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Cosmological tests of gravity

Imprints of cosmological tensions in reconstructed gravity

Kazuya Koyama

Institute of Cosmology and Gravitation,
University of Portsmouth

Pogosian, Raveri, Martinelli, KK, Silvestri, Zhao *Nature Astron.* 6 (2022) 2107.12990,
Raveri, Pogosian, Martinelli, KK, Silvestri, Zhao et.al. *JCAP* 02 (2023) 061 2107.12992

Standard model of cosmology

- Lambda (L) CDM model

  Einstein equations and matter conservation
  (isotropy and homogeneity)

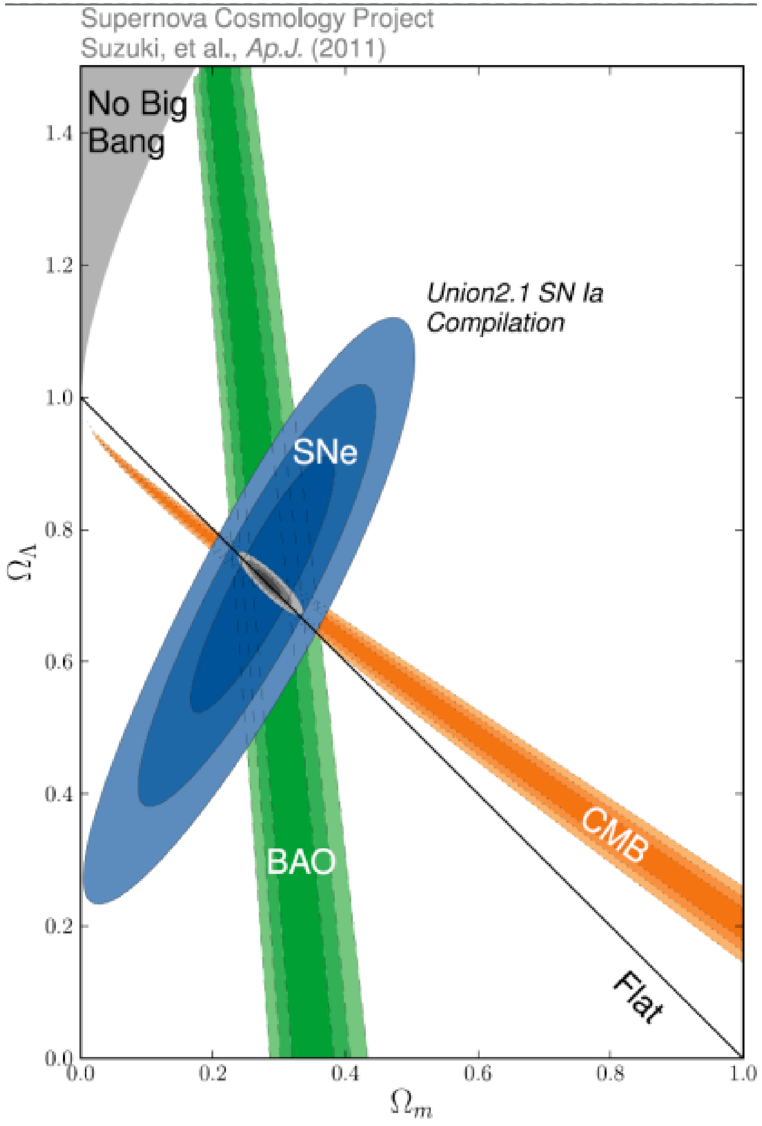
$$H(t)^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3} - \frac{K}{a^2}$$

$$\dot{\rho} + 3H(\rho + P) = 0, \quad \rho = \sum_i \rho_i$$

  The background expansion history

$$E(z) = \frac{H(z)}{H_0} \qquad 1 + z = \frac{a_0}{a}$$

$$E(z)^2 = \Omega_m(1+z)^3 + \Omega_r(1+z)^4 + \Omega_\Lambda$$



Linear perturbations

- Geometry (FRW metric + perturbations)

$$ds^2 = a(\eta)^2[-(1+2\Psi)d\eta^2 + (1-2\Phi)d\vec{x}^2]$$

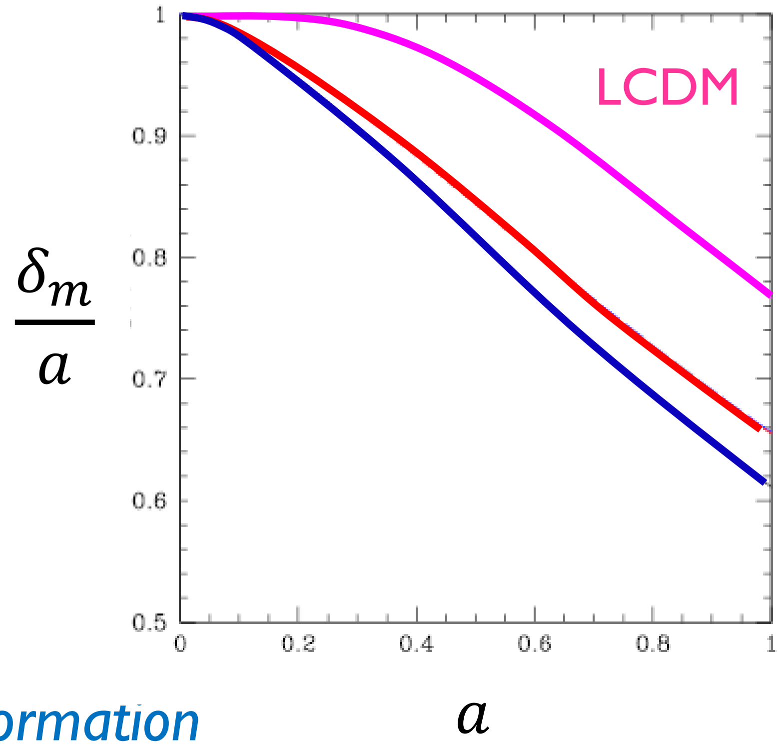
- Matter

$$T^0_0 = -\rho_m(1+\delta_m)$$
$$T^0_i = \rho_m v_{m_i}, \quad \partial^i v_{mi} = \theta_m$$

Energy-momentum conservation (no interaction)

$$\dot{\delta}_m - \frac{1}{a}\theta_m - 3\dot{\Phi} = 0$$
$$\dot{\theta}_m + H\theta_m - \frac{k^2}{a^2}\Psi = 0$$

$$\Rightarrow \quad \ddot{\delta}_m + 2H\dot{\delta}_m = \frac{k^2}{a^2}\Psi$$



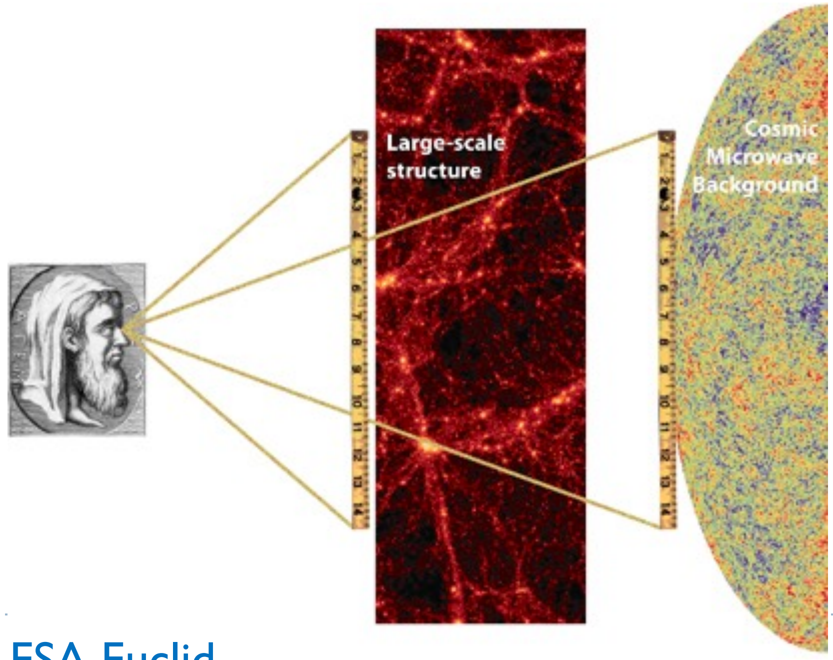
dark energy/modified gravity change the growth of structure formation

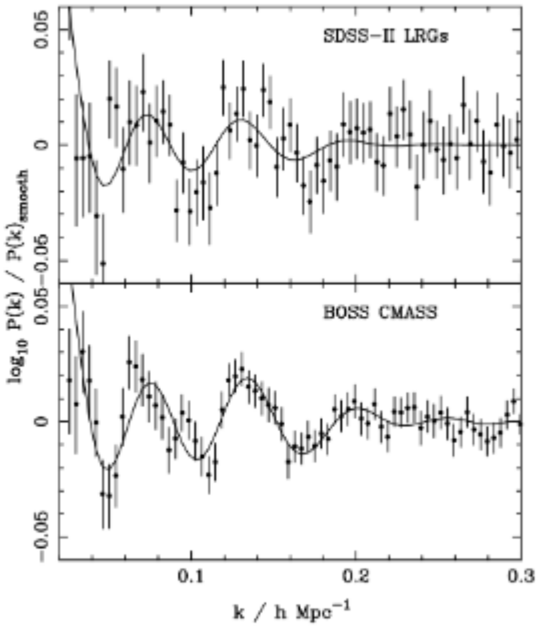
Observations –background

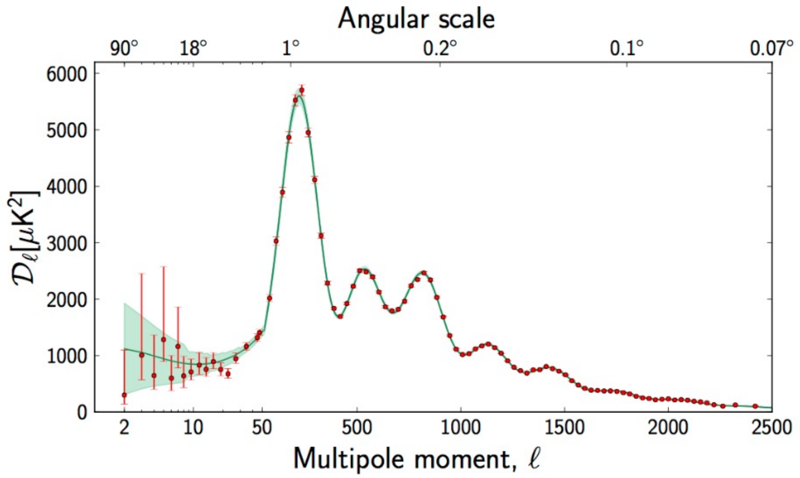
- Background $H(z)$

  Supernovae: luminosity distance

  CMB/Baryon Acoustic Oscillation (BAO): angular diameter distance
  (the sound horizon as a standard ruler)







