



# Magnifying the gravitational-wave Universe

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[ezquiaga.github.io](https://ezquiaga.github.io)

VILLUM FONDEN

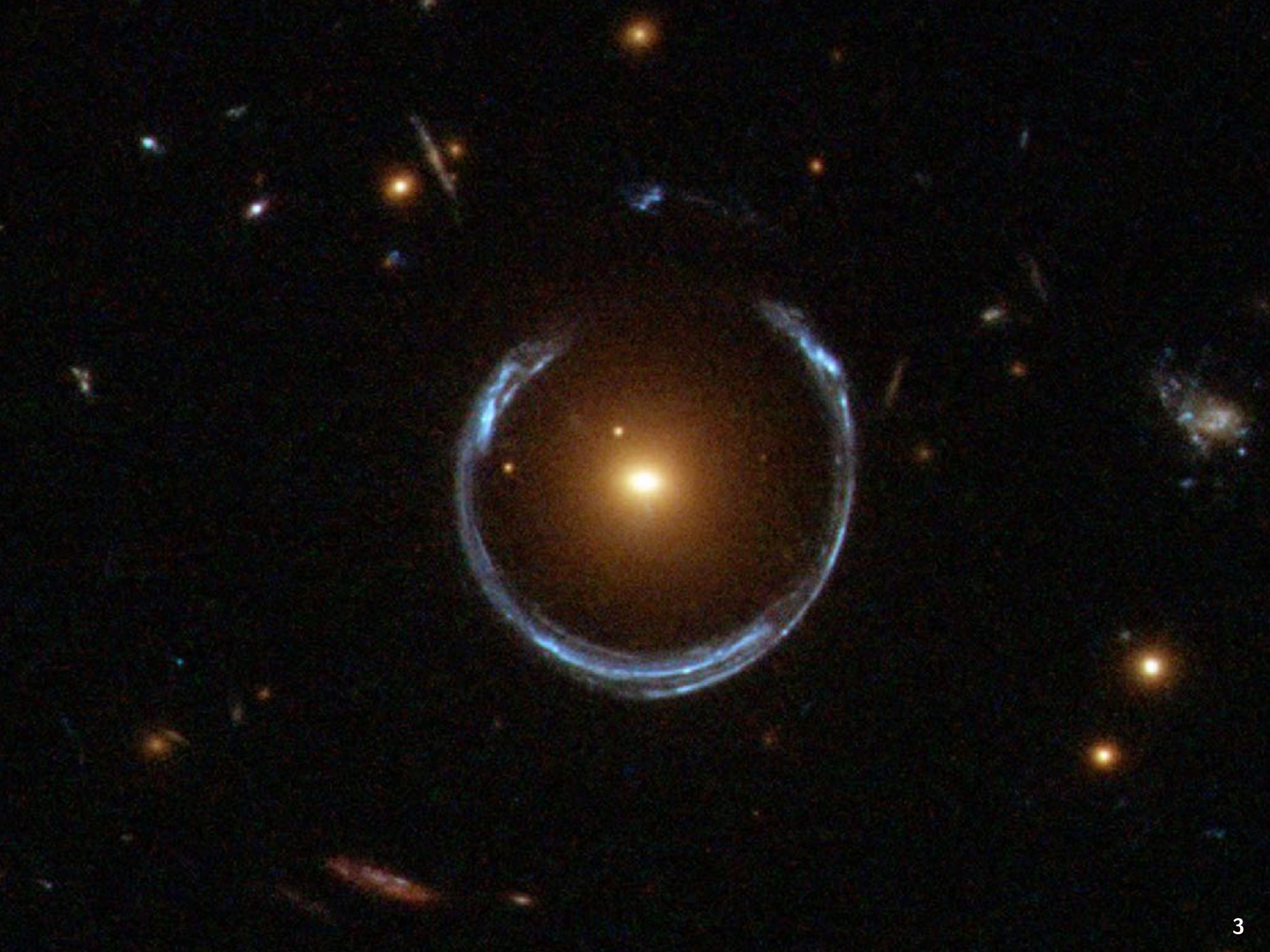


KØBENHAVNS  
UNIVERSITET

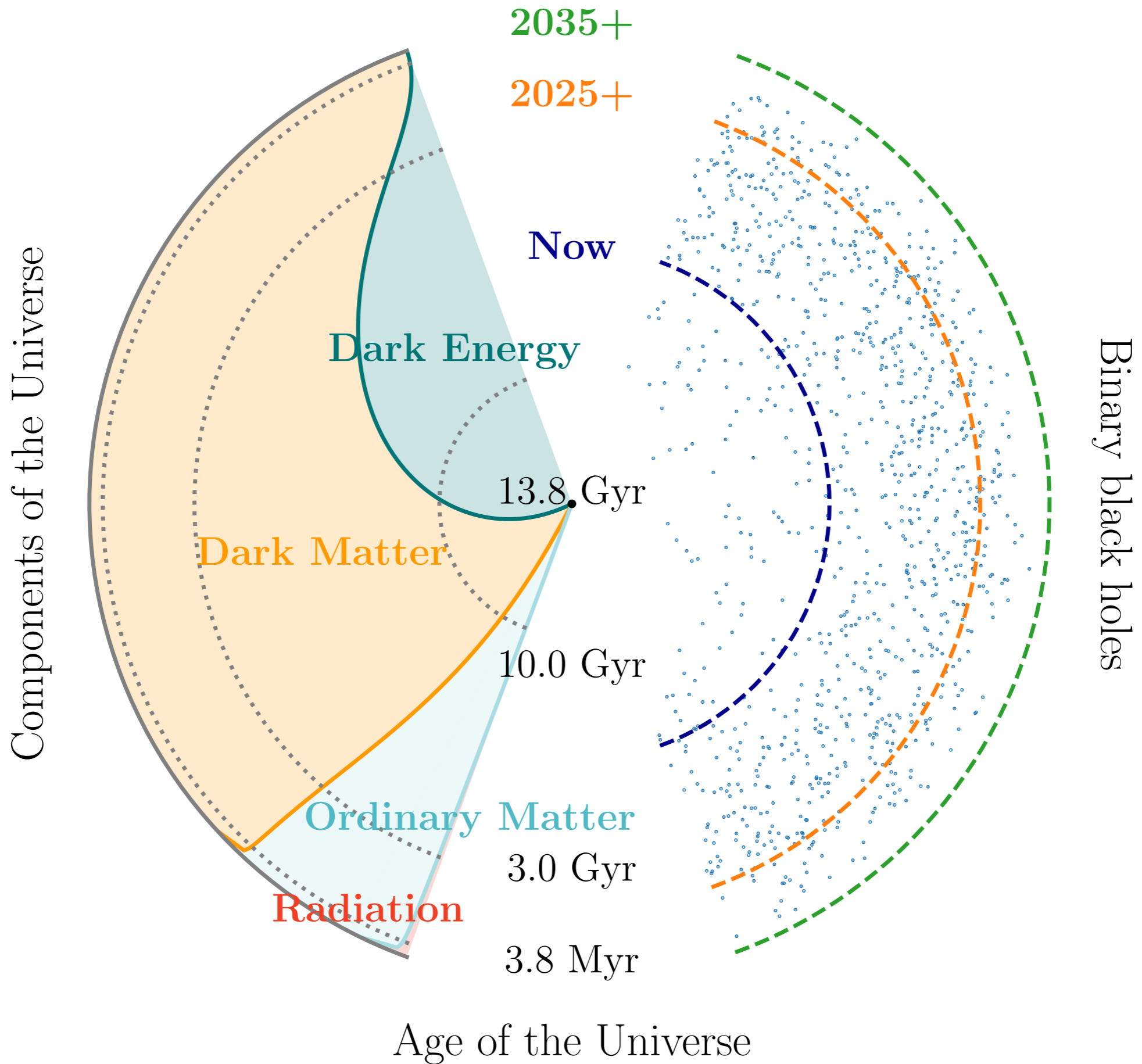
[Gustave Vidal, 1930]



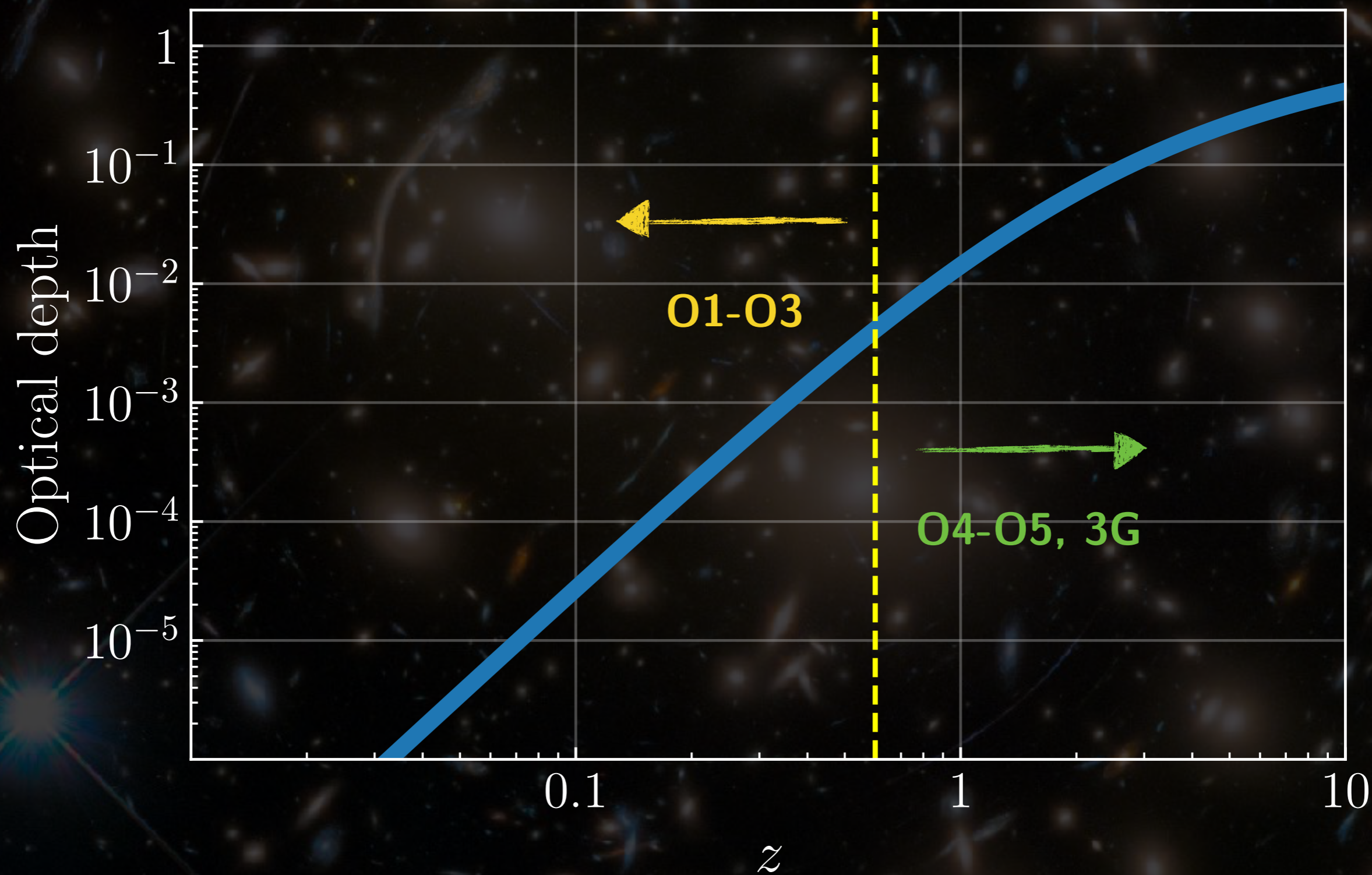
[JWST Deep field]



# Gravitational Wave horizons



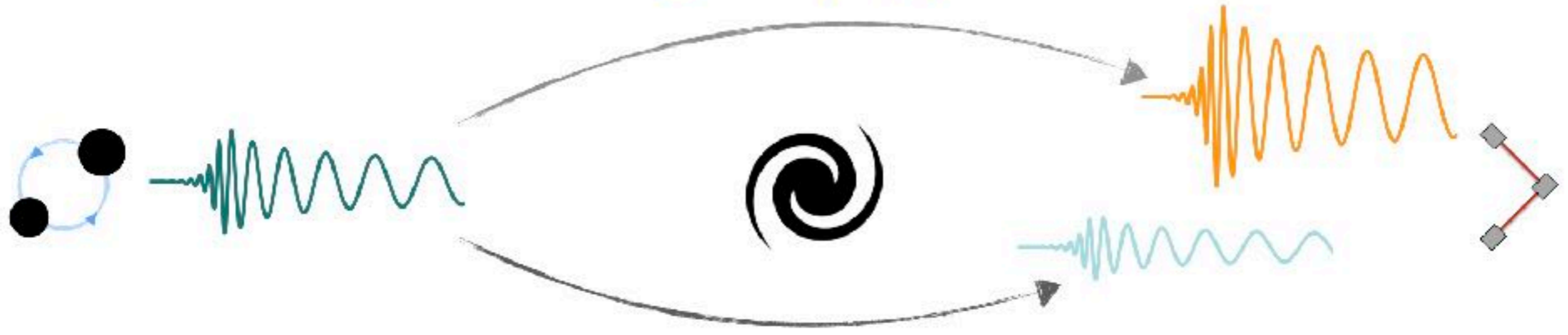
# Gravitational lensing



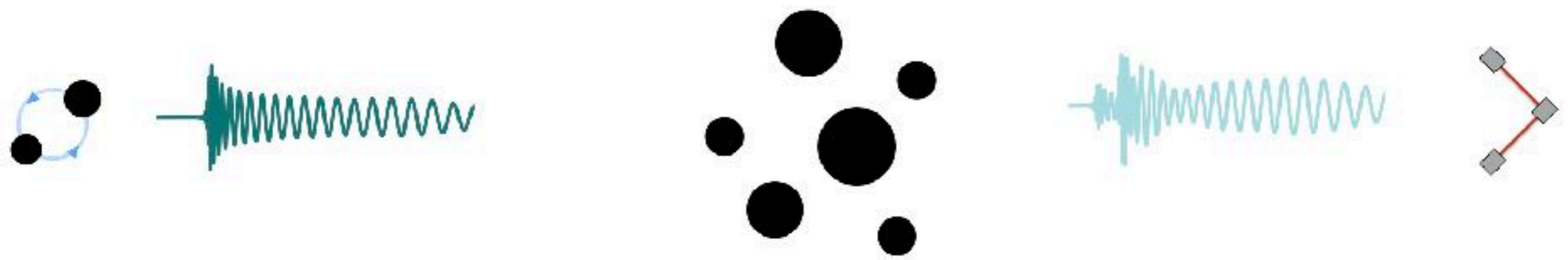
# Gravitational lensing of gravitational waves

- Clean signals, not affected by the medium
- All-sky coverage, high redshifts, large wavelengths
- Well understood selection effects of detectors

