PONT/2023-05

Dynamical Tunneling-Induced Cosmological Bounce

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I. MOTIVATION

A. Null Energy Condition

$$p = \omega\rho$$

NULL ENERGY CONDITION

$$\rho + p \geq 0$$

Matter/Radiation

$$\omega > -1 \implies \rho + p > 0$$

Dark Energy

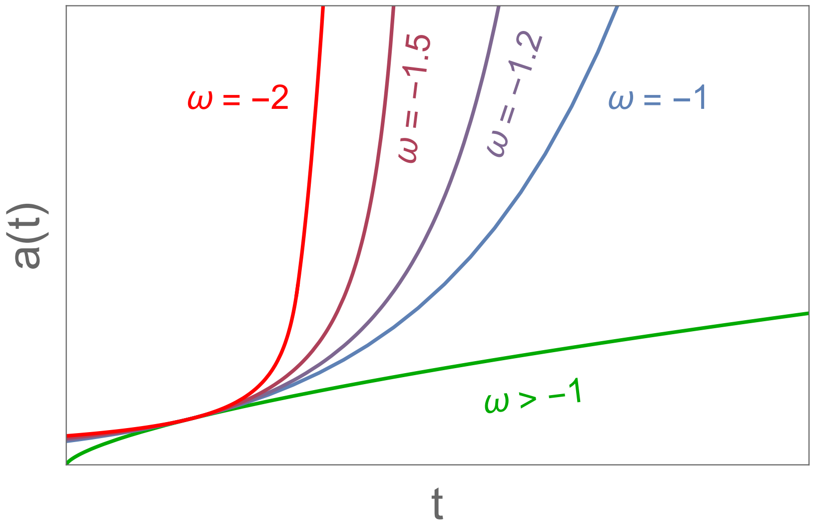
$$\omega = -1 \implies \rho + p = 0$$

Phantom

$$\omega < -1 \implies \rho + p < 0$$

I. MOTIVATION

B. Phantom Dark Energy



Matter/Radiation

$$a(t, \omega > -1) = \left[(1+\omega)\frac{t}{t_0} - \omega\right]^{\frac{2}{3(1+\omega)}}$$

Dark Energy

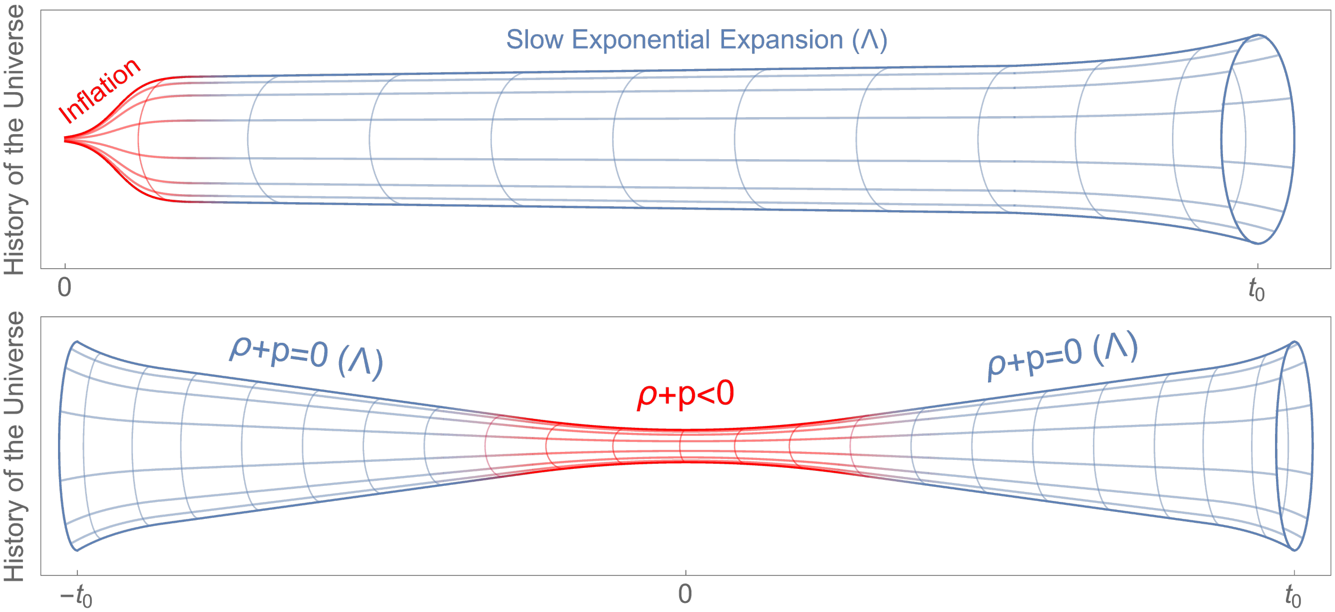
$$a(t, \omega = -1) = e^{\frac{2}{3t_0}(t-t_0)}$$

Phantom

$$a(t, \omega < -1) = \left[(1+\omega)\frac{t}{t_0} - \omega\right]^{\frac{2}{3(1+\omega)}}$$

I. MOTIVATION

C. The Cosmological Bounce



[1] S. W. Hawking and R. Penrose (1970)

II. INTRODUCTION

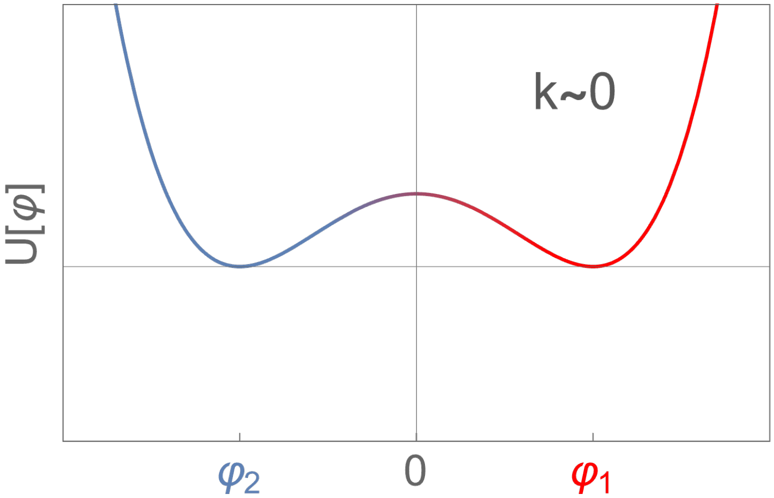
A. Finite Volume

Time Boundary Conditions

$\varphi(0, \mathbf{x}) = \varphi(\beta, \mathbf{x}) \quad , \quad \beta \equiv 1/T$

