The Cosmological Flow

of Primordial Correlators

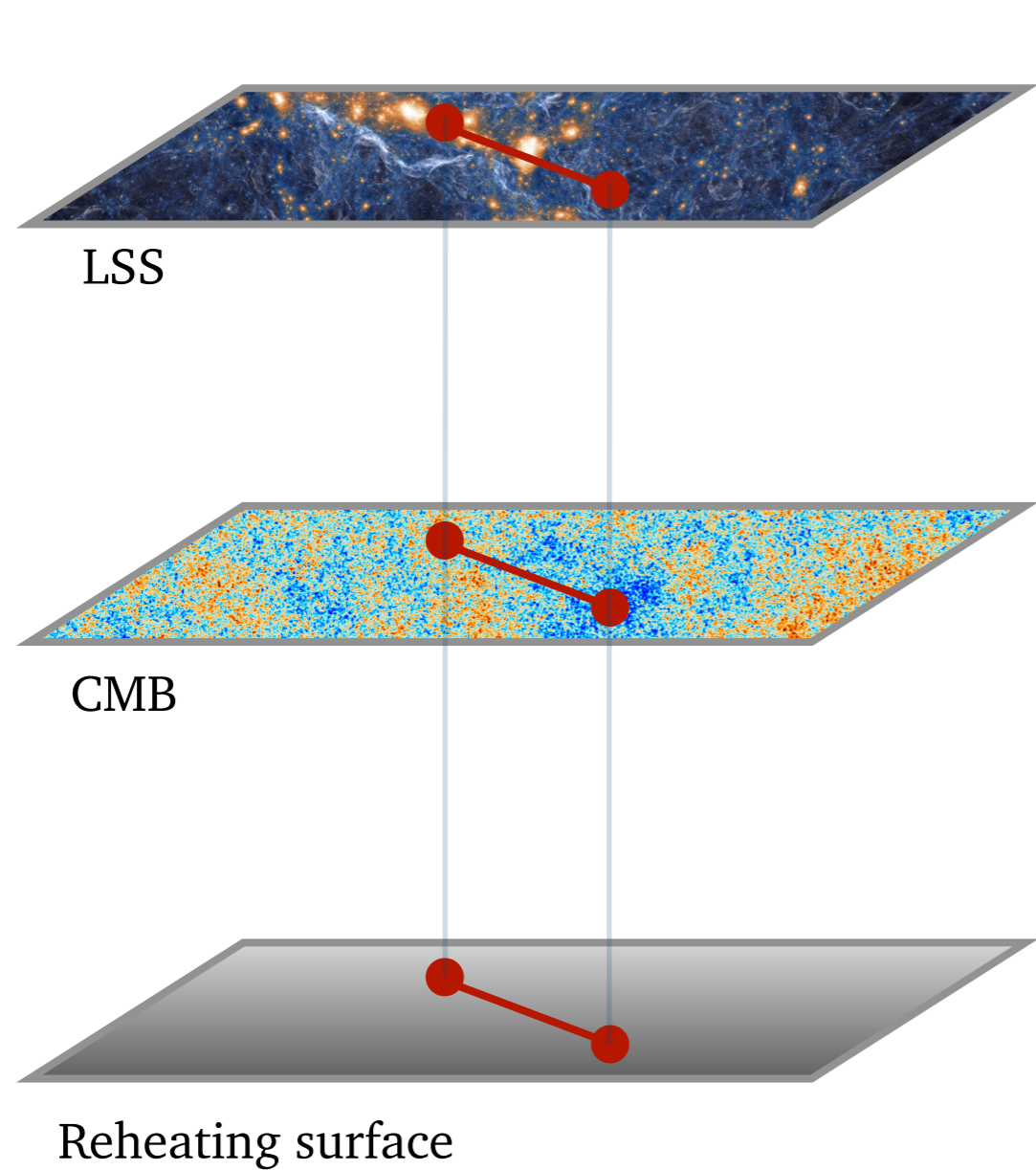**Denis Werth**

SORBONNE UNIVERSITÉ

IAP

erc — European Research Council — Established by the European Commission — GEODESI

Cosmology: A History of Time

Cosmological fluctuations are **correlated** on large scales

Time

LSS

CMB

Reheating surface

Transfer function

$$\langle \delta X^2(\boldsymbol{k}, t) \rangle = \mathbf{T}^2(\boldsymbol{k}; t, t_0) \langle \delta X^2(\boldsymbol{k}, t_0) \rangle$$

- Linear clustering
- Dark matter, photons, baryons
- ...

