Probing inflation:

precision physics, exploratory physics, and formal aspects

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European Research Council
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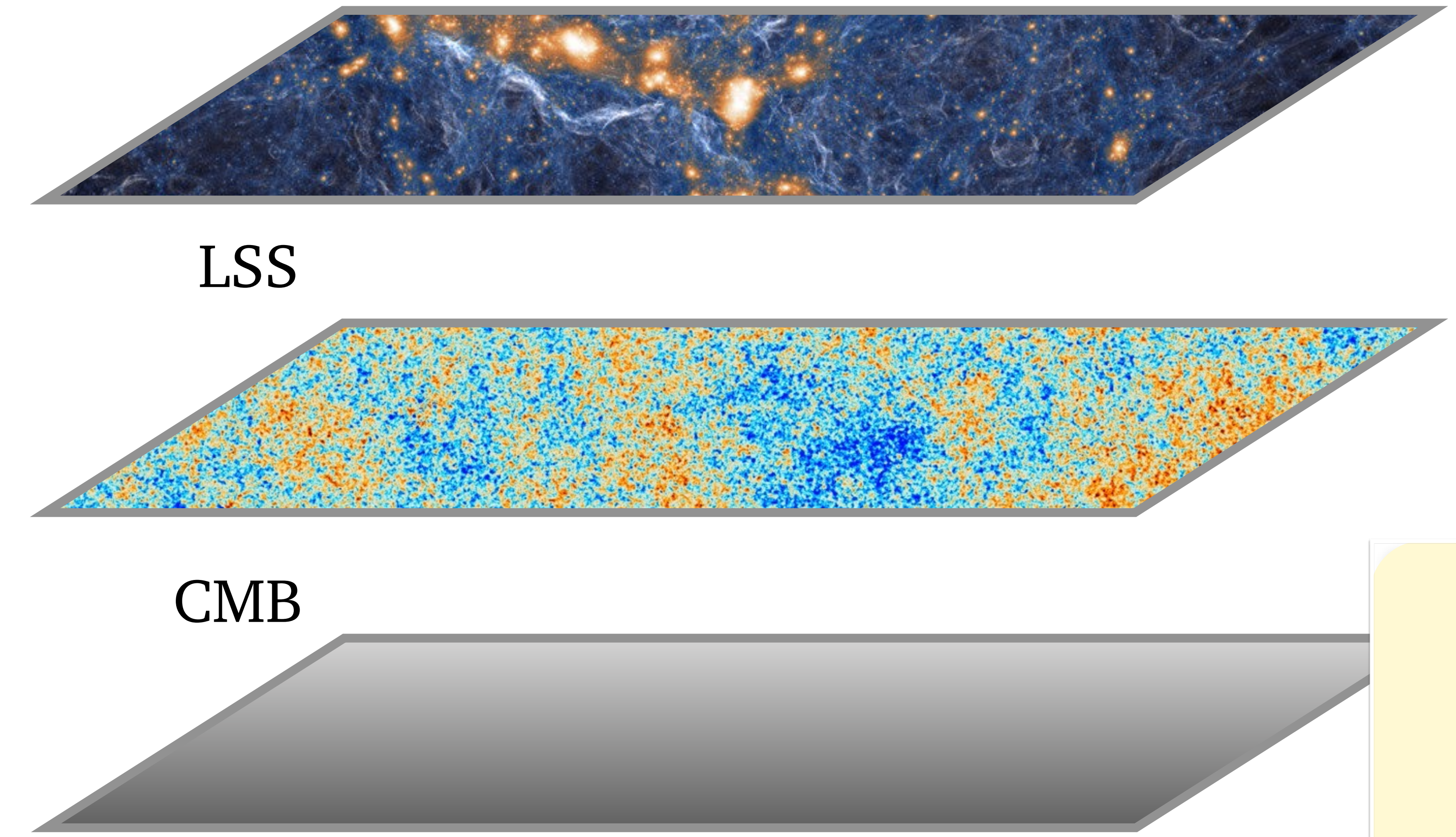
GEODESI

A detective's work

Time



LSS

CMB

Reheating surface

Observations

observational data

Statistical properties

$$\mathbb{P}\left(\frac{\delta\rho}{\rho}, h_{ij}\right)$$

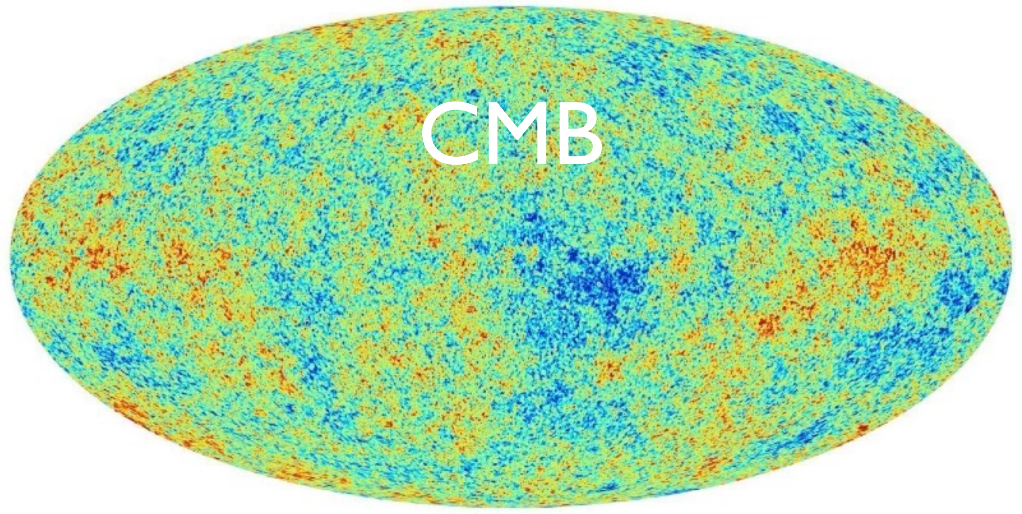
Physics of inflation?

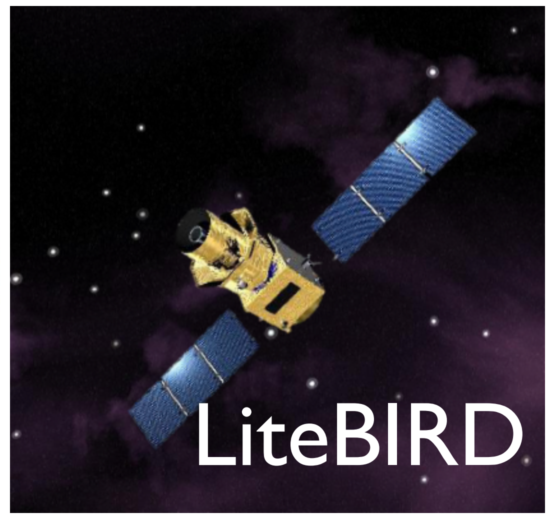
theoretical data

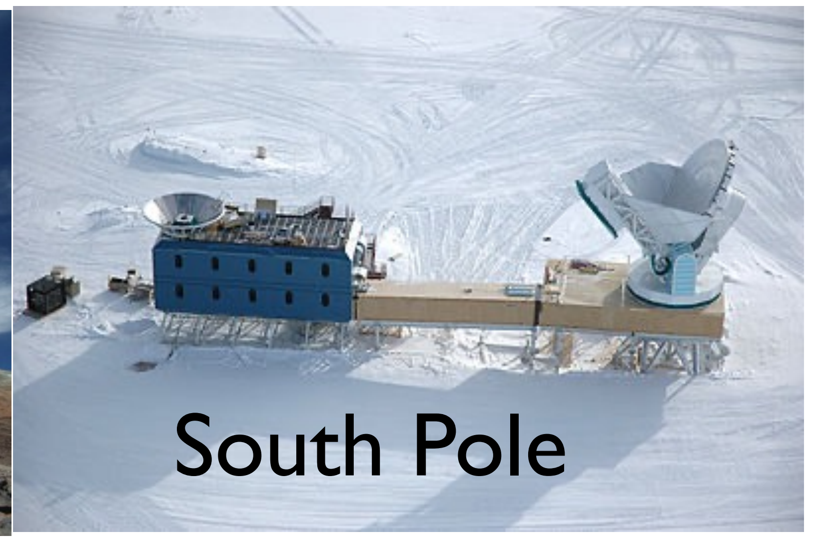
"Data! data! data!"

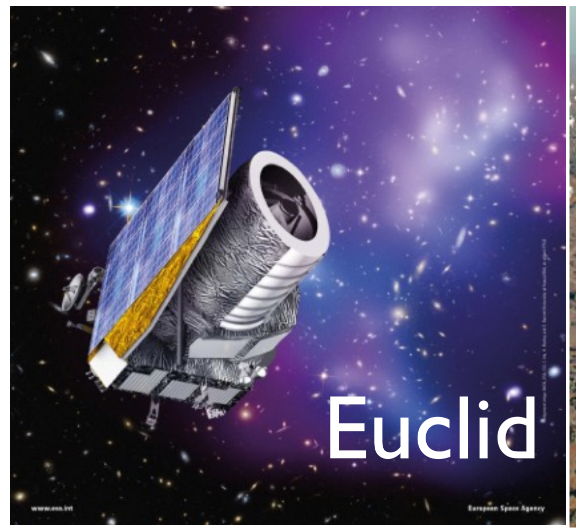
Quantum + gravitational physics, tested observationally!

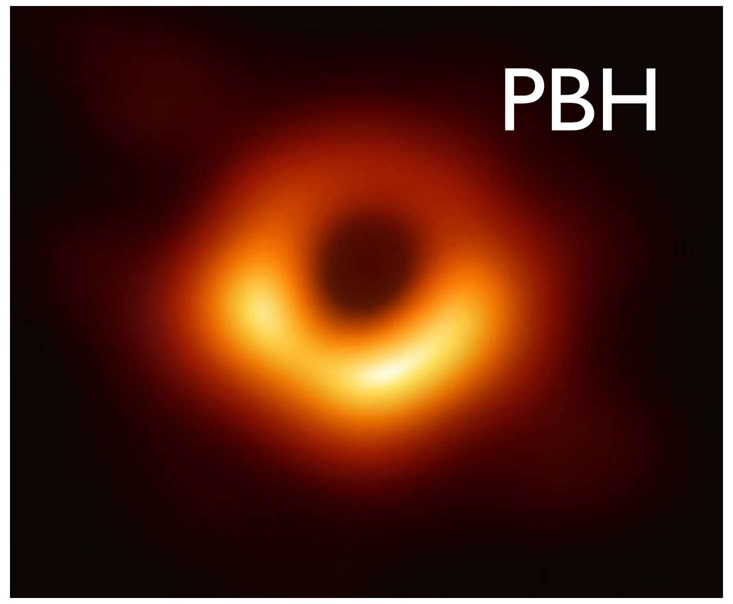
+

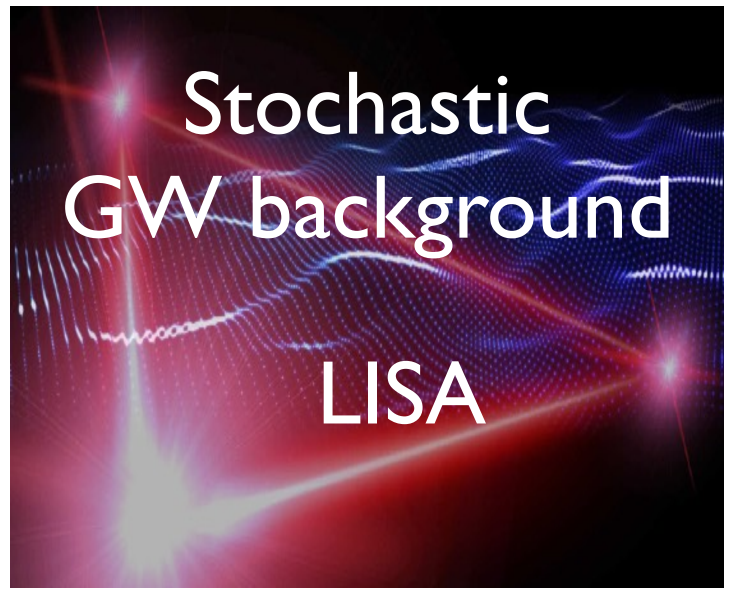












Physics of inflation?

What is the mechanism driving inflation?

At which energy did inflation occur?

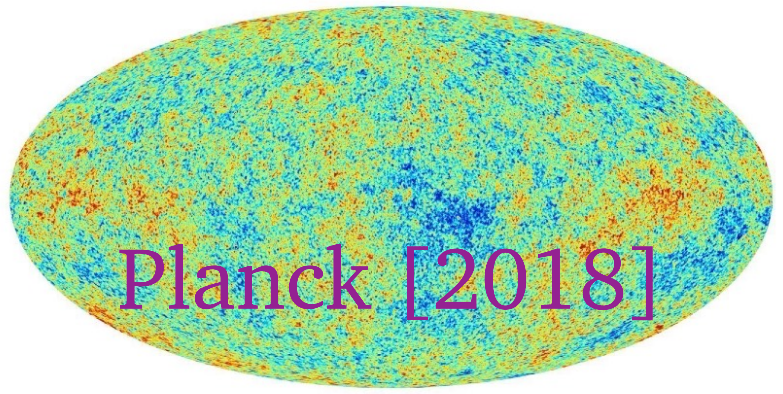
What is the particle content of inflation?

How is the inflationary energy transferred to Standard Model particles?

Did inflation happen in one go?

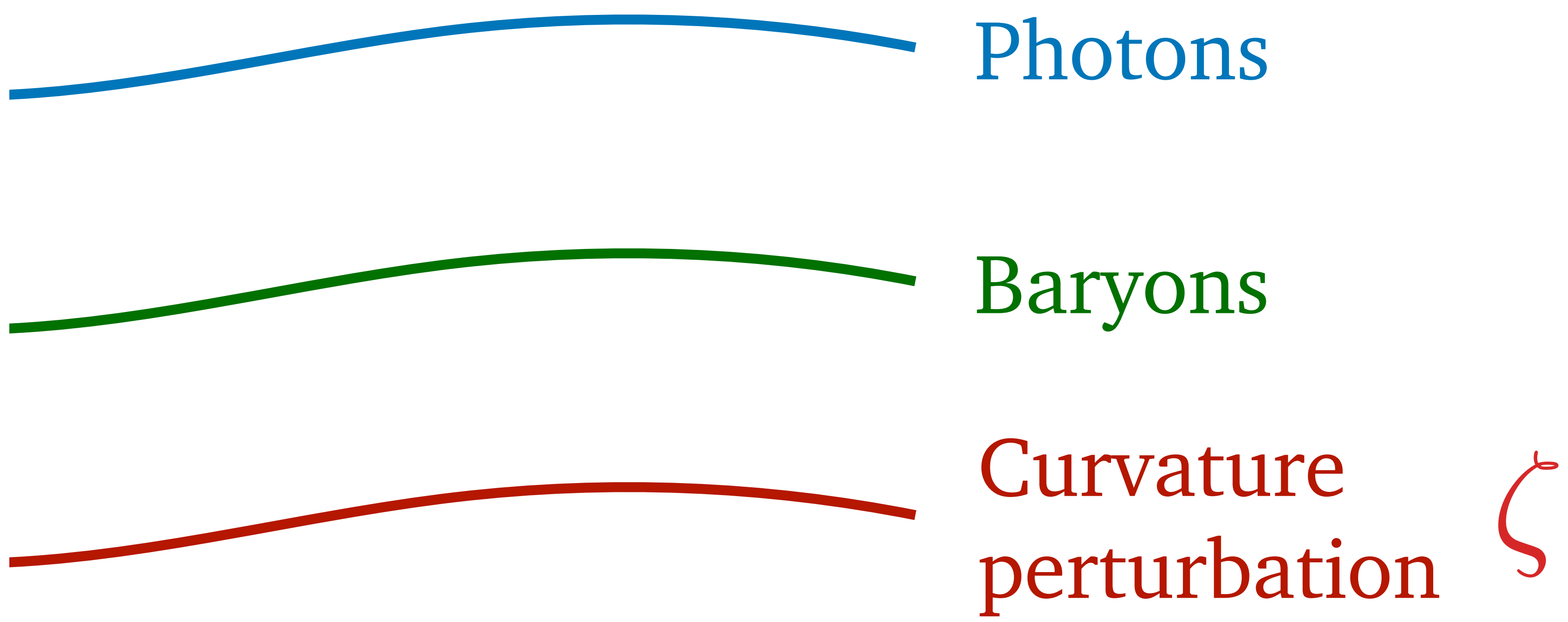
Alternatives to inflation?

