Resonant Features in Inflation Beyond Perturbation Theory

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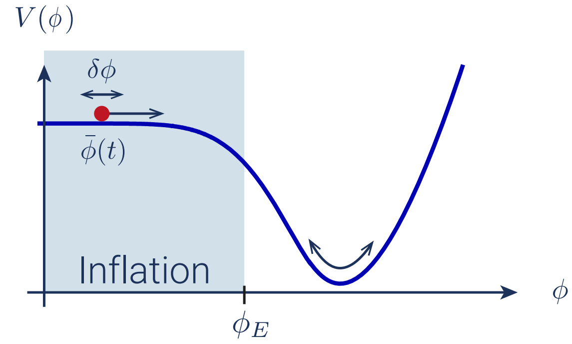
Outline

- Perturbation theory (PT) and beyond in Inflation
- Action for curvature perturbations
- Resonant features
- Results and conclusions

Slow-roll Inflation

- Inflation: period of early acceleration



- Inflaton ϕ rolls down its potential.

  Approximate de Sitter expansion:

$$ds^2 = \frac{-d\eta^2 + d\boldsymbol{x}^2}{\eta^2}$$

- Curvature perturbations ζ freeze outside of the horizon

$$h_{ij} = a^2 e^{2\zeta(\boldsymbol{x},\eta)}\delta_{ij} \ , \qquad \langle \zeta_{\boldsymbol{k}} \zeta_{-\boldsymbol{k}} \rangle' = \frac{P_\zeta}{k^3}$$

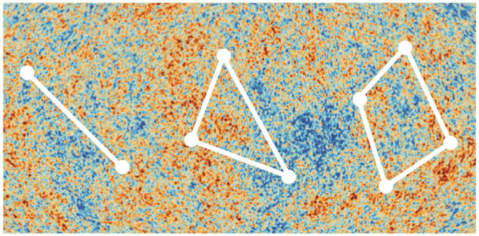
- At CMB scales the typical fluctuations are

$$P_\zeta \equiv H^2/(2\epsilon M_{\rm Pl}^2) \sim 10^{-10}, \ \ \zeta \sim 10^{-5}$$

Power spectrum

Perturbation theory

Statistics of ζ is almost perfectly Gaussian, with corrections characterized by $\langle\zeta^3\rangle, \langle\zeta^4\rangle$



$\langle\zeta^2\rangle$ $\langle\zeta^3\rangle$ $\langle\zeta^4\rangle$

- Inflationary correlators computed in PT using in-in or the **wavefunction of the universe (WFU)** approach [Maldacena `02]

$$\Psi[\zeta,\eta] = \exp\left[\int_{\boldsymbol{k}} \psi_2(\boldsymbol{k})\zeta_{\boldsymbol{k}}\zeta_{-\boldsymbol{k}} + \int_{\boldsymbol{k}} \psi_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3} + \dots\right]$$

- At tree level the coefficients ψ_n are related to correlators

$$\langle\zeta_{\boldsymbol{k}}\zeta_{-\boldsymbol{k}}\rangle = \frac{1}{2\,\mathrm{Re}\,\psi_2(\boldsymbol{k})} \qquad \langle\zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle \propto \mathrm{Re}\,\psi_3(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)$$

Why going beyond PT

- PT: corrections close to the peak of the probability distribution $P(\zeta) = |\Psi|^2$

[Celoria, Creminelli, GT, Yingcharoenrat `21]

$$P(\zeta) \sim \exp\left[-\frac{\zeta^2}{2P_\zeta} + \frac{\langle\zeta^3\rangle}{P_\zeta^3}\zeta^3 + \frac{\langle\zeta^4\rangle}{P_\zeta^4}\zeta^4 + \dots\right]$$

$$\sim \exp\left[-\frac{\zeta^2}{2P_\zeta}\left(1 + \frac{\langle\zeta^3\rangle}{P_\zeta^2}\zeta + \frac{\langle\zeta^4\rangle}{P_\zeta^3}\zeta^2 + \dots\right)\right]$$

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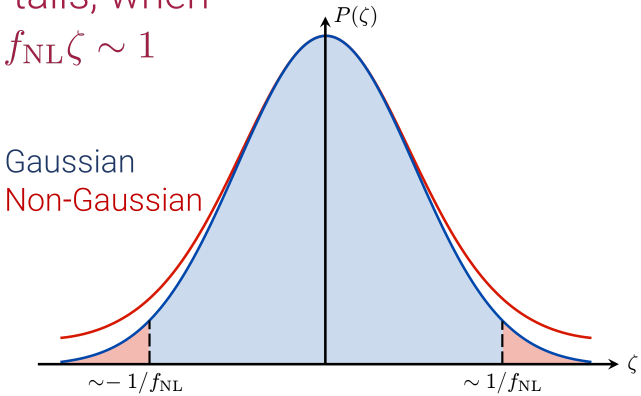
- Large corrections on the tails, when $f_{\rm NL}\zeta \sim 1$