Spatial Boundary Conditions

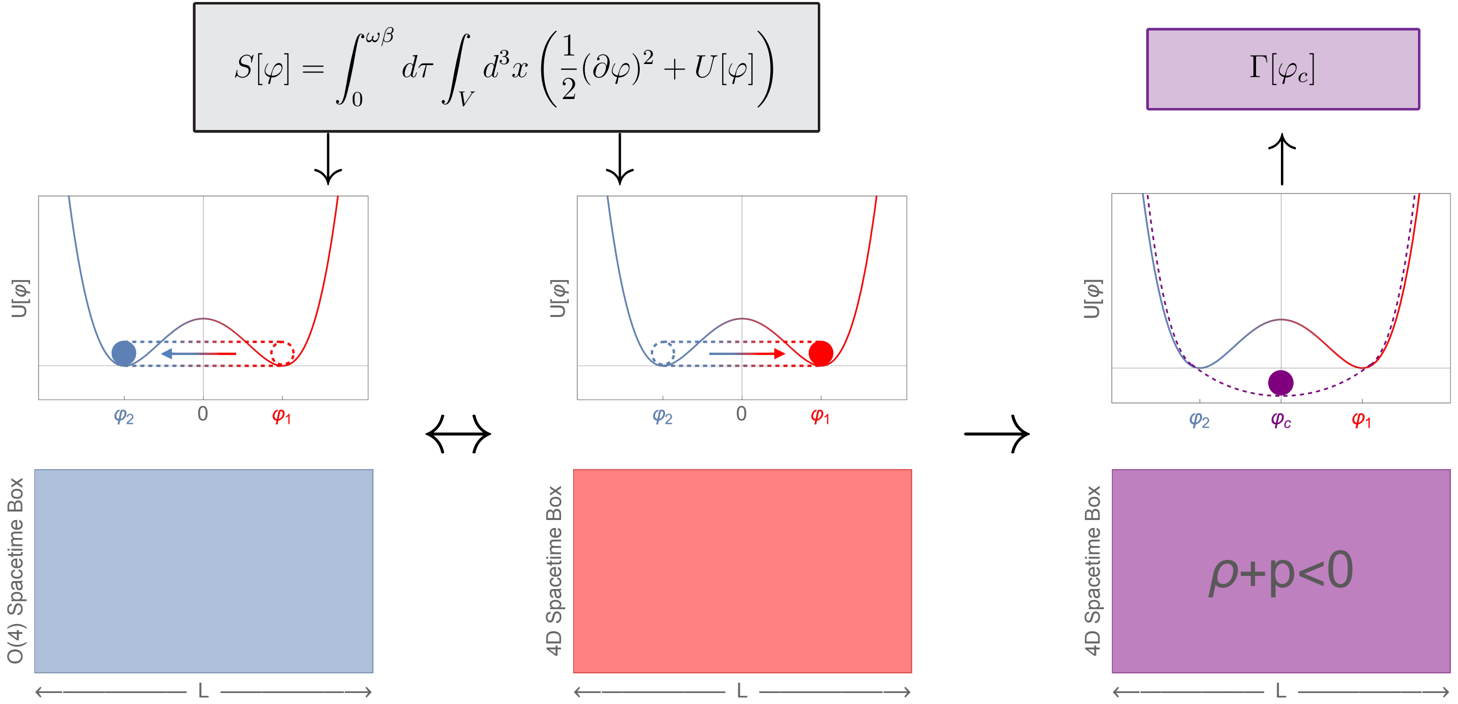
Assume continuous momentum



$$S[\varphi] = \int_0^{\omega\beta} d\tau \int_V d^3x \left(\frac{1}{2}(\partial\varphi)^2 + U[\varphi]\right)$$

$$U[\varphi] = \frac{\lambda v^4}{24}\Big((\varphi^2-1)^2 + 4k\varphi\Big)$$

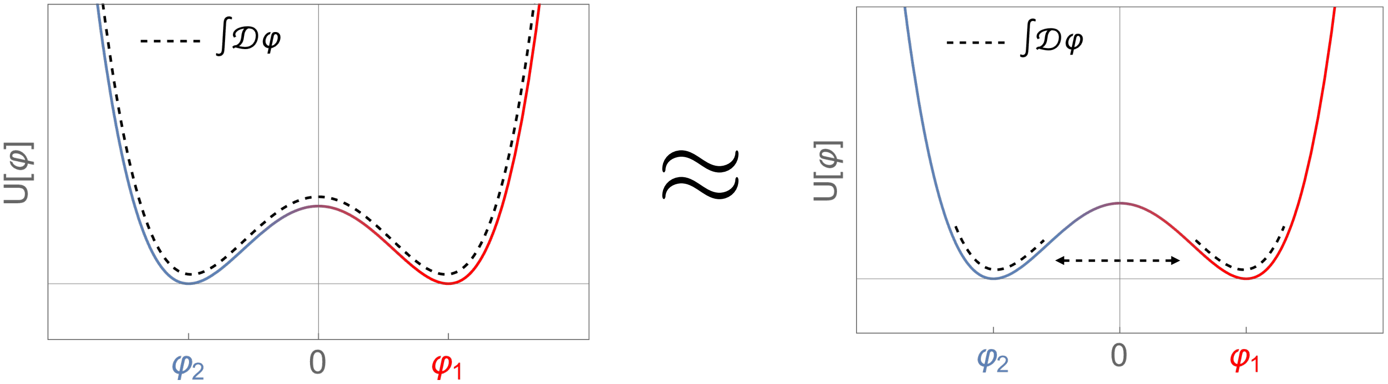
C. Overview



$S[\varphi] = \int_0^{\omega\beta} d\tau \int_V d^3x \left(\frac{1}{2}(\partial\varphi)^2 + U[\varphi]\right)$
$\Gamma[\varphi_c]$
U[φ]
φ2
0
φ1
φc
O(4) Spacetime Box
4D Spacetime Box
ρ+p<0
L