## Strong lensing by galaxies



## Interference effects by compact lenses

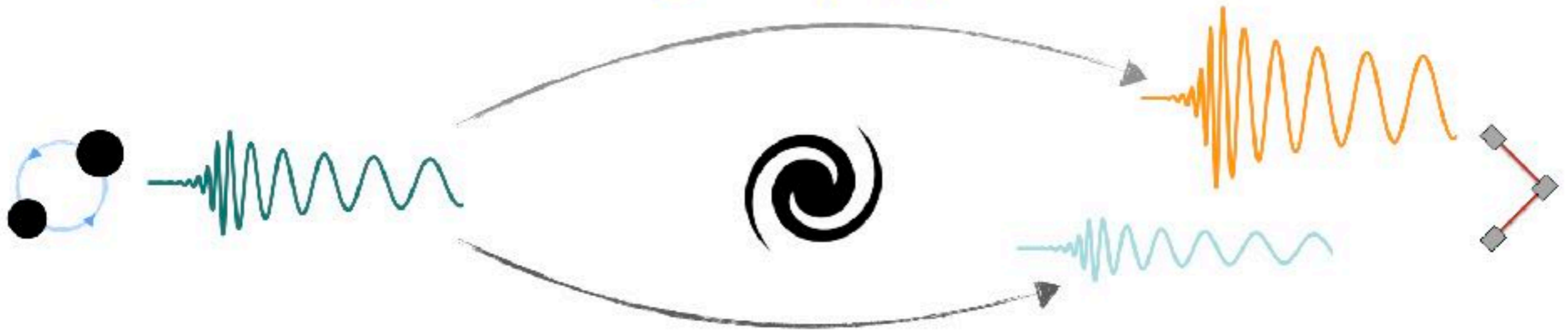


Source

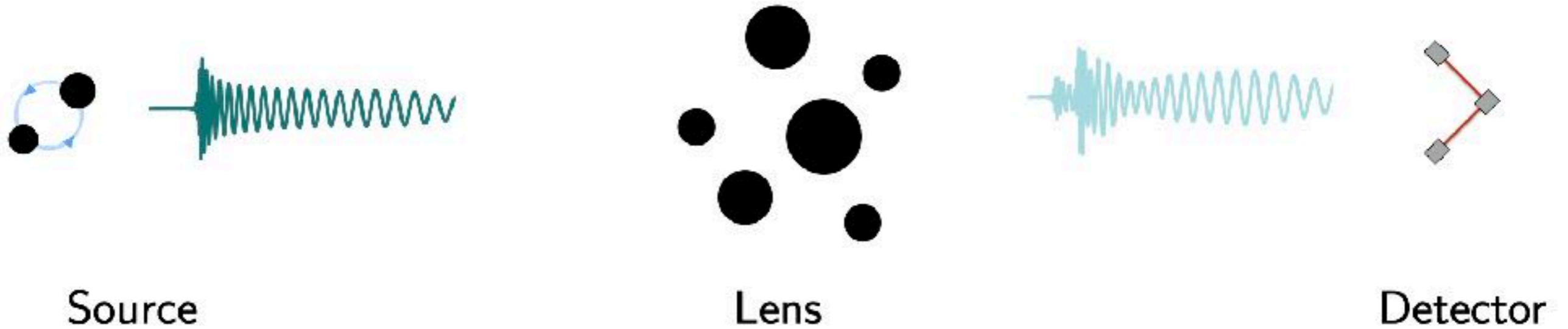
Lens

Detector

## Strong lensing by galaxies

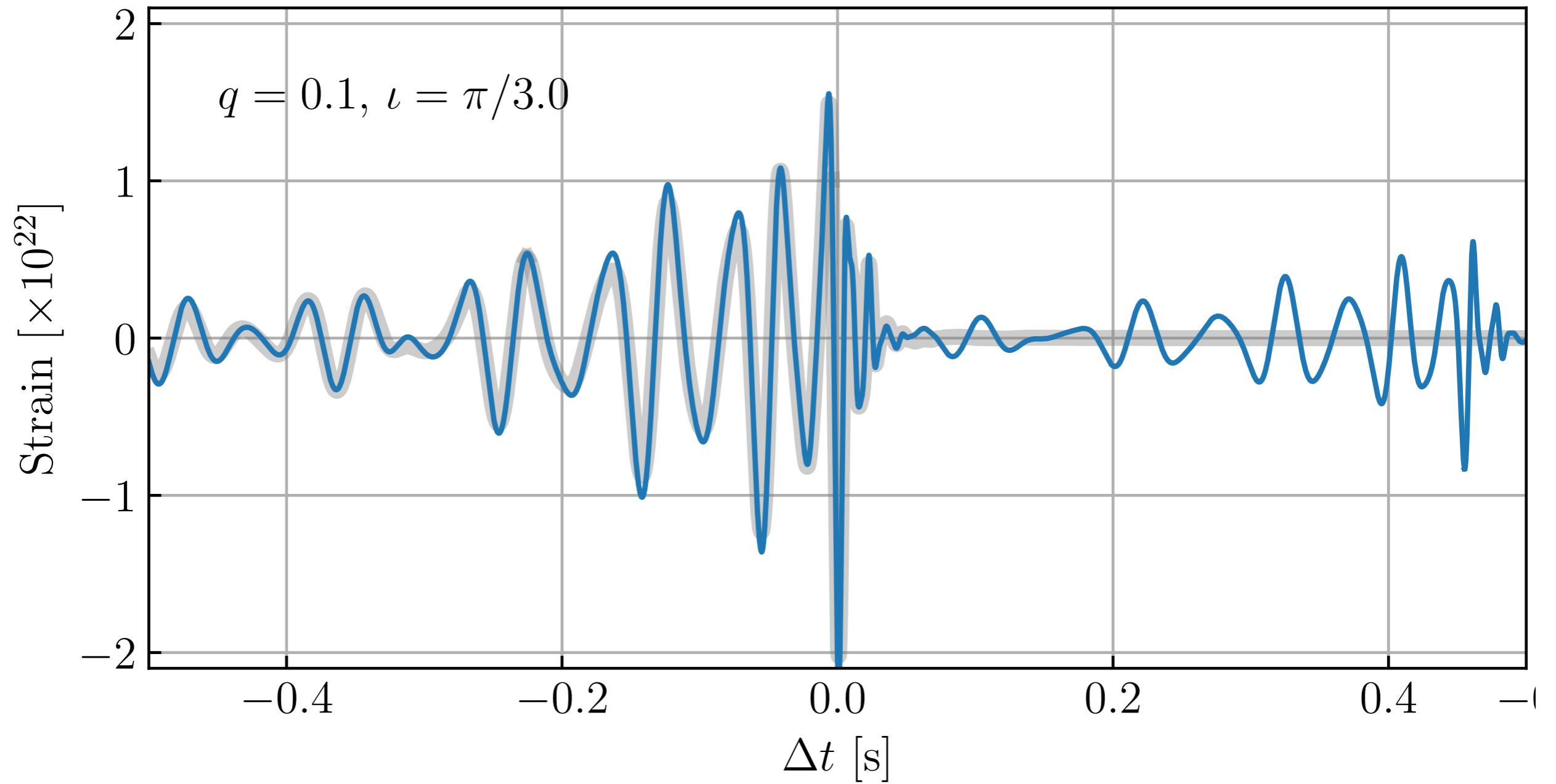


## Interference effects by compact lenses



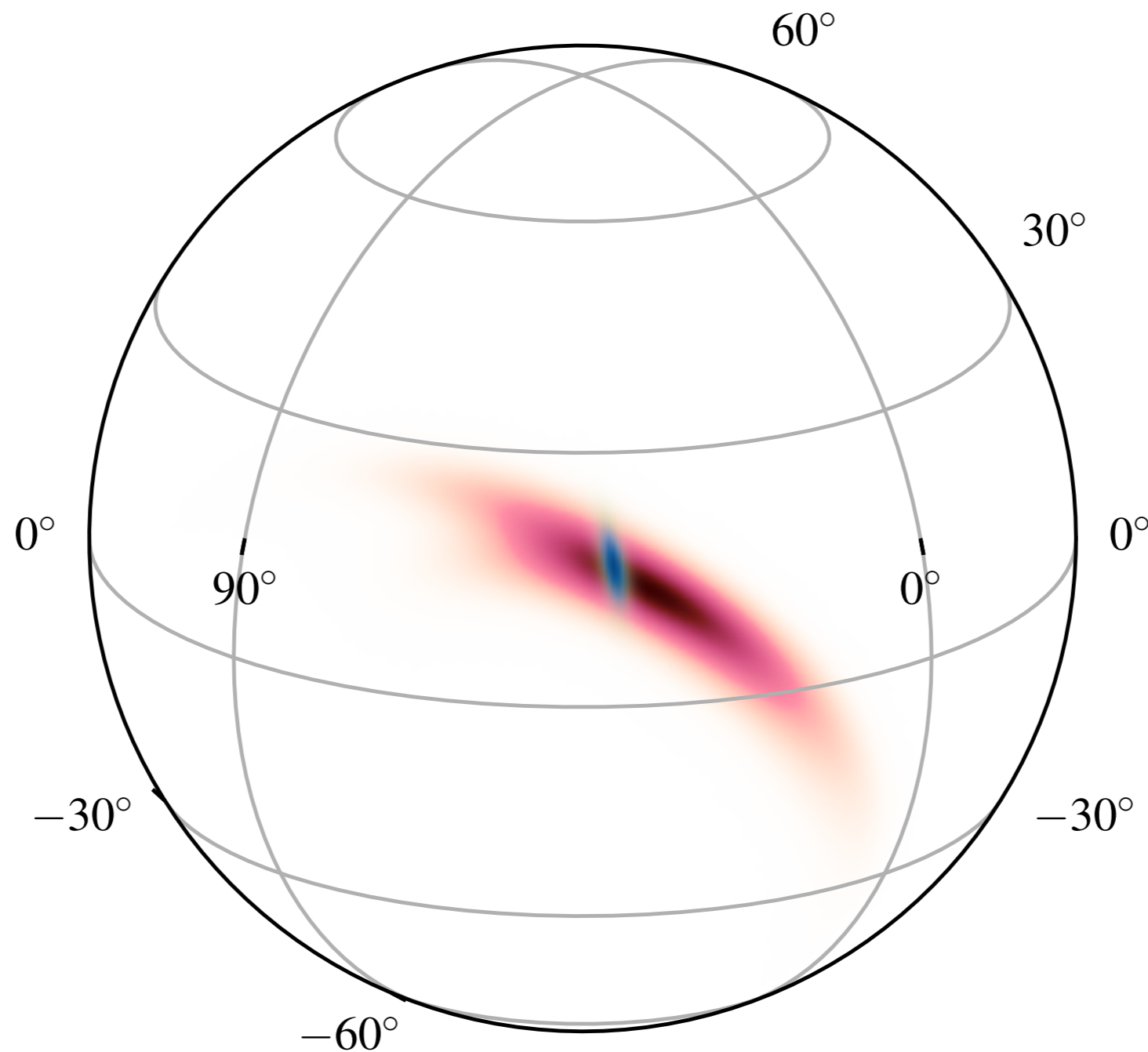


# Repeated signals



# GW sky localization

$$\theta_E \sim 1'' \sqrt{\frac{M}{10^{12} M_\odot}} \sqrt{\frac{1 \text{ Gpc}}{D}}$$



# Gravitational lensing of gravitational waves

- Clean signals, not affected by the medium
- All-sky coverage, high redshifts, large wavelengths
- Well understood selection effects of detectors
- Precise timing information
- Poor sky localization
- Coherent detection of waveform

# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega t_j - i\pi n_j)$$

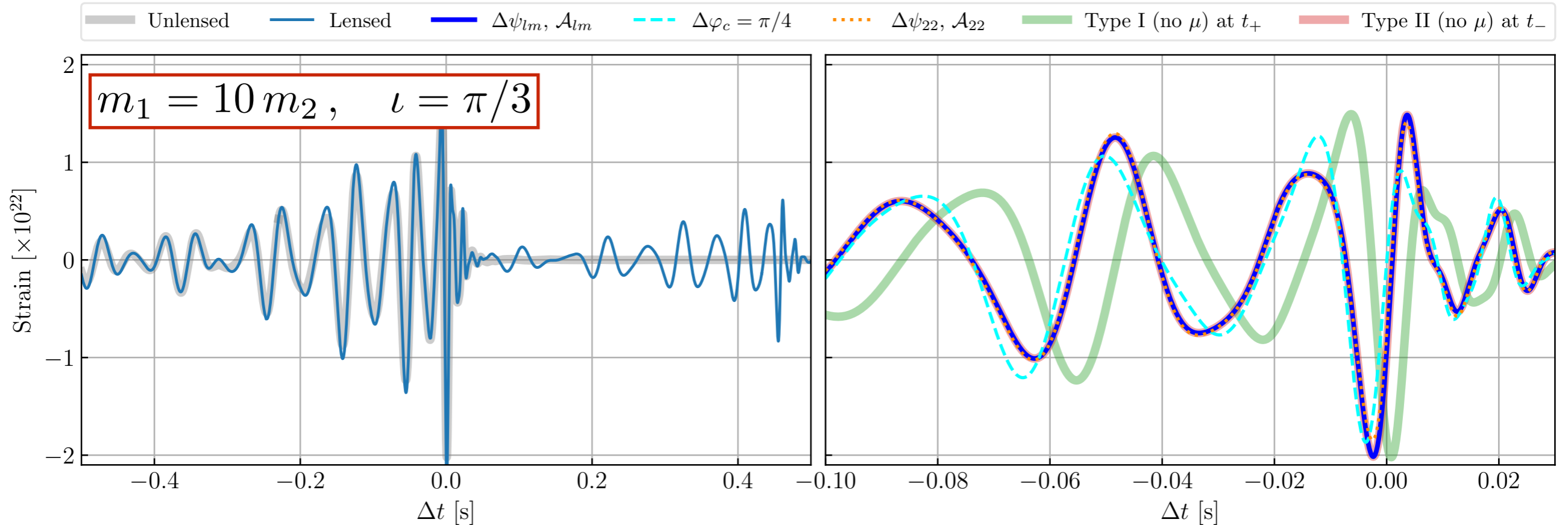
Magnification  
Time delay  
Phase shift

- Each image type (I, II and III) acquire a different phase shift

$$n_j = 0, 1/2, 1$$

# Waveform distortions in **type II** images

$$h = \sum_{l,m} \mathcal{A}_{lm} \cos [m(\Omega t + \varphi_c) - 2\chi_{lm}] \quad \Delta t = n_j \pi / |\omega|$$



# Strong lensing

$$\Delta t_d \cdot \omega \gg 1$$

$$h_L(\omega) = F(\omega, \theta_S) \cdot h(\omega)$$

$$F \approx \sum_j |\mu_j|^{1/2} \exp(i\omega t_j - i\pi n_j)$$

Magnification  
Time delay  
Phase shift

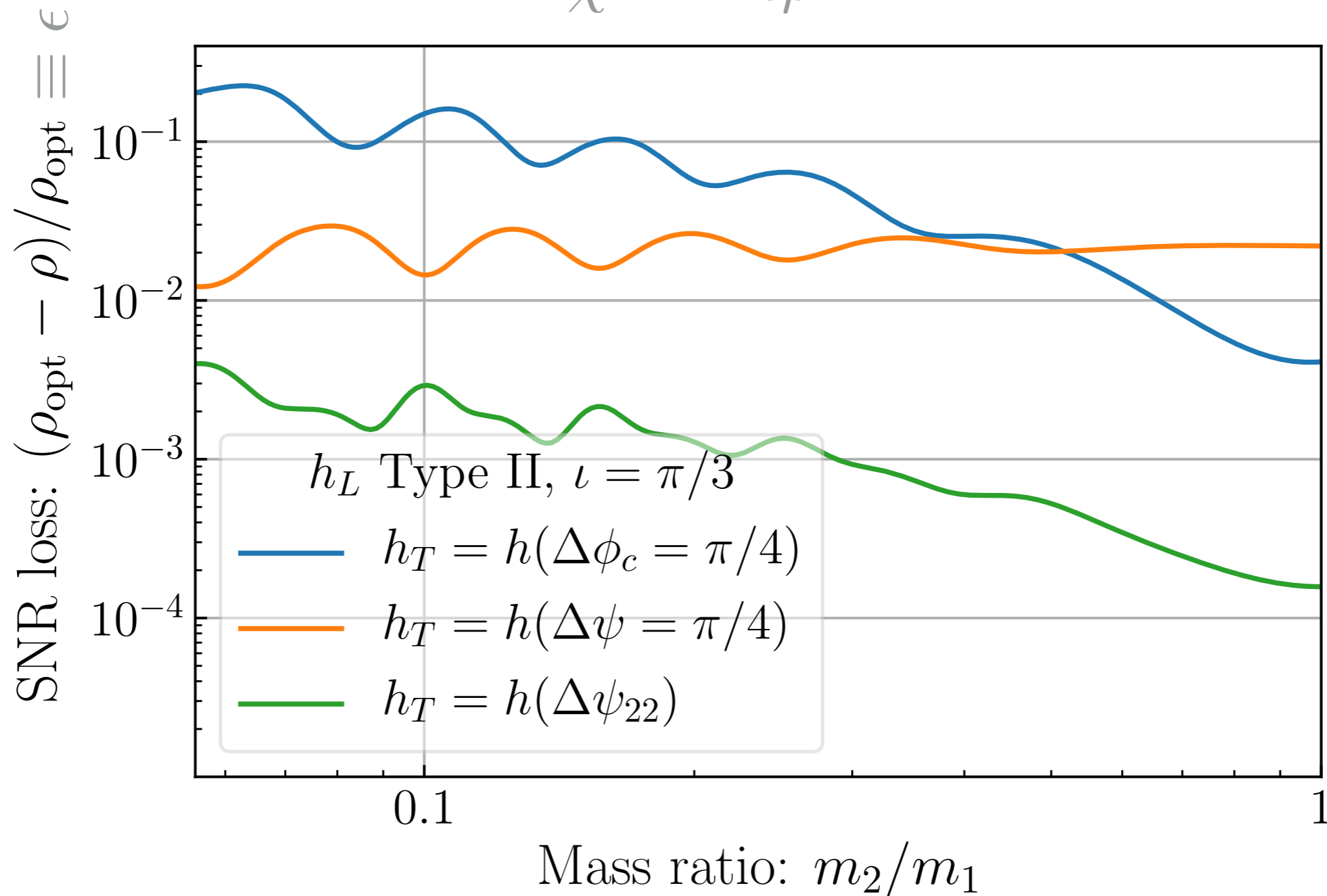
- Each image type (I, II and III) acquire a different phase shift

$$n_j = 0, 1/2, 1$$

- Lensed GWs *can differ* from (unlensed) GR wave-forms
- *Identify* strong lensing with *single image*