Clues so far



Adiabatic

$$\delta_X(\boldsymbol{x}) \propto \delta_Y(\boldsymbol{x})$$

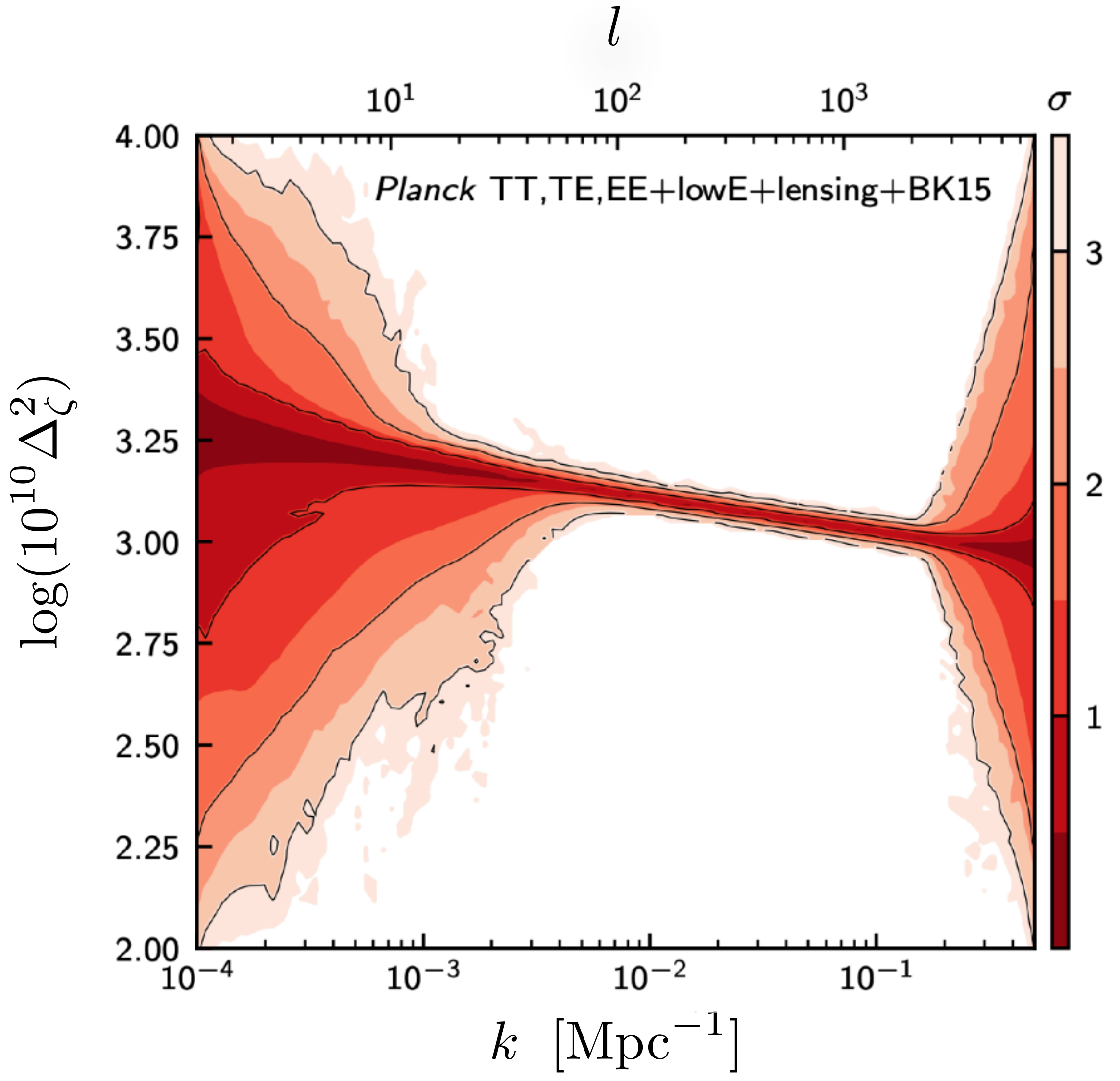


$$g_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

Single fluctuating scalar degree of freedom left over

Almost scale-invariant

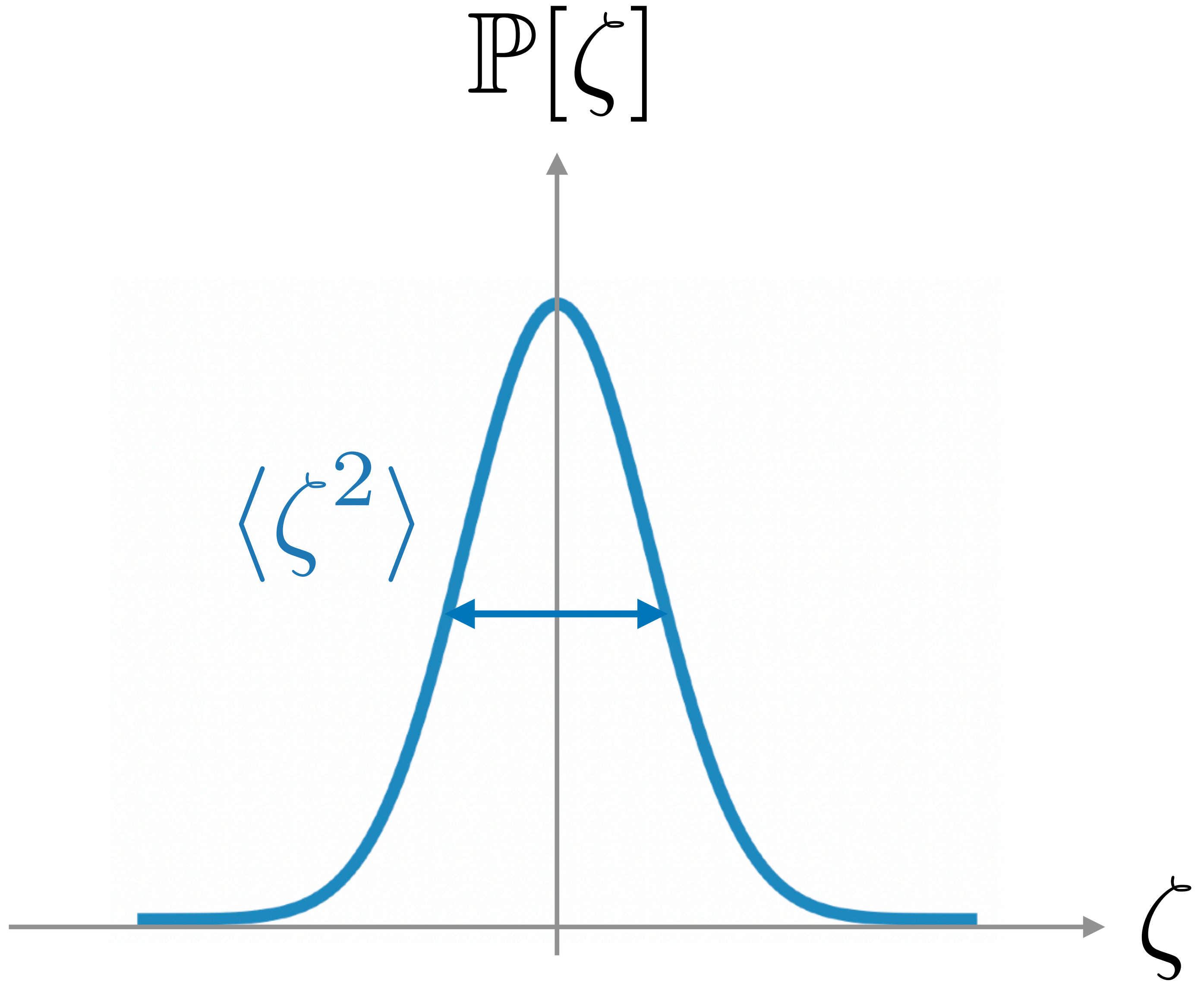
$$\Delta_\zeta^2 = \frac{k^3}{2\pi^2} \langle \zeta_{\boldsymbol{k}} \zeta_{-\boldsymbol{k}} \rangle' = A_s \left(\frac{k}{k_\star} \right)^{n_s - 1}$$



$$n_s = 0.9652 \pm 0.0042$$

Approximate time-translation invariance

Very gaussian



$$\frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} < 10^{-3}$$

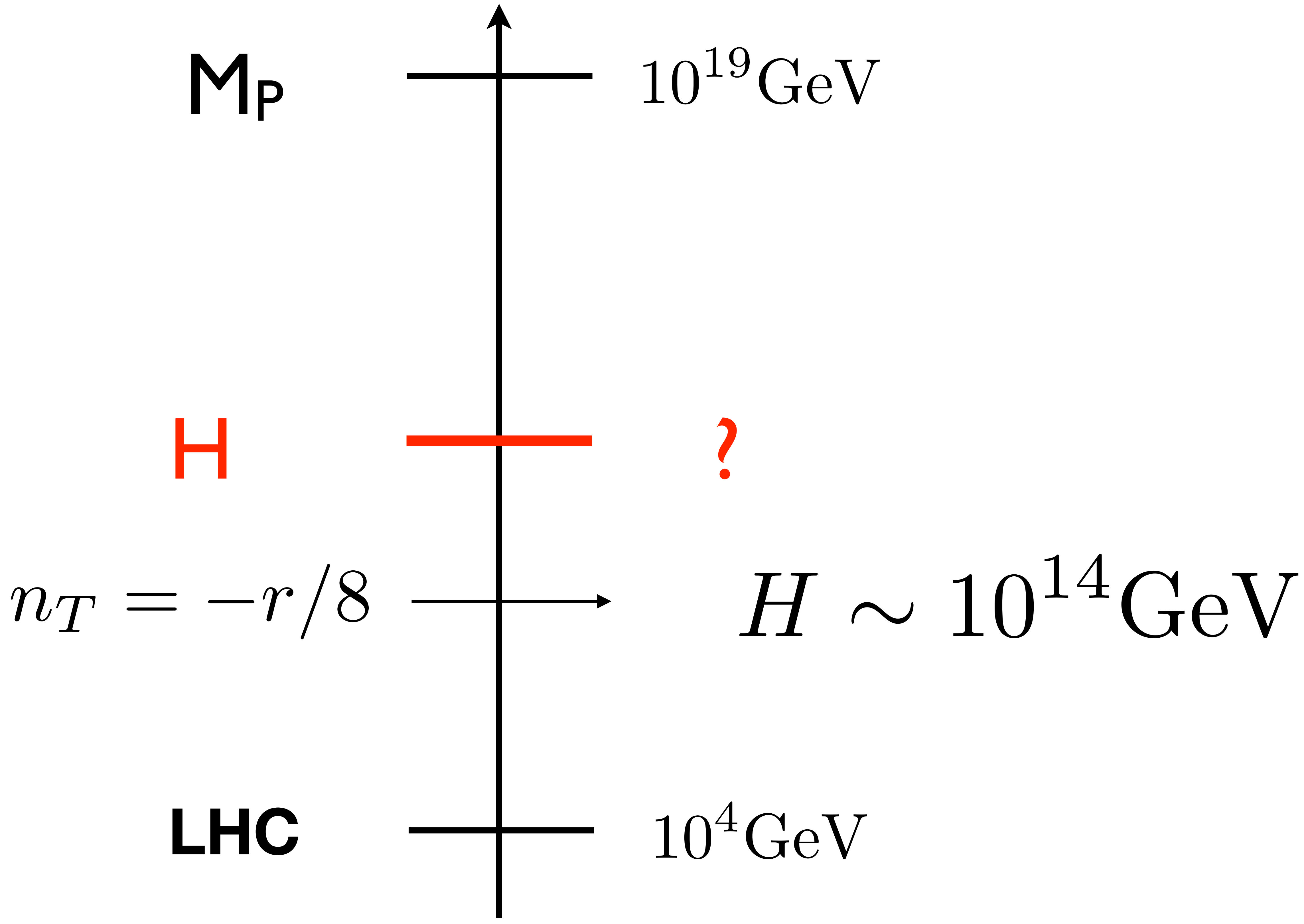
Weakly coupled theory

Clues so far

Planck + BICEP

Primordial gravitational waves
from B-modes polarization of CMB

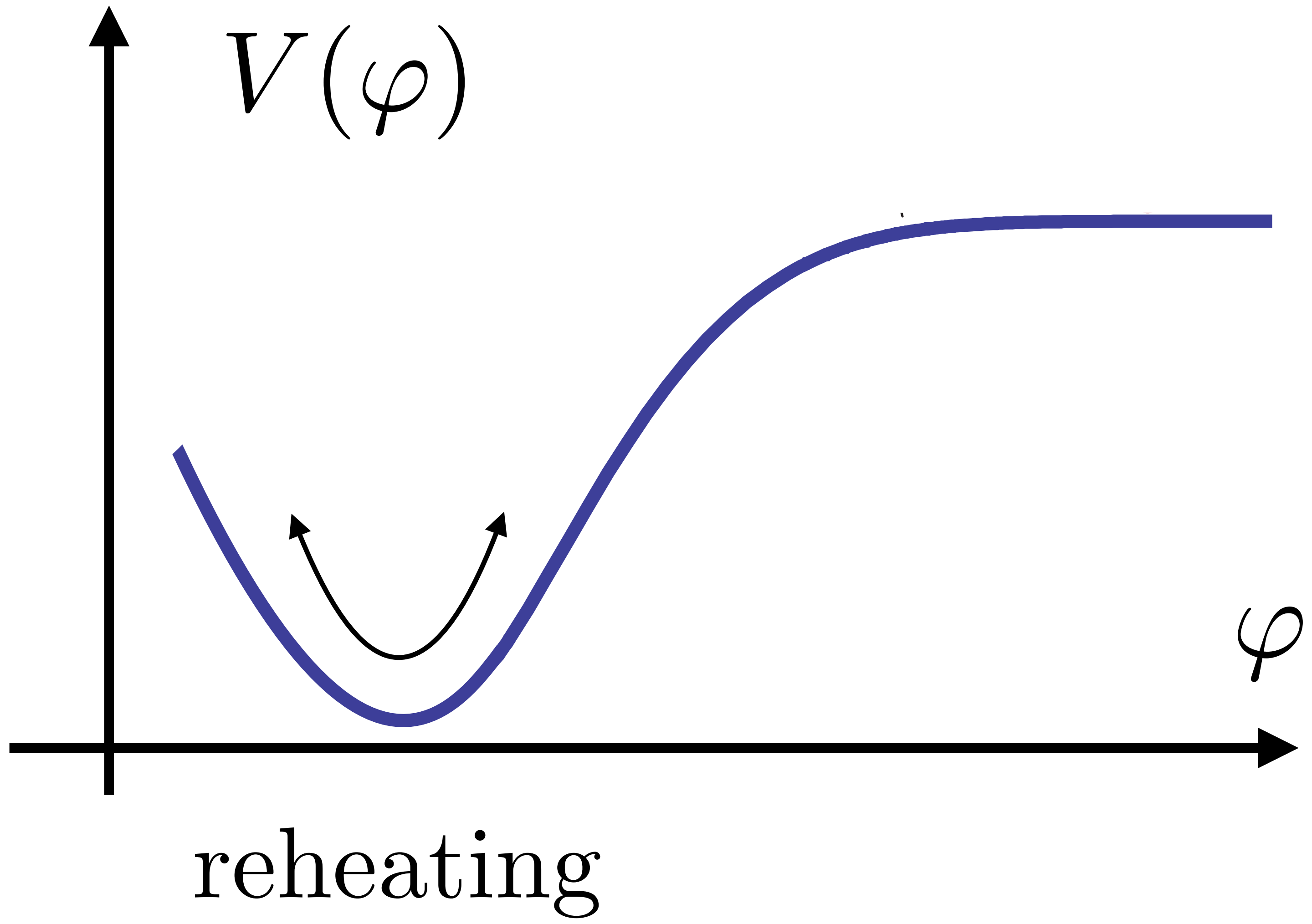
$$\frac{\langle h_{ij} h^{ij} \rangle}{\langle \zeta \zeta \rangle} \lesssim 10^{-2}$$



Detection would be spectacular
(hint about gravity at Planck scale)

No useful theoretical lower bound:
B-modes may be forever out of reach

Simple fit: single-field slow-roll inflation, but it is not natural



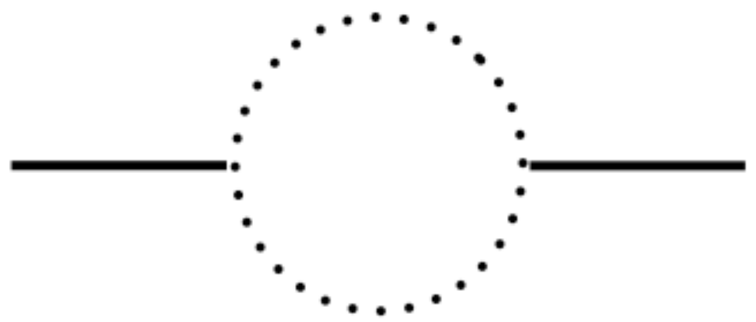
$$\eta \equiv M_{\rm pl}^2 \frac{V_{,\phi\phi}}{V} \ll 1$$

Prolonged phase of inflation

Why is the inflaton so light?

$$\eta \approx \frac{m_\phi^2}{H^2} \ll 1$$

like the Higgs
hierarchy problem



$$m_\phi^2 \sim \Lambda_{\text{cut-off}}^2 \gg H^2$$

Simple fit: single-field slow-roll inflation, but it is not natural

$$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 - V_0(\phi) + \sum_\delta \frac{\mathcal{O}_\delta(\phi)}{M^{\delta-4}}$$

Slow-roll action

Corrections to the low-energy effective potential

$$\frac{\Delta m_\phi^2}{H^2} \sim \left(\frac{M_{\rm Pl}}{M}\right)^2$$

$$\Delta\eta \gtrsim 1$$

Planck-scale physics does not decouple

Symmetries do not help

Outline

I. Precision physics

II. Exploratory physics

III. Formal aspects

I. Precision physics

- Non-Gaussianities and Effective Field Theory of Inflationary fluctuations
- Imprints of extra fields
- Cosmological collider and low-speed collider

Primordial non-Gaussianities

Introductory review:
Renaux-Petel, 1508.06740

Higher-order correlators: beyond free fields → measure of interactions

Cosmology

Particle physics

Goal: establish a standard model of inflation

Identify degrees of freedom, mass, dispersion relation, spin, interactions

! Additional difficulty compared to particle physics:

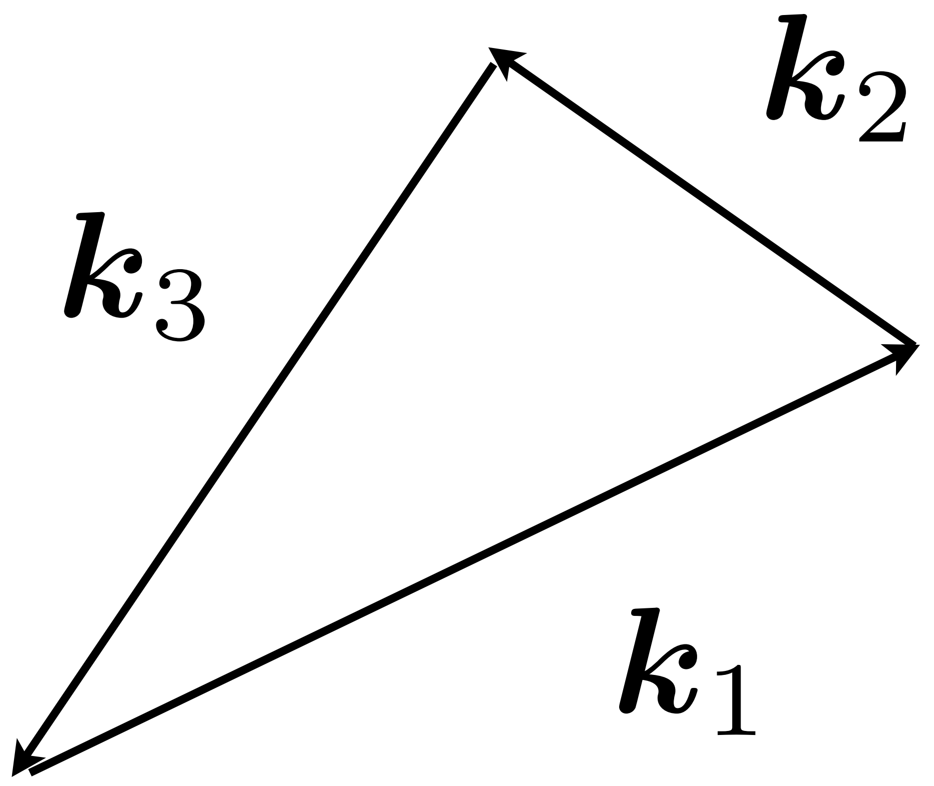
everything is, a priori, time-dependent

Bispectrum

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B_\zeta(k_1, k_2, k_3)$$

Homogeneity

Isotropy



$$B_\zeta \equiv (2\pi)^4 \frac{S(k_1, k_2, k_3)}{(k_1 k_2 k_3)^2} A_s^2$$

Amplitude $S \sim f_{\rm NL}$

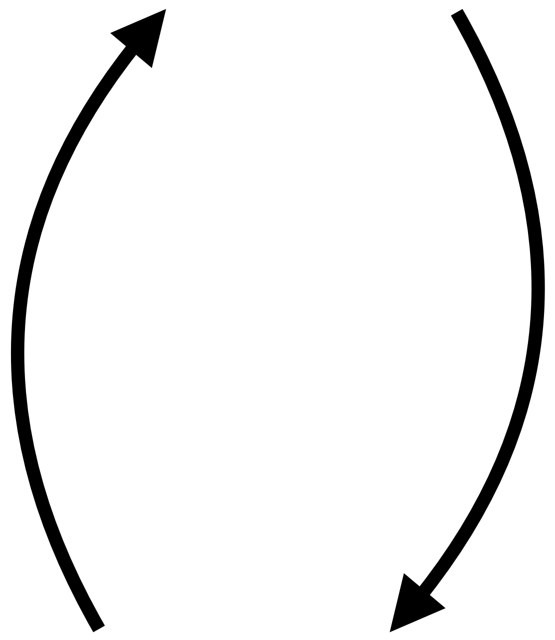
Scale-dependence (overall size)