The physics is encoded in the **time evolution** of these fluctuations
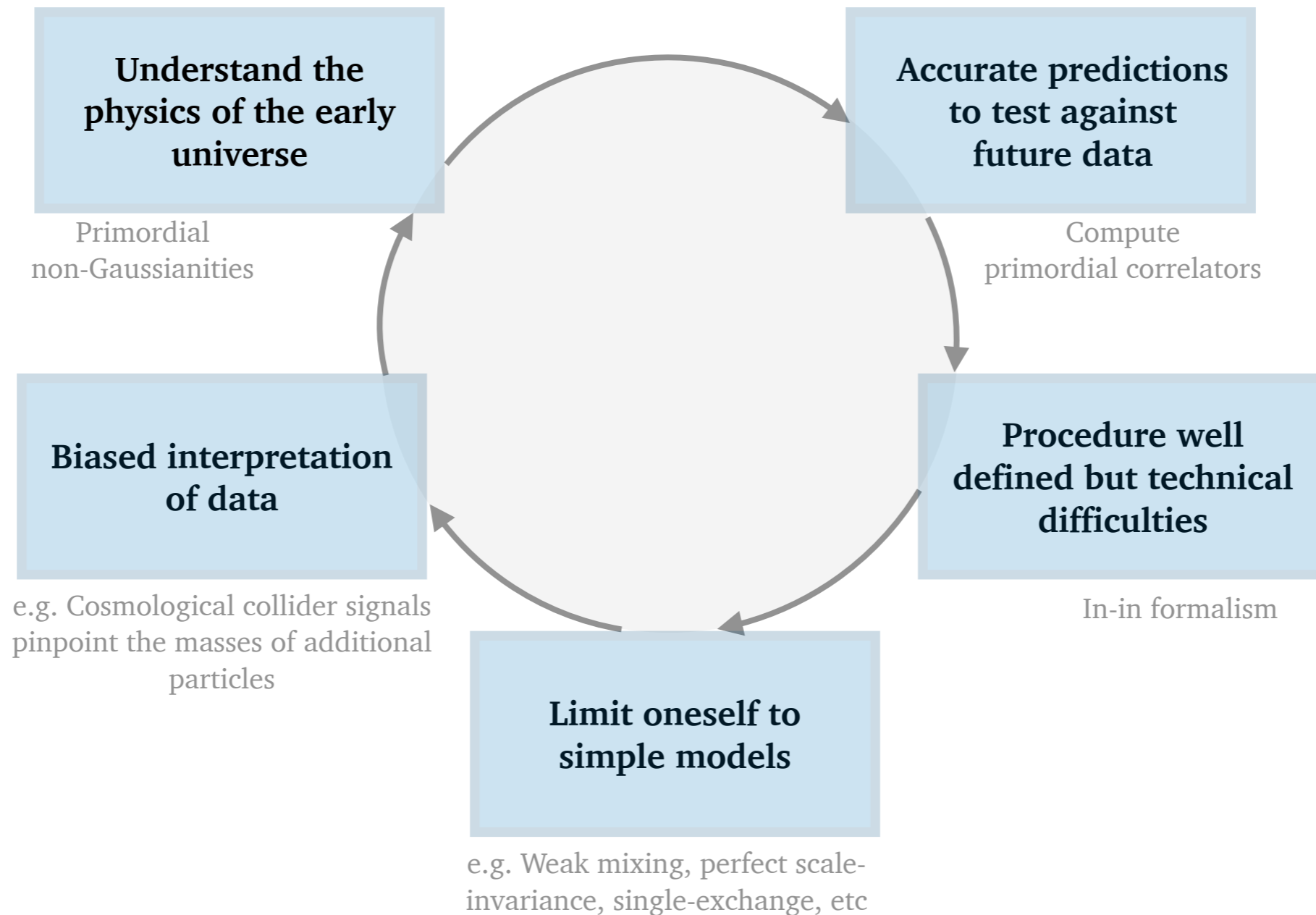
The Cosmological Flow Philosophy

Follow the **time evolution** of primordial fluctuations from their origin as **quantum vacuum fluctuations** to the reheating surface

Time

Reheating surface

Physics encoded in the time evolution

Theory → **Cosmological Flow** → **Observables**
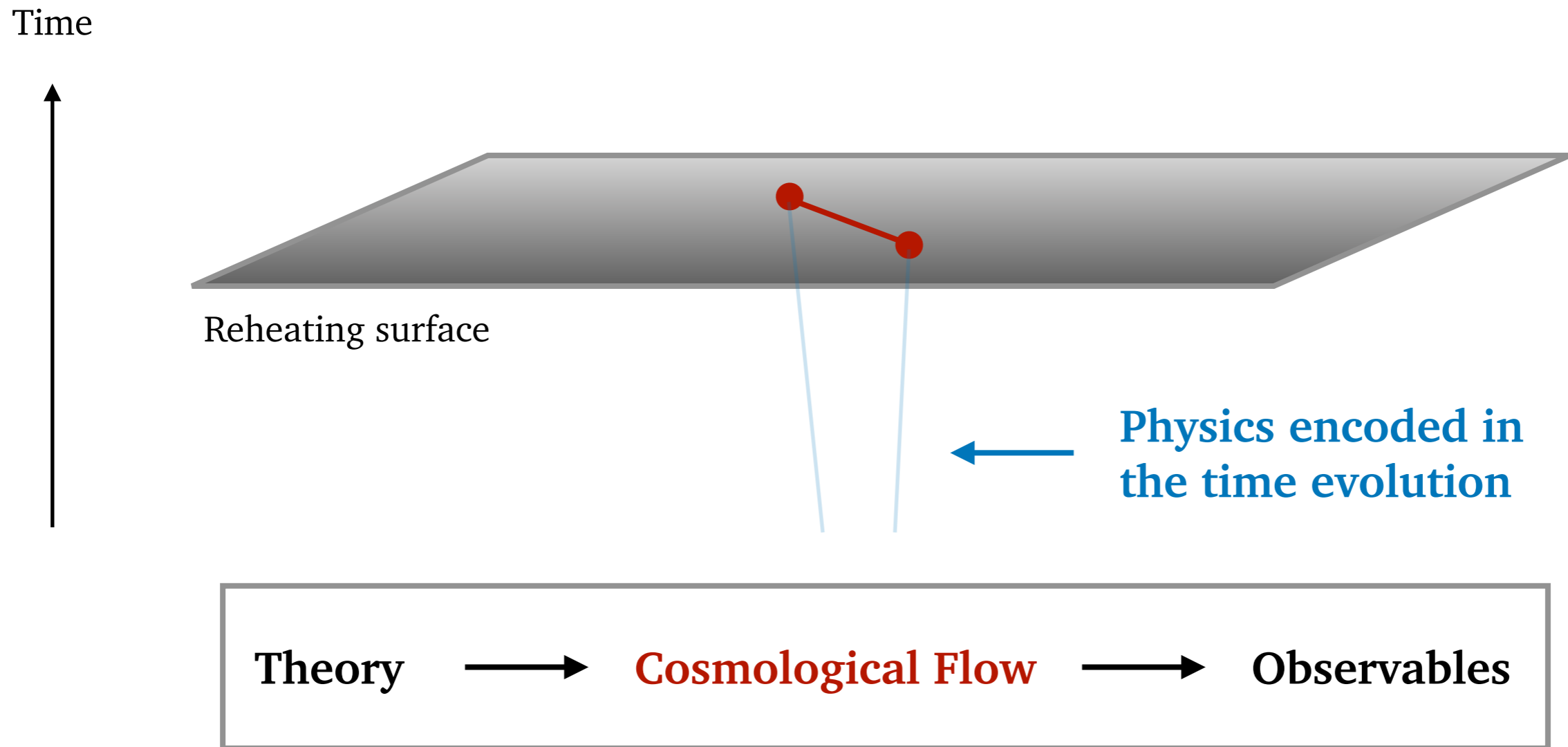
By studying primordial correlators, we can reconstruct a "**standard model of inflation**" (identify new particles, dispersion relations, masses, spins, interactions)