# type II image: Effect on parameter estimation

$$\Delta\chi^2 \sim 2\epsilon\rho^2$$



**Ezquiaga et al.**; *Phase effects from strong lensing of GWs* (PRD, [arXiv 2008.12814](https://arxiv.org/abs/2008.12814))

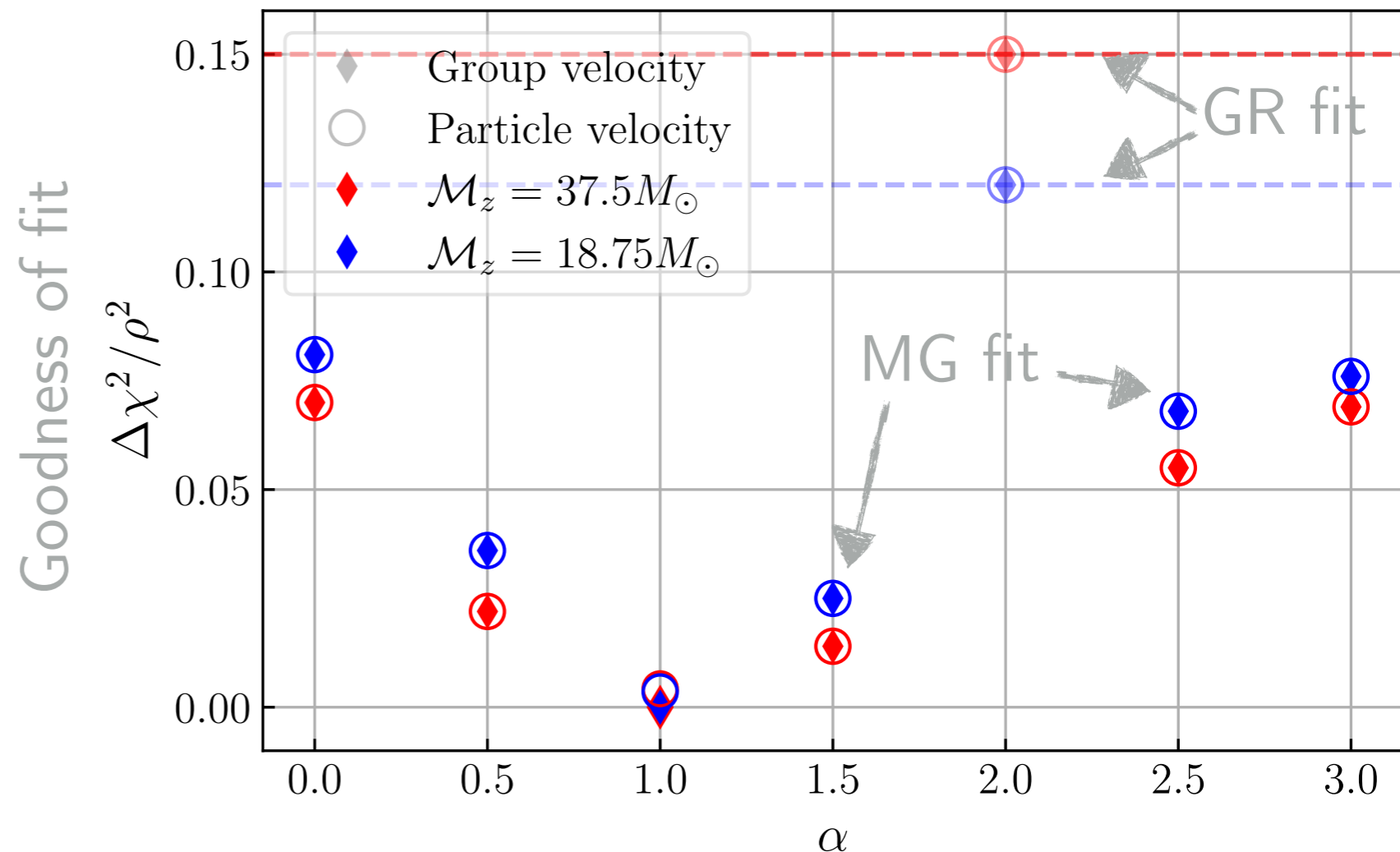
Implications for next-generation detectors: [Wang et al.'21](#), [Janquart et al.'21](#), [Vijaykumar et al.'22](#)

# Note: degeneracies with modified gravity

- Strong lensing:  $F \approx \sum_j |\mu(\vec{\theta}_j)|^{1/2} \exp\left(i\omega t(\vec{\theta}_j) - i\pi \text{sign}(\omega)n_j\right)$
- Modified GW propagation:  $h_{\text{mg}} \sim h_{\text{gr}} e^{-i \int \Delta\omega dz/H(z)} \equiv h_{\text{gr}} e^{i\Delta\Psi}$

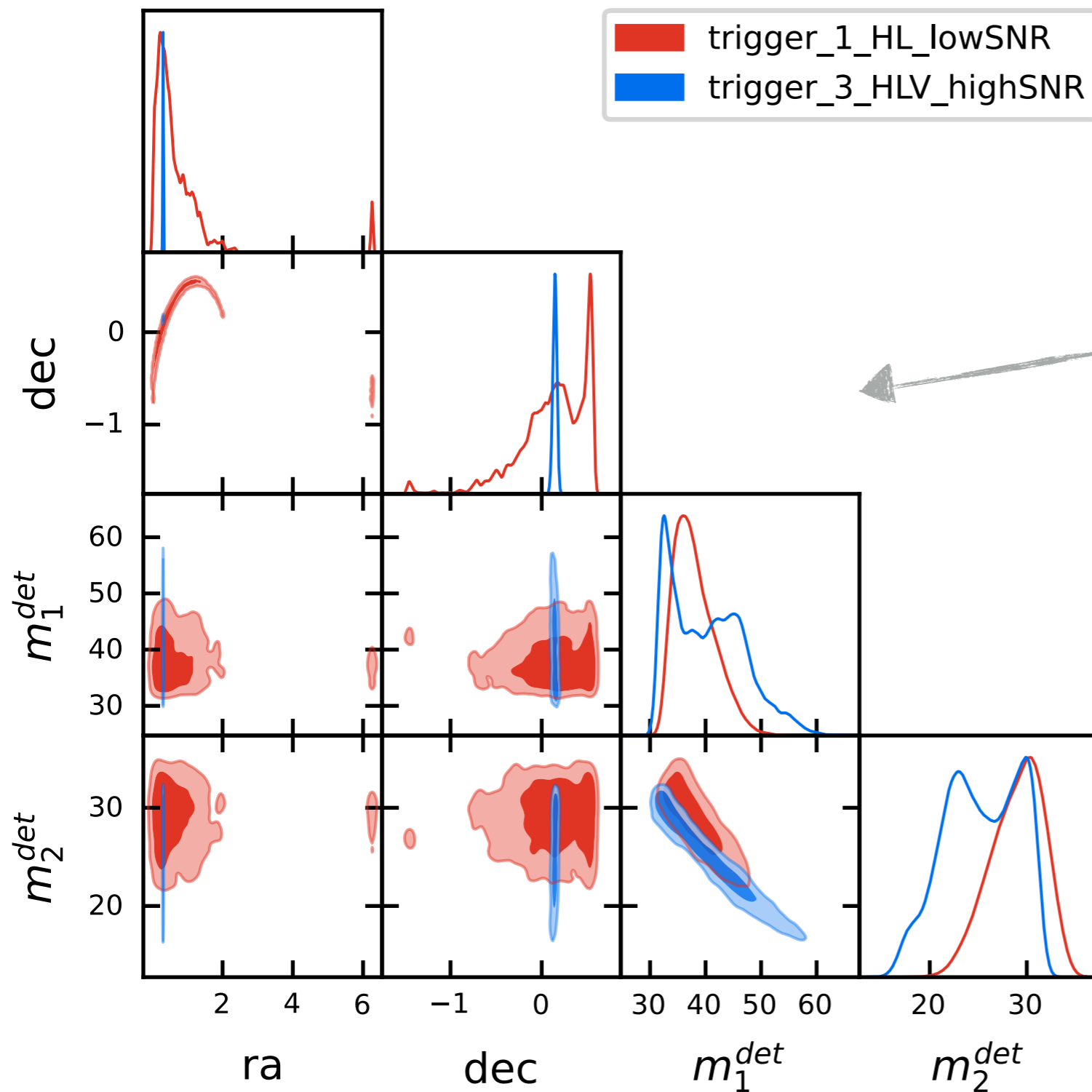
$$\omega^2 = c^2 k^2 + \Lambda (ck)^\alpha$$

$$\Delta\Psi(f) = -\frac{\Lambda}{2} D_\alpha (2\pi f_s)^{1-\alpha}$$





# Searching for strongly lensed GWs



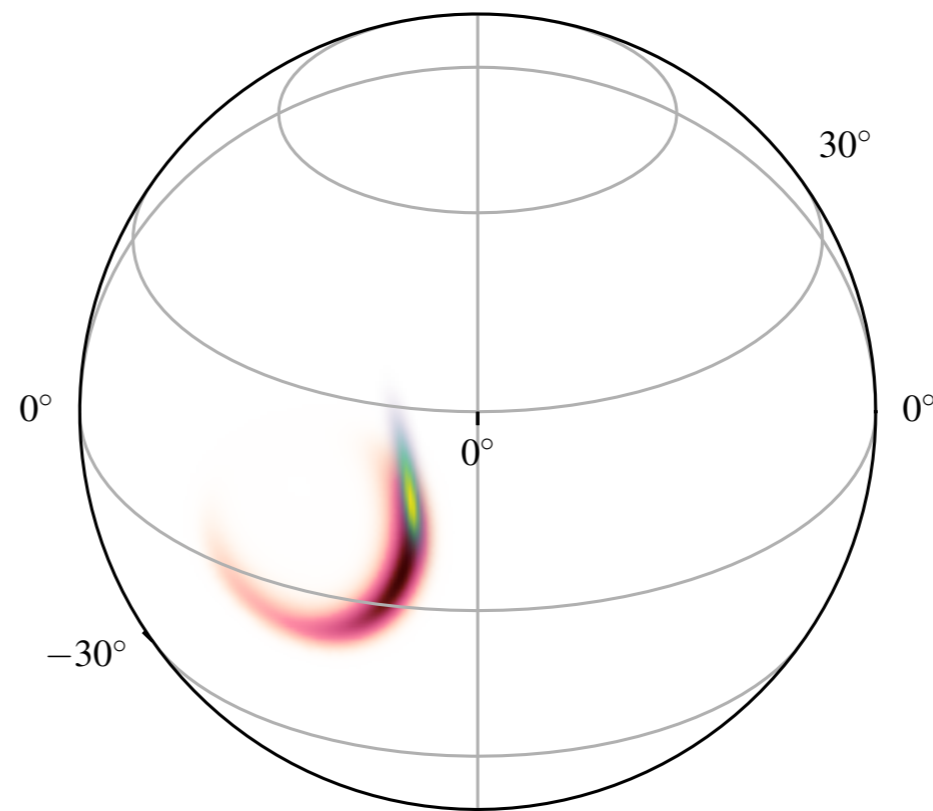
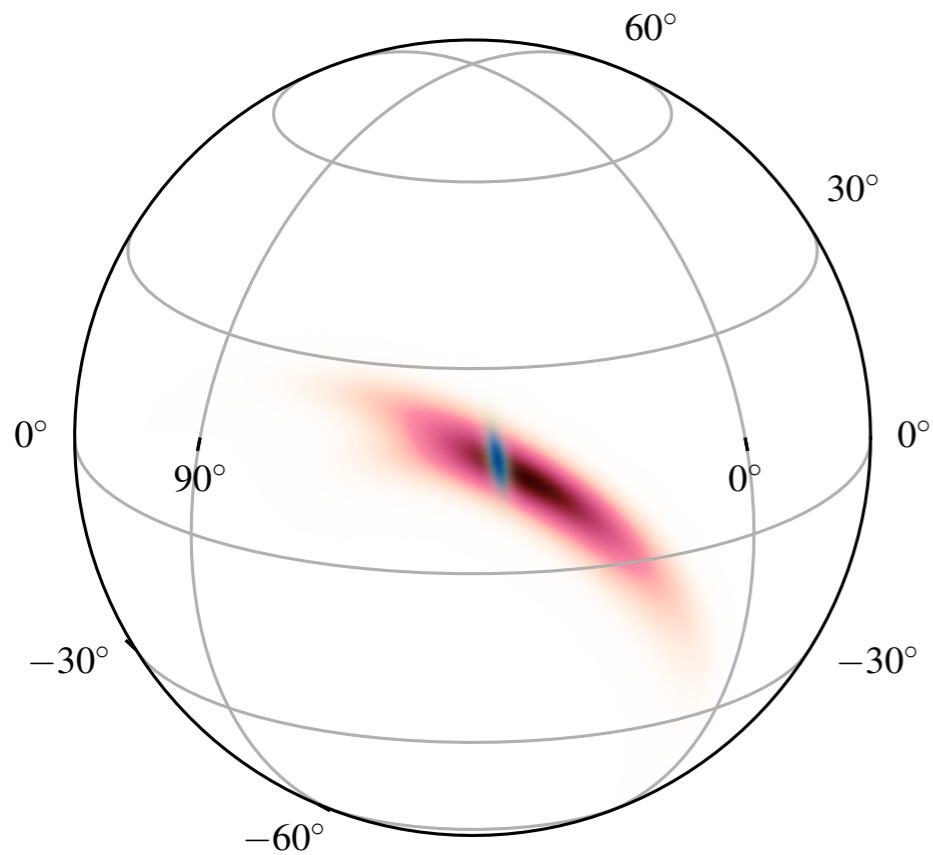
Look for overlaps in the  
(detector frame) masses,  
sky positions and spins  
+  
Joint parameter estimation

# Lensing or luck?

$$N_{\text{false alarm}} \sim N^2$$

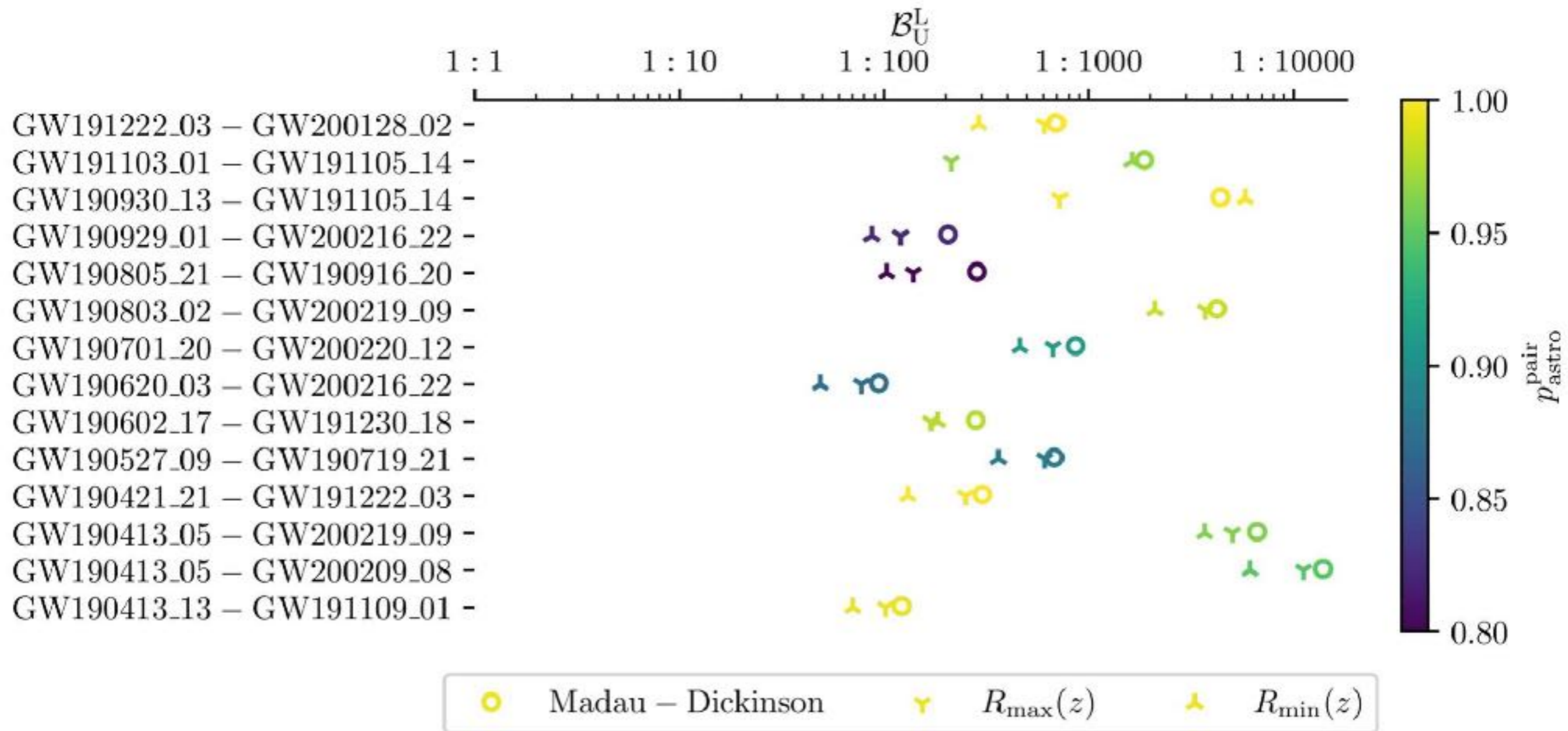
■ Lensed Image 1  
■ Lensed Image 2

■ Event 12  
■ Event 89



Mesut Çalışkan  
(UChicago/JHU)