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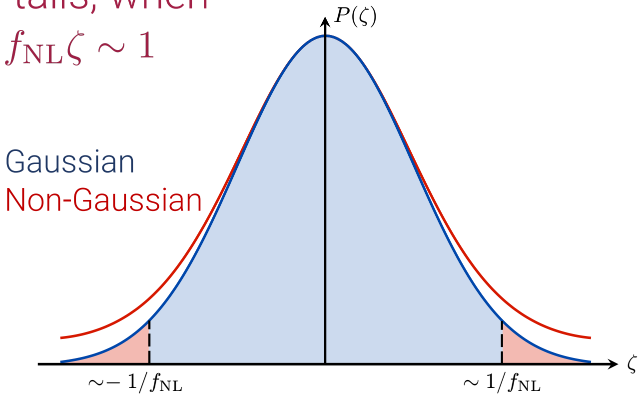
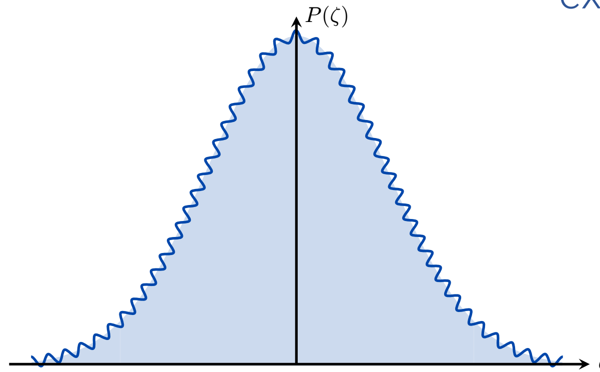


- Small corrections, no Taylor expansion



How to go beyond PT

[Celoria, Creminelli, GT, Yingcharoenrat `21]

The tail of the distribution is amenable to a semiclassical calculation

$$\Psi[\bar{\zeta}(\boldsymbol{x})] = \int_{\mathrm{BD}}^{\bar{\zeta}(\boldsymbol{x})} \mathcal{D}\zeta \; e^{iS[\zeta]} \simeq e^{iS[\zeta_{\mathrm{cl}}]}$$

- Here, ζ_{cl} is the classical (non-linear) solution of the equation of motion
- Loops are usually suppressed: $\zeta_{\mathrm{QM}} \sim P_\zeta^{1/2} \ll \zeta_{\mathrm{cl}}$
- Different from other non-perturbative approaches (e.g. stochastic approach)

[Starobinsky `86]

Action for ζ

- We work in the EFT of inflation, in the decoupling limit $\epsilon \ll 1\,, \quad M_{\rm Pl} \gg H$

  [see S. Renaux-Petel and G. Cabass review talks] [Cheung '08, Pajer '17, Behbahani '12]

- At leading order, the metric is dS and non-dynamical
- The EFT action can be re-casted as:

$$S = M_{\rm Pl}^2 H^{-2} \int \mathrm{d}^4x\, a(t)^3 \dot{H}(t - \zeta/H)\, (\partial_\mu \zeta)^2 + \mathcal{O}(\epsilon^2)$$

- Novel expression, valid non-perturbatively
- Makes clear that $\zeta = \mathrm{const.}$ is a solution

Resonant features

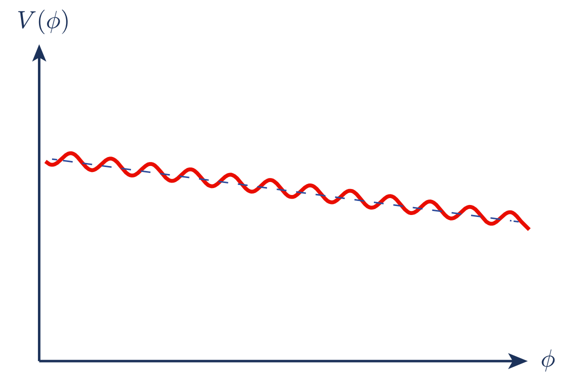
[Chen+ '08, Hannestad+ '09, Flauger+ '09; Flauger, Pajer '10; Leblond, Pajer '11]

- **Resonant features**: small but fast oscillations in $\dot{H}$

$$V(\phi) = V_{\rm sr}(\phi) + \Lambda^4 \cos(\phi/f)$$

$$\dot{H}(t) = \dot{H}_\star(1 - b\cos(\omega t))$$

$$b \ll 1\,,\ \alpha \equiv \omega/H \gg 1$$



- Non-Gaussianities: enhanced and with peculiar shape

$$\langle \zeta_{\boldsymbol{k}_1}\zeta_{\boldsymbol{k}_2}\zeta_{\boldsymbol{k}_3}\rangle \propto b\sqrt{\alpha}\,\alpha^2 \sin(\alpha \log K)$$

$$K \equiv k_1 + k_2 + k_3$$

- All (tree level) correlators are known analytically

WFU for the resonant model

WFU becomes non-perturbative when $\alpha^2\zeta > 1$

We focus on the tails of the WFU ($\zeta \gg P_\zeta^{1/2}$) with $\alpha \gg 1\,,\ b \ll 1$