III. SEMI-CLASSICAL APPROXIMATION

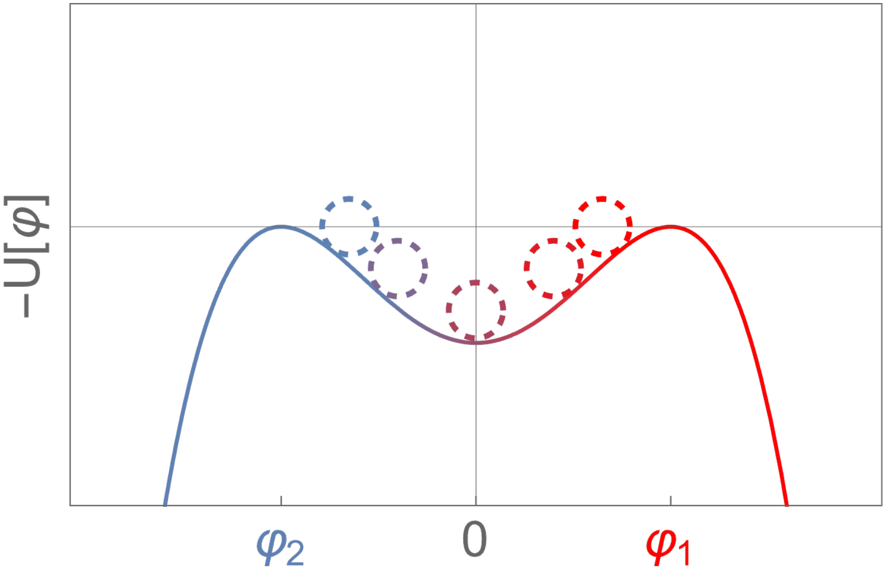
A. Model



$$Z[j] = \int \mathcal{D}\varphi \exp\Big(- S[\varphi]\Big) \approx \sum_i F_i \exp\Big(- S[\varphi_i]\Big)$$

III. SEMI-CLASSICAL APPROXIMATION

B. Saddle Points

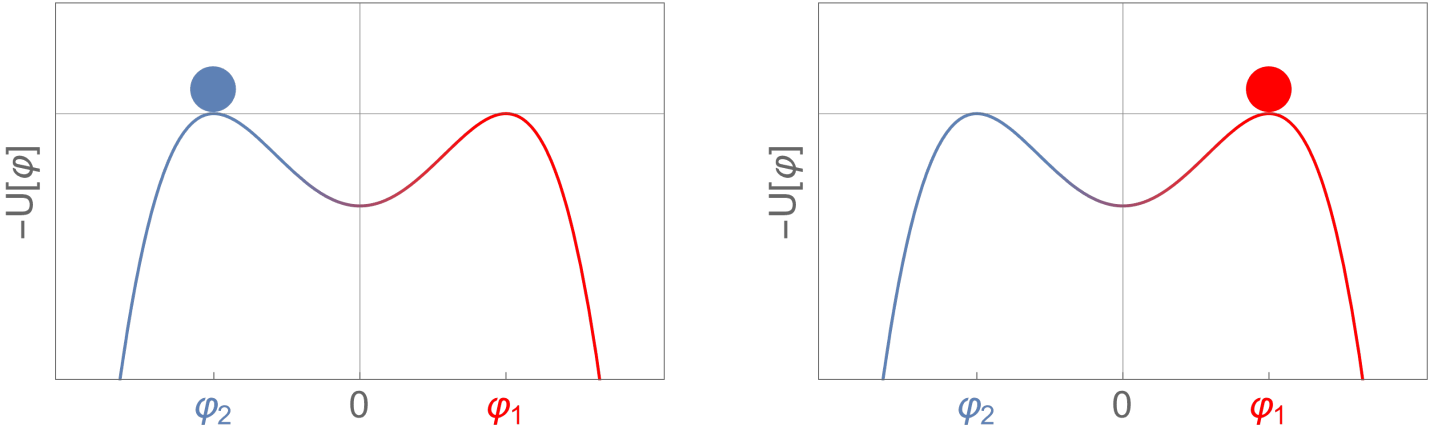


$$\varphi'' \cancel{- \frac{1}{\omega^2}\nabla^2\varphi} - \varphi^3 + \varphi - \cancel{k} = 0$$

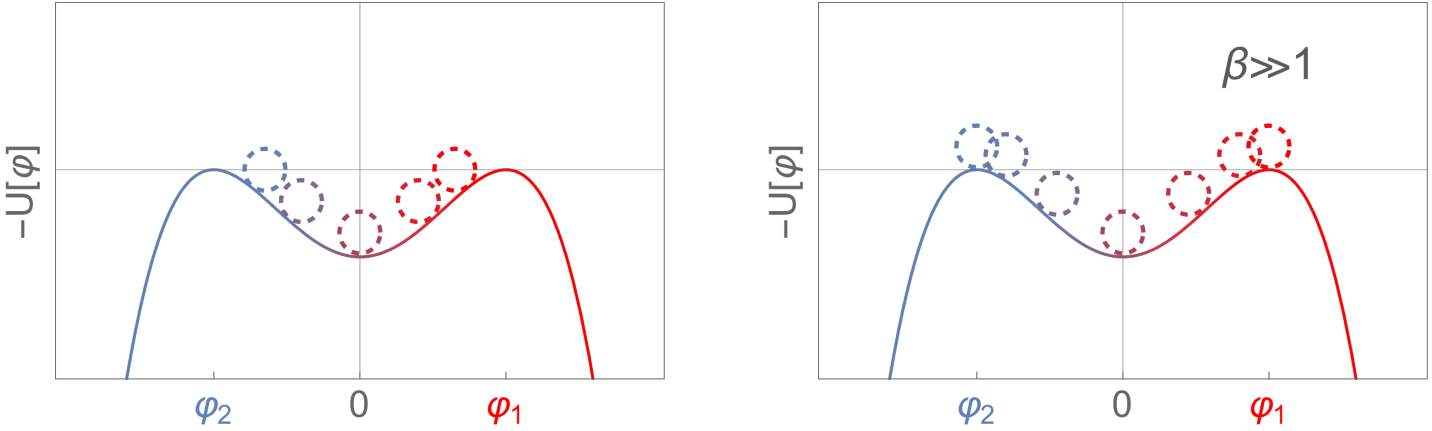
$$-U[\varphi] = -\frac{\lambda v^4}{24}\left(\varphi^2 - 1\right)^2$$

[2] S. R. Coleman (1977); [3] C. G. Callan, Jr. and S. R. Coleman (1977)