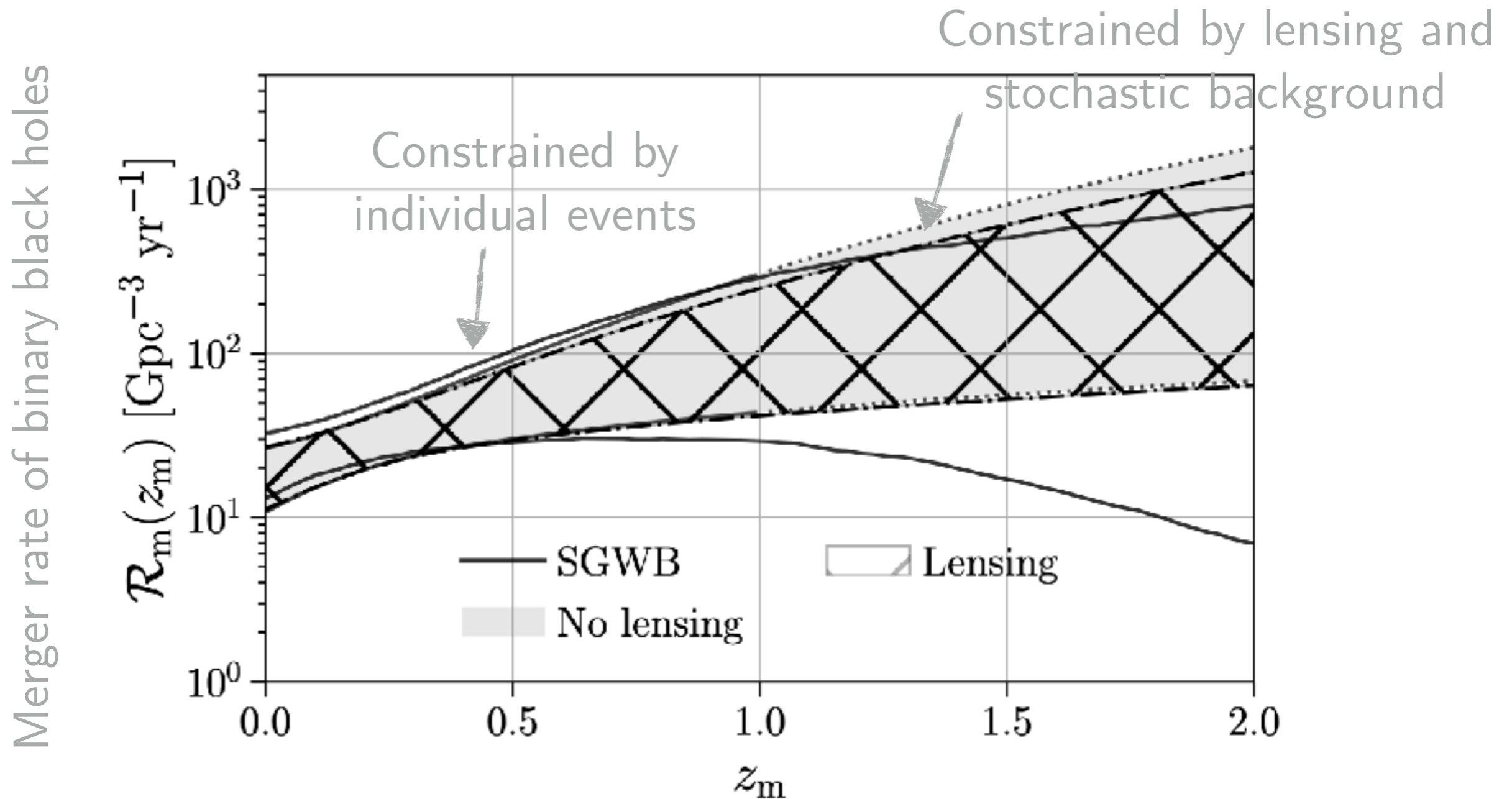
# LVK: no evidence of strong lensing so far...



LVC (incl. **Ezquiaga**); *Search GW lensing O3a* (ApJ, [arXiv 2105.06384](https://arxiv.org/abs/2105.06384), [science summary](#))

LVK (incl. **Ezquiaga**); *Search GW lensing full O3* ([arXiv 2304.08393](https://arxiv.org/abs/2304.08393), [science summary](#))

# LVK: no evidence of strong lensing so far...

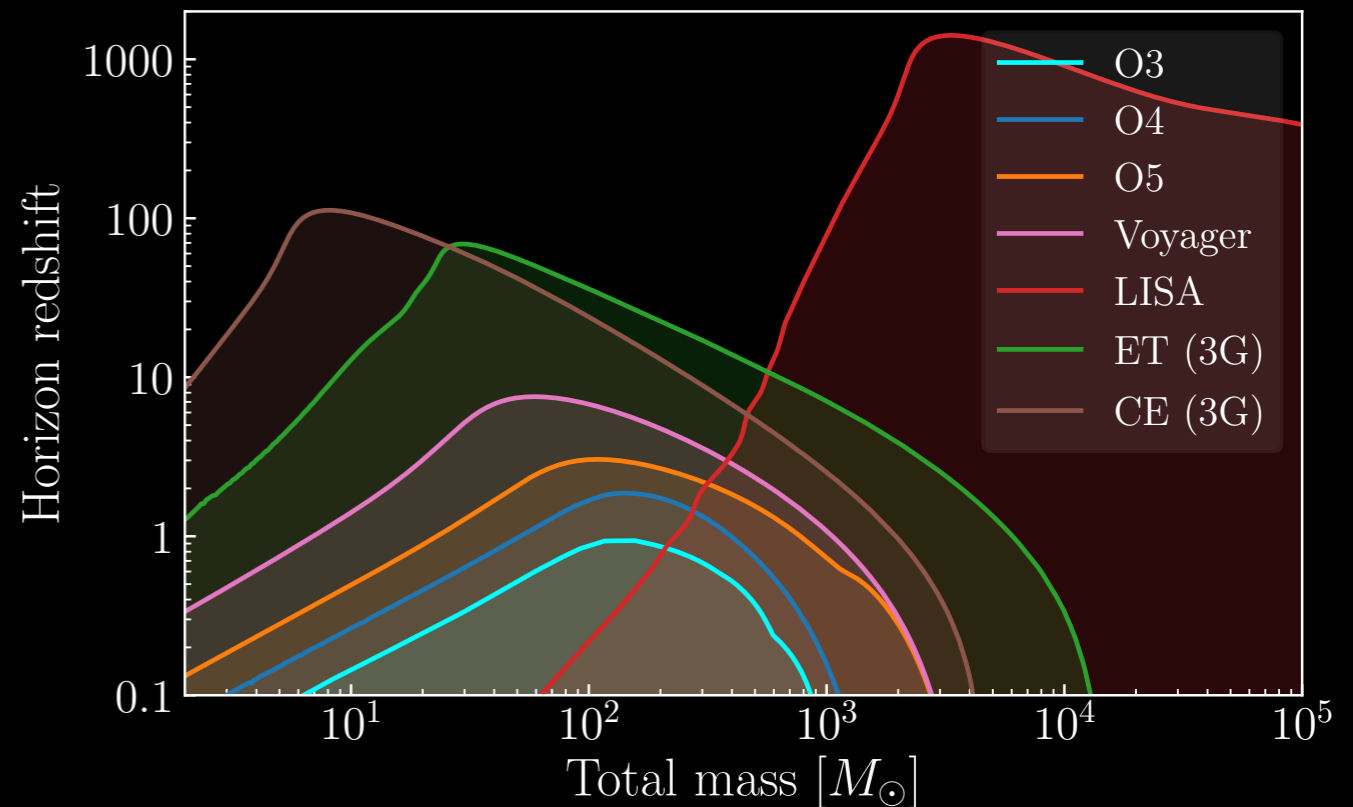


LVC (incl. **Ezquiaga**); *Search GW lensing O3a* (ApJ, [arXiv 2105.06384](https://arxiv.org/abs/2105.06384), [science summary](#))

LVK (incl. **Ezquiaga**); *Search GW lensing full O3* ([arXiv 2304.08393](https://arxiv.org/abs/2304.08393), [science summary](#))

# Looking ahead

**Now** **2G**: current generation ground-based GW detectors  
**3G**: next generation ground-based GW detectors



**Approx. 100**  
 events typically at  
 $z < 0.6$

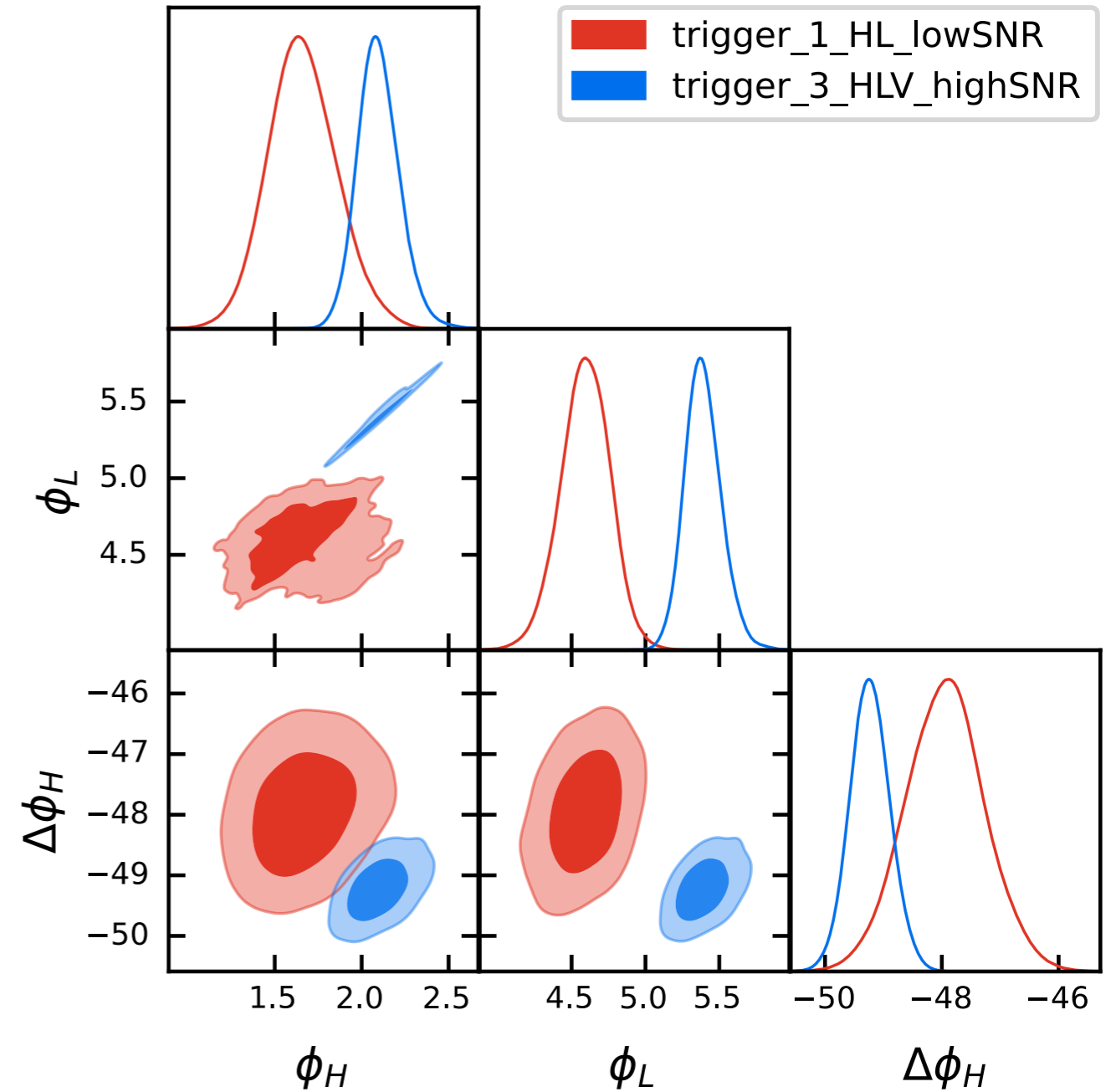
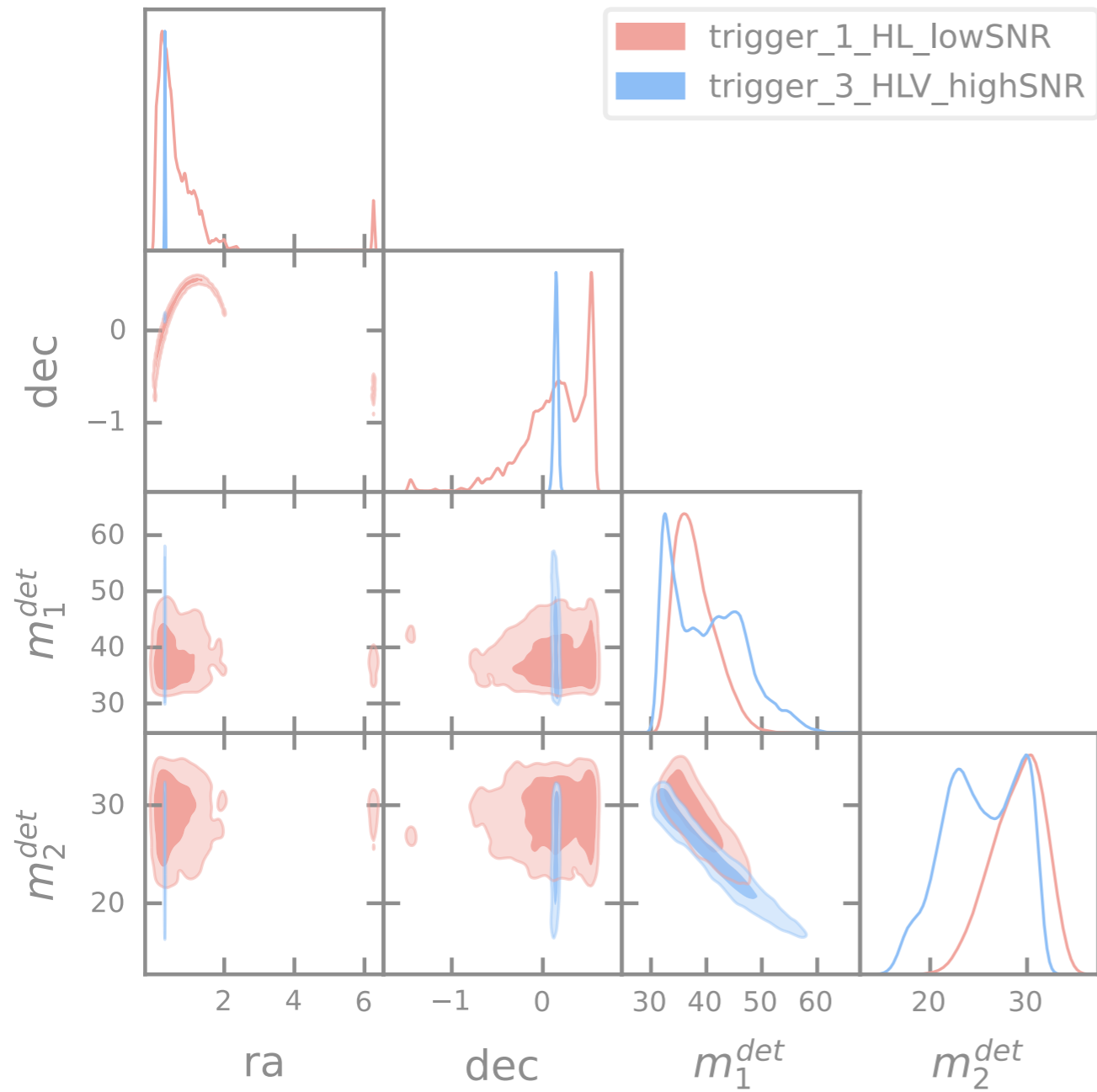