Shape dependence (configuration of triangles)

Effective Field Theory of Inflationary Fluctuations

Formulation of theories
straight at the level of fluctuations

motivates



informs

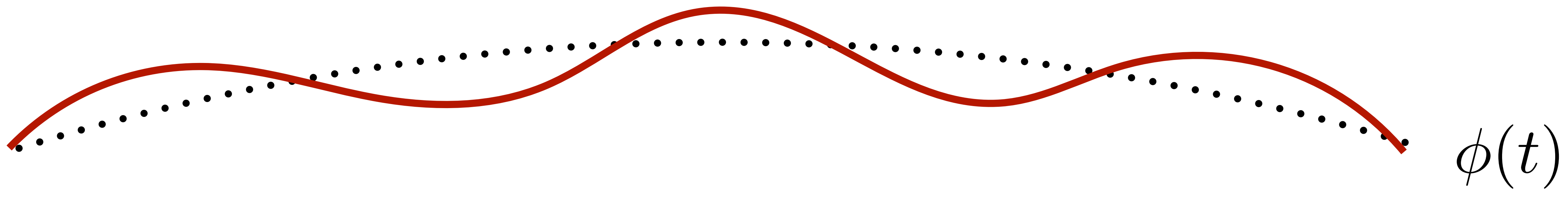
Source of inflation

Direct link with observations,
sytematic, symmetries manifest

Preferred space-like foliation (existence of clock)
breaks time reparametrization invariance

Guaranteed: Goldstone boson

$\pi(\boldsymbol{x}, t)$ fluctuation of the clock field



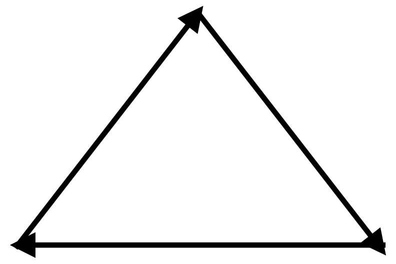
$$\zeta = -H\pi + \ldots$$

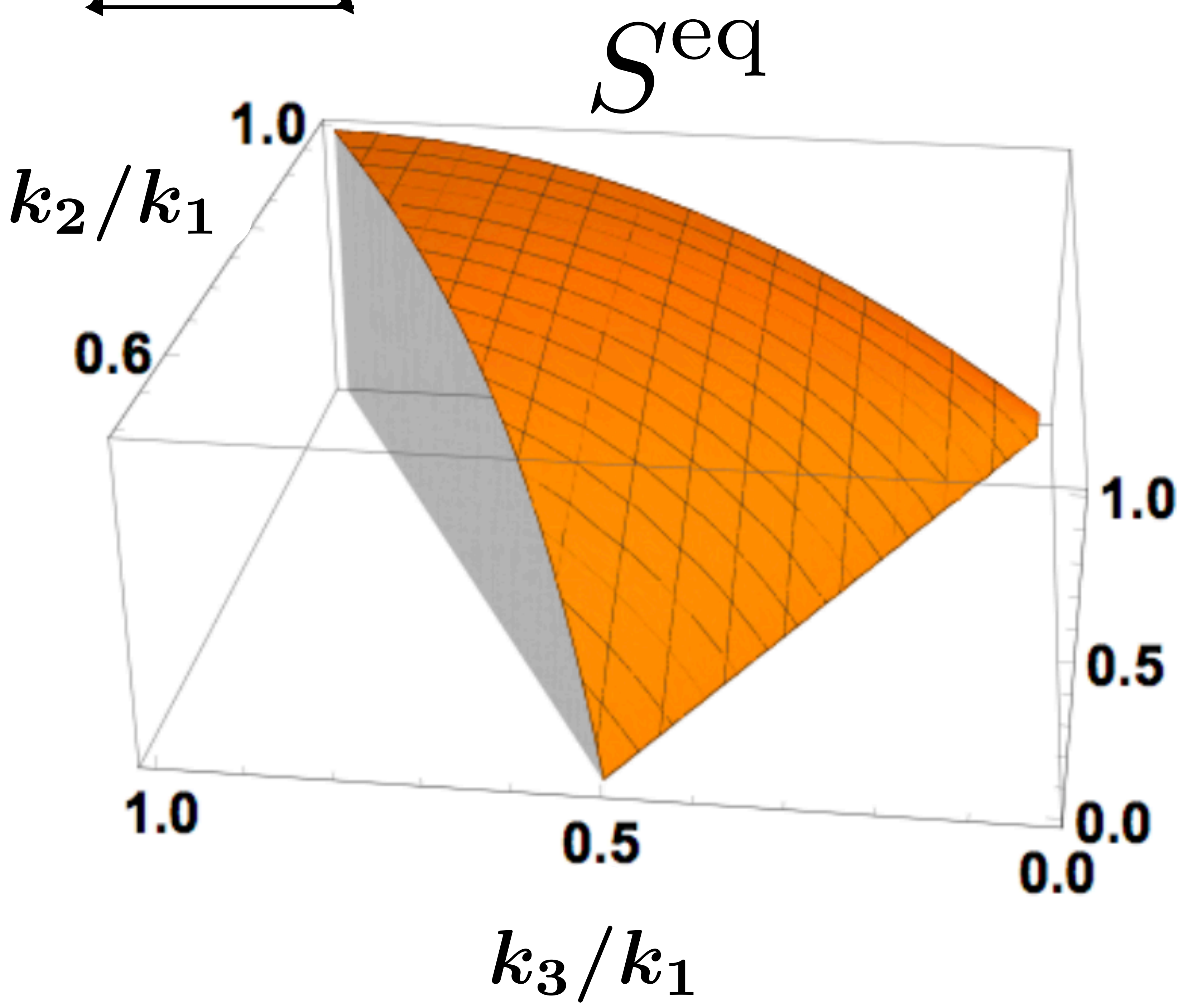
Cheung, Creminelli, Fitzpatrick,
Kaplan, Senatore [2008]

Vanilla EFT and equilateral/orthogonal non-Gaussianities

$$\mathcal{L}_\pi/a^3 = \frac{M_{\rm pl}^2|\dot H|}{c_s^2}\left[\dot\pi^2 - c_s^2\frac{(\partial_i\pi)^2}{a^2} + (1-c_s^2)\left(\dot\pi^3 - \dot\pi\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{4}{3}M_3^4\frac{c_s^2}{M_{\rm pl}^2|\dot H|}\dot\pi^3\right]$$

Non-linearly realised symmetry → $f_{\rm NL}^{\rm eq} \sim \frac{1}{c_s^2} - 1$





$f_{\rm NL}^{\rm eq} = -26 \pm 47$

$f_{\rm NL}^{\rm orth} = -38 \pm 24$

$c_s \geq 0.021$

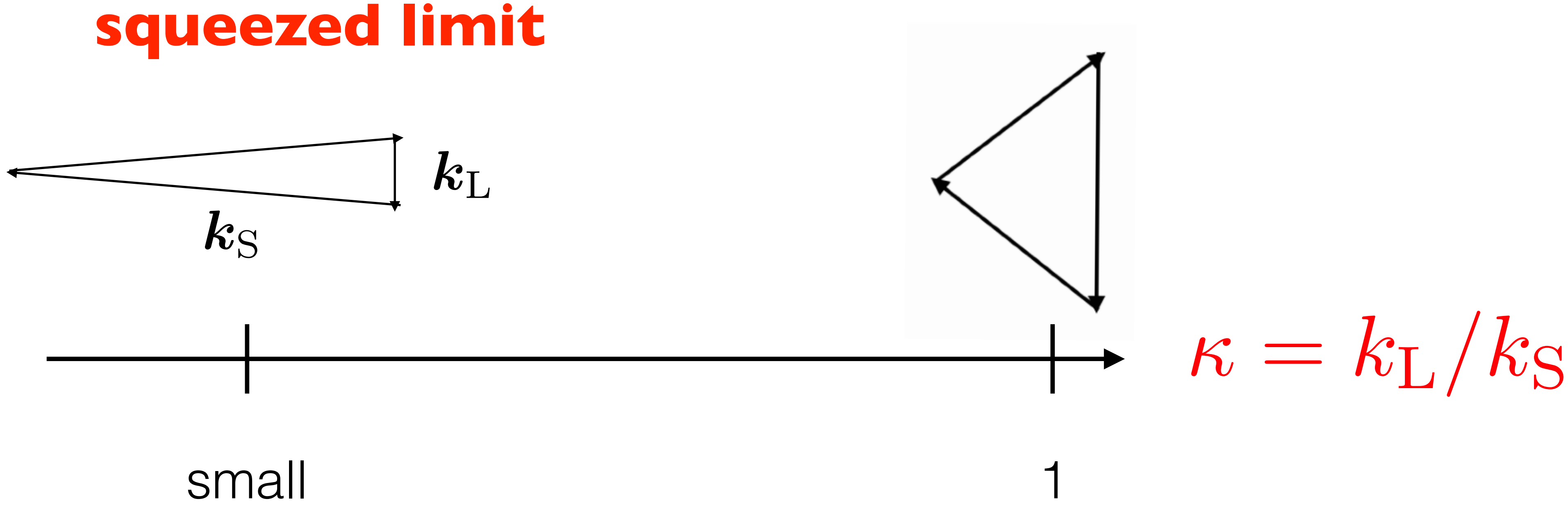
Planck 2018 (68% CL)

$f_{\rm NL}^{\rm eq} \sim 1$ threshold for slow-roll dynamics

$f_{\rm NL} = \mathcal{O}(\epsilon, \eta) \sim 10^{-2}$ gravitational floor Maldacena (03)

Imprints of additional degrees of freedom

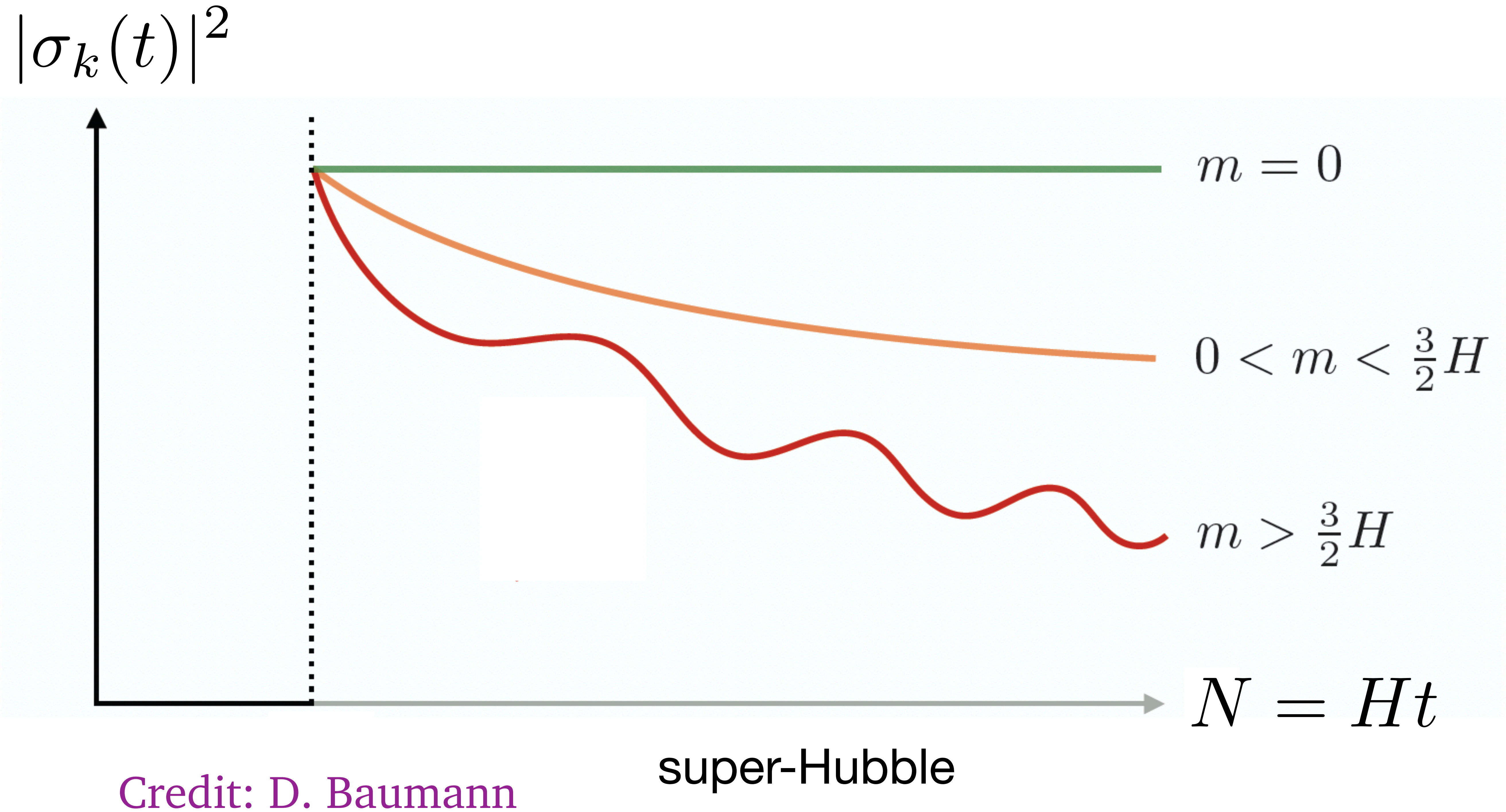
Probed in
squeezed limit



Hierarchies of scales → Hierarchies of times → Probe super-Hubble evolution of fields → Mass

Imprints of additional degrees of freedom

$$\ddot{\sigma}_k + 3H\dot{\sigma}_k + m^2\sigma_k \simeq 0$$



Credit: D. Baumann

$$S \propto \kappa^{1/2-\nu}$$

$$\sigma \propto \frac{1}{a^{\frac{3}{2}-\nu}}$$

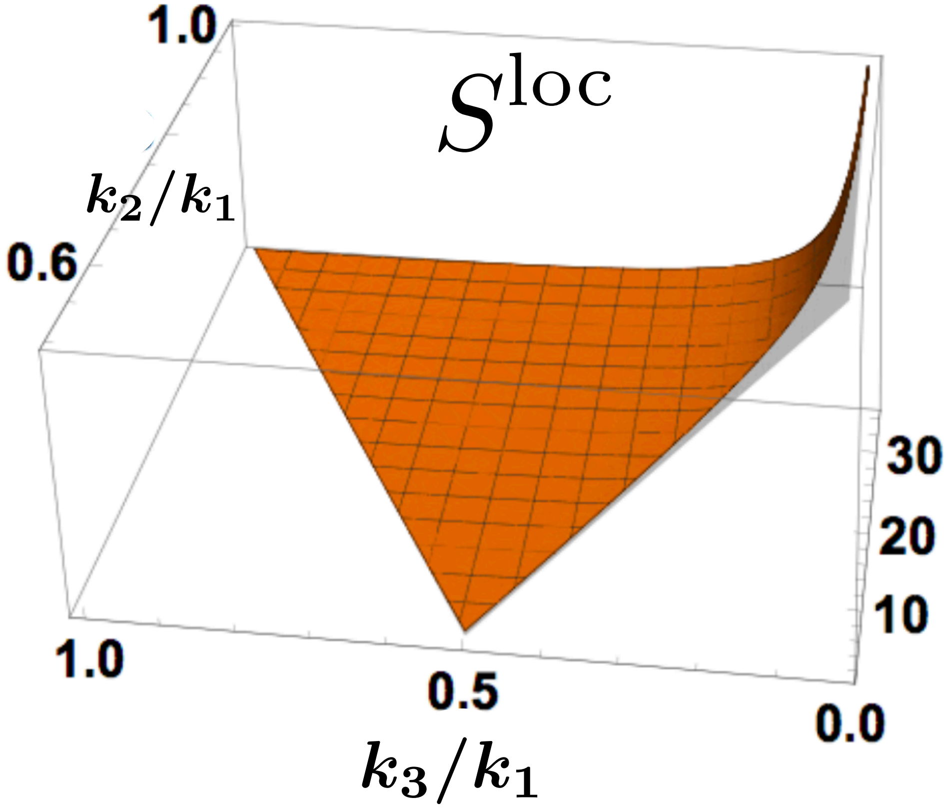
$$\nu = \sqrt{9/4 - m^2/H^2}$$

$$\sigma \propto \frac{\cos(\mu N + \phi)}{a^{\frac{3}{2}}}$$

$$\mu = \sqrt{m^2/H^2 - 9/4}$$

$$S \propto \kappa^{1/2} \cos(\mu \log(\kappa) + \varphi)$$

Local shape



Special role of massless fields
they do not decay

$$f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$$

Detection via scale-dependent bias

Dalal et al [2008]