- **Crucial simplification**: at linear order in b the action is obtained using the free solution for $\zeta_{\rm cl}$

$$\zeta_{\rm cl}(\eta, \boldsymbol{k}) = \zeta_{\rm cl}(\eta, \boldsymbol{k})|_{b=0} = (1 - ik\eta)e^{ik\eta}\bar{\zeta}_{\boldsymbol{k}}$$

$$\Psi[\bar{\zeta}(\boldsymbol{x})] = \int_{\rm BD}^{\bar{\zeta}(\boldsymbol{x})} \mathcal{D}\zeta\; e^{iS[\zeta]} \simeq e^{iS[\zeta_{\rm cl}]}$$

$$S = S_0 + b\Delta S_1 + \mathcal{O}(b^2)$$

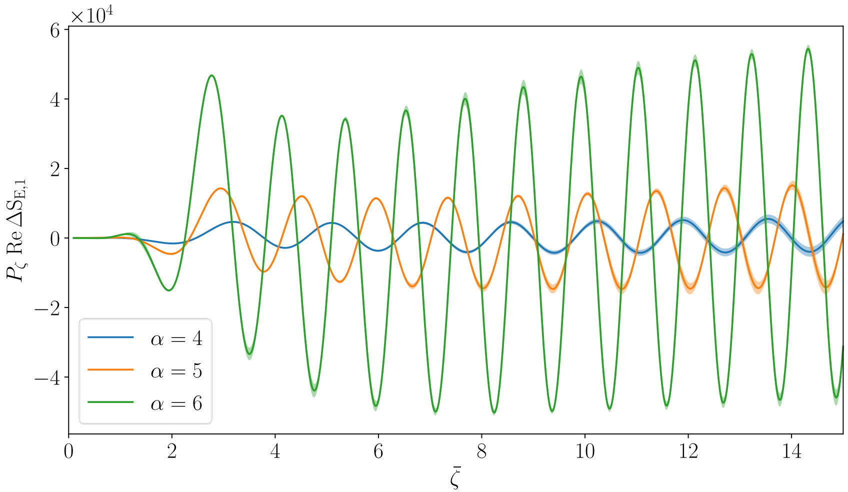
$$\Delta S_1 \equiv -\int {\rm d}\eta\, {\rm d}^3x \frac{1}{2\eta^2 P_\zeta}\left[\zeta'^2 - (\partial_i\zeta)^2\right] \cos\left(\alpha \log\eta + \alpha\zeta\right)$$

WFU for the resonant model: results

- We choose a spherically-symmetric profile at late times $\bar{\zeta}(r)$
- The integral can be solved analytically in saddle point approximation

$$\Delta S_1 \propto \frac{e^{\frac{\alpha\pi}{2}}}{\alpha^2 P_\zeta}\, e^{i\alpha\bar{\zeta}}\, e^{-i\alpha/2\log\left(-\nabla^2\bar{\zeta}\right)}$$

- We also compute the action numerically for *gaussian profile* at late times $\bar{\zeta}(r) = \bar{\zeta}\, e^{-k^2 r^2}$



Conclusions and future directions

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- We studied the tails of the WFU for ζ with resonant features
- First analytical example of non-perturbative features from inside-the-horizon interactions in single field inflation

Near-future directions:

- **Implications for observations** need to be explored ($\zeta \sim P_\zeta^{1/2}$)
- Apply our formalism to **localized features**

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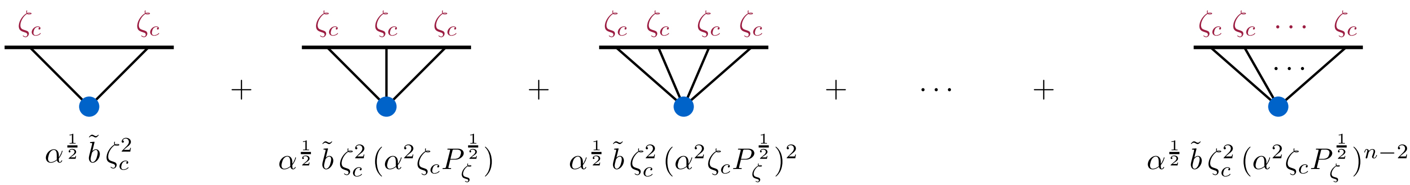
Thank you for listening

Backup slides

Connection with PT diagrams

- The leading-order semiclassical method re-sums all tree-level Witten diagrams
- At linear order in $b \ll 1$ only a subset remains



$$\zeta_c \equiv \zeta / P_\zeta^{1/2}$$