**1000s / year**  
 with some  $z > 1$



**100,000s / year**  
 with most  $z > 1$

# Fight false alarms: **phase consistency**

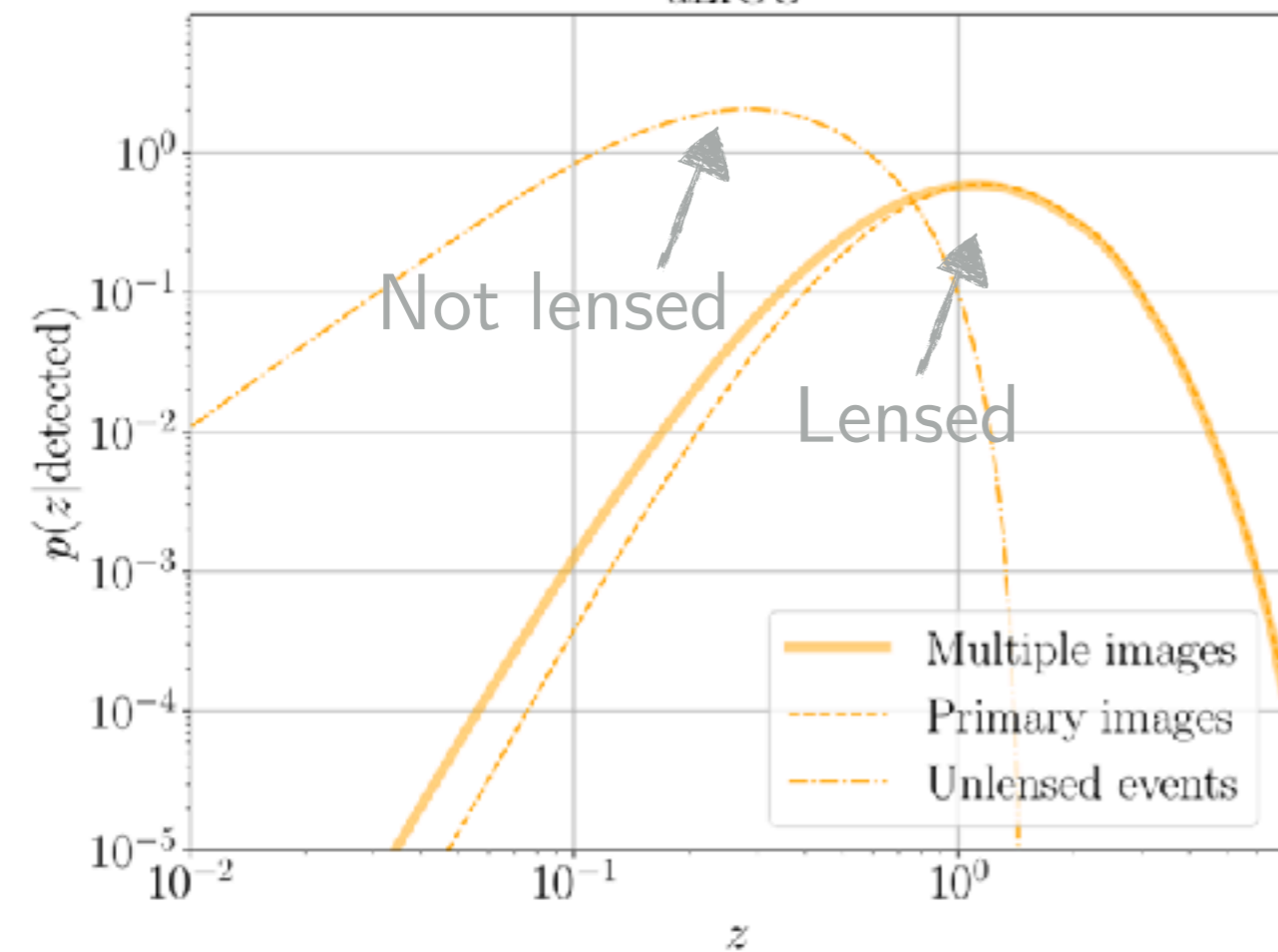


# Probing source and lens populations

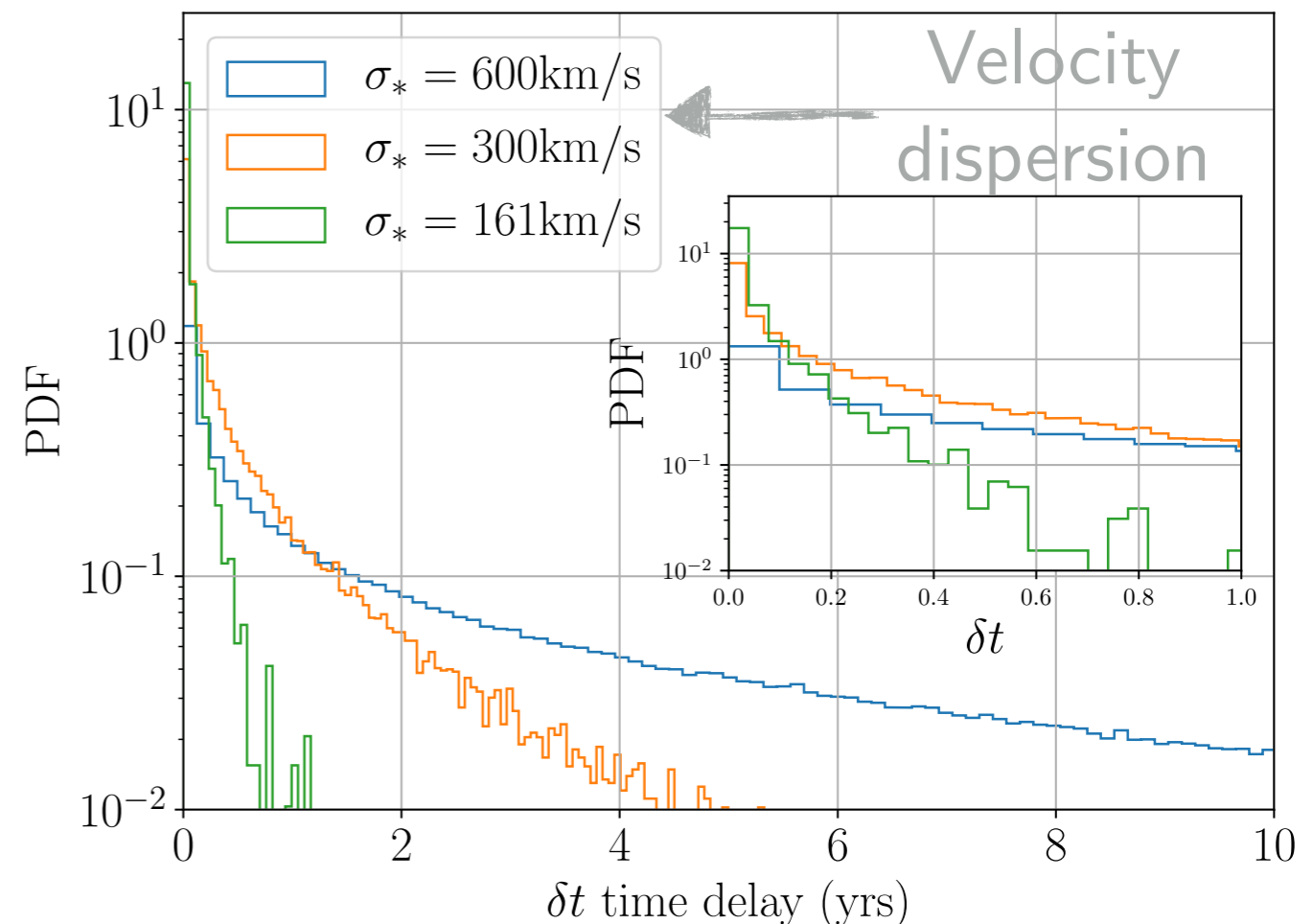


Fei Xu  
(UChicago)

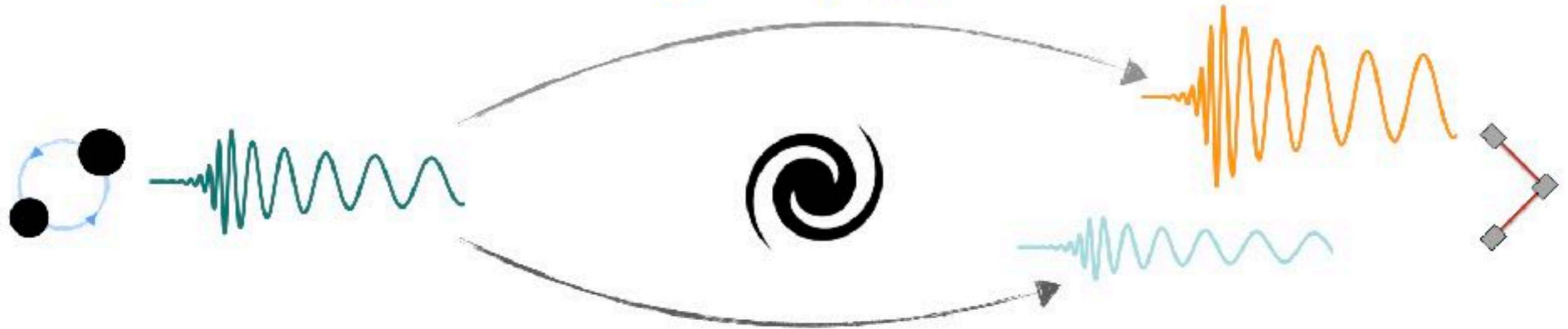
Detected populations  
aLIGO



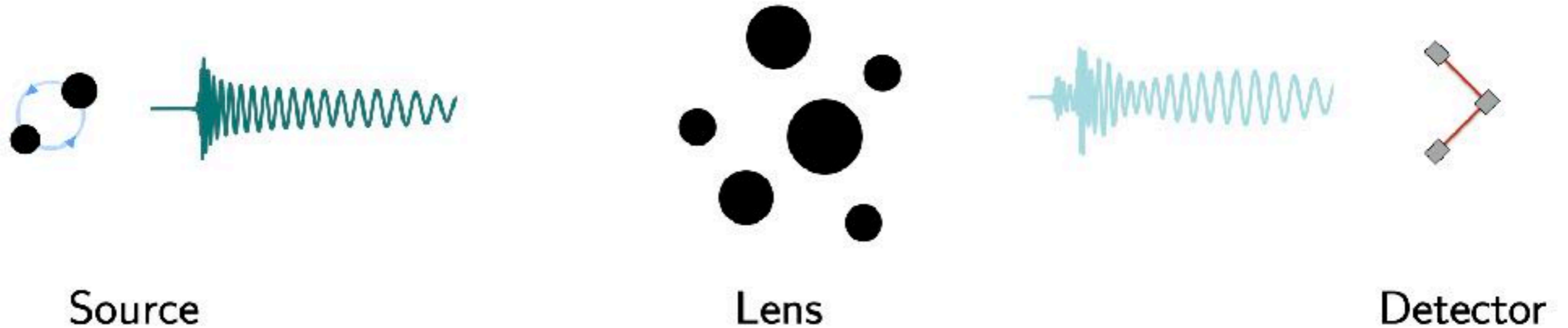
Time delay distributions



## Strong lensing by galaxies



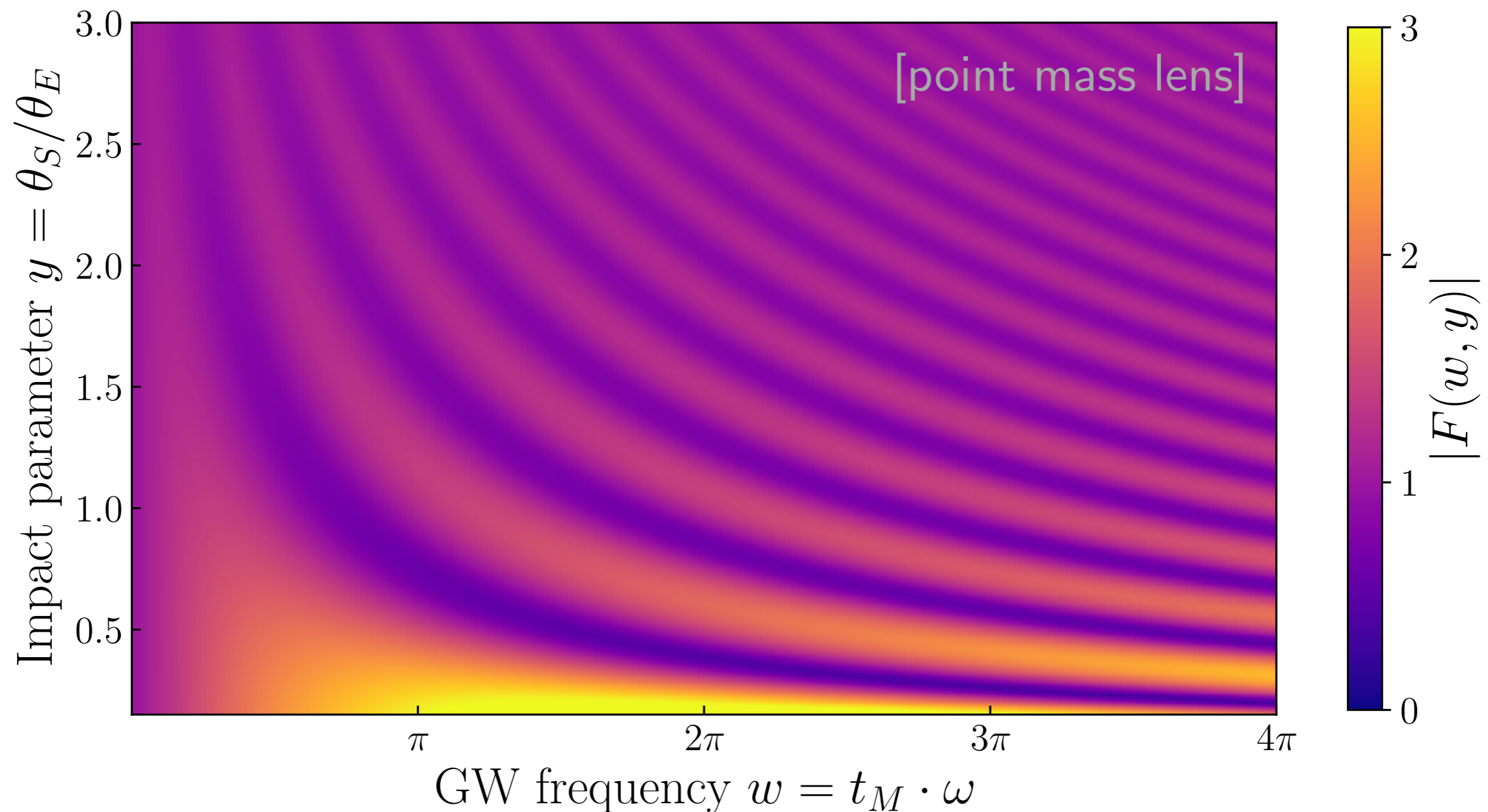
## Interference effects by compact lenses