Not possible in single-clock inflation :

$$\underset{\kappa \ll 1}{S \, \propto \, \kappa}$$

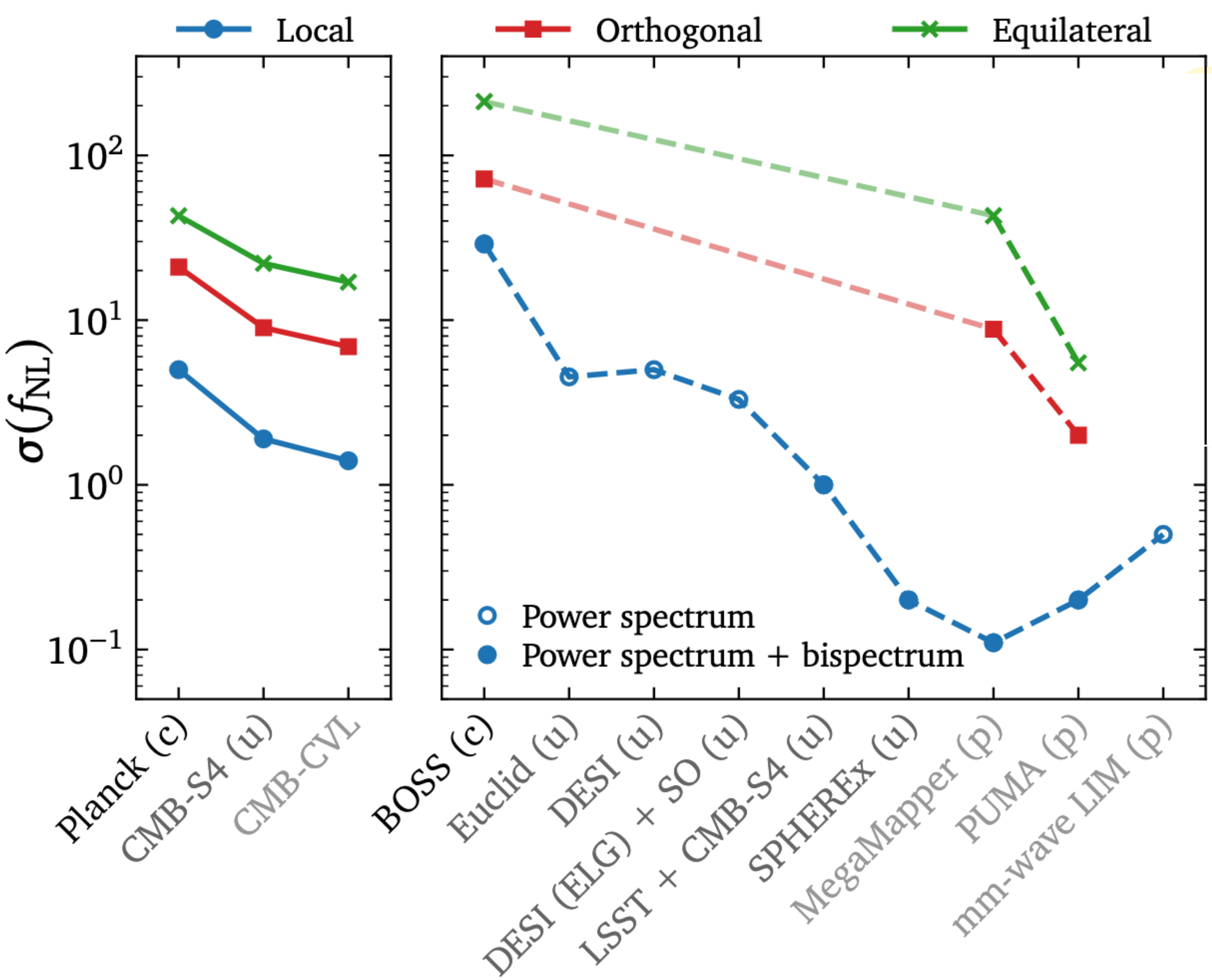
Maldacena, Creminelli, Zaldarriaga,
Tanaka, Urakawa, Pajer, Schmidt …

Prospects

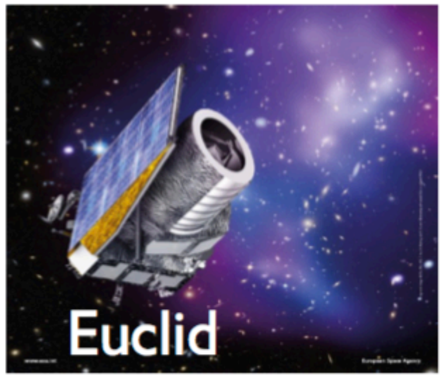
→

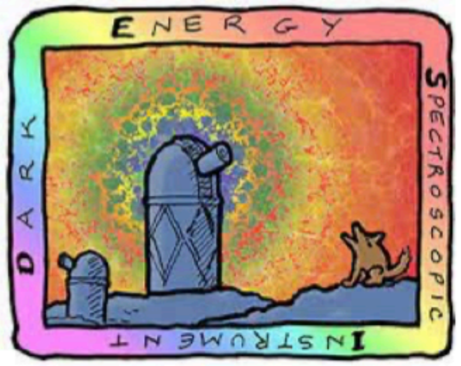
Huge efforts with CMB-S4 & large-scale structure surveys

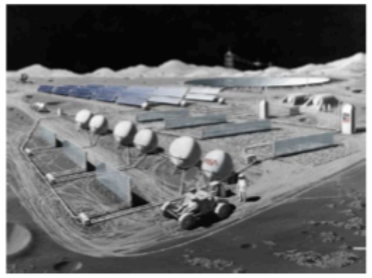
(scale-dependent bias, EFT of LSS, position space maps, simulation based inference etc)



Snowmass 2203.08128





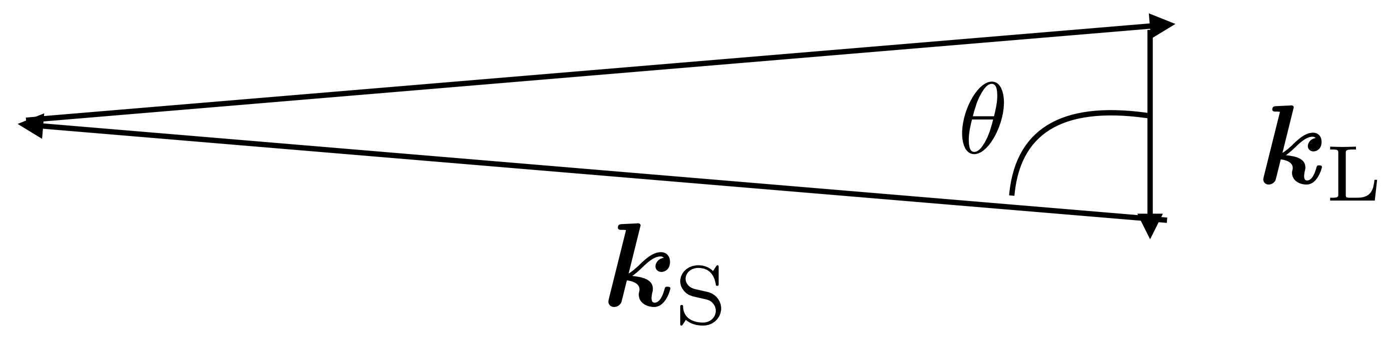


Long-term: 21cm radio-astronomy from the far side of the moon! (dark ages)

See talks by Matthieu Tristam, Giovanni Cabass, Guido D'Amico

Cosmological collider physics

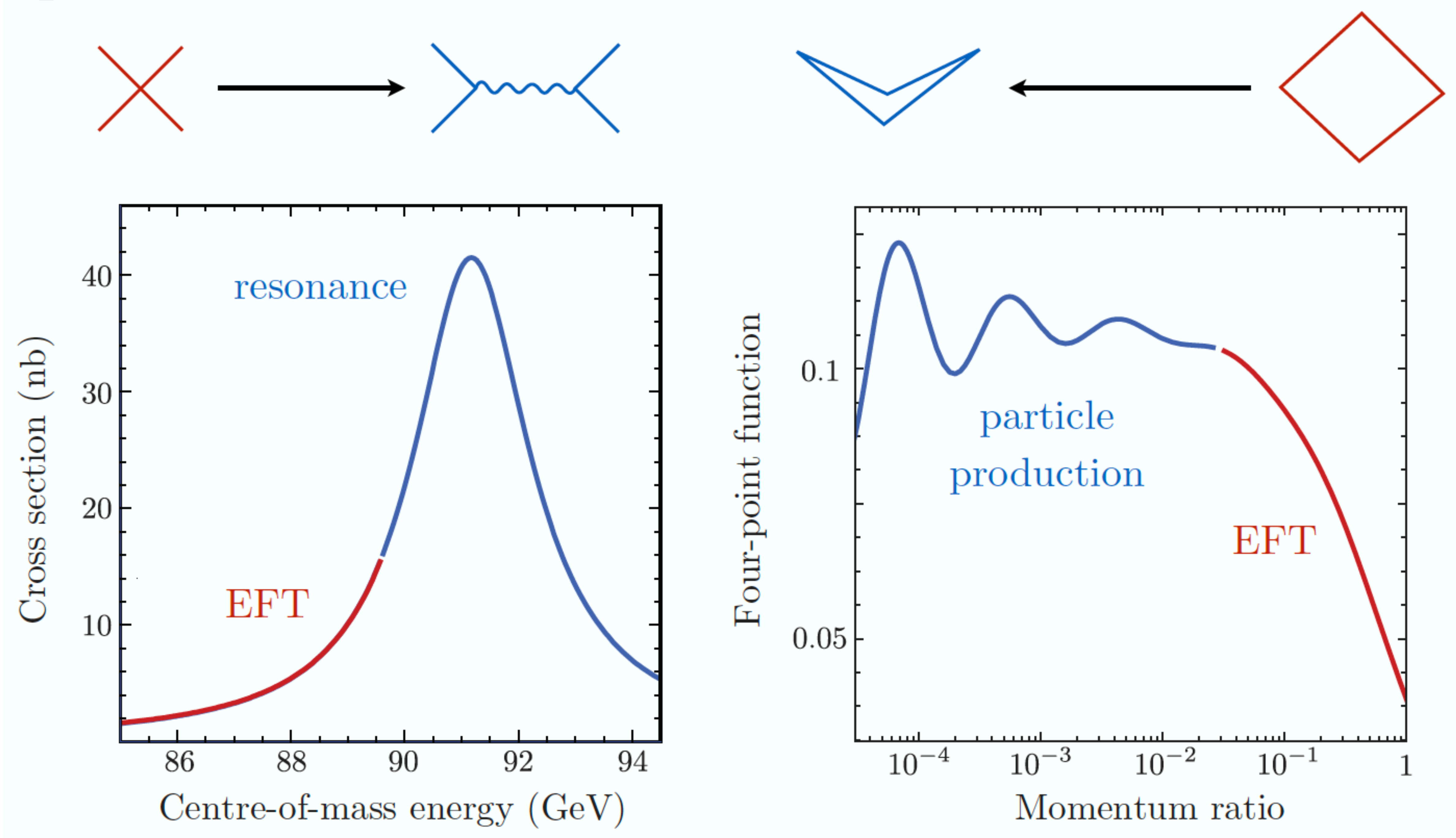
3pt



$$S \propto \kappa^{1/2} e^{-\pi\mu} \cos(\mu \log(\kappa) + \varphi) P_S(\cos\theta)$$

Mass & Spin of heavy particle

4pt



From 1811.00024

Chen, Wang 2009
Baumann Green 2011
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
Lee, Bauman, Pimentel 2016
Arkani-Hamed, Baumann, Lee, Pimentel 2018
+ many works

Cosmological collider: « a robust probe of field content of inflation »?

$$S \propto \kappa^{1/2} e^{-\pi\mu} \cos(\mu \log(\kappa) + \varphi) P_S(\cos\theta)$$

Chen, Wang 2009
Noumi, Yamaguchi, Yokohama 2012
Arkani-Hamed, Maldacena 2015
+ many works

As robust probe as assumptions are restrictive:
unique additional dof, weakly mixed, scale-invariant

Observed π

Dispersion relation

← **mixing** →

Type

Strength

Time-dependence

Hidden sector

Several dofs

Mass and spin

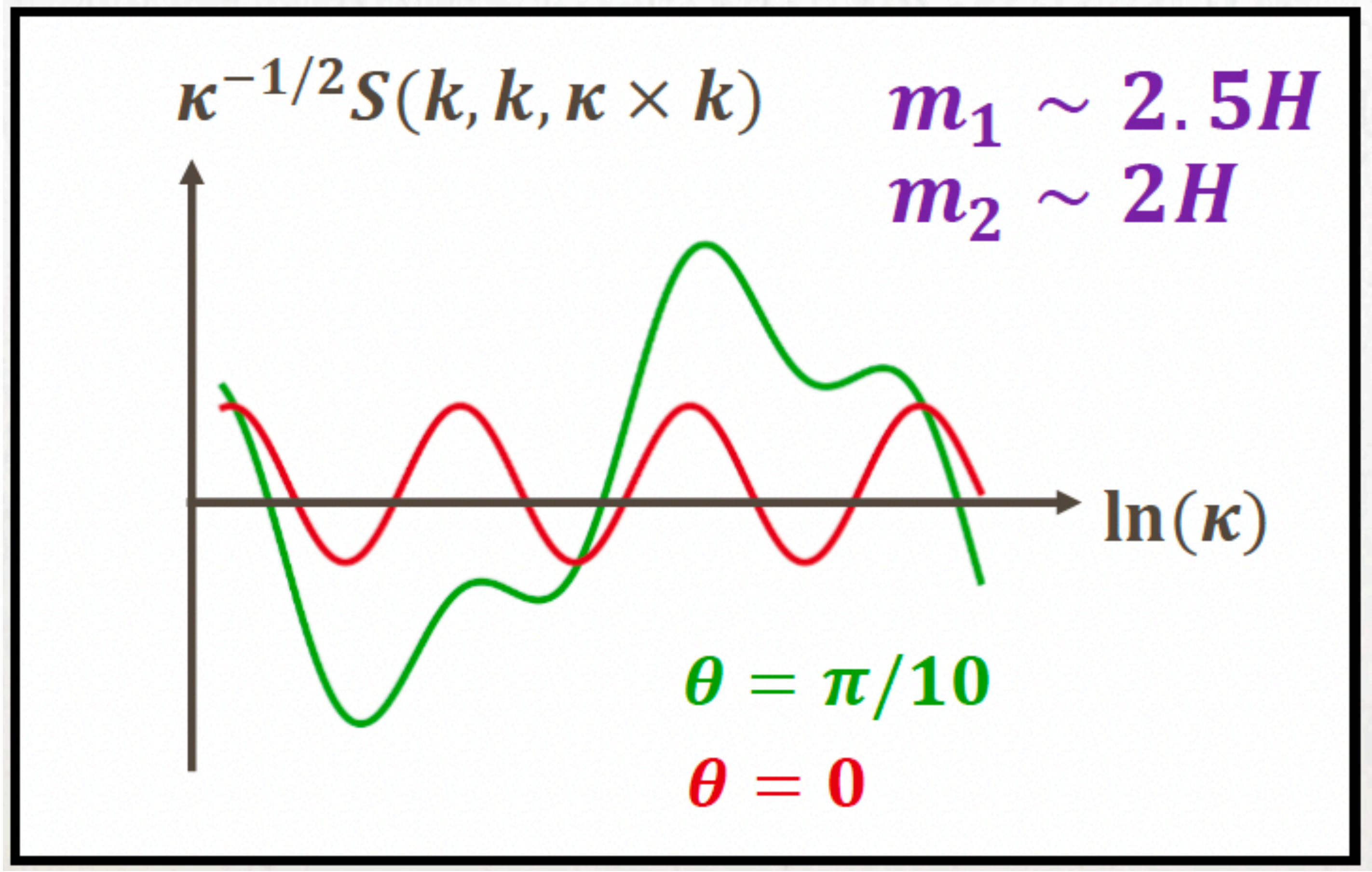
Interactions

Time-dependence…

Cosmological collider signatures beyond restrictive assumptions

(personal works)

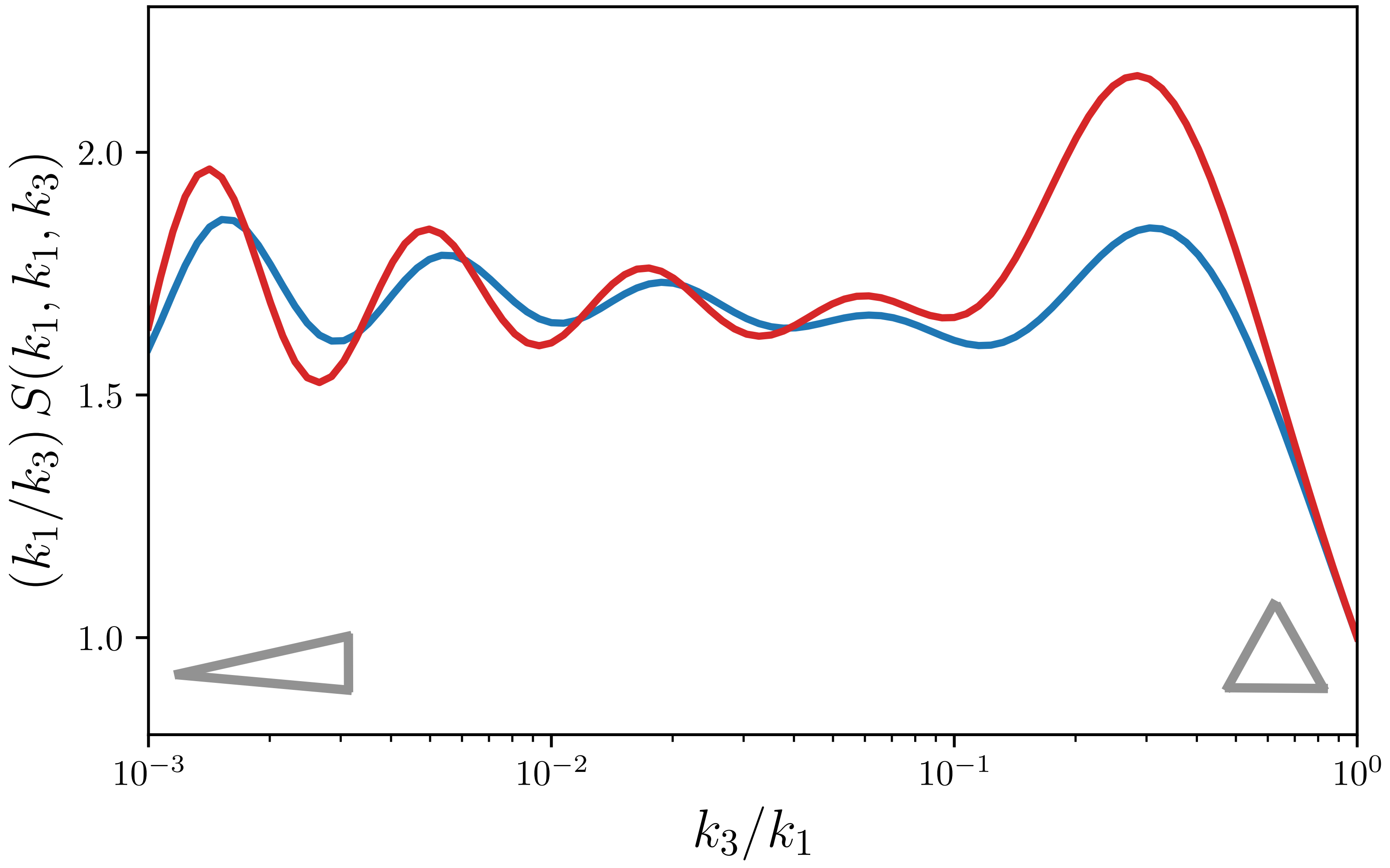
Mass mixing



Inflationary flavor oscillations

Pinol, Aoki, Renaux-Petel, Yamaguchi [2021]