ESA Planck

- ESA Euclid

BOSS

Observations

- Weak lensing

Bartelmann & Schneider astro-ph/9912508

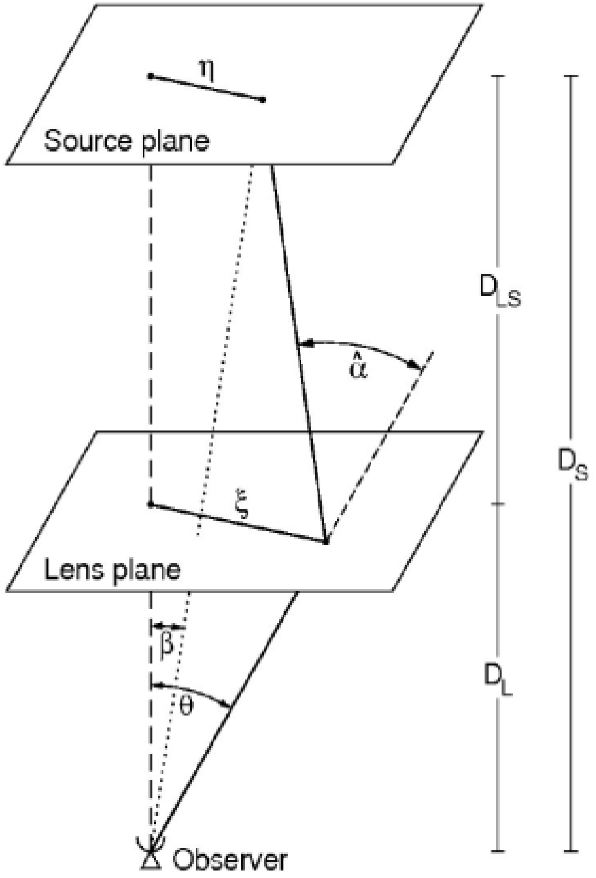
$$ds^2 = a^2\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

Convergence

$$\kappa(\vec{n}) = \int d\chi \boxed{\frac{D_{sL}D_L}{D_s}} \nabla_\perp{}^2 \phi_W(\eta_0 - \chi, \chi\vec{n}), \quad \phi_W = \frac{1}{2}(\Psi + \Phi)$$

geometry

Galaxy shape is determined by shear which can be computed from convergence



Observations

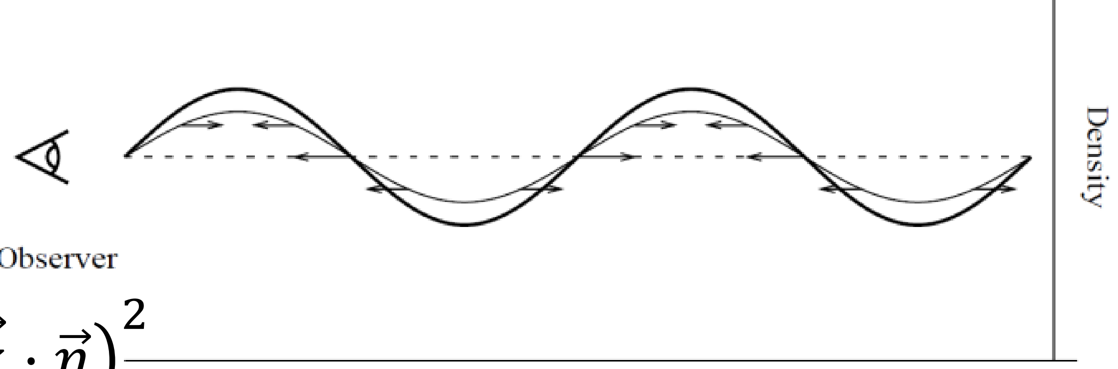
- Redshift distortions

  galaxies have peculiar velocities

  clustering of galaxies in redshift space

  is enhanced along the line of sight

$$\vec{s} = \vec{r} + (\vec{v}\cdot\vec{n})\vec{n}/H, \quad \vec{n} = \vec{r}/r$$



$$\delta^s(k,\mu) = \delta_m(k) - \mu^2\theta_m(k), \quad \mu^2 = \frac{\left(\vec{k}\cdot\vec{n}\right)^2}{k^2}$$

Hamilton astro-ph/9708102

If the continuity equation holds, the velocity dispersion is related to the growth rate

$$\delta^s(k,\mu) = \delta_m(k)\left(1 - \mu^2\frac{\theta_m(k)}{\delta_m(k)}\right) = \delta_m(k)(1+\mu^2 f) \qquad f = \frac{d\ln\delta_m}{d\ln a}$$

Consistency relation

Zhang & Jain Phys. Rev. D78 (2008) 063503
Song & KK JCAP 0901(2009) 048

- In GR, gravitational equations are given by

$$H^2 = \frac{8\pi G}{3}\rho_T, \quad \rho_T = \sum_i \rho_i$$

$$\frac{k^2}{a^2}\Phi = 4\pi G a^2 \rho_T \delta_T, \quad \rho_T\delta_T = \sum_i \rho_i \delta_i$$

- Consistency relation

$$\alpha(k,t) = \frac{2k^2}{3a^2H^2}\frac{(\Phi+\Psi)-\Psi}{\delta_T} = 1$$

background: H^2

Weak lensing: $(\Phi+\Psi)$

Redshift Space Distortion: Ψ

$$k^2\Psi = \frac{d(a\theta_m)}{dt}$$

Galaxy distribution: δ_T

$$\delta_g = b_T\delta_T$$

We have just enough number of observations to check the relation

Parametrisation

Amendola et.al JCAP 0804 (2008) 013
Zhao et.al. Phys. Rev. Lett. 103 (2009) 241301

- Background

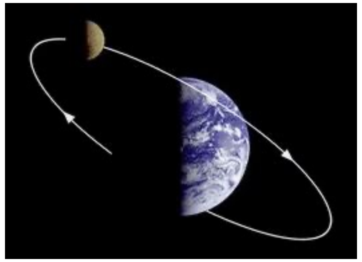
$$F(H^2) = \frac{8\pi G}{3}\rho_m \quad \Rightarrow \quad H^2 = \frac{8\pi G}{3}(\rho_m + \rho_{DE})$$

Equation of state $w_{DE}(z) = \frac{P_{DE}}{\rho_{DE}}$ can be ill-defined for modified gravity as ρ_{DE} can vanish

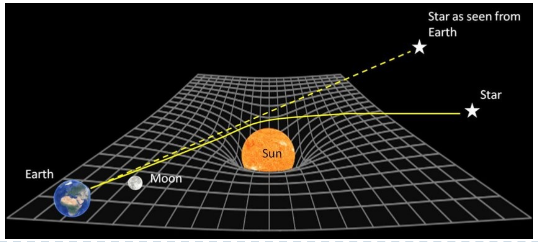
Instead, we can parametrise the effective dark energy density directly $\Omega_{DE}(z) = \frac{\rho_{DE(z)}}{\rho_{crit}}$