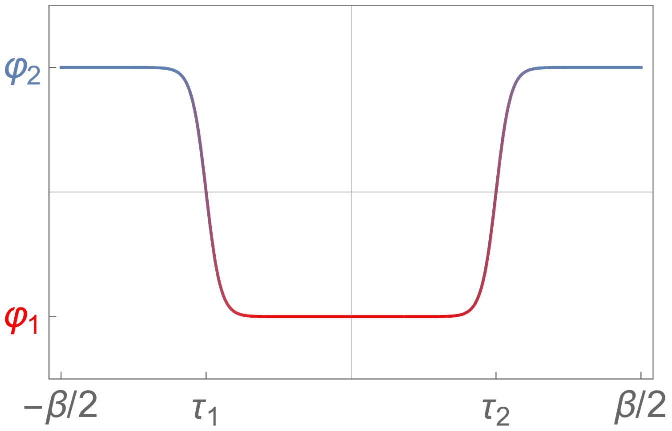
$$\cancel{\varphi''} - \varphi^3 + \varphi = 0$$

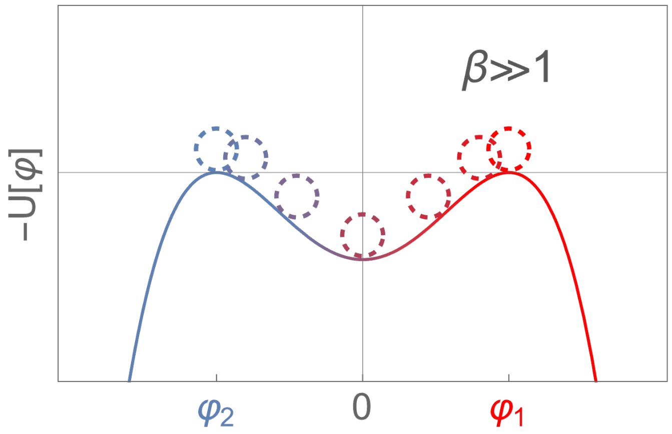


$$\varphi'' - \varphi^3 + \varphi = 0, \quad \text{where: } \varphi(0, \mathbf{x}) = \varphi(\beta, \mathbf{x})$$



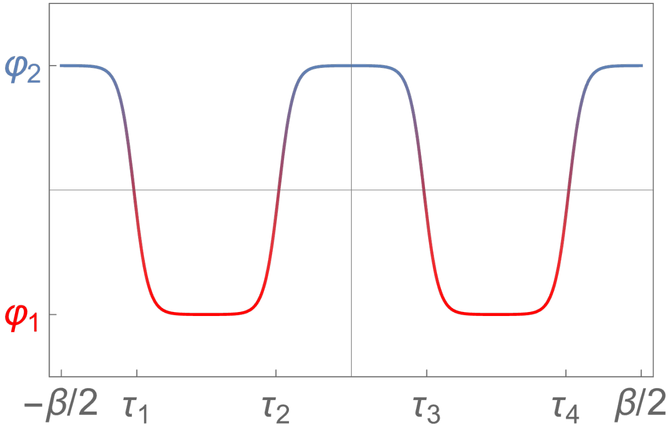
[4] T. W. B. Kibble (1976); [5] W. H. Zurek (1985)

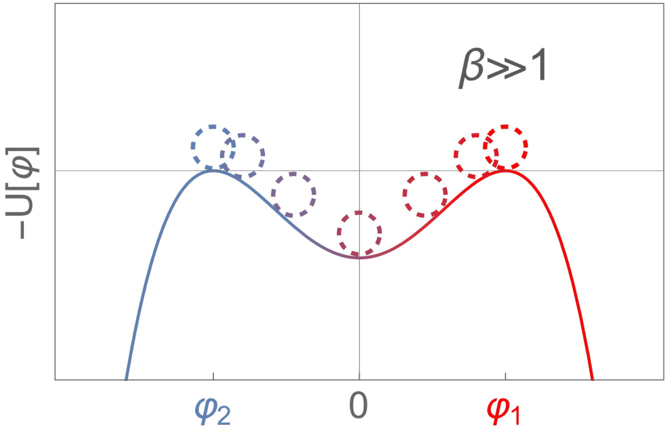




$$\varphi_{\text{pair}}\left(\tau\right)\simeq\tanh\left(\frac{\tau-\tau_1}{\sqrt{2}}\right)\tanh\left(\frac{\tau-\tau_2}{\sqrt{2}}\right)$$