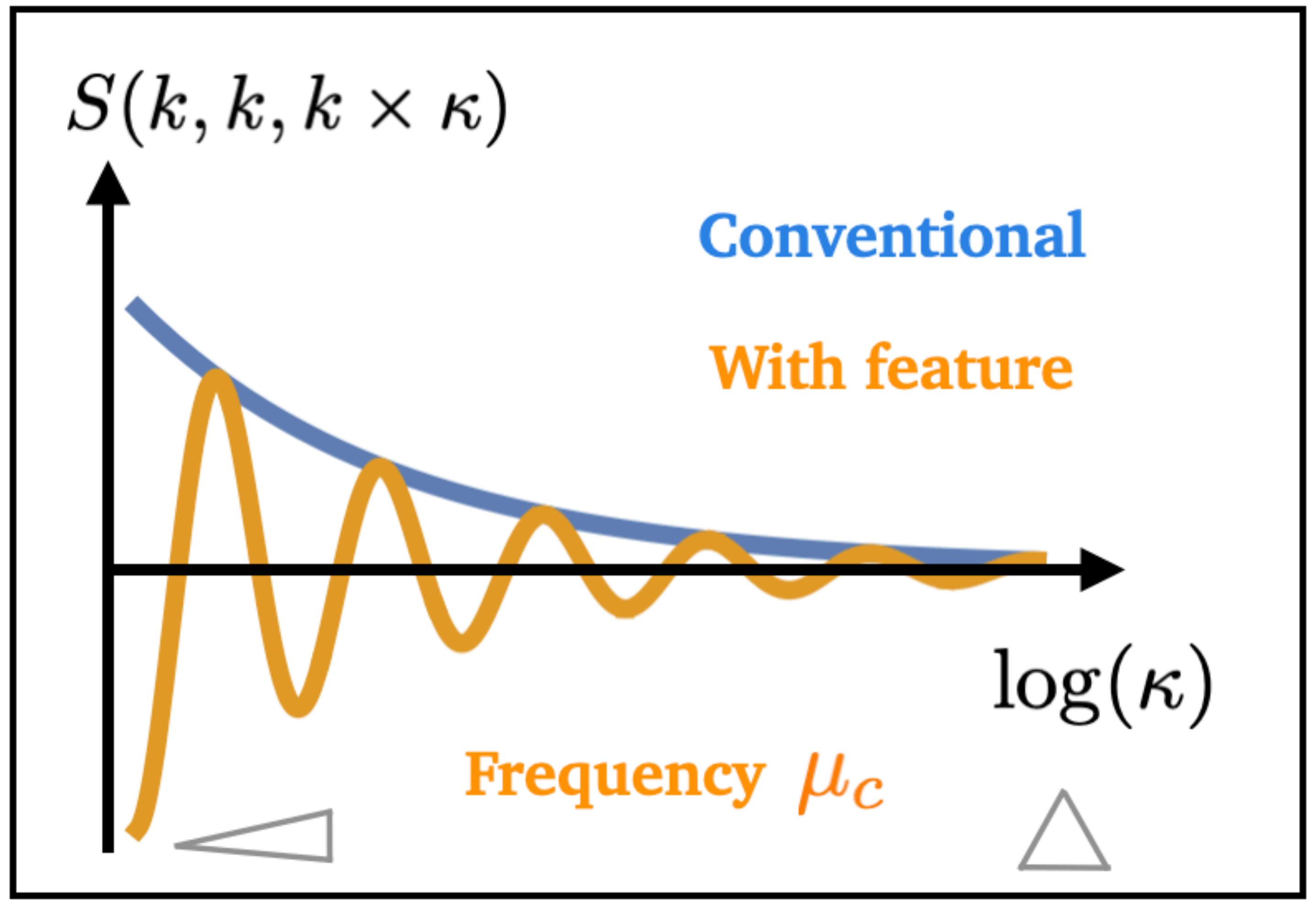
Strong mixing



Breaking degeneracies weak/strong mixing

Werth, Pinol, Renaux-Petel [2023 a,b]

Time-dependent mixing



Soft limits complementary to equilateral to diagnose features

Werth, Pinol, Renaux-Petel [2023 a,b]

Is that it for theorists?

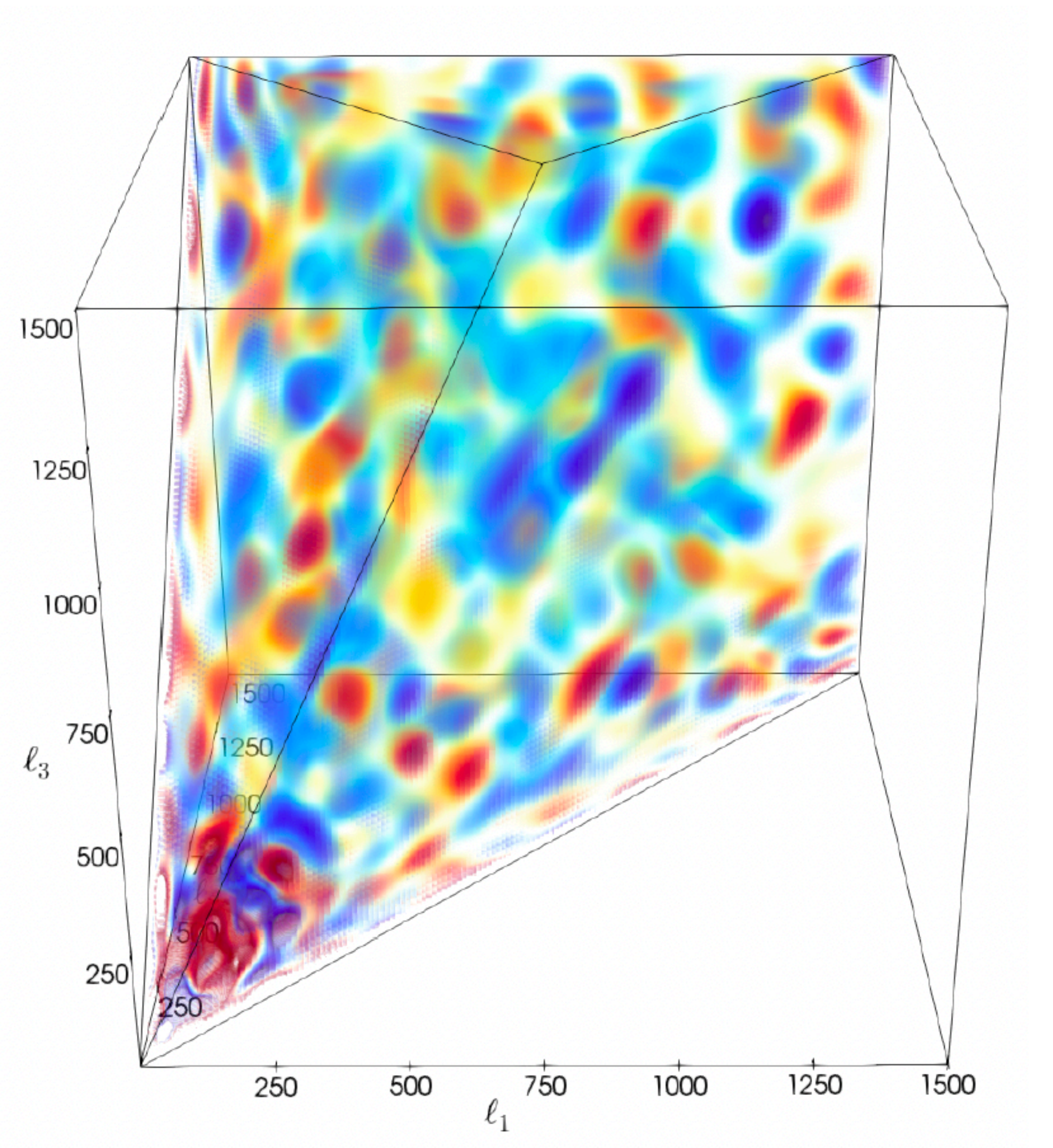
Beyond what I said: features, excited states …

The Planck team constrained a high number of shapes

Physics of inflation?

Statistical properties

$$\mathbb{P}(\zeta)$$



Fergusson, Shellard
modal decomposition

Theorists' task

Building dictionary

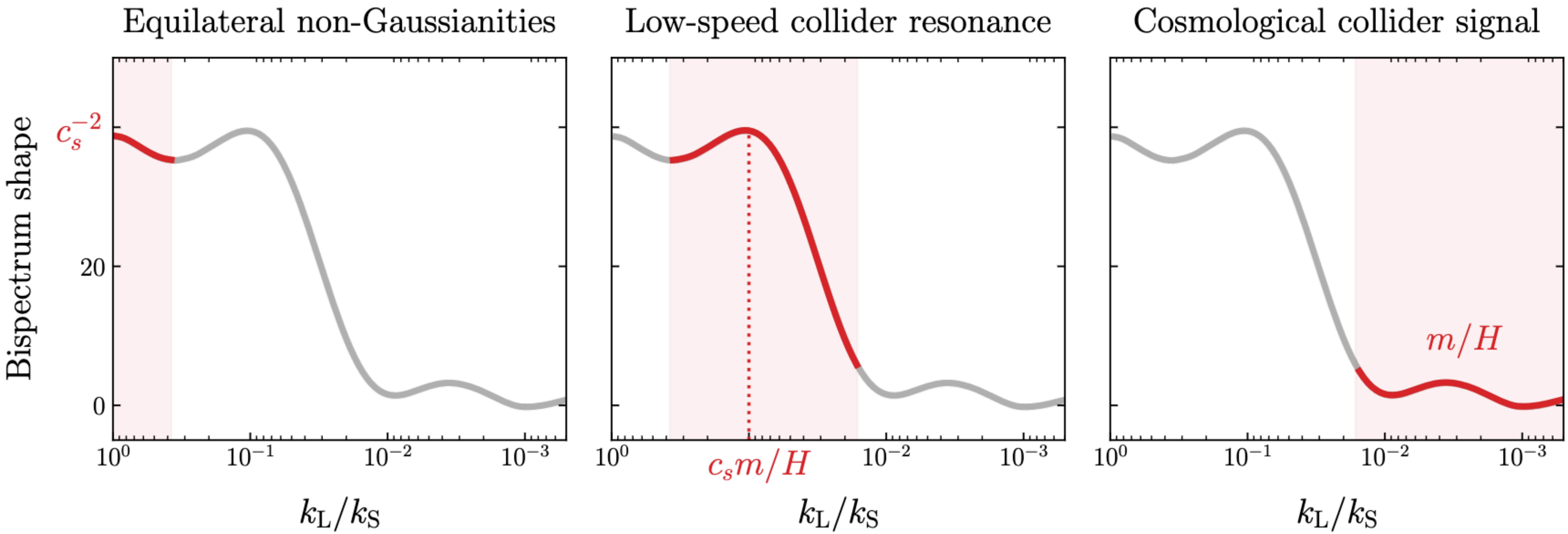
Identifying targets worth measuring

Is the dictionary complete?!

No: cosmological flow approach.
see Denis Werth's talk

Interesting targets not yet identified?!

Detection of large Equilateral NG, what is next?
Low-speed collider as next target



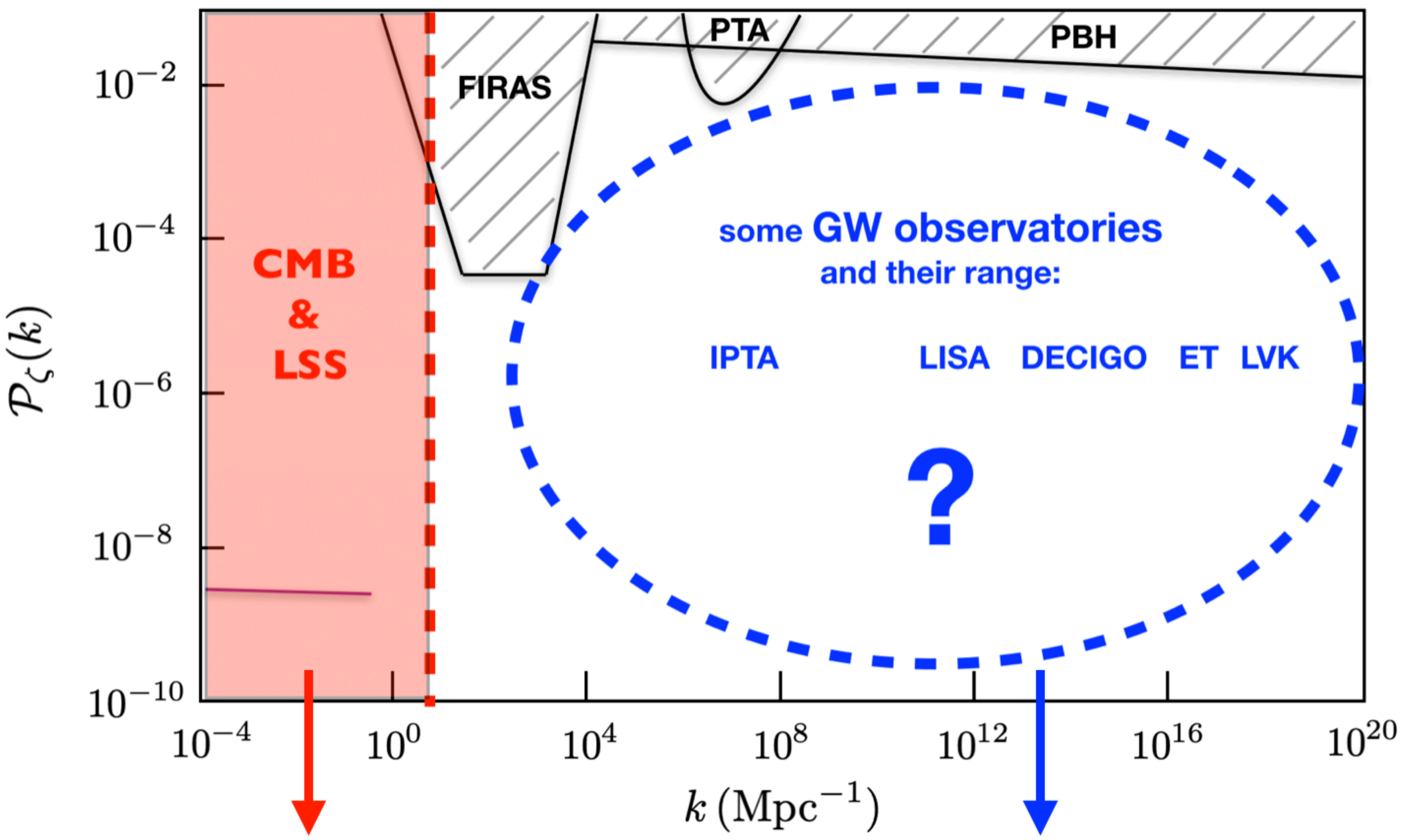
New discovery channel of heavy fields with $m < H/c_s$

Jazayeri, Renaux-Petel [2022]
Jazayeri, Renaux-Petel, Werth [2023]

II. Exploratory physics

- Primordial Black Holes(PBH) and Stochastic Gravitational Wave Backgrounds (SGWB) as probes of dark era of inflation
- Motivations for non-vanilla inflation
- SGWB from active sources