- Perturbations

$$k^2\Psi = -4\pi G a^2 \mu(z,k)\rho_m\delta_m$$: Newton potential

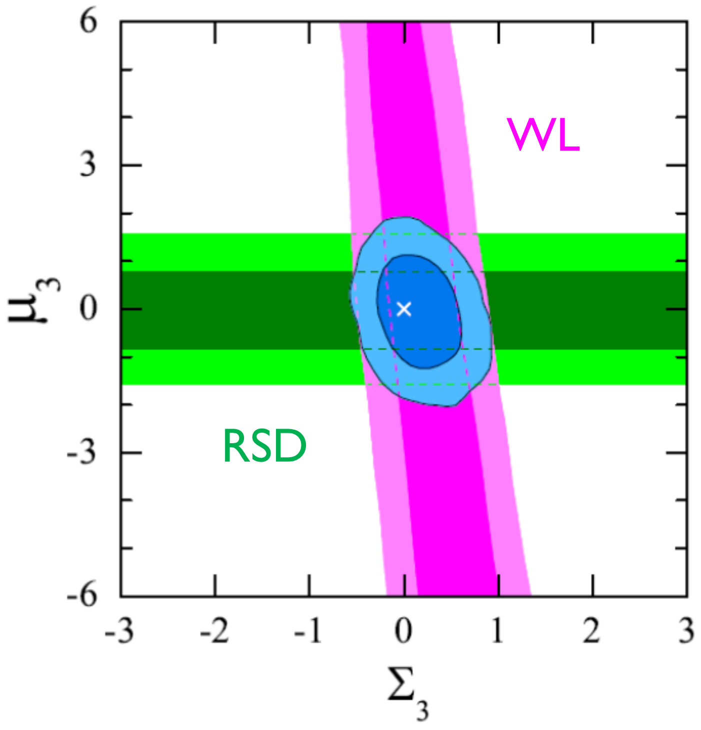


$$k^2(\Psi + \Phi) = -8\pi G a^2 \Sigma(z,k)\rho_m\delta_m$$: lensing potential



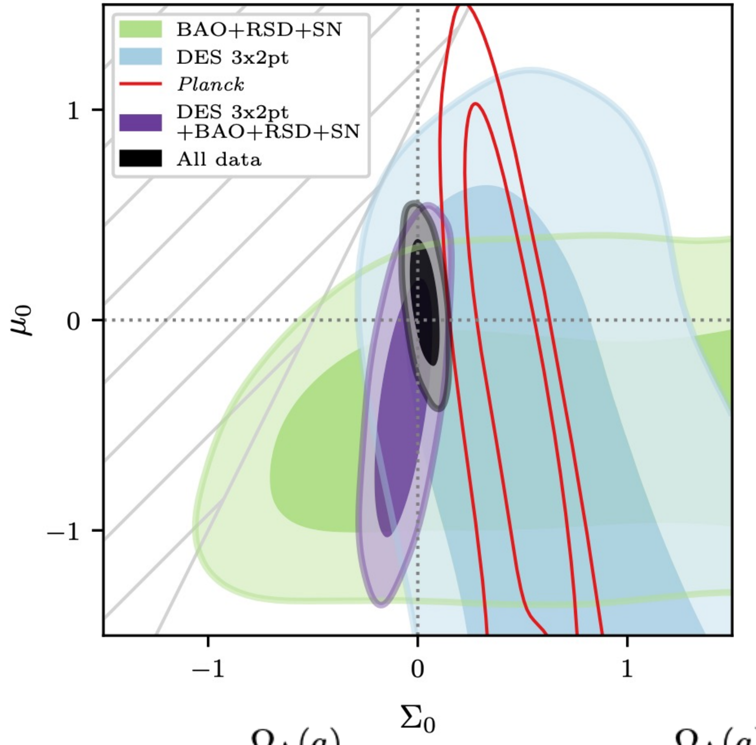
Current constraints

- Weak Lensing +Redshift space distortion



$$\Sigma = 1 + \Sigma_s a^s, \quad \mu = 1 + \mu_s a^s$$

Song et.al. PRD84 (2011) 083523



$$\Sigma(a,k) = \Sigma_0 \frac{\Omega_\Lambda(a)}{\Omega_{\Lambda,0}}, \qquad \mu(a,k) = \mu_0 \frac{\Omega_\Lambda(a)}{\Omega_{\Lambda,0}}.$$

DES Y3 arXiv: 2207.05766

From theory to data

Peirone et.al. arXiv:1712.00444
Espejo et.al. arXiv:1809.01121

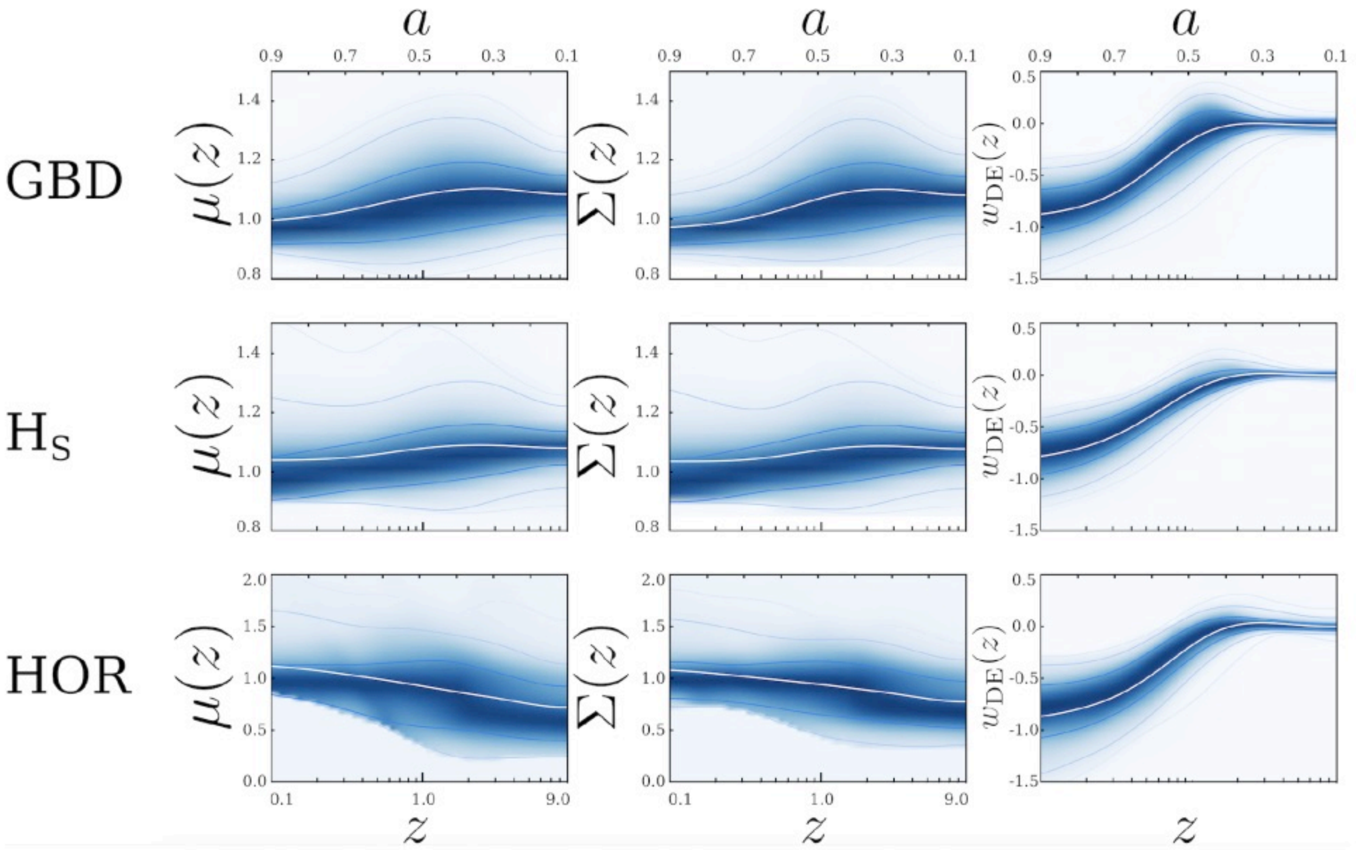
- Effective theory of dark energy

General description of the background and linear perturbations in a scalar-tensor theory

$$\mathcal{S} = \int d^4x\sqrt{-g}\Big\{\frac{m_0^2}{2}\Omega(t)R + \Lambda(t) - c(t)\,a^2\delta g^{00} + \frac{M_2^4(t)}{2}\left(a^2\delta g^{00}\right)^2 - \frac{\bar{M}_1^3(t)}{2}\,a^2\delta g^{00}\,\delta K^\mu_\mu + \frac{\bar{M}_2^2(t)}{2}\left[\left(\delta K^\mu_\mu\right)^2 - \delta K^\mu_\nu\,\delta K^\nu_\mu - \frac{a^2}{2}\delta g^{00}\,\delta\mathcal{R}\right] + \Big\} + S_m[g_{\mu\nu},\chi_m],$$

a random sample of these functions

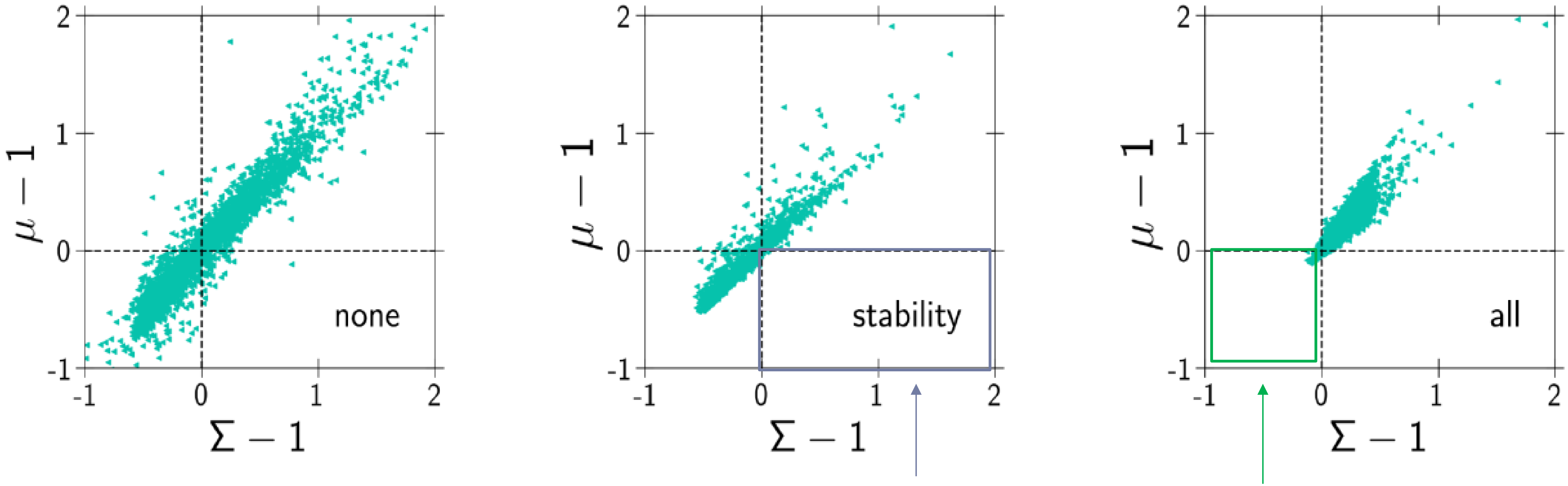
$$f(a) = \frac{\sum_{n=0}^{N}\alpha_n\,(a-a_0)^n}{1+\sum_{m=1}^{M}\beta_m\,(a-a_0)^m}$$



The role of stability and observational prior

Peirone et.al. arXiv:1712.00444

- Horndeski theory (the most general scalar tensor theory with 2[nd] order e.o.m)