Why the Cosmological Flow: Break the Vicious Circle

The **Cosmological Flow** is an **efficient** and **systematic** approach
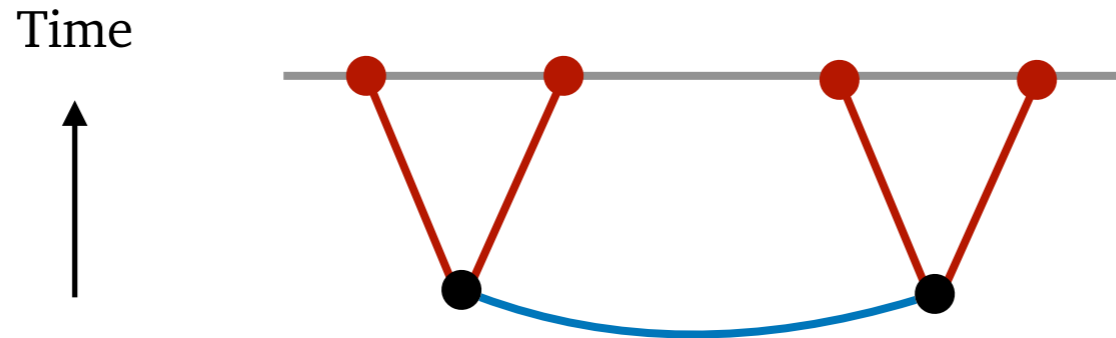to compute primordial correlators

The Cosmological Flow

In practice, we compute Feynman-Witten **diagrams**

Time

$$\langle \boldsymbol{X}^4 \rangle = \int_{-\infty}^{0} \mathrm{d}t \, \mathrm{d}t' V(t) V(t') \, K(k_1, t) \, K(k_2, t) \, \mathcal{G}(k_{12}, t, t') K(k_3, t') K(k_4, t')$$

Nested time integrals

Complicated
hypergeometric functions

Late-time correlators
receive contributions
from all times

Recent Analytical Developments

dS Cosmological Bootstrap

Arkani-Hamed, Baumann, lee, Pimentel, Joyce,
Duaso Pueyo [2019, 2020, 2022]

Boostless Cosmological Bootstrap

Pajer [2020]
Pimentel and Wang [2022]
Jazayeri and Renaux-Petel [2022]

AdS-inspired Mellin Space

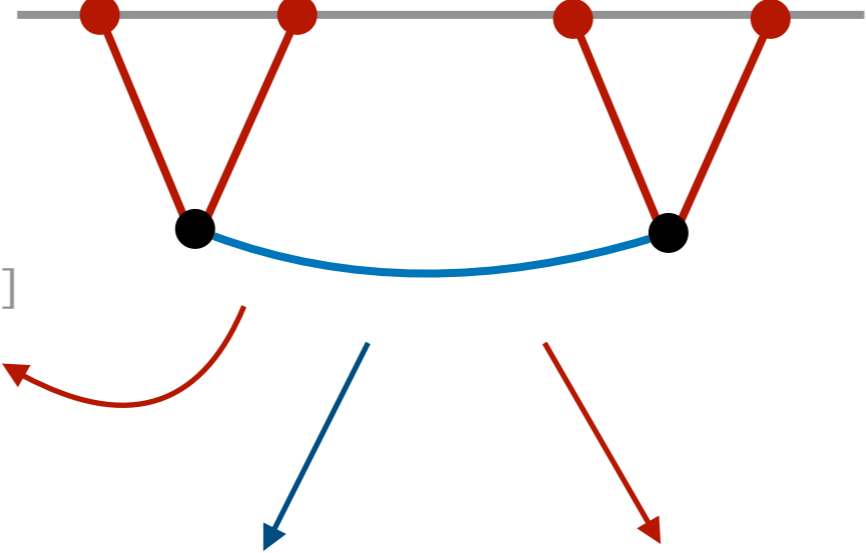Sleight and Taronna [2019, 2021, 2022]