Context



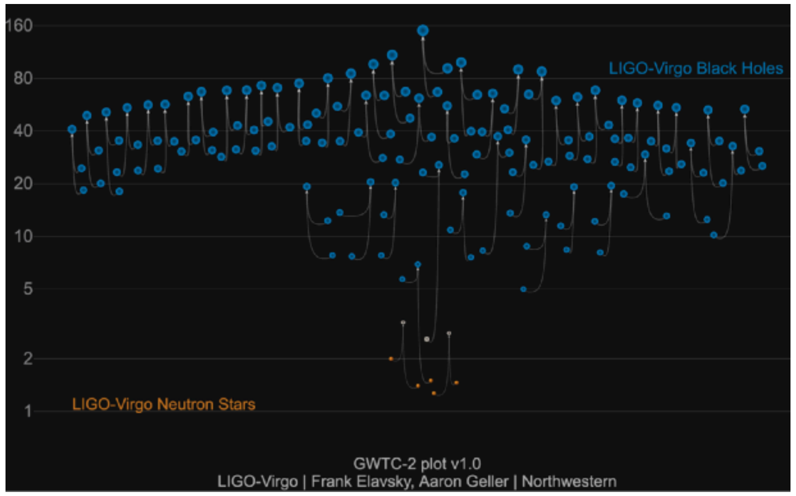
Precision physics

Inflation as a cosmological collider

Exploratory physics

Probing dark inflationary era with gravitational waves and PBH

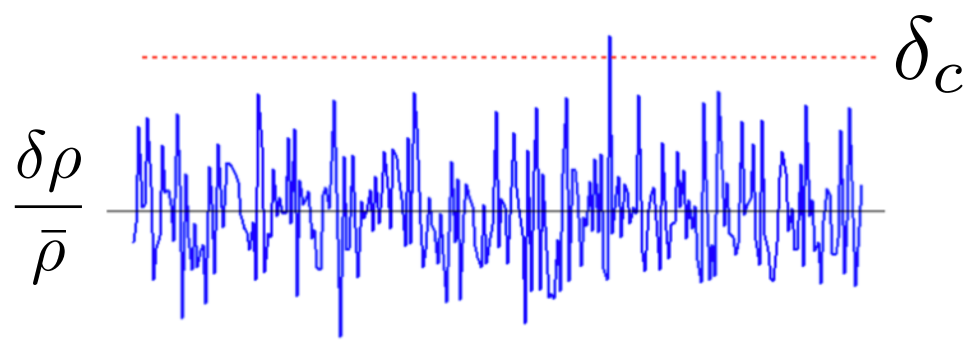
Context: GW astronomy



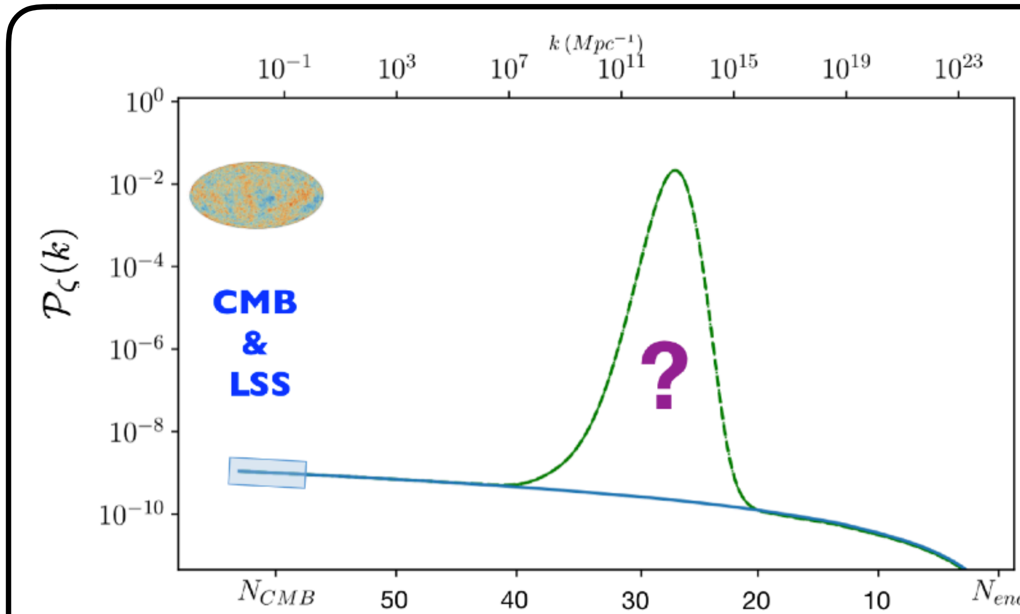
Have **Primordial Black Holes** been already detected? Data will tell

Large activity.
Soon PBH review from LISA CosWG

Large primordial overdensities



Boosted power spectrum on small scales and **nontrivial dynamics of inflation**

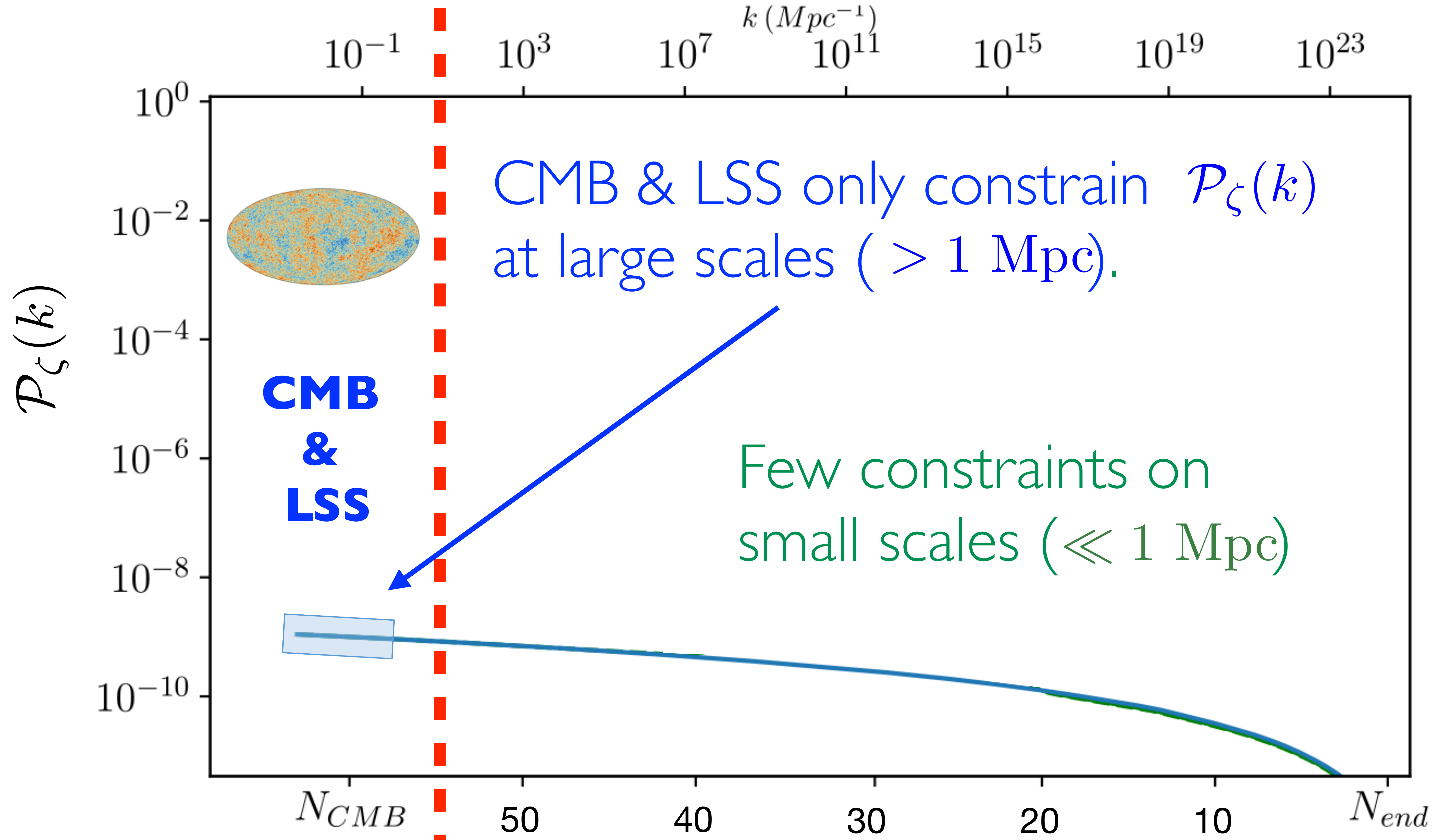


Scalar-induced **SGWB**

$$M \simeq 10^{-12} M_\odot \left(\frac{f_{\rm LISA}}{f}\right)^2$$

Bartolo et al 2019

Inflation on small scales?

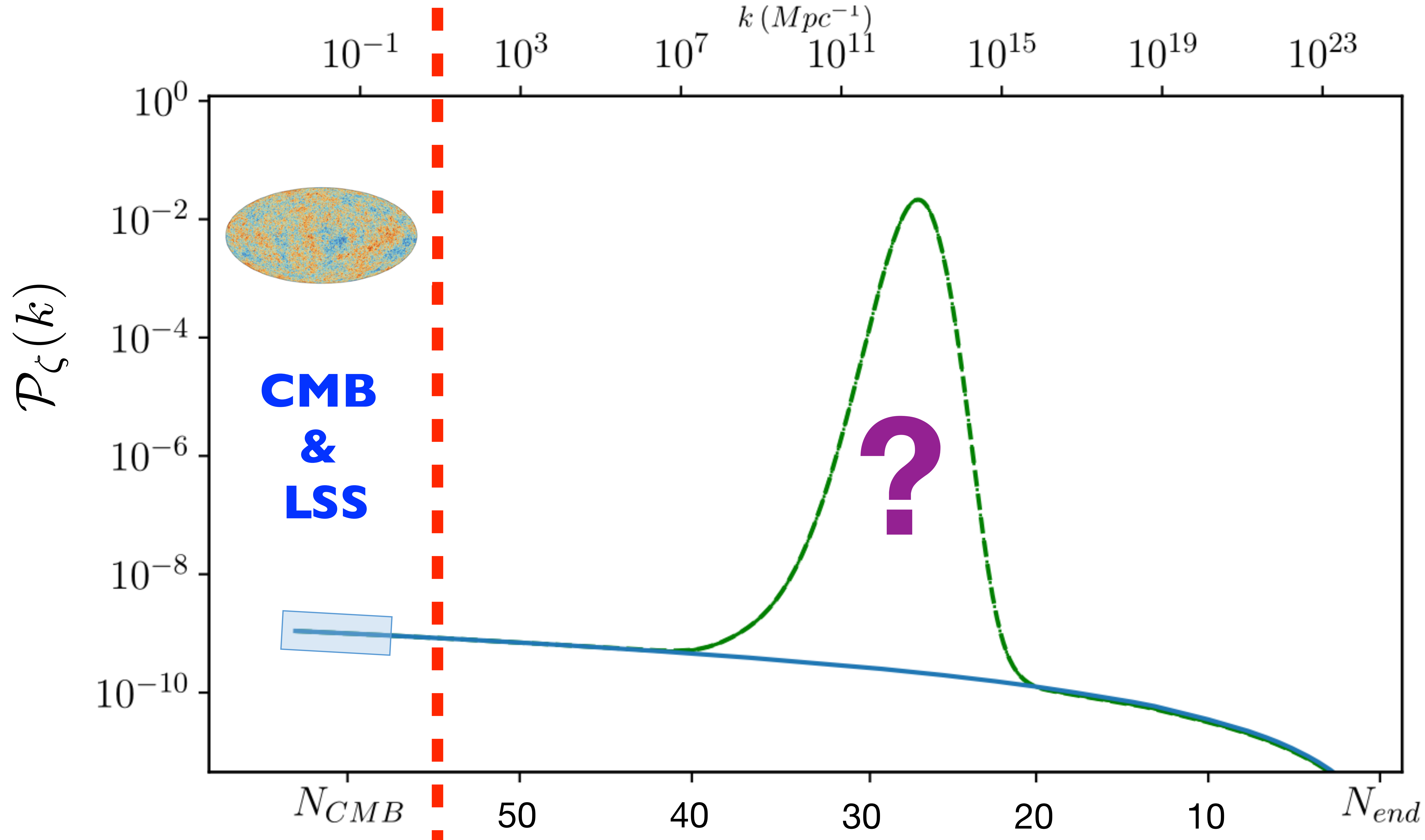


Emergent simplicity

+

Corrections

Inflation on small scales?

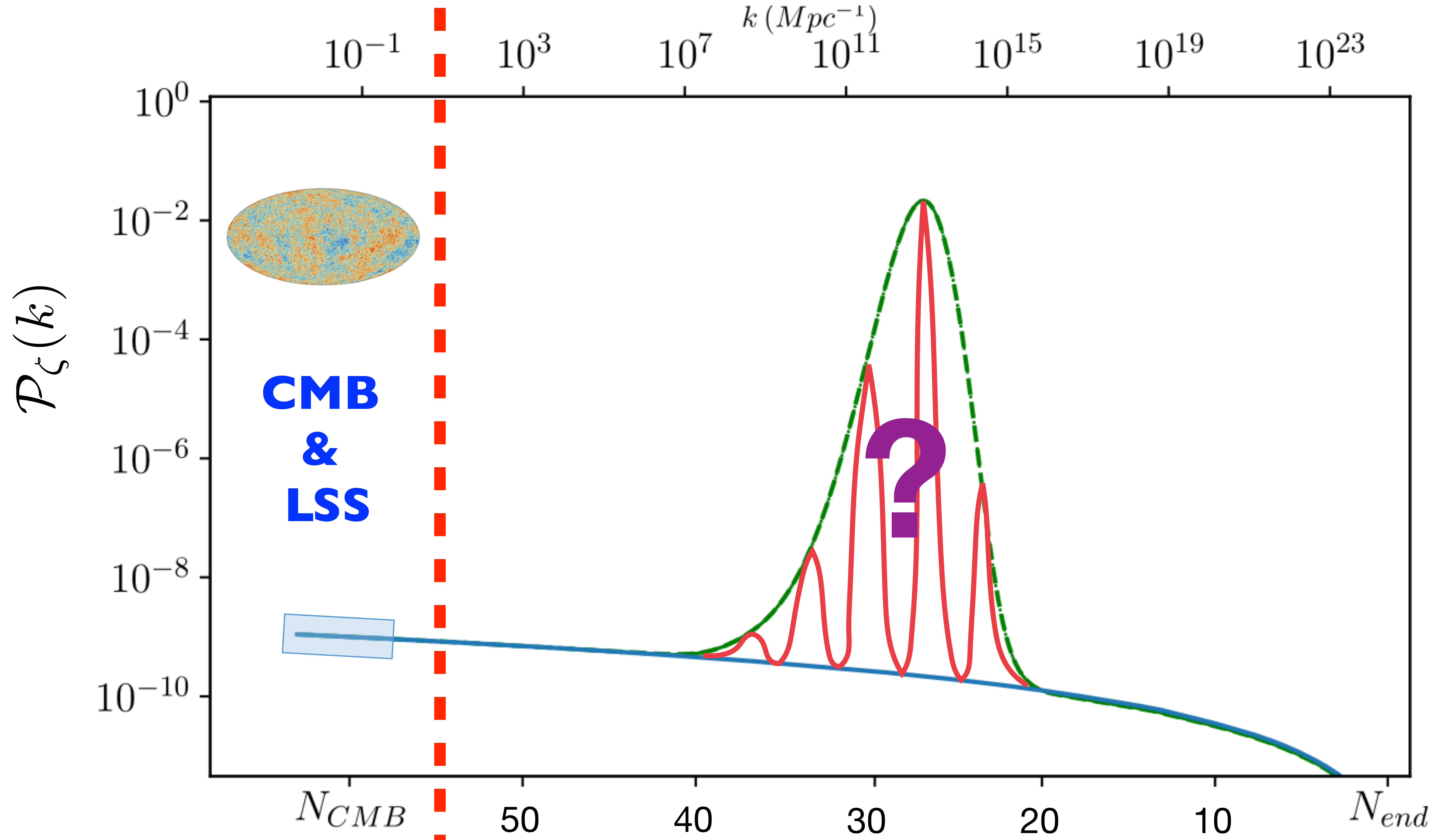


Emergent simplicity
+
Corrections

Drastically different?

Naturally unnatural

Inflation on small scales?



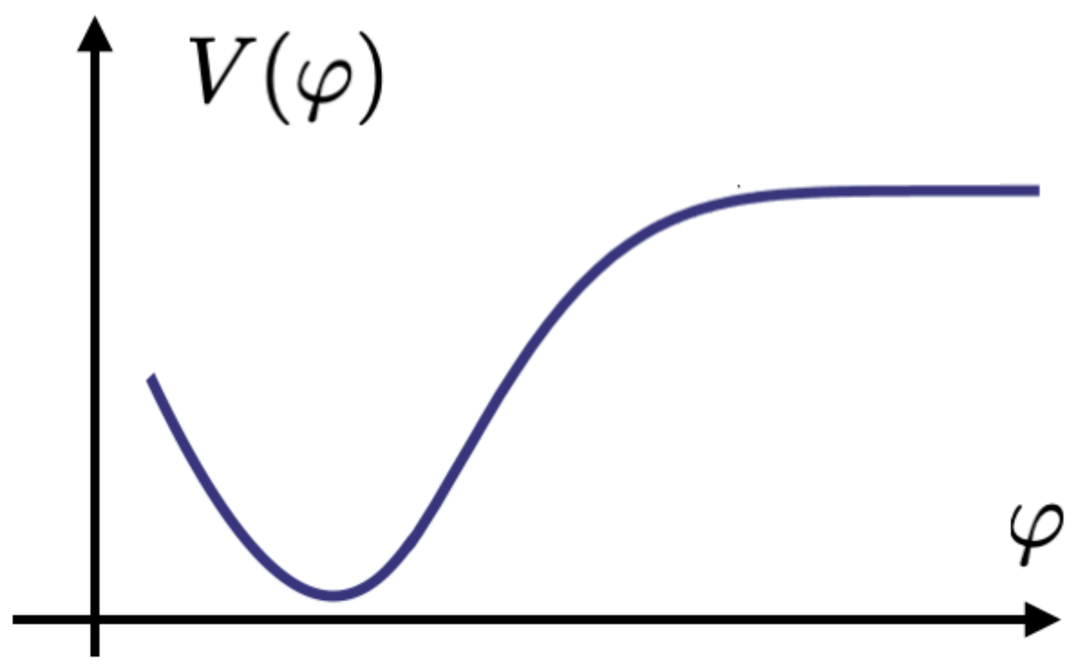
Emergent simplicity
+
Corrections

Drastically different?