Stability condition removes this part

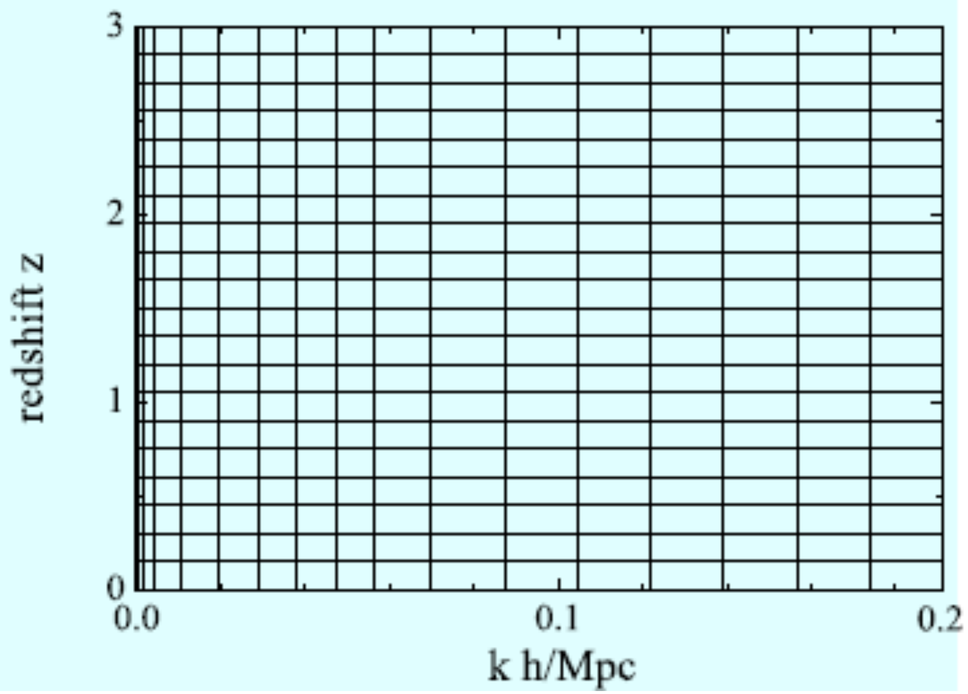
Observational prior removes this part

Model independent constraints

- Make bins

  treat $\mu(k_i, z_i)$, $\Sigma(k_i, z_i)$ in each bin as parameters

  Errors on these parameters are highly correlated



- Principal component analysis

  Diagonalise the covariance matrix

$$C_p = W\ \Lambda^{-1}\ W^T, \quad W = (\vec{e}_1, \vec{e}_2, \ldots,)$$
$$p = \{\mu_{1,}\ldots, \Sigma_1, \ldots.\}$$

  Uncorrelated parameter

$$q_i = -1 + \sum_j W_{ij} p_j / \sum_j W_{ij} \qquad \text{GR: } q_i = 0$$

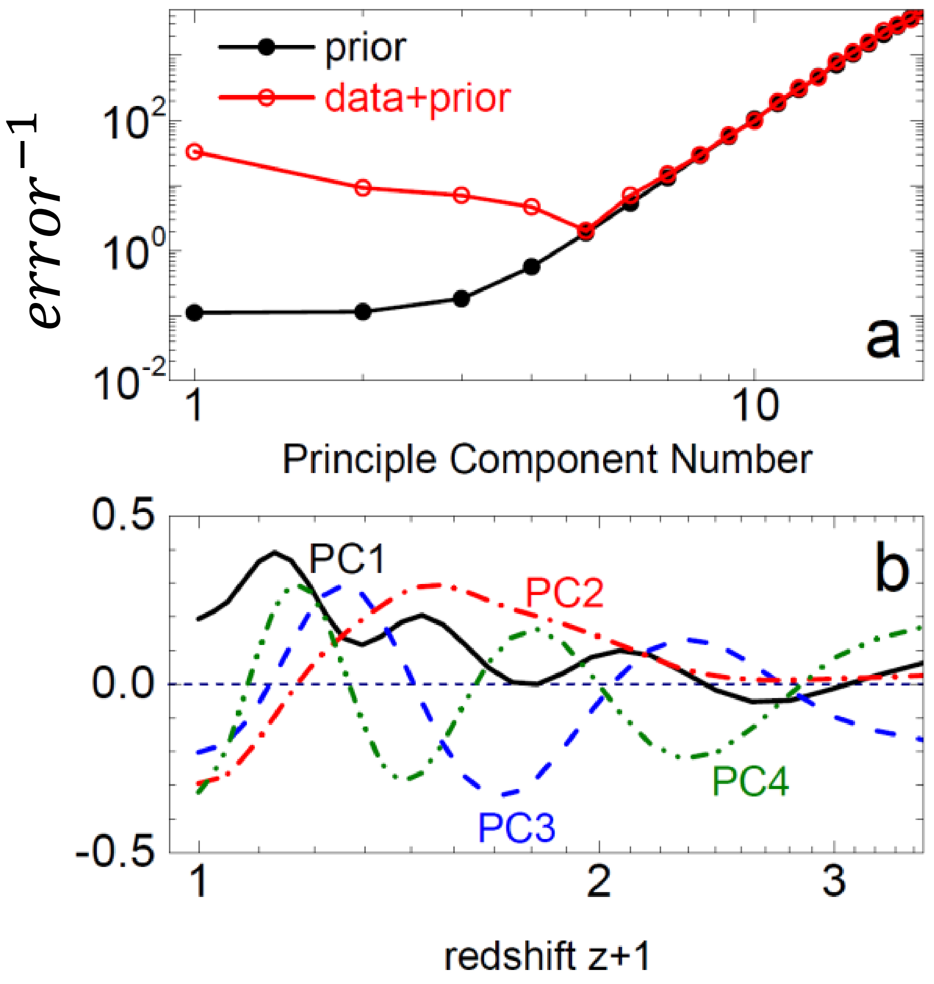
Theoretical prior

- High frequency modes

  Reconstruction is prone to ill-constrained oscillating modes that depend on the size of bins

  It can also suffer from over-fitting, i.e. reconstructed functions try to wiggle through data points

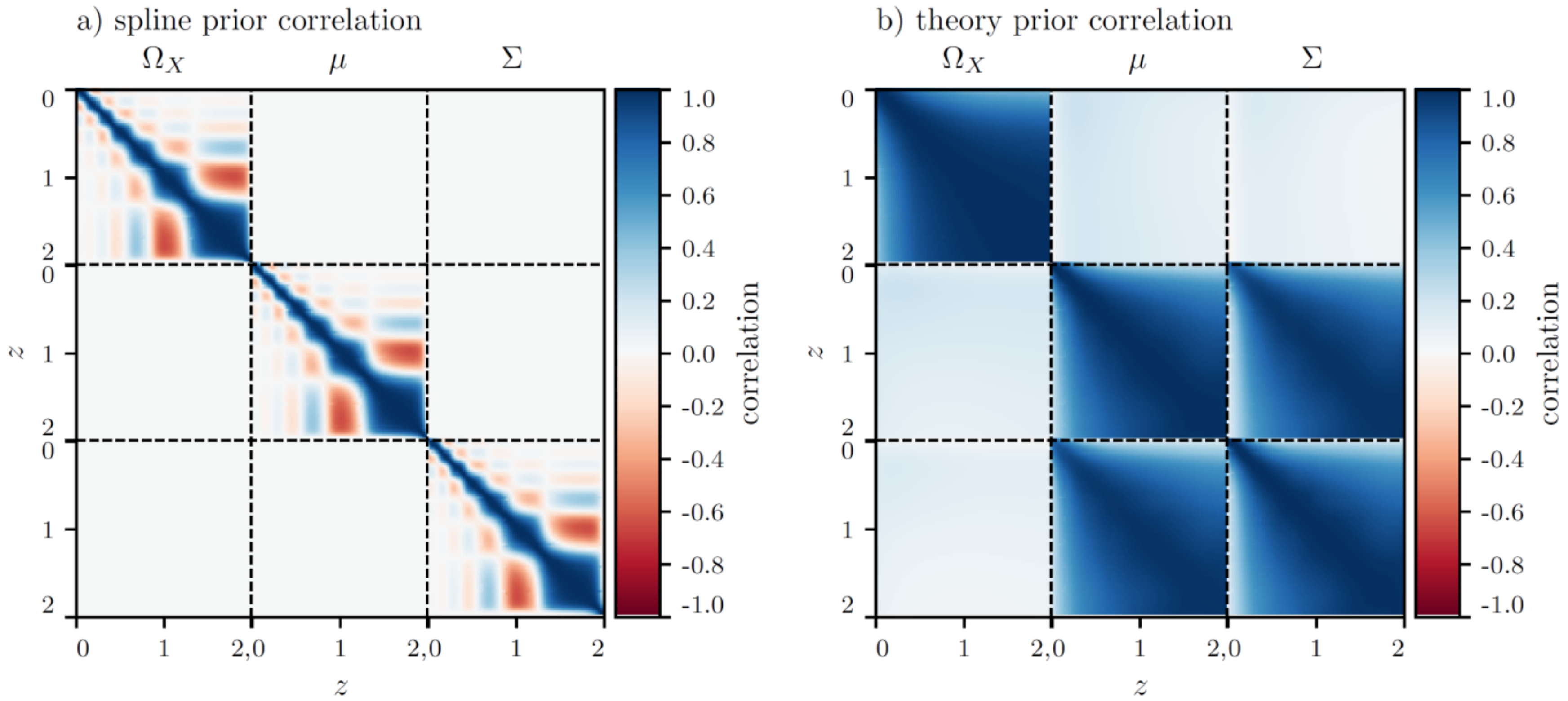
  It requires a theoretical prior to set the smoothness of these functions