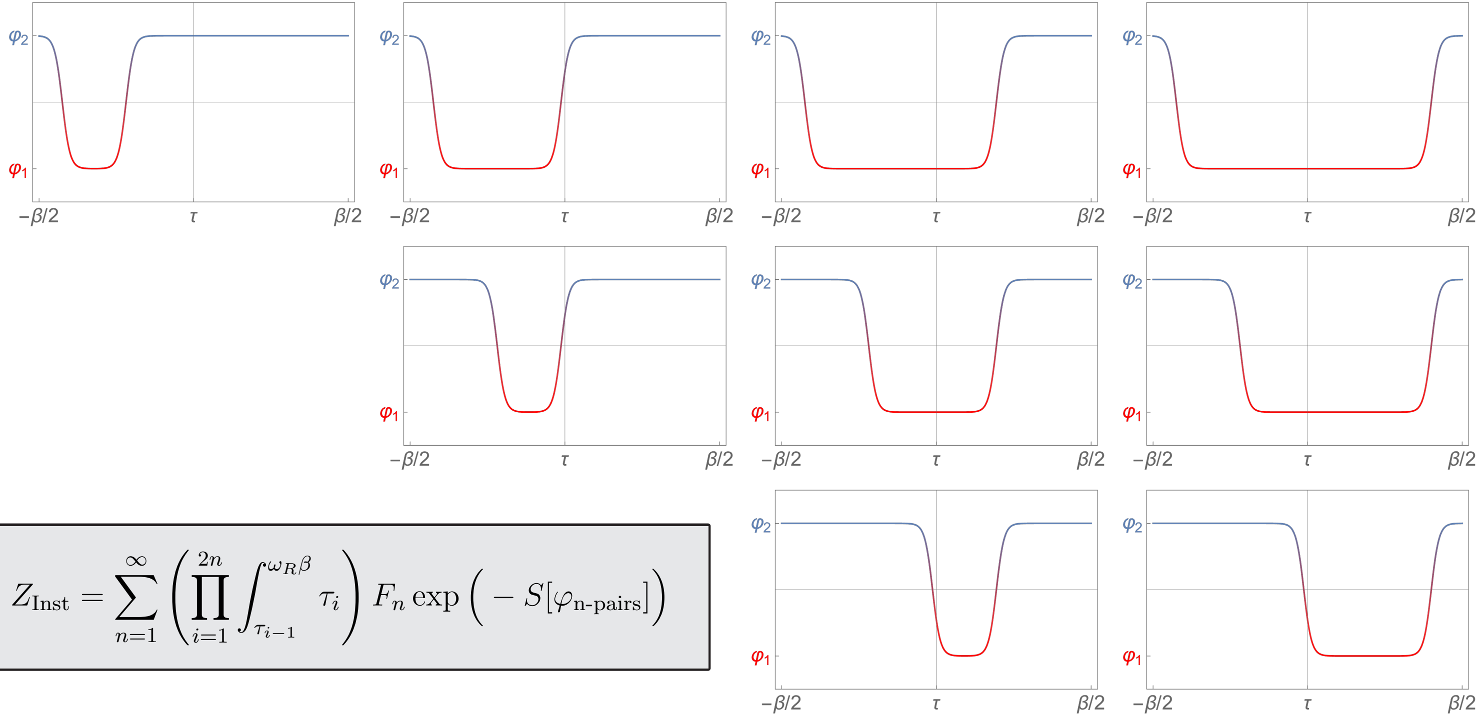
F. Instanton/Anti-Instanton Gas





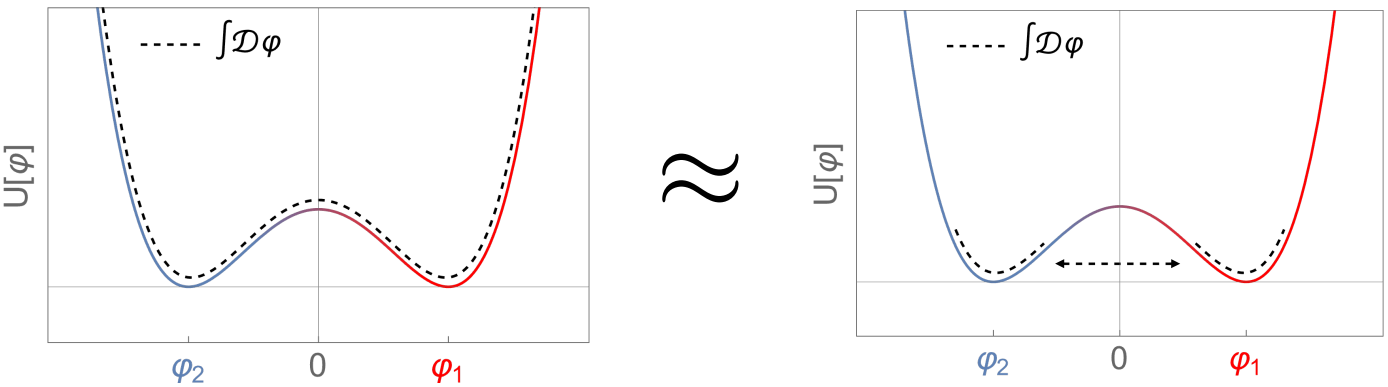
$$\varphi_{\text{n-pairs}}(\tau) = \prod_{i=1}^{2n} \tanh\left(\frac{\tau - \tau_i}{\sqrt{2}}\right)$$

G. Translational Invariance



$$Z_{\text{Inst}} = \sum_{n=1}^{\infty} \left(\prod_{i=1}^{2n} \int_{\tau_{i-1}}^{\omega_R \beta} \tau_i \right) F_n \exp\Big(- S[\varphi_{\text{n-pairs}}] \Big)$$

H. Partition Function

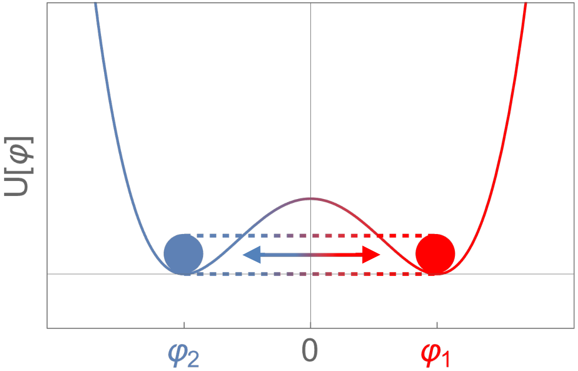


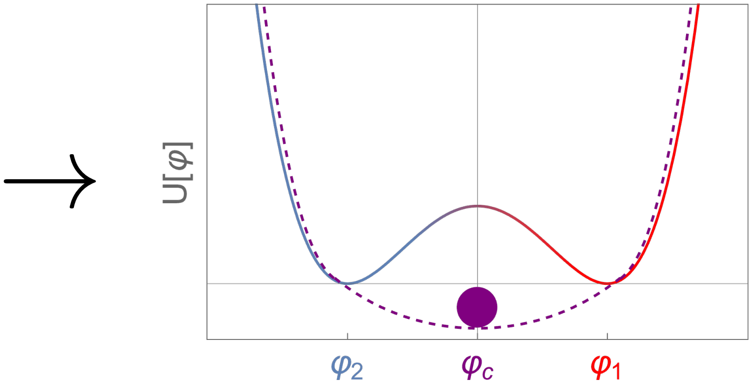
$$Z[k] \approx F_2 \exp\Big(-S[\varphi_2]\Big) + F_1 \exp\Big(-S[\varphi_1]\Big) + \sum_{n=1}^{\infty}\left(\prod_{i=1}^{2n}\int_{\tau_{i-1}}^{\omega_R\beta}\tau_i\right) F_n \exp\Big(-S[\varphi_{\text{n-pairs}}]\Big)$$

IV. NON-EXTENSIVE GROUND STATE

A. Legendre Transformation

$$Z[k] \approx F_2 \exp\Big(-S[\varphi_2]\Big) + F_1 \exp\Big(-S[\varphi_1]\Big) + \sum_{n=1}^{\infty}\left(\prod_{i=1}^{2n}\int_{\tau_{i-1}}^{\omega_R\beta}\tau_i\right) F_n \exp\Big(-S[\varphi_{\text{n-pairs}}]\Big) = \exp\Big(-W[k]\Big)$$





$$\Gamma(\varphi_c) = -\ln Z\Big(k(\varphi_c)\Big) - 4B_r\omega_r\beta\int k(\varphi_c)\; d\varphi_c$$

CONVEX, NON-EXTENSIVE

$$\Gamma(\varphi_c) = \Gamma(0) + B_r\omega_r\beta\left(g_0 + \frac{\lambda}{16\pi^2}g_1\right)\varphi_c^2 + \mathcal{O}(\varphi_c^4)$$

$$g_0 \equiv \frac{4\left(1+\cosh(\bar{N})\right)}{1+16B_r\omega_r\beta+\cosh(\bar{N})}$$