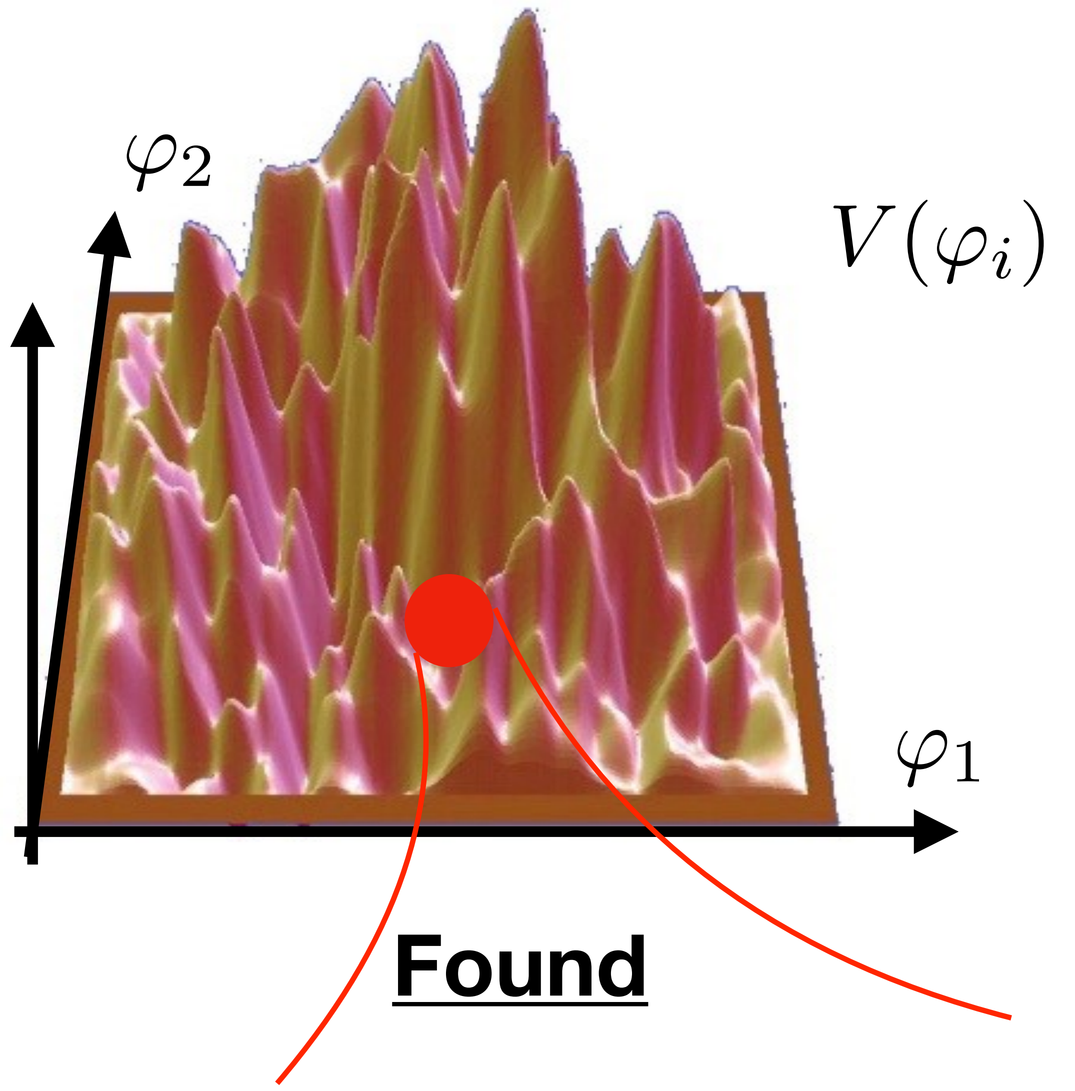
Naturally unnatural

Taking theory seriously

A prolonged phase of smooth 60 e-folds of inflation is not natural (eta-problem)



Hoped



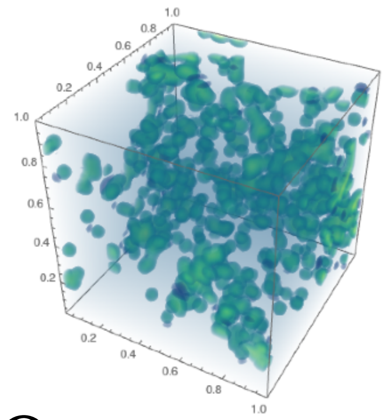
Found

More natural for inflation to have occurred in successive phases with different properties

Ultra slow-roll, hybrid-type transitions, sharp turns, etc

GW from inflation, which ones?

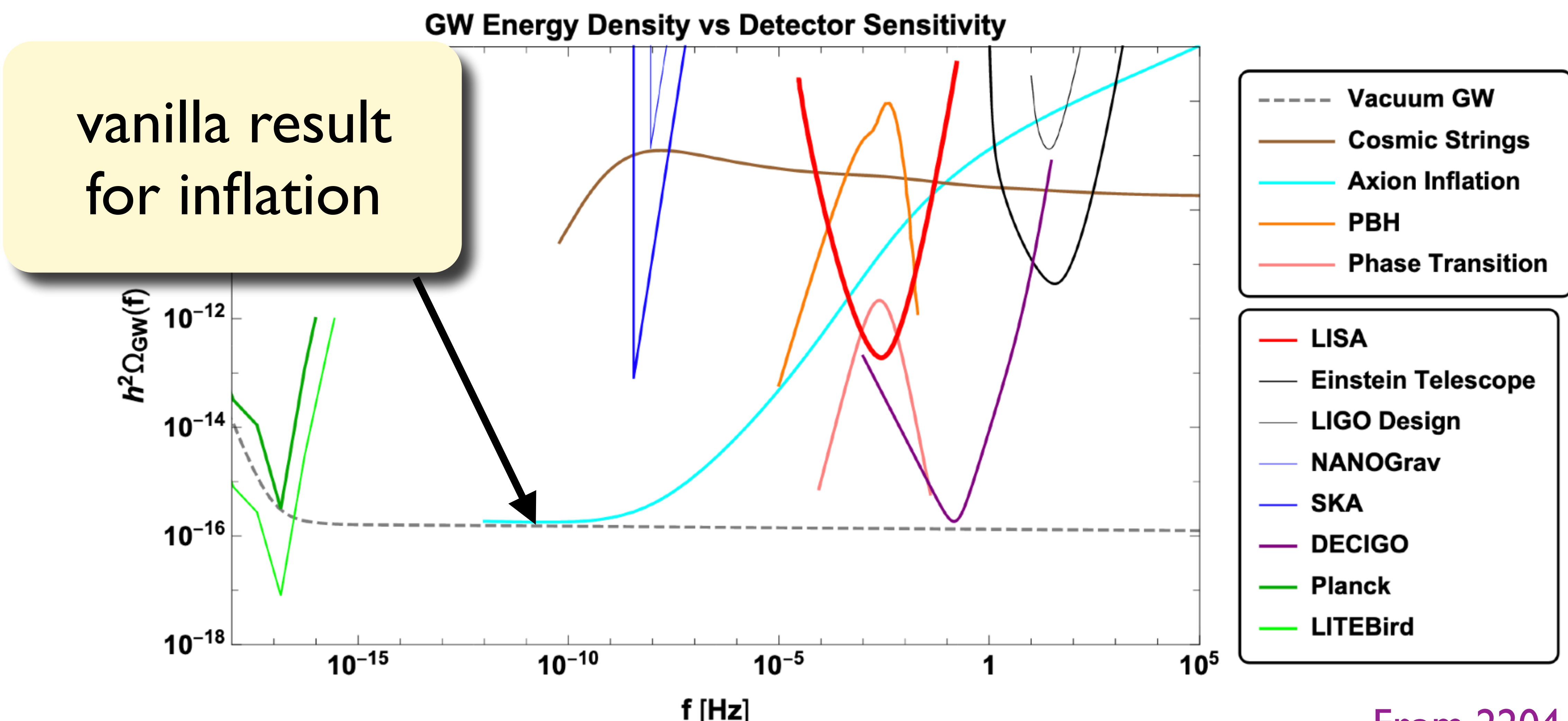
Vacuum quantum fluctuations



$$h^2\Omega_{\rm GW} \sim 10^{-6}\,\frac{H^2}{M_{\rm Pl}^2}$$

$$\Box h = 0$$

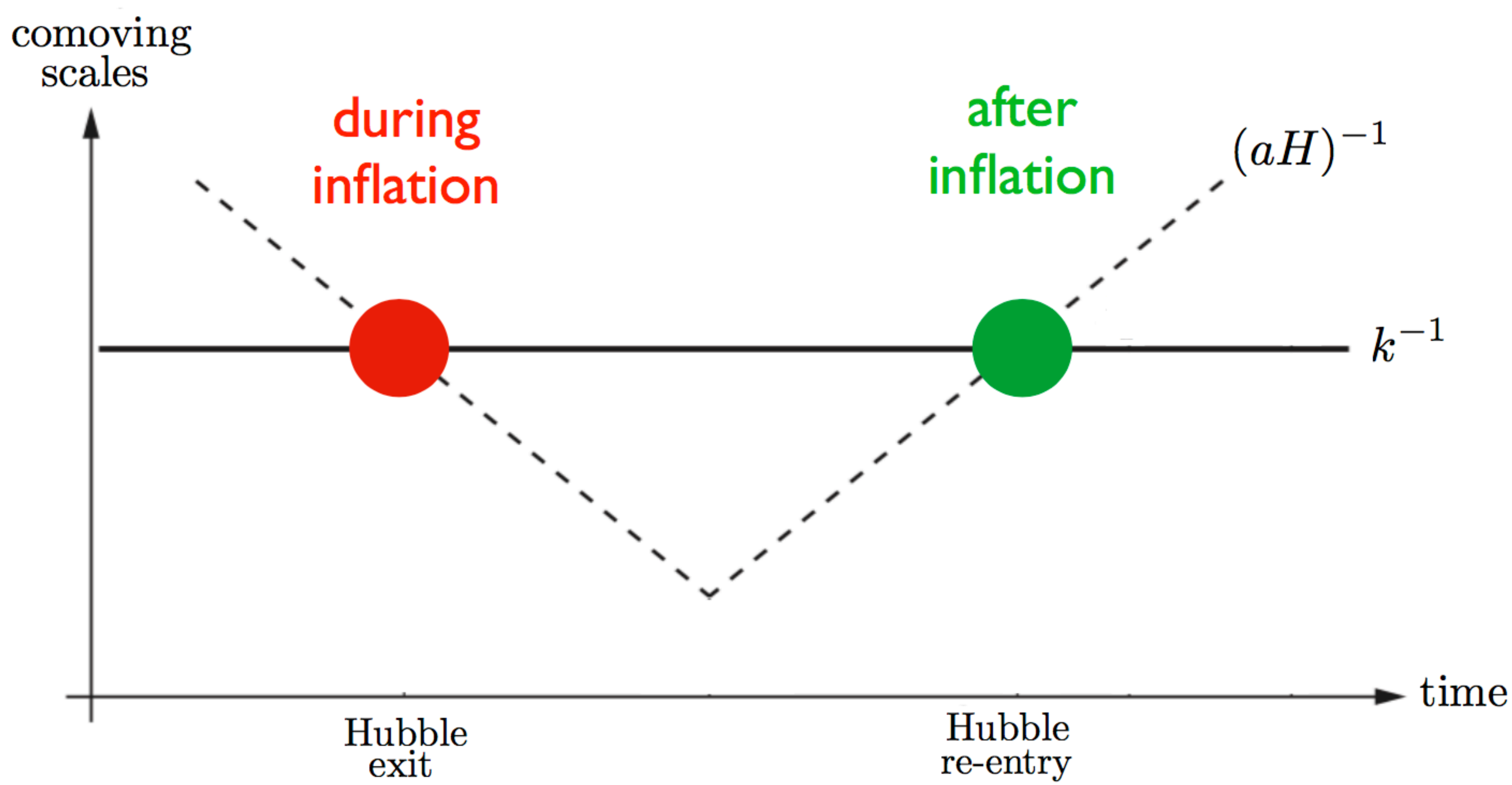
Tiny signals for interferometers



From 2204.05434

GW from active sources

GW generated by nonlinear processes $\Box h \sim (\partial\zeta)^2$



Sensitive to field content and dynamics of inflation

Only sensitive to primordial fluctuations left over after inflation (+ thermal history)

Less studied, more model-dependent
Not less relevant

Studied in detail

Review Domenech 2021

Vast activity

GW
Primordial Cosmology

Correlation with other probes

$$\mathrm{GW} \times (\mathrm{CMB},\, \mathrm{LSS})$$

Frequency profile

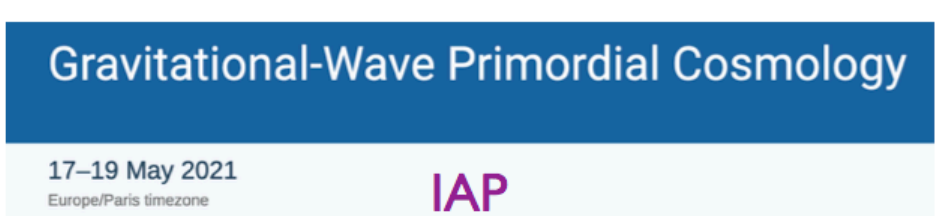
$$\Omega_{\mathrm{GW}}(f)$$

SGWB anisotropies

$$\Omega_{\mathrm{GW}}(f, \hat{n})$$

Chirality

$$h_+ \neq h_\times$$



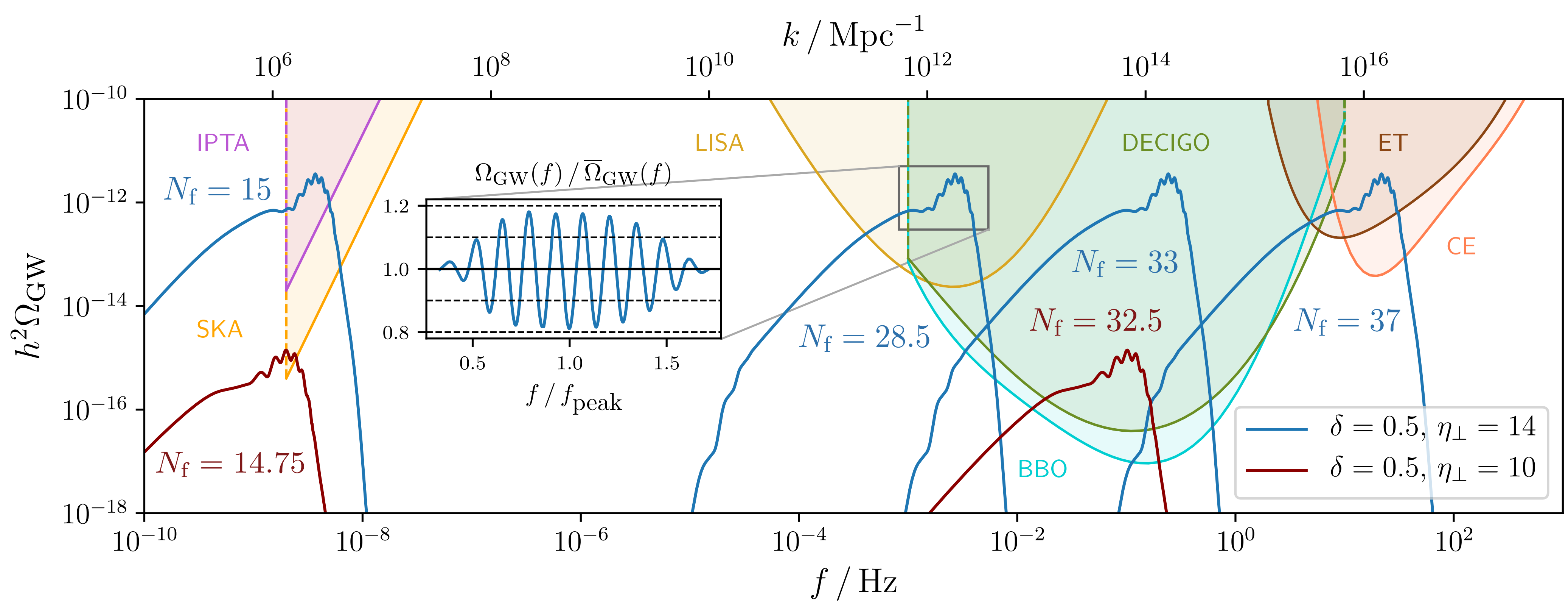
Post-inflationary scalar-induced GWs

$$\Box h \sim (\partial\zeta)^2 \longrightarrow \Omega_{\rm GW}(f) \sim \int\int \mathcal{P}_\zeta \otimes \mathcal{P}_\zeta \sim 10^{-5}\,\mathcal{P}_\zeta^2$$

$$\mathcal{P}_\zeta \sim 10^{-4} \longrightarrow \Omega_{\rm GW} \gtrsim 10^{-13}$$ **LISA**

GW frequency → temperature @ production (non-inflation source) Caprini's talk

GW frequency → time during inflation (inflation source)

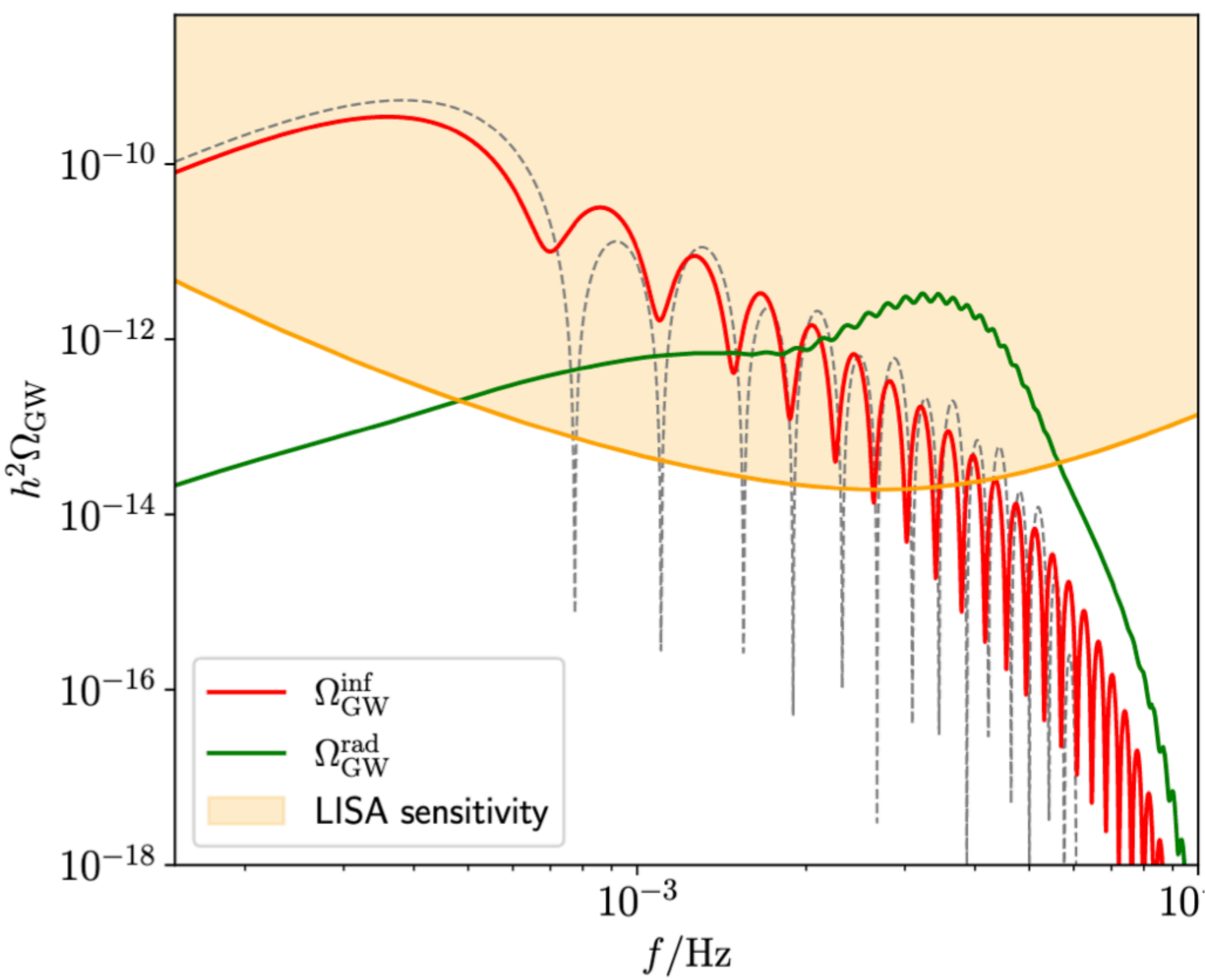


Fumagalli, RP, Witkowski 20

$N_{\rm f}$ = time of feature after horizon exit of CMB modes

Example of SGWB from sharp features

A sharp event during inflation leads to smoking gun oscillatory signatures for the two types of scalar-induced GWs



Oscillatory patterns in SGWB frequency profile

Model-independent: frequencies indicate time of feature

Motivated target of new physics

Fumagalli, RP, Witkowski et al (2020,2021)
several more aspects:

general formalism for GW during inflation
resonant features
expansion history after inflation
detectability with LISA

Some open questions

- Impact of trispectrum on post-inflationary scalar-induced GW?