Cosmological Potytopes

Arkani-Hamed, Benincasa, Postnikov,
McLeod [2017, 2018, 2019, 2020, 2022]

**Fundamental Principles
(Unitarity & Locality)**

Pajer, Stefanyszyn, Supeł, Goodhew, Jazayeri, Melville,
Gordon Lee, Bonifacio, Wang [2020, 2021, 2022]

Partial Mellin-Barnes Representation
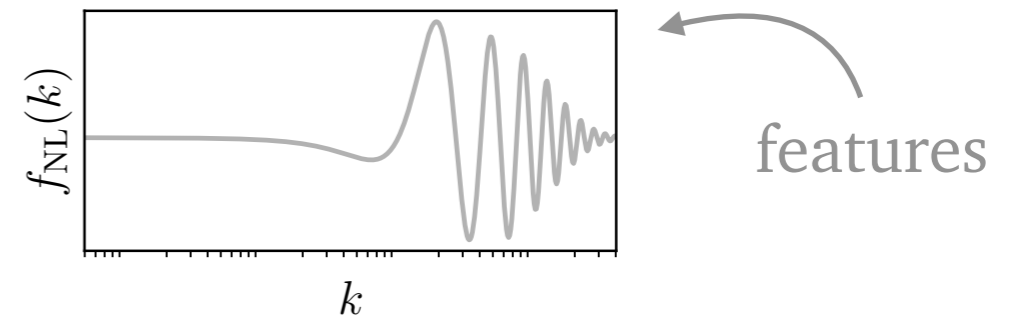
Qin and Xianyu [2022]

Limitations of Analytical Methods

Weak Quadratic Mixing

$$\mathcal{L}^{(2)} \supset \rho \dot{\phi} \sigma$$

treated perturbatively

(Near) Scale-Invariance

features

Only Single-Exchange Diagram
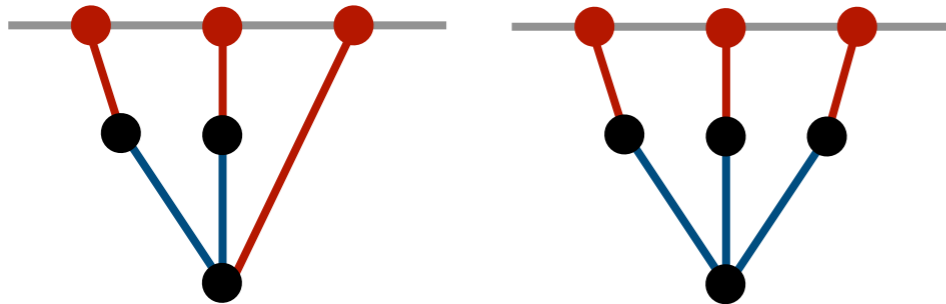
**Large hierarchy of masses/couplings
but not the intermediate regimes**

energy

$$\rho \quad \ll \quad H \quad \ll \quad m$$

Often only 1 or 2 Fields

energy

$$m_\phi \quad m_1 \quad m_2 \quad H \quad m_3$$
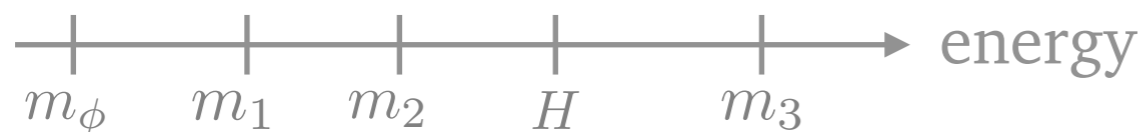
**Treatment of Equilateral and
Squeezed Configurations Separately**

$$\rhd \ \sim e^{-\pi\mu} \qquad \triangle \ \sim \frac{1}{\mu^2}$$

Aside from isolated examples...

The Cosmological Flow

From first principles, the time evolution of primordial correlators
is encoded in the **flow equations** (Ehrenfest theorem)