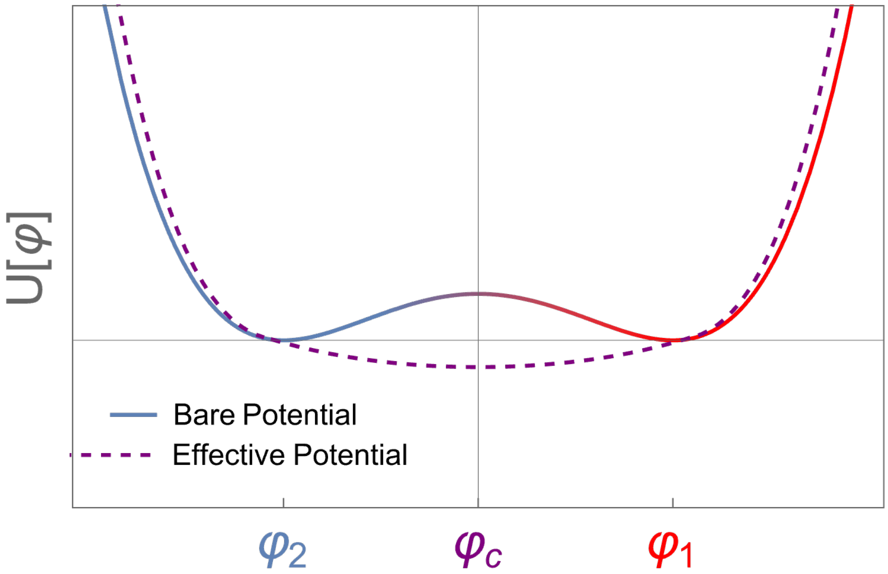
$$\bar{N} \equiv \omega_r\beta\sqrt{\frac{6}{\pi}S_{\text{inst}}}\; e^{-S_{\text{inst}}}, \quad S_{\text{inst}} \equiv \frac{8\sqrt{2}}{3}B_r$$

$$g_1 \equiv \frac{\left(1+\cosh(\bar{N})\right)\left(7+32B_r\omega_r\beta+7\cosh(\bar{N})\right)}{\left(1+16B_r\omega_r\beta+\cosh(\bar{N})\right)^2}$$

$$B_r \equiv \frac{\lambda_r v_r^4 V}{24\omega}$$

[6] J. Alexandre and D. Backhouse (2023)

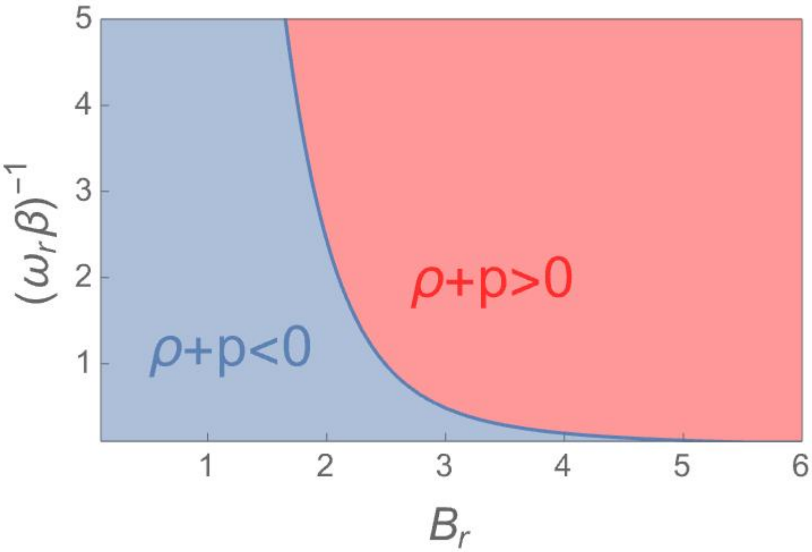
C. Effective Potential



$$U[\varphi] = \frac{\lambda v^4}{24}(\varphi^2 - 1)^2$$

$$U_{\text{eff}}(\varphi_c) = \frac{1}{V\beta}\Gamma[\varphi_c]$$

D. NEC Violation



$\rho + p > 0$: Finite T, Infinite V

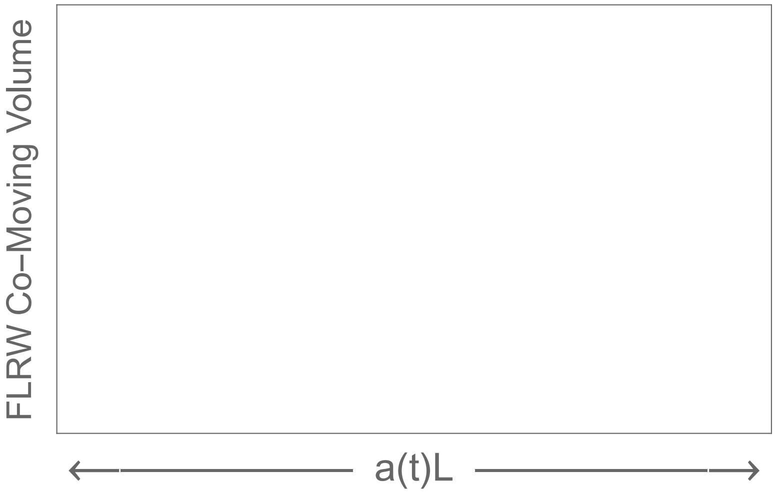
$\rho + p < 0$: Zero T, Finite V

$\rho + p = 0$: Zero T, Infinite V

For Low T: $\rho + p \approx \frac{4\omega_R^{5/2}}{(\sqrt{2}\pi\beta)^{3/2}} e^{-\omega_R\beta/\sqrt{2}} - \frac{\omega_R}{V}\left(S_{\rm inst} + \frac{1}{2}\right)\sqrt{\frac{6}{\pi}S_{\rm inst}}\; e^{-S_{\rm inst}}$