# Wave effects:

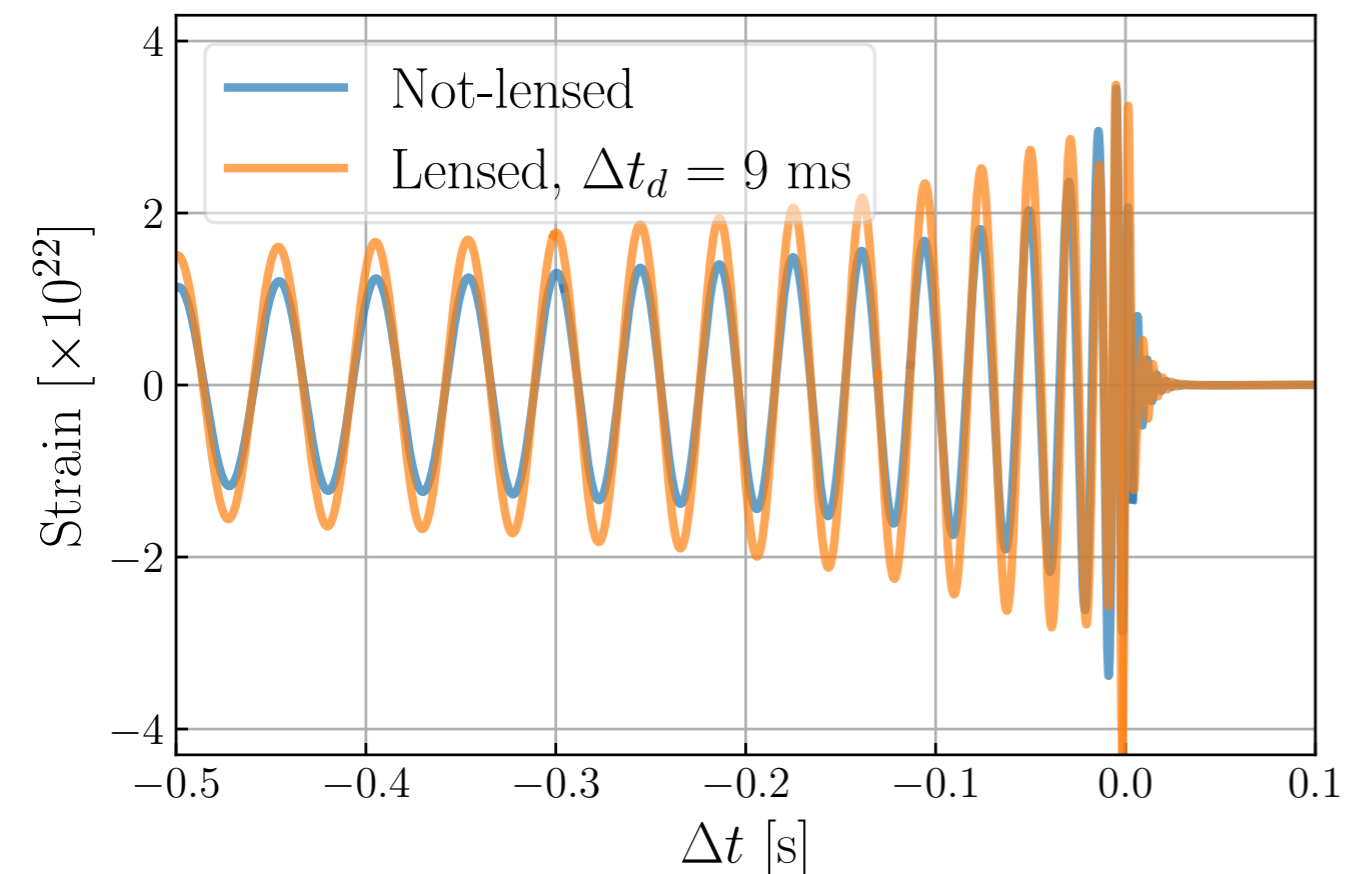
$$F(\omega, \vec{y}) = \frac{\omega}{2\pi i} \int d^2x \exp [i\omega T(\vec{x}, \vec{y})]$$



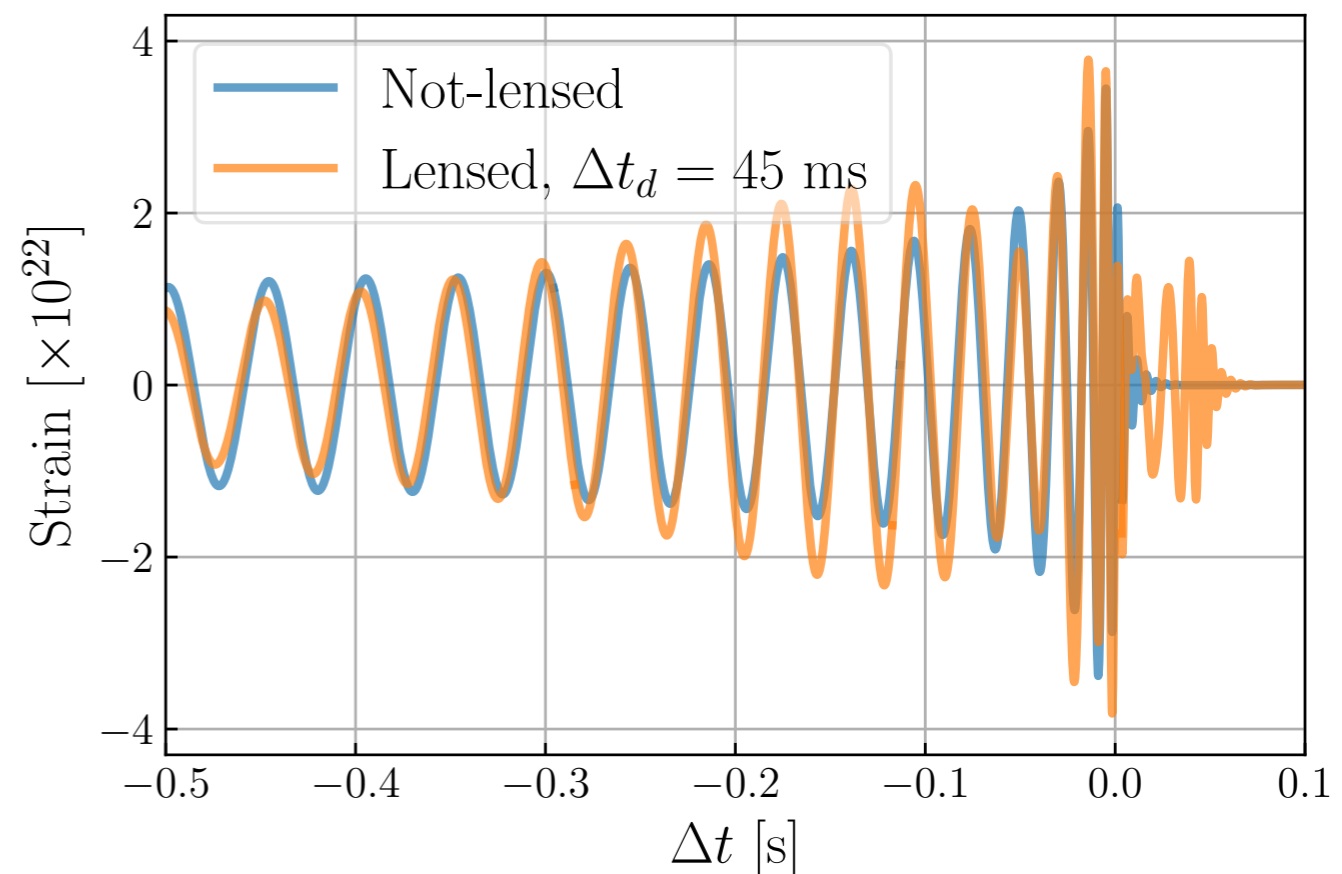
# Wave effects:

$$\Delta t_d(y = 1) \simeq 4 \left( \frac{(1 + z_L) M_L}{100 M_\odot} \right) \text{ ms}$$

## Diffraction

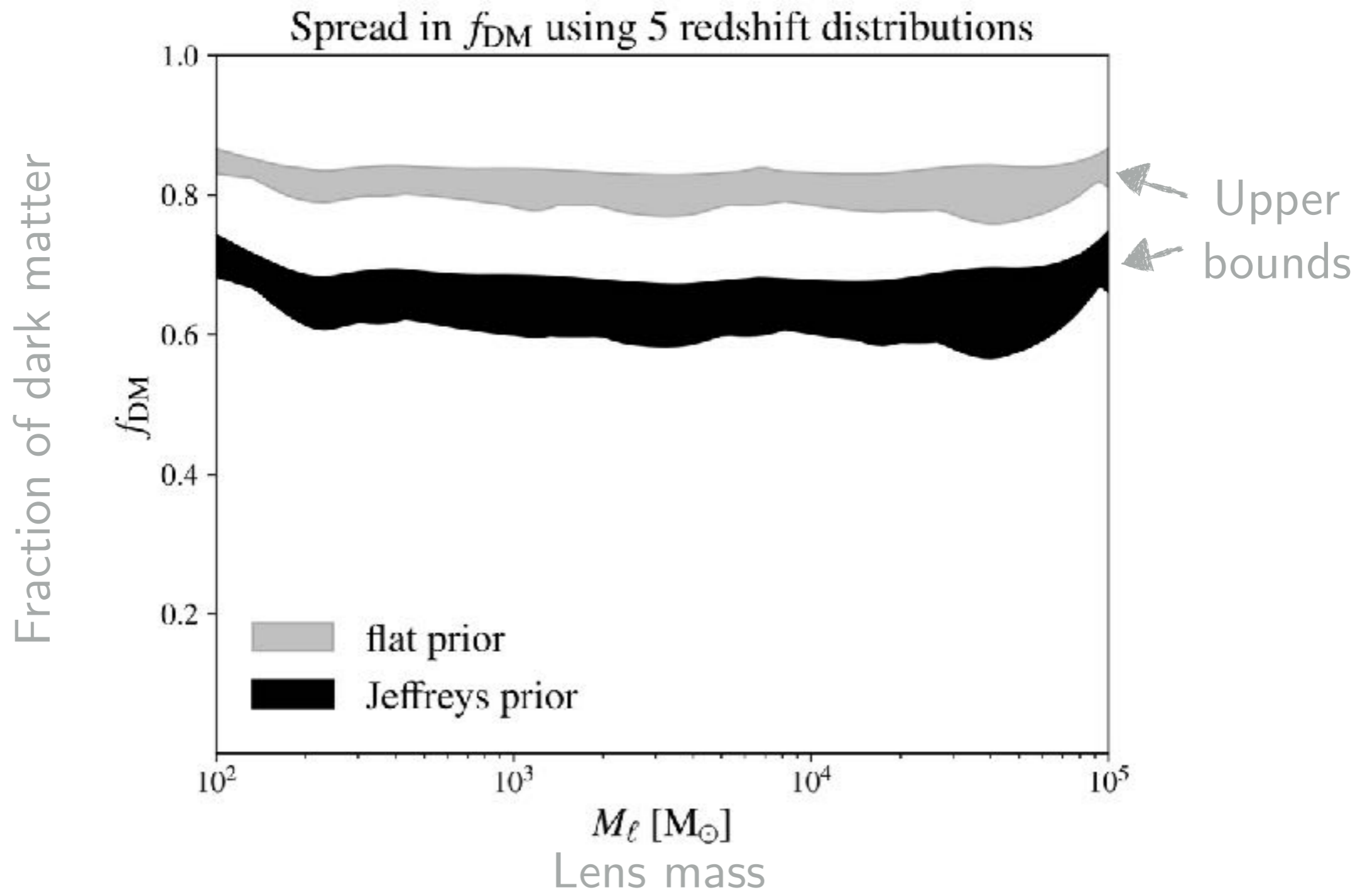


## Interference



Probing compact objects ([Dai et al.'18](#), [Diego'19](#), [Tambalo et al.'22](#), ...), strong lensing + microlensing ([Seo et al.'21](#), [Mena et al.'22](#), ...), breaking mass-sheet degeneracy ([Cremonese, Ezquiaga, Salzano'21](#)), solving diffraction integral ([Feldbrugge&Turok'20](#), [Tambalo et al.'22](#))

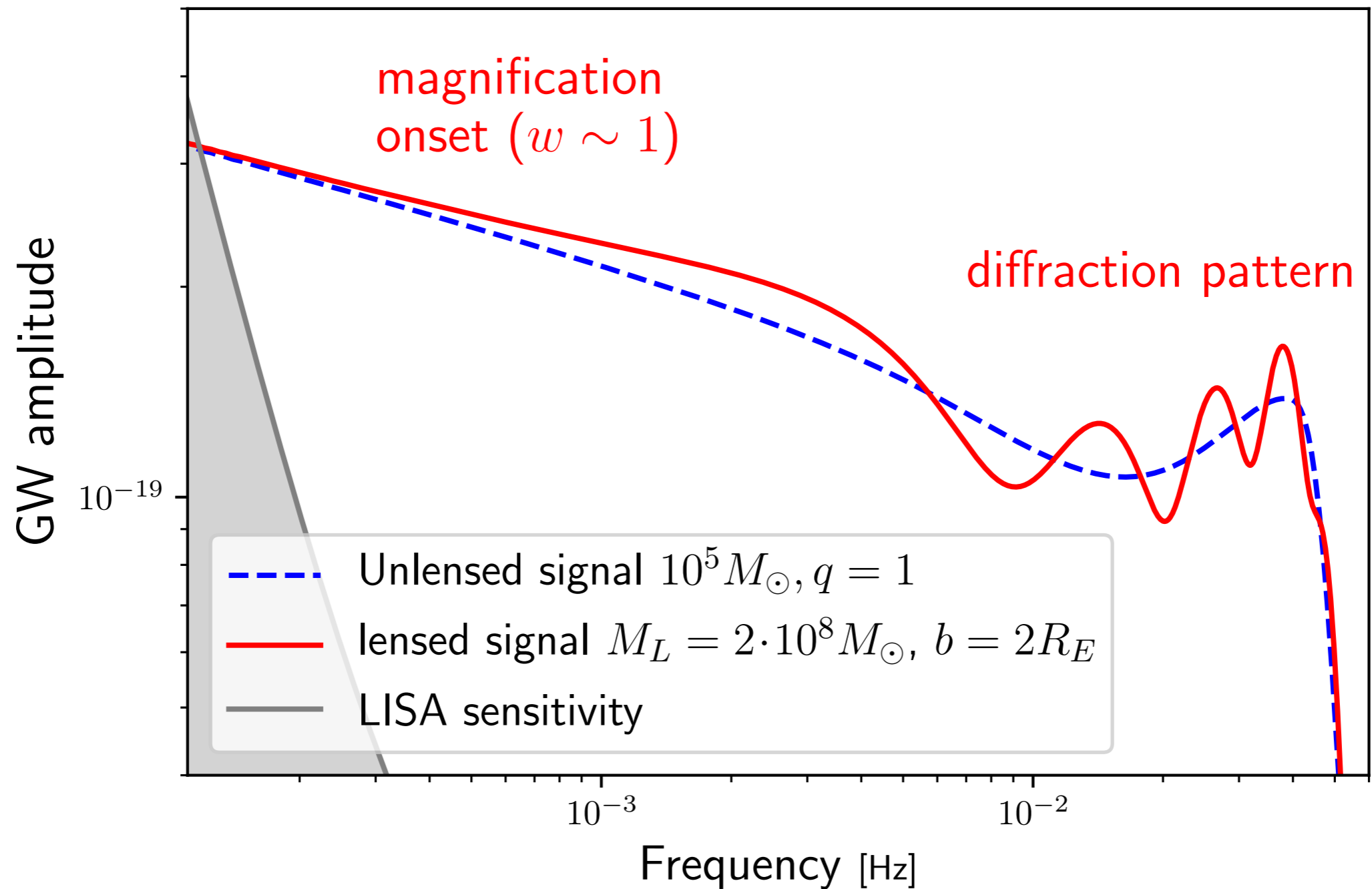
# LVK: no evidence of wave effects so far...



LVC (incl. **Ezquiaga**); *Search GW lensing O3a* (ApJ, [arXiv 2105.06384](#), [science summary](#))

LVK (incl. **Ezquiaga**); *Search GW lensing full O3* ([arXiv 2304.08393](#), [science summary](#))

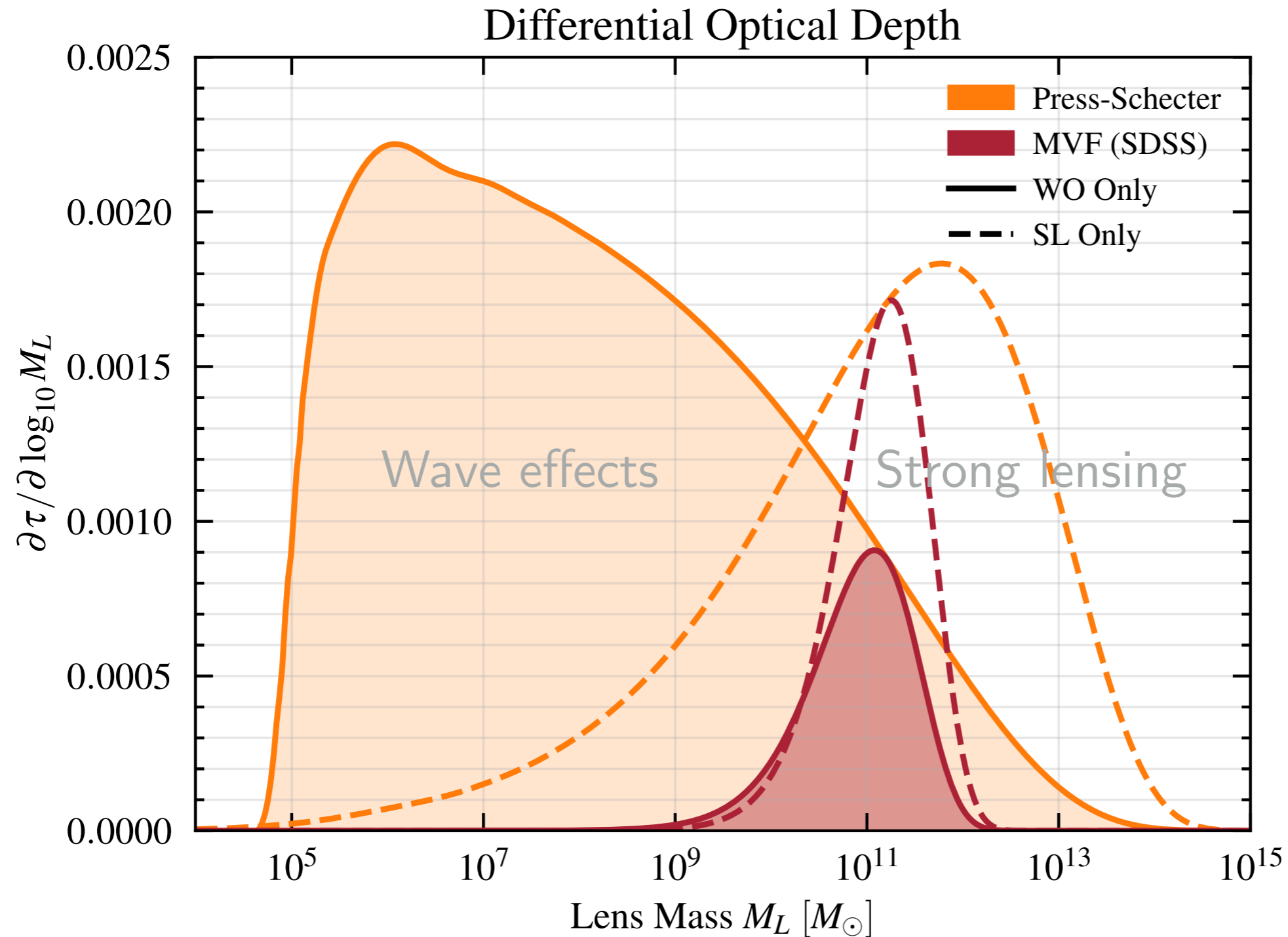
# Wave effects: LISA



# Probing dark matter sub-halos



Mesut Çalışkan  
(JHU)