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\rangle = u^{\mathsf{a}}{}_{\mathsf{c}}\langle \boldsymbol{X}^{\mathsf{c}}\boldsymbol{X}^{\mathsf{b}}\rangle + u^{\mathsf{b}}{}_{\mathsf{c}}\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{c}}\rangle$$
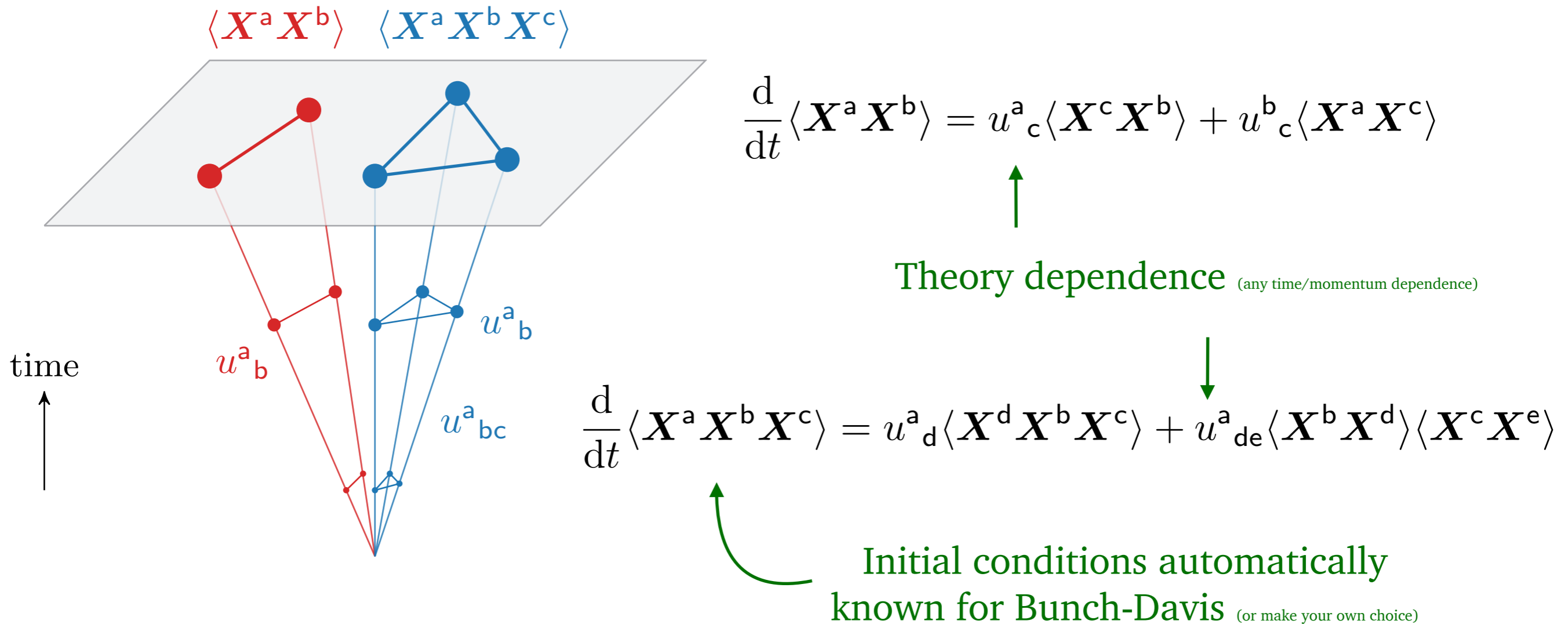
Theory dependence (any time/momentum dependence)

$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{c}}\rangle = u^{\mathsf{a}}{}_{\mathsf{d}}\langle \boldsymbol{X}^{\mathsf{d}}\boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{c}}\rangle + u^{\mathsf{a}}{}_{\mathsf{de}}\langle \boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{d}}\rangle\langle \boldsymbol{X}^{\mathsf{c}}\boldsymbol{X}^{\mathsf{e}}\rangle$$

**Initial conditions automatically
known for Bunch-Davis** (or make your own choice)

We have converted the problem of computing nested time
integrals to **solving a set of coupled differential equations**

Dias, Fazer, **Mulryne**, **Seery**, Ronayne
[2010, 2011, 2012, 2013, 2015, 2016, 2018]

Werth, Pinol, Renaux-Petel [2023 a, b]

Applications

Goldstone Boson coupled to an Additional Field

We couple the **Goldstone boson** to an additional **massive scalar field**
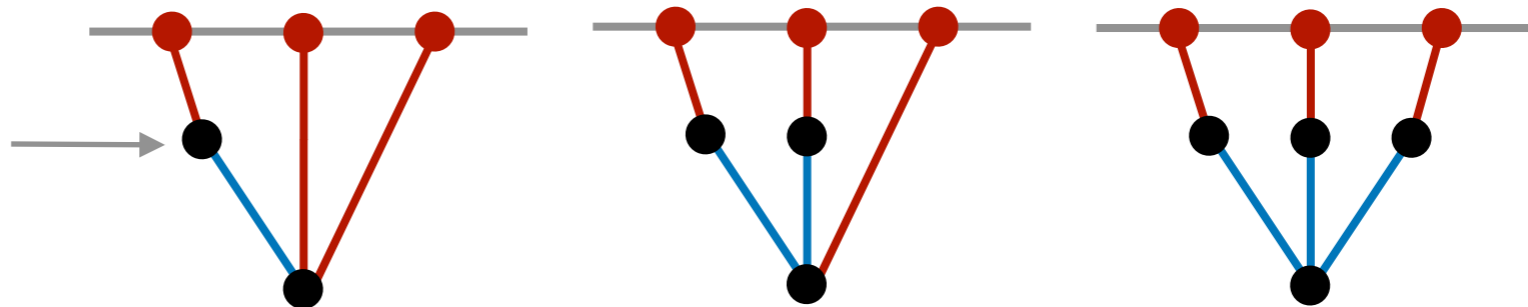$$\zeta = -H\pi$$

π quadratic sector $\qquad\qquad$ σ quadratic sector

**linear mixing
(broken Lorenz invariance)**

$$\mathcal{L}/a^3 = -\frac{1}{2}\left[-\dot{\pi}_c{}^2 + c_s^2 \frac{(\partial_i \pi_c)^2}{a^2}\right] - \frac{1}{2}\left[(\partial_\mu \sigma)^2 + m^2 \sigma^2\right] + \rho(t)\dot{\pi}_c \sigma$$

$$-\mu\sigma^3 - \frac{1}{2}\alpha \dot{\pi}_c \sigma^2 - \frac{1}{2\Lambda_1}\frac{(\partial_i \pi_c)^2}{a^2}\sigma - \frac{1}{2\Lambda_2}\dot{\pi}_c{}^2 \sigma \quad \longleftarrow \quad \text{Cubic interactions}$$

