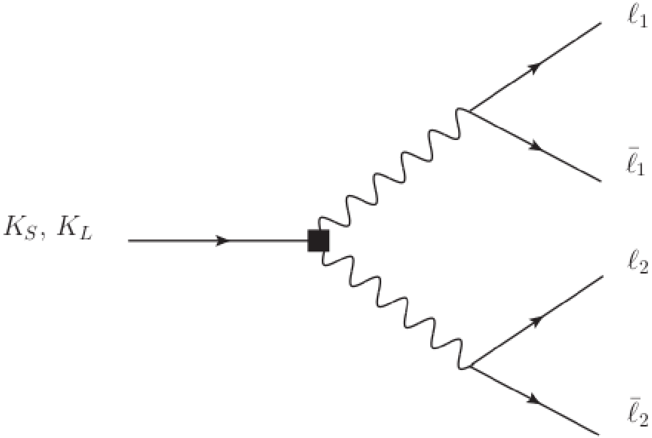
[Buras Venturini 2109.11032]

- Combination of $K_S \to \mu^+\mu^-$ and $K_L \to \pi^0\nu\bar{\nu}$:

$$\frac{\mathcal{B}(K_S \to \mu^+\mu^-)_{l=0}}{\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})} = 1.55 \times 10^{-2} \left(\frac{\lambda}{0.225}\right)^2 \left(\frac{Y(x_t)}{X(x_t)}\right)^2$$

- Depends only on Wolfenstein-λ ($|V_{us}|$) and m_t.
- Does not depend on $|V_{cb}|$.
- $K \to \mu^+\mu^-$ and $K_L \to \pi^0\nu\bar{\nu}$ are sensitive to different NP operators.

$K \to \mu^+\mu^-\mu^+\mu^-$



[D'Ambrosio Greynat Vulvert 1309.5736]

$K \to \mu^+\mu^-\mu^+\mu^-$

Experiment [LHCb, Kaon'22]

$$\mathcal{B}^{\text{EXP}}(K_S \to \mu^+\mu^-\mu^+\mu^-) \leq 5.1 \cdot 10^{-12}$$
$$\mathcal{B}^{\text{EXP}}(K_L \to \mu^+\mu^-\mu^+\mu^-) \leq 2.3 \cdot 10^{-9}$$

Theory: SM [D'Ambrosio Greynat Vulvert 1309.5736]

$$\mathcal{B}^{\text{SM}}(K_S \to \mu^+\mu^-\mu^+\mu^-) \lesssim O(4 \cdot 10^{-14})$$
$$\mathcal{B}^{\text{SM}}(K_L \to \mu^+\mu^-\mu^+\mu^-) \lesssim O(1 \cdot 10^{-12})$$

- Time-dependent interference effects determine $sgn\,(\mathcal{A}(K_L \to \gamma\gamma))$
  ➥Related to discrete ambiguities of φ_0.

NP Models with Massive Dark Photons [white paper 2201.07805]

$$\mathcal{B}^{\text{NP}}(K_S \to \text{2 dark photons} \to \mu^+\mu^-\mu^+\mu^-) \simeq 2.2 \cdot 10^{-12}$$
$$\mathcal{B}^{\text{NP}}(K_L \to \text{2 dark photons} \to \mu^+\mu^-\mu^+\mu^-) \simeq 2.5 \cdot 10^{-10}$$

Almost Conclusion

- Within the SM, time-dependent $K \to \mu^+\mu^-$ gives the same information as $K_L \to \pi^0 \nu\bar{\nu}$.
- They have complementary NP sensitivity: Combination can be used to distinguish models.
- "Theoretically clean, experimentally hard": Can we do it?
- Expect even more improved measurements of $\mathcal{B}(K_S \to \mu^+\mu^-)$ and $\mathcal{B}(K_{S,L} \to \mu^+\mu^-\mu^+\mu^-)$ at LHCb. $\Rightarrow$ Constrain or discover BSM physics.

What can LHCb do?

- Can you get a beam that is not equal mix of K^0 , $\overline{K}^0$?

$$\left(\frac{d\Gamma}{dt}\right) \propto C_L e^{-\Gamma_L t} + C_S e^{-\Gamma_S t} + \frac{N_{K^0} - N_{\overline{K}^0}}{N_{K^0} + N_{\overline{K}^0}} \cdot 2\left(C_{sin}\sin(\Delta m t) + C_{cos}\cos(\Delta m t)\right) e^{-\Gamma t}$$

- Measuring C_{sin}, C_{cos} challenged by $\sigma(pp \to K^0 X) \simeq \sigma(pp \to \overline{K}^0 X)$.
- Can it be done with tagging?
- E.g. tag accompanying particles in: [D'Ambrosio Kitahara 1707.06999]

$$pp \to K^0 K^- X$$
$$pp \to K^0 \Lambda^0 X$$
$$pp \to K^{*+} X \to K^0 \pi^+ X \qquad \text{(similar to } D^{*+} \to D^0 \pi^+\text{)}$$

- Do any jet machine learning ideas help?