$\langle h^2 \rangle$ sourced by $\langle \zeta^4 \rangle \sim \langle \zeta^2 \rangle^2 + \langle \zeta^4 \rangle_c$

Unal 18
Atal, Domenech 21
Adshead, Lozanov, Weiner 21

No-go: theoretical consistency imposes negligible effect in simple situations

Garcia-Saenz, Pinol, Renaux-Petel, Werth 22

- Theoretical consistency of scenarios with boosted power spectrum: back reaction, perturbative control, loops

Fumagalli, Renaux-Petel, Witkowski 20
Inomata, Braglia, Chen 22

III. Formal aspects

- Cosmological bootstrap
- Beyond perturbation theory

The Cosmological bootstrap

Snowmass review, Baumann et al 2022

Study of primordial correlators using basic physical principles of **locality, analyticity and unitarity** (and symmetries). Surprisingly constraining! Active field.

$$\langle \varphi(\boldsymbol{x}_1) \ldots \varphi(\boldsymbol{x}_n) \rangle = \int \mathcal{D}\varphi \, \varphi(\boldsymbol{x}_1) \ldots \varphi(\boldsymbol{x}_n) \big| \Psi(\varphi) \big|^2$$

Correlators ⟷ Wavefunction

$$\Psi(\varphi) = \exp\Big[\sum_{n \geq 2} \frac{1}{n!} \int \prod \mathrm{d}\boldsymbol{k}_i \delta\big(\sum \boldsymbol{k}_i\big) \, \varphi_{\boldsymbol{k}_1} \ldots \varphi_{\boldsymbol{k}_n} \psi_n(\boldsymbol{k}_i) \Big]$$

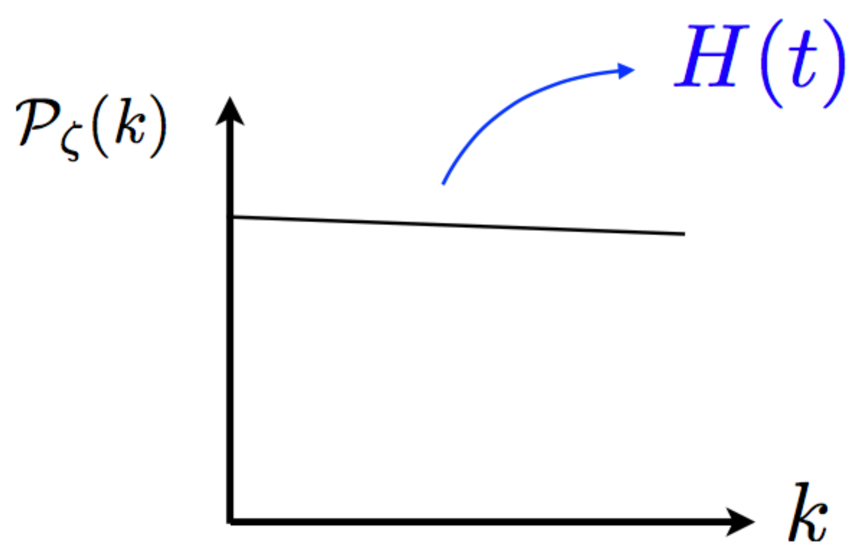
↓

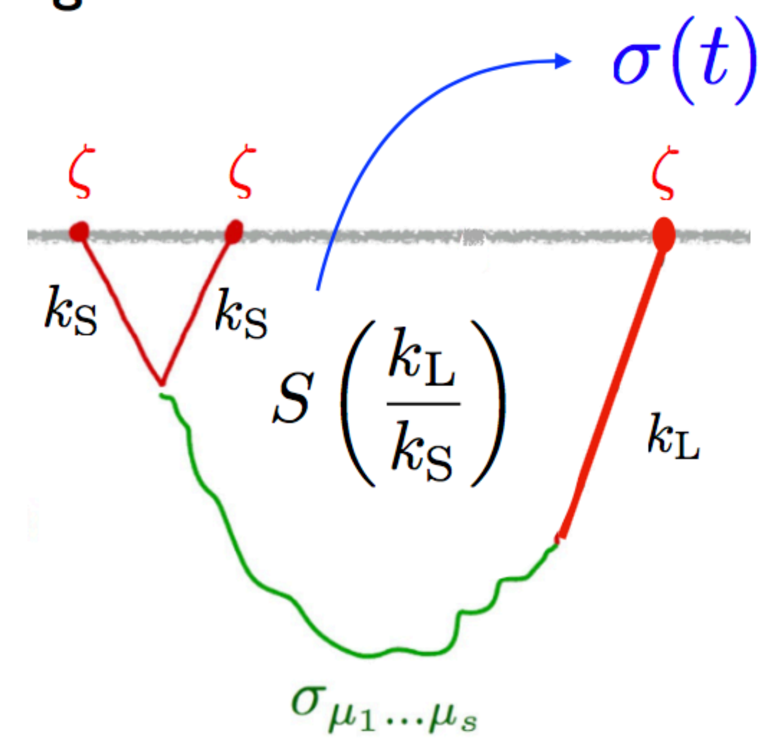
Wavefunction similar to S-matrix for particle physics

Wavefunction coefficients (perturbation theory)

Locality

Well known: Momentum dependence of correlation functions encode time dependence during inflation





Differential equation in time verified by mode functions

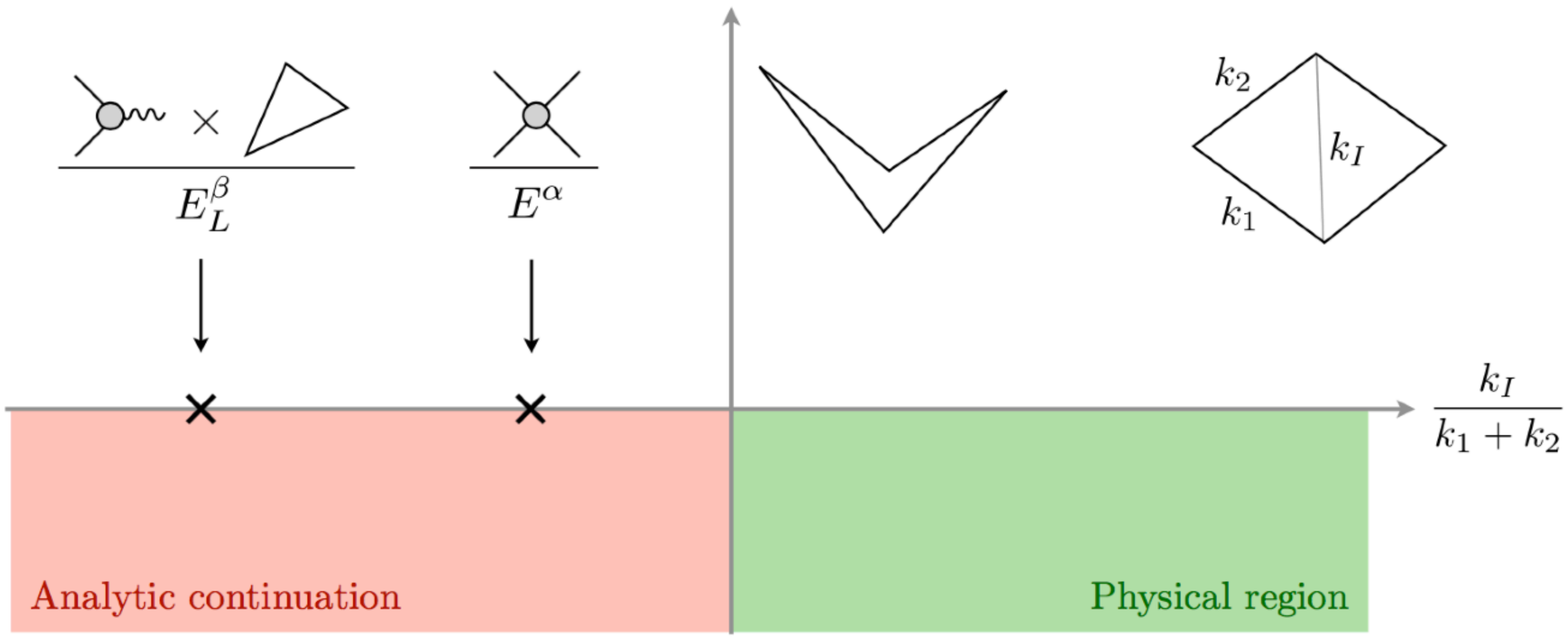
$$O(\partial_\eta)\varphi^a(k\eta) = 0$$

→

Differential equation in momentum space verified by correlators

$$\tilde{O}(\partial_{k_i})\langle \zeta_{\boldsymbol{k}_1} \ldots \zeta_{\boldsymbol{k}_n} \rangle = 0$$

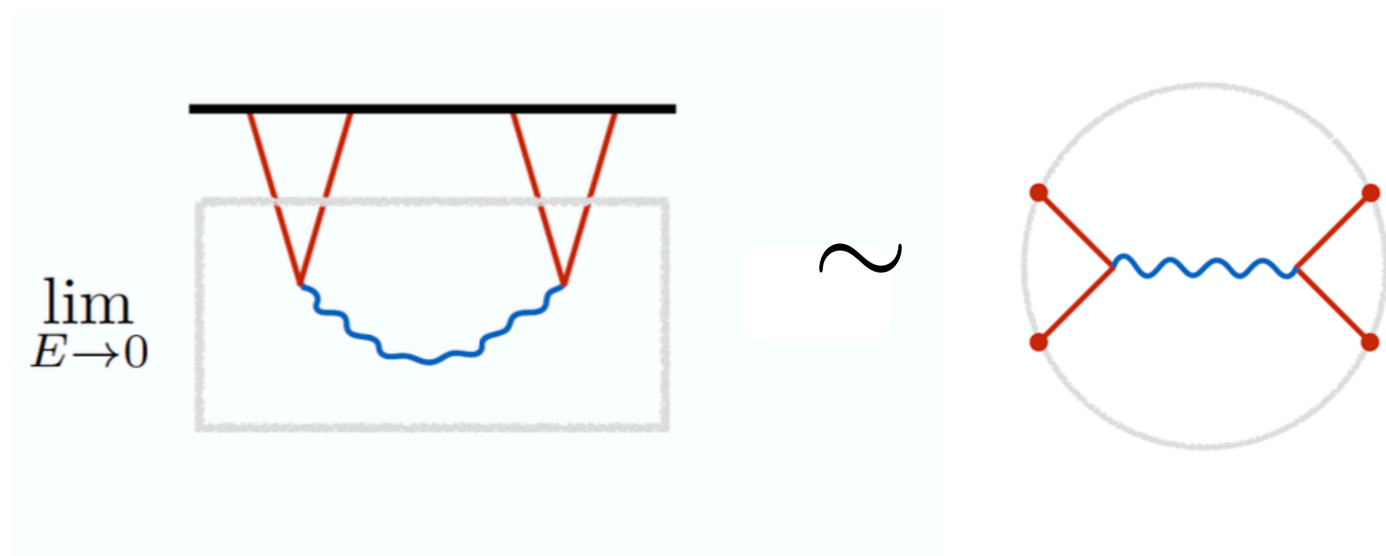
Analyticity/Singularities



Singularities of analytically continued wavefunction coefficients constrain physical region

Total energy singularity

$$\lim_{E=\sum k_i \to 0} \psi_n = \frac{iA_n}{E^\alpha}$$



Interactions pushed back at infinite past ⟷ flat space amplitude

Maldacena, Pimentel 2011
Raju, 2012

Also partial energy singularities

Unitarity

« **Conservation of probabilities** »

Unitary time evolution

$$UU^\dagger = \mathbb{1}$$

→

Identities relating different orders of wavefunction coefficients

Cosmological optical theorem and cutting rules

$$U = \mathbb{1} + \delta U$$

$$\delta U + \delta U^\dagger = -\delta U \delta U^\dagger$$

Goodhew, Jazayeri, Pajer 2020
Baumann et al 2021
Melville, Pajer 2023
...

Some bootstrap results

4 pt of graviton
GR in dS space

Bonifacio et al 2022

3 and 4 pt single-exchange
de Sitter invariant

Arkani-Hamed, Baumann, Lee, Pimentel 2018

3 and 4 pt single-exchange
de Sitter boost broken (large NGs)

Pimentel, Wang 2022
Jazayeri, Renaux-Petel 2022

3 pt single-field
EFT of inflation

Pajer 2020

Infrared divergences

Secular divergences $$\lambda N^2 + (\lambda N^2)^2 + \dots$$

e.g. $\lambda\varphi^4$

Small coupling constant

number of e-folds

Perturbation theory fails at late times

Resummation with stochastic formalism

Starobinsky, 86

$$\frac{\mathrm{d}\varphi}{\mathrm{d}N} = -\frac{V'(\varphi)}{3H^2} + \frac{H}{2\pi}\xi$$

super-Hubble
coarse-grained field

Classical

Quantum
(gaussian white noise)

Some IR divergence developments

Tail of pdf
(useful for PBH)

Ezquiaga, Garcia-Bellido, Vennin 2019
Achucarro, Cespedes, Davis, Palma 2021
Chen, Palma, Riquelme, Hitschfeld, Sypsas 2018

Stochastic $\delta\mathcal{N}$
1 pt pdf $\mathbb{P}[\zeta]$

Fujita, Kawasaki, Tada, Takesako 2013, 2014
Vennin, Starobinsky 2015

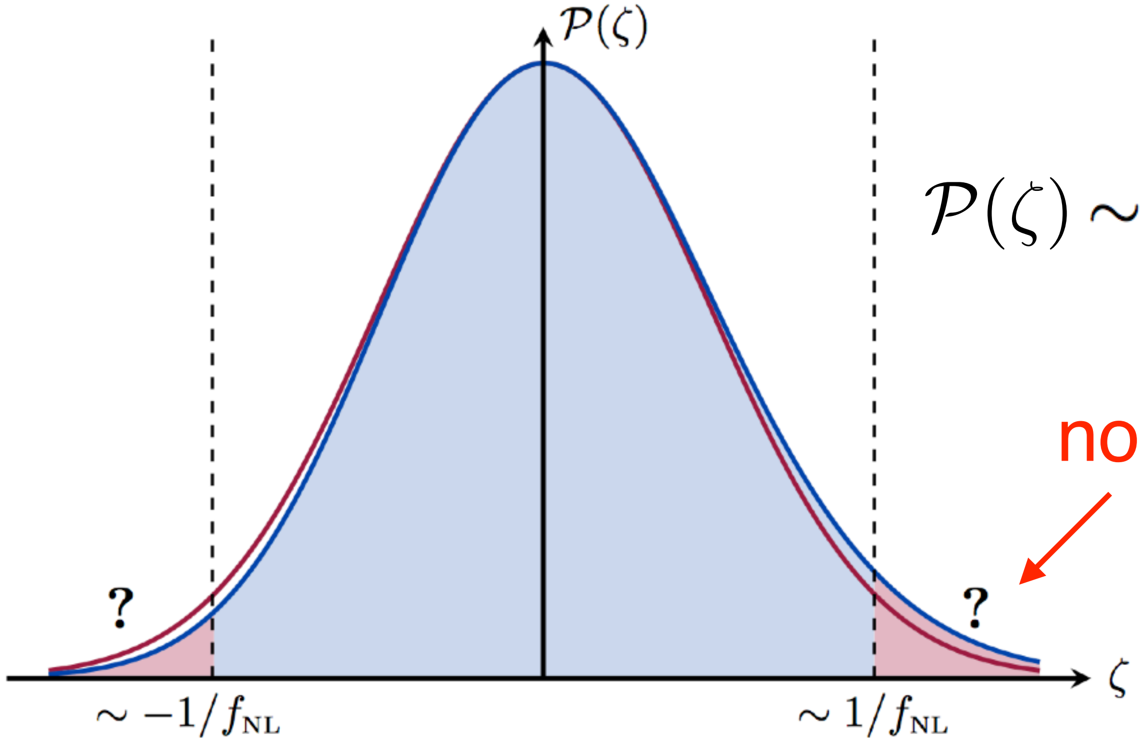
Systematic corrections
to stochastic formalism

Gorbenko, Senatore, Baumgart, Sundrum, Mirbabayi, Cohen, Green, Premkumar 2019-2022

Manifestly covariant
formulation

Pinol, Renaux-Petel, Tada 2019, 2020

Full non-perturbative tail of wavefunction



$$\mathcal{P}(\zeta) \sim \exp\left[-\frac{\zeta^2}{2P_\zeta}\left(1 + \frac{\langle\zeta\zeta\zeta\rangle}{P_\zeta^2}\zeta + \frac{\langle\zeta\zeta\zeta\zeta\rangle}{P_\zeta^3}\zeta^2 + \dots\right)\right]$$

non-perturbative method needed

Full profile! Not only 1 pt pdf

Rare-event limit is semi-classical

$$\Psi[\zeta(\boldsymbol{x})] \sim e^{iS[\zeta_{\rm cl}]/\hbar}$$

Celoria, Creminelli, Tambalo, Yingcharoenrat 2021

Resummation of all tree-level non-linearities, with negligible loop effects

Oscillations on the tail from resonant features

Creminelli, Renaux-Petel, Tambalo, Yingcharoenrat 2023

See Giovanni Tambalo's talk

Conclusion

No immediate motivation from observations has led to burst of new ideas

Precision physics

Non-Gaussianities as cosmological collider to establish standard model of inflation

Exploratory physics

SGWB and PBH as probe of unknown dark era of inflation

Formal aspects

Many developments building bridges with other fields

New approaches, new targets, new questions

Exciting times for inflationary cosmology


Thank you