Assuming weak mixing

Relax assumptions: beyond weak mixing, time-dependent mixing

Werth, Pinol, Renaux-Petel [2023 a, b]

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore [2008]

Cosmological Collider Signal at Strong Mixing

The cosmological collider signal of **heavy but weakly mixed** particle oscillates at the same frequency than that of a **light but strongly mixed** particle

$$\mu_{\text{eff}}^2 = m_{\text{eff}}^2/H^2 - 9/4$$

<u>Frequency</u>
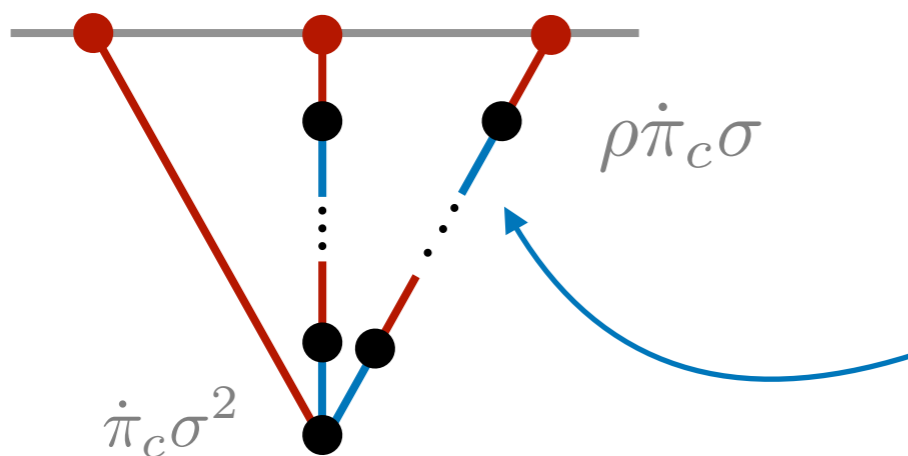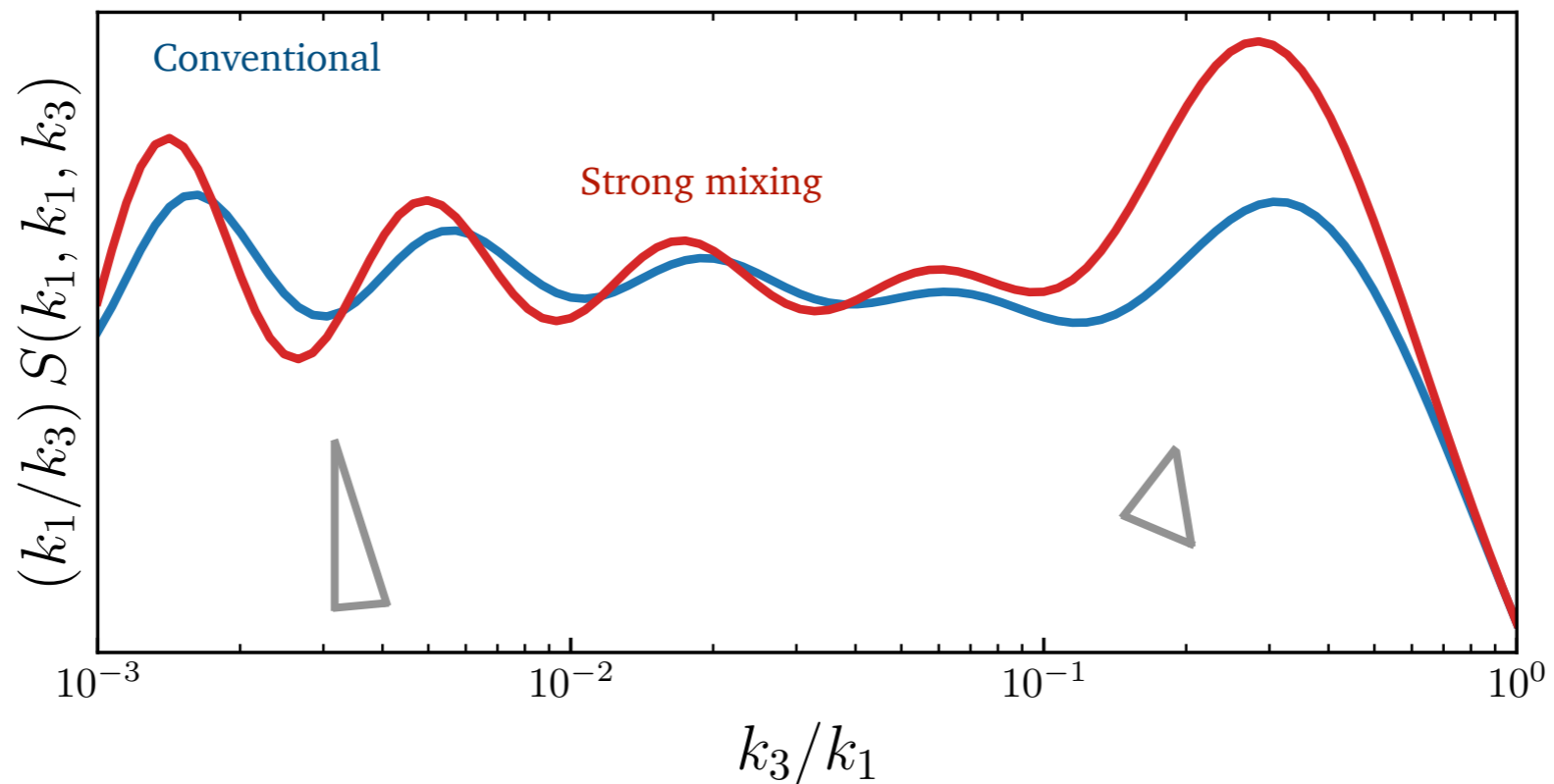
Effective mass for the heavy field
$$m^2 \rightarrow m_{\text{eff}}^2 = m^2 + \rho^2$$