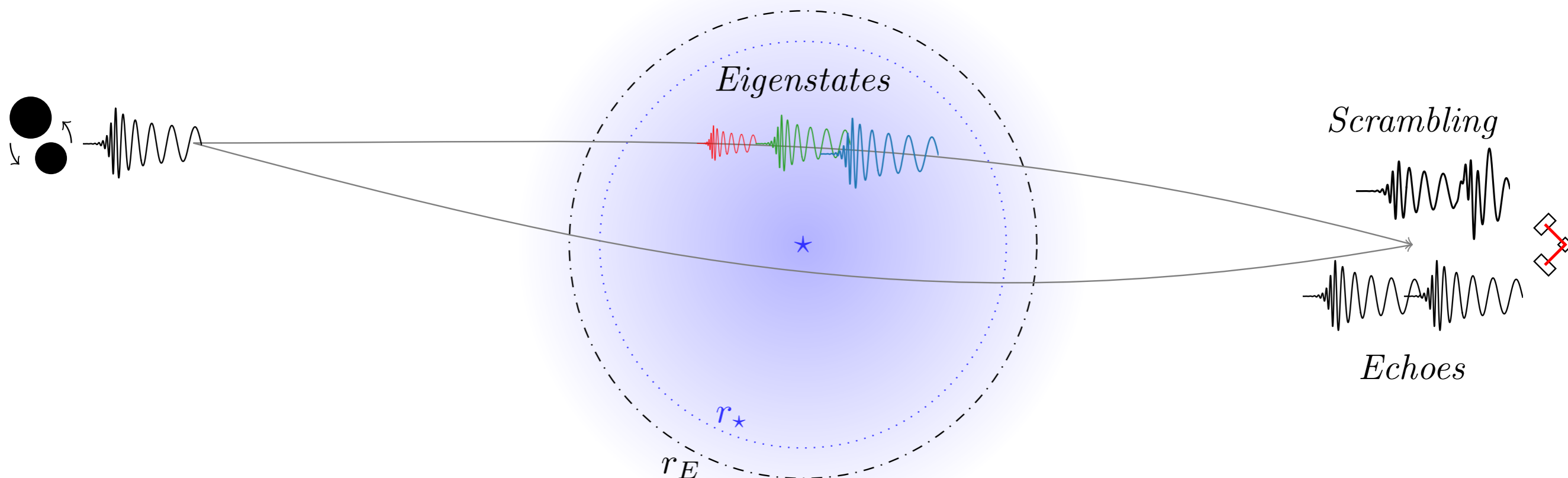
# Gravitational lensing beyond general relativity



# GW lensing beyond GR



- Beyond GR the background of the additional fields  $\phi(r)$  modify propagation (besides the change in gravitational potential)



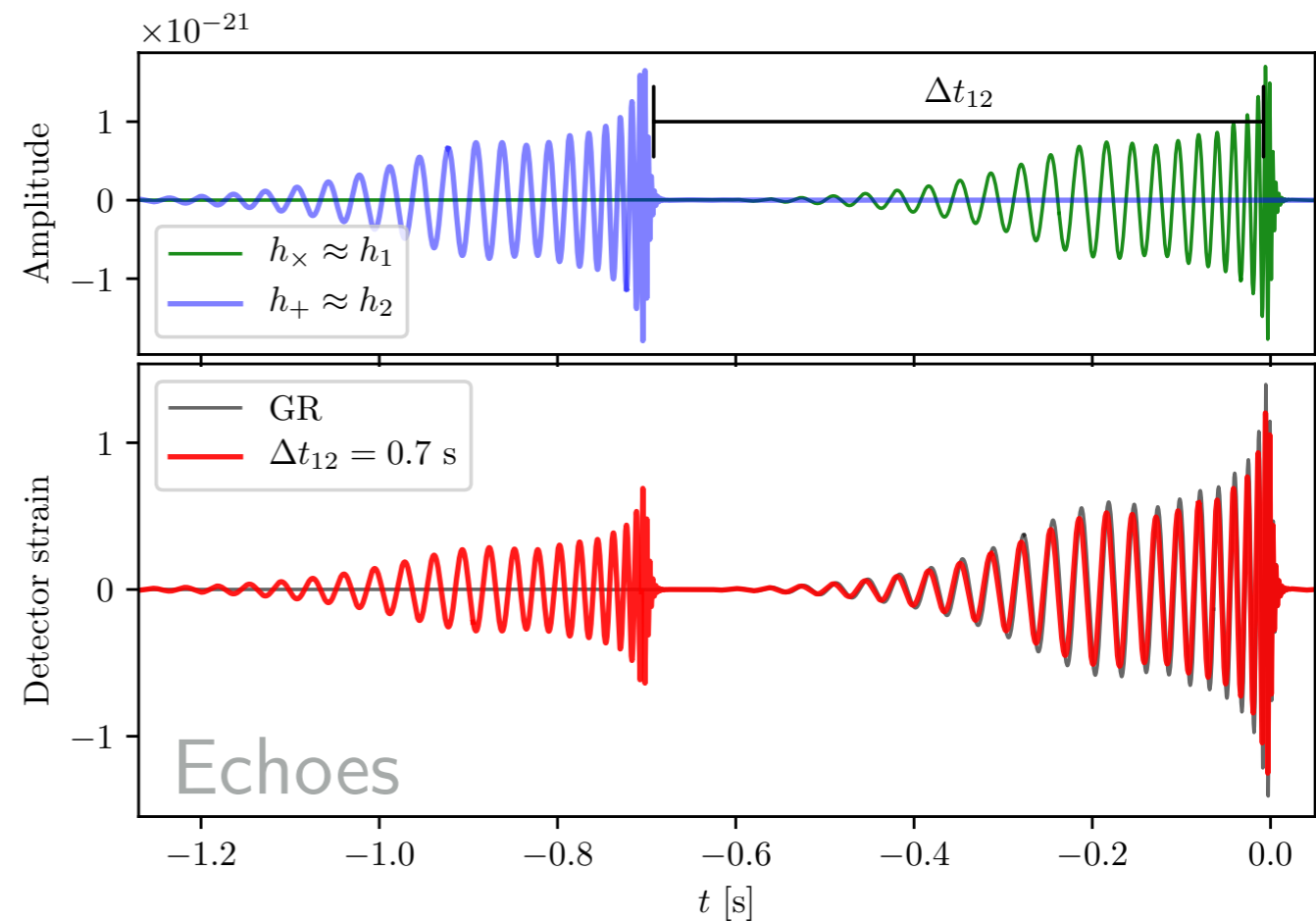
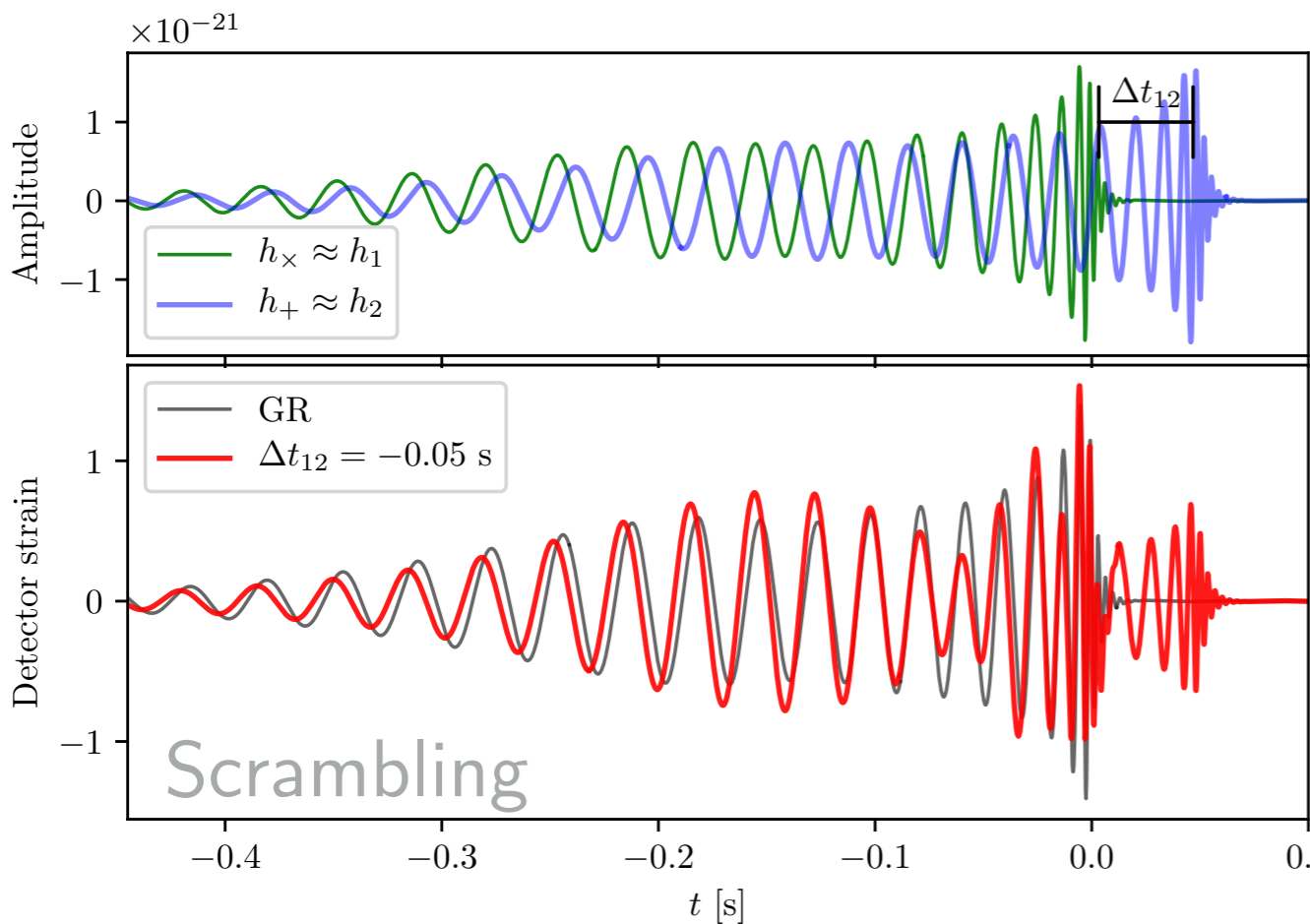
GWs can **mix** with the additional fields. The propagation **eigenstates** may have different speeds, **splitting** or **distorting** each image

# GW lensing beyond GR

Modified effective metric for each eigenstate and polarization mixing

Time delays

Birefringence

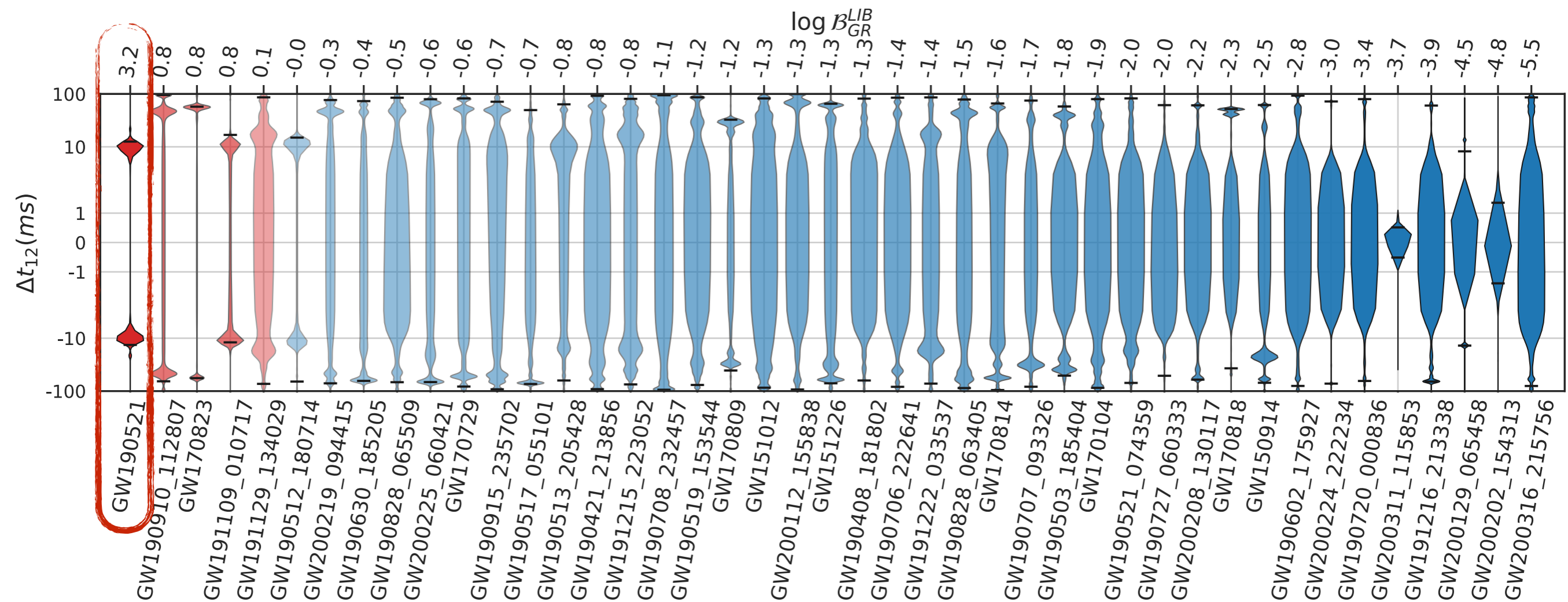
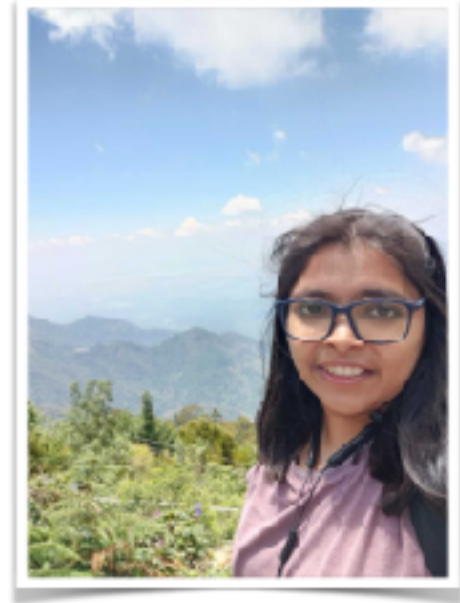


- No need of EM counterpart! Extend cosmological test GW propagation!