V. COSMOLOGY

A. FLRW Volume



$V \to a^3(t)V$

$T \to 0$

Assume Adiabatic Expansion

$\Longrightarrow$ $a =$ constant in QFT calculations

FLAT SPACETIME

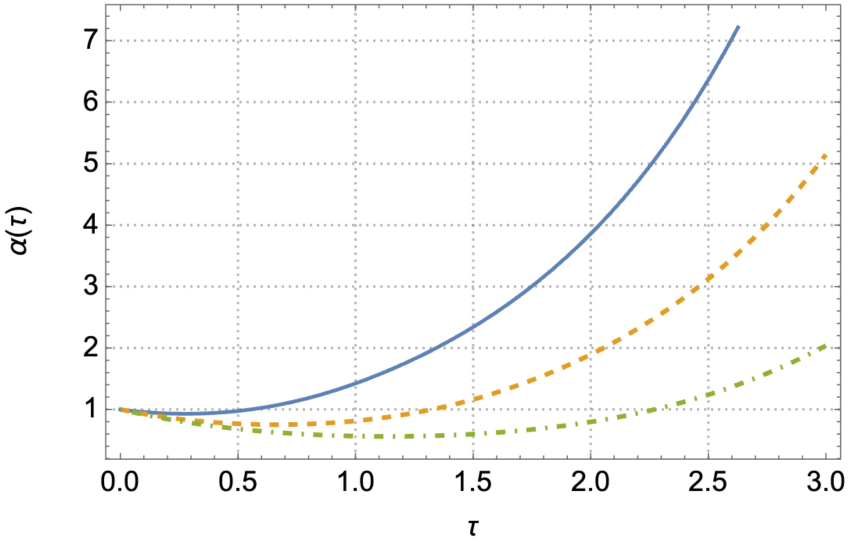
$$\rho + p = -\frac{\omega_R}{V}\left(S_{\text{inst}} + \frac{1}{2}\right)\sqrt{\frac{6}{\pi}S_{\text{inst}}}\; e^{-S_{\text{inst}}} < 0$$

FLRW SPACETIME

$$\rho + p = -\frac{\omega_R}{a^3 V}\left(a^3 S_{\text{inst}} + \frac{1}{2}\right)\sqrt{\frac{6}{\pi}a^3 S_{\text{inst}}}\; e^{-a^3 S_{\text{inst}}} < 0$$

$$\mathrm{V} \to a^3(t)V \quad , \quad S_{\text{inst}} \propto V$$

[7] J. Alexandre and S. Pla (2023)



$$H^2 = \frac{\kappa_R}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{\kappa_R}{6}(\rho + 3p)$$

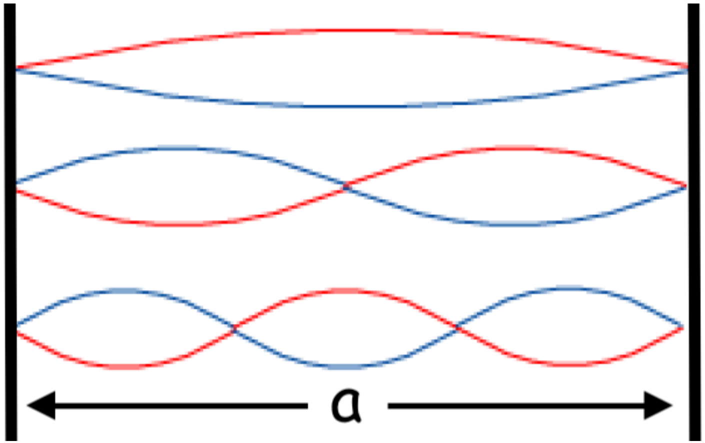
Larger NEC Violation

Smaller NEC Violation

[7] J. Alexandre and S. Pla (2023)

VI. DISCRETE MOMENTUM CORRECTIONS

A. The Casimir Effect



$$E_{\text{Discrete}}(a) = \frac{\hbar}{2} \sum_{n=1}^{\infty} \omega_n$$

$$E_{\text{Continuum}}(a) = \frac{\hbar a}{2\pi} \int_0^{\infty} \omega dk$$

$$E_{\text{Casimir}}(a) = E_{\text{Discrete}}(a) - E_{\text{Continuum}}(a) = -\frac{\pi \hbar c}{24 a}$$

[8] M. Bordag, U. Mohideen and V. M. Mostepanenko (2001)

B. The Casimir Effect in Different Geometries

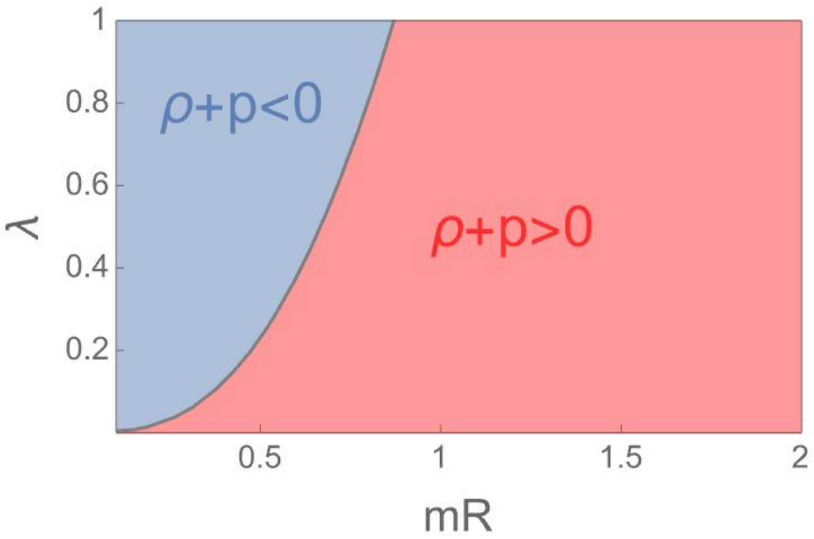
Dirichlet plates of separation a: $E_{\text{Cas}} \simeq -\frac{A}{8\sqrt{2}}\left(\frac{m}{\pi a}\right)^{3/2} e^{-2ma}$

Periodic Box of length L: $E_{\text{Cas}} \simeq -\frac{(mL)^{3/2}}{L}\exp(-mL)$

3-sphere with radius of curvature R: $E_{\text{Cas}} \simeq +\frac{(mR)^{5/2}}{R}\exp(-2\pi mR)$

$$a, L, R \gg 1/m$$

C. NEC Violation



Casimir: $\rho + p \sim e^{-R}$

Tunneling: $\rho + p \sim e^{-R^3}$

$\rho + p = 0 : mR \sim \sqrt{\lambda}$

$$\rho + p = \frac{E_{\rm Cas}}{V} - \frac{\partial E_{\rm Cas}}{\partial V} \quad - \frac{\omega_R}{V}\left(S_{\rm inst} + \frac{1}{2}\right)\sqrt{\frac{6 S_{\rm inst}}{\pi}}\; e^{-S_{\rm inst}}$$

VI. CONCLUSIONS

Dynamical NEC Violation via Tunneling at Finite Temperature

NEC Violation Occurs at Sufficiently Low Temperatures at Finite Volumes

Tunneling in an FRLW Volume Dynamically Generates a Cosmological Bounce

Tunneling and the Casimir Effect Compete on de Broglie Length Scales

VII. REFERENCES

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[6] J. Alexandre and D. Backhouse, (2023), arXiv:2301.02455 [hep-th].