Resummation of quadratic mixings
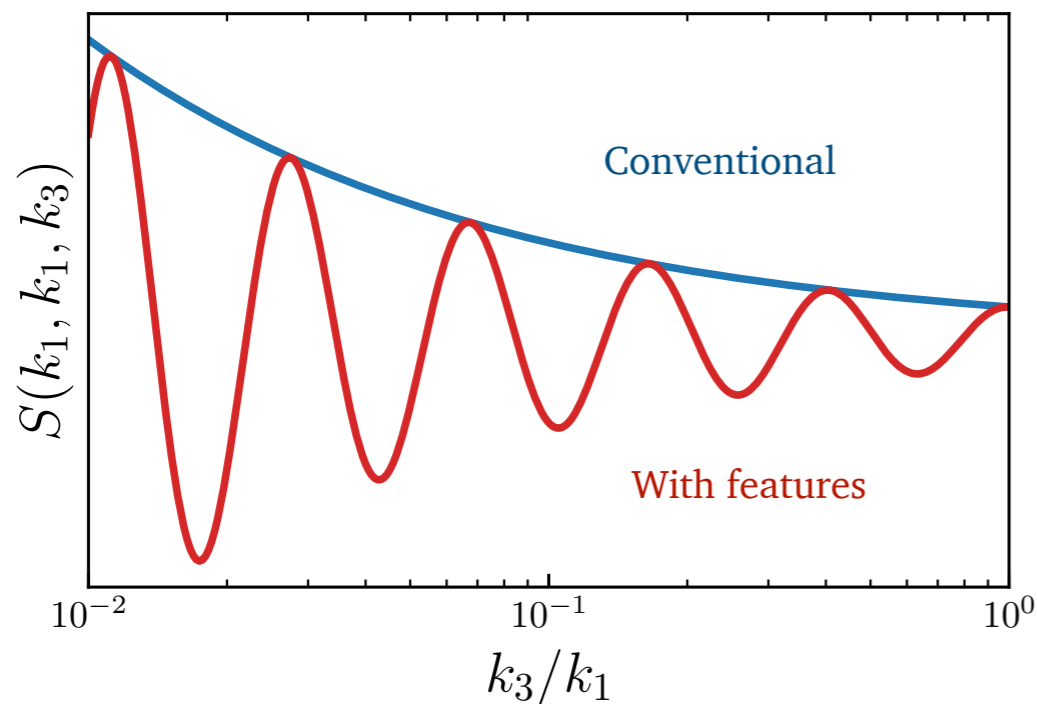
Cosmological Collider Signal with Features

The cosmological collider signal of an **oscillating linear mixing** exhibits new features that were not appreciated before

$$\rho(t) = \rho_0 (a_0/a)^n \sin[\omega_c(t - t_0)]$$
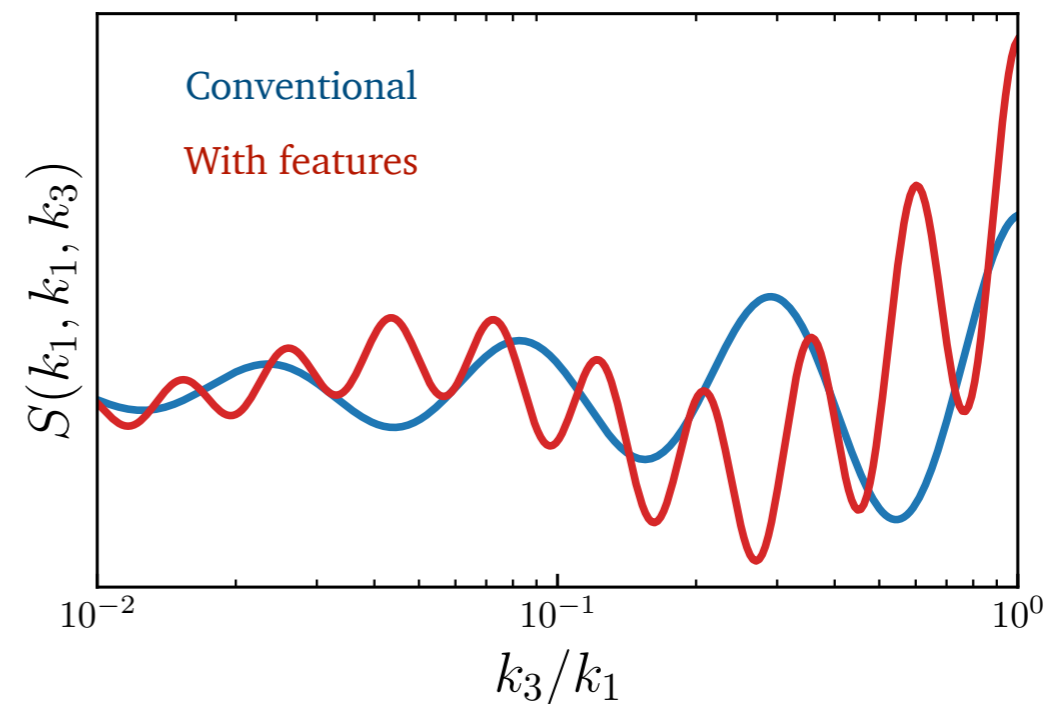
$$\mu^2 = m^2/H^2 - 9/4$$
$$\mu_c = \omega_c/H$$