Theoretical prior

Espejo et.al. arXiv:1809.01121

- Correlation between bins as well as functions from Horndeski theory

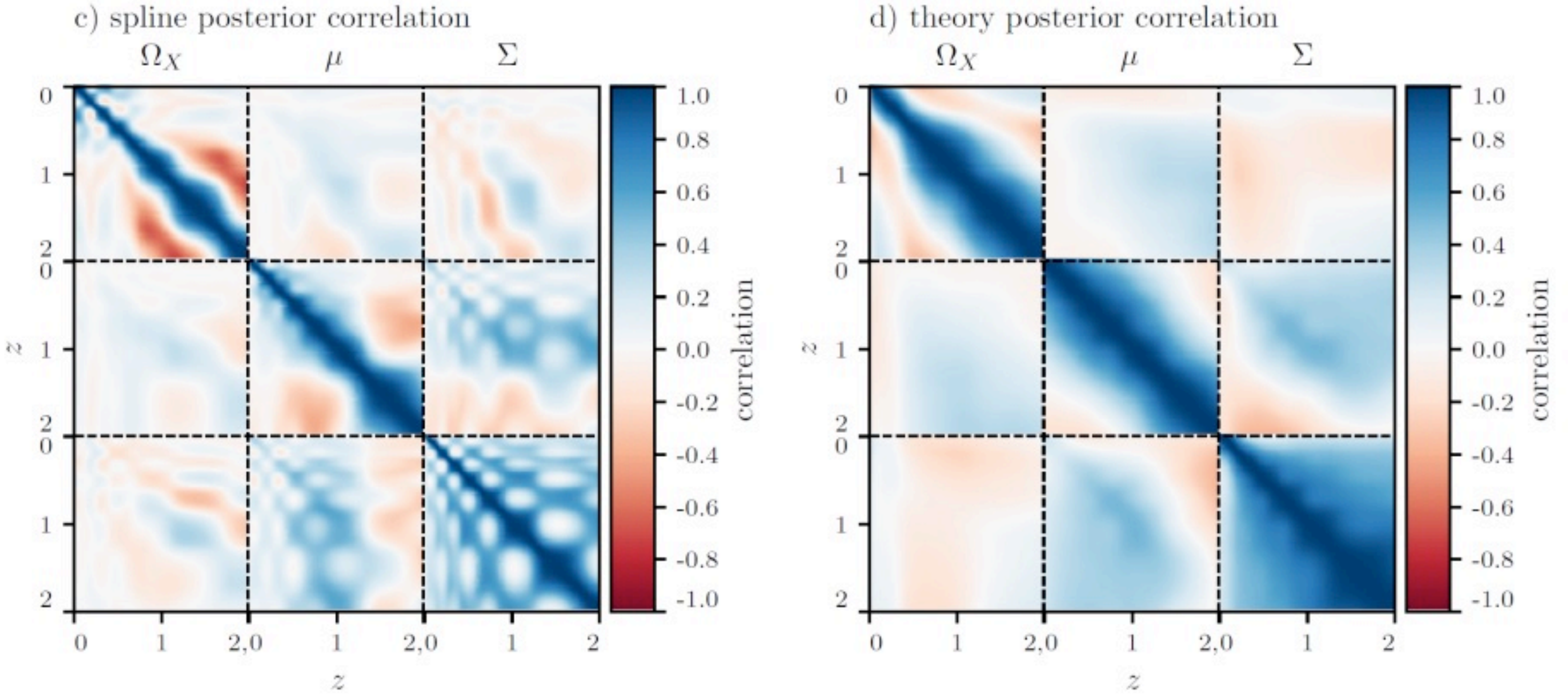


Correlation without prior introduced by binning

Theoretical prior

Pogosian, Raveri, Martinelli KK, Silvestri, Zhao 2107.12990, 2107.12992

- Correlation after including observational constraints



"Tensions" with LCDM – Hubble constant

Riess et.al. arXiv:2012.08534

- Local measurement of Hubble constant

$$m = M + 25 + 5\log_{10} D_L(z),$$
$$= -5a + 5\log_{10} c\hat{d}_L(z)$$

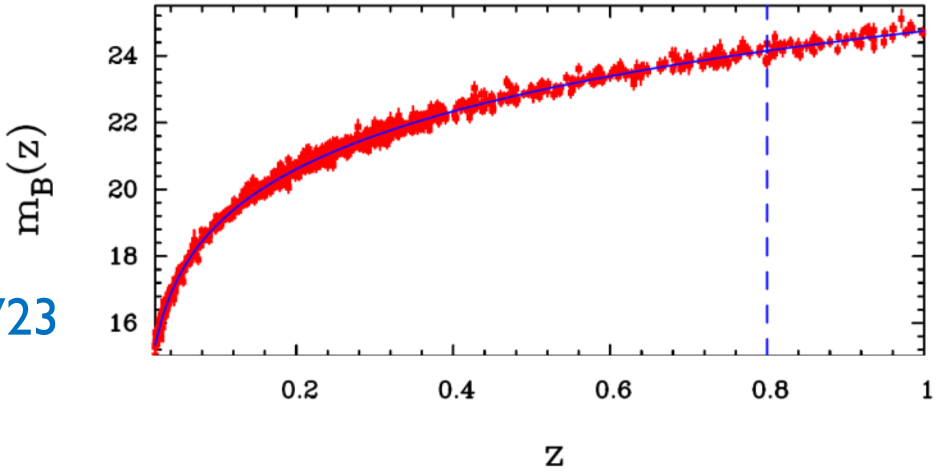
$$5a = -(M + 25 - 5\log_{10} H_0)$$
$$\hat{d}_L(z) = H_0 D_L(z)/c$$

m: apparent magnitude, M: absolute magnitude

- Pantheon SNe

$$a_B = 0.71273 \pm 0.00176$$

Efstathiou arXiv:2103.08723



- Local distance ladder measurement (SH0ES)

$$M_B^0 = -19.253 \pm 0.027 \text{ mag}$$

$$\Rightarrow \mathrm{H}_0 = 73.04 \pm 1.04 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

Riess et.al. arXiv:2112.04510

“Tensions” with LCDM – weak lensing

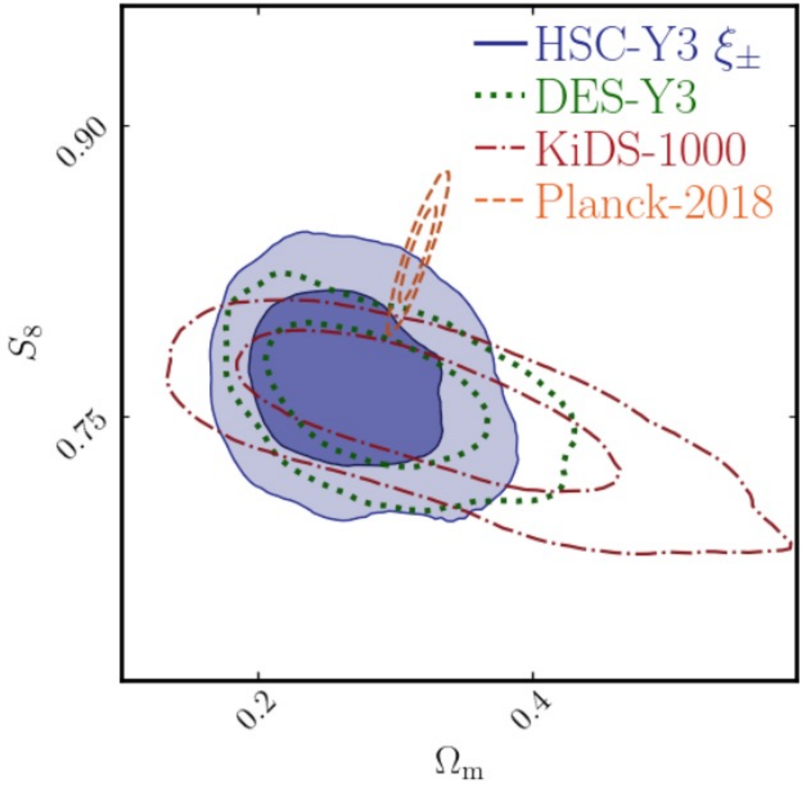
- Weak lensing

The amplitude of weak lensing is determined by the S8 parameters in LCDM

$$S_8 \equiv \sigma_8 \sqrt{\Omega_{\rm m}/0.3}$$

σ_8 : amplitude of fluctuations

The prediction from CMB is slightly larger than the values from WL surveys.
Again, CMB constraint here assume LCDM



HSC Y3. arXiv:2304.00702

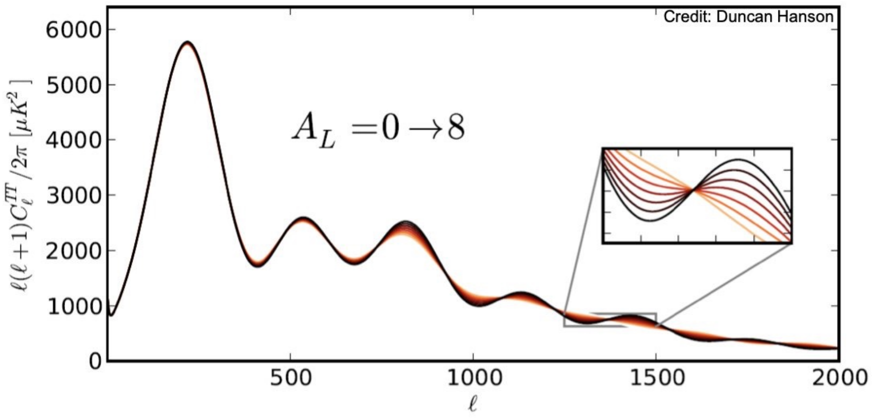
DES: https://www.darkenergysurvey.org/; HSC: https://www.naoj.org/Projects/HSC/;
KiDS: http://kids.strw.leidenuniv.nl/DR3/lensing.php

“Tensions” with LCDM – CMB lensing in TT

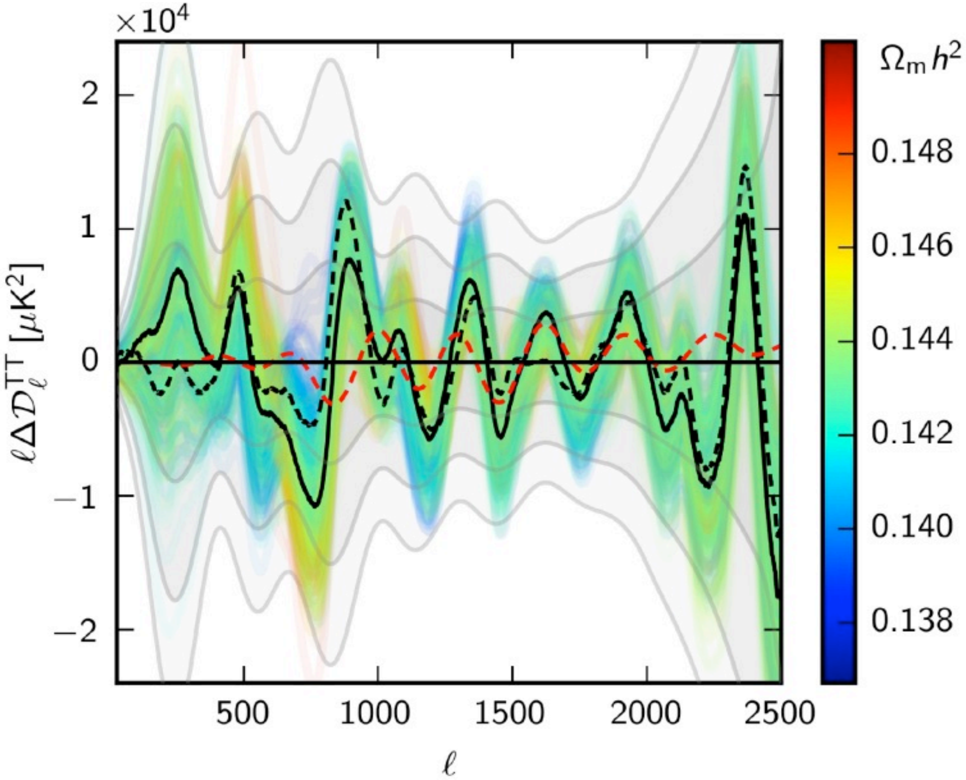
- **CMB temperature power spectrum**

  The Planck CMB temperature power spectrum is well fitted by LCDM but at high ell, there are residual oscillations