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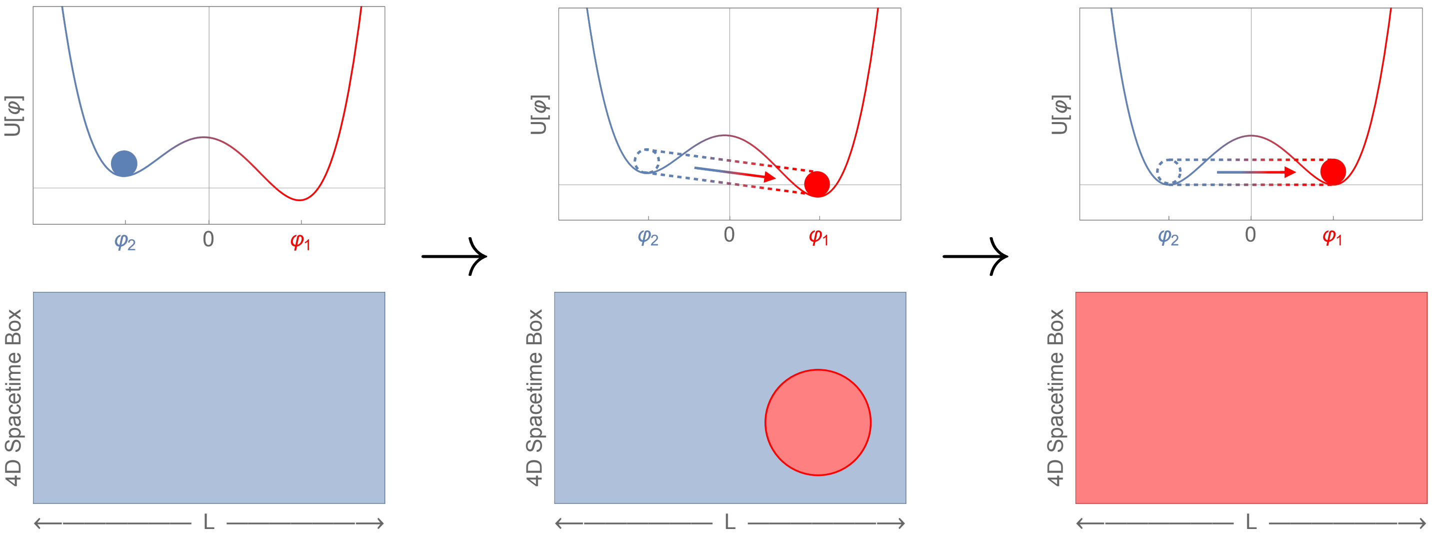
[9] H. Kleinert, (2004), 10.1142/5057.

. APPENDIX A: SPATIALLY-DEPENDENT SADDLE POINTS



$$\varphi'' + \cancel{\frac{1}{\omega^2}\nabla^2\varphi} - \varphi^3 + \varphi - \cancel{k} = 0$$

$$\phi_c = \langle \phi \rangle = \frac{1}{Z} \int \mathcal{D}\phi \; \phi \exp\left(-S[\phi]\right) = -\frac{\hbar}{Z} \frac{\delta Z[j]}{\delta j(x)}$$

$$\varphi_c(k) = \left(-f_0 + \frac{\lambda_r}{128\pi^2} f_1\right) k + \mathcal{O}(k^3)$$

$$k(\varphi_c) = -\frac{1}{2} \left(g_0 + \frac{\lambda}{16\pi^2} g_1\right) \varphi_c + \mathcal{O}(\varphi_c^3)$$

$$\Gamma(\varphi_c) = -\ln Z\Big(k(\varphi_c)\Big) - 4 B_r \omega_r \beta \int k(\varphi_c) \; d\varphi_c$$

. APPENDIX C: FLUCTUATION FACTORS

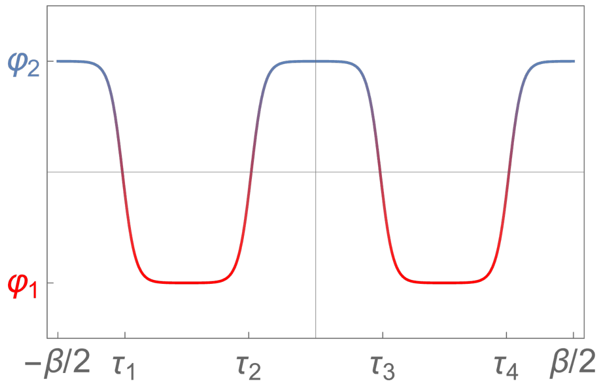
A. Static Saddle Points

$$Z[j] \approx F_2 \exp\left(-S[\varphi_2]\right) + F_1 \exp\left(-S[\varphi_1]\right) + \sum_{n=1}^{\infty}\left(\prod_{i=1}^{2n}\int_{\tau_{i-1}}^{\omega_R\beta}\tau_i\right) F_n \exp\left(-S[\varphi_{\text{n-pairs}}]\right)$$

$$\ln\left(F_{L,R}\right) = B_r\omega_r\beta\left(+\frac{\lambda_r}{96\pi^2}(3\varphi_{L,R}^2-1)^2\ln\left(\frac{3}{2}\varphi_{L,R}^2-\frac{1}{2}\right)-\frac{\lambda_r(3\varphi_{L,R}^2-1)}{3\pi^2}\sum_{l=1}^{\infty}\frac{K_2\left(l\omega_r\beta\sqrt{3\varphi_{L,R}^2-1}\right)}{(l\omega_r\beta)^2}\right)$$

[6] J. Alexandre and D. Backhouse (2023)

$$Z[j] \approx F_2 \exp\left(-S[\varphi_2]\right) + F_1 \exp\left(-S[\varphi_1]\right) + \sum_{n=1}^{\infty} \left(\prod_{i=1}^{2n} \int_{\tau_{i-1}}^{\omega_R \beta} \tau_i \right) F_n \exp\left(-S[\varphi_{\text{n-pairs}}]\right)$$



$$F_n = F_L(\beta/2) F_R(\beta/2) \left(\frac{6S_{int}}{\pi}\right)^n$$

[9] H. Kleinert (2004)