Light field

$$S = (k_3/k_1)^{\frac{1}{2}+n-\nu} \mathcal{A} \cos(\mu_c \log(k_3/k_1) + \varphi)$$

Heavy field

$$S = (k_3/k_1)^{\frac{1}{2}+n} \sum_{\pm} \mathcal{A}_{\pm} \cos[(\mu \pm \mu_c) \log(k_3/k_1) + \varphi_{\pm}]$$

New templates

Werth, Pinol, Renaux-Petel [2023 a, b]

Conclusions and Take-Home Messages

Primordial **non-Gaussianities** to understand the **physics of inflation**, primary target for future missions

Cosmological Collider: probe the laws of physics at the highest reachable energies

The Cosmological Flow

Concentrating on **exploring** and **understanding** the physics in motivated scenarios **in full generality**

Efficient and **systematic** approach to compute inflationary correlators, avoiding technical difficulties
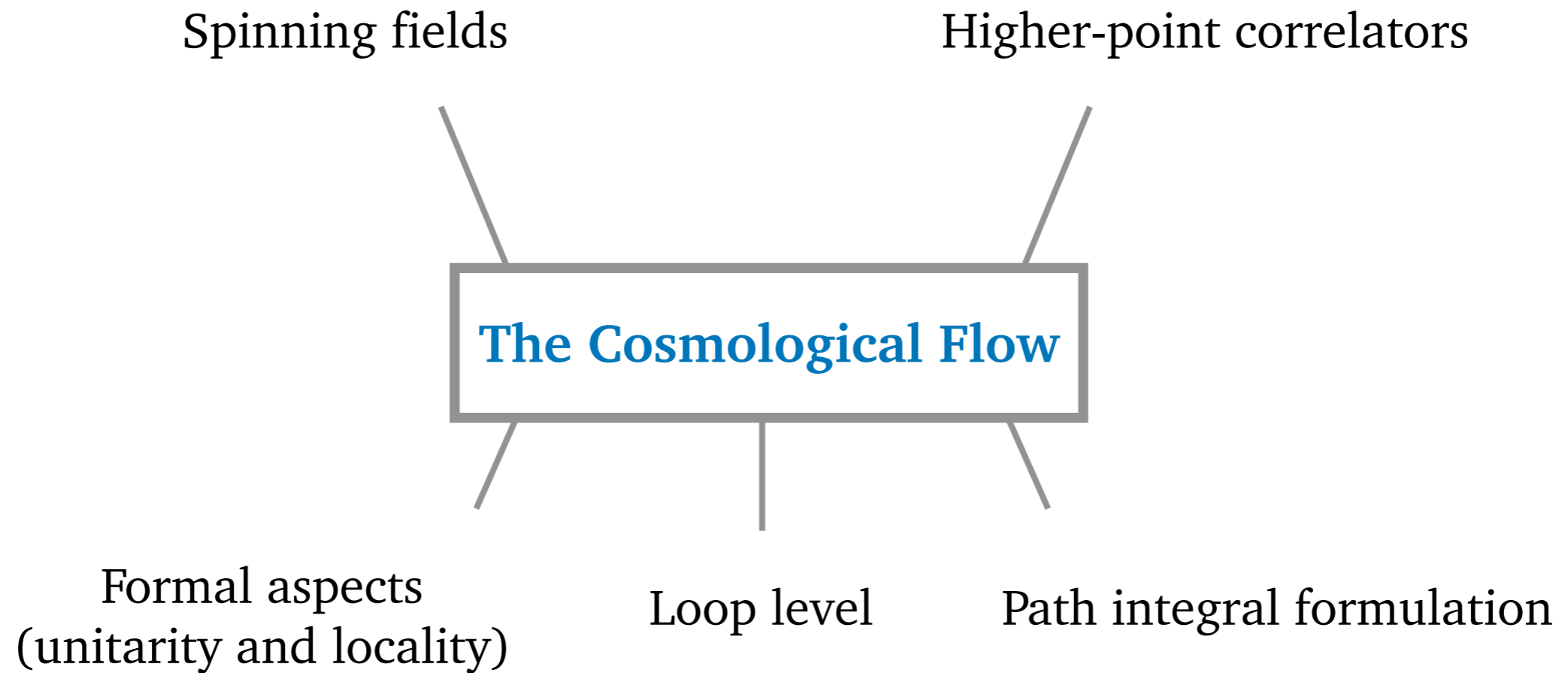
We have only scratched the tip of the iceberg …

Thank you

Outlook

The cosmological flow offers straight **extensions**

Spinning fields   Higher-point correlators

<div style="text-align:center">

The Cosmological Flow

</div>

Formal aspects
(unitarity and locality)   Loop level   Path integral formulation

We have paved the way to a systematic investigation of the
rich and fascinating subject of inflationary correlators