  CMB peaks are smeared out by CMB lensing. These residuals are well fitted if CMB lensing amplitude is larger than that in LCDM



$$\tilde{C}_\ell^{\phi\phi} = A_L C_\ell^{\phi\phi}$$



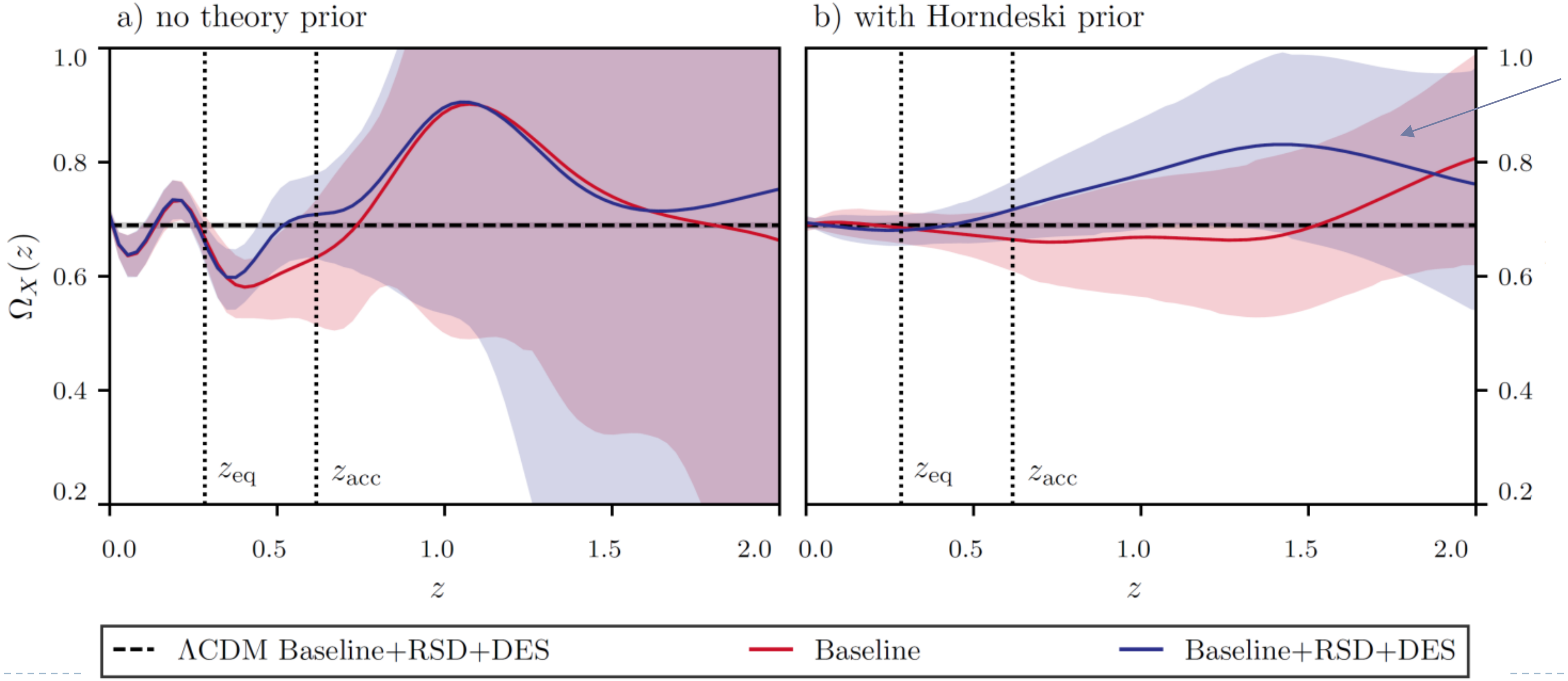
$$A_L = 1.243 \pm 0.096$$

Planck 2018 arXiv:1807.06209

Reconstructed functions (late time modifications)

- Data Baseline: Planck 2018 (T, E, lensing) + BAO (eBOSS+) + SNe (Pantheon)
  RSD: eBOSS + BOSS
  DES: Dark Energy Survey year 1

10 values (nodes) uniformly spaced in $a \in [1, 0.25]$
$z \in [0, 3]$



$\Omega_{DE}(z) > \Omega_{DE}(0)$

Larger energy density of DE suppresses the growth of structure leading to smaller S8

Theoretical prior suppresses oscillations

Pogosian, Raveri, Martinelli, KK, Silvestri, Zhao 2107.12990, 2107.12992

Hubble constant tension

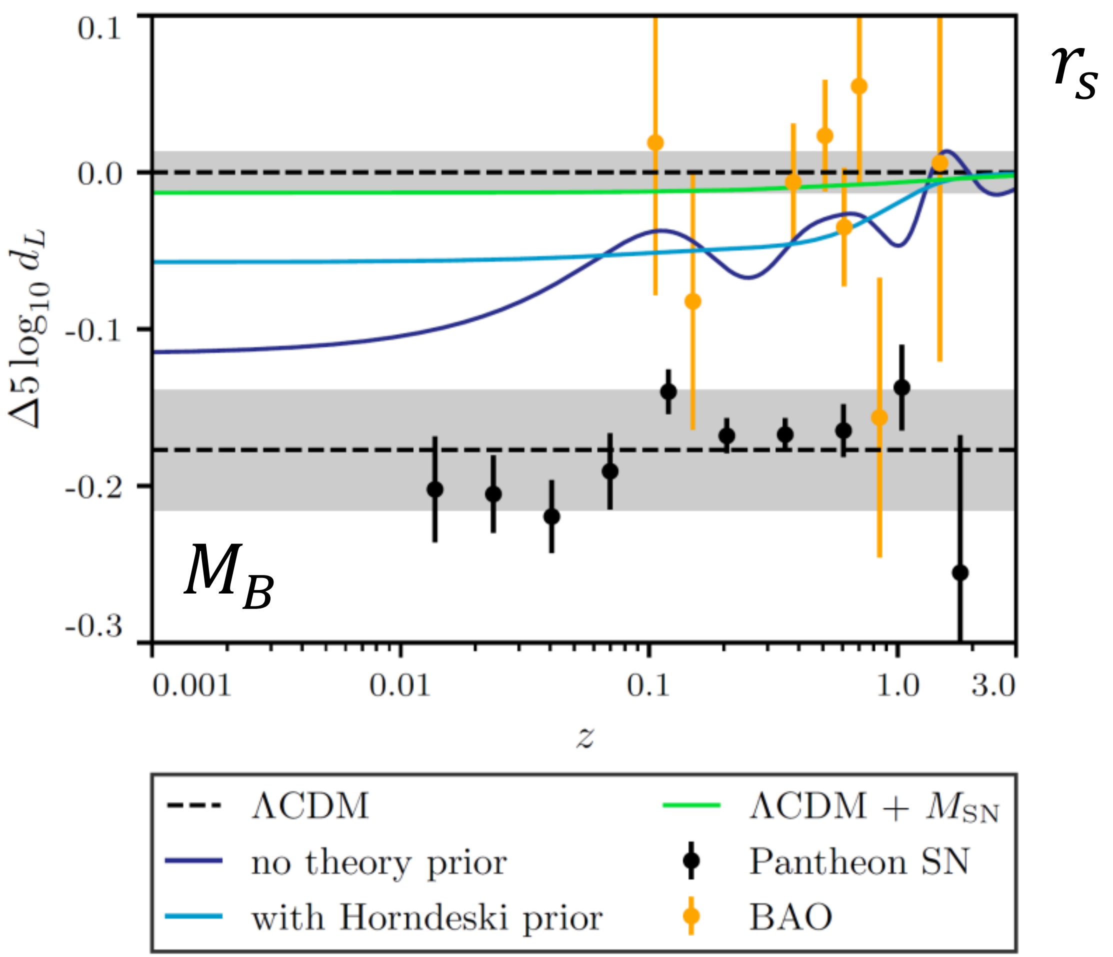
Pogosian, Raveri, Martinelli, KK, Silvestri, Zhao 2107.12990, 2107.12992

- Hubble constant tension

  The luminosity distance inferred from CMB and BAO does not agree with the one calibrated from SNe with the prior on the absolute magnitude from the local distance ladder

  This makes it hard for late time modifications to fully resolve the tension even though the fit can be improved from LCDM

  Distance measurements from CMB and BAO assumes the sound horizon in LCDM at early times