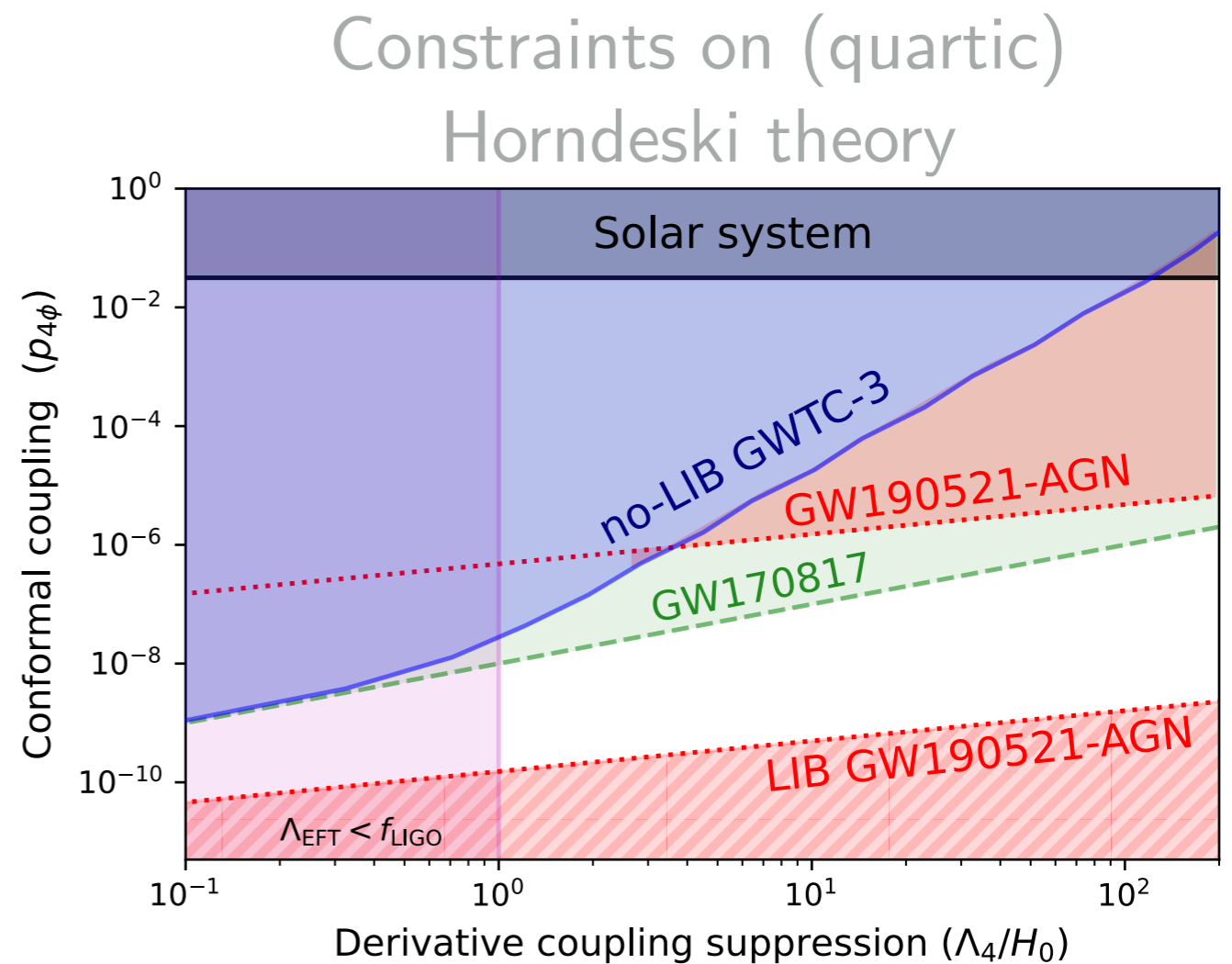
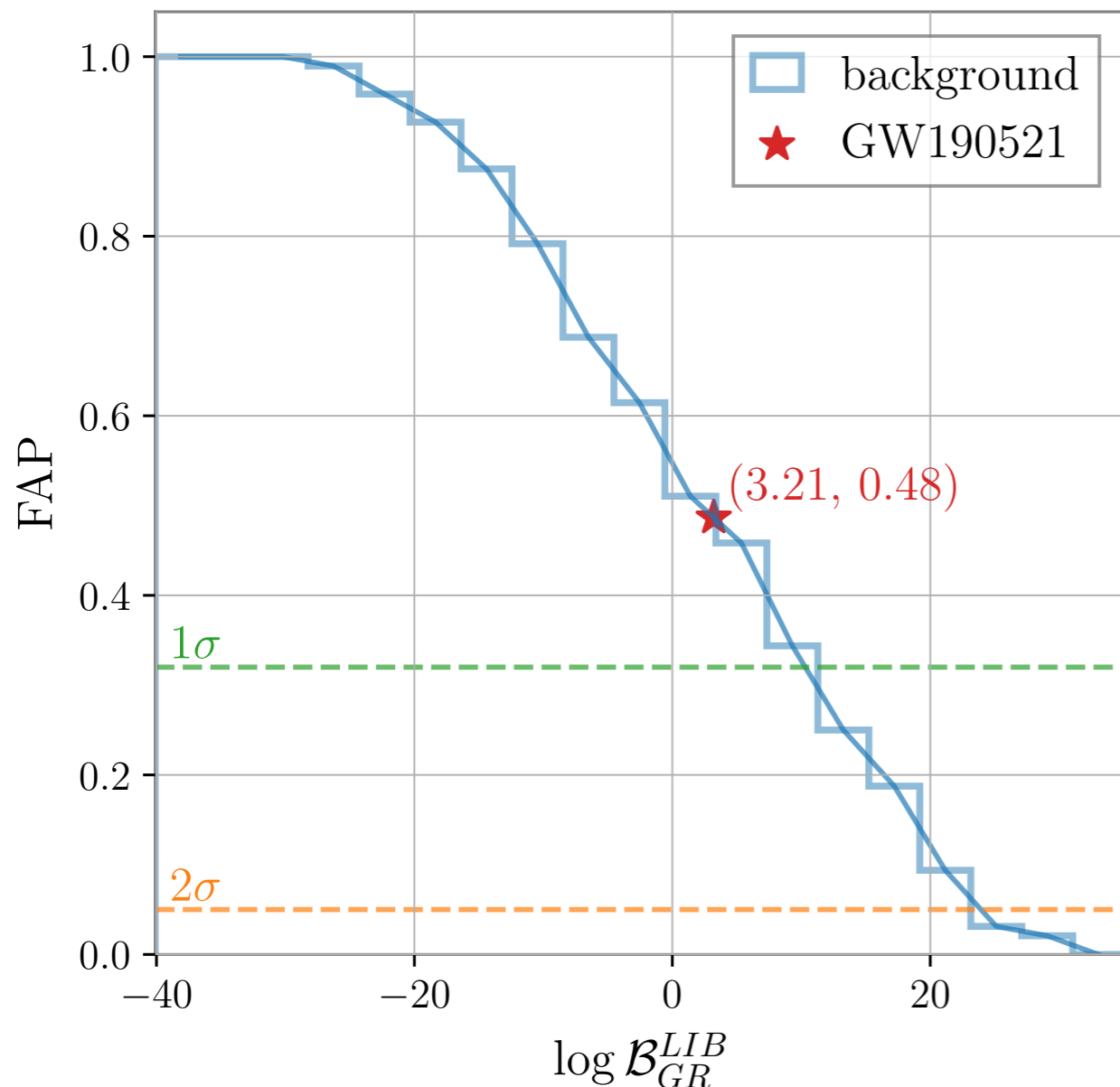
# Probing lens-induced birefringence (LIB)

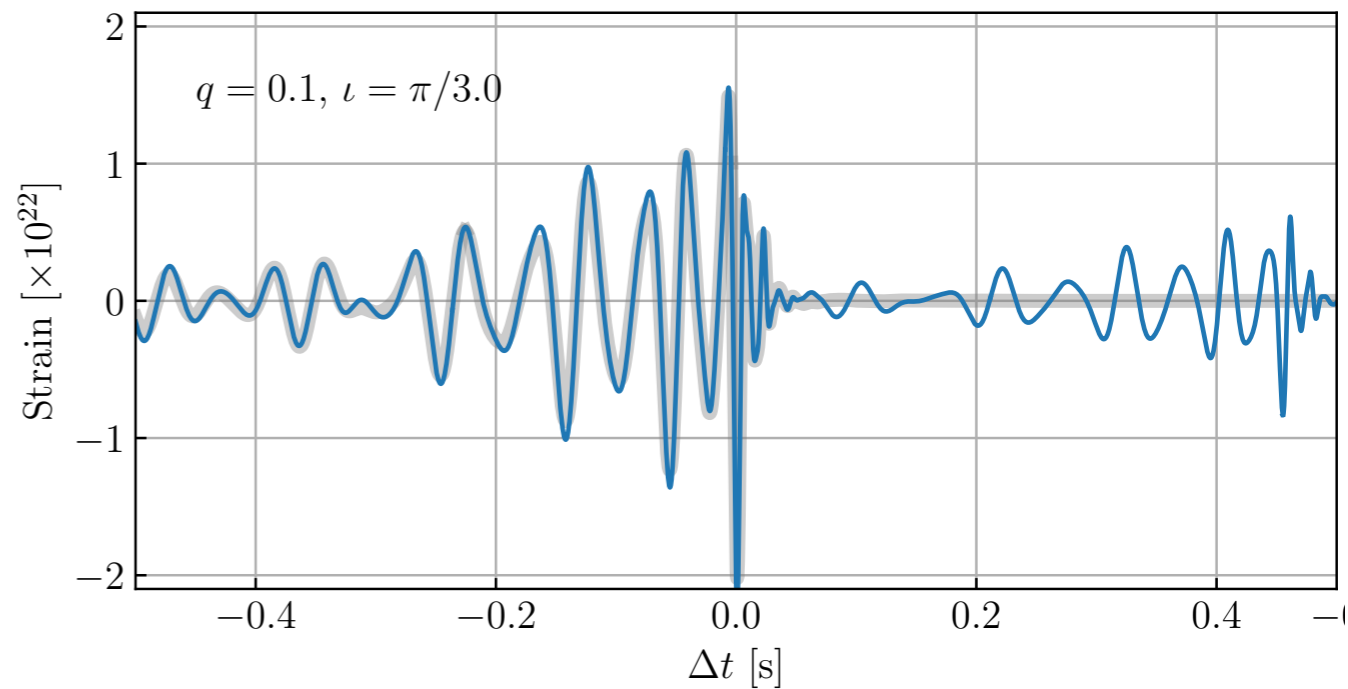
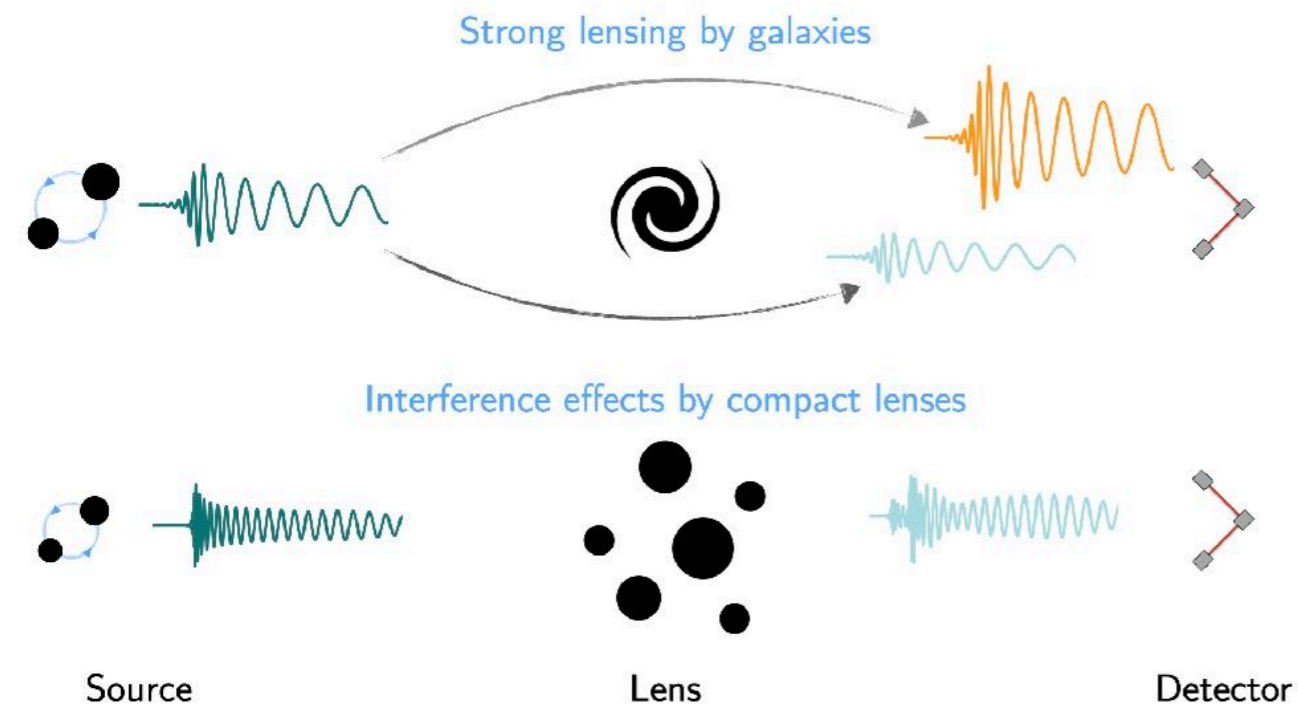
Srashti Goyal  
(ICTS)



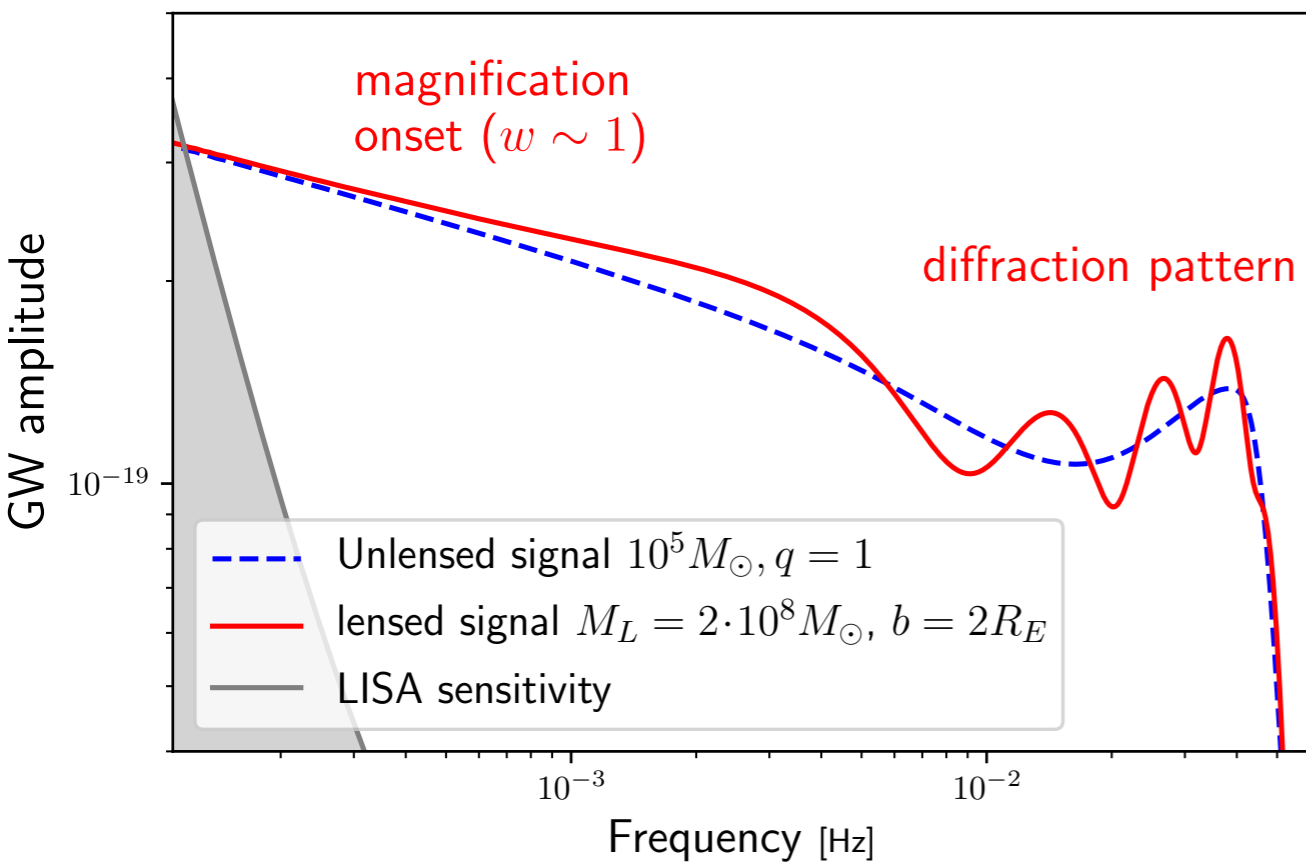
# Probing lens-induced birrefringence (LIB)

Srashti Goyal  
(ICTS)





***Multiple images, type II distortion, expected soon!***



***Wave effects, PBH, sub-halos***



***Lensing beyond general relativity***



Medfinansieret af Den Europæiske  
Unions Connecting Europe-facilitet

# Join us!

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[ezquiaga.github.io/joinus](https://ezquiaga.github.io/joinus)