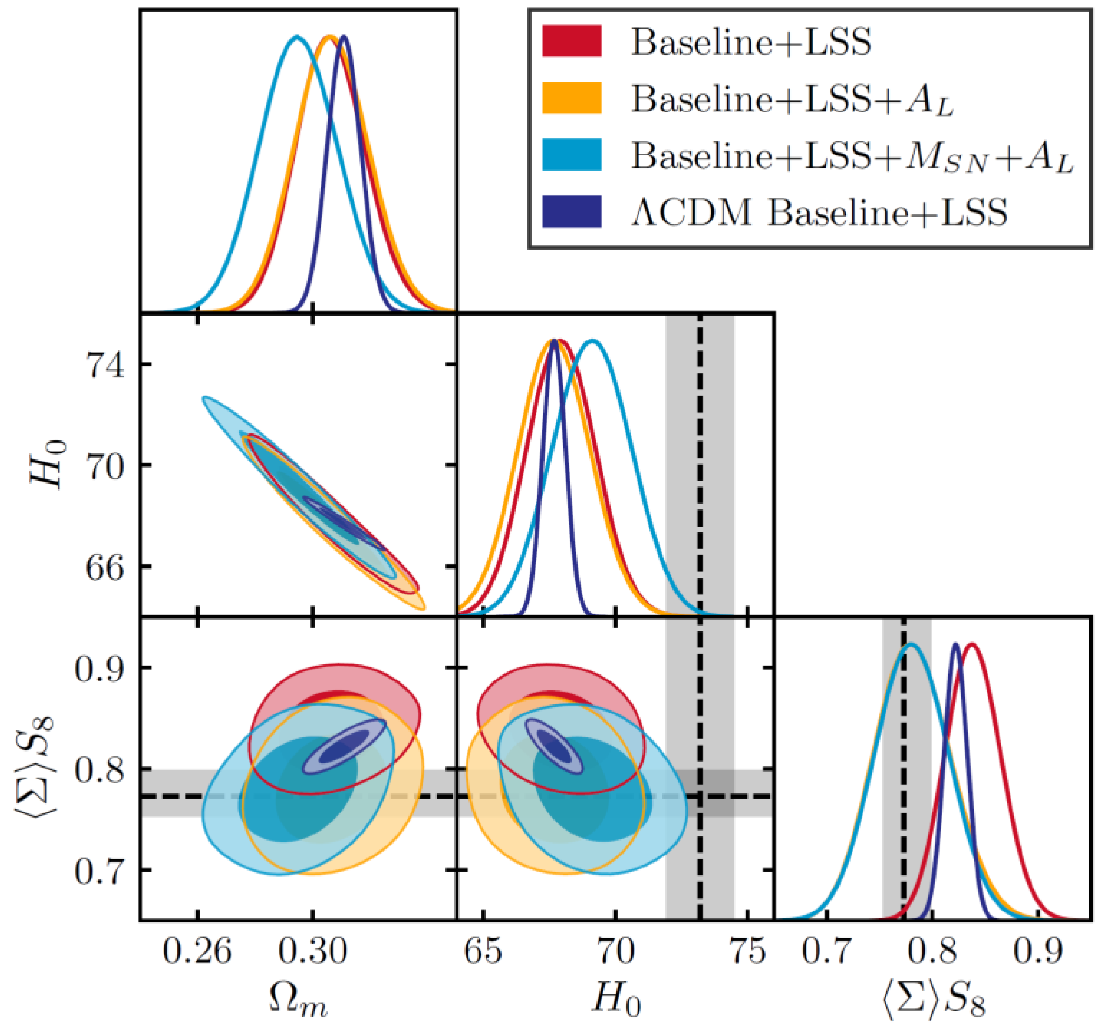


Implications for tensions in extended cosmologies

- Extended cosmologies

  $\Omega_{DE}(z), \mu(z), \Sigma(z)$

- Hubble constant tension
  - It is not possible to resolve the tension fully due to the inconsistency with BAO
- Lensing anomalies
  - CMB lensing anomaly can be resolved either by $\Sigma > 1$ or $A_L > 1$.
  - Fit to DES cannot be improved even if S_8 is lower if $\Sigma > 1$ as $\Sigma \times S_8$ stays the same.
  - We need $A_L > 1$ to improve fit to DES



- Pogosian, Raveri, Martinelli, KK, Silvestri, Zhao 2107.12990, 2107.12992

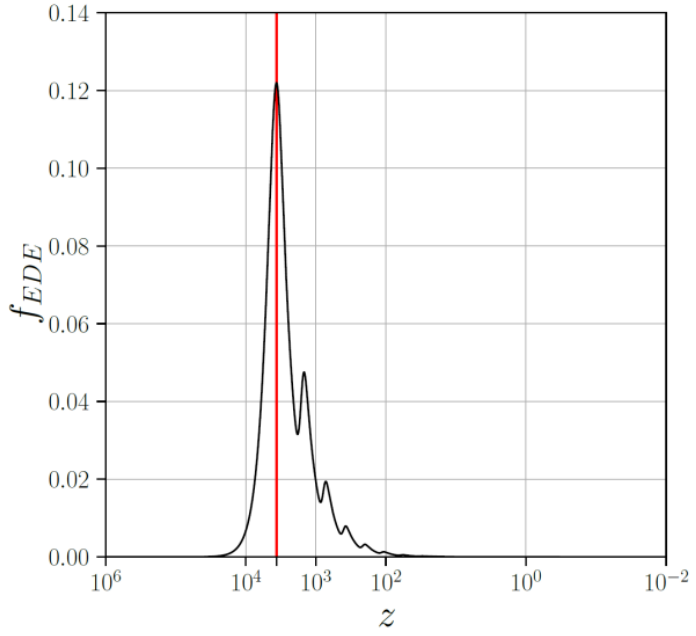
Hubble constant tension – early time solutions

- Reducing sound horizon

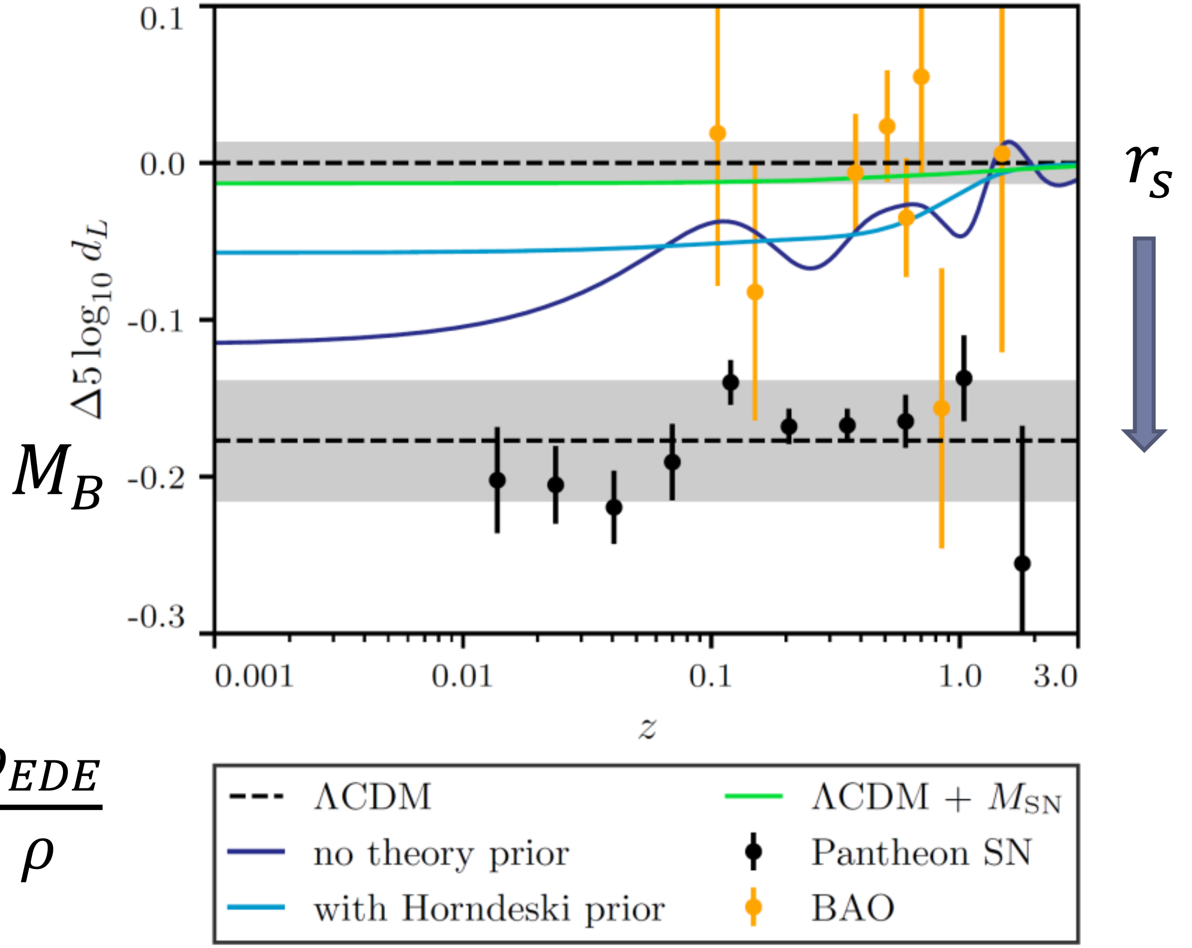
$$r_\star = \int_{z_\star}^{\infty} c_s(z)\mathrm{d}z/H(z)$$

early dark energy to increase $H(z)$

Poulin et.al. arXiv:1811.04083



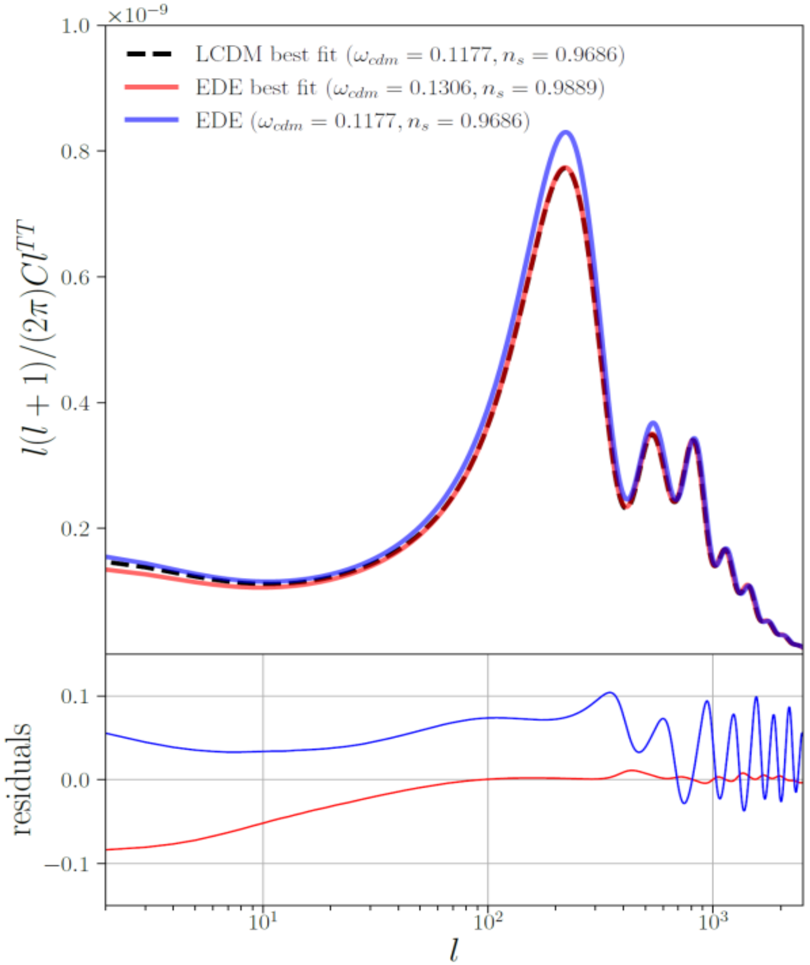
$$f_{EDE} = \frac{\rho_{EDE}}{\rho}$$

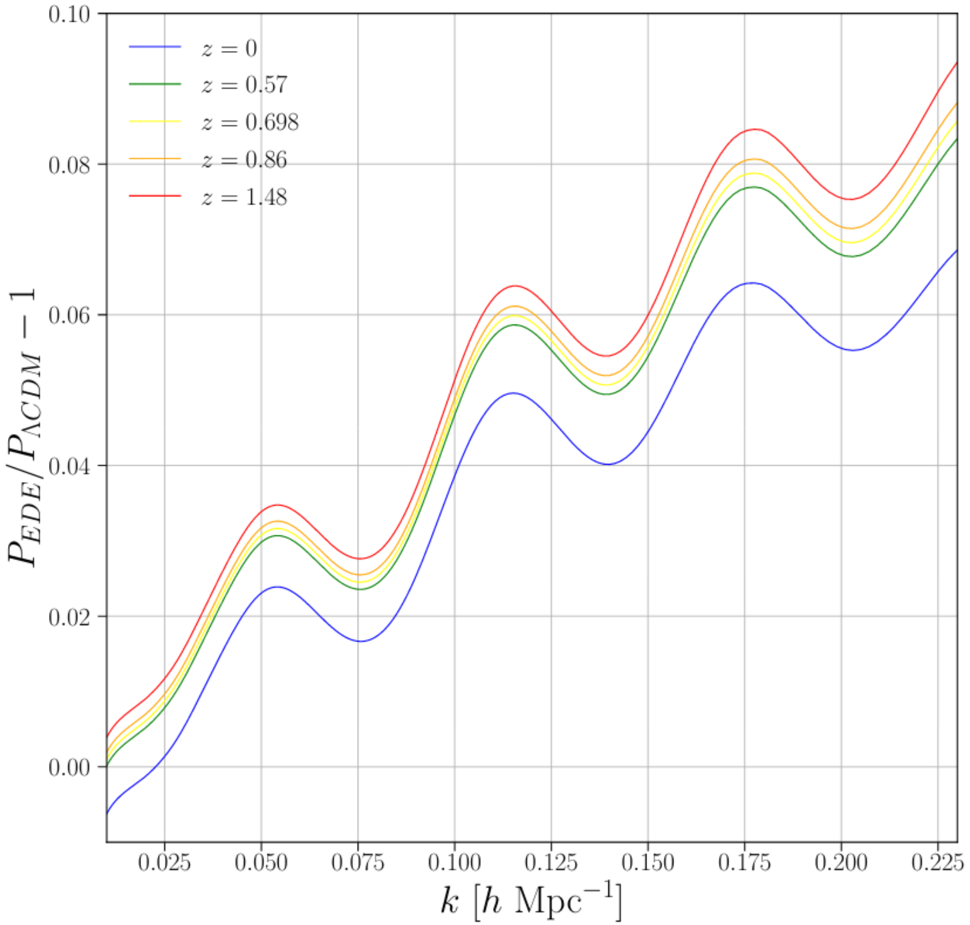


Early dark energy

- CMB and Large Scale Structure

  larger Ω_m, n_s are required to fit CMB, which leads to a larger amplitude of P(k)

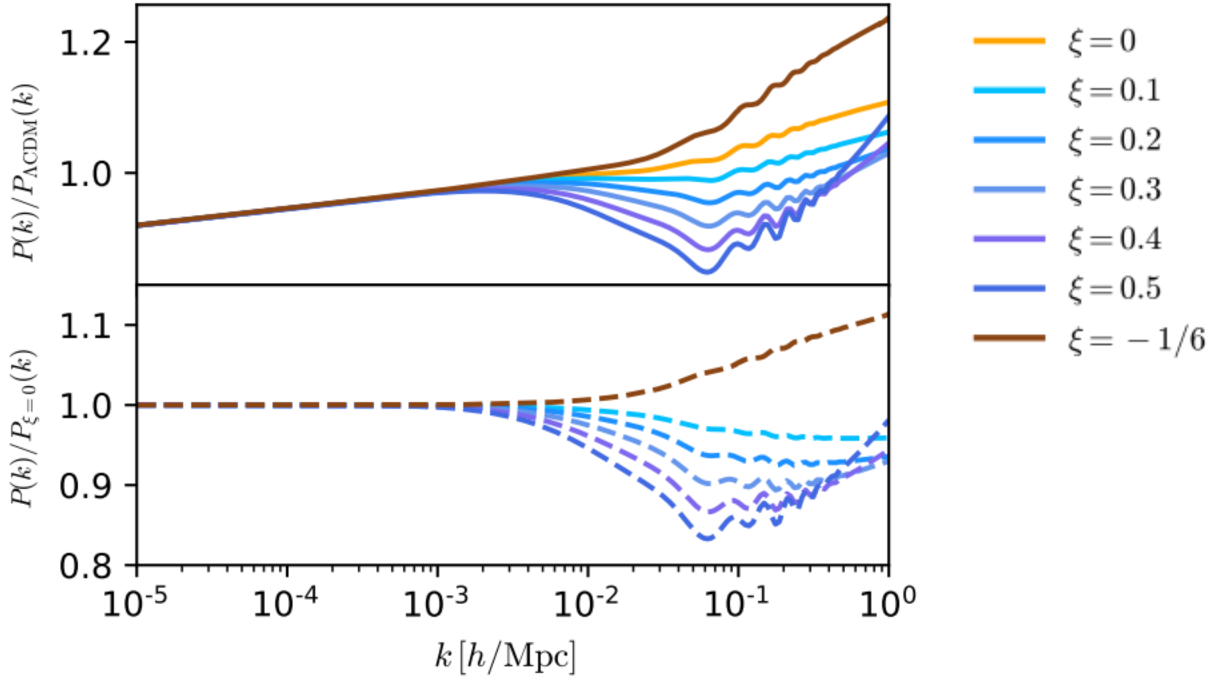


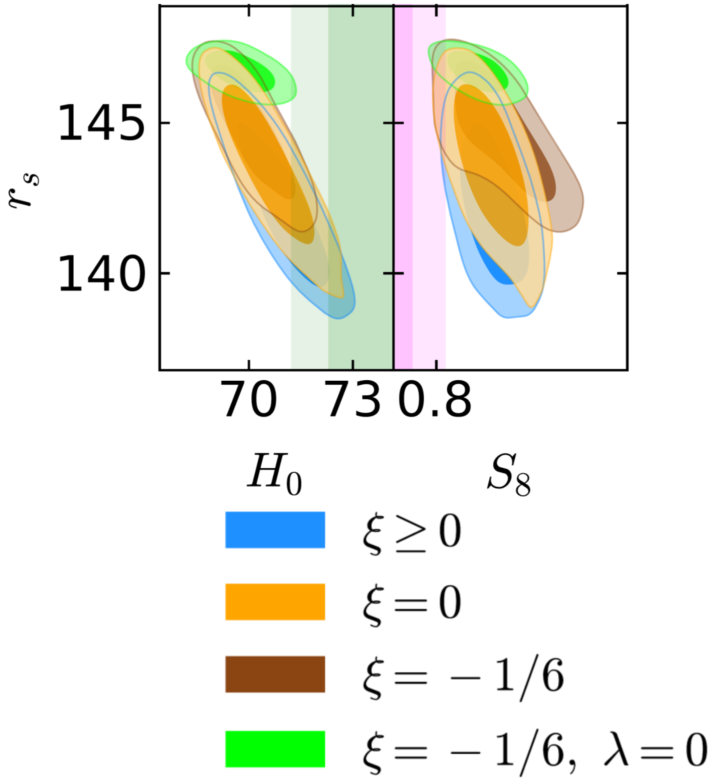


Early dark energy/modified gravity

- Example

$$S=\int d^4x\sqrt{-g}\left[\frac{F(\sigma)}{2}R-\frac{g^{\mu\nu}}{2}\partial_\mu\sigma\partial_\nu\sigma-\Lambda-V(\sigma)\right]\qquad F(\sigma)=M_{pl}^2+\xi\sigma^2\qquad V(\sigma)=\lambda\sigma^4/4$$





Braglia, Ballardini, Finelli, KK 2011.12934

Conclusion

- Cosmological "tensions" in reconstructed gravity $\Omega_X(z), \mu(z), \Sigma(z)$
  - Hubble constant tension (H_0)
  - Lensing anomalies in CMB (A_L)
  - Weak lensing amplitude (S_8)

Late-time dynamical dark energy and modifications of gravity are not likely to offer a solution to the Hubble constant (H_0) tension, or simultaneously solve the A_L and S_8 tensions.

Early time modifications are required to fully resolve the tensions (H_0 and A_L)

(some early time modifications, i.e. early DE make S_8 